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THE ARCHITECTURE OF ECONOMIC SYSTEMS: HIERARCHIES AND POLYARCHIES

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ABSTRACT

This paper presents some new perspectives on the structure and performance of alternative economic organizations. We posit that decision makers make errors of judgement (for example, they sometimes select bad projects while rejecting good projects), and that how these errors are aggregated within different organizations depends on their architecture (for example, on how individuals are organized together). Using this framework, we compare the performances of two polar forms of organizations: hierarchies and polyarchies.

Assuming judgmental abilities of individuals are similar in the two systems, we show that polyarchies accept a larger proportion of bad projects (compared to hierarchies) whereas hierarchies reject a larger proportion of good projects. We then determine the conditions under which polyarchies have higher or lower expected profit. The conditions under which polyarchies perform better appear to be more plausible and, moreover, this conclusion holds also in the case where the rules for accepting or rejecting projects are rationally determined based on the information available to individuals. The architecture of organizations also affects their portfolio of available projects; we determine conditions under which polyarchies have better or worse portfolios compared to those available to hierarchies.

There are many possible extensions of our approach. Among them are the analysis of internal structure of firms, selection of managers (by other managers) and the reproduction and self-perpetuation of organizations over time.

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THE ARCHITECTURE OF ECONOMIC SYSTEMS:

HIERARCHIES AND POLYARCHIES

There is a widespread belief that the internal organization of an economic system has an important effect on its performance. Yet, there is very little in traditional economic analysis which investigates such a relationship.[1] In this paper, we present some new ways of looking at economic systems. We motivate our discussion in the context of economic systems, but it has implications for the internal organization of large corporations as well.

The thesis of this paper is that economic systems behave fundamentally differently under different forms of organization, and that central to an understanding of the performance of an economic system is an understanding of its architecture. The architecture (like that of a computer or electrical system) describes, among other things, how the constituent decision making units are arranged together in a system (i.e., who makes which decisions) and who conveys what information to whom. Our attempt is to relate the architecture of alternative economic systems, along with their attendant rules for meting out rewards and punishments, to their performance.

The axiom of human behavior which plays a basic role in our analysis is that all decision makers make errors of judgment. For concreteness we focus on simple decisions which involve accepting or rejecting certain projects. Individuals (or the constituents of economic systems) make these decisions based on the information available to them. In any event, because of the errors in judgment, some projects which get accepted should have been rejected, and some projects which are rejected should have been accepted. Using the analogy from the classical theory of statistical inference, these errors correspond to Type-II and Type-I errors.

The typology of economic systems on which we focus in this paper arises from the differences in how individuals are organized together in a system. We think of a polyarchy as a system in which there are independent (and possibly competing) sources of decision making. In contrast, a hierarchy is visualized as a system in which the decision making authority is more concentrated. In Section I, we present simple (polar) models of these two systems.[2]

The most important consequence of how the individuals are arranged together is that the aggregation of errors is different in different economic systems. The aggregation of errors, in turn, determines the performance of a system. For example, in a market economy, if one firm rejects a profitable idea (say, for a new product), then there is a possibility that some other firm might accept it. In contrast, if a single agency makes such decisions and this agency rejects the idea, then the idea must remain unused. The same, however, is also true for those ideas which are unprofitable. As a result, one would expect a greater incidence of Type-II error in a polyarchy, and a greater incidence of Type-I error in a hierarchy.

It should be apparent, however, that the overall performance of a system (for example, its profit level) will depend not only on its architecture, but also on the mix of projects that is available to its decision makers, and on the nature of errors that the decision making entails.

Initially, in Section II, we assume that individuals make similar errors in the two systems, and that the nature of errors is exogenously specified.

Moreover, the mix of projects available to the two systems is also identical. The performance of economic systems is thus attributable primarily to what we have called their architecture.

We then examine, in Section III, how the portfolio of projects available to an economic system is influenced by its architecture. This represents more of a general equilibrium view: the differences in the architecture will affect the chances that different types of projects have of being accepted or rejected. This will, in turn, influence the incentives of those who conceptualize and invent projects, and will thus affect the kinds of projects which are invented. For example, one would expect that the inventors would attempt to generate those projects which are more likely to be accepted.

The above analysis takes the errors in judgment (that is, the probabilities of good projects being rejected and those of bad projects being selected by evaluators) as exogenous, as well as identical in the two systems. In Section IV, we analyze an endogenous determination of these errors. This we do by determining rational screening rules for project acceptance, where the constituents in the economic systems take into account whatever information is available to them.

In this paper, we analyze only one, albeit an important one, aspect of the architecture of economic systems. Some other important aspects are sketched out in the concluding section.

I. THE MODEL

In the following model, a polyarchy consists of two firms, and a hierarchy consists of two bureaus. The task of a bureau or a firm is to screen projects. Each project has a scalar (net) benefit, which can be positive, negative, or zero. A screen (i.e., a bureau or a firm) evaluates every project and accepts or rejects the project.

The decision process in a polyarchy and a hierarchy are depicted in Figures 1 and 2 respectively. In a polyarchy, the two firms screen the projects independently. For specificity, one may think of projects arriving randomly (with probability one-half) at one of the two firms. If a particular project is accepted by a firm, then it is no longer available to the other firm. If the project is rejected, then it goes to the other firm where, once again, it can be accepted or rejected. Neither firm screens the same project twice, so that a project can not cycle back and forth between firms.[3] The portfolio of projects selected in a polyarchy therefore consists of the projects accepted separately by each of the two firms.

In contrast, in a hierarchy, all projects are first evaluated by the lower bureau (bureau 1); those which are accepted are forwarded to the higher bureau (bureau 2) and others are discarded. The projects selected by the system then are those which pass through the higher bureau. Drawing an analogy from the design of electrical circuits, the screens are placed in series in a hierarchy whereas they are placed in parallel in a polyarchy.

Throughout the paper, the superscripts P and H represent a polyarchy and a hierarchy respectively. Also, for brevity, we use the superscript s, where s = P or H. Let x denote the net profit (benefit) from a project,[4] and let $p^s(x)$ denote the probability that this project will pass through a screen in the system. We refer to $p^s(x)$ as the screening function.[5] Then the probabilities that the project x will be accepted in the system s, denoted by f^s, are given by

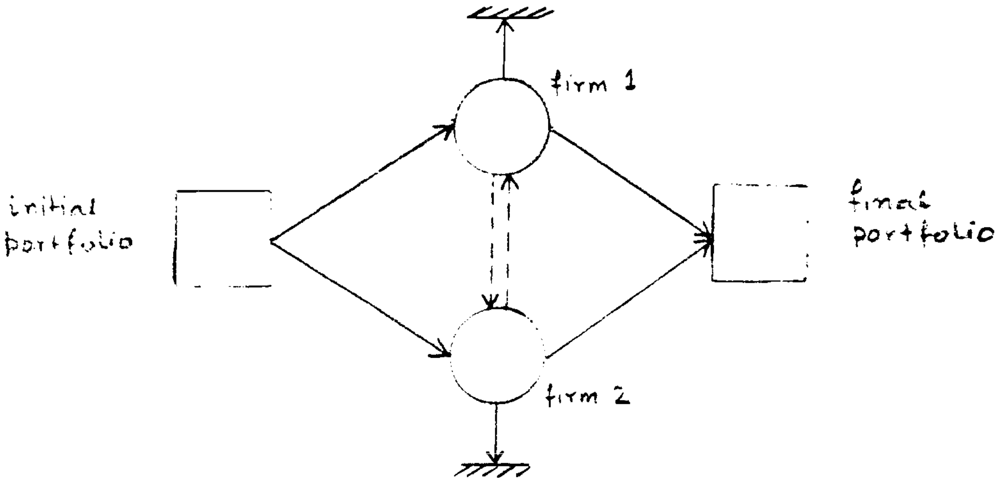

FIGURE 1 : POLYARCHY

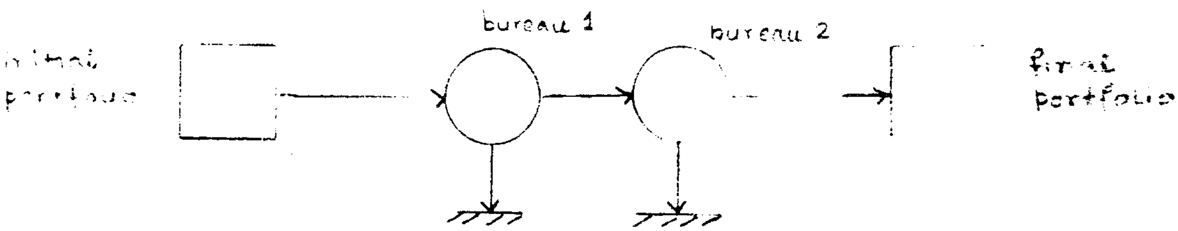

FIGURE 2 : HIERARCHY

(1) $$f^P \equiv p^P(x)\ (2 - p^P(x))$$

(2) $$f^H \equiv (p^H\ (x))^2$$

We refer to the portfolio of projects available to an economic system as the initial portfolio and the portfolio that it selects as the final portfolio. For system s, N^s denotes the number of projects in the initial portfolio, $g^s(x)$ denotes its pdf, and $G^s(x)$ denotes its cdf. The initial portfolio contains both profitable and unprofitable projects, i.e., there are projects with positive as well as negative x's.[6]

Our interest is in examining the final portfolios in the two systems. These can be represented using many different summary statistics. For example, the fraction of initial projects selected by the systems, denoted by n^s, is given by

(3) $$n^s = E[f^s],$$

in which E denotes the expectation operator; the expectation is calculated with respect to the pdf for the system.

On the other hand, if we are interested in studying the profitability of alternative systems, then an important statistic is the expected profit. We denote this by Y^s, which is[7]

(4) $$Y^s = N^s E[xf^s].$$

One might also be interested in other statistics; we discuss these later.

Screening Function: The screening function $p(x)$ is the probability that projects with different levels of profit are accepted by a bureau or a firm. It summarizes the error making properties of a screen. It can take any form, provided

$$(5) \qquad 1 \geq p(x) \geq 0,$$

for all x, and the strict inequalities hold for at least some x.[8]

Two properties of the screening function are of special interest. The first is its slope, i.e., $p_x(x) = \frac{\partial p(x)}{\partial x}$. If $p_x(x)$ is positive then a project with higher profit has a higher local probability of being accepted by a screen. If $p_x(x) = 0$, then the screening is indiscriminate, since it does not distinguish between a better and a worse project.

While in certain cases it is possible that the sign of p_x changes over the range of projects, we consider here only those screens which have at least some, but not complete, discriminating ability throughout the range of projects. That is, $p_x > 0$, for all x. Further, if p and $\tilde{p}$ represent two screens, and if $p_x(\tilde{x}) > \tilde{p}_x(\tilde{x})$, then we refer to the former screen as locally more discriminating at $x = \tilde{x}$.

The second important property of screens is the level of $p(x)$. If $p(\tilde{x}) > \tilde{p}(\tilde{x})$, then we call the former screen locally slacker, and the latter locally tighter, at $x = \tilde{x}$.

In some cases examined in this paper, we employ linear screening functions. The corresponding conclusions will approximately hold for all those screens for which the curvature of $p(x)$ is small. In such cases, $p(x)$

is expressed as

$$(6) \qquad p(x) = p_\mu + p_x (x - \mu),$$

where μ is the mean of the initial portfolio, i.e., $\mu = E[x]$, and $p_\mu = p(\mu)$ is the probability that the average project will pass through a screen. Clearly, a higher p_μ and p_x imply globally higher slackness and discriminating capability.

It is useful here to ask: what is it that the economics literature typically assumes concerning the screening functions? In the absence of uncertainty in the outcome of projects (as in the case of the present model)[9], much of the literature makes no distinction between the projects which are worth selecting, and those which are actually selected. This assumption, in the context of the present model, implies that: $p(x) = 1$ if $x > 0$, and $p(x) = 0$ if $x \leq 0$. In this case, the performances of a polyarchy and a hierarchy are identical in every respect, as can be easily verified.

Thus, the architecture of economic systems ceases to be a relevant issue, if one assumes that human decision making is absolutely faultless. Even a casual observation of actual functioning of business and public organizations, in contrast, makes it abundantly clear that errors of judgment are an inescapable feature of human decision making. This realization not only makes it necessary to recast the traditional view of the literature, but it also provides a potential cornerstone for unraveling certain hitherto unrecognized differences among different types of organizational systems.

II. PORTFOLIOS SELECTED IN ALTERNATIVE SYSTEMS

In this section, we investigate two questions. First, how is the portfolio selected in each of the two systems affected by the exogenous parameters representing the initial portfolios and the characteristics of the screens? Second, what is the relative performance of the final portfolios in the two systems, and how is this influenced by the exogenous parameters? While answering the second question, we assume that the two systems have the same initial portfolios and screening mechanisms; the differences in their performance are therefore solely due to the difference in their architecture. This assumption is not required for answering the first question since it does not involve any comparison across the systems.

The statistics of the portfolio selected which we examine in some detail are: the proportion of original projects selected and the (expected) profit from the portfolio selected. We also point out how some other statistics can be analyzed.

A. The Size of Portfolios Selected

The proportion of the initial portfolio selected by the two systems, n^S, is given in (3). Denoting the difference in these proportions by Δn, we find that

(7) $$\Delta n = n^P - n^H > 0, \text{ since}$$

(8) $$f^P - f^H = 2p(x)\{1 - p(x)\} \geq 0,$$

and it is strictly positive for some x.

Therefore: <u>A polyarchy always selects a larger proportion of initial projects than a hierarchy.</u>

The reason behind this result is quite intuitive. Consider a hypothetical situation in which the second firm in a polyarchy does not accept any project, whereas the higher bureau in a hierarchy accepts all of the projects forwarded to it by the lower bureau. The proportion of projects accepted in the two systems, then, would be the same, namely, $E[p(x)]$. It follows then that the actual proportion of projects accepted in a polyarchy will always exceed that in a hierarchy.

In fact, this intuition can be extended further. To see this, let good (bad) projects be denoted by an arbitrary non-empty set A within the range of x. Then the screening is completely faultless (erroneous) if $p(x) = 1$ for $x \varepsilon A$, and it is completely erroneous (faultless) if $p(x) = 0$, for $x \varepsilon A$. Now, from (1) and (2), $\int_A f^P g dx > \int_A f^H g dx$, unless $p(x)$ equals either 1 or 0, for $x \varepsilon A$.

It follows that: <u>A polyarchy accepts a larger number of good as well as bad projects than a hierarchy, no matter how one defines good and bad projects, provided the screening is neither completely faultless nor completely erroneous</u>. Therefore, <u>the incidence of Type-I error is relatively higher in a hierarchy, whereas the incidence of Type-II error is relatively higher in a polyarchy</u>.

To understand the impact of initial portfolios on the size of portfolios selected, let β represent a parameter representing the initial portfolio, i.e., the pdf of the initial portfolio is $g(x, \beta)$. Then, from (3)

$$(9) \qquad \frac{\partial n^S}{\partial \beta} = \int f^S g_\beta dx = - \int f_x^S G_\beta dx$$

where $g_\beta = \frac{\partial g}{\partial \beta}$, $G_\beta = \frac{\partial G}{\partial \beta}$ and $f^s_x = \frac{\partial f^s}{\partial x}$; and the last expression in (9) is obtained by integration by part.[10] In (9), $f^s_x > 0$, from (1) and (2).

Next, integrating (9) by parts again, we obtain

$$(10) \qquad \frac{\partial n^s}{\partial \beta} = \int f^s_{xx} \, [\int^x G_\beta(z)dz]dx$$

where $f^s_{xx} = \frac{\partial^2 f^s}{\partial x^2}$. Using our assumption that p(x) is approximately linear, we find from (1) and (2) that $f^P_{xx} < 0$, and $f^H_{xx} > 0$. The signs of (9) and (10) can now be evaluated using the standard properties of stochastic dominance.[11] The key results are as follows.

An improvement (worsening) in the initial portfolio in the sense of first-order stochastic dominance leads to a larger (smaller) proportion of initial projects being selected in both a polyarchy and a hierarchy. An improvement (worsening) in the initial portfolio in the sense of second-order stochastic dominance leads to a larger (smaller) proportion of initial projects being selected in a polyarchy, and a smaller (larger) proportion being selected in a hierarchy.

These results can be seen in Figure 3. In this figure, f^P and f^H are concave and convex quadratics in x, since p is linear. n^S is the area above the x-axis bounded by the product of f^S and g. Naturally, this area corresponding to f^P is higher than that to f^H; and this area expands, for both a polyarchy and a hierarchy, if the probability weight g(x) shifts from lower x to higher x. Also, if the probability weight shifts from the mean to the two sides (due to a mean preserving spread, for example) then the area representing n^S decreases in a polyarchy and it increases in a hierarchy.

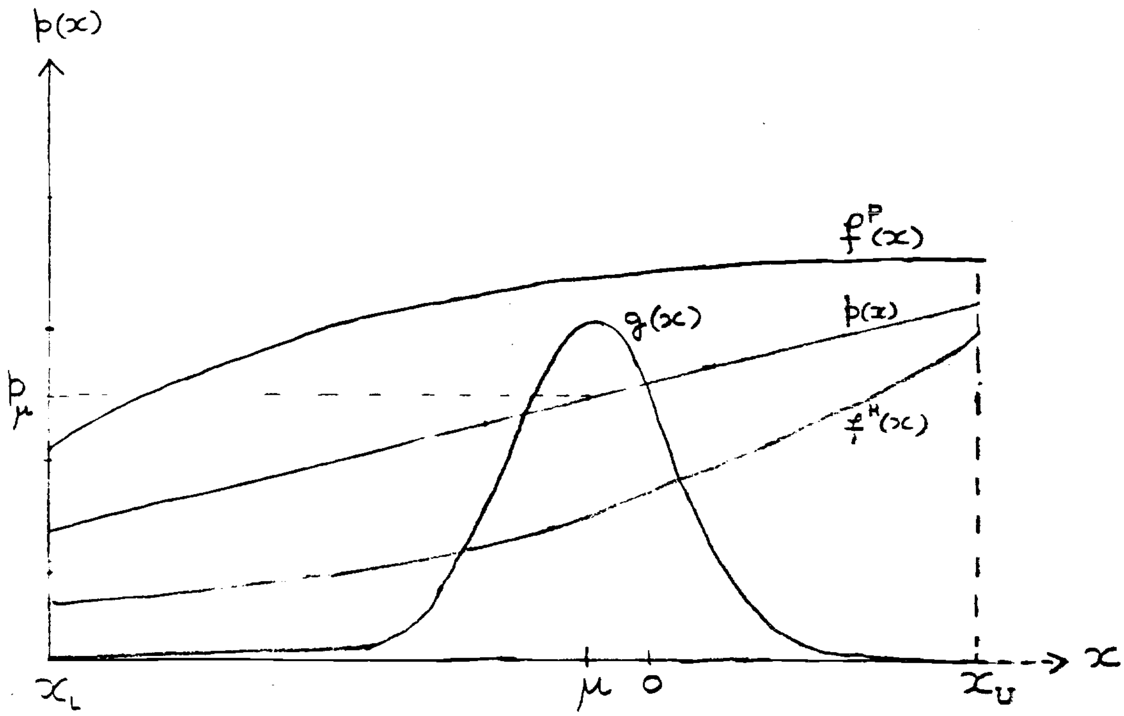

FIGURE 3

Probabilities of Acceptance in Alternative Economic Systems.

Explicit expressions for n^s are derived in the Appendix I. From these expressions, one can obtain bounds on the magnitudes of n^s, and also one can ascertain how n^s is influenced by changes in the screening function. As one can see in Figure 3: <u>A higher (lower) slackness in screening raises (lowers) the proportion of projects selected in both systems</u>. We also show that: <u>A higher (lower) discriminating ability in screening lowers (raises) the proportion selected in a polyarchy, whereas it raises (lowers) the proportion selected in a hierarchy</u>.

B. Profit in Alternative Systems

Probably the single most important indicator of the performance of a system is its (expected) profit. We begin our analysis of profit with simple initial portfolios which contain two types of projects; more general portfolios are examined later.

<u>Two Types of Projects</u>: Consider an initial portfolio consisting of two types of projects, good and bad, with respective net profits x_1 and $-x_2$, where x_1 and x_2 are positive. The probabilities of passing through a screen are denoted by parameters p_1 and p_2 respectively. The initial portfolio contains a fraction α of good projects. If $\Delta Y = Y^P - Y^H$ is the indicator to compare the profit levels in the two systems, then from (4)

$$\Delta Y = 2N[p_1(1 - p_1)\,\alpha x_1 - p_2(1 - p_2)\,(1 - \alpha)x_2] \tag{11}$$

The above expression allows us to demarcate the parameters' space into two regions: one in which a polyarchy has a higher profit than a hierarchy, and the other in which the reverse holds. To see this in its simplest form, first assume that $x_1 = x_2$, i.e., a good and a bad project have symmetric gain and loss. The parameters which determine the sign of (11) are p_1 , p_2 , and α.

Figure 4 summarizes the results. We are concerned only with the area below the 45^o line, since $p_1 > p_2$. Now take, for a moment, the case in which the initial portfolio contains good and bad projects in equal proportions, i.e., $\alpha = 1/2$. Then a polyarchy has a higher profit in the region ODA, whereas the reverse holds in the region ADB. That is, polyarchy has a higher profit if

$$(12) \qquad p_1 + p_2 < 1,$$

and the reverse holds otherwise.

The intuition behind this result is as follows. Recall that a polyarchy selects more projects, good as well as bad, than a hierarchy. A polyarchy's profit from good projects is greater, but so is its loss from bad projects. For a polyarchy to perform better, therefore, its higher profit from good projects must outweigh its higher loss from bad projects. Now, the expression (11) shows that a polyarchy's relative profit from good projects is highest when $p_1 = 1/2$, and this decreases when p_1 diverges from one-half. Similarly, a polyarchy's relative loss from bad projects is highest when $p_2 = 1/2$, and it decreases as p_2 diverges from this point. When we add the gain and the loss, then we find that a polyarchy has a higher profit level if (12) is satisfied.

If the initial portfolio contains a smaller proportion of good projects, that is if $\alpha < 1/2$, then from (11), we find that the parameter space is

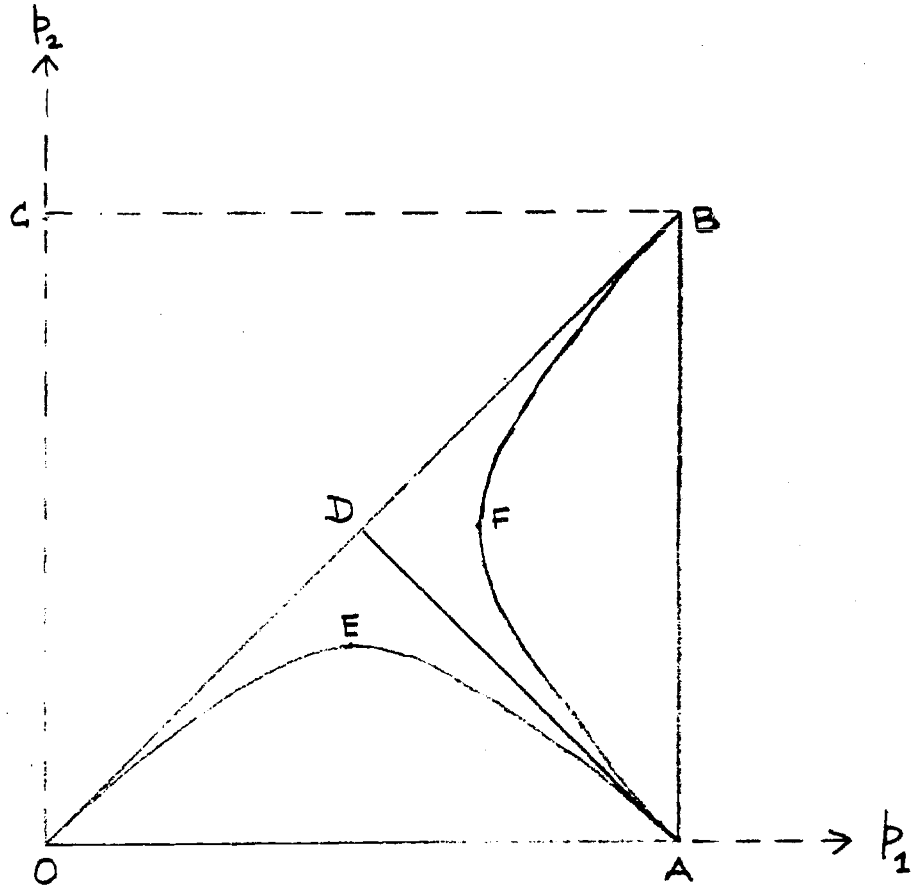

FIGURE 4

COMPARISON OF A POLYARCHY
AND A HIERARCHY

separated by a hyperbola like OEA, which is inside the region ODA[12]. A polyarchy has a higher profit within the region OEA, and the reverse holds outside of it. The region OEA shrinks if the proportion of good projects in the initial portfolio is smaller, and it coincides with the line OA if $\alpha \to 0$.

The opposite case, in which the initial portfolio contains a greater proportion of good project has a parallel implication. A polyarchy then has a higher profit outside of the region AFB, and the reverse holds inside it. Not surprisingly, the two regions OEA and AFB coincide with the triangles ODA and ADB respectively, as α tends to one-half.

The same figure allows us to interpret the case in which good and bad projects have different gain and loss, that is when $x_1 \neq x_2$. As is apparent from (11), a higher x_1/x_2 leads to a higher relative profit in a polyarchy, i.e., a higher gain from a good project or a lower loss from a bad project is relatively advantageous to a polyarchy. It is intuitive, therefore, that x_1/x_2 plays a role analogous to α. Specifically, the line AD separates the two relevant regions in Figure 4 if $\alpha = x_2/(x_1 + x_2)$. A hyperbola like OEA is the boundary if either α or x_1/x_2 is smaller than what would satisfy the last equation. If α or x_1/x_2 is larger, on the other hand, then a hyperbola like AFB is the relevant boundary.[13]

What we find concerning the screening probabilities, therefore, is that if p_1 is small (and significantly larger than p_2) then a polyarchy has higher profit even if, within a range, the initial portfolio has fewer good projects and if the gain from a good project is smaller than the loss from a bad project. On the other hand, within a range of initial portfolios, a hierarchy has higher profit if p_1 is large, and if it is close to p_2.

A qualitative summary of the results is as follows. If the initial portfolio improves (because of a larger proportion of good projects, a higher gain from a good project, or a lower loss from a bad project), then the relative profit in a polyarchy (compared to a hierarchy) improves. The reverse happens if the initial portfolio worsens. Within a range of initial portfolios, however, a polyarchy has a higher profit than a hierarchy if the screening of projects is tight and if it has high discriminating ability. The opposite conclusion holds if the screening is slack and if it has low discriminating ability.

A General Project Portfolio: Now consider initial project portfolios consisting of a spectrum of projects. Recall that the number of projects is N^s. If $p^s(x)$ is a polynomial of order m then, from (4), Y^s is a function of up to 2m + 1 moments of the distribution of initial portfolio. If the screening function is approximately linear, then

$$(13) \qquad Y^s = N^s \phi(\mu^s, \sigma^s, \eta^s, p_\mu^s, p_x^s)$$

where: $\sigma^2 = E[(x - \mu)^2] > 0$ is the variance and $\eta = -E[x - \mu)^3]/\sigma^3$ is the negative of the coefficient of skewness of the initial portfolio. ϕ^s is the expected profit per project in the initial portfolio, i.e., $\phi^s = Y^s/N^s$.

Explicit expressions for ϕ^s are presented in Appendix I. Though these expressions can be interpreted for arbitrary values of μ and η, it might be useful to hypothesize about the relevant range of these parameters. Common experience suggests that the world is full of unprofitable ideas, whereas profitable ideas are quite scarce. Also, the (net) profit that can be typically made from even the best project is limited. In contrast, the losses from those projects which are truly the worst can be much larger. It is reasonable therefore to assume that the mean and the skewness of the universe of projects are nonpositive. That is,

(14a) $\mu \leq 0$, and

(14b) $\eta \geq 0$.

The relative profit level is given by (see Appendix I)[14]

(15) $$\Delta Y = 2N\mu[p_\mu(1 - p_\mu) - p_x^2 \sigma^2] + 2N[(1 - 2p_\mu)p_x \sigma^2 + p_x^2 \sigma^3 \eta].$$

The sign of the above expression depends critically on p_μ which, it should be recalled, is the probability that the average project (which according to (14a) has a nonpositive profit) will pass through a screen. For brevity in exposition in what follows, we call the screening tight if $p_\mu < 1/2$. We note the following qualitative results.

First, the intuition developed in the earlier section concerning the screening function is confirmed in the present case as well. Specifically, a greater tightness or a greater discriminating ability in screens corresponds to a higher relative profit in a polyarchy, provided the screening is tight.

Second, we obtain some new insights concerning the initial portfolio: A larger negative skewness in the initial portfolio implies a higher relative profit in a polyarchy. Also, a larger variance implies a higher relative profit in a polyarchy, provided the screening is tight.

Third, we note some of the circumstances in which the profit level in a polyarchy is greater or smaller than that in a hierarchy. It is obvious from (15) that if the screening of projects is indiscriminate, then the profit in a polyarchy can not be larger than that in a hierarchy.

Therefore: For a polyarchy to have a greater profit than a hierarchy, it is necessary that the screens should have some discriminating ability.

Further, a polyarchy has a lower profit if the initial portfolio is symmetric and the screening is slack. On the other hand, a polyarchy has a higher profit if the mean of the initial portfolio is close to zero, and if the screening is tight.

C. Other Characteristics of Portfolios Selected

Obviously, additional summary statistics might be of some interest in comparing the overall performance of the two systems. Among these are the mean and the higher moments of the final portfolios. To obtain these, let $h^s(x)$ denote the probability that a project with profit x is among the projects selected in system s. Then from Bayes' theorem

$$(16) \qquad h^s(x) = \frac{f^s(x)g(x)}{n^s} .$$

Moments of the final portfolios can be derived using the above pdf.

For example, the mean of the final portfolio in the system s is $\int xh^s(x)dx$. Denoting the mean by Π^s, we have

$$(17) \qquad \Pi^s = \mu + \frac{\psi^s}{n^s} , \text{ where}$$

$$(18) \qquad \psi^s = E[(x - \mu)f^s]$$

is the covariance between x and $f^s(x)$. Variance and other higher moments of the final portfolios can be obtained similarly.

Note that the covariance (18) is positive because $p_x(x)$ is positive and, therefore, x and f_x^s are positively related. Thus: the final portfolios in both a polyarchy as well as in a hierarchy have higher means than the means of their initial portfolios. This happens because both systems improve upon their initial portfolios due to the discriminating ability of their screens.[15]

III. INVENTORS' INCENTIVES

A. Determinants of the Initial Project Portfolio

Thus far we have taken the initial portfolio as given, and examined how the performance of economic systems is affected by exogenous changes in this portfolio. This is clearly a partial equilibrium view, since it ignores the responses and incentives of those who invent projects. In a more general equilibrium view, which we explore in this section, we explicitly take such responses into account.

There are many ways to represent the incentives which influence inventors' behavior, the constraints which inventors face, and how these two aspects are related to the relevant features of the alternative economic systems. What we emphasize here is the commonly observed fact that the acceptance of an idea or invention is often the most significant reward to its inventor, and that inventors would attempt to invent those projects which have greater chances of being accepted in an economic system.

Specifically, we assume that the gain to an inventor from a project depends on whether the project is accepted or not. The utilities to an inventor from an accepted and a rejected project are denoted by U_a and U_r respectively, where $U_a \geq U_r$, and U's are exogenous parameters.[16] We make this assumption partly for simplicity, but it is also true that, in the present problem, pecuniary reward to inventors can be linked only tenuously with the actual values of projects.[17]

We visualize the project creation process as follows. There are a large number of inventors. Each of them selects a technology of invention from a family of such technologies. A particular technology determines the number of projects which will be generated, and it also determines the statistical distribution from which these projects would be drawn. If inventors are identical then, in a symmetric equilibrium, they will choose the same technology and the resulting collection of projects will constitute the initial project portfolio.

One can therefore represent the inventor's choice as being exercised through parameters (N, β), where N is the number of projects and the vector β influences the pdf of the initial project portfolio. The pdf, thus, is represented as $g(x, \beta)$. The maximand of inventors can then be written as

$$\text{(19)} \qquad N[\int U_a f^s g(x, \beta)dx + \int U_r(1 - f^s)g(x, \beta)dx],$$

which can be rearranged in a simpler form as

$$\text{(20)} \qquad Nn^s + NU ,$$

where $U = U_r/(U_a - U_r)$, and $n^s = \int f^s g(x, \beta)dx$.

A heuristic way to interpret the above expression is as follows: Inventors gain a bonus of one dollar for each project selected and a fixed fee of U dollars for each project invented. Two special cases are worth noting. If there is no fixed fee (that is, $U_r \to 0$, and the second term in (20) drops out) then the inventors maximize the number of projects selected. On the other hand, if there is no bonus (that is, $U_a = U_r$, and only the second term in (20) matters) then the inventors maximize the number of projects invented. These responses are in accord with the intuition.

Further, the combinations of N and β that can be chosen is restricted by what is feasible. We represent the technological feasibility as

$$(21) \qquad (N, \beta) \in T$$

where T is the technologically feasible set of N and the elements of β.[18] Maximization of (20), under the constraint (21) on the choice variables, yields N^s and β^s which, in turn, characterize the initial portfolio of projects available to each of the two systems.

B. <u>Two Types of Projects</u>

In this case, the inventors' choice variables are the numbers of good and bad projects, which are denoted by N_1 and N_2. From (20), the maximand of inventors in the system s is

$$(22) \qquad (f_1^s N_1 + f_2^s N_2) + (N_1 + N_2)U,$$

in which recall that f_1^s and f_2^s are respectively the probabilities that a good and a bad project will be accepted in the system. These probabilities are related to the screening probabilities, p_1 and p_2, through (1) and (2). Further, the constraint (21) can now be represented in a simple form as

$$(23) \qquad N_2 = N_2(N_1).$$

Clearly, we need to consider only those cases in which there is a trade-off between N_1 and N_2; otherwise the two systems have identical initial portfolios.

Dividing (22) by a constant, $f_2^s + U$, the maximand of inventors can be written as

$$(24) \qquad F^s N_1 + N_2 ,$$

where, $F^s = (f_1^s + U)/(f_2^s + U)$. The above maximand implies that the number of good projects in a polyarchy is larger (or smaller) than that in a hierarchy if F^P is greater (or smaller) than F^H. To examine this issue further, we obtain the following from (1) and (2).

$$(25) \qquad F^P - F^H \sim 2(p_1 - p_2)[(1 - p_1 - p_2) U - p_1 p_2]$$

Now, recall that $p_1 > p_2 > 0$. Therefore, the expression (25) is positive if: $1 > p_1 + p_2$, that is, the screening probabilities fall in the region OAD of Figure 4, and if

$$(26) \qquad U > p_1 p_2/(1 - p_1 - p_2).$$

Therefore, the initial portfolio in a polyarchy contains a larger number of good projects (compared to that in a hierarchy) if the screening of projects is tight and discriminating, and if the fixed gains to inventors from the invented projects are significant (so that (26) is satisfied). The reverse holds if the above conditions are not satisfied.[19]

IV. RATIONAL SCREENING RULES

A. Determinants of Screening Rules

Although there are many possible sources of error in screening, the one on which we focus here is that due to errors in the assessment of the value of projects. We posit that the observations on the value of projects are contaminated by random errors, such that a perfect inference is not possible. If the observations which an evaluator makes on a project with profit x are represented by y, then

$$y = y(x, \theta) \tag{27}$$

where θ represents random errors. The screening rule then is a binary function (accept or reject) of observations y.

In addition, the screening rules will depend on the nature of information flows within economic systems. For example, if firms in a polyarchy do not share information, then a firm can not distinguish between the projects which are being evaluated for the first time from those projects which were rejected by the other firm.[20] A firm, therefore, will use the same screening rule for all projects that it receives.

On the other hand, if firms provide to one another their own observations on the projects which they reject, then they might use two screening rules: one for the projects being evaluated for the first time and another for the rest of the projects. In fact, the same could happen even at an intermediate level of information flow in which firms label their rejected projects.

It is important to note here that the architecture of the economic system itself conveys some information to its constituents, which they will use in setting Bayesian decision rules. For example, even when there is no information sharing, the two firms would not act independently, since each firm knows that some of the projects it receives are those rejected by the other firm and, consequently, the portfolio of projects faced by a firm is not a replica of the initial portfolio, but it has been modified by the other firm.

Similarly, in a hierarchy, an important question is whether the upper bureau uses the entire information collected in the organization (i.e., observations made by both bureaus) in its own screening rule, or that only a part of the entire information is available at the top. In the case of no information sharing between the bureaus, the upper bureau's screening will be based on its own information, and on the knowledge of how the portfolio has been modified by the lower bureau, but not on what value was observed by the lower bureau for a particular project.[21]

In the section below we examine the case in which there is noinformation sharing between screens in either of the two economic systems. This is partly for simplicity, but it also represents a base case which brings out the trade-offs which would remain important regardless of the nature of information flows.

B. Cut-off Levels for Observed Project Values

Project evaluators often use cut-off levels for screening; a project is accepted if its observed profit is above the cut-off level, and it is rejected otherwise. Suppose that every screen makes one scalar observation; that the errors in the observations made by different screens are identically and independently distributed; and that errors are additive,[22] i.e.,

$$y(x, \theta) = x + \theta. \tag{28}$$

Denote the pdf of θ by $\ell(\theta)$, and the corresponding cdf by $L(\theta)$. Then

$$p(x, R) = \mathrm{Prob}[y(x, \theta) > R] = 1 - L(R - x), \text{ and} \tag{29}$$

$$p_x(x) = -p_R(x) = \ell(R - x) \geq 0, \tag{30}$$

where $p_R = \frac{\partial p}{\partial R}$.

In a polyarchy, denote the two firms by superscripts i and j, such that i, j = 1 and 2; and $i \neq j$. If R^i is the cut-off level for firm i, then $p^i \equiv p(x, R^i)$ denotes the corresponding screening function. For firm i, then, the probability that a given project with profit x will arrive to be evaluated for the first time is 1/2, and the probability that the same project will arrive after being rejected from firm j is $(1 - p^j)/2$. Since these two kinds of arrivals can not be distinguished, the probability that a given (undifferentiated) project with value x will arrive at firm i is

$\frac{1}{2}(2 - p^j)$, and the probability that it will be selected is $\frac{1}{2}p^i(2 - p^j)$. The expected profit of firm i is[23]

$$(31) \qquad \phi^{iP} = \frac{1}{2} E[xp^i(2 - p^j)],$$

in which we have suppressed the number of initial projects, N.

The firms maximize the above with respect to R^i, taking R^j as given. In a symmetric equilibrium, the optimal R^1 and R^2 will be the same, which we denote by R^P. The unique internal optimum, R^P, is obtained from

$$(32) \qquad E[xp_R(2 - p)] = 0$$

In a hierarchy, let R^1 and R^2 denote the cut-off levels for the lower and upper bureau respectively. The profit is represented as

$$(33) \qquad \phi^H = E[xp^1p^2]$$

Its derivatives with respect to R's are

$$(34) \qquad \frac{\partial \phi^H}{\partial R^i} = E[xp_R^i\, p^j],$$

for i = 1 and 2.

Now, from (33), ϕ^H remains unchanged if R^1 and R^2 are interchanged by one another. It follows therefore that <u>if the optimal cut-off levels are unique, then a hierarchy uses the same cut-off level in both bureaus.</u>[24] Denote this cut-off point by R^H. If this optimum is internal, then it is characterized by

(35) $E[xpp_R] = 0$

<u>**Selection Externality**</u>: Consider for a moment, a hypothetical situation in which the cut-off levels for firms in a polyarchy are being set to maximize the economy's total profit. In this case, (31) is maximized with respect to R^i and R^j. Call this the 'social optimum'. Clearly, this optimum will entail, in general, different cut-off points than what independent firms would set; for example, in the optimum derived earlier. Also, the level of profit in the economy will be higher (or at least no smaller) in the social optimum than in the private optimum. What we note then, is that <u>the selection of projects in a polyarchy might generate an externality</u>, although the firms are identical (even concerning the information they have).[25] <u>This externality could prevent the firms from using those screening rules which maximize the economy's profit</u>. In contrast, <u>there is no selection externality in a hierarchy</u>; since the optimal screening rules maximize the economy's profit.

C. Two Types of Projects

Consider a portfolio consisting of two types of projects. Denote the (random) observations on good and bad projects by y_1 and y_2. A general property of cut-off points is seen as follows. Suppose for a moment that the cut-off point R in a screen is set at the highest observation that a bad project can yield, i.e., $R = \max y_2$. Then, the screen will block the bad projects completely, though several good projects might be rejected as a consequence. It is obvious that there is no gain in setting the cut-off point at a level higher than $\max y_2$, since by doing so one loses additional good projects, without affecting bad projects (which are not being selected in any case).

By a similar logic, it is never advantageous to set R below the lowest observation that a good project can yield, at which level all good projects are accepted.[26] Thus

(36) $$\max y_2 > R > \min y_1$$

The trade-off within the above range of R is obvious: A higher cut-off level leads to fewer bad projects being accepted, but also to more good projects being rejected. For brevity, we refer to the upper and lower limit in (36) as the highest and the lowest cut-off points.

In Appendix II, we have derived explicit solutions for symmetric projects, i.e., $x_1 = x_2 = x$, when the observation errors are uniformly distributed with mean zero. The qualitative results are summarized below:

(i) A lower (higher) proportion of good projects in the initial portfolio corresponds to a tighter (slacker) screening in both systems. For instance, if the proportion of good projects is greater than two-thirds, then both systems adopt the lowest cut-off level, ensuring that no good project is rejected. This certainly implies a significant probability of accepting a bad project, but this screening rule is optimal since there are fewer bad projects in the initial portfolio.

(ii) The screening rule in a polyarchy is more conservative than that in a hierarchy. For example, if the proportion of good projects is less than one-half, then a polyarchy adopts the highest cut-off level, ensuring that no bad project is selected, even though it loses many good projects by adopting this screening rule. In contrast, a hierarchy never finds it optimal to adopt the highest cut-off level.

(iii) A polyarchy has a higher (lower) profit than a hierarchy if the proportion of good projects in the initial portfolio is less (more) than one-half. Obviously, if one hypothesizes (as we did earlier) that unprofitable ideas typically outnumber the profitable ones in a portfolio, then the present model predicts that a polyarchy is a superior institutional arrangement. This result can be understood in two parts. First, as noted above, a small proportion of good projects prompts the decision makers to set tighter screening rules. Second, according to the intuition developed earlier in the paper, tighter screening implies a relatively better performance in a polyarchy.

V. CONCLUDING REMARKS

In this paper, we have proposed a typology of economic systems based on how their constituents are arranged together. We have compared the performance of polar economic systems (decentralized versus centralized)[27]. Errors in individuals' judgment, which are an inescapable feature of human decision making, play a key role in our analysis; since how mistakes add up in an economic system depends critically on how individual decision making units are organized together.

We began this paper by assuming that the nature of individuals' errors is similar in the two different architectures under examination, and focused our attention on what these errors imply for systems' performance. Subsequently, we analyzed how the nature of errors (the screening functions) would differ between the two systems, when individuals use optimal decision rules based on the information available to them.

There are other important reasons as well why the nature of errors associated with different architectures may differ. For instance, it is widely believed that the quality of decision making (i.e., who does what) varies greatly across organizations; some individuals are better able to screen projects than others. Two questions that can then be posed are: (i) How does a system's performance depend on the assignment of individuals of different abilities to different positions within the system, and (ii) How are these decisions affected by the system's architecture.

Decisions about who is to occupy what position within an organization are usually made by other individuals within the system, and these decisions are affected, in turn, by factors which are similar to those which affect decisions on projects. Thus, the architecture of economic systems affects the quality of those who make judgments about people as well as about projects. This provides a basis for analyzing the "rules of succession", and what we call (in Sah and Stiglitz (1984b)) the self-perpetuating aspects of economic systems.

Another characteristic of economic systems which affects their performance is the nature of rewards and punishments meted out to the decision makers. This has, of course, been the question around which much of the recent literature on incentives has focused. In the present paper we have not emphasized this aspect as much as it deserves.[28] This, however, is not because we think that incentive problems are unimportant, but because we think that some of them are rather well understood in the literature, and the new aspects upon which we focus here (for example, aggregation of errors) have received insufficient attention.

On the other hand, we should point out that the architecture of a system may be critical in determining what incentive structures are feasible. For example, in a hierarchical structure, promotions constitute an important part of the rewards, a kind of reward which may not always be desirable in a polyarchy. On the other hand, in polyarchical structures, several parallel units perform similar functions, and it is possible to devise reward structures, based on relative performance. These reward structures have a number of desirable properties concerning incentive, risk, and flexibility,[29] and these may not be feasible in a hierarchical system. Different architectures may also differ in the degree of individual accountability which is feasible within them. One criticism of modern bureaucracies is, for example, that collective decision making makes it difficult to reward and punish bureaucrats individually.

Although we have motivated the present analysis in the context of economic systems, our approach also has implications for the economics of internal organizations. There are two main differences. First, certain kinds of externalities (such as the selection externality we pointed out earlier) might be internalized by a corporation in setting its internal rules. Second, if one internal architecture is better than another (based on whatever the corporate criterion might be), then one might expect a corporation to adopt the better architecture.[30] Whether a similar response arises in an economic system is not obvious, since the theories of societal response have not reached the same level of articulation as those of corporate response.

Moreover, the problems of the design of economic systems are so complex that it might not be reasonable to expect that one would find the best of all possible systems. The analogy to computer architecture is then suggestive: the standard question in this case is not to find the "best" architecture, since it is nearly impossible to find it, but to analyze the properties of alternative structures, with a view to making some possible improvements.

The architecture of an economic system affects the behavior of the organization in other ways as well; some of which have been the subject of extensive study outside economics. Social psychologists have emphasized, for example, that an individual's behavior may differ if he has participated in the decision making process compared to when he has been ordered to undertake a particular task. Though these aspects of human behavior have not traditionally been incorporated into economic analysis, if they are important determinants of <u>economic</u> behavior, e.g., of the effort exerted by individuals or of the quality of their decision making, then they should be.

The above discussion is by no means exhaustive. Yet, it is clear that to pursue all, or even a few, of these facets simultaneously would be nearly impossible. We have chosen, therefore, to examine a few limited aspects in the present paper, and have attempted to pursue other important facets in our subsequent research.

Appendix I

Proportion of Projects Accepted: A direct evaluation of n^s based on (1), (2), (3), and (6) yields

(A1) $$n^P = p_\mu(2 - p_\mu) - p_x^2\sigma^2 \text{ , and}$$

(A2) $$n^H = p_\mu^2 + p_x^2\sigma^2.$$

It is obvious that

(A3) $$\frac{\partial n^S}{\partial p_\mu} > 0, \quad \frac{\partial n^P}{\partial p_x} < 0, \quad \text{and} \quad \frac{\partial n^H}{\partial p_x} > 0.$$

Further, substitution of (6) in (8) leads to

(A4) $$p_\mu(1 - p_\mu) > p_x^2\sigma^2$$

Expressions (A1) and (A2), in conjunction with (A4), yield the following bounds on n^s

(A5) $$p_\mu(2 - p_\mu) > n^P > p_\mu, \text{ and}$$

(A6) $$p_\mu > n^H > p_\mu^2$$

Expected Profit: From (1), (2), (4), and (6), we obtain the following expressions:

(A7) $$\phi^P = \mu[p_\mu(2 - p_\mu) - p_x^2 \sigma^2] + [2(1 - p_\mu)p_x \sigma^2 + p_x^2 \sigma^3 \eta],$$

(A8) $$\phi^H = \mu[p_\mu^2 + p_x^2 \sigma^2] + [2p_\mu p_x \sigma^2 - p_x^2 \sigma^3 \eta], \text{ and}$$

(A9) $$\Delta Y = 2N\mu[p_\mu(1 - p_\mu) - p_x^2 \sigma^2] + 2N[(1 - 2p_\mu)p_x \sigma^2 + p_x^2 \sigma^3 \eta],$$

since $\Delta Y = N(\phi^P - \phi^H)$.

Expressions (A9), (14a) and (14b) yield

(A10) $$\frac{\partial \Delta Y}{\partial \eta} > 0, \text{ and}$$

(A11) $$\frac{\partial \Delta Y}{\partial \sigma} > 0, \ \frac{\partial \Delta Y}{\partial p_\mu} < 0, \text{ and } \frac{\partial \Delta Y}{\partial p_x} > 0, \text{ provided } p_\mu < \frac{1}{2}.$$

The derivatives of ϕ^P with respect to the parameters can be signed directly. Signing the derivatives of ϕ^H is easier if (A4) is used.

APPENDIX II

Optimal cut-off levels: From (4), the expected profit in the system s is given by

$$\text{(A12)} \qquad Y^s = Nx[f_1^s \alpha - f_2^s(1 - \alpha)].$$

The constant Nx can be dropped from (A12) since it does not play any role in the analysis below. Now, since the two firms in a polyarchy set the same cut-off point; from (1), $f_k^P = p_k(2 - p_k)$, for k = 1 and 2. Also, it turns out in the present case that the two bureaus in a hierarchy set the same cut-off level. Thus, from (2), $f_k^H = p_k^2$. Further, from (29), $p_1 = 1 - L(R - x)$, and $p_2 = 1 - L(R + x)$.

Using these expressions, the internal optima in a polyarchy and a hierarchy are obtained, from (32) and (35), as solutions to the following equations:

$$\text{(A13)} \qquad \frac{\alpha}{1 - \alpha} \frac{\ell(R - x)}{\ell(R + x)} = \frac{1 + L(R + x)}{1 + L(R - x)}$$

$$\text{(A14)} \qquad \frac{\alpha}{1 - \alpha} \frac{\ell(R - x)}{\ell(R + x)} = \frac{1 - L(R + x)}{1 - L(R - x)}$$

Further, if the left hand side exceeds the right hand side in the above expressions, then the cut-off level is at the lower corner of (36). If the opposite is true, then the cut-off level is at the upper corner.

The observation errors are uniformly distributed with mean zero. That is: $\ell = \frac{1}{2u}$, for $u > \theta > -u$; and $\ell = 0$, otherwise. From (36), therefore, the range of R is: $-x + u > R > x - u$. Let $\delta = x/u$. Clearly, δ is less than one from the last inequality, and it is positive. We find that in a polyarchy

(A15) $$R^P = x - u, \text{ and } Y^P = \alpha - (1 - \alpha)(1 - \delta^2), \text{ if } \alpha > \frac{1 + \delta}{2 + \delta},$$

(A16) $$= \frac{x}{2\alpha - 1} - 3u, \text{ and } Y^P = \delta^2\alpha(1 - \alpha)/(2\alpha - 1),$$

$$\text{if } \frac{1 + \delta}{2 + \delta} > \alpha > \frac{2}{4 - \alpha}, \text{ and}$$

(A17) $$= -x + u, \text{ and } Y^P = \delta\alpha(2 - \delta), \text{ if } \alpha < \frac{2}{4 - \delta}.$$

In a hierarchy, on the other hand, the optimum is described as

(A18) $$R^H = x - u, \text{ and } Y^H = \alpha - (1 - \alpha)(1 - \delta)^2, \text{ if } \alpha > \frac{1 - \delta}{2 - \delta}, \text{ and}$$

(A19) $$= u - \frac{x}{1 - 2\alpha}, \text{ and } Y^H = \delta^2\alpha(2\alpha^2 - 3\alpha + 1)/(1 - 2\alpha)^2, \text{ otherwise.}$$

Note the following sufficient conditions concerning the cut-off points. According to (A15) and (A17), a polyarchy uses the lowest cut-off point if $\alpha > 2/3$, and it uses the highest cut-off point if $\alpha < 1/2$. A hierarchy, on the other hand, uses the lowest cut-off point (see (A18)) if $\alpha > 1/2$, but it never uses the highest cut-off point. Also, it is obvious from the above expressions that the cut-off point is raised in both systems (either continuously, or in steps, depending on the region of α) if α goes down.

The relative profit level is calculated as $\Delta Y = Y^P - Y^H$. Take the range: $0 < \alpha < (1 - \delta)/(2 - \delta)$. In this range, expressions (A17) and (A19) hold, and $\Delta Y = \alpha\delta(2 - \delta - \delta\xi)$, where $\xi = (2\alpha^2 - 3\alpha + 1)/(1 - 2\alpha)^2$. Further, in the present range of α, ξ is an increasing function of α. Evaluating ξ at $\alpha = (1 - \delta)/(2 - \delta)$, we find that $\xi = 1/\delta$. Consequently, $\Delta Y > 0$ within this range of α.

Next take the range: $\frac{2}{4 - \delta} > \alpha > \frac{1 - \delta}{2 - \delta}$. In this range, (A17) and (A18) hold, and $\Delta Y = (1 - 2\alpha)(1 - \delta)^2$. Thus, $\Delta Y \gtrless 0$, if $1/2 \gtrless \alpha$. Within the remaining range of α, (A18) holds for a hierarchy, whereas (A15) and (A16) hold for a polyarchy, and it turns out that $\Delta Y < 0$. Therefore, the result: $\Delta Y \gtrless 0$ if $1/2 \gtrless \alpha$ applies to the entire range of α.

FOOTNOTES

1. In fact, quite the contrary view has frequently been put forward: the Lange-Lerner-Taylor equivalence theorem, for example, argues that a market economy and a bureaucratic economy using a price system behave in an essentially identical manner. See Hayek (1935), Lange and Taylor (1964), Mises (1935), among others, on this issue.

2. Of course, this is not the only useful typology. We discuss some other typologies later.

3. We assume that the projects are being evaluated sequentially. The probability of a project being accepted in a polyarchy remains the same, however, even if the projects are being evaluated simultaneously. (An example of simultaneous screening is the evaluation of manuscripts by publishers, in contrast to the sequential evaluation of manuscripts by journal editors.) What differs in these two cases is the cost of evaluation, and the time taken in decision making.

4. This scalar valuation takes into account all of the relevant benefits and costs. Also, we are assuming that the inter-project externalities are not significant (that is, the value of one project does not depend significantly on whether some other projects are undertaken or not), and that there is no restriction on the number of projects that can be undertaken. Of course, this model is not the only one which can be used to compare alternative economic systems. We have investigated two other formulations. In the first, only a fixed number of projects can be

undertaken, e.g., due to a fixed supply of investment capital, and the project evaluators' attempt to select the best projects. In the second, the number of evaluators is fixed, and we determine the best way of organizing them. See Sah and Stiglitz (1984a).

5. We are assuming that the screening functions corresponding to the two screens within a system are identical. This allows us to focus on the effect of architecture; also, as we shall see in Section IV, this turns out to be the case when optimal screening rules are determined in a variety of circumstances.

6. There is not much point in considering portfolios which contain only profitable projects, or only unprofitable projects. It is highly unlikely that real world portfolios have such features. In any event, if there are such portfolios, and if project evaluators know the relevant feature, then the issue of screening disappears altogether.

7. We are abstracting from the screening costs which may differ in the two systems. See footnote 24, however, on this issue.

8. The last condition merely rules out those uninteresting cases in which all projects are either accepted, or rejected.

9. The only uncertainty in the present model is in project acceptance. See Sah and Stiglitz (1984a) for a discussion of project uncertainty and some of its implications.

10. We also assume that the end-points of the projects' distribution are fixed.

11. That is: $G_\beta(x) \leq (\geq) 0$, with strict inequality holding for at least some x, implies an improvement (worsening) in the sense of first-order stochastic dominance. And: $\int^x G_\beta(z)dz \leq (\geq) 0$, with strict inequality holding for at least some x, implies an improvement (worsening) in the sense of second-order stochastic dominance.

12. The center of these hyperbolas is $p_1 = 1/2$, and $p_2 = 1/2$, as is evident in Figure 4. The slopes of their asymptotes are $\pm [\alpha x_1/(1 - \alpha)x_2]^{1/2}$. For a hyperbola like OEA, the length of the transverse axis (which is twice the distance DE) is $[\{(1 - \alpha)x_2 - \alpha x_1\}/4(1 - \alpha)x_2]^{1/2}$. The corresponding length for a hyperbola like AFB is $[\{\alpha x_1 - (1 - \alpha)x_2\}/4\alpha x_1]^{1/2}$. The special case in which $x_1 = x_2$ follows from these expressions.

13. The same approach is useful even when the initial portfolios differ in the two systems. Take the general case in which, for $i = 1$ and 2, N_i^s is the number of projects, and x_i^s is the (positive number) denoting the profit or loss from a project of type i in the system s. Then the center of hyperbolas is given by $p_i = N_i^P x_i^P/[N_i^P x_i^P + N_i^H x_i^H]$. Other relevant details can be readily worked out.

14. The expressions in Appendix I can also be used to ascertain the impact on profit (in each of the two systems) of changes in the initial portfolio (as reflected in the changes in the first three moments, since these are the only relevant parameters), and in the screening function. Sah and

Stiglitz (1983) derive several propositions on the above comparative statics. For brevity, we do not discuss these results here.

15. Explicit expressions for Π^S can easily be derived. Qualitatively, many of the properties of the mean are similar to those of expected profit. See Sah and Stiglitz (1983) for detailed results on the mean and the variance of final portfolios.

16. The assumption that $U_a \geq U_r$ is consistent with there being higher pecuniary rewards to inventors from an accepted project than from a rejected project. It is also consistent with the common observation that inventors partly measure their success in non-pecuniary terms (such as prestige), so that an accepted project is more desirable than a rejected project, even if the financial rewards are independent of project acceptance. The assumption that U's are exogenous parameters implies that the inventors' effort is not significantly sensitive to the parameters of their choice. Also, we assume that the financial rewards are negligibly small compared to the (net) profit in a system. These assumptions can be relaxed by endogenizing inventors' efforts and the pecuniary rewards.

17. Such a link is nearly impossible for projects which are rejected, since it is difficult to ascertain the profit from a project which has never been undertaken. Even for the projects which are accepted, the actual values are known much later in time than when the decisions are made. Moreover, the actual outcomes of accepted projects are often contaminated by other factors over which inventors of projects have little or no

control. Even then, some dependence between pecuniary rewards to inventors and the actual values of accepted projects might be desirable if monitoring costs are small and if there are no impediments to arbitrarily complex contracts. Also, it is possible to link the rewards to the (ex-ante) observed values of projects. A full analysis of such incentive schemes, however, is not attempted in the present paper.

18. If there is a possibility of significant variation in the level of effort, then there would be one such constraint for each level of effort.

19. Depending on parameters of the function (23) and on what the magnitude of U is, one can demarcate the space of acceptance probabilities (p_1 and p_2) into regions in which one system has a higher or lower profit than another. See Footnote 13.

20. It is assumed, however, that a firm has the information whether a project was evaluated earlier by them or not, and the firm uses this information to prevent a project from being evaluated more than once.

21. Which one of the possible information flows actually exists would depend on the costs involved. In addition, information is often contaminated in the process of transfer. (Not only is it generally difficult for one individual to communicate to another all that he knows, but also there are incentive problems associated with whether it is in his interest to do so). These factors are important in both polyarchies and hierarchies. It is not obvious, therefore, that information sharing is more easily achieved within a bureaucracy than among business firms, as is sometimes believed.

22. Each one of these aspects can be generalized and can be made endogenous. The number of observations, for example, would depend on the cost and the technology of observation making.

23. A more complete determination of screening rules will, of course, take into account its impact on the incentives of inventors. For brevity, we do not present here the details of such an analysis.

24. Obviously, this might not hold if screening costs are significant, since in this case: $\phi^H = E[xp^1p^2] - CE[p^1] - C$, where C is the cost of evaluating a project. We discuss the implications of evaluation costs in Sah and Stiglitz (1984a).

25. Some aspects of this externality are parallel to those arising in the context of screening and sorting. Also, note that the statements concerning screening externalities will hold in more general models as well.

26. If y_1 and y_2 do not overlap, i.e., if $\min y_1 > \max y_2$, then the decision problem disappears. This is because one can achieve perfect selection by selecting any R in the non-overlapping region. We do not concern ourselves with situations in which perfect inference is possible.

27. The nature of questions in which we are interested are not new. Ever since Plato (1968) stated the problem of defining a typology of alternative state systems, and that of comparing them, these questions

have been the centers of controversial debates in the literature on political theory. See Popper (1950) for an appraisal of some of the earlier literature concerning state systems. The differences between our approach and the approaches taken in the political theory literature are briefly sketched in Sah and Stiglitz (1984c).

28. For example, our model for determining screening rules can be extended to include the possible effect of individuals' effort on errors, and the effort-reward trade off which is optimal in each of the two systems. Note, however, that the incentive structure may not always affect individuals' performance; it may not take much more effort to make a good decision than a bad one, but it may take much more ability.

29. See Nalebuff and Stiglitz (1983).

30. We have examined here only two polar architectures. More generally, organizations (as well as economic systems) combine hierarchical and polyarchical features. Committees with alternative voting rules and different information structures are simple, but important, examples. Also note that the problem of finding the "best" architecture (given exogenous parameters) is methodologically the same as the one of comparing alternative architectures (which we have adopted in this paper); though, in certain circumstances, it might be possible to use standard optimization techniques by positing parametrically defined architectures.

REFERENCES

Chandler, Alfred D. Jr., Strategy and Structure, Cambridge: The M.I.T. Press, 1962.

Hayek, Frederick A. von, The Present State of the Debate, in Federick A. von Hayek, ed., Collectivist Economic Planning, London: Routledge, 1935.

Lange, Oskar and Taylor, Fred M., On the Economic Theory of Socialism, New York: McGraw-Hill, 1964.

Lindblom, Charles E., Politics and Markets, New York: Basic Books, 1977.

Mises, Ludwig von, Economic Calculation in the Socialist Commonwealth, in Frederick A. von Hayek, ed., Collectivist Economic Planning, London: Routledge, 1935.

Nalebuff, Barry J. and Stiglitz, Joseph E., Information, Competition and Markets, American Economic Review, 1983, 73, 278-283.

Plato, The Republic, Oxford: Clarendon Press, 1968.

Popper, Karl, The Open Society and Its Enemies, Part I - The Spell of Plato, Part II - The High Tide of Prophesy: Hegel, Marx and the Aftermath, Princeton: Princeton University Press, 1950.

Sah, Raaj Kumar and Stiglitz, Joseph E., The Architecture of Economic Systems - I: Hierarchies and Polyarchies, mimeo, Department of Economics, Princeton University, 1983.

Sah, Raaj Kumar and Stiglitz, Joseph E., 1984a, The Architecture of Economic Systems - II: Resource Generation and Resource Constraints in Hierarchies and Polyarchies, mimeo, Department of Economics, Princeton University, 1984.

Sah, Raaj Kumar and Stiglitz, Joseph E., 1984b, Perpetuation and Self-Reproduction of Economic Systems: The Selection and Performance of Managers, mimeo, Department of Economics, Princeton University, 1984.

Sah, Raaj Kumar and Stiglitz, Joseph E., 1984c, Towards a General Theory of Alternative Economic Systems, paper to be presented at the American Economic Association meeting, Dallas, 1984.

Weber, Max, The Theory of Social and Economic Organizations, New York: Oxford University Press, 1947.

Weitzman, Martin L., Prices vs. Quantities, Review of Economic Studies, 1974, 41, 477-491.