GOODS TRADE AND INTERNATIONAL EQUITY PORTFOLIOS

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ABSTRACT

We show that international trade in goods is the main determinant of international equity portfolios
and offers a compelling -- theoretically and empirically -- resolution of the portfolio home bias puzzle.
The model implies that investors can achieve full international risk diversification if the share of wealth
invested in foreign equity matches their country's degree of openness (the imports to GDP share). The
empirical evidence strongly supports this implication.

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Introduction

In a typical country, consumers tend to consume mostly goods that are produced domestically. This phenomenon has been termed home bias in consumption. Similarly, investors tend to invest most of their wealth in domestic assets, and most of the capital in any country is owned by the domestic residents. This phenomenon has been termed portfolio home bias. Are these two biases linked[1]?

Stockman and Dellas, 1988, (henceforth, SD) argue that this is indeed the case[2]: the presence of consumption home bias –due to the existence of non–traded goods– can induce a similar bias in portfolio holdings. In SD's endowment economy with a *separable* utility function between traded and non–traded goods, the optimal portfolio of a domestic investor involves full ownership of the firms that produce the domestic non–traded goods and a fully diversified ownership of the firms that produce the tradable goods. If the share of non–traded goods in consumption is large, the model can generate substantial portfolio home bias.

The existing literature has not viewed the SD model as offering a full resolution to the puzzle[3]. The conventional wisdom at the time put the share of non–traded goods in the consumption basket in the US at around 50%. With this share, the SD model implies that 75% of domestic wealth will be invested in domestic assets. But this figure falls significantly short of the actual degree of portfolio home bias observed in the US[4]. Recent work by Burstein et al., 2005, however, suggests that the true CPI share of non traded goods is much higher (close to 80%). With this

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[1]Naturally, there is also a large literature that attempts to explain portfolio home bias without making use of consumption home bias (see, for instance, the survey paper by Lewis, 1996). However, it has been met with limited success so far.

[2]See also Obstfeld and Rogoff, 2000, who suggest that these two biases may not only have a common source but they may also be connected to a number of other biases –puzzles– in international finance. Coeurdacier, 2006, disputes this and claims that introducing trade costs in goods market alone, as suggested by Obstfeld and Rogoff, is not sufficient to explain these two biases simultaneously. This finding obtains under a very special form of trade costs and it is not valid in general. SD and especially the present paper paper establish that trade costs (as manifested in the existence of non-traded goods) can account for both biases quite well.

[3]There exists a large literature that examines whether the portfolio home bias that emanates from non–traded goods can be augmented with bias arising from traded goods when the SD assumption of separability between traded and non–traded goods is lifted (Tesar, 1993, Baxter, Jermann, and King, 1998, Serrat, 2001, Pesenti and Van Wincoop, 2002). In the models of Tesar and Pesenti–Van Wincoop there may be home bias in the shares of the tradeable sector depending on the value of the preferences and covariance parameters. Unfortunately, these papers rely on partial equilibrium analysis and thus their results may not hold in general equilibrium. Baxter, Jermann, and King, 1998, on the other hand, use a general equilibrium, two-period exchange economy. They argue that the model *cannot* generates a home bias in the traded goods portfolio. And that equity holdings in the non–traded goods may well be home biased yet not sufficiently so in order to make the total portfolio exhibit home bias. But this result owes to the perfect substitutability between traded goods as well as the the absence of a demand for dynamic hedging. Serrat, 2001, uses a dynamic, general equilibrium model and finds that the domestic investors fully own the equity of the firms that produce the domestic non–traded good and that there is also home bias in the equity positions in traded goods. Kollmann, 2006a, has disputed the latter claim, arguing that the correct solution to the model of Serrat does not involve any portfolio home bias in *traded* goods equity. Obstfeld, 2007, provides a thorough review of these as well as other related issues in his Ohlin lectures.

[4]See Tesar and Werner, 1995, French and Poterba, 1991.

figure, the SD model implies that the share of domestic wealth invested in domestic equity is around 90%, a number very close to its real world counterpart.

The evidence thus offers support to the SD model. But this test relies heavily on our ability to separate goods into the traded and non–traded categories, a rather controversial enterprize. An alternative empirical strategy (e.g., Heathcote and Perri, 2007) for taking international portfolio models to the data is to work with the degree of openness[5]. The SD model has a sharp implication on this front: it implies that investors can achieve full international risk diversification if the share of wealth invested in foreign equity is equal to the share of imports in GDP.

Table 1 reports these two shares for the G5 (US, Japan, UK, Germany and France). The first column of the table gives the share of foreign portfolio equity holdings in total domestic —equity— portfolio over the period 1995–2004. The second column reports the imports to GDP share over the same period. The match between the figures in these two columns is very high. The correlation coefficient is 0.92. The fact that a similarly good match is obtained when considering other sub–periods or even individual years or when looking at a larger set of countries (see section 3.1) suggests that this stylized fact is quite robust.

Nonetheless, as we elaborate below, the empirical strategy of casting a model's implications in terms of the import share also faces challenges: the model abstracts from some important, relevant elements present in the real world. In our model, trade flows and value added coincide. But this is not so in the data. Namely, the standard imports to GDP ratio overstates the degree of openness because it does not correct for re-exports (the foreign value added contained in domestic exports). On the asset side, the measure used in Figure 1 ignores housing in the calculation of domestic equity wealth (thus, it overstates the share of wealth invested in foreign assets). And it does not include the *indirect* holdings of foreign assets that arise from holding stocks in domestic multinationals (thus, it understates the share of wealth invested in foreign assets). In section 3, we make the appropriate corrections of the trade and the asset data of the US (unfortunately, there exists no data that would allow such correction in other countries' data). The close match between the degree of openness and the foreign asset share of domestic wealth remains (11.7% and 10% respectively).

Having established that the SD model offers a compelling explanation of the home portfolio puzzle, we examine the effects of two popular refinements. In particular, we examine how deviations from non-separability and symmetry in traded goods consumption affect the properties of the model. This exercise is important for two reasons. First, it may suggest ways for further improving the performance of the model. And second and more importantly, it can provide

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[5]Of course, there exists a correspondence between these two categories in the model. In SD, a share of 11% for imports (the average US figure during the last twenty five years) translates into a 78% share for non traded goods in the CPI. This is very similar to the figure reported by Burstein et al., 2005.

Table 1: Foreign Equity and Import Shares, 1995-2004

	Equity Share	Import Share
US	0.1203	0.1334
JAP	0.0829	0.0923
UK	0.2623	0.2888
GER	0.3676	0.2906
FR	0.2033	0.2437
Corr = 0.922		

Note: See section 3.1 for details on the computatin of the shares. Data sources: Lane–Milesi-Feretti, 2006 for foreign equity assets and liabilities. World Development indicators for stock market capitalization. And IFS for import shares.

an important robustness check. Typically, the theoretical implications reported in the portfolio literature tend to be very sensitive to even small variation in the key parameters of the model (see Obstfeld, 2007). We find that our model does not suffer from this weakness. Namely, departures from separability as well as plausible variation in the main parameters of the model do not affect materially the properties of optimal portfolios. An additional finding of interest is that the model can also deliver home bias in the equities of the traded goods industries if the consumption of traded goods has a foreign bias[6]. There exists no empirical evidence on the presence or absence of home bias in traded goods equity and hence it is not known whether portfolio bias in traded goods represents a desirable feature or not. If it were, our model indicates how it could be obtained.

The rest of the paper is structured as follows. Section1 contains a description of the model as well as the solution for country portfolios. The solution method is quite simple and is closely related to that of Kollmann, 2006b. Section 2 presents the parametrization of the model. Section 3 describes and discusses the main results, while 4 offers extensions and a sensitivity analysis. Section 5 concludes. A technical appendix provides a formal description of the properties of portfolios.

1   The Model

The world consists of two countries, indexed by $i = 1, 2$. In each period, each country receives an exogenous endowment of a traded, $Y_{it} > 0$ and a non–traded $Z_{it} > 0$, good. The goods are perishable. We use $\mathscr{Y}_t = \{Y_{it}, Z_{it}; i = 1, 2\}$ to denote the vector of endowments.

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[6]A foreign bias in the consumption of traded goods is a standard implication of trade theory. It is also a likely feature of the CPI; see the discussion in section 3

1.1 The household

Country i is inhabited by a representative agent whose preferences are described by

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \frac{C_{it}^{1-\sigma} - 1}{1 - \sigma} \text{ with } \sigma > 0 \tag{1}$$

C_{it} denotes total consumption in country i. It consists of traded and non traded goods according to the specification

$$C_{it} = \left(\omega_i^{\frac{1}{\rho}} C_{it}^{y\ \frac{\rho-1}{\rho}} + (1 - \omega_i)^{\frac{1}{\rho}} C_{it}^{z\ \frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \quad \omega_i \in (0,1) \text{ and } \rho > 0 \tag{2}$$

where C_{it}^y (resp. C_{it}^z) denotes the consumption of traded (resp. non–traded) goods in country i at period t.

The traded good aggregate combines domestic and foreign goods according to

$$C_{it}^y = \left(\alpha_i^{\frac{1}{\varphi}} C_{iit}^{y\ \frac{\varphi-1}{\varphi}} + (1 - \alpha_i)^{\frac{1}{\varphi}} C_{ijt}^{y\ \frac{\varphi-1}{\varphi}} \right)^{\frac{\varphi}{\varphi-1}} \quad \alpha_i \in (0,1) \text{ and } \varphi > 0 \tag{3}$$

where C_{ijt}^y denotes the consumption of the traded good j in country i at period t.

The individuals have access to an equity market where the shares of the firms that own the endowments of the four goods (the four "trees") can be traded. The budget constraint of the representative household in country i takes the form

$$\sum_{j=1}^{2} \left[Q_{jt}^y S_{ijt+1}^y + Q_{jt}^z S_{ijt+1}^z P_{jt}^y C_{ijt}^y \right] + P_{it}^z C_{it}^z = \sum_{j=1}^{2} \left[(Q_{jt}^y + P_{jt}^y Y_{jt}) S_{ijt}^y + (Q_{jt}^z + P_{jt}^z Z_{jt}) S_{ijt}^z \right] \tag{4}$$

where P_{jt}^y and P_{jt}^z are the prices of the traded and non–traded good j respectively. S_{ijt}^y denotes the number of shares of traded good j owned by the households in country i at the beginning of period t while S_{ijt}^z is the number of shares of the non–traded good. The price of traded goods shares is Q_{jt}^y and that of non–traded is Q_{jt}^z. The traded goods shares yield a dividend of $P_{jt}^y Y_{jt}$ and the non–traded ones $P_{jt}^z Z_{jt}$. Note that there are four assets (equities) in the model and four independent sources of uncertainty. This implies that the equity markets in this model can support the complete asset markets allocation of resources up to a linear approximation[7]. As in Kollmann, 2006b, we will use this equivalence to determine asset holdings.

The household's consumption/portfolio choices are determined by maximizing (1) subject to (2)–(4). The domestic traded good will be used as the numéraire good. Then the evolution of asset prices is given by the standard Euler equations

$$Q_{jt}^y \lambda_t^i = \beta \mathbb{E}_t \lambda_{t+1}^i (Q_{jt+1}^y + P_{jt+1}^y Y_{jt+1}) \tag{5}$$

$$Q_{jt}^z \lambda_t^i = \beta \mathbb{E}_t \lambda_{t+1}^i (Q_{jt+1}^z + P_{jt+1}^z Z_{jt+1}) \tag{6}$$

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[7]A confirmation of this claim can be found in the accompanying technical appendix.

where $i, j = 1, 2$. Since asset markets are complete and the two economies are perfectly symmetric,[8] we have $\lambda_t^1 = \lambda_t^2$.

Market clearing requires that

$$Z_{1t} = C_{1t}^z \tag{7}$$

$$Z_{2t} = C_{2t}^z \tag{8}$$

$$Y_{1t} = C_{11t}^y + C_{21t}^y \tag{9}$$

$$Y_{2t} = C_{12t}^y + C_{22t}^y \tag{10}$$

The equilibrium satisfies the FOCs of the optimization problems of the representative agents in the two countries and the market clearing conditions. Since asset markets are effectively complete, the solution of the model can be determined without any need to know equity shares. The model is solved using a perturbation method and takes the form

$$X_t^e = G_x(\mathscr{Y}_t)$$

where $X_t^e \in \{C_{ijt}^{ye}, C_{ijt}^{ze}, P_{jt}^{ye}, P_{jt}^{ze}, Q_{jt}^{ye}, Q_{jt}^{ze}; i, j = 1, 2\}$. We will explain below how to use $X_t^e = G_x(\mathscr{Y}_t)$ to determine $S_{ijt+1}^{ye}, S_{ijt+1}^{ze}$.

1.2 Specification of the endowments

The endowment process for the traded goods takes the form[9]

$$y_{1t} - \overline{y}_1 = \rho_{11}^y(y_{1t-1} - \overline{y}_1) + \rho_{12}^y(y_{2t-1} - \overline{y}_2) + \varepsilon_{1t}^y$$

$$y_{2t} - \overline{y}_2 = \rho_{21}^y(y_{1t-1} - \overline{y}_1) + \rho_{22}^y(y_{2t-1} - \overline{y}_2) + \varepsilon_{2t}^y$$

where the eigenvalues of the matrix $A_y = \begin{pmatrix} \rho_{11}^y & \rho_{12}^y \\ \rho_{21}^y & \rho_{22}^y \end{pmatrix}$ all lie inside the unit circle, and $(\varepsilon_{1t}^y, \varepsilon_{2t}^y) \rightsquigarrow \mathscr{N}(0, \Sigma_y)$.

Similarly for the non–traded goods

$$z_{1t} - \overline{z}_1 = \rho_{11}^z(z_{1t-1} - \overline{z}_1) + \rho_{12}^z(z_{2t-1} - \overline{z}_2) + \varepsilon_{1t}^z$$

$$z_{2t} - \overline{z}_2 = \rho_{21}^z(z_{1t-1} - \overline{z}_1) + \rho_{22}^z(z_{2t-1} - \overline{z}_2) + \varepsilon_{2t}^z$$

where the eigenvalues of the matrix $A_z = \begin{pmatrix} \rho_{11}^z & \rho_{12}^z \\ \rho_{21}^z & \rho_{22}^z \end{pmatrix}$ all lie inside the unit circle, and $(\varepsilon_{1t}^z, \varepsilon_{2t}^z) \rightsquigarrow \mathscr{N}(0, \Sigma_z)$.

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[8]Relaxing the perfect symmetry assumption implies that $\lambda_t^1 \propto \lambda_t^2$ where the proportionality factor is given by the relative initial wealth ratio.

[9]We denote $y_{it} = \log(Y_{it})$, i=1,2. Likewise for z.

1.3 Solving for asset holdings

Asset holdings are indeterminate in the deterministic steady state around which the system is log-linearized and solved. This creates a difficulty. Our solution method is a variation of that of Kollmann, 2006b (which in turn is related to Baxter et al., 1998).

Let us focus on the domestic economy ($i = 1$). We define wealth, in utility terms, as

$$\Omega_t^1 \equiv \lambda_t^1 \left(Q_{1t}^y S_{11t+1}^y + Q_{2t}^y S_{12t+1}^y + Q_{1t}^z S_{11t+1}^z + Q_{2t}^z S_{12t+1}^z \right) \tag{11}$$

where λ_t^1 are the Lagrange multipliers associated with the budget constraints (4).

Using the definition of wealth, the budget constraint of the household can be rewritten as

$$\Omega_t^1 + \lambda_t^1 (C_{11t}^y + P_{2t}^y C_{12t}^y + P_{1t}^z C_{1t}^z) = \lambda_t^1 \frac{Q_{1t}^y + Y_{1t}}{Q_{1t-1}^y} Q_{1t-1}^y S_{11t}^y + \lambda_t^1 \frac{Q_{2t}^y + P_{2t}^y Y_{2t}}{Q_{2t-1}^y} Q_{2t-1}^y S_{12t}^y \tag{12}$$

$$+ \lambda_t^1 \frac{Q_{1t}^z + P_{1t}^z Z_{1t}}{Q_{1t-1}^z} Q_{1t-1}^z S_{11t}^z + \lambda_t^1 \frac{Q_{2t}^z + P_{2t}^z Z_{2t}}{Q_{2t-1}^z} Q_{2t-1}^z S_{12t}^z$$

$$\tag{13}$$

Updating equation (13) by one period, and making use of the household's Euler equations for asset decisions and the fact that shares are predetermined leads to a stochastic difference equation in wealth

$$\Omega_t^1 = \beta \mathbb{E}_t \left[\Omega_{t+1}^1 + \lambda_{t+1}^1 (C_{11t+1}^y + P_{2t+1}^y C_{12t+1}^y + P_{1t+1}^z C_{1t+1}^z) \right]$$

This equation determines wealth, Ω_t^1.

Let us define the asset returns as

$$R_{1t}^y = \frac{Q_{1t}^y + Y_{1t}}{Q_{1t-1}^y}, \qquad R_{2t}^y = \frac{Q_{2t}^y + P_{2t}^y Y_{2t}}{Q_{2t-1}^y},$$

$$R_{1t}^z = \frac{Q_{1t}^z + P_{1t}^z Z_{1t}}{Q_{1t-1}^z}, \qquad R_{2t}^z = \frac{Q_{2t}^z + P_{2t}^z Z_{2t}}{Q_{2t-1}^z}.$$

Let also define each asset's wealth share in country i as

$$\alpha_{i1t+1}^y = \frac{\lambda_t^1 Q_{1t}^y S_{11t+1}^y}{\Omega_t^1} \qquad \qquad \alpha_{i2t+1}^y = \frac{\lambda_t^1 Q_{2t}^y S_{12t+1}^y}{\Omega_t^1}$$

$$\alpha_{i1t+1}^z = \frac{\lambda_t^1 Q_{1t}^z S_{11t+1}^z}{\Omega_t^1} \qquad \alpha_{i2t+1}^z = 1 - \alpha_{i1t+1}^y - \alpha_{i2t+1}^y - \alpha_{i1t+1}^z$$

Using these definitions in the domestic budget constraint gives

$$\frac{\Omega_t^1}{\lambda_t^1} + C_{11t}^y + P_{2t}^y C_{12t}^y + P_{1t}^z C_{1t}^z = \frac{\Omega_{t-1}^1}{\lambda_{t-1}^1} ((R_{1t}^y - R_{2t}^z)\alpha_{11t}^y + (R_{2t}^y - R_{2t}^z)\alpha_{12t}^y + (R_{1t}^z - R_{2t}^z)\alpha_{11t}^z) + R_{2t}^z \frac{\Omega_{t-1}^1}{\lambda_{t-1}^1}$$

or, equivalently

$$(R_{1t}^y - R_{2t}^z)\alpha_{11t}^y + (R_{2t}^y - R_{2t}^z)\alpha_{12t}^y + (R_{1t}^z - R_{2t}^z)\alpha_{11t}^z = \frac{\lambda_{t-1}^1}{\Omega_{t-1}^1} \left(\frac{\Omega_t^1}{\lambda_t^1} + C_{11t}^y + P_{2t}^y C_{12t}^y + P_{1t}^z C_{1t}^z \right) - R_{2t}^z$$

The last equation can be written in more compact form as

$$M(\mathscr{Y}_t) \bullet \alpha_t = L(\mathscr{Y}_t) \text{ with } \alpha_t = \{\alpha_{11t}^y, \alpha_{12t}^y, \alpha_{11t}^z\} \quad (14)$$

Since shares are predetermined, equation (14) implies

$$(M(\mathscr{Y}_t) - \mathbb{E}_{t-1}M(\mathscr{Y}_t)) \bullet \alpha_t = L(\mathscr{Y}_t) - \mathbb{E}_{t-1}L(\mathscr{Y}_t) \quad (15)$$

Market completeness implies that the last relation should hold for all realizations of shocks. Projecting equation (15) on each of the shocks gives a system of four equations in three unknowns, $\{\alpha_{1t}^y, \alpha_{2t}^y, \alpha_{1t}^z\}$. Using any of these three equations to determine the three wealth shares and equation $\alpha_{i2t+1}^z = 1 - \alpha_{i1t+1}^y - \alpha_{i2t+1}^y - \alpha_{i1t+1}^z$, to determine the fourth one gives the solution for wealth and portfolio shares[10]. To a first order approximation, this linear system delivers constant shares.[11] In the technical appendix we report how to check whether the four stocks effectively complete the markets in our model.

2 Parametrization

Our baseline parametrization corresponds to the model of SD. It is reported in Table 2. Setting $\sigma\rho = 1$ implies that the marginal utility of traded good is not affected by variation in the consumption of the non–traded good (and vice versa). The value of ρ used by Stockman and Tesar, 1995, is $\rho = 0.44$. Hence, a natural choice in this case is to set $\sigma = 2$ and $\rho = 0.5$. As in SD, we set $\alpha_1 = \alpha_2 = 0.5$ so there is no bias in the consumption of traded goods. The average degree of openness (import share) in the US over 1970–2005 is 10.5%. This implies a value for the share of traded goods in the CPI,[12] $\omega = 0.21$ (the imports share is $\omega(1-\alpha) = 0.21 \times 0.50 = 0.105$). This share is virtually identical to that reported by Burstein et al. 2005. It is considerably smaller than that used in the earlier literature because it takes into account the non–traded services and goods associated with the distribution of traded goods (distribution costs associated with wholesale and retail services, marketing and advertising, and local transportation services). It is computed as follows: In the US, the share of traded goods in the CPI, computed the traditional way, is 0.429. Burstein et al, calculate that approximately 50% of that involves non–traded distribution costs. Hence, the true share of traded goods in CPI is only about 0.21 (50% of

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[10]We have checked that it does not make any difference for the results which of the three projections are used or which wealth shares we solve for first. The remaining equation is always satisfied. See the appendix.

[11]This first order approximation with time invariant shares will prove useful for an intuitive exposition of our results. Nevertheless, we have also computed a second order approximation to equation (15). In this case, there is time variation in share holdings, but the average values are virtually identical to the ones from the first order approximation. The interested reader is referred to a companion technical appendix to this paper. The results strongly indicate that low order approximations have very good accuracy properties when computing asset holdings.

[12]Note that this result assumes that the relative price of the non–traded good is equal to one in steady state. This is ensured by selecting the ratio of the averages of the endowments as $\overline{z}/\overline{y} = (1 - \omega)/\omega$.

0.429) of which half approximately comes from imported goods and the other half comes from what Burstein et al. call local goods[13].

The elasticity of substitution between domestic and foreign traded goods is set to 1.5 as in Backus et al., 1995. The discount factor is set to 0.99. The endowment processes are assumed to be identical. In fact, the form of the processes does not matter for the results.[14] More precisely, neither persistence nor volatilities are of any consequences for the determination of wealth and equity shares. Naturally, the average level of the process does matter. In the benchmark, traded and non–traded endowments are such that the relative price of non–traded good is equal to unity.

We also report results with an alternative parametrization that involves a bias in the consumption of traded goods. Traded goods consumption –home– bias has been used in the literature as a shortcut for the non-traded goods specification. As mentioned above, Burstein et al., 2005, assign half of the traded goods in the CPI to imports (10.5% in the US case) and the other half to local (exportable) goods. But it seems reasonable to assume that some of these local goods may actually be non–tradeables[15]. Let us —arbitrarily— assume that about one–fourth of this 10.5% of local goods is indeed non–traded (say, 2.5%) rather than exportables. This implies that the true share of domestic exportables in the domestic consumption basket is only 8% rather that the 10.5% we used earlier (10.5% − 2.5%), and consequently, there exists foreign bias in domestic consumption. With this assumption, the true share of traded goods decreases from $\omega = 0.21$ to $\omega = 0.21 - 0.025 = 0.185$ and $\alpha_1 = 0.08/0.185 = 0.43$.[16] We call this the case of bias in the consumption of the traded goods.

3   The results

Table 3 reports wealth shares, *i.e.* the share of total wealth of a domestic agent that is held in the form of one of the four assets available, and equity shares *i.e.* the share of the value of equity in a particular industry that is owned by domestic agents, in the separable case of SD. As detailed below in section (3.1.1), if utility is separable and there is no consumption bias in traded goods, then the model implies that the –average– share of foreign equity held in the domestic equity portfolio is equal to the average share of imports in GDP. That is, the model

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[13]Burstein et al, 2005, report a total import content in US consumption of 9.1%. Local goods are mostly exportable goods that are consumed locally.

[14]This is a standard results in models where all diversifiable risk is perfectly shared.

[15]The behavior of the prices of these goods lies in between that of non–traded and traded. See Burstein et al., 2005.

[16]In both cases, the steady state levels of the endowments are adjusted so that the relative price of the foreign traded good is one.

Table 2: Parametrization

Parameter		Separable	Consumption Bias
Preferences			
Discount Factor	β	0.99	0.99
Risk aversion	σ	2.00	2.00
Total Consumption Bundle			
Share of traded	ω	0.21	0.185
Substitution traded/nontraded	ρ	0.50	0.50
Traded Goods Bundle			
Share of Domestic Traded	$\alpha_1 = \alpha_2$	0.50	0.43
Substitution Domestic/Foreign	φ	1.50	1.50
Endowments			
Persistence	$\rho_{11}^y = \rho_{22}^y = \rho_{11}^z = \rho_{22}^z$	0.85	0.85
Spillover	$\rho_{12}^y = \rho_{21}^y = \rho_{12}^z = \rho_{21}^z$	0.05	0.05
Volatility	$\sigma_1^y = \sigma_2^y = \sigma_1^z = \sigma_2^z$	0.01	0.01
Correlation	$\mathrm{Corr}(\varepsilon_1^y, \varepsilon_2^y),\ \mathrm{Corr}(\varepsilon_1^z, \varepsilon_2^z)$	0.00	0.00

predicts that

$$s_{ij} = \frac{A_{ij}}{A_{ii} + A_{ij}} = t_i$$

where s_{ij} is the share of the equity wealth of the domestic investors that is invested in foreign equity and t_i is the imports shares. s_{ij} can be re-written as

$$s_{ij} = \frac{A_{ij}}{A_{ii} + A_{ij}} = \frac{A_{ij}/y_i}{A_{ii}/y_i + A_{ij}/y_i} = \frac{A_{ij}/y_i}{(A_{ii}/A_i)(A_i/y_i) + A_{ij}/y_i}$$

or

$$s_{ij} = \frac{\text{FOREIGN ASSETS}}{\text{STOCK MARKET CAPITALIZATION - FOREIGN LIABILITIES + FOREIGN ASSETS}}$$

where all of these quantities are measures as a % of GDP.

We have used data on foreign portfolio equity assets and liabilities from Lane–Milesi–Feretti, 2006, and data on market capitalization from the World Development Indicators to compute s_{ij} for financially developed countries. Figure 3 reports this share as well as the imports–GDP share. As in Table 1 the match is nearly perfect. A similarly good match obtains in individual years, in subperiods and so on. The result is very robust. The fit is not as good when less developed countries are included but this is not surprising given the widespread use of capital controls and the presence of severe official and unofficial financial impediments in those countries. Our model does not apply to such cases. Similarly, the fit is lower when very open economies, such as Ireland or Singapore, are included in the sample. This is due to the fact that the reported degree of openness overstates the true one due to re-exporting. This issue is taken up below.

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It should be noted that ours is not the only paper to explore the link foreign asset holding and the degree of openness. Heathcote and Perri, 2007, have performed a related exercise. They use different measures of openness and, more importantly, of the share of asset holdings. Their results are broadly consistent with the notion that trade in goods is an important determinant of asset trade[17].

Figure 1: Portfolio equity assets and imports (share of GDP, average 1995–2004, developed countries)

Table 3: Shares: The separable case

	Wealth Shares				Equity Shares			
	α_{11}^y	α_{11}^z	α_{12}^y	α_{12}^z	S_{11}^y	S_{11}^z	S_{12}^y	S_{12}^z
Separable	0.1050	0.7900	0.1050	0.0000	0.5000	1.0000	0.5000	0.0000

Nevertheless, there are good reasons for suspecting that the match seen in table 1 and in figure 3 may be implausibly good. This is due to the existence of important discrepancies between the measures of trade and wealth in the model and in the data. In particular, gross trade flows and

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[17]See also, Aviat and Coeurdacier, 2007, for a test that confirms this relationship in the context of the gravity model.

value added coincide in our model but not in the data. A great deal of trade involves intermediate products, and some of these goods are used in the production of exportables. Subsequently, the imports share needs to be adjusted for re-exports in order to arrive at an empirical measure of openness that corresponds to that in the model.

On the asset side, there are two problems with the wealth share used above. First, the model abstracts from the most important form of equity for households, namely, housing. In the appendix, we include housing by adding another equity that generates a flow of non-traded services and which is not traded in international equity markets[18]. We show that the share of foreign equity in domestic wealth (inclusive of housing) is equal to the imports share even in the extended model (under separability).

The second problem is that the model abstracts from multinational firms. Investing in domestic firms that operate abroad and/or own foreign assets is an indirect way for domestic investors to obtain international portfolio diversification. Hence, focusing only on the direct domestic and foreign equity components of the portfolio overestimates the degree of portfolio home bias.

Before proceeding to describe how we have dealt with these issues, it is instructive to derive the implied share of foreign equity in the domestic portfolio in the model that includes housing equity as a component of the domestic wealth (the modified model can be found in Appendix C.2).

Computing the share of foreign equity in wealth

The share of foreign equity in total domestic equity wealth, $\tilde{s}_{ij}$, is given by

$$\tilde{s}_{ij} = \frac{A_{ij}}{A_{ij} + A_{ii} + A_i^H} \tag{16}$$

where A_{ij} is domestic claims on –tradeable– foreign capital, A_{ii} is domestic claims on –tradeable– domestic capital, and A_i^H is the value of domestic housing.

Let re-write the share as

$$\tilde{s}_{ij} = \frac{1}{\frac{A_{ij} + A_{ii}}{A_{ij}} + \frac{A_i^N}{A_{ij}}} = (s_{ij}^{-1} + \frac{A_i^H}{A_i} \frac{A_i}{A_{ii}} \frac{A_{ii}}{A_{ij}})^{-1} \tag{17}$$

where A_i is the capitalization of the stock market in country i and s_{ij} is defines as

$$s_{ij} = \frac{A_{ij}}{A_{ij} + A_{ii}} \tag{18}$$

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[18]In the separable case it does not matter whether equity in housing is internationally traded or not. In equilibrium, all domestic housing is owned by domestic agents.

Solving equation (18) for A_{ii}/A_{ij} gives

$$\frac{A_{ii}}{A_{ij}} = s_{ij}^{-1} - 1 \tag{19}$$

We now determine A_i/A_{ii}. Using the definitions of the shares, we have

$$\frac{A_{ij}}{A_{ii}} = (1 - s_{ij})^{-1} - 1 \equiv k_1 \tag{20}$$

$$\frac{A_{ji}}{A_{jj}} = (1 - s_{ji})^{-1} - 1 \equiv k_2 \tag{21}$$

Let $A_i = zA_j$, where $z > 0$. z captures the relative size of the two stock markets, i and j. Using (20)-(21) to substitute for A_{ij} and A_{ji} in the equation $A_{ii} + A_{ji} = z(A_{jj} + A_{ij})$ leads to the following:

$$\frac{A_{ii}}{A_{jj}} = \frac{z - k_2}{1 - z * k_1} \equiv m \tag{22}$$

Consequently

$$s_{ij} = \frac{A_{ij}}{A_{ii} + A_{ij}} = \frac{A_{ij}}{m * A_{jj} + A_{ij}} = \frac{A_{ij}}{m * (A_j - A_{ij}) + A_{ij}} = \frac{1}{m * (\frac{A_i}{z*k_1 A_{ii}} - 1) + 1} \tag{23}$$

and so

$$\frac{A_i}{A_{ii}} = (\frac{s_{ij}^{-1} - 1}{m} + 1) * z * k_1 = L(s_{ij}, s_{ji}, z, \frac{A_i^H}{A_i}) \tag{24}$$

Making the corrections

We need information on –properly measured– s_{ij}, s_{ji} as well as on A_i^H/A_i and z. And on the trade side, we need to adjust imports for the foreign content of domestic exports.

Let us start with re-exports. Hummels et al., 2001, use input-output tables to compute the share of imported goods that is used as input to produce export goods for 10 OECD countries over the period 1970-1990. They find a substantial amount of re-exporting, ranging from a low of 11% of exports for the US and Japan to a high of 35% for Netherlands (in 1990). Chen et al., 2005, offer updated figures for these countries, which reveal that the degree of re-exporting increased further during the 1990s. For the US, this figure was 12.3% in 1997, the last year reported in Chen et al. Note that these figures refer to goods trade, there exists no information on the foreign valued added component of exports of domestic services[19].

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[19]Services are about 27% of total (goods and services) US trade.

The calculation of the market value of the housing stock is notoriously difficult, specially in an international context, due to the existence of serious measurement problems. We will rely on three alternative measures. The first is due to Case-Quigley-Shiller, 2005. They estimate real housing market wealth per capita for owner occupied housing in the US[20], up until 1999 using the Case-Shiller approach. They also report the ratio of the owner-occupied housing stock to the total housing stock for 1999, so one can compute the total value of housing. The ratio of the value of the housing stock to that of the stock market (stock market capitalization) in the 1990s falls in the interval[21] $0.8 - 1.0$.

The second measure uses the market value of the real estate holdings of US households[22] reported in the Federal Reserve Flow of Funds. The A_i^H/A_i ratio is constructed by dividing this value by stock market capitalization in the US (AMEX, NYSE, NASDAQ). The average value over 1995-2004 is 0.93.

The third measure utilizes the value of US housing constructed by Davis and Heathcote (2008). With this measure, the average ratio of housing to stock market capitalization over 1995-2004 is 1.18.

The relative size of stock markets, z, can be computed using data on stock market capitalization from the World Federation of Exchanges Members ($http : /www.world - exchanges.org$). The average ratio of US (AMEX, NYSE, NASDAQ) versus non-US stock markets, A_i/A_j, is 0.90 (for 1995-2004).

The third issue concerns the measurement of the share of foreign stocks in the total portfolio of stocks ($\tilde{s}_{ij}, \tilde{s}_{ji}$). The problem with the standard measures (such as those used in Table 4) is that they only contain the direct shares, which may be different from the effective shares. These shares coincide in the model but not in the data. It is well known that holding stocks of multinational firms may represent an indirect way of holding foreign equity. Determining the degree to which stocks in multinationals contribute to international portfolio diversification and thus constructing the effective share of foreign assets in domestic wealth is a difficult task. In a recent paper, Cai and Warrock, 2004, attempt this for the US. They run a factor model to calculate –for each firm– the extent to which its foreign beta varies with the amount of its foreign operations. And they then use this information to compute indirect foreign equity holdings. Adding the indirect to the direct holdings allows them to calculate the effective holdings. Namely, the share of foreign equities in US equity portfolios, s_{ij}, and the share of US equities in foreign equity portfolios, s_{ji}. The average value of the former over 1995-2003 is

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[20]They also compute the value of housing for some other OECD countries, but these values are not comparable across countries.

[21]There is some time variation. For instance, it is 0.83 in 1995-99, 0.96 over 1992-1999 and 1.05 over 1990-1999.

[22]Including also the real estate holdings of non-profit institutions has a very small effect on A_i^H/A_i.

around 19% and of the latter about 16%.

There exists no comparable information that would allow us to make these important corrections on the asset side (housing and effective foreign assets) for countries[23] other than the US. So we limit ourselves to the US. Table 4 reports the implications of the model for the share of foreign equity assets in domestic wealth as a function of A_i^H/A_i. Due to the uncertainty about A_i^H/A_i we compute this share for various values within the range of available estimates.

Table 4: Implied Foreign Equity Share in US Portfolios, 1995-2004

A_i^H/A_i	0.80	0.90	1.00	1.10	1.20
$\tilde{s}_{ij}$	0.107	0.100	0.097	0.092	0.088
Net US Imports/GDP= 0.117					

Note: Net US imports are calculated by subtracting the foreign content of domestic exports (re-exports) from imports. The share of foreign value added in US goods exports was 12.3% in 1997 (the last year reported by Chen et al., 2005).

The simple model with traded and non traded goods thus implies a share of foreign equity in wealth that is remarkably close to that observed in the real world. Admittedly, this represents only one observation, so it is only suggestive rather than conclusive. Nevertheless, we view this finding as confirming the view, first elaborated[24] by Stockman and Dellas, 1988, that trade in goods is the key determinant of international equity portfolios. The fact that the underlying composition of the consumption basket between traded and non traded goods is also satisfied by the data –as Burstein et al. 2005 establish– increases confidence in the theory.

Having argued that that trade in goods is sufficient for understanding international equity portfolios we now turn to the discussion of the underlying intuition. This will also help shed some light on the driving forces behind portfolio allocations in general. We then proceed to investigate the conditions under which the model also generates home bias in traded goods equity.

3.1 Discussion

3.1.1 Separable utility ($\sigma\rho = 1$), no consumption bias ($\alpha = 0.5$)

Observation 1: Investors choose to hold 100% and 0% of the domestic and foreign non–traded good equity respectively.

Variation in the endowment of the domestic non–traded good affects domestic residents who

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[23]Constructing internationally comparable measures of housing wealth, determining the effective share of foreign assets in domestic portfolios and calculating re-exports shares for countries other than the US and/or for more recent periods constitute important research projects on their own, that go beyond the scope of this paper.

[24]But already implicit in Lucas, 1982.

own equity in the non–traded goods sector in two ways. First, it affects the value of the stream of dividends in proportion to the number of shares held. And second, it affects the expenditure needed to finance the consumption of the non–traded good (because of the price change). If a domestic investor holds 100% of the domestic non–traded good equity and also consumes 100% of that good then the gain (loss) as an investor exactly offsets the loss (gain) as a consumer.

If utility is separable ($\sigma\rho = 1$) then traded and non–traded goods are neither substitutes nor complements. That is, variation in consumption of the non–traded good does not change the agents' utility from consumption of the traded good (and vice versa). In this case it is optimal to hold 100% of the domestic non–traded good equity (and 0% of the foreign) in order to prevent any international wealth redistribution following shocks to the non–traded good endowment. There is no need to have any redistribution —which would lead to changes in the relative shares of the traded good consumption bundle across the two countries— as the marginal utility of the consumption of the traded goods is independent of the amount of the non–traded good consumed.

Observation 2: Investors choose to hold a 50% share in each of the two traded goods –domestic and foreign– industries.

With separability between traded and non traded goods and similar preferences over traded goods across countries, SD show that the efficient level of consumption of traded goods is $c^y_{11t} = c^y_{21t} = y_{1t}/2$ and $c^y_{12t} = c^y_{22t} = y_{2t}/2$ (see also the solutions for c_{iit} in the Appendix A.4). This consumption pattern can be supported if each country holds a 50% share in each of the two traded goods.

We can now establish that such a portfolio implies that the average share of domestic wealth invested in foreign equity is equal to the average degree of trade openness (imports to GDP share). The average degree of openness in country 1 is

$$t_1 = \frac{P^y_2 C^y_{12}}{Y_1 + P^z_1 Z_1} = \frac{P^y_2 C^y_{12}/Y_1}{1 + P^z_1 Z_1/Y_1}$$

Recall that the steady state levels of the endowments were selected such that the relative prices are equal to unity. We show in appendix A.2 that this implies $P^y_2 C^y_{12}/Y_1 = C^y_{12}/Y_1 = 1-\alpha = 0.5$. The share of imports is then

$$t_1 = \frac{0.5}{1 + Z_1/Y_1}$$

The average share of country 1 wealth invested in foreign equity is

$$s_{12} = \frac{Q^y_2 S^y_{12} + Q^z_2 S^z_{12}}{Q^y_1 S^y_{11} + Q^y_2 S^y_{12} + Q^z_1 S^z_{11} + Q^z_2 S^z_{12}}$$

Using the equity shares reported above ($S^z_{12} = 0$ and $S^y_{12} = S^y_{11} = 0.5$) and the fact that in a

symmetric equilibrium $Q_1^y = Q_2^y = Q^y$ and $Q_1^z = Q_2^z = Q^z$, s_{12} reduces to

$$s_{12} = \frac{0.5}{1 + Q^z/Q^y}$$

As shown in the appendix A.2, the average ratio of stock prices Q^z/Q^y is equal to the average ratio of the dividends associated with these stocks, Z_1/Y_1. Consequently, the model implies $s_{12} = t_1$.

It is worth mentioning that we have also solved an augmented version of the model that also contains explicitly a distribution sector[25]. The details are described in Appendix C.1, where we also show that the introduction of a distribution sector does not alter the implication of the model regarding the equality of the foreign equity share and the imports/GDP share.

4   Extensions

In this section we modify the two key assumptions of the model studied above: Namely, that utility is separable in traded and non-traded goods. And that there exists no bias in the consumption basket of the traded goods. There exist three motivations for undertaking these modifications. First, to examine whether the –small– gap between the model implied foreign equity share and that observed in the data (table 4) can be bridged by departing from these two assumptions. Second, to discover the conditions under which the model can also generate biases within the traded goods equity sub-portfolio. And third, given the uncertainty surrounding the values of some important parameters (for instance, the elasticity of substitution between traded and non-traded goods), to check the sensitivity of the results to plausible variation in parameter values.

4.0.2   Non-separability

We first explore the implications of non-separability for optimal portfolios.

Non–separable utility ($\sigma\rho \neq 1$), no consumption bias ($\alpha = 0.5$)

We relax the assumption of separability by allowing $\sigma\rho$ to depart from unity. Moreover, we will assume that $\rho < 1$. In this case, the domestic and foreign traded goods are substitutes ($d^2u/dc_{11}^y dc_{12}^y < 0$) if the traded goods are more substitutable among themselves than they are with the non–traded goods, that is, if $\varphi > \rho$. This is an assumption that we will maintain throughout this section[26].

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[25]Recall that Burstein et al., 2005, emphasize that the omission of this sector is the source of the mismeasurement of the contribution of non-traded goods to consumption.

[26]Standard calibrations typically set $\varphi > 1$ and $\rho < 1$.

We now study the effects of a non–traded shock on consumption. The following proposition describes the changes in efficient consumption[27].

Proposition 1 *In equilibrium, the impact effect of a non–traded goods shock satisfies*

$$\frac{\partial c_{iit}^y}{\partial z_{it}} \gtreqless 0, \ \frac{\partial c_{iit}^y}{\partial z_{jt}} \lesseqgtr 0, \ \frac{\partial c_{ijt}^y}{\partial z_{it}} \gtreqless 0, \ \frac{\partial c_{ijt}^y}{\partial z_{jt}} \lesseqgtr 0, \ \frac{\partial c_{it}^y}{\partial z_{it}} \gtreqless 0, \ \frac{\partial c_{it}^y}{\partial z_{jt}} \lesseqgtr 0, \ \Longleftrightarrow \sigma\rho \lesseqgtr 1$$

The effects of traded goods shocks on efficient consumption allocations as well as the supporting portfolios can be studied in a similar fashion (see the appendix A.5).

Let us assume that $\sigma\rho < 1$, so that non–traded and traded are *complements*. In an efficient equilibrium, we want consumption of traded goods to be higher in the country that has experienced a positive endowment shock in its non–traded sector, because the increase in the consumption of non–traded goods increases the marginal utility of consumption of the traded good. But this implies that we want this country to experience a redistribution of income in its favor. This would happen if some of the shares of firms producing domestic non–traded goods were held by foreign investors. In this case, the gain to the domestic agents as consumers of the non–traded good would exceed their loss as investors in that good, because while they consume 100% of it they hold less than 100% of its equity. The foreign agents suffer an investment loss without reaping any consumption benefit. The resulting income redistribution allows the domestic agents to claim a larger proportion of the traded goods bundle. Hence, the optimal portfolio here involves holding a large share –almost unity– in the domestic non–traded sector and a small[28] share in the foreign non–traded sector. This is illustrated in the top row of Table 5. Here we have used the same parametrization as in the separable case (with $\sigma = 2$) except for the elasticity of substitution between traded and non traded which takes the value $\rho = 0.4$.

Table 5: Shares: The non-separable case

	Wealth Shares				Equity Shares			
	α_{11}^y	α_{11}^z	α_{12}^y	α_{12}^z	S_{11}^y	S_{11}^z	S_{12}^y	S_{12}^z
$\rho = 0.4$	0.1050	0.7741	0.1050	0.0159	0.5000	0.9798	0.5000	0.0202
$\rho = 0.6$	0.1050	0.8073	0.1050	-0.0173	0.5000	1.0219	0.5000	-0.0219

Exactly the same type of reasoning establishes that the domestic agents will want to hold more than 100% of the domestic non–traded equity and will want to short foreign non traded goods equity if traded and non–traded goods are substitutes, that is, if $\sigma\rho > 1$. The second row of Table 6 reports asset shares under the assumption that $\rho = 0.6$.

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[27]The interested reader is referred to Appendix A.6 for a formal proof of all the propositions.

[28]At least for small deviations from $\sigma\rho = 1$.

4.0.3 Consumption bias

Separable utility ($\sigma\rho = 1$), consumption bias ($\alpha \neq 0.5$)

We now turn to the issue of portfolio bias in the traded goods equity. One way of accomplishing this is by introducing a bias in the consumption of the traded goods[29]. A standard implication of trade theory –see below– is that there should be foreign bias in traded goods consumption. Suppose that this is the case ($\alpha_i < 0.5$). Assume also that $\sigma\rho = 1$. *Does this make the domestic investors want to hold an equity share in the domestic traded that exceeds or falls short of 0.5?*

Consider a positive shock to the domestic traded endowment. Let us hold consumption of the foreign traded good constant and allow the consumption of the domestic traded good to increase by the same proportion in the two countries. Because of the foreign bias in traded consumption, foreign consumption of traded goods increases by more than domestic consumption of traded goods. If $\varphi > \rho$, the marginal utility of both the domestic and foreign traded good is *decreasing* in total traded good consumption. This implies that the marginal utility of both traded goods has decreased more abroad that at home, violating the risk sharing principle. In order for the marginal utilities to be equalized, the domestic consumption of the traded goods must increase by a larger proportion than foreign consumption. In other words, the ratio of domestic to foreign expenditures of traded goods must increase. This result is described in the next proposition.

Proposition 2 *Let Θ_t denote the ratio of domestic to foreign traded expenditures*

$$\Theta_t = \frac{C_{11t}^y + P_{2t}^y C_{12t}^y}{C_{21t}^y + P_{2t}^y C_{22t}^y}$$

and let us assume $\sigma\rho = 1$, we then have

$$\frac{\partial \log(\Theta_t)}{\partial y_{1t}} \lesseqqgtr 0 \iff \rho \lesseqqgtr 1 \ and \ \alpha \gtreqqless \frac{1}{2}$$

In order to support the efficient equilibrium, asset holdings must be such that dividend income at home increases by more than dividend income abroad following a positive shock to the endowment of the domestic traded good. *What portfolio shares will deliver this?* The next proposition addresses this question.

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[29]The role of consumption bias for generating portfolio home bias has been investigated by Kollmann, 2006b in a model *without* any non-traded goods and *home* consumption bias. Kollmann finds that, in order for his model to generate an overall degree of portfolio home bias that *exceeds* the degree of consumption home bias, it must be the case that the elasticity of substitution between domestic and foreign traded goods falls within a narrow range strictly *below* unity (see Kollmann, 2006b). This requirement may be problematic as typical estimates of the elasticity of substitution between domestic and foreign traded goods not only exceed unity but they can be quite high. A specification with non traded goods does not suffer from this weakness.

Proposition 3 *When the elasticity of substitution between traded and non traded goods is less than unity ($\rho < 1$), traded goods are suitably substitutable among themselves ($\varphi > \rho + (1-\rho)/4\alpha(1-\alpha)$) and there is foreign bias in traded goods ($\alpha < 0.5$), then the optimal portfolio allocation exhibits home portfolio bias in traded goods industries.*

In order to understand this result, recall that the holdings of non–traded goods equity are not affected by the value of α as long as $\sigma\rho = 1$, hence, these shares remain at 100% and 0% respectively. In contrast, the appropriate holdings of traded goods equity depend on the value of α. If $\alpha < 0.5$ (foreign bias in the consumption of traded goods) then the supporting portfolio of traded goods equity must have a home bias ($S_{11}^y > 0.5, S_{12}^y < 0.5$), as long as $\varphi > 1$. With $\varphi > 1$, the increase in the endowment of the domestic traded good will lower its relative price but less than one for one. Holding shares $S_{21}^y < 0.5$ still allows the foreign consumer to consume more of the domestic tradeable because of its lower relative price. But, at the same time, it makes relative dividend incomes move in favor of the domestic agents, which allows them to claim a larger share of world tradeables (we argued above that this is a property of an efficient equilibrium).

The numerical results corresponding to the case of consumption bias and separability under the parametrization in the second column of Table 2 appear in Table 6.

Table 6: Shares: The separable case

	Wealth Shares				Equity Shares			
	α_{11}^y	α_{11}^z	α_{12}^y	α_{12}^z	S_{11}^y	S_{11}^z	S_{12}^y	S_{12}^z
Cons. Bias	0.1060	0.8150	0.0790	0.0000	0.5729	1.0000	0.4271	0.0000

Foreign consumption bias generates home bias in the traded goods equity sub-portfolio[30] and it also increases the degree of portfolio home bias. Nevertheless, as the comparison of Tables 6 and 3 reveals, this effect is rather small quantitatively.

Under home bias in the consumption of tradeables, $\alpha > 0.5$, the opposite pattern obtains. That is, there is *foreign bias* in the portfolio of traded goods equity. Nevertheless, overall portfolio home bias may still obtain. If the share of non–traded goods in the CPI is not too much below 50%, there will be home bias in consumption and portfolio independent of whether α is greater

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[30]Kollmann, 2006b, draws on Kang and Stulz, 1997, who report the existence of home bias in the shares of Japanese manufacturing, to claim the existence of home bias in the equity of traded goods. This interpretation is not justified as domestic consumption of foreign manufactures contains a significant domestic non–traded component (see Berstein et al., 2005). Interestingly, Kang and Stulz report that foreign investors hold larger equity positions in the manufacturing sector and lower ones in the wholesale and retail distribution as well as in services. This finding highlights the importance of the distinction between traded and non–traded goods for portfolio choice and offers support to the key thesis of our paper.

or smaller than 0.5.

The link between consumption and portfolio bias suggested above may seem paradoxical when looked through the prism of some interesting, extreme cases. Suppose, for instance, that only the domestic agents consume the foreign traded good (and vice versa). And that utility is separable between traded and non-traded goods. Under these circumstances, optimal hedging would have the domestic agents hold all of the foreign traded good equity. Or, suppose instead, that only the domestic agents consume the domestic traded good (and analogously for the foreign agents). Under these circumstances, optimal hedging would have the domestic agents hold all of the domestic traded good equity. Both of these cases seem to violate the spirit of Proposition 3 as they involve a positive rather than negative link between consumption and portfolio bias. Nonetheless, there is no contradiction between this proposition and these examples. When α is driven to zero (the first example) or to unity (the second example) then the right hand side in the second condition in Proposition 3 is driven to infinity, the proposition does not hold and it cannot be used to study optimal portfolios. One should view Proposition 3 as being a useful tool for determining optimal portfolios within an appropriate range of parameter values. Fortunately, for most economies in the real world, this appropriate range is also the empirically relevant one[31].

For the sake of assessing the model's prediction for bias in the traded goods equity, it is of interest to consider whether α is likely to exceed or to fall short of 0.5 in the real world and by how much.

We have already discussed how the CPI decomposition creates a presumption[32] that α is below (but not too much) 0.5. We have also carried a detailed study of the Swiss CPI. The documentation of the CPI only provides —precise— information on the domestic (or imports) content of each item that enters the basket. A possible —but arbitrary— classification that does not rely on an *ad hoc* assignment of items to the various categories (non–traded, domestic trade, foreign traded) is to assign an item to the non–traded category if the domestic content is 100%, to the foreign tradeable if the import content is more than 50% and to the domestic tradeable in all other cases. Such a classification produces the values $60\%, 11\%, 29\%$ for non–traded, domestic tradeable and foreign tradeable respectively. While it is clear that this classification is far from ideal as something may have 100% domestic content and still be a tradeable good, we view these figures as indicating that the likelihood of foreign bias in traded goods is not negligible.

Finally, a strong case for foreign bias in the consumption of traded goods can be made based

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[31]Under the benchmark values of $\rho = 0.5$ and $phi = 1.5$, the second condition in Proposition 3 is violated when $alpha$ is either below 0.15 or above 0.85.

[32]As Burstein et al., 2005, discuss, there exists no direct information on the domestic exportable goods component of the CPI.

on trade theory. Standard trade theory implies the existence of international specialization, with countries typically producing a small range of traded goods and exporting most of their tradeables production in exchange for a much broader set of foreign produced traded goods. A similar prediction arises in the new trade models, to the extent that there are gains from specialization, as it would be the case in the presence of returns to scale, country specific factors and so on.

4.1 Sensitivity Analysis

We now turn to the investigation of how variation in the key parameters of the model affects the composition of optimal portfolios. Table 7 provides information on the role of deviations from separability and from symmetry in the consumption of traded goods. Tables 8 and 9 provide information on the sensitivity of the results around the two baseline parametrizations reported in Table 2. The results are quite robust. In the range of the parameter values considered there are only two cases –both of them empirically unlikely– where the model fails to generate home bias in portfolio. Namely, when the elasticity of substitution between traded and non–traded goods, ρ, is very high (say, $\rho = 5$). And when the share of imports[33]. in the domestic consumption basket is very high ($\omega = 0.75$), see Table 8. Note also that higher values of the elasticity of substitution between domestic and foreign traded (as suggested by Obstfeld and Rogoff, 2000) do not undermine home bias. This is encouraging because there is great uncertainty regarding the precise value of this elasticity. The robustness of our results in the presence of plausible, large variation in the parameters values is another advantage of our proposed solution to the portfolio bias relative to other approaches. Those other approaches produce results that are extremely sensitive to even slight but plausible variation in key parameters.

5 Conclusions

Investors tend to invest most of their wealth in domestic assets, and most of the capital in any country is owned by the domestic residents. This is true even in countries that appear to be well integrated within the world capital markets. A large literature has attempted to provide an explanation to this phenomenon, with rather limited success so far. We show that the degree of international trade in goods is the main determinant of international equity portfolios. A simple model with traded and non–traded goods implies that international equity positions should match import shares. Subsequently, to the extent that true –re-exports adjusted– import shares fall short of 50%, international equity portfolios will exhibit a home bias.

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[33]We have already discussed what values of α make it unlikely for proposition 3 to be satisfied.

We compute the share of foreign equities in total domestic wealth for the US households, taking into account their equity in housing and also their indirect foreign equity holdings that arise from holding the stock of US multinationals. This share comes out at around 10% for the period 1995-2004. The corresponding "true" US imports to GDP share, after correcting for the foreign value added of US exports is around 11.7%. We interpreting this finding as suggesting that the model offers a compelling explanation not only to the portfolio bias puzzle but also, and more generally, to the determination of international portfolios.

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Table 7: The role of separability and of consumption bias

	Wealth Shares				Equity Shares			
	α_{11}^{y}	α_{11}^{z}	α_{12}^{y}	α_{12}^{z}	S_{11}^{y}	S_{11}^{z}	S_{12}^{y}	S_{12}^{z}
$\sigma\rho < 1$								
$\alpha < 0.5$	0.1365	0.7502	0.0735	0.0398	0.6499	0.9496	0.3501	0.0504
$\alpha = 0.5$	0.1050	0.7525	0.1050	0.0375	0.5000	0.9525	0.5000	0.0475
$\alpha > 0.5$	0.0735	0.7502	0.1365	0.0398	0.3501	0.9496	0.6499	0.0504
$\sigma\rho = 1$								
$\alpha < 0.5$	0.1278	0.7900	0.0822	0.0000	0.6087	1.0000	0.3913	0.0000
$\alpha = 0.5$	0.1050	0.7900	0.1050	0.0000	0.5000	1.0000	0.5000	0.0000
$\alpha > 0.5$	0.0822	0.7900	0.1278	0.0000	0.3913	1.0000	0.6087	0.0000
$\sigma\rho > 1$								
$\alpha < 0.5$	0.1175	0.8374	0.0925	-0.0474	0.5596	1.0601	0.4404	-0.0601
$\alpha = 0.5$	0.1050	0.8363	0.1050	-0.0463	0.5000	1.0587	0.5000	-0.0587
$\alpha > 0.5$	0.0925	0.8374	0.1175	-0.0474	0.4404	1.0601	0.5596	-0.0601

Note: Here we assume that $\omega = 0.21$ as in our first benchmark calibration. $\rho\sigma < 1$ corresponds to the case $\rho = 0.25$ while $\rho\sigma > 1$ denotes $\rho = 0.75$. The case $\alpha < 1$ assumes $\alpha = 0.4$ and $\alpha > 0.5$ assumes $\alpha = 0.6$.

Table 8: Shares: Sensitivity analysis (Benchmark: Separable case)

	Wealth Shares				Equity Shares			
	α_{11}^y	α_{11}^z	α_{12}^y	α_{12}^z	S_{11}^y	S_{11}^z	S_{12}^y	S_{12}^z
Benchmark	0.1050	0.7900	0.1050	0.0000	0.5000	1.0000	0.5000	0.0000
Consumption share of traded goods: ω								
0.10	0.0222	0.9556	0.0222	0.0000	0.5000	1.0000	0.5000	0.0000
0.50	0.3950	0.2100	0.3950	0.0000	0.5000	1.0000	0.5000	0.0000
0.75	0.4857	0.0287	0.4857	0.0000	0.5000	1.0000	0.5000	0.0000
Elasticity of substitution traded vs nontraded: ρ								
0.25	0.1050	0.7525	0.1050	0.0375	0.5000	0.9525	0.5000	0.0475
0.75	0.1050	0.8363	0.1050	-0.0463	0.5000	1.0587	0.5000	-0.0587
5.00	0.1050	-0.0488	0.1050	0.8388	0.5000	-0.0618	0.5000	1.0618
Consumption share of domestic traded good in consumption of traded: α								
0.25	0.2100	0.7900	0.0000	0.0000	1.0000	1.0000	0.0000	0.0000
0.75	0.0000	0.7900	0.2100	0.0000	0.0000	1.0000	1.0000	0.0000
Elasticity of substitution between traded goods: φ								
0.60	0.1050	0.7900	0.1050	0.0000	0.5000	1.0000	0.5000	0.0000
2.00	0.1050	0.7900	0.1050	0.0000	0.5000	1.0000	0.5000	0.0000
5.00	0.1050	0.7900	0.1050	0.0000	0.5000	1.0000	0.5000	0.0000
Elasticity of intertemporal substitution: σ								
0.50	0.1050	0.9716	0.1050	-0.1816	0.5000	1.2299	0.5000	-0.2299
4.00	0.1050	0.8197	0.1050	-0.0297	0.5000	1.0376	0.5000	-0.0376

Table 9: Shares: Sensitivity analysis (Benchmark: Consumption Bias)

	Wealth Shares				Equity Shares			
	α_{11}^y	α_{11}^z	α_{12}^y	α_{12}^z	S_{11}^y	S_{11}^z	S_{12}^y	S_{12}^z
Benchmark	0.1060	0.8150	0.0790	0.0000	0.5729	1.0000	0.4271	0.0000
Consumption share of traded goods: ω								
0.10	0.0295	0.9484	0.0220	0.0000	0.5729	1.0000	0.4271	0.0000
0.50	0.4669	0.1850	0.3481	0.0000	0.5729	1.0000	0.4271	0.0000
0.75	0.5588	0.0246	0.4166	0.0000	0.5729	1.0000	0.4271	0.0000
Elasticity of substitution traded vs nontraded: ρ								
0.25	0.1111	0.7795	0.0739	0.0355	0.6008	0.9565	0.3992	0.0435
0.75	0.0999	0.8570	0.0851	-0.0420	0.5398	1.0515	0.4602	-0.0515
5.00	0.3017	-0.5284	-0.1167	1.3434	1.6309	-0.6483	-0.6309	1.6483
Consumption share of domestic traded good in consumption of traded: α								
0.25	0.1850	0.8150	0.0000	0.0000	1.0000	1.0000	0.0000	0.0000
0.75	0.0000	0.8150	0.1850	0.0000	0.0000	1.0000	1.0000	0.0000
Elasticity of substitution between traded goods: φ								
0.60	0.0764	0.8150	0.1086	0.0000	0.4129	1.0000	0.5871	0.0000
2.00	0.0992	0.8150	0.0858	0.0000	0.5361	1.0000	0.4639	0.0000
5.00	0.0942	0.8150	0.0908	0.0000	0.5089	1.0000	0.4911	0.0000
Elasticity of intertemporal substitution: σ								
0.50	0.1113	0.9760	0.0737	-0.1610	0.6016	1.1975	0.3984	-0.1975
4.00	0.1069	0.8424	0.0781	-0.0274	0.5777	1.0336	0.4223	-0.0336

A First Order Conditions and the log-linearized version

A.1 Efficient Allocation

Let us focus on the efficient allocation problem, we have the following set of conditions in equilibrium

$$C_t^1 = \left(\omega^{\frac{1}{\rho}} C_{1t}^{y}{}^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} C_{1t}^{z}{}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{25}$$

$$C_t^2 = \left(\omega^{\frac{1}{\rho}} C_{2t}^{y}{}^{\frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} C_{2t}^{z}{}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{26}$$

$$C_{1t}^y = \left(\alpha^{\frac{1}{\varphi}} C_{11t}^{y}{}^{\frac{\varphi-1}{\varphi}} + (1-\alpha)^{\frac{1}{\varphi}} C_{12t}^{y}{}^{\frac{\varphi-1}{\varphi}} \right)^{\frac{\varphi}{\varphi-1}} \tag{27}$$

$$C_{2t}^y = \left((1-\alpha)^{\frac{1}{\varphi}} C_{21t}^{y}{}^{\frac{\varphi-1}{\varphi}} + \alpha^{\frac{1}{\varphi}} C_{22t}^{y}{}^{\frac{\varphi-1}{\varphi}} \right)^{\frac{\varphi}{\varphi-1}} \tag{28}$$

$$\Lambda_t P_{1t}^z = (1-\omega)^{\frac{1}{\rho}} C_{1t}^{z}{}^{-\frac{1}{\rho}} C_t^1{}^{\frac{1}{\rho}-\sigma} \tag{29}$$

$$\Lambda_t = \omega^{\frac{1}{\rho}} \alpha^{\frac{1}{\varphi}} C_{11t}^{y}{}^{-\frac{1}{\varphi}} C_{1t}^{y}{}^{\frac{1}{\varphi}-\frac{1}{\rho}} C_t^1{}^{\frac{1}{\rho}-\sigma} \tag{30}$$

$$\Lambda_t P_{2t}^y = \omega^{\frac{1}{\rho}} (1-\alpha)^{\frac{1}{\varphi}} C_{12t}^{y}{}^{-\frac{1}{\varphi}} C_{1t}^{y}{}^{\frac{1}{\varphi}-\frac{1}{\rho}} C_t^1{}^{\frac{1}{\rho}-\sigma} \tag{31}$$

$$\Lambda_t P_{2t}^z = (1-\omega)^{\frac{1}{\rho}} C_{2t}^{z}{}^{-\frac{1}{\rho}} C_t^2{}^{\frac{1}{\rho}-\sigma} \tag{32}$$

$$\Lambda_t P_{2t}^y = \omega^{\frac{1}{\rho}} \alpha^{\frac{1}{\varphi}} C_{22t}^{y}{}^{-\frac{1}{\varphi}} C_{2t}^{y}{}^{\frac{1}{\varphi}-\frac{1}{\rho}} C_t^2{}^{\frac{1}{\rho}-\sigma} \tag{33}$$

$$\Lambda_t = \omega^{\frac{1}{\rho}} (1-\alpha)^{\frac{1}{\varphi}} C_{21t}^{y}{}^{-\frac{1}{\varphi}} C_{2t}^{y}{}^{\frac{1}{\varphi}-\frac{1}{\rho}} C_t^2{}^{\frac{1}{\rho}-\sigma} \tag{34}$$

$$Y_{1t} = C_{11t}^y + C_{21t}^y \tag{35}$$

$$Y_{2t} = C_{12t}^y + C_{22t}^y \tag{36}$$

$$Z_{1t} = C_{1t}^z \tag{37}$$

$$Z_{2t} = C_{2t}^z \tag{38}$$

As can be immediately seen from (29)–(34), setting $\rho\sigma = 1$ corresponds to a separable utility function as in this case

$$\Lambda_t P_{1t}^z = (1-\omega)^{\frac{1}{\rho}} C_{1t}^{z}{}^{-\frac{1}{\rho}}$$

$$\Lambda_t = \omega^{\frac{1}{\rho}} \alpha^{\frac{1}{\varphi}} C_{11t}^{y}{}^{-\frac{1}{\varphi}} C_{1t}^{y}{}^{\frac{1}{\varphi}-\frac{1}{\rho}}$$

$$\Lambda_t P_{2t}^y = \omega^{\frac{1}{\rho}} (1-\alpha)^{\frac{1}{\varphi}} C_{12t}^{y}{}^{-\frac{1}{\varphi}} C_{1t}^{y}{}^{\frac{1}{\varphi}-\frac{1}{\rho}}$$

$$\Lambda_t P_{2t}^z = (1-\omega)^{\frac{1}{\rho}} C_{2t}^{z}{}^{-\frac{1}{\rho}}$$

$$\Lambda_t P_{2t}^y = \omega^{\frac{1}{\rho}} \alpha^{\frac{1}{\varphi}} C_{22t}^{y}{}^{-\frac{1}{\varphi}} C_{2t}^{y}{}^{\frac{1}{\varphi}-\frac{1}{\rho}}$$

$$\Lambda_t = \omega^{\frac{1}{\rho}} (1-\alpha)^{\frac{1}{\varphi}} C_{21t}^{y}{}^{-\frac{1}{\varphi}} C_{2t}^{y}{}^{\frac{1}{\varphi}-\frac{1}{\rho}}$$

A.2 Some steady state results

Let us focus on the steady state of the problem, we have the following set of conditions in equilibrium

$$C^1 = \left(\omega^{\frac{1}{\rho}} C_1^{y \frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} C_1^{z \frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{39}$$

$$C^2 = \left(\omega^{\frac{1}{\rho}} C_2^{y \frac{\rho-1}{\rho}} + (1-\omega)^{\frac{1}{\rho}} C_2^{z \frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \tag{40}$$

$$C_1^y = \left(\alpha^{\frac{1}{\varphi}} C_{11t}^{y \frac{\varphi-1}{\varphi}} + (1-\alpha)^{\frac{1}{\varphi}} C_{12}^{y \frac{\varphi-1}{\varphi}} \right)^{\frac{\varphi}{\varphi-1}} \tag{41}$$

$$C_2^y = \left((1-\alpha)^{\frac{1}{\varphi}} C_{21}^{y \frac{\varphi-1}{\varphi}} + \alpha^{\frac{1}{\varphi}} C_{22}^{y \frac{\varphi-1}{\varphi}} \right)^{\frac{\varphi}{\varphi-1}} \tag{42}$$

$$\Lambda P_1^z = (1-\omega)^{\frac{1}{\rho}} C_1^{z-\frac{1}{\rho}} C^{1 \frac{1}{\rho}-\sigma} \tag{43}$$

$$\Lambda = \omega^{\frac{1}{\rho}} \alpha^{\frac{1}{\varphi}} C_{11t}^{y-\frac{1}{\varphi}} C_1^{y \frac{1}{\varphi}-\frac{1}{\rho}} C^{1 \frac{1}{\rho}-\sigma} \tag{44}$$

$$\Lambda P_2^y = \omega^{\frac{1}{\rho}} (1-\alpha)^{\frac{1}{\varphi}} C_{12}^{y-\frac{1}{\varphi}} C_1^{y \frac{1}{\varphi}-\frac{1}{\rho}} C^{1 \frac{1}{\rho}-\sigma} \tag{45}$$

$$\Lambda P_2^z = (1-\omega)^{\frac{1}{\rho}} C_2^{z-\frac{1}{\rho}} C^{2 \frac{1}{\rho}-\sigma} \tag{46}$$

$$\Lambda P_2^y = \omega^{\frac{1}{\rho}} \alpha^{\frac{1}{\varphi}} C_{22}^{y-\frac{1}{\varphi}} C_2^{y \frac{1}{\varphi}-\frac{1}{\rho}} C^{2 \frac{1}{\rho}-\sigma} \tag{47}$$

$$\Lambda = \omega^{\frac{1}{\rho}} (1-\alpha)^{\frac{1}{\varphi}} C_{21}^{y-\frac{1}{\varphi}} C_2^{y \frac{1}{\varphi}-\frac{1}{\rho}} C^{2 \frac{1}{\rho}-\sigma} \tag{48}$$

$$Y_1 = C_{11t}^y + C_{21}^y \tag{49}$$

$$Y_2 = C_{12}^y + C_{22}^y \tag{50}$$

$$Z_1 = C_1^z \tag{51}$$

$$Z_2 = C_2^z \tag{52}$$

$$Q_1^y = \beta(Q_1^y + Y_1) \tag{53}$$

$$Q_2^y = \beta(Q_2^y + P_2^y Y_2) \tag{54}$$

$$Q_1^z = \beta(Q_1^z + P_1^z Z_1) \tag{55}$$

$$Q_2^z = \beta(Q_2^z + P_2^z Z_2) \tag{56}$$

Note that defining $\overline{P}_1^y = \left(\alpha + (1-\alpha)P_2^{y 1-\varphi} \right)^{\frac{1}{1-\varphi}}$ and $\overline{P}_2^y = \left(\alpha P_2^{y 1-\varphi} + (1-\alpha) \right)^{\frac{1}{1-\varphi}}$, equations (44)–(45) and (47)–(48) reduce to

$$\Lambda \overline{P}_1^y = (1-\omega)^{\frac{1}{\rho}} C_1^{y-\frac{1}{\rho}} C^{1 \frac{1}{\rho}-\sigma} \tag{57}$$

$$\Lambda \overline{P}_2^y = (1-\omega)^{\frac{1}{\rho}} C_2^{y-\frac{1}{\rho}} C^{2 \frac{1}{\rho}-\sigma} \tag{58}$$

Let focus on the case of a symmetric equilibrium: $Y_1 = Y_2 = Y$, $Z_1 = Z_2 = Z$, $C_1^z = C_2^z$, $C^1 = C^2$, $C_{11}^y = C_{22}^y$ and $C_{12}^y = C_{21}^y$. We first give conditions such that $\overline{P}_1^y = \overline{P}_2^y = P_2^y = P_1^z = P_2^z = 1$. Since we will restrict ourselves to a symmetric economy, it will be sufficient to only

consider the domestic economy. In this case, using (43) and (57), we get

$$\frac{P_1^z}{\overline{P}_1^y} = \frac{(1-\omega)^{\frac{1}{\rho}}C_1^{z-\frac{1}{\rho}}}{(1-\omega)^{\frac{1}{\rho}}C_1^{y-\frac{1}{\rho}}}$$

Hence, we have

$$\frac{P_1^z}{\overline{P}_1^y} = 1 \iff \frac{C_1^z}{C_1^y} = \frac{1-\omega}{\omega}$$

Note that once we have $\overline{P}_1^y = 1$, it follows from its definition that $P_2^y = 1$, and therefore $\overline{P}_2^y = 1$. Also note that, in equilibrium, we have $C_1^z = Z_1$, and $C_1^y = C_{11}^y + P_2^y C_{12}^y = C_{11}^y + C_{12}^y = C_{11}^y + C_{21}^y$ (the last step follows from symmetry). Since in equilibrium $C_{11}^y + C_{21}^y = Y_1$, it follows that $C_1^y = Y_1$. Hence, in order for relative prices to be equal to unity in the steady state it is sufficient that the ratio of endowment be given by

$$\frac{Z_1}{Y_1} = \frac{Z_2}{Y_2} = \frac{Z}{Y} = \frac{1-\omega}{\omega}$$

This is the assumption we make in the paper.

With relative prices equal to unity, in a symmetric equilibrium equations (44)–(45) and (47)–(48) imply

$$\frac{\alpha}{C_{11}^y} = \frac{1-\alpha}{C_{12}^y} = \frac{\alpha}{C_{22}^y} = \frac{1-\alpha}{C_{21}^y}$$

which produces

$$\frac{\alpha}{C_{11}^y} = \frac{1}{C_{11}^y + C_{21}^y} = \frac{1}{Y_1} \iff \frac{C_{11}^y}{Y_1} = \alpha$$

and similarly,

$$\frac{C_{11}^y}{Y_1} = \frac{C_{22}^y}{Y_2} = \alpha \text{ and } \frac{C_{12}^y}{Y_1} = \frac{C_{21}^y}{Y_2} = 1 - \alpha$$

Asset prices in the deterministic steady state can be easily determined. With relative prices equal to unity, we have

$$Q_i^x = \frac{\beta}{1-\beta}x_i$$

where $i = \{1, 2\}$ and $x = \{y, z\}$. A direct consequence is then that $Q^z/Q^y = Z/Y$.

A.3 Log–linear Representation

The log-linear version of this system is given by

$$c_t^1 = \varpi c_{1t}^y + (1 - \varpi)z_{1t} \tag{59}$$

$$c_t^2 = \varpi c_{2t}^y + (1 - \varpi)z_{2t} \tag{60}$$

$$c_{1t}^y = \alpha c_{11t}^y + (1 - \alpha)c_{12t}^y \tag{61}$$

$$c_{2t}^y = (1 - \alpha)c_{21t}^y + \alpha c_{22t}^y \tag{62}$$

$$\frac{1 - \sigma\rho}{\rho}c_{1t} - \frac{z_{1t}}{\rho} = \lambda_t + p_{1t}^z \tag{63}$$

$$\left(\frac{1}{\varphi} - \frac{1}{\rho}\right)c_{1t}^y + \frac{1 - \sigma\rho}{\rho}c_{1t} - \frac{c_{11t}^y}{\varphi} = \lambda_t \tag{64}$$

$$\left(\frac{1}{\varphi} - \frac{1}{\rho}\right)c_{1t}^y + \frac{1 - \sigma\rho}{\rho}c_{1t} - \frac{c_{12t}^y}{\varphi} = \lambda_t + p_{2t}^y \tag{65}$$

$$\frac{1 - \sigma\rho}{\rho}c_{2t} - \frac{z_{2t}}{\rho} = \lambda_t + p_{2t}^z \tag{66}$$

$$\left(\frac{1}{\varphi} - \frac{1}{\rho}\right)c_{2t}^y + \frac{1 - \sigma\rho}{\rho}c_{2t} - \frac{c_{22t}^y}{\varphi} = \lambda_t + p_{2t}^y \tag{67}$$

$$\left(\frac{1}{\varphi} - \frac{1}{\rho}\right)c_{2t}^y + \frac{1 - \sigma\rho}{\rho}c_{2t} - \frac{c_{21t}^y}{\varphi} = \lambda_t \tag{68}$$

$$y_{1t} = \alpha c_{11t}^y + (1 - \alpha)c_{21t}^y \tag{69}$$

$$y_{2t} = (1 - \alpha)c_{12t}^y + \alpha c_{22t}^y \tag{70}$$

where $\varpi = \omega^{\frac{1}{\rho}}\bar{y}^{\frac{\rho-1}{\rho}}/(\omega^{\frac{1}{\rho}}\bar{y}^{\frac{\rho-1}{\rho}} + (1 - \omega)^{\frac{1}{\rho}}\bar{z}^{\frac{\rho-1}{\rho}})$

A.4 Solving the Log–linearized Version

From (69) and (70) we get

$$c_{21t}^y = \frac{y_{1t} - \alpha c_{11t}^y}{1 - \alpha} \tag{71}$$

$$c_{12t}^y = \frac{y_{2t} - \alpha c_{22t}^y}{1 - \alpha} \tag{72}$$

Plugging this last result in (61) and (62), we get

$$c_{1t}^y = y_{2t} + \alpha c_{11t}^y - \alpha c_{22t}^y \tag{73}$$

$$c_{2t}^y = y_{1t} + \alpha c_{22t}^y - \alpha c_{11t}^y \tag{74}$$

therefore, using (59) and (60), we obtain

$$c_t^1 = \varpi\alpha c_{11t}^y - \varpi\alpha c_{22t}^y + \varpi y_{2t} + (1 - \varpi)z_{1t} \tag{75}$$

$$c_t^2 = \varpi\alpha c_{22t}^y - \varpi\alpha c_{11t}^y + \varpi y_{1t} + (1 - \varpi)z_{2t} \tag{76}$$

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Using (71)–(76) in (64) and (68), we get

$$\lambda_t = \left(\frac{1}{\varphi} - \frac{1}{\rho}\right)(y_{2t} + \alpha c_{11t}^y - \alpha c_{22t}^y) + \frac{1-\sigma\rho}{\rho}(\varpi\alpha c_{11t}^y - \varpi\alpha c_{22t}^y + \varpi y_{2t} + (1-\varpi)z_{1t}) - \frac{c_{11t}^y}{\varphi}$$

$$\lambda_t = \left(\frac{1}{\varphi} - \frac{1}{\rho}\right)(y_{1t} + \alpha c_{22t}^y - \alpha c_{11t}^y) + \frac{1-\sigma\rho}{\rho}(\varpi\alpha c_{22t}^y - \varpi\alpha c_{11t}^y + \varpi y_{1t} + (1-\varpi)z_{2t}) - \frac{c_{21t}^y}{\varphi}$$

computing the difference, we get

$$\left(\frac{2\alpha(\rho-\varphi)}{\rho\varphi} + \frac{2\alpha\varpi(1-\sigma\rho)}{\rho} - \frac{1}{\varphi(1-\alpha)}\right)c_{11t}^y - \left(\frac{2\alpha(\rho-\varphi)}{\rho\varphi} + \frac{2\alpha\varpi(1-\sigma\rho)}{\rho}\right)c_{22t}^y$$

$$= \left(\frac{(\rho-\varphi)}{\rho\varphi} + \frac{\varpi(1-\sigma\rho)}{\rho} - \frac{1}{\varphi(1-\alpha)}\right)y_{1t} - \left(\frac{(\rho-\varphi)}{\rho\varphi} + \frac{\varpi(1-\sigma\rho)}{\rho}\right)y_{2t}$$

$$+ \frac{(1-\varpi)(1-\sigma\rho)}{\rho}(z_{2t} - z_{1t}) \tag{77}$$

Likewise, using (71)–(76) in (65) and (67), we get

$$\lambda_t + p_{2t}^y = \left(\frac{1}{\varphi} - \frac{1}{\rho}\right)(y_{2t} + \alpha c_{11t}^y - \alpha c_{22t}^y) + \frac{1-\sigma\rho}{\rho}(\varpi\alpha c_{11t}^y - \varpi\alpha c_{22t}^y + \varpi y_{2t} + (1-\varpi)z_{1t}) - \frac{c_{12t}^y}{\varphi}$$

$$\lambda_t + p_{2t}^y = \left(\frac{1}{\varphi} - \frac{1}{\rho}\right)(y_{1t} + \alpha c_{22t}^y - \alpha c_{11t}^y) + \frac{1-\sigma\rho}{\rho}(\varpi\alpha c_{22t}^y - \varpi\alpha c_{11t}^y + \varpi y_{1t} + (1-\varpi)z_{2t}) - \frac{c_{22t}^y}{\varphi}$$

computing the difference, we get

$$\left(\frac{2\alpha(\rho-\varphi)}{\rho\varphi} + \frac{2\alpha\varpi(1-\sigma\rho)}{\rho} - \frac{1}{\varphi(1-\alpha)}\right)c_{22t}^y - \left(\frac{2\alpha(\rho-\varphi)}{\rho\varphi} + \frac{2\alpha\varpi(1-\sigma\rho)}{\rho}\right)c_{11t}^y$$

$$= \left(\frac{(\rho-\varphi)}{\rho\varphi} + \frac{\varpi(1-\sigma\rho)}{\rho} - \frac{1}{\varphi(1-\alpha)}\right)y_{2t} - \left(\frac{(\rho-\varphi)}{\rho\varphi} + \frac{\varpi(1-\sigma\rho)}{\rho}\right)y_{1t}$$

$$+ \frac{(1-\varpi)(1-\sigma\rho)}{\rho}(z_{1t} - z_{2t}) \tag{78}$$

Let us denote $\gamma = \frac{(\rho-\varphi)}{\rho\varphi} + \frac{\varpi(1-\sigma\rho)}{\rho}$, $\psi = 1/\varphi(1-\alpha)$ and $\zeta = (1-\varpi)(1-\sigma\rho)/\rho$, c_{11t}^y and c_{22t}^y are solution to the system

$$\begin{pmatrix} 2\alpha\gamma - \psi & -2\alpha\gamma \\ -2\alpha\gamma & 2\alpha\gamma - \psi \end{pmatrix}\begin{pmatrix} c_{11t}^y \\ c_{22t}^y \end{pmatrix} = \begin{pmatrix} \gamma - \psi & -\gamma & -\zeta & \zeta \\ -\gamma & \gamma - \psi & \zeta & -\zeta \end{pmatrix}\begin{pmatrix} y_{1t} \\ y_{2t} \\ z_{1t} \\ z_{2t} \end{pmatrix}$$

which has solution

$$\begin{pmatrix} c_{11t}^y \\ c_{22t}^y \end{pmatrix} = \frac{1}{(\psi - 4\alpha\gamma)}\begin{pmatrix} \psi - \gamma(1+2\alpha) & (1-2\alpha)\gamma & \zeta & -\zeta \\ (1-2\alpha)\gamma & \psi - \gamma(1+2\alpha) & -\zeta & \zeta \end{pmatrix}\begin{pmatrix} y_{1t} \\ y_{2t} \\ z_{1t} \\ z_{2t} \end{pmatrix}$$

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We have

$$\psi - 4\alpha\gamma = \frac{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{\rho\varphi(1-\alpha)}$$

$$\psi - \gamma(1+2\alpha) = \frac{(\rho - (1+2\alpha)(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{\rho\varphi(1-\alpha)}$$

$$(1-2\alpha)\gamma = \frac{(1-2\alpha)(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{\rho\varphi(1-\alpha)}$$

$$\zeta = \frac{(1-\varpi)(1-\sigma\rho)}{\rho}$$

Therefore, we have

$$
c_{11t}^y = \frac{\rho - (1+\alpha(1-2\alpha))(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{1t} + \frac{(1-2\alpha)(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{2t}
$$
$$
+ \frac{(1-\varpi)(1-\sigma\rho)\varphi(1-\alpha)}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{1t} - \frac{(1-\varpi)(1-\sigma\rho)\varphi(1-\alpha)}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{2t}
$$

and

$$
c_{22t}^y = \frac{(1-2\alpha)(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{1t} + \frac{\rho - (1+\alpha(1-2\alpha))(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{2t}
$$
$$
- \frac{(1-\varpi)(1-\sigma\rho)\varphi(1-\alpha)}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{1t} + \frac{(1-\varpi)(1-\sigma\rho)\varphi(1-\alpha)}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{2t}
$$

Using 71 and 72, we get

$$
c_{21t}^y = \frac{\rho - (1-(1-\alpha)(1-2\alpha))(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{1t} - \frac{\alpha(1-2\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{2t}
$$
$$
- \frac{\alpha(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{1t} + \frac{\alpha(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{2t}
$$

and

$$
c_{12t}^y = - \frac{\alpha(1-2\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{1t} + \frac{\rho - \alpha(1-(1-\alpha)(1-2\alpha))(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{2t}
$$
$$
+ \frac{\alpha(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{1t} - \frac{\alpha(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{2t}
$$

The traded good aggregates are then given by

$$
c_{1t}^y = \frac{\alpha(\rho - 2(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{1t} + \frac{(1-\alpha)(\rho - 2\alpha(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{2t}
$$
$$
+ \frac{2\alpha(1-\alpha)(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{1t} - \frac{2\alpha(1-\alpha)(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{2t}
$$
$$
c_{2t}^y = \frac{(1-\alpha)(\rho - 2\alpha(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{1t} + \frac{\alpha(\rho - 2(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{2t}
$$
$$
- \frac{2\alpha(1-\alpha)(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{1t} + \frac{2\alpha(1-\alpha)(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{2t}
$$

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and the consumption aggregates take the form

$$
\begin{aligned}
c_t^1 =& \frac{\varpi\alpha(\rho - 2(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{1t} + \frac{\varpi(1-\alpha)(\rho - 2\alpha(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{2t} \\
&+ \frac{(1-\varpi)(\rho(1 - 4\alpha(1-\alpha)) + 2\alpha(1-\alpha)\varphi(2 - \varpi(1-\sigma\rho)))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{1t} \\
&- \frac{2\varpi\alpha(1-\alpha)(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{2t} \\
c_t^2 =& \frac{\varpi(1-\alpha)(\rho - 2\alpha(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{1t} + \frac{\varpi\alpha(\rho - 2(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{2t} \\
&- \frac{2\varpi\alpha(1-\alpha)(1-\varpi)(1-\sigma\rho)\varphi}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{1t} \\
&+ \frac{(1-\varpi)(\rho(1 - 4\alpha(1-\alpha)) + 2\alpha(1-\alpha)\varphi(2 - \varpi(1-\sigma\rho)))}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{2t}
\end{aligned}
$$

Prices are then given by

$$
\begin{aligned}
p_{2t}^y =& \frac{1 - \varpi(1-\sigma\rho)}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}(y_{1t} - y_{2t}) + \frac{(1-2\alpha)(1-\varpi)(1-\sigma\rho)}{(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}(z_{1t} - z_{2t}) \\
p_{1t}^z =& \frac{\rho(\alpha\varphi + (1-\alpha)\rho) - (1-\alpha)(\rho + 2\alpha\varphi)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{\rho\varphi(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{1t} \\
&+ \frac{(1-\alpha)((\rho - 2\alpha\varphi)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)) - \rho(\rho - \varphi))}{\rho\varphi(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{2t} \\
&+ \frac{(1-\alpha)(1-\sigma\rho)((1-2\alpha)(1-\varpi)\rho + 2\alpha\varphi(1+\varpi)) - (4\alpha(1-\alpha)\varphi + (1 - 4\alpha(1-\alpha))\rho)}{\rho(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{1t} \\
&- \frac{((1-2\alpha)\rho + 2\alpha\varphi)(1-\alpha)(1-\varpi)(1-\sigma\rho)}{\rho(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{2t} \\
p_{2t}^z =& \frac{\rho(\alpha\rho + (1-\alpha)\varphi) - \alpha(\rho + 2(1-\alpha)\varphi)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{\rho\varphi(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{1t} \\
&- \frac{\alpha(\rho(\rho - \varphi) - (\rho - 2(1-\alpha)\varphi)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{\rho\varphi(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{2t} \\
&- \frac{\alpha(1-\varpi)(1-\sigma\rho)(2(1-\alpha)\varphi + (1 - 2(1-\alpha))\rho)}{\rho(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{1t} \\
&+ \frac{\alpha(1-\sigma\rho)((1 - 2(1-\alpha))(1-\varpi)\rho + 2(1-\alpha)\varphi(1+\varpi)) - (4\alpha(1-\alpha)\varphi + (1 - 4\alpha(1-\alpha))\rho)}{\rho(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{2t}
\end{aligned}
$$

We can finally get λ_t

$$
\begin{aligned}
\lambda_t =& -\frac{\varphi(1 - \varpi(1-\sigma\rho))(\rho - 2\alpha(1-\alpha))(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{\rho\varphi(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{1t} \\
&+ \frac{2\alpha(1-\alpha)\varphi(1 - \varpi(1-\sigma\rho))(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}{\rho\varphi(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}y_{2t} \\
&+ \frac{\alpha(1-\varpi)(1-\sigma\rho)(\rho - 2(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{\rho(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{1t} \\
&+ \frac{(1-\alpha)(1-\varpi)(1-\sigma\rho)(\rho - 2\alpha(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}{\rho(\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho)))}z_{2t}
\end{aligned}
$$

A.5 Properties of the Solution

In the following, we review some properties of the log–linear solution of the model and report the proofs of the main propositions reported in the main text.

Lemma 1 *The dominator of each coefficient involved in the solution of the efficient allocation is positive.*

Proof: The denominator of each coefficient is always proportional to

$$\Delta \equiv \rho - 4\alpha(1 - \alpha)(\rho - \varphi + \varpi\varphi(1 - \sigma\rho))$$

which rewrites

$$\Delta = \rho(1 - 4\alpha(1 - \alpha)) + \rho\sigma\varpi\varphi 4\alpha(1 - \alpha) + 4\alpha(1 - \alpha)\varphi(1 - \varpi)$$

By definition, $0 \leqslant \varpi \leqslant 1$ and since $0 \leqslant \alpha \leqslant 1$, we have $4\alpha(1 - \alpha) \leqslant 1$. The result follows.
Q.E.D. □

Result 1 *The effects of traded goods shocks on efficient consumption: In equilibrium, the impact effect of a traded goods shock satisfies*

$$\frac{\partial c_{iit}^y}{\partial y_{it}} > 0 \iff \alpha > \frac{1}{2} \ or \ \varphi > \frac{\alpha(1 - 2\alpha)\rho}{(1 + \alpha(1 - 2\alpha))(1 - \varpi(1 - \sigma\rho))} \ and \ \alpha < \frac{1}{2} \tag{79}$$

$$\frac{\partial c_{iit}^y}{\partial y_{jt}} > 0, \ \frac{\partial c_{ijt}^y}{\partial y_{it}} < 0 \iff \varphi \lessgtr \frac{\rho}{1 - \varpi(1 - \sigma\rho)} \ and \ \alpha \lessgtr \frac{1}{2} \tag{80}$$

Proof: Let us first prove the first part of the result. Given that the denominator is positive, and looking at the solution for c_{iit}^y, the sign of the response to y_{it} is given by the sign of

$$\rho - (1 + \alpha(1 - 2\alpha))(\rho - \varphi(1 - \varpi(1 - \sigma\rho)))$$

This quantity is strictly positive as long as

$$\varphi > \frac{\alpha(1 - 2\alpha)\rho}{(1 + \alpha(1 - 2\alpha))(1 - \varpi(1 - \sigma\rho))}$$

When $\alpha > 0.5$, the right hand side of the inequality is negative. Since $\varphi \geqslant 0$ by assumption, the inequality is always satisfied in that case. When $\alpha < 0.5$, the inequality must hold.

Let us now prove the second part of the result. Given that the denominator of the solution is positive, $\alpha \in (0, 1)$, $\varpi \in (0, 1)$, $\varphi > 0$ and $\rho > 0$, the sign of the coefficient in front of y_{jt} in the solution of c_{iit}^y and c_{ijt}^y is entirely determined by the sign of $(1 - 2\alpha)(\rho - \varphi(1 - \varpi(1 - \sigma\rho)))$. The result then follows.
Q.E.D. □

Result 2 *The effects of endowment shocks on the relative price of traded goods: The impact effect of an endowment shock on the relative price of the foreign traded good satisfies*

1. *Shocks on traded*

$$\frac{\partial p_{2t}^y}{\partial y_{1t}} > 0, \ \frac{\partial p_{2t}^y}{\partial y_{2t}} < 0$$

2. *Shocks on non–traded*

$$\frac{\partial p_{2t}^y}{\partial z_{1t}} > 0, \ \frac{\partial p_{2t}^y}{\partial z_{2t}} < 0 \Longleftrightarrow \sigma\rho \lesseqgtr 1 \ and \ \alpha \lesseqgtr \frac{1}{2}$$

Proof:

1. We proved in result 1 that the denominator of the coefficient on endowments is positive. Furthermore, $1 - \varpi(1 - \sigma\rho)$ is positive. The result then follows.

2. Since the denominator of the coefficient is positive, the sign of the impact effect of a shock on non–traded is given by the sign of $(1 - 2\alpha)(1 - \rho\sigma)$. The result follows.

Q.E.D. □

Lemma 2 *Assume that $\sigma\rho = 1$ and that the elasticity of substitution between traded and non traded goods is less than unity ($\rho < 1$), then the relative price of the foreign traded good increases less than one for one following an increase in the domestic endowment, provided domestic and foreign traded goods are sufficiently good substitutes ($\varphi > \rho + \frac{1-\rho}{4\alpha(1-\alpha)}$)*

Proof:Lemma 2 Since $\rho\sigma = 1$, $\partial p_{2t}^y/\partial y_{1t}$ reduces to

$$\frac{\partial p_{2t}^y}{\partial y_{1t}} = \frac{1}{\rho - 4\alpha(1-\alpha)(\rho-\varphi)}$$

In the case where $\rho < 1$ (the case we consider) we have

$$\frac{\partial p_{2t}^y}{\partial y_{1t}} < 1 \Longleftrightarrow \rho - 4\alpha(1-\alpha)(\rho-\varphi) > 1$$

which amounts to

$$\varphi > \rho + \frac{1-\rho}{4\alpha(1-\alpha)}$$

since $\alpha \in (0,1)$ and $\rho < 1$ the second term is positive. Note that in a neighborhood of $\alpha = 0.5$, $4\alpha(1-\alpha) \simeq 1$, it is sufficient that $\varphi > 1$.

Q.E.D. □

A.6 Proofs of Propositions

Proof (Proof of proposition 1): Given that the denominator is positive, $\alpha \in (0,1)$, $\varpi \in (0,1)$, $\varphi > 0$ and $\rho > 0$, the sign of the coefficient in front of $z_{.t}$ in the solution is entirely determined by the sign of $\sigma\rho - 1$. The result then follows.

Q.E.D. □

Proof (Proof of proposition 2): Let us now consider the expenditures ratio

$$\Theta_t = \frac{C_{11t}^y + P_{2t}^y C_{12t}^y}{C_{21t}^y + P_{2t}^y C_{22t}^y}$$

its log–linear approximation is given by

$$\vartheta_t = \alpha c_{11t}^y + (1-\alpha)(p_{2t}^y + c_{12t}^y) - (1-\alpha)c_{21t}^y - \alpha(p_{2t}^y + c_{22t}^y)$$

Making use of (71)–(72), this rewrites as

$$\vartheta_t = 2\alpha(c_{11t}^y - c_{22t}^y) + (1-2\alpha)p_{2t}^y - (y_{1t} - y_{2t})$$

Plugging the solution of the log–linear version of the model, one gets

$$\frac{\partial \vartheta_t}{\partial y_{1t}} = \frac{(1-2\alpha)(1-\rho-\varpi(1-\sigma\rho))}{\rho - 4\alpha(1-\alpha)(\rho - \varphi + \varpi\varphi(1-\sigma\rho))}$$

Let us then consider the case where $\rho\sigma = 1$, this reduces to

$$\frac{\partial \vartheta_t}{\partial y_{1t}} = \frac{(1-2\alpha)(1-\rho)}{(1-4\alpha(1-\alpha))\rho + 4\alpha(1-\alpha)\varphi}$$

Since we established in Result 1 that the denominator is positive, the sign of the latter derivative is given by the sign of

$$(1-2\alpha)(1-\rho)$$

The result follows.
Q.E.D. □

Proof (Proof of proposition 3): As noted in the text, share holdings are constant to a first order approximation and the budget constraints write

$$\text{Home: } S_{11}^y y_{1t} + S_{12}^y P_{2t}^y y_{2t} = C_{11t}^y + P_{2t}^y C_{12t}^y$$
$$\text{Abroad: } S_{21}^y y_{1t} + S_{22}^y P_{2t}^y y_{2t} = C_{21t}^y + P_{2t}^y C_{22t}^y$$

Computing the ratio of the two budget constraints, we get

$$\Theta_t = \frac{S_{11}^y y_{1t} + S_{12}^y P_{2t}^y y_{2t}}{S_{21}^y y_{1t} + S_{22}^y P_{2t}^y y_{2t}}$$

which admits the log–linear version

$$\vartheta_t = \frac{S_{11}^y y_1}{S_{11}^y y_1 + S_{12}^y P_2^y y_2} y_{1t} + \frac{S_{12}^y P_2^y y_2}{S_{11}^y y_1 + S_{12}^y P_2^y y_2}(p_{2t}^y + y_{2t}) - \frac{S_{21}^y y_1}{S_{21}^y y_1 + S_{22}^y P_2^y y_2} y_{1t} - \frac{S_{22}^y P_2^y y_2}{S_{21}^y y_1 + S_{22}^y P_2^y y_2}(p_{2t}^y + y_{2t})$$

Making use of equilibrium on equity markets

$$S_{11}^y + S_{21}^y = 1$$
$$S_{12}^y + S_{22}^y = 1$$

and assuming symmetry ($y_1 = y_2 = y$, $p_2^y = 1$ and $S_{11}^y = S_{22}^y = s$) this reduces to

$$\vartheta_t = (1-2s)(p_{2t}^y + y_{2t} - y_{1t})$$

then

$$\frac{\partial \vartheta_t}{\partial y_{1t}} = (1-2s)\left(\frac{\partial p_{2t}^y}{\partial y_{1t}} - 1\right)$$

From Proposition 2, it is clear then in the case of complementary goods ($\rho < 1$) and home bias ($\alpha > 0.5$)

$$\frac{\partial \vartheta_t}{\partial y_{1t}} < 0$$

Assuming that traded goods are highly substitutable ($\varphi > \rho + (1-\rho)/4\alpha(1-\alpha)$), we have from lemma 2 that $\partial p_{2t}^y / \partial y_{1t} < 1$. Then the condition for ϑ_t to decrease with y_{1t} is that , s, the share of domestic traded firms held by domestic agents be lower than 0.5.
Q.E.D. □

B Accuracy Issues

B.1 Higher Order Approximations

We first investigate the accuracy of the wealth and equity shares derived in the log-linear model by computing the same shares under a second order perturbation. Table 10 reports the shares under a first and a second order approximation for the two benchmark cases respectively. In the case of a second order approximation, we report the average over 1000 simulations of the model. For each simulation, we generate a time series of 1000 realizations of the four productivity shocks. For each realization, we solve the model and compute the equity shares. We then average the shares over the 1000 realizations to get the average share for a particular simulation. From the resulting distribution of shares we compute the average across simulations as well as the standard deviation of the shares. This allows us to judge whether the constant shares obtained in a first order approximation are a good approximation.

Table 10: Shares: Accuracy check

	Wealth Shares				Equity Shares			
	α_{11}^y	α_{11}^z	α_{12}^y	α_{12}^z	S_{11}^y	S_{11}^z	S_{12}^y	S_{12}^z
Separable								
P1	0.1050	0.7900	0.1050	0.0000	0.5000	1.0000	0.5000	0.0000
P2	0.1050	0.7900	0.1050	0.0000	0.5000	1.0000	0.5000	0.0000
	(1.85e-4)	(2.69e-4)	(1.35e-4)	(3.85e-5)	(5.85e-6)	(1.76e-6)	(1.86e-5)	(1.05e-7)
Cons. Bias								
P1	0.1060	0.8150	0.0790	0.0000	0.5729	1.0000	0.4271	0.0000
P2	0.1060	0.8150	0.0790	0.0000	0.5729	1.0000	0.4271	0.0000
	(1.97e-4)	(2.40e-4)	(1.22e-4)	(4.21e-5)	(7.79e-6)	(1.92e-6)	(2.34e-5)	(1.10e-7)

Note: Standard errors into parenthesis.

Two main results emerge from the table. First, solving the model at a higher order approximation does not make any difference for the level of wealth and equity shares. In a companion paper, using a related model we show that the same is true even when one uses much higher order approximations. Second, the shares when computed from a higher order approximation method do not display any volatility. For instance, in the consumption bias case, the largest volatility observed in wealth shares is about 0.002%. Consequently, working with constant shares in this economy does not compromise the ability of the model to address the international portfolio bias problem.

B.2 Checking for Market Completeness

Recall that we determine the wealth shares by projecting the budget constraint of the household on each of the four shocks (see equation 14). Under a first order approximation, this method delivers constant shares. Since the four wealth shares must sum to unity, we actually have five equations in these four unknowns. Markets are effectively complete if there is a solution for the wealth shares satisfying all five equations — thus making the following equation hold for all realizations of the shocks

$$M(\mathscr{Y}_t) \bullet \alpha_t = L(\mathscr{Y}_t) \tag{81}$$

where $\mathscr{Y}_t$ contains the four shocks of the model. When solving for the four shares, we use three of the projections together with the condition that the wealth shares add up to one. One way of checking for market completeness is by inspecting the residuals of the remaining –omitted– projection. In particular, we obtain three wealth share from the projection of equation (81) on the first three shocks and then obtain the fourth share from the requirement that the shares add up to unity. We then substitute the shares in the –omitted– projection of equation (81) on the last shock and check the size of its residual from zero. We repeat this for every possible combination of the four projections. The results are reported in Table 11.

Table 11: Checking for Market Completeness: Residuals

Projection	Separable		Consumption Bias	
	P1	P2	P1	P2
$(y, y^\star, z)$	0.1854e-8	0.6397e-5	0.1343e-8	0.6998e-5
$(y, y^\star, z^\star)$	0.1854e-8	0.6384e-5	0.1343e-8	0.6983e-5
$(y, z, z^\star)$	0.1854e-8	0.6399e-5	0.1343e-8	0.6999e-5
$(y^\star, z, z^\star)$	0.1854e-8	0.6385e-5	0.1343e-8	0.6984e-5

The first column tells which of the four projections were used in the calculation of the shares. The second and third columns report the absolute value of the residual from the missing projection for, respectively, the first order (P1) and the second order (P2) approximation in the separable case. The last two columns report the same information in the consumption bias case. Under a first order approximation, the residuals of the last equation are essentially zero in both cases, hence, up to a first order approximation, markets are effectively complete in this economy. Under a second order approximation, we need to solve each system of equations (projections) for each realization of the shocks. We therefore simulate the model and solve the system for each realization of the shocks. We then compute the average of the absolute value of the residuals of the missing equation across simulations. We simulate the model 1000 times and generate a time series of 1000 observations for each draw. As can be seen from the table, the residuals are negligible also in the case of a second order approximation, an indication that

market completeness extends to higher orders of approximation.

An alternative to this approach is to check whether the risk sharing condition is satisfied. In a perfectly symmetric world, perfect risk sharing implies that the marginal utility of consumption is equated across countries.[34] An equivalent statement is that the Lagrange multipliers associated with the foreign and domestic households' budget constraint are equal.

Due to the constancy of equity shares the budget constraint of the household takes the simple form

$$\sum_{j=1}^{2} P_{jt}^{y} C_{ijt}^{y} + P_{it}^{z} C_{it}^{z} = \sum_{j=1}^{2} P_{jt}^{y} Y_{jt} S_{ij}^{y} + P_{jt}^{z} Z_{jt} S_{ij}^{z}$$

Using the equity shares from Table 3 in the budget constraint and solving the optimization problem of the domestic and foreign household allows us to compute the Lagrange multipliers. Perfect risk sharing means that the domestic and foreign Lagrange multipliers are perfectly correlated. We draw a time series for each shock and use the solution of the model to compute the average and the standard deviation of the Lagrange multiplier in each country as well as their correlation. We also report the maximal absolute deviation between the two Lagrange multipliers. The moments are averaged across simulations. As in the preceding case, 1000 draws of length 1000 are simulated. The results are reported in Table 12. These results establish that

Table 12: Risk Sharing: Checking Lagrange Multipliers (λ_1, λ_2)

	$E(\lambda_1)$	$100 \times \sigma(\lambda_1)$	$E(\lambda_2)$	$100 \times \sigma(\lambda_2)$	$\text{Corr}(\lambda_1, \lambda_2)$	$\max(|\lambda_1 - \lambda_2|)$
Separable	0.0441	0.1454	0.0441	0.1454	1.00	1.5960e-7
Cons. Bias	0.0342	0.1129	0.0342	0.1129	1.00	2.4037e-7

the Lagrange multipliers are the same across the two economies: They have the same average and standard deviation are perfectly positively correlated. The maximal absolute deviation is about 1e-7, less than the tolerance criterion used when solving the non–linear problem. It is worth noting that this test is extremely demanding as it actually tests for the joint hypothesis that *(i)* there is perfect risk sharing and *(ii)* equity shares are constant over time.

Finally, we carried out one more test. We imposed three of the four equity shares and let the households determine their optimal quantity of the fourth share. An advantage of this procedure is that it does not require the imposition of perfect risk sharing and also does not suppress the dynamics. In this case, the budget constraint takes the form

$$Q_{1t}^{y} S_{i1t+1}^{y} + \sum_{j=1}^{2} P_{jt}^{y} C_{ijt}^{y} + P_{it}^{z} C_{it}^{z} = (Q_{1t}^{y} + P_{1t}^{y} Y_{1t}) S_{i1t}^{y} + P_{2t}^{y} Y_{2t} S_{i2}^{y} + \sum_{j=1}^{2} P_{jt}^{z} Z_{jt} S_{ij}^{z}$$

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[34]In a non symmetric world, perfect risk sharing implies that two marginal utilities be proportional across countries— the proportionality factor being given by the ratio of initial wealths.

Again we then solve the model at the first and second order and simulate it. The same experiment is repeated for each possible three share subset of the four equities. Table 13 reports moments on the derived equity shares. Tables 14 and 15 report moments for the Lagrange multipliers using respectively a first and a second order approximation. Again the results establish that the model exhibits market completeness, that is, risk is perfectly shared across the two countries. Note that the "free" equity share, as computed by this method, is exactly equal to that obtained under perfect risk sharing and is also constant over time.

Table 13: Equity Shares

| | | P1 | | P2 | |
	$S^\star$	$E(S)$	$\sigma(S)$	$E(S)$	$\sigma(S)$
Separable					
S_{11}^y	0.5000	0.5000	2.14335e-9	0.5000	2.14335e-9
S_{11}^z	0.5000	0.5000	2.56456e-9	0.5000	2.56459e-9
S_{12}^y	1.0000	1.0000	4.07866e-10	1.0000	4.07896e-10
S_{12}^z	0.0000	0.0000	6.46943e-10	0.0000	6.46944e-10
Cons. Bias					
S_{11}^y	0.5729	0.5729	8.67822e-9	0.5729	8.67822e-9
S_{11}^z	0.4271	0.4271	7.33902e-9	0.4271	7.33903e-9
S_{12}^y	1.0000	1.0000	5.05901e-10	1.0000	5.05903e-10
S_{12}^z	0.0000	0.0000	1.95961e-9	0.0000	1.95961e-9

Table 14: Lagrange Multipliers (λ_1, λ_2), P1

	$E(\lambda_1)$	$100 \times \sigma(\lambda_1)$	$E(\lambda_2)$	$100 \times \sigma(\lambda_2)$	$\text{Corr}(\lambda_1, \lambda_2)$	$\max(|\lambda_1 - \lambda_2|)$
Separable						
S_{11}^y	0.0441	0.1452	0.0441	0.1452	1.00	3.77699e-9
S_{11}^z	0.0441	0.1452	0.0441	0.1452	1.00	4.49921e-9
S_{12}^y	0.0441	0.1452	0.0441	0.1452	1.00	2.08635e-9
S_{12}^z	0.0441	0.1452	0.0441	0.1452	1.00	4.22181e-9
Cons. Bias						
S_{11}^y	0.0342	0.1128	0.0342	0.1128	1.00	9.50948e-9
S_{11}^z	0.0342	0.1128	0.0342	0.1128	1.00	8.06487e-9
S_{12}^y	0.0342	0.1128	0.0342	0.1128	1.00	2.4594e-9
S_{12}^z	0.0342	0.1128	0.0342	0.1128	1.00	9.07048e-9

Table 15: Lagrange Multipliers (λ_1, λ_2), P2

	$E(\lambda_1)$	$100 \times \sigma(\lambda_1)$	$E(\lambda_2)$	$100 \times \sigma(\lambda_2)$	$\mathrm{Corr}(\lambda_1, \lambda_2)$	$\max(|\lambda_1 - \lambda_2|)$
Separable						
S_{11}^y	0.0441	0.1452	0.0441	0.1452	1.00	3.78899e-9
S_{11}^z	0.0441	0.1452	0.0441	0.1452	1.00	4.55496e-9
S_{12}^y	0.0441	0.1452	0.0441	0.1452	1.00	2.13807e-9
S_{12}^z	0.0441	0.1452	0.0441	0.1452	1.00	4.21243e-9
Cons. Bias						
S_{11}^y	0.0342	0.1128	0.0342	0.1128	1.00	9.4976e-9
S_{11}^z	0.0342	0.1128	0.0342	0.1128	1.00	8.05011e-9
S_{12}^y	0.0342	0.1128	0.0342	0.1128	1.00	2.51532e-9
S_{12}^z	0.0342	0.1128	0.0342	0.1128	1.00	9.0204e-9

C Model Extensions

C.1 A Model with a Distribution Sector

The world consists of two countries, indexed by $i = 1, 2$. In each period, each country receives an exogenous endowment of a traded, $Y_{it} > 0$ and a non–traded $Z_{it} > 0$, good. The goods are perishable. We use $\mathscr{Y}_t = \{Y_{it}, Z_{it}; i = 1, 2\}$ to denote the vector of endowments.

Country i is inhabited by a representative agent whose preferences are described by

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \frac{C_{it}^{1-\sigma} - 1}{1 - \sigma} \text{ with } \sigma > 0 \tag{82}$$

C_{it} denotes total consumption in country i. It consists of traded and non traded goods according to the specification

$$C_{it} = \left(\omega_i^{\frac{1}{\rho}} C_{it}^{y \frac{\rho-1}{\rho}} + (1 - \omega_i)^{\frac{1}{\rho}} C_{it}^{z \frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \omega_i \in (0, 1) \text{ and } \rho > 0 \tag{83}$$

where C_{it}^y (resp. C_{it}^z) denotes the consumption of traded (resp. non–traded) goods in country i at period t.

The traded goods aggregate combines domestic and foreign goods according to

$$C_{it}^y = \left(\alpha_i^{\frac{1}{\varphi}} T_{iit}^{\frac{\varphi-1}{\varphi}} + (1 - \alpha_i)^{\frac{1}{\varphi}} T_{ijt}^{\frac{\varphi-1}{\varphi}} \right)^{\frac{\varphi}{\varphi-1}} \alpha_i \in (0, 1) \text{ and } \varphi > 0 \tag{84}$$

where T_{ijt} denotes the flow of services associated with the traded good j in country i in period t. The flow of services of good j in economy i, T_{ijt}, is produced by combining the traded good and the non–traded good, according to

$$T_{ijt} = \left(\gamma_{ij}^{\frac{1}{\zeta}} C_{ijt}^{y \frac{\zeta-1}{\zeta}} + (1 - \gamma_{ij})^{\frac{1}{\zeta}} Z_{ijt}^{y \frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}} \gamma_{ij} \in (0, 1) \text{ and } \zeta > 0 \tag{85}$$

where C^y_{ijt} (resp. Z^y_{ijt}) denotes the consumption of the traded (resp. non–traded) good j in country i at period t.

The individuals have access to an equity market where shares of the firms that own the endowments of the four goods (the four "trees") can be traded. The budget constraint of the representative household in country i takes the form

$$\sum_{j=1}^{2} \left[Q^y_{jt} S^y_{ijt+1} + Q^z_{jt} S^z_{ijt+1} + P^y_{jt} C^y_{ijt} \right] + P^z_{it} C^z_{it} = \sum_{j=1}^{2} \left[(Q^y_{jt} + P^y_{jt} Y_{jt}) S^y_{ijt} + (Q^z_{jt} + P^z_{jt} Z_{jt}) S^z_{ijt} \right]$$

(86)

where P^y_{jt} and P^z_{jt} are the prices of the traded and non–traded good j respectively. S^y_{ijt} denotes the number of shares of traded good j owned by the households in country i at the beginning of period t while S^z_{ijt} is the number of shares of the non–traded good. The price of traded goods shares is Q^y_{jt} and that of non–traded is Q^z_{jt}. The traded goods shares yield a dividend of $P^y_{jt} Y_{jt}$ and the non–traded ones $P^z_{jt} Z_{jt}$. Note that there are four assets (equities) in the model and four independent sources of uncertainty. This implies that the equity markets in this model can support the complete asset markets allocation of resources up to a linear approximation. As in Kollmann, 2006b, we will use this equivalence to determine asset holdings.

The household's consumption/portfolio choices are determined by maximizing (82) subject to (83)–(86). The domestic traded good will be used as the numéraire good. Then the evolution of asset prices is given by the standard Euler equations

$$Q^y_{jt} \lambda^i_t = \beta \mathbb{E}_t \lambda^i_{t+1} (Q^y_{jt+1} + P^y_{jt+1} Y_{jt+1})$$
$$Q^z_{jt} \lambda^i_t = \beta \mathbb{E}_t \lambda^i_{t+1} (Q^z_{jt+1} + P^z_{jt+1} Z_{jt+1})$$

where $i, j = 1, 2$. Since asset markets are complete and the two economies are perfectly symmetric,[35] we have $\lambda^1_t = \lambda^2_t$.

Market clearing requires

$$Z_{1t} = C^z_{1t} + Z^y_{11t} + Z^y_{12t}$$
$$Z_{2t} = C^z_{2t} + Z^y_{21t} + Z^y_{22t}$$
$$Y_{1t} = C^y_{11t} + C^y_{21t}$$
$$Y_{2t} = C^y_{12t} + C^y_{22t}$$

The equilibrium satisfies the FOCs of the optimization problems of the representative agents in the two countries and the market clearing conditions. Since asset markets are effectively complete, the solution of the model can be determined without any need to know equity shares.

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[35]Relaxing the perfect symmetry assumption would simply have implied that $\lambda^1_t \propto \lambda^2_t$ where the proportionality factor is given by the relative initial wealth ratio.

We use the same endowment process as in the main text.

The parametrization of the model is the same as in the benchmark model. We only need to assign values to the two parameters pertaining to the production of flow of services, γ and ζ. We consider a separable version of the model with no home bias. Given the corresponding calibration for our benchmark model, this case corresponds to a situation where $\gamma = 0.5$ and $\zeta = 0.5$. As a second experiment, we consider a version in which we relax separability, while maintaining the assumption of no home bias, by assuming that the two traded goods are complements. In this case, we keep $\gamma = 0.5$, and set $\zeta = 0.25$. We finally —arbitrarily— assume that about one–fourth of the local goods is non–traded (say, 2.5%) rather than exportables. This implies that the true share of domestic exportables in the domestic consumption basket is only 8% rather that the 10.5% we used earlier ($10.5\% - 2.5\%$). This leads us to set $\alpha = 0.4683$. We call this the case of bias in traded consumption. Table 16 then reports the wealth and equity shares for these calibrations, as weel as the import share (s_m). As can be seen from the table, in the

Table 16: Model with distribution sector

	s_m	Wealth Shares				Equity Shares			
		α_{11}^y	α_{11}^z	α_{12}^y	α_{12}^z	S_{11}^y	S_{11}^z	S_{12}^y	S_{12}^z
Separable case	0.105	0.105	0.790	0.105	0.000	0.500	1.000	0.500	0.000
Non–Separable case	0.105	0.105	0.765	0.105	0.025	0.500	0.969	0.500	0.031
Bias in Traded	0.105	0.068	0.780	0.129	0.023	0.346	0.972	0.654	0.028

separable case, the total share of foreign equities in total wealth ($\alpha_{12}^y + \alpha_{12}^z$) is exactly equal to the import share. In other words, taking the distribution sector into account does not alter the key implication of the model. Note that this still holds under departures from the separable case (lowering ζ). Similarly, even withe home bias, the total share of foreign equities in total wealth (0.1520) remains close to the import share (0.1050).

C.2 A Model with Housing

The world consists of two countries, indexed by $i = 1, 2$. In each period, each country receives an exogenous endowment of a traded, $Y_{it} > 0$ and a non–traded $Z_{it} > 0$, good. The goods are perishable. We use $\mathscr{Y}_t = \{Y_{it}, Z_{it}; i = 1, 2\}$ to denote the vector of endowments.

Country i is inhabited by a representative agent whose preferences are described by

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \frac{C_{it}^{1-\sigma} - 1}{1 - \sigma} \text{ with } \sigma > 0 \tag{87}$$

C_{it} denotes total consumption in country i. It consists of traded and non traded goods and

housing according to the specification

$$C_{it} = \left(\omega_{1i}^{\frac{1}{\rho}} C_{it}^{y}{}^{\frac{\rho-1}{\rho}} + \omega_{2i}^{\frac{1}{\rho}} C_{it}^{h}{}^{\frac{\rho-1}{\rho}} + (1 - \omega_{1i} - \omega_{2i})^{\frac{1}{\rho}} C_{it}^{z}{}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}} \quad \omega_i \in (0,1) \text{ and } \rho > 0 \qquad (88)$$

where C_{it}^{y} (resp. C_{it}^{h}, C_{it}^{z}) denotes the consumption of traded (resp. housing services and non–traded) goods in country i at period t.

The traded good aggregate combines domestic and foreign goods according to

$$C_{it}^{y} = \left(\alpha_i^{\frac{1}{\varphi}} C_{iit}^{y}{}^{\frac{\varphi-1}{\varphi}} + (1 - \alpha_i)^{\frac{1}{\varphi}} C_{ijt}^{y}{}^{\frac{\varphi-1}{\varphi}} \right)^{\frac{\varphi}{\varphi-1}} \quad \alpha_i \in (0,1) \text{ and } \varphi > 0 \qquad (89)$$

where C_{ijt}^{y} denotes the consumption of the traded good j in country i at period t.

The individuals have access to an equity market where the shares of the firms that own the endowments of the four goods (the four "trees") can be traded. The budget constraint of the representative household in country i takes the form

$$\sum_{j=1}^{2} \left[Q_{jt}^{y} S_{ijt+1}^{y} + Q_{it}^{h} S_{it+1}^{h} + Q_{jt}^{z} S_{ijt+1}^{z} + P_{jt}^{y} C_{ijt}^{y} \right] + P_{it}^{z} C_{it}^{z} + P_{it}^{h} C_{it}^{h}$$

$$= \sum_{j=1}^{2} \left[(Q_{jt}^{y} + P_{jt}^{y} Y_{jt}) S_{ijt}^{y} + (Q_{it}^{h} + P_{it}^{h} H_{it}) S_{it}^{h} + (Q_{jt}^{z} + P_{jt}^{z} Z_{jt}) S_{ijt}^{z} \right] \qquad (90)$$

where P_{jt}^{y}, P_{it}^{h} and P_{jt}^{z} are the prices of the traded goods, housing services, and non–traded good j respectively. S_{ijt}^{y} denotes the number of shares of traded good j owned by the households in country i at the beginning of period t while S_{ijt}^{z} is the number of shares of the non–traded good. Likewise, S_{it}^{h} is the number of shares of housing. The price of traded goods shares is Q_{jt}^{y} and that of non–traded is Q_{jt}^{z}. Q_{it}^{h} is the price of a share in housing. The traded goods shares yield a dividend of $P_{jt}^{y} Y_{jt}$, housing yields $P_{it}^{h} H_{it}$ and the non–traded ones $P_{jt}^{z} Z_{jt}$. Note that there are six assets (equities) in the model and six independent sources of uncertainty. This implies that the equity markets in this model can support the complete asset markets allocation of resources up to a linear approximation. As in Kollmann, 2006b, we will use this equivalence to determine asset holdings.

The household's consumption/portfolio choices are determined by maximizing (87) subject to (88)–(90). The domestic traded good will be used as the numéraire good. Then the evolution of asset prices is given by the standard Euler equations

$$Q_{jt}^{y} \lambda_t^i = \beta \mathbb{E}_t \lambda_{t+1}^i (Q_{jt+1}^{y} + P_{jt+1}^{y} Y_{jt+1})$$
$$Q_{it}^{h} \lambda_t^i = \beta \mathbb{E}_t \lambda_{t+1}^i (Q_{it+1}^{h} + P_{it+1}^{h} H_{it+1})$$
$$Q_{jt}^{z} \lambda_t^i = \beta \mathbb{E}_t \lambda_{t+1}^i (Q_{jt+1}^{z} + P_{jt+1}^{z} Z_{jt+1})$$

where $i, j = 1, 2$. Since asset markets are complete and the two economies are perfectly symmetric we have $\lambda_t^1 = \lambda_t^2$.

Market clearing requires that

$$Z_{1t} = C_{1t}^z$$
$$Z_{2t} = C_{2t}^z$$
$$H_{1t} = C_{1t}^h$$
$$H_{2t} = C_{2t}^h$$
$$Y_{1t} = C_{11t}^y + C_{21t}^y$$
$$Y_{2t} = C_{12t}^y + C_{22t}^y$$

The equilibrium satisfies the FOCs of the optimization problems of the representative agents in the two countries and the market clearing conditions. Since asset markets are effectively complete, the solution of the model can be determined without any need to know equity shares.

We use the same endowment process as in the main text. We however have to add an extra endowment process for housing. We set the same process for housing as for the other goods.[36]

The parametrization of the model is the same as our benchmark model. We however have to assign values to the two parameters[37] pertaining to housing. In particular, we have to select a value for ω_2. We set it such that it matches the ratio of housing to equity wealth in household balance sheets, which is about 0.75 on US data. This led us to set $\omega_2 = 0.42856$. We then ran the same experiments as in the main text. Table 17 reports the wealth and equity shares for these calibrations, as well as the import share (s_m).

Table 17: Model with housing

	s_m	Wealth Shares					Equity Shares				
		α_{11}^y	α_{11}^z	α_{11}^h	α_{12}^y	α_{12}^z	S_{11}^y	S_{11}^z	S_{11}^h	S_{12}^y	S_{12}^z
Separable case	0.105	0.105	0.361	0.429	0.105	0.000	0.500	1.000	1.000	0.500	0.000
Cons. Bias	0.105	0.106	0.386	0.429	0.079	0.000	0.570	1.000	1.000	0.430	0.000

As can be seen from the table, introducing housing does not affect our main results.

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[36]We have also experimented with alternative processes. The results are left unaffected.

[37]It should be noted that in the separable case these parameter values for not matter for the main implication of the model regarding the match between the wealth share of foreign assets and the share of imports .