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# FACTOR ANALYSIS IN A MODEL WITH RATIONAL EXPECTATIONS

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## ABSTRACT

DSGE models are characterized by the presence of expectations as explanatory variables. To use these models for policy evaluation, the econometrician must estimate the parameters of expectation terms. Standard estimation methods have several drawbacks, including possible lack or weakness of identification of the parameters, misspecification of the model due to omitted variables or parameter instability, and the common use of inefficient estimation methods. Several authors have raised concerns over the implications of using inappropriate instruments to achieve identification. In this paper we analyze the practical relevance of these problems and we propose to combine factor analysis for information extraction from large data sets and GMM to estimate the parameters of systems of forward looking equations. Using these techniques, we evaluate the robustness of recent findings on the importance of forward looking components in the equations of a standard New-Keynesian model.

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# 1 Introduction

This paper is about the estimation of New-Keynesian models of the monetary transmission mechanism. We evaluate a number of recent findings obtained using single equation methods and we develop a system approach that makes use of additional identifying information extracted using factor analysis from large data sets. The combination of factor analysis and GMM to estimate the parameters of systems of forward looking equations is one of the distinctive features of our work, which extends the univariate analysis in Favero, Marcellino and Neglia (2005). The latter paper has also stimulated theoretical research on the properties of the factor-GMM estimators, see Bai and Ng (2006b) and Kapetanios and Marcellino (2006b), which provide a sound theoretical framework for our empirical analysis.

Following the influential work of Galí and Gertler (1999, GG), a number of authors have used instrumental variable methods to estimate one or more equations of the New-Keynesian model of the monetary transmission mechanism. GG used the New-Keynesian paradigm to explain the behavior of U.S. inflation as a function of its first lag, expected first lead, and the marginal cost of production. Their work stimulated considerable debate, much of which has focused on the size and significance of future expected inflation in the New-Keynesian Phillips curve. Similar arguments have been made over the role of expected future variables in the other equations of the New-Keynesian model: for example Clarida, Galí and Gertler (1998) estimate a Taylor rule in which expected future inflation appears as a regressor and Fuhrer and Rudebusch (2002) have estimated an Euler equation for output in which expected future output appears on the right-hand-side.

The estimation of models that include future expectations has revived a debate that began in the 1970's with the advent of rational expectations econometrics. In this context, a number of authors have raised economet-

ric issues that relate to the specification and estimation of single equations with forward looking variables. For example, Rudd and Whelan (2005, RW) showed that the GG parameter estimates for the coefficient on future inflation may be biased upward if the equation is mis-specified due to the omission of relevant regressors that are instead used as instruments. With regard to the estimation of the coefficients of future variables they pointed out that this problem can yield differences between estimates that are based on the following two alternative estimation methods. The first (direct) method estimates the coefficient directly using GMM; the second (indirect) method computes a partial solution to the complete model that removes the expected future variable from the right-hand-side and substitutes an infinite distributed lag of all future expected forcing variables. RW use their analysis to argue in favor of Phillips curve specifications that favor backward lags of inflation over the New-Keynesian specification that includes only expected future inflation as a regressor.

Galí, Gertler and Lopez-Salido (2005, GGLS) have responded to the RW critique by pointing out that, in spite of the theoretical possibility of omitted variable bias, estimates obtained by direct and indirect methods are fairly close, and when additional lags of inflation are added as regressors in the structural model to proxy for omitted variables, they are not significant. While the Rudd-Whelan argument is convincing, the CGLS response is less so since other (contemporaneous) variables might also be incorrectly omitted from the simple GG inflation equation. Even if additional lags of inflation were found to be insignificant, their inclusion could change the parameters of both the closed form solution and the structural model. We argue, in this paper, that these issues can only be resolved by embedding the single equation New-Keynesian Phillips curve in a fully specified structural model.

Other authors, e.g. Fuhrer and Rudebusch (2002), Lindé (2005) and Jon-

deau and Le Bihan (2003) have pointed out that the Generalized Method of Moment (GMM) estimation approach followed by GG could be less robust than maximum likelihood estimation (MLE) in the presence of a range of model mis-specifications such as omitted variables and measurement error, typically leading to overestimation of the parameter of future expected inflation. GGLS correctly replied that no general theoretical results are available on the relative merits of GMM and MLE under mis-specification, that the comparison could be biased by the use of an inappropriate GMM estimator, and that other authors such as Ireland (2001) provided evidence in favor of a (pure) forward looking equation for US inflation when using MLE. In this paper we hope to shed additional light on the efficiency and possible bias of GMM estimation by comparing alternative estimation methods on the *same* data set and the same model specification.

A different and potentially more problematic critique of the GG approach comes from Mavroeidis (2005), Bårdsen, Jansen and Nymoen (2003), and Nason and Smith (2005), building upon previous work on rational expectations by Pesaran (1987). Pesaran (1987) stressed that the conditions for identification of the parameters of the forward looking variables in an equation of interest should be carefully checked prior to single equation estimation. To check identification conditions one must specify a model for all of the right-hand-side variables. Even if there are enough instruments available such that conventional order and rank conditions are fulfilled and parameters are not underidentified there might nevertheless be a problem of weak identification. The articles cited above have shown that in the presence of weak identification, estimation by GMM yields unreliable results. Weak identification is related to the quality of the instruments when applying GMM. When instruments are only weakly correlated with the corresponding endogenous variables they might not be particularly useful for forecasting e.g. future

expected inflation. The resulting GMM estimators might then suffer from weak identification, which leads to non-standard distributions for the estimators. As a consequence, this can yield misleading inference, see e.g. Stock, Wright and Yogo (2002) for a general overview on weak instruments and weak identification.

In summary, the recent literature on the New-Keynesian Phillips curve has highlighted four main problems with the single equation approach to estimation by GMM. First, parameter estimates may be biased due to correlation of the instruments with the error term. Second, an equation of interest could be mis-specified because of omitted variables or parameter instability within the sample. Third, parameters of interest may not be identified because there are not enough instruments available. Fourth, parameters may be weakly identified if the correlation of the instruments with the target variable is low.

In this paper we analyze the practical relevance of these problems, propose remedies for each of them, and evaluate whether the findings on the importance of the forward looking component are robust when obtained within a more general econometric context. In Section 2 we compare single equation and system methods of estimation for models with forward looking regressors. In Section 3 we conduct a robustness analysis for a full forward looking system. In Section 4 we analyze the role of information extracted from large data sets to reduce the risk of specification bias and weak instruments problems. In Section 5 we summarize the main results of the paper and conclude.

# 2 Single Equation versus System Approach

We begin this Section with a discussion of the estimation of the New-Keynesian Phillips curve. This will be followed by a discussion of single-equation esti-

mation of the Euler equation and the policy rule. We then contrast the single equation approach to a closed, three-equation, New-Keynesian model. We estimate simultaneously a complete structural model which combines the three previously estimated single-equation models for the Phillips curve, the Euler equation and the policy rule and we compare system estimates of parameters with those of the three single-equation specifications.

Our starting point is a version of the New-Keynesian Phillips curve inspired by the work of Galí and Gertler (GG 1999),

$$\pi_t = \alpha_0 + \alpha_1 \pi^e_{t+1} + \alpha_2 x_t + \alpha_3 \pi_{t-1} + e_t, \tag{1}$$

where $\pi_t$ is the GDP deflator, $\pi^e_{t+1}$ is the forecast of $\pi_{t+1}$ made in period $t$, $x_t$ is a real forcing variable (e.g. marginal costs as suggested by GG, unemployment - with reference to Okun's law - as in e.g. Beyer and Farmer (2007$a$), or any version of an output gap variable). The error term $e_t$ is assumed to be i.i.d. $(0, \sigma^2_e)$ and is, in general, correlated with the non-predetermined variables (i.e, with $\pi^e_{t+1}$ and $x_t$). Since we want to arrive at the specification of a system of forward looking equations, we prefer to use as a real forcing variable the output gap[1], measured as the deviation of real GDP from its one-sided HP filtered version as widely used in the literature.[2]

To estimate equation (1) we replace $\pi^e_{t+1}$ with $\pi_{t+1}$, such that (1) becomes

$$\pi_t = \alpha_0 + \alpha_1 \pi_{t+1} + \alpha_2 x_t + \alpha_3 \pi_{t-1} + v_t. \tag{2}$$

[1] The forward looking IS curve is usually specified in terms of the output or unemployment gap.

[2] Notice that although common practice in the applied literature the use of the HP filtered version of the output gap is by no means unproblematic. For example, Nelson (2006) finds that the HP cyclical component of U.S. real GDP has no predictive power for future changes in output growth and Fukac and Pagan (2006) provide an example in which HP filtering produces biased coefficient estimates within a New-Keynesian model.

Equation (2) can be estimated by GMM, with HAC standard errors to take into account the MA(1) structure of the error term $v_t = e_t + \alpha_1(\pi^e_{t+1} - \pi_{t+1})$.[3] All data is for the US, quarterly, for the period 1970:1-1998:4, where the constraint on the end date is due to the large data set we use in Section 4.[4]

In the first panel of the first column of Table 1 we report the single-equation estimation results. As in GG (1999) and Galí et al. (2003), we find a larger coefficient on $\pi^e_{t+1}$, about 0.78, than on $\pi_{t-1}$, about 0.23. The coefficient on the forcing variable is very small and not statistically significant at the 5% level, again in line with previous results.

There are at least two related problems with this single equation approach: first, the appropriateness or availability of the instruments cannot be judged in isolation without reference to a more complete model, and therefore, second, the degree of over, just, or under-identification is undefined.

The issue of identification and the use of appropriate instruments in rational expectations models is a very subtle one, see e.g. Pesaran (1987), Mavroeidis (2005), Bårdsen et al. (2003) or Beyer and Farmer (2003*a*). In linear backward looking models, such as conventional simultaneous equation models, rank and order conditions can be applied in a mechanical way (see e.g. Fisher, 1966). In rational expectation models, however, the conditions for identification depend on the solution of the model, i.e. whether the solution of the model is determinate or indeterminate, see Beyer and Farmer (2007*b*).

[3] In particular, to compute the GMM estimates we start with an identity weighting matrix, get a first set of coefficients, use these to update the weighting matrix and finally iterate coefficients to convergence. To compute the HAC standard errors, we adopt the Newey West (1994) approach with a Bartlett kernel and fixed bandwidth. These calculations are carried out with Eviews 5.0.

[4] We have estimated the models using the output gap and unemployment as real forcing variables. To save space we present here only the output gap results.

In our case, as it is common in this literature, we have used (three) lags of $\pi_t$, $x_t$ and the interest rate, $i_t$ as instruments where $i_t$ is the 3-month US Federal funds interest rate. However, since $i_t$ does not appear in (1), both $\pi^e_{t+1}$ and $x_t$ may not at all or may only weakly depend on lags of $i_t$, which would make $i_t$ an irrelevant or a weak instrument. To evaluate whether or not lagged interest rates are suitable instruments, we estimated the following sub-VAR model:

$$\begin{aligned} x_t &= b_0 + b_1\pi_{t-1} + b_2 x_{t-1} + b_3 i_{t-1} + u_{xt}, \\ i_t &= c_0 + c_1\pi_{t-1} + c_2 x_{t-1} + c_3 i_{t-1} + u_{ct}, \end{aligned} \tag{3}$$

where $u_{xt}$ and $u_{ct}$ are i.i.d. error terms, which are potentially correlated with $e_t$. If $b_3 = 0$, i.e., $i_t$ does not Granger cause $x_t$, then lags of $i_t$ are not relevant instruments for the endogenous variables in (1).

Whether lagged values of inflation and the real variable beyond order one (i.e., $\pi_{t-2}$, $\pi_{t-3}$, $x_{t-2}$ and $x_{t-3}$) are relevant instruments for $\pi^e_{t+1}$ is also questionable. If the solution for $\pi_t$ only depends on $\pi_{t-1}$ and $x_{t-1}$, which is the case when the solution is determinate, then the additional lags are not relevant instruments. However, in case of indeterminacy additional lags of $\pi_t$ and $x_t$ matter, which may re-establish the relevance of $\pi_{t-2}$, $\pi_{t-3}$, $x_{t-2}$ and $x_{t-3}$ as instruments.[5]

As a consequence of the model dependence with respect to the number of available *and* relevant instruments, the Hansen's $J$-statistic, a popular measure for the relevance of the instruments and overidentifying restrictions that we also present for conformity to the literature, can be potentially un-

[5] Beyer and Farmer (2003*a*) conduct a systematic search of the parameter space in a model closely related to the one studied in this paper. They sample from the asymptotic parameter distribution of the GMM estimates and find, for typical identification schemes, that point estimates lie in the indeterminate region, but anywhere from 5% to 20% of the parameter region may lie in the non-existence or determinate region.

informative and even misleading when applied in a forward looking context.

Estimating (1) and (3) using only one lag of $\pi$, $x$, and $i$ as instruments, we find that $b_3 \neq 0$ but the null hypothesis $b_3 = 0$ cannot be rejected. In this case, since the instruments are only weakly correlated with their targets, the resulting GMM estimators can suffer from weak identification, see Mavroeidis (2005, 2006). This might lead to non-standard distributions for the estimators and can yield misleading inference, see e.g. Stock, Wright and Yogo (2002). Empirically, we find that the size of the standard errors for the estimators of the parameters $\alpha_1$ and $\alpha_2$ in (1) matches the estimated values for $\alpha_1$ and $\alpha_2$.

However, when we estimate (1) and (3) using three lags of $\pi$, $x$, and $i$ as instruments, we find that $b_3 \neq 0$ but the null hypothesis $b_3 = 0$ is strongly rejected. The estimated parameters for (1) are reported in the first panel in column 2 of Table 1. Compared with the corresponding single equation estimates we find that the point estimates of the parameters are basically unaffected (there is a non-significant decrease of about 5% in the coefficient of $\pi^e_{t+1}$ and a corresponding increase in that of $\pi_{t-1}$). Yet, there is a substantial reduction in the standard errors of 30-40%. Similar results are obtained when (3) is substituted for a VAR(3) specification. These findings suggest that the model is identified, but the solution could be indeterminate. Intuitively, indeterminacy arises because the sum of the estimated parameters $\alpha_1$ and $\alpha_3$ in (1) is very close to one.

So far the processes for the forcing variables was assumed to be purely backward looking. As an alternative we consider a forward looking model also for $x_t$. For example, Fuhrer and Rudebusch (2002) estimated a model for a representative agent's Euler equation (in their notation)

$$x_t = \beta_0 + \beta_1^* x^e_{t+1} + \beta_2^* (\frac{1}{k} \sum_{j=0}^{k-1} (i^e_{t+j} - \pi^e_{t+j+1})) + \beta_3^* x_{t-1} + \beta_4^* x_{t-2} + \eta_t, \quad (4)$$

where $x_t$ is real output (detrended in a variety of ways), $x^e_{t+1}$ is the forecast of $x_{t+1}$ made in period $t$, $i_t - \pi^e_{t+1}$ is a proxy for the real interest rate at time $t$, and $\eta_t$ is an i.i.d. $(0, \sigma^2_\eta)$ error term. In our sample period, the second lag of $x$ is not significant and only the current interest rate matters. Hence, the model becomes

$$x_t = \beta_0 + \beta_1 x^e_{t+1} + \beta_2(i_t - \pi^e_{t+1}) + \beta_3 x_{t-1} + \eta_t. \tag{5}$$

Replacing the forecast with its realized value, we get

$$x_t = \beta_0 + \beta_1 x_{t+1} + \beta_2(i_t - \pi_{t+1}) + \beta_3 x_{t-1} + \mu_t, \tag{6}$$

where $\mu_t = \beta_1(x^e_{t+1} - x_{t+1}) + \beta_2(\pi^e_{t+1} - \pi_{t+1})$.

As in the case of the New-Keynesian Phillips curve, this equation can be estimated by GMM, appropriately corrected for the presence of an MA component in the error term $\mu_t$. As in our estimates of the New-Keynesian Phillips curve, we use three lags of $x$, $i$ and $\pi$ as instruments. The results are reported in the first column of the second panel of Table 1 The coefficient on $x^e_{t+1}$ is slightly larger than 0.5 and significant, and the coefficient on $x_{t-1}$ is also close to 0.5 and significant. These values are in line with those in Fuhrer and Rudebusch (2002), who found lower values when using ML estimation rather than GMM and the positive sign of the real interest in the equation for the output gap is similar to the Fuhrer-Rudebusch results when they used HP de-trending.

As with the New-Keynesian Phillips curve, we estimate Equation (6) simultaneously together with a sub-VAR(1) as in (3), but here for the forcing variables $\pi_t$ and $i_t$. Again, the significance of the coefficients in the VAR(1) equations (in particular those for lagged $\pi_t$ in the $i_t$ equation) lends support to their relevance as instruments. The numerical values of the estimated parameters for the Euler equation remain nearly unchanged. However, as in

the case of the Phillips curve above, the precision of the estimators increases substantially. These results are reported in the second column of the second panel in Table 1.

In order to complete our building blocks for a forward looking system we finally also model the interest rate with a Taylor rule as in Clarida, Galí and Gertler (1998, 2000). Our starting point here is the equation

$$i_t^* = \bar{i} + \gamma_1(\pi_{t+1}^e - \pi_t^*) + \gamma_2(x_t - x_t^*), \tag{7}$$

where $i_t^*$ is the target nominal interest rate, $\bar{i}$ is the equilibrium rate, $x_t$ is real output, and $\pi_t^*$ and $x_t^*$ are the desired levels of inflation and output. The parameter $\gamma_1$ indicates whether the target real rate adjusts to stabilize inflation ($\gamma_1 > 1$) or to accommodate it ($\gamma_1 < 1$), while $\gamma_2$ measures the concern of the central bank for output stabilization.

Following the literature, we introduce a partial adjustment mechanism of the actual rate to the target rate $i^*$ :

$$i_t = (1 - \gamma_3)i_t^* + \gamma_3 i_{t-1} + v_t, \tag{8}$$

where the smoothing parameter $\gamma_3$ satisfies $0 \leq \gamma_3 \leq 1$, and $v_t$ is an i.i.d. $(0, \sigma_v^2)$ error term. Combining (7) and (8), we obtain

$$i_t = \gamma_0 + (1 - \gamma_3)\gamma_1(\pi_{t+1}^e - \pi_t^*) + (1 - \gamma_3)\gamma_2(x_t - x_t^*) + \gamma_3 i_{t-1} + v_t \tag{9}$$

where $\gamma_0 = (1 - \gamma_3)\bar{i}$, which becomes

$$i_t = \gamma_0 + (1 - \gamma_3)\gamma_1(\pi_{t+1} - \pi_t^*) + (1 - \gamma_3)\gamma_2(x_t - x_t^*) + \gamma_3 i_{t-1} + \epsilon_t, \tag{10}$$

with $\epsilon_t = (1 - \gamma_3)\gamma_1(\pi_{t+1}^e - \pi_{t+1}) + v_t$, after replacing the forecasts with their realized values

The results for single equation GMM estimation (with 3 lags as instruments) are reported in the first column of the third panel of Table 1. As in

Clarida *et al* (1998, 2000), the coefficient on future inflation is larger than one. We also found the coefficient on output to be larger than one, although the standard errors around both point estimates are rather large. Again, as in the cases of single equation estimations of the Phillips curve and the Euler equation, we are able to reduce the variance of our point estimates by adding sub-VAR(1) equations for the forcing variables $\pi_t$ and $x_t$ when estimating the resulting system by GMM (see column 2). As above, for both approaches we have used up to three lags for the intrument variables.

We are now in a position to estimate the full forward looking system, composed of Equations (1), (5) and (9):

$$\begin{aligned} \pi_t &= \alpha_0 + \alpha_1 \pi^e_{t+1} + \alpha_2 x_t + \alpha_3 \pi_{t-1} + e_t, \qquad (11) \\ x_t &= \beta_0 + \beta_1 x^e_{t+1} + \beta_2 (i_t - \pi^e_{t+1}) + \beta_3 x_{t-1} + \eta_t, \\ i_t &= \gamma_0 + (1-\gamma_3)\gamma_1(\pi^e_{t+1} - \pi^*_t) + (1-\gamma_3)\gamma_2(x_t - x^*_t) + \gamma_3 i_{t-1} + v_t \end{aligned}$$

The results are reported in column 3 of Table 1. For each of the three equations the estimated parameters are very similar to those obtained either in the single equation case or in the systems completed with VAR equations. Furthermore, the reductions in the standard errors of the estimated parameters are similar to those obtained with sub-VAR(1) specifications. Since the VAR equations can be interpreted as reduced forms of the forward looking equations, this result suggests that completing a single equation of interest with a reduced form may be enough to achieve as much efficiency as within a full system estimation. However, the full forward looking system represents a more coherent choice from an econometric point of view, and the finding that the forward looking variables have large and significant coefficients in all the three equations lends credibility to the complete rational expectations model.

The nonlinearity of our system of forward looking equations makes the

evaluation of global identification impossible. However, if we linearize the model around the estimated parameters and focus on local identification, we can show that the model is (at least) exactly identified, see Beyer *et al* (2005). Exact identification holds when the point estimates imply a determinate solution. The model would be potentially overidentified in case of an indeterminate equilibrium.

# 3 Robustness analysis

While system estimation increases efficiency, the full forward looking model in (11) could still suffer from mis-specification problems, see e.g. Canova and Sala (2006). To evaluate this possibility, we conducted four types of diagnostic tests. First, we ran an LM test on the residuals of each equation to check for additional serial correlation i.e. serial correlation beyond the one that is due to the MA(1) error structure of the model. Second, we ran the Jarque and Bera normality test on the estimated errors. Although our GMM estimation approach is robust to the presence of non-normal errors,[6] rejection of normality could signal other problems, such as the presence of outliers or parameter instability. Third, we ran an LM test to check for the presence of ARCH effects; rejection of the null of no ARCH effects might more generally be a signal of changes in the variance of the errors. Finally, we checked for parameter constancy by running recursive estimates of the forward looking system.

The results of our mis-specification tests are reported in the bottom lines of each panel in Table 2. For convenience, we also present in column 1 again the estimated parameters. There are only minor problems of residual correlation in the inflation equation, but normality is strongly rejected for

[6] Note that this is not the case for maximum likelihood estimation.

the inflation and interest rate equation and the interest rate equation also fails the test for absence of serial correlation and absence of ARCH.

The rejection of correct specification could be due to parameter instability in the full sample 1970:3 - 1998:4. Instability might be caused by a variety of sources including external events such as the oil shocks, internal events, such as the reduction in the volatility of output (e.g. McConnell and Perez-Quiros (2000)), or changes in the monetary policy targets. Since we had more faith in the second part of our sample, we implemented a backward recursion by estimating the system first for the subsample 1988:1-1998:4, and recursively reestimating the system by adding one quarter of data to the beginning of the sample, i.e. our second subsample consisted of the quarters 1987:4–1998:4, our third was 1987:3 – 1998:4 and so on until 1970:3-1998:4.

In Figure 1, we report recursive parameter estimates. These graphs confirm that the likely source of the rejection of ARCH, normality and serial correlation tests is the presence of parameter change. Although the parameter estimates are stable back to 1985:1, going further back than this is associated with substantial parameter instability in all three equations, and particularly in the estimated Taylor rule. Although parameter instability is more pronounced when we use unemployment as a measure of economic activity, it is also present in estimates obtained when using the output gap.

Overall, these mis-specification tests cast serious doubts on results obtained for the full sample, and they suggest that a prudent approach would be to restrict our analysis to a more homogeneous sample. For this reason, in the subsequent analysis, we report results only for the subperiod 1985:1-1998:4.

Our subsample results are presented in the second column of Table 2. It is interesting to note that the values of the estimated parameters of the New-Keynesian Phillips curve and the Euler equation are similar to those obtained

for the full sample. However, parameter estimates of the coefficients of the Taylor rule differ substantially from the single equation estimates. Most prominently, there is a marked decrease in the estimated coefficient on the output gap. In the post 1985 subsample we fail to reject the null hypothesis for all four of our diagnostic tests, thereby lending additional credibility to our estimation results.

The final issue we briefly consider is the role of the method of estimation. Fuhrer and Rudebusch (2002), Lindé (2005) and Jondeau and Le Bihan (2003) have suggested that GMM may lead to an upward bias in the parameters associated with the forward looking variables, while maximum likelihood (ML) produces better results. For example Lindé (2005) finds that estimated parameters were not heavily biased from true parameters. In case of exact identification ML coincides with indirect least squares where the reduced form parameters of the model are mapped back into those of the structural form. We compared our estimates with the point estimates from GMM by computing the indirect least squares estimates from the reduced form. Using this approach, we find that our GMM estimates are similar to the ML values.

For the subsample 1985-1998, the estimated coefficient on $\pi^e_{t+1}$ in the inflation equation is 0.76 and 0.62 for that on future expected output gap in the Euler equation whereas the GMM estimates of these parameters are, respectively, 0.61 and 0.47. The differences are slightly larger for the coefficient on future inflation in the Taylor rule, in the range $2.1 - 2.4$ with ML. Overall we are reassured that our finding of significant coefficients on future expected variables is robust to alternative system estimation methods.

# 4 Enlarging the information set

The analysis in Sections 2 and 3 supports the use of a system approach to the estimation of forward looking equations. For the 1985:1–1998:4 sample, our estimated system passes a wide range of mis-specification tests. Moreover, the Hansen's $J$-statistic, reported at the foot of Table 2, is unable to reject the null of relevant instruments for this period (but it is worth recalling the caveats on the use of the $J$-test in this context). However, there could still be problems of weak instruments and/or omitted variables which are hardly detectable using standard tests, (see e.g. Mavroeidis (2005)). This section proposes a method that can potentially address both of these issues.

Our approach is to augment our data by adding information extracted from a large set of 146 macroeconomic variables as described in Stock and Watson (2002a, 2002b, SW). We assume that these variables are driven by a few common forces, i.e. the factors, plus a set of idiosyncratic shocks. This assumption implies that the factors provide an exhaustive summary of the information in the large dataset, so that they may alleviate omitted variable problems when used as additional regressors in our small system. Moreover, the factors extracted from the Stock and Watson data are known to have good forecasting performance for the macroeconomic variables in our small dataset and they are therefore likely to be useful as additional instruments that may alleviate weak instrument problems, too. Bernanke and Boivin (2003) and Favero, Marcellino and Neglia (2005) showed that when estimated factors are included in the instrument set for GMM estimation of Taylor rules, the precision of the parameter estimators increases substantially. The economic rationale for inclusion of these variables is that central bankers rely on a large set of indicators in the conduct of monetary policy; our extracted factors may provide a proxy for this additional information. An additional reason for being interested in the inclusion of factors in our analysis is that,

the inclusion of factors in small scale VARs has been shown to remove the "price puzzle" suggesting that factors may be used to reduce or eliminate the estimation bias, that arises from the omission of relevant right-hand-side variables.[7]

In the following subsection, we present a brief overview on the specification and estimation of factor models for large datasets. Following this discussion, we evaluate whether the use of the estimated factors changes the size and or the significance of the coefficients of the forward looking components in the New Keynesian model.

## 4.1 The factor model

Equation (12) represents a general formulation of the dynamic factor model

$$z_t = \Lambda f_t + \xi_t, \tag{12}$$

where $z_t$ is an $N \times 1$ vector of variables and $f_t$ is an $r \times 1$ vector of common factors. We assume that $r$ is much smaller than $N$, and we represent the effects of $f_t$ on $z_t$ by the $N \times r$ matrix $\Lambda$. $\xi_{it}$ is an $N \times 1$ vector of idiosyncratic shocks.

Stock and Watson require the factors, $f_t$, to be orthogonal although they may be correlated in time and with the idiosyncratic components for each factor.[8] Notice that the factors are not identified since Equation (12) can be rewritten as

$$z_t = \Lambda G G^{-1} f_t + \xi_t = \Psi p_t + \xi_t,$$

where $p_t$ is an alternative set of factors and $G$ is an arbitrary invertible $r \times r$ matrix. This fact makes it difficult to form a structural interpretation of the

[7] For a definition and discussion of this issue the reader is referred to Christiano, Eichenbaum and Evans (1999) pages 97–100.

[8] Precise moment conditions on $f_t$ and $\xi_t$, and requirements on the loading matrix $\Lambda$, are given in SW.

factors, but it does not prevent their use as a summary of the information contained in $z_t$.

SW define the estimators $\widehat{f}_t$ as minimizing the objective function

$$V_{N,T}(f,\Lambda) = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(z_{it} - \Lambda_i f_t)^2.$$

Under the hypothesis of $r$ common factors, they show that the optimal estimators of the factors are the $r$ eigenvectors corresponding to the $r$ largest eigenvalues of the $T \times T$ matrix $N^{-1}\sum_{i=1}^{N} \underline{z}_i \underline{z}_i^{'}$, where $\underline{z}_i = (z_{i1}, ..., z_{iT})$. Moreover, the $r$ eigenvectors corresponding to the $r$ largest eigenvalues of the $N \times N$ matrix $T^{-1}\sum_{t=1}^{T} z_t z_t^{'}$ are the optimal estimators of $\Lambda$. These eigenvectors coincide with the principal components of $z_t$; they are also the OLS estimators of the coefficients in a regression of $z_{it}$ on the $k$ estimated factors $\widehat{f}_t$, $i = 1, ..., N$.[9] Although there are alternative estimation methods available such as the one by Forni and Reichlin (1998) or Forni et al (2000), we chose the SW approach since there is some evidence to suggest that it dominates the alternatives in this context.[10]

No statistical test is currently available to determine the optimal number of factors. SW and Bai and Ng (2002) suggested minimizing a particular information criterion, however its small sample properties in the presence of heteroskedastic idiosyncratic errors deserves additional investigation. In their

[9] SW prove that when $r$ is correctly specified, $\widehat{f}_t$ converges in probability to $f_t$, up to an arbitrary $r \times r$ transformation matrix, $G$. When $k$ factors are assumed, with $k > r$, $k - r$ estimated factors are redundant linear combinations of the elements of $f_t$, while even when $k < r$ consistency for the first $k$ factors is preserved (because of the orthogonality hypothesis). See Bai (2003) for additional inferential results.

[10] Kapetanios and Marcellino (2006a) found that SW's estimator performs better in simulation experiments, and Favero et al. (2005) reached the same conclusion when using the estimated factors for the estimation of Taylor rules and VARs.

empirical analysis with this data set, SW found that the first 2-3 factors are the most relevant for forecasting key US macroeconomic variables. In the following analysis we however evaluate the role of up to six factors to make sure sufficient information is captured. Finally, Bai and Ng (2006a) have shown that the estimated factors when used in subsequent econometric analyses do not create any generated regressor problem when $\sqrt{T}/N$ is $o_p(1)$. This condition requires the number of variables to grow faster than the sample size, which basically guarantees faster convergence of the factor estimators than of the estimators of the other parameters of interest. We assume that this condition is satisfied in our context, where $\sqrt{T}/N = 0.055$.

## 4.2 The role of the estimated factors

As we mentioned, the estimated factors can proxy for omitted variables in the specification of the forward looking equations. In particular, we use up to six contemporaneous factors as additional regressors in each of the three structural equations, and retain those which are statistically significant.

Since the factors are potentially endogenous, we use their first lag as additional instruments. These lags are likely to be useful also for the other endogenous variables in each structural equation. Bai and Ng (2006b) and Kapetanios and Marcellino (2006b) provide a detailed derivation of the properties of factor-based GMM estimators.

In column 3 of Table 2 we report the results of GMM estimation of the forward looking system over the period 1985-1998.

First, a few factors are strongly significant in the equations for inflation and the real variable. While it is difficult to provide an economic interpretation for this result, it does point to the omission of relevant regressors in the Phillips curve and Euler equation. In contrast, no factors are significant in the Taylor rule, which indicates that output gap and inflation expectations

are indeed the key driving variables of monetary policy over this period.

Second, in general the estimated parameters of the forward looking variables are 10 to 20% lower than those without factors, but they remain strongly statistically significant.

Third, the precision of the estimators systematically increases, as the standard errors of the estimated parameters are 10 to 50% lower than those without the factors. This confirms the usefulness of the additional information contained in the factors.

Fourth, since the highest lag order of the regressors in the structural model is one, it could suffice to include one lag of $\pi_t$, $x_t$, and $i_t$ in the instrument set instead of three lags. In this case, the point estimates are unaffected, as expected, but the standard errors increase substantially. This finding suggests that the solution of the system could be indeterminate, in which case more lags would indeed be required.

Finally, since there is no consensus on the best way to compute robust standard errors in this context, we verified the robustness of our findings based on Newey West (1994) comparing them with those based on Andrews (1991). The latter are in general somewhat lower, but the advantages resulting from the use of factors are still systematically present.

# 5 Conclusions

In this paper we provided a general econometric framework for the analysis of models with rational expectations, focusing in particular on the hybrid version of the New-Keynesian Phillips curve that has attracted considerable attention in the recent period.

First, we showed that system estimation methods where the New-Keynesian Phillips curve is complemented with equations for the interest rate and ei-

ther unemployment or the output gap yield more efficient parameter estimates than traditional single equation estimation, while there are only minor changes in the point estimates and the expected future variables play an important role in all the three equations. The latter result remains valid even if MLE is used rather than system GMM.

Second, we stressed that it is important to evaluate the correct specification of the model, and we showed that our systems provide a proper statistical framework for the variables over the 1985-1998 period, while during the '70s there is evidence of parameter changes, in particular in the interest rate equation.

Third, we analyzed the role of factors that summarize the information contained in a large data set of U.S. macroeconomic variables. Some factors were found to be significant as additional regressors in the New-Keynesian Phillips curve and in the Euler equation, alleviating potential omitted variable problems. Moreover, using lags of the factors as additional instruments in our small New-Keynesian system, the standard errors of the GMM estimates systematically decrease for all the estimated parameters; the gains are particularly large for the coefficients of forward looking variables.

In conclusion, using the factors, data after 1985 is not inconsistent with the New-Keynesian interpretation of a determinate equilibrium driven by three fundamental shocks. The estimated parameters of the complete model form a consistent picture which coincides with New-Keynesian economic theory. We should note that while our results support the relevance of forward looking variables in our estimated equations there is a large variety of alternative models compatible with the observed data which can have very different properties both in terms of the relevance of the forward looking variables and of the characteristics of their dynamic evolution. This has been demonstrated in Beyer and Farmer (2003*b*). A more detailed analysis of this issue

represents an interesting topic for further research in this field.

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# Appendix: Tables and Figures

**Table 1. Single equation vs sytem estimation, GDP gap**

| Eqn. | Var. (coeff.) | Estimation method Single | Sub-VAR | System |
|---|---|---|---|---|
| | | [1] | [2] | [3] |
| $\pi_t$ | $\pi^e_{t+1}$ $(\alpha_1)$ | 0.778*** | 0.726*** | 0.672*** |
| | | (0.139) | (0.090) | (0.084) |
| | $gap_t$ $(\alpha_2)$ | −0.067* | −0.051** | −0.038* |
| | | (0.034) | (0.025) | (0.023) |
| | $\pi_{t-1}$ $(\alpha_3)$ | 0.231* | 0.281*** | 0.334*** |
| | | (0.128) | (0.082) | (0.072) |
| | $Adj.\ R^2$ | 0.860 | 0.899 | 0.870 |
| | $J-stat$ | 4.809 (6) | 17.993(18) | 13.780(18) |
| | $p-value$ | 0.56 | 0.45 | 0.74 |
| $x_t = gap_t$ | $gap^e_{t+1}$ $(\beta_1)$ | 0.544*** | 0.538*** | 0.540*** |
| | | (0.046) | (0.033) | (0.034) |
| | $ri_t$ $(\beta_2)$ | 0.017 | 0.016 | 0.015 |
| | | (0.013) | (0.011) | (0.011) |
| | $gap_{t-1}$ $(\beta_3)$ | 0.487*** | 0.486*** | 0.484*** |
| | | (0.040) | (0.030) | (0.029) |
| | $Adj.\ R^2$ | 0.954 | 0.954 | 0.954 |
| | $J-stat$ | 5.778 (6) | 18.453(18) | – |
| | $p-value$ | 0.44 | 0.42 | |
| $i_t$ | $\pi^e_{t+1}$ $(\gamma_1)$ | 1.103** | 1.072*** | 1.186*** |
| | | (0.458) | (0.362) | (0.308) |
| | $gap_t$ $(\gamma_2)$ | 1.675* | 1.659** | 1.476** |
| | | (0.922) | (0.691) | (0.704) |
| | $i_{t-1}$ $(\gamma_3)$ | 0.921*** | 0.922*** | 0.920*** |
| | | (0.027) | (0.020) | (0.022) |
| | $Adj.\ R^2$ | 0.885 | 0.885 | 0.885 |
| | $J-stat$ | 10.702* (6) | 15.574(18) | – |
| | $p-value$ | 0.098 | 0.62 | |

Note: The instrument set includes the constant and three lags of $gap$, $\pi$, $i$. Sample is 1970:1-1998:4. The columns report results for single equation estimation (Single), system estimation where the completing equations are Sub-VARs (Sub-VAR), and full forward looking system (System). HAC s.e. (no pre-whitening, Bartlett kernel, fixed bandwith Newey West) in (). *, **, and *** indicate significance at 10%, 5% and 1%.
J-stat is $\chi^2(p)$ under the null hypothesis of $p$ valid over-identifying restrictionns

**Table 2. Alternative forward looking sytems, GDP gap**

| | | 1970-1998 | 1985-1998 | |
|---|---|---|---|---|
| Eqn. | Var. (coeff.) | No factors | No factors | Significant Factors as regressors |
| | | [1] | [2] | [3] |
| $\pi_t$ | $\pi^e_{t+1}$ $(\alpha_1)$ | 0.672*** | 0.605*** | 0.435*** |
| | | (0.083) | (0.093) | (0.081) |
| | $gap_t$ $(\alpha_2)$ | −0.038* | −0.012 | 0.067 |
| | | (0.023) | (0.018) | (0.049) |
| | $\pi_{t-1}$ $(\alpha_3)$ | 0.334*** | 0.319*** | 0.343*** |
| | | (0.072) | (0.067) | (0.042) |
| | $Adj.\ R^2$ | 0.870 | 0.481 | 0.531 |
| | $No\ corr$ (4) | 1.599 | 2.007 | 2.302* |
| | $Norm$ | 6.907** | 1.877 | 0.065 |
| | $No\ ARCH$ (4) | 2.990 | 0.565 | 1.505 |
| $x_t = gap_t$ | $gap^e_{t+1}$ $(\beta_1)$ | 0.540*** | 0.466*** | 0.623*** |
| | | (0.034) | (0.044) | (0.033) |
| | $ri_t$ $(\beta_2)$ | 0.015 | −0.021 | 0.057*** |
| | | (0.011) | (0.015) | (0.010) |
| | $gap_{t-1}$ $(\beta_3)$ | 0.484*** | 0.558*** | 0.491*** |
| | | (0.029) | (0.047) | (0.040) |
| | $Adj.\ R^2$ | 0.954 | 0.966 | 0.953 |
| | $No\ corr$ (4) | 0.177 | 0.884 | 0.453 |
| | $Norm$ | 2.721 | 2.198 | 0.440 |
| | $No\ ARCH$ (4) | 1.510 | 0.896 | 1.270 |
| $i_t$ | $\pi^e_{t+1}$ $(\gamma_1)$ | 1.186*** | 1.123** | 1.098*** |
| | | (0.308) | (0.458) | (0.178) |
| | $gap_t$ $(\gamma_2)$ | 1.476** | 0.771*** | 0.846*** |
| | | (0.704) | (0.160) | (0.012) |
| | $i_{t-1}$ $(\gamma_3)$ | 0.920*** | 0.867*** | 0.841*** |
| | | (0.022) | (0.024) | (0.090) |
| | $Adj.\ R^2$ | 0.885 | 0.945 | 0.947 |
| | $No\ corr$ (4) | 2.172* | 0.237 | 0.227 |
| | $Norm$ | 525.0*** | 1.833 | 3.257 |
| | $No\ ARCH$ (4) | 2.743** | 0.765 | 0.583 |
| | $J-stat$ | 13.780(18) | 12.824 (18) | 12.942 (18) |
| | $p-value$ | 0.74 | 0.80 | 0.99 |

Note: The instrument set includes the constant and three lags of $gap$, $\pi$, $i$ (no factors) plus the first lag of the six estimated factors (other case).
The regressors are either as in Table 1 (no factors) or include some contemporaneous factors (see text for details)
HAC s.e. (as in Tab.1) in (). *, **, and *** indicate significance at 10%, 5% and 1%; The mis-specification tests (No corr, Norm, No ARCH) are conducted on the residuals of an MA(1) model for the estimated errors. No corr is LM(4) test for no serial correlation,Norm is Jarque-Bera statistic for normality, and ARCH in LM(4) test for no ARCH effects.
J-stat is $\chi^2(p)$ under the null hypothesis of $p$ valid over-identifying restrictionns

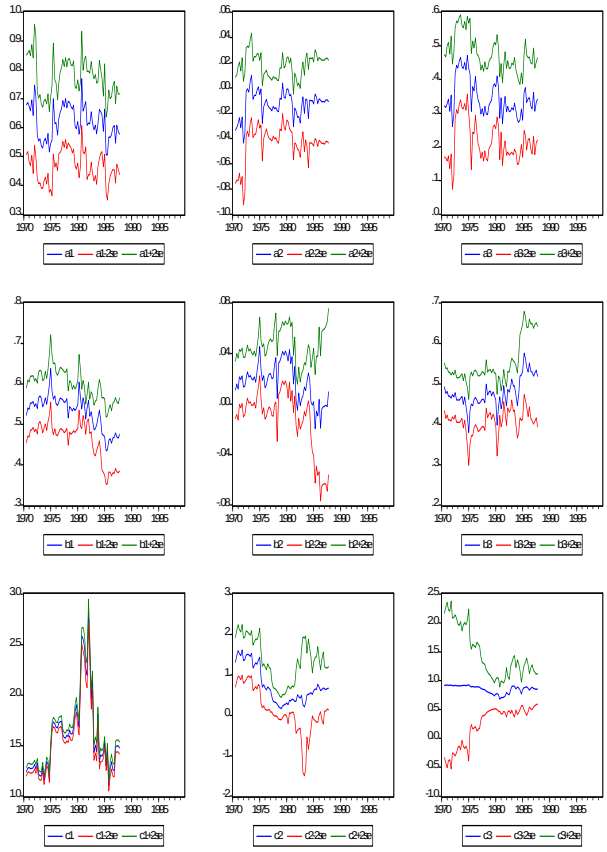


Figure 1: Backward recursive estimation, 1988:1 - 1970:1, system with GDP gap