INCOME TAX DESIGN AND THE DESIRABILITY OF SUBSIDIES TO SECONDARY
WORKERS IN A HOUSEHOLD MODEL WITH JOINT AND NON-JOINT TIME

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ABSTRACT

In this paper we analyze income tax design in a two member household labor supply model where time spent on consumption together by the two household members is valued differently from time spent apart. We treat consumption as a non excludable public good to members of the household; one example would be where all household members or one alone can watch TV. When jointly consumed, however, TV services are valued more highly than the same consumption undertaken separately. We use this model to numerically investigate the welfare implications of different tax structures. In sharp contrast to existing literature, our results suggest the desirability of subsidizing secondary worker's labor supply. We also relate our discussion to existing individual-household tax unit literature.

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1 Introduction

In this paper, we consider optimal income tax design in a labour supply model where the household values jointly consumed leisure differently than non-joint leisure time. We present a two member household optimization model of consumption and labour supply with household preferences which treat joint and sole consumption time differently. The choice of which household member supplies joint or sole time to the market is endogenous. The model offers no closed form solution and is not amenable to qualitative algebraics. It can nonetheless be used as an analytical framework for numerical simulation. And we use conventional Cobb Douglas and CES functional forms for preferences.A model formulation embodying these features appears not to have been considered in existing tax literature. Our numerical simulations clearly suggest the desirability of subsidizing secondary worker whose labour supply if increased can generate more joint time via reduced primary labour supply. This stands in sharp contrast to prior public finance literature.

The entry point for our analysis is that if members of the household value time spent together in consumption differently from time spent apart, the optimal tax structure will tax joint and non-joint time at different rates. We show this by calibrating our model to quasi-realistic data on primary and secondary labour supply behaviour and relative wage rates. We then use numerical simulation to explore the implications of different tax structures and find optimal tax rates. Subsidies to secondary workers are desirable since increased labour supply by them will allow primary workers to reduce labour supply and increase more highly valued joint time. In addition, neither taxing the household as a unit nor taxing the individual members based on a simple Ramsey rule (as in Boskin and Sheshinski (1983)) is the best approach. We assess the size of gains from moving to an optimal tax scheme from both an initial uniform household based and individual Ramsey rule based tax schemes. The size of gains depends on the revenue requirement of the government as well as key model

parameters and data; particularly the wage rate spread across household members, and shares and elasticities in preferences.

Relative to both tax literature and that on household labour supply (see Chiaporri (1988, 1992), Fortin and Lacroix (1997), and Apps and Rees (1988, 1996)) we model households with primary workers who supply considerably more time to the market than secondary workers, but who sacrifice time spent together in joint consumption if they exploit the difference in labour market opportunities across household members that different wage rates imply. Consistent with empirical observation, in this framework secondary workers will have larger labour supply elasticities than primary workers since the secondary worker only foregoes non-joint leisure time when working, while the primary worker forgoes the more valuable joint leisure.

A broader implication of our analysis is that the Haig-Simons tradition emphasizing the use of broadly based income taxes which values all time equally reflects narrow and overly simple analytics. Income tax design should instead seek to exploit heterogeneity in the value of time and also across the activities individuals are engaged when using time (such as time of day which is not considered explicitly here). The key to efficient tax design is to recognize how and why such heterogeneity arises and correspondingly restructure income and other taxes. Here the quantification gains stemming from such design exploitation are large; probably larger than many existing estimates of the welfare costs of tax distortions of labour supply (see Whalley (1988).

2 A Simple Model of Household Consumption and Labour Supply with Joint and Non Joint Time to Consume

The idea behind our paper is that consumption jointly engaged in by both household members is typically valued more highly than consumption undertaken by one family member alone. Thus, joint consumption yields benefits which exceed the sum of the separate benefits from sole consumption, and shared time together in consumption yields benefits to household members which go beyond the direct consumption of the good. For example, if a household buys a TV and both members of the household view programs together, joint consumption benefits accrue to both household members which may exceed the separate benefits to each if they view programs apart.

Much consumption within the household embodies some form of intra-household jointness. Housing and many durable goods are analogous to pure public goods for which there is non-excludability of other household members. Excludable goods, such as food consumed by household members, might also have higher values if they are jointly consumed since dinner enjoyed together might be preferred to eating separately.

The idea that joint leisure consumption is preferred to time spent alone finds support in available evidence on joint retirement decisions. Pozzebon and Mitchell (1990) were the first to observe that "family considerations are more in wives' retirement decisions than own economic opportunities". Since then a series of papers (Gustman and Steimeier 1994, Measter 2001, Gustman and Steinmeyer 2002, and Coile 2004) all report evidence of co-ordinated retirement decisions. Maestas (2001) concludes that leisure complementarity is the main cause of joint retirement, and finds no support for the alternative view that correlated pension incentives cause joint retirement. Similarly, Gustman and Steinmeier (2002) find that leisure complementarity accounts

for a good portion of the apparent interdependence of the retirement decisions of husbands and wives.

This notion has broader application than to the retirement decision, for example, the added work-life stress associated with overtime work (see Golden and Wiens-Tuers (2005)). Since household members value joint consumption strongly this will be an important factor in their decisions about time allocation to the market. Specifically household members will try to organize their labour supply to overlap wherever possible and demand higher compensation when forced to work non-jointly.

For the purposes of the analytics we present here, we assume both that all household consumption takes this form and that goods consumed with joint leisure time are valued over consumption of the same good occurring with sole leisure time. We know of no prior literature embodying such features, but the structure we set out is analytically well defined and can be used in numerical simulation, even if there are no closed form solutions. We use specific functional forms and parameter values and generate results, which, even if parameter dependent, point towards a different design of the income tax compared to existing public finance literature.

We assume a household utility function defined over a single consumption good C and two types of leisure time enjoyed by household members which we denote as joint, L_J, and non-joint L_{NJ}. Joint leisure time L_J requires that both household members are not supplying their time to the market and non-joint leisure requires that only one household member is supplying time to the market. Since there are no labour market frictions and joint leisure is preferred to non-joint leisure the household will always coordinate so that the secondary worker works a subset of the primary worker's hours. Joint leisure time is therefore $L_J = \bar{L} - \max(M_1, M_2)$ where $\bar{L}$ denotes the time endowment and M_1 and M_2 represents the hours worked by individual 1 and 2 respectively. Non-joint leisure will therefore be $L_{NJ} = \max(M_1, M_2) - \min(M_1, M_2)$.

Our budget constraint is therefore $pC + w_i L_{NJ} + (w_1 + w_2)L_J = (w_1 + w_2)\bar{L}$, where

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i indicates the individual who supplies less time to the market.

We use a Cobb Douglas household utility function over nested CES subfunctions defined over consumption and type of leisure, i.e. joint or non-joint. This treatment allows for different utility weights to be placed on the joint and non joint [1]. consumption composites. Using CES nested subfunctions enables us to calibrate the model to literature based elasticity estimates.

In the no tax case, household utility maximization subject to a budget constraint implies

$$\max_{i\in\{1,2\},C,L_{NJ},L_J} (\gamma_J C^{\rho_J} + (1-\gamma_J)L_J^{\rho_J})^{\frac{\alpha}{\rho_J}} (\gamma_{NJi} C^{\rho_{NJi}} + (1-\gamma_{NJi})L_{NJ}^{\rho_{NJi}})^{\frac{1-\alpha}{\rho_{NJi}}} \quad (1)$$

$$\text{subject to:} \quad pC + w_i L_{NJ} + (w_1 + w_2)L_J = (w_1 + w_2)\bar{L}$$

where C is the quantity of the consumption good enjoyed by both household members. γ_J and γ_{NJi} are CES share parameters on joint and non-joint leisure time for the household member i. ρ_J and ρ_{NJi} are the substitution elasticity parameters across consumption and joint and non-joint time within the nested CES function. α and $(1-\alpha)$ are the Cobb Douglas weights over joint and non-joint nested consumption aggregates. In the budget constraint, p is the price of the consumption good, and w_1 and w_2 are the two wage rates faced by the individual household members (assumed exogenously given). Determining which individual, i, ends up consuming non-joint leisure (working less) depends on knowledge of both the value of market time (w_i) as well as preferences for non-joint time.

If the individual with the higher wage were to work full time this would maximize household income; but if this strategy were pursued all time devoted to leisure

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[1]Ideally, these preferences should imply that if both household members face the same wage, then consumption of non-joint time should be zero. However, with these preferences, some non-joint time will always be preferred, but the weighting parameters in the CES or C-D preferences we use apply to a composite of the consumption good common to both joint and non-joint time, and types of leisure.

would be non joint and this would reduce the utility benefits of consumption to the household. On the other hand, if all leisure were to be in joint form, this would forgo opportunities to exploit the comparative advantage of household members in generating income, given wage differentials across family members. Some optimal allocation of joint and non joint time is thus implied by (1) whenever wages vary across individuals.

If we assume, for now, that household member 2 is the one who consumes non joint leisure time (the secondary worker) and member 1 supplies non joint time to the market (the primary worker), the first order conditions to (1) imply:

$$\frac{p}{w_1 + w_2} = \frac{\gamma_J C^{\rho_J - 1}}{(1 - \gamma_J)L_J^{\rho_J - 1}} + \frac{(1 - \alpha)\gamma_{NJ}C^{\rho_{NJ} - 1}(\gamma_J C^{\rho_J} + (1 - \gamma_J)L_J^{\rho_J})}{\alpha(1 - \gamma_J)L_J^{\rho_J - 1}(\gamma_{NJ}C^{\rho_{NJ}} + (1 - \gamma_{NJ})L_{NJ}^{\rho_{NJ}})} \quad (2)$$

$$\frac{p}{w_2} = \frac{\gamma_{NJ}C^{\rho_{NJ} - 1}}{(1 - \gamma_{NJ})L_{NJ}^{\rho_{NJ} - 1}} + \frac{\alpha\gamma_J C^{\rho_J - 1}(\gamma_{NJ}C^{\rho_{NJ}} + (1 - \gamma_{NJ})L_{NJ}^{\rho_{NJ}})}{(1 - \alpha)(1 - \gamma_{NJ})L_{NJ}^{\rho_{NJ} - 1}(\gamma_J C^{\rho_J} + (1 - \gamma_J)L_J^{\rho_J})} \quad (3)$$

These, together with the budget constraint from (1), yield a system of three equations in three unknowns, C, L_J, L_{NJ}. Assuming a prior regime selection, computing a solution to (2), (3) along with the budget constraint from (1), allows for an evaluation of household utility. Comparing this household utility to that computed with household member 2 being the primary worker yields an optimal solution to (1).

Tax rates t_1 and t_2 on household members can then easily be incorporated into this analysis. The first order conditions (2) and (3) become:

$$\frac{p}{(w_1(1 - t_1) + w_2(1 - t_2))} = \frac{\gamma_J C^{\rho_J - 1}}{(1 - \gamma_J)L_J^{\rho_J - 1}} + \frac{(1 - \alpha)\gamma_{NJ}C^{\rho_{NJ} - 1}(\gamma_J C^{\rho_J} + (1 - \gamma_J)L_J^{\rho_J})}{\alpha(1 - \gamma_J)L_J^{\rho_J - 1}(\gamma_{NJ}C^{\rho_{NJ}} + (1 - \gamma_{NJ})L_{NJ}^{\rho_{NJ}})} \quad (4)$$

$$\frac{p}{w_2(1 - t_2)} = \frac{\gamma_{NJ}C^{\rho_{NJ} - 1}}{(1 - \gamma_{NJ})L_{NJ}^{\rho_{NJ} - 1}} + \frac{\alpha\gamma_J C^{\rho_J - 1}(\gamma_{NJ}C^{\rho_{NJ}} + (1 - \gamma_{NJ})L_{NJ}^{\rho_{NJ}})}{(1 - \alpha)(1 - \gamma_{NJ})L_{NJ}^{\rho_{NJ} - 1}(\gamma_J C^{\rho_J} + (1 - \gamma_J)L_J^{\rho_J})} \quad (5)$$

and the budget constraint becomes

$$pC + w_2(1 - t_2)L_{NJ} + (w_1(1 - t_1) + w_2(1 - t_2))L_J = (w_1(1 - t_1) + w_2(1 - t_2))\bar{L} + R \quad (6)$$

where $R = t_2 w_2(\bar{L} - L_J - L_{NJ}) + t_1 w_1(\bar{L} - L_J)$ is the revenue from the taxes returned to the household in lumpsum payment.

In similar fashion to above, solving (4), (5) and the budget constraint yields C, L_J and L_{NJ} in the presence of taxes, and for these solution values the revenue R associated with the optimal solution can be found. Optimal tax rates can be found by a grid search over tax rate pairs constrained to yield the same revenue.

3 Numerical Analysis of Optimal Tax Structures in a Simple Joint Leisure Time Model

We have used the model above to evaluate optimal tax rates for a quasi-realistic model parameterization generated by calibration to a base case data set. We also use the model to compare the welfare implications of optimal tax structures to both household based uniform and individual based Ramsey rule tax schemes. We go beyond the conventional exact levels calibration used in micro models developed in Shoven and Whalley (1972) and discussed in Dawkins, Srinivasan and Whalley (2003). We use both a level (base case data) calibration and a changes calibration to literature based labour supply elasticities.

Model parameters are calibrated so that under a uniform tax scheme the model solution reproduces a base case dataset and literature based labour supply elasticities. The data we use for this purpose are reported in Table 1 where base case data and implied labour supply elasticities are displayed. Primary household members (individual 1) work 40 hours a week, and secondary members work 25 hours per week at half the wage of the primary worker. The wage of the primary worker and commodity prices are both normalized to one implying a value of $C = 52.5$ and $w_2 = 0.5$.[2] The values for L_J and L_{NJ} are directly implied by the labour supplies M_1 and M_2.

Specifying values for ρ_J, ρ_{NJ} and α we can calibrate the model to the in Table 1 find unique values of γ_J and γ_{NJ} which solve equations (4) and (5). This then allows us to iterate on values of ρ_J and ρ_{NJ}, recalculating the γs at each iteration of the calibration, to find the model parameter values for which the point estimates of the labour supply elasticities for the primary and secondary workers are equal to set values of $\eta_{11} = 0.15$ and $\eta_{22} = 0.5$. These are taken as estimates consistent with empirical literature (see Hill and Killingsworth (1989) and Piggott and Whalley

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[2]This normalization of choice of units of commodities as the amount selling for one dollar in a benchmark (or reference) equilibrium is often attributed to Harberger (1962).

(1996)). This leaves the unobservable and unestimated parameter α which reflects the weight of jointly consumed leisure in relation to non-joint leisure. Since primary and secondary workers are assumed to organize their time spent at work to minimize non-joint leisure (the secondary worker always works a subset of the hours of the primary worker) only values for α that weight joint leisure more highly than non-joint leisure are admissible to the model. This implies a lower bound for α of 0.66 since joint leisure represents two peoples time. To identify the model a value of $\alpha = 0.8$ is chosen as the appropriate midpoint of model admissible values.[3]

Table 1
Base Case Quasi Realistic Data, Labour Supply
Elasticities and Calibrated Model Parameters

1. Base Case Dataset				
$C = 52.5$	$L_J = 30.0$	$L_{NJ} = 15.0$	$M_1 = 40.0$	$M_2 = 25.0$
$w_1 = 1.0$	$w_2 = 0.5$	$p = 1$		
$t_1 = 0.3$	$t_2 = 0.3$	$\bar{L} = 70$		

2. Own Price Elasticities Used in Calibration
$\eta_{11} = 0.15 \qquad \eta_{22} = 0.50$

3. Identifying assumptions
$\alpha = 0.8$

4. Calibrated Parameters
$\gamma_J = 0.50235 \qquad \gamma_{NJ} = 0.69209 \qquad \rho_J = 0.40560 \qquad \rho_{NJ} = 0.05259$

5. Implied Cross Wage Elasticities, Utility, and Government Revenue Requirement
$\eta_{21} = -0.1747 \qquad \eta_{12} = 0.07613 \qquad U = 39.46 \qquad R = 15.75$

We then use this calibrated model parameterization to assess the welfare costs of a range of revenue equivalent tax structures. Table 2 reports results from a comparison of different equal yield tax rates for the model parameterization set out in Table 1. The different tax rates schemes all yield the same revenue as the uniform tax rate structure in table 1. For each pair of tax rates we also evaluate the Hicksian equivalent variation of removing the taxes, moving from the uniform tax scheme and moving from the Ramsey rule tax scheme.[4]

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[3]Sensitivity analysis revealed that although the magnitudes used for α change the model prediction numerically a subsidy is optimal across all values for α.

[4]The Ramsey rule for optimal taxation is that the ratio of taxes should be equal to the inverse

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Table 2
Welfare Implications of Revenue Equivalent
Individual Based Tax Regimes

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$w_1 = 1$	$w_2 = 0.5$	$M_1 = 40$	$M_2 = 25$	$\alpha = 0.8$
		$\eta_{11} = 0.15$	$\eta_{22} = 0.5$	
Tax Regime		EV(%) of removing tax	EV(%) of moving from uniform	EV(%) of moving from Ramsey Rule
t_1	t_2		tax structure	tax structure
0.459	-0.20	1.519	0.875	0.199
0.439	-0.15	1.513	0.881	0.204
0.421	-0.10	1.521	0.873	0.196
0.402	-0.05	1.541	0.854	0.177
0.384	0.000	1.576	0.818	0.142
0.368	0.050	1.632	0.763	0.088
0.351	0.100	1.711	0.685	0.009
0.336	0.150	1.820	0.577	-0.10
0.323	0.200	1.966	0.433	-0.24
0.310	0.250	2.157	0.245	-0.43
0.300	0.300	2.407	0.000	-0.67

By visual inspection, the tax rate configuration that minimizes the Hicksian equivalent variation of removing the taxes involves a tax rate on the primary worker of 0.439 and a negative tax rate of -0.150 on the secondary worker. Moving to this optimal configuration from the uniform tax scheme of 0.3 results in a non trivial welfare gain of 0.881% of household income. Similarly, moving to the optimal configuration from the Ramsey rule tax scheme of $t_1 = 0.351$, $t_2 = 0.1$ is 0.2% of household income. Here, the desirability of subsidizing secondary workers labour supply emerges as a clear and central theme. This occurs since a subsidized increase in secondary workers labour supply allows for a reduction in primary workers labour supply, increasing more highly valued joint time consumption. The orders of magnitude of the quantitative gains involved are seemingly of the same order of magnitude of several existing estimates of the welfare costs of tax distortions of labour supply (see Whalley (1988).

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ratio of simple demand elasticities. In this case $\frac{t_1}{t_2} = \frac{\eta_{22}}{\eta_{11}}$.

4 Summary and Conclusion

In this paper we present a model of household consumption and labour supply behaviour in which household members value time spent together in consumption more highly than time spent apart. Implicitly we treat consumption within the household as a pure public good. This analytical structure appears not to have been employed in previous literature, in part because of its endogenous regime choice by the household involved and hence the absence of closed form solutions. The model does, however, have intuitive appeal and can be worked with numerically.

We use the model to analyze optimal tax design, arguing in this case that the conventional public finance literature focus on whether to tax individuals or households is misfocused, since the issue is optimal tax treatment of jointly and non-jointly supplied time to the market. In turn optimal tax rates on joint and non-joint time can imply different tax rates for household members depending on their relative labor supplies. Our insights have application to the structure of social security systems, although a more detailed life-cycle model would be required for analysis.

We perform numerical simulation with the model for using calibrated to a quasi-realistic base case data set, with further calibration to literature based labor supply elasticities. Results clearly point strongly to the desirability of subsidizing secondary workers labor supply, since this allows for increased labor supply by them, reduced primary worker labor supply, and increased joint consumption time over sole consumption time. Results also show a welfare gain from optimal joint/non-joint time taxation compared to both uniform taxation and individual Ramsey rule taxation. Our modelling has application not only to the optimal income tax literature per se, but to other issues, such as overtime pay and joint retirement.

References

[1] Apps, P. F. and R. Rees (1988) "Taxation and the Household", *Journal of Public Economics*, 35(4), pp. 355-69.

[2] Apps, P. F. and R. Rees (1996) "Labour Supply, Household Production and Intra-Family Welfare Distribution", *Journal of Public Economics*, 60(5), pp. 199-220.

[3] Boskin, M. J. and E. Sheshinski (1983) "Optimal Tax Treatment of the Family: Married Couples", *Journal of Public Economics*, 20(4), pp. 281-97.

[4] Chiaporri, P-A. (1988) "Rational Household Labour Supply",*Econometrica*, 56(1), pp. 63-90.

[5] Chiaporri, P-A. (1992) "Collective Labour Supply and Welfare", *Journal of Political Economy*, 100(6), pp. 437-67.

[6] Coile, C. (2004) "Retirement Incentives and Couples' Retirement decisions", *Topics in Economic Analysis & Policy*, 4(1).

[7] Dawkins, C., T. N. Srinivasan and J. Whalley (2001) "Calibration" in (eds), *Handbook of Econometrics*, Volume 5 New York, N.Y.: Elsevier Science Publishing, Co. 2001, pp. 3653-3703.

[8] Fortin, B. and G. Lacroix (1997) "A Test of the Unitary and Household Models of Collective Labour Supply", *The Economic Journal*, 107(443), pp. 933-55.

[9] Golden, L and B. Wiens-Tuers (2005) "To your happiness? Extra hours of labor supply and worker well-being", *The Journal of Socio-Economics*, 35, pp. 382397.

[10] Gustman, A. L. and T. L. Steinmeier (1994) "Retirement in a Family Context: A Structural Model for Husbands and Wives" (January 1994). NBER Working Paper No. W4629. Available at SSRN: http://ssrn.com/abstract=226973.

[11] Gustman, A. L. and T. L. Steinmeier (2002) "Social Security, Pensions and Retirement Behavior Within the Family", NBER Working Paper No. 8772 February 2002.

[12] Hill, M. A. and M. R. Killingsworth (eds.) (1989) *Comparable worth: Analyses and evidence*, Ithaca, N.Y.: ILR Press.

[13] Maestas, N. (2001) "Labor, Love & Leisure: Complementarity and the Timing of Retirement by Working Couples," mimeo, UC Berkeley, Department of Economics, 2001. In preparation for re-submission.

[14] Musgrave, R. A. and P. B. Musgrave (1989) *Public Finance in Theory and Practice*, New York: McGraw Hill Book Company, 5th edition.

[15] Piggott, J. and J. Whalley (1996) "The Tax Unit and Household Production", *Journal of Political Economy*, 104(2), pp. 398-418.

[16] Pozzebon, S. and O. S. Mitchell (1989) "Married Women's Retirement Behavior," *Journal of Population Economics*, Springer, 2(1), pp. 39-53.

[17] Whalley, J. (1988) "Lessons from General Equilibrium Models" in H. Aaron, H. Galper and J. Pechman (eds.), Uneasy Compromise: Problems of a Hybrid Income-Consumption Tax, Washington, DC: Brookings Institution, 1988, pp. 15-50.