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CYCLICAL BEHAVIOR OF PRICES AND QUANTITIES IN THE AUTOMOBILE MARKET

Olivier J. Blanchard

Angelo Melino

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ABSTRACT

This paper has a simple goal, that of understanding the joint behavior of prices and quantities in a particular market. More precisely, it examines whether we can find decision problems for suppliers and buyers, together with a market equilibrium structure, which are consistent with the observed price and quantity time series. Because of the relative homogeneity of the product, of the size of the market, and of the quality of the data, the market chosen is the automobile market.

The first conclusion we reach is that this goal is difficult to achieve. The behavior of prices appears inconsistent with simple -- competitive, monopolistically competitive or monopolistic -- market structures. Prices appear, in a well defined sense, to be too "sticky".

We then consider potential explanations and extensions. None appears completely satisfactory. In particular, the introduction of costs of changing prices does not seem able to explain the joint behavior of prices and quantities.

Olivier J. Blanchard
Dept. of Economics
M.I.T.
Cambridge, MA 02139
(617) 253-8891

Angelo Melino
Institute for Quantitative Analysis
150 George Street
Toronto ON M5S1A1
(416) 978-6541

This paper examines the dynamic behavior of prices and quantities in the automobile market. The macroeconomic tradition has been to estimate reduced form equations, that is price and quantity equations. Research on prices and research on quantities have however proceeded largely independently and, as a result, variables explaining prices are sometimes different from variables explaining quantities. It appears more useful to focus on the supply and demand schedules themselves, or even, to borrow an expression coined by Sargent [1983], to go beyond supply and demand and to determine what decision problems for suppliers and buyers are consistent with the observed price and quantity time series.

Although our interest is ultimately in macroeconomic fluctuations, we have decided to study a particular market. Only by looking at a market in which a relatively homogeneous product is traded can we have some hope of using a reliable specification and reliable data on prices and quantities. The automobile market, or more precisely the North American market for American automobiles, is such a market: good data on production and sales as well as decent data for prices are available monthly for a long period of time.

As automobiles are durable goods, both sides of the market face intertemporal decision problems. Consumers must decide not only whether to purchase a car or not, but also when to purchase it. Faced for example with a decline in prices, they have to assess whether the decline is temporary, in which case they will change the timing of their purchases, or whether it is permanent. Suppliers must decide not only how much to sell, but also how much to produce. Faced with a decline in demand for example, they have to assess whether it is temporary, in which case they may not change their production schedule much, but use inventories instead, or whether it is permanent. Section I formalizes the decision problem of consumers and shows its implications. Section II does the same for suppliers.

We then characterize market equilibrium. The structural model, derived from the first order conditions of consumers and suppliers tells us that current production, sales and prices depend on three sets of variables: they depend on exogenous variables, on lagged state variables, such as the stocks of cars held by suppliers and by consumers, and on their own expected values in the following period. Thus, to perform estimation, we must first derive a reduced form expressing production, sales and prices as a function of the state variables, and of current and lagged exogenous variables. This is done in Section III under the assumption of rational expectations.

Not only does the model tell us which current and which lagged variables enter the reduced form, it has also strong implications for the signs of the coefficients. Thus it tells us how to look at the time series properties of prices and quantities, and what to look for. Unconstrained reduced forms are estimated and presented in Section IV. Most qualitative implications of the model are confirmed except one: There is substantial serial correlation in the residuals of the price equation, thus evidence of what is usually referred to as "price stickiness". This suggests to us two possibilities. The first is that the model is correctly specified but that our price series is a mediocre proxy for the correct price series. The second is that the model is incorrectly specified, that the serial correlation hides the role of prices as lagged dependent variables and that firms really face some form of costs of adjusting either nominal prices, real prices, or mark ups. The rest of the paper examines these two possibilities.

Section V considers the case of measurement error. We may either drop the price equation and do estimation using only quantity equations, or we may assume that the measurement error is uncorrelated with exogenous variables and lagged disturbances, and keep the price equation allowing for measurement error. We perform estimation for both cases. In the first case, dropping the price equation prevents us from learning about absolute convexities of the various components of cost and utility (heuristically, about the absolute slopes of demand and supply curves), but we can still learn about their relative convexities. The structural estimates we obtain are reasonable, and for the most part, consistent with previous studies. In the second case, if we keep the price equation, we can learn about absolute convexities. The estimates imply implausibly small convexities (heuristically, implausibly flat demand and supply curves); this is how the model "explains" the weak effect of state and exogenous variables on prices.

In Section VI, we consider the alternative possibility that prices are correctly measured and do indeed adjust more slowly than predicted by the initial model. We want to see whether the presence of costs of adjustment for prices could explain the behavior of prices and quantities. Of course, even if they did, this would clearly leave open the issue of what these costs really stand for as physical costs of changing prices appear to be very small. As suppliers are now price setters, we cannot characterize the equilibrium as competitive. Thus we characterize suppliers as time consistent monopolists. Section VII estimates the modified model. Perhaps not surprisingly, this model suffers from a problem opposite to that of the original model. As lagged prices are now state variables, we can explain the price equation satisfactorily . The model however implies an effect of lagged prices on both sales and production which does not appear in the data. Section VIII summarizes what we have and have not learned and shows some implications of our estimated structural parameters.

Section I. The Behavior of Consumers

We formalize the automobile market as a market in which consumers buy only American cars and in which suppliers, that is producers and dealers, sell cars only in America. We also assume initially that the market is competitive. The adequacy of these assumptions for the sample period we consider will be discussed in Section IV.

Each consumer faces each period a discrete choice, that of buying or not buying a car. As we do not want to formalize this discrete choice problem, we formalize the behavior of a fictional aggregate consumer. He maximizes the expected present value of utility and his decision problem is stated as follows:[2]

$$\max E\left(\sum_{i=0}^{\infty} \beta^{i} U_{t+i} | \Omega_t\right), \ \beta < 1 \quad \text{where}$$

$$U_t \equiv A_0(C_t, \varepsilon_{0t}) + A_1(X_t, \varepsilon_{1t}) + A_2(S_t, \varepsilon_{2t})$$

$$A_{0c} > 0, \ A_{0cc} < 0; \ A_{1x} > 0, \ A_{1xx} < 0; \ A_{2s} < 0, \ A_{2ss} < 0$$

subject to:

$$X_t = S_t + \Theta X_{t-1} \quad \text{and}$$

$$A_{t+1} = \beta^{-1}(A_t + Z_t - P_t S_t - C_t) \ .$$

where

C_t is consumption, excluding car services, which will, for short, be referred to as "consumption".

X_t is the stock of cars, and by appropriate normalization, the flow of car services.

S_t is the quantity of cars purchased.

P_t is the relative price of cars in terms of C.

Z_t is income.

A_t is financial wealth.

β is equal to both $(1+r)^{-1}$ and $(1+\delta)^{-1}$, where r is the interest rate and δ the subjective discount rate.

ε_0, ε_1, ε_2 are disturbances; each is the sum of three components, a deterministic time trend, a deterministic seasonal and a white noise disturbance.

Ω_t is the information set at time t.

What are the main characteristics of this decision problem?

First, utility is additively separable in time. We exclude interest rate effects, as we assume that the interest rate is constant and equal to the subjective discount rate.

In each period, utility is the sum of three components. The first two give the utility derived from consumption (C) and car services (X), respectively; marginal utility is a decreasing function of each. The third captures costs of adjustment. Although it is not clear what costs of adjustment a consumer actually faces, there is substantial evidence of slow adjustment as would be implied by costs of adjustment (see for example Bernanke [1981] for estimation of a model similar to this one, also for automobiles but using panel data). For convenience, we formalize them as a negative utility of purchases, rather than as a cost in the budget constraint. Marginal disutility is an increasing function of the quantity of cars purchased (S).

Finally, the consumer faces two accumulation equations. The first is for the stock of cars and assumes constant exponential depreciation. Empirical evidence (Wykoff [1973]) suggests that the depreciation rate is higher in the first year than in later years; we have not taken this into account. The second is the accumulation equation for financial wealth.

We allow for disturbances in each of the three utility terms. They, together with supply disturbances will allow us to justify the presence of trends, seasonals and disturbances in the reduced form.

First order conditions

Deriving the first order conditions at time t and rearranging gives two conditions:

$$A_{0c}\left(C_t, \varepsilon_{0t}\right) = E\left[\ A_{0c}\left(C_{t+1}, \varepsilon_{0t+1}\right) \mid \Omega_t\ \right]$$

and:

$$A_{1x}\left(X_t, \varepsilon_{1t}\right) + A_{2s}\left(X_t - \theta X_{t-1}, \varepsilon_{2t}\right) - \theta\beta E\left[\ A_{2s}\left(X_{t+1} - \theta X_t, \varepsilon_{1t+1}\right) \mid \Omega_t\ \right]$$
$$= P_t\, A_{0c}\left(C_t, \varepsilon_{0t}\right) - \theta\beta E\left[\ P_{t+1}\, A_{0c}\left(C_{t+1}, \varepsilon_{0t+1}\right) \mid \Omega_t\ \right]$$

The first condition is the standard one: As the interest rate, in terms of C, equals the discount rate, the level of C must be such that there is no expected change in the marginal utility of consumption.

To understand the second one, consider first the case where A_{2s} is identically equal to zero. Consumers choose a stock X_t so as to equalize marginal utility of car services to the user cost. The user cost is defined as the difference between the current price in terms of marginal utility of consumption, and the expected price, also in terms of marginal utility and allowing for discounting and depreciation. If the marginal utility of consumption is constant, the user cost is simply $P_t - \theta\beta E\left(P_{t+1} \mid \Omega_t\right)$. In the presence of adjustment costs, the desired stock depends not only on the user cost but also on itself lagged and itself expected one period ahead.

Linearized first order conditions

As we want to have a linear model, in order to solve it under rational expectations, we have to linearize the first order conditions. We linearize around sample mean values. Deviations from sample means are composed of both deterministic (trend and seasonal) and stochastic components. Let the same letters as before denote now the stochastic components of the variables, so that S_t for example is now the stochastic component of car purchases. Let us also ignore, for the moment and for notational simplicity, the deterministic components in the linearized first order conditions.

Define:

$$a_1 \equiv -A_{1xx}/A_{0c};\ a_2 \equiv -A_{2ss}/A_{0c};\ a_0 \equiv -\overline{P}\ (1-\beta\Theta)\ A_{0cc}/A_{0c}$$

$$\xi_t \equiv (+A_{1\varepsilon}\ \varepsilon_{1t} + A_{2\varepsilon}\ \varepsilon_{2t} - \overline{P}\ (1-\beta\theta)\ A_{0c}\ \varepsilon_{0t})/A_{0c}$$

we have:

$$(1)\quad (a_1 + (1+\Theta^2\beta)a_2)X_t - \Theta a_2 X_{t-1} - \Theta\beta a_2\ E(X_{t+1}|\Omega t) = -P_t + \Theta\beta E\ (P_{t+1}|\Omega_t) + a_0 C_t + \xi_t$$

$$S_t = X_t - \Theta X_{t-1}$$

The parameters a_0, a_1, a_2 are all positive and measure the curvature of each of the three components of utility. Equation (1) tells us that the stock depends on itself lagged and itself expected next period, on the user cost, on the level of consumption and on a composite disturbance term, which as structural disturbances are white, is also white. The "structural" parameters of the consumer problems are therefore $(\Theta, \beta, a_0, a_1, a_2)$. Heuristically, the level of (a_1, a_2) determines the elasticity of demand to price. More precisely, equation (1) implies that doubling (a_1, a_2) will simply halve the size of the response of purchases to prices, but leave the shape of the dynamic response unchanged. The shape of this dynamic response depends on the ratio of a_1 to a_2.

Section II. The Behavior of Firms[3]

For the sample period we consider, American automobiles are produced by four companies, organized in ten divisions. In each division, manufacturers produce the cars and do not hold substantial inventories; inventories are held by dealers.

Because of data limitations, in particular the absence of reliable price series at the division or the company level, we aggregate all divisions together. Furthermore, we assume that for each division, there is a shadow competitive market between dealers and producers. As a result, we formalize suppliers as one firm taking both production and sales decisions. This firm maximizes the expected present value of cash flows and its decision problem is stated as follows:

$$\max E\left(\sum_{i=0}^{\infty} \beta^{i} \Pi_{t+i} \,|\Omega_{t}\right) , \beta < 1 \quad \text{where:}$$

$$\Pi_{t} \equiv P_{t}S_{t} - W_{t}B_{0}(Y_{t}, \eta_{0t}) - B_{1}(Y_{t}-Y_{t-1}, \eta_{1t}) - B_{2}(I_{t-1} - b_{3}S_{t}, \eta_{2t})$$

$$B_{0y} > 0 , \quad B_{0yy} > 0$$

$$\delta B_{1}/\delta(Y-Y_{-1}) \equiv B_{1y} \gtreqless 0 \text{ as } Y - Y_{-1} \gtreqless 0; \; B_{1yy} > 0$$

$$\delta B_{2}/\delta(I_{-1}-B_{3}S) \equiv B_{2i} \gtreqless 0 \text{ as } I_{-1} - b_{3}S \gtreqless 0; \; B_{2ii} > 0$$

$$b_{3} \geq 0$$

subject to:

$$I_{t} = I_{t-1} + Y_{t} - S_{t}$$

where the new symbols introduced are:

Y_t the production of cars

I_t the (producer-dealers) inventory of cars

W_t the real wage

η_0, η_1, η_2 are disturbances. Each is the sum of three components, a deterministic trend, a seasonal, and a white noise disturbance.

Cash flow is the difference between revenues and costs. There are three types of costs. The first, B_0, is the standard convex cost of production, multiplied by the real wage. The second, B_1, is a convex cost associated with changes in the level of production. The third, B_2, is a convex cost of being away from target inventory; target inventory is assumed to be a linear function of sales, with marginal desired inventory to sales ratio of b_3. These three types of costs have been found to be important in previous studies (Blanchard [1983]). The first two costs imply that, ceteris paribus, the firm would prefer a constant level of production and thus tend to stabilize production. The third cost implies that, ceteris paribus, the firm would prefer to adjust production so as to maintain a constant marginal inventory to sales ratio. This creates an accelerator effect of sales on production and thus tends to destabilize production.

Firms face one accumulation equation, giving the behavior of inventories of new cars. It is assumed that cars do not depreciate until they are sold to consumers.

First order conditions.

Define λ_t as the lagrange multiplier associated with the accumulation equation at time t. λ_t is a shadow price of inventories and is therefore, under our assumptions about dealers and producers, the price at which dealers purchase cars from producers. Deriving the first order conditions at time t gives three conditions:

$$W_t B_{0y}(Y_t,\eta_{0t}) + B_{1y}(Y_t-Y_{t-1},\eta_{1t}) - \beta E\left[B_{1y}(Y_{t+1}-Y_t,\eta_{1t+1})|\Omega_t\right] = \lambda_t$$

$$P_t + b_3 B_{2i}(I_{t-1}-b_3 S_t,\eta_{2t}) = \lambda_t$$

$$\lambda_t = -\beta E\left[B_{2i}(I_t-b_3 S_{t+1},\eta_{2t+1})|\Omega_t\right] + \beta E\left[\lambda_{t+1}|\Omega_t\right]$$

The first equation characterizes production. Consider the case where there are no costs to changing production, so that $B_{1y} \equiv 0$. Then production is such that the marginal cost equals the shadow price of inventories. If B_{1y} is differen from zero, then production depends also on itself lagged and itself expected one period ahead. The second condition characterizes sales. Sales must be such that the sum of the price and the marginal cost of being away from target inventory equals the shadow price. The last condition characterizes the dynamics of this shadow price of end of period inventory. The shadow price equals the expected discounted marginal benefit (which can be positive or negative) of having that level of inventory at the beginning of the next period, plus the expected discounted value of the shadow price next period.

Linearized first order conditions.

We now linearize these first order conditions around the sample means. If we now define:

$$b_0 \equiv \bar{W}\, B_{0yy};\ b_1 \equiv B_{1yy};\ b_2 \equiv B_{2ii};\ b \equiv B_{0y}$$

$$\mu_t \equiv -\bar{W}\, B_{0\eta}\eta_{0t} - B_{1\eta}\eta_{1t} + b_3 B_{2\eta}\eta_{2t}$$

$$\sigma_t \equiv -b_3 B_{2\eta}\eta_{2t}$$

we have, after elimination of λ_t and linearization, and ignoring again, for notational convenience, deterministic components:

$$(2)\quad (b_1(1+\beta)+b_0)Y_t - b_1 Y_{t-1} - \beta b_1 E\left[Y_{t+1}|\Omega_t\right] = P_t - bW_t + b_2 b_3(I_{t-1} - b_3 S_t) + \mu_t$$

$$(3)\quad \beta b_2(1-b_3)(I_t - b_3 E[S_{t+1}|\Omega_t]) + b_2 b_3(I_{t-1} - b_3 S_t) = \beta E\left[P_{t+1}|\Omega_t\right] - P_t + \sigma_t$$

The parameters b_0, b_1, b_2 are all positive and measure the curvature, or convexity of each of the three components of costs. b is also positive and measures the sample mean marginal cost of production. As μ_t and σ_t are linear combinations of structural disturbances which have white noise stochastic components, they also have white noise stochastic components. The elimination of the shadow price λ_t from the first order conditions makes equations (2) and (3) more difficult to interpret.

The "structural" parameters of the firm's problem are therefore $(\beta, b, b_0, b_1, b_2, b_3)$. Heuristically, the level of (b_0, b_1, b_2) determines the elasticity of supply with respect to prices; more precisely, equations (2) and (3) implies that doubling (b_0, b_1, b_2) will halve the size of the response of sales and production to prices, leaving the shape of the dynamic response unchanged. This shape depends on b_3 and the ratios of b_0 and b_1 to b_2.

Section III. Market equilibrium, structural and reduced forms.

Given market equilibrium, and given income for consumers, real wages for firms, and disturbances, our model allows us to solve for the behavior of car sales, car production, car prices, and consumption of non-car services. We shy away from estimating the complete model and characterize only the behavior of car sales, production and prices, given consumption of non-car services, real wages and disturbances. (Taking consumption (C) as given raises econometric issues to which we shall return.)

Market equilibrium is then characterized by the three linearized first order conditions, equations (1) to (3), and the two accumulation equations for stocks, X and I. If b_1 is different from zero, which we shall assume, the system can be written in the following matrix form:

$$\begin{bmatrix} X_t \\ I_t \\ Y_t \\ \text{-----} \\ E(S_{t+1}|\Omega_t) \\ E(Y_{t+1}|\Omega_t) \\ E(P_{t+1}|\Omega_t) \end{bmatrix} = \underset{(6x6)}{A} \begin{bmatrix} X_{t-1} \\ I_{t-1} \\ Y_{t-1} \\ \text{----} \\ S_t \\ Y_t \\ P_t \end{bmatrix} + \underset{(6x2)}{B} \begin{bmatrix} C_t \\ W_t \end{bmatrix} + \underset{(6x3)}{D} \begin{bmatrix} \xi_t \\ \mu_t \\ \sigma_t \end{bmatrix}$$

The elements of the matrices A, B, D depend on the structural parameters, namely $\alpha \equiv (\Theta, \beta;\ a_0, a_1, a_2, b_0, b_1, b, b_2, b_3)$. The matrices are given in Appendix A. The first two equations are the accumulation equations; the third is simply the identity $Y_t = Y_t$, introduced for convenience. The next three equations are the first order conditions, which give (S_t, Y_t, P_t) as functions of their expected value in the next period, of the state variables X_{t-1}, I_{t-1}, Y_{t-1}, of the current values of C_t, W_t and of the disturbances ξ_t, μ_t, σ_t. We shall refer to this matrix system as the structural form of the model. To do estimation, we must now solve it to obtain an observable reduced form. We proceed in two steps.

Derivation of the reduced form.

The first step is to derive (S_t, Y_t, P_t) as functions of the state variables $(X_{t-1}, I_{t-1}, Y_{t-1})$, of current and expected future values of C, W and of current disturbances. This is done as follows (this part relies on Blanchard-Kahn [1980]):

Partition A, B, D such that:

$$A = \begin{bmatrix} \underset{(3\text{x}3)}{A_{11}} & \underset{(3\text{x}3)}{A_{12}} \\ \underset{(3\text{x}3)}{A_{21}} & \underset{(3\text{x}3)}{A_{22}} \end{bmatrix}, \quad B = \begin{bmatrix} \underset{(3\text{x}2)}{B_1} \\ \underset{(3\text{x}2)}{B_2} \end{bmatrix}, \quad D = \begin{bmatrix} \underset{(3\text{x}3)}{0} \\ \underset{(3\text{x}3)}{D_2} \end{bmatrix}$$

and assume that the information set Ω_t includes at least current and lagged values of all variables in the above matrix system.

Let Π and J be the eigenvector and the eigenvalue matrices associated with A. Order the diagonal elements of J by increasing absolute value. Partition J and Π conformably to A so that:

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_2 \end{bmatrix} \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix}$$

If (C,W) follows a stationary process, (S,Y,P) will follow a stationary process if and only if A has exactly three roots on each side of the unit circle. We know that this condition holds in this case because the market solution is equivalent to the solution of a central planning problem (this equivalence will not hold when we extend the model; whether the condition holds will then become an empirical question). In this case, the solution is given by:

$$(4)\quad \begin{bmatrix} S_t \\ Y_t \\ P_t \end{bmatrix} = -\Pi_{22}^{-1}\Pi_{21}\begin{bmatrix} X_{t-1} \\ I_{t-1} \\ Y_{t-1} \end{bmatrix} - \Pi_{22}^{-1}\sum_{i=0}^{\infty} J_2^{-i-1}(\Pi_{21}B_1 + \Pi_{22}B_2)E\left(\begin{bmatrix} C_{t+i} \\ W_{t+i} \end{bmatrix} \Big| \Omega_t\right)$$

$$- \Pi_{22}^{-1} J_2^{-1}\Pi_{22} D_2 \begin{bmatrix} \xi_t \\ \mu_t \\ \sigma_t \end{bmatrix}$$

The next step is to solve for the unobservable expectations of future C and W. We make the following joint hypothesis: (1) The information set Ω_t includes only current and lagged values of the variables in the structural form. (2) C_t and W_t are uncorrelated with current and lagged disturbances ξ_t, μ_t, σ_t.

We know that this joint hypothesis cannot be exactly correct. In particular, we know that consumption will in general depend on the utility disturbances, ε_0, ε_1, ε_2, of which ξ is a linear combination. Whether this implies a large correlation between C and ξ depends on the relative size of the variances of the ε's and the consumer "structural" parameters. We shall maintain the hypothesis because it might hold approximately and is convenient. We shall however test the exogeneity of (C,W) with respect to (S,Y,P). Rejection of exogeneity would imply rejection of the joint hypothesis.

Under the joint hypothesis, the projection of C_{t+i} and W_{t+i} on Ω_t is the same as the projection on current and lagged values of C_t and W_t. We assume that (C_t, W_t) has a kth order autoregressive bivariate representation, which we write directly in quasi-first order form:

Define $\phi'_t = [C_t, W_t]$ and $\Phi'_t = [\phi'_t \cdots \phi'_{t-k+1}]$

then:

$$\begin{bmatrix} \phi_t \\ \vdots \\ \\ \phi_{t-k+1} \end{bmatrix} = \begin{bmatrix} H_1 & \cdots & H_k \\ & & 0 \\ & I & \vdots \\ & & 0 \end{bmatrix} \begin{bmatrix} \phi_{t-1} \\ \vdots \\ \\ \phi_{t-k} \end{bmatrix} + \begin{bmatrix} u_t \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

or using more concise notation:

(5) $\Phi_t = H\,\Phi_{t-1} + U_t \; ; \qquad \phi_t = [I\ 0..0]\Phi_t \equiv \delta\,\Phi_t$

then:

$$E\,(\Phi_{t+i}|\Omega\ ) = E\,(\Phi_{t+i}|\Phi_t) = H^i\,\Phi_t$$

Replacing in (4) gives the following observable reduced form:

(6)
$$\begin{bmatrix} S_t \\ Y_t \\ P_t \end{bmatrix} = -\Pi_{22}^{-1}\Pi_{21} \begin{bmatrix} X_{t-1} \\ I_{t-1} \\ Y_{t-1} \end{bmatrix} -(\Pi_{22}^{-1}\sum_{i=0}^{\infty} J_2^{-i-1}(\Pi_{21}B_1 + \Pi_{22}B_2)\delta\ H^i)\,\Phi_t$$

$$- \Pi_{22}^{-1} J_2\,\Pi_{21} D_2 \begin{bmatrix} \xi_t \\ \mu_t \\ \sigma_t \end{bmatrix}$$

The observable reduced form gives (S_t, Y_t, P_t) as a function of $(X_{t-1}, I_{t-1}, Y_{t-1})$, current and lagged values, up to (k-1) lags, of C_t and W_t, and of white noise disturbances. What are the restrictions imposed by the model?

If, as we assume, there are no restrictions on the contemporaneous covariance matrix of structural disturbances, then there are no restrictions on the contemporaneous covariance matrix of the disturbances of the reduced form.

We have, for notational convenience, ignored the deterministic components. They are however present in the reduced form. Again, our assumption that disturbances may have different trends and have unconstrained seasonal components implies that the structural model imposes no restrictions on trends or seasonals in the reduced form.

The restrictions of the model are on the coefficients of $(X_{t-1}, I_{t-1}, Y_{t-1})$ and Φ_t. The coefficients on the state variables are function only of the set of structural parameters, $\alpha \equiv (\beta, \theta;\ a_0, a_1, a_2, b, b_0, b_1, b_2, b_3)$. The coefficients on current and lagged C and W depend on both α and H. This dependence is clearly a complicated one.

From reduced form to structural parameters: identification.

Given the non-linearity of the mapping from structural parameters to the reduced form, we must discuss the issue of identification. Parameters of H, that is of the (C,W) process, are clearly identified, as (5) can be estimated directly. We shall assume values for θ and β of .98 in both cases.[4] The issue is thus identification of $(a_0, a_1, a_2, b, b_0, b_1, b_2, b_3)$. This was studied as an example in another paper on identification (Blanchard [1982]) and the results are reported here:

All parameters in $(a_0, a_1, a_2, b, b_0, b_1, b_2, b_3)$ are almost always identified (the rank condition may not be satisfied if some of them have value of zero). This remains true for all parameters except a_0 and b if coefficients on current and lagged C and W are left unconstrained.

If we ignore the price equation and estimate only the production and sales equations, b_3 is still identified but $(a_0, a_1, a_2, b, b_0, b_1, b_2)$ are identified only up to a scale factor. That is, two markets in which these parameters differ only by a scale factor will, if we limit ourselves to quantities, be observationally equivalent. A lower scale factor corresponds to "flatter demand and supply curves": it will imply less movement in prices but will not affect the behavior of quantities.

Section IV. Evidence from the Reduced Form

Before turning to the reduced form evidence, we briefly describe the data, describe our treatment of deterministic components and test for exogeneity of C and W.

The Data

Our assumption that endogenous and exogenous variables follow stable stationary processes is justified only in the absence of major changes in market structure. Thus, we start the sample period in 1966-1, after a major reorganization in production, and end in 1979-12, before (or at least not long after the beginning of) the large increase in the share of imports.

As production is rather arbitrarily distributed between the U.S. and Canada, we must look at North America as a whole.[5] Our production and sales series are thus for North America. Our theoretical model ignores the possibility that cars may be sold outside of America. In fact, during the sample period, sales outside of America averaged 1.8% of total sales, with no apparent trend in this ratio. Thus, in our empirical work, we do not explicitly include these sales; we treat them as if they were sales to American consumers.

As we do monthly estimation, we need monthly series for the other variables. These series are however available for the U.S. and not for Canada. We therefore use U.S. series as proxies for American series. The price series is the new car price component of the CPI, constructed by BLS, which attempts to measure the transaction price rather than the list price; in particular it takes into account dealer concessions, either in the form of discounts or over-allowance of trade-ins; the deflator we use to get a relative price is the PCE deflator.

C is personal consumption expenditures; although C is conceptually consumption of non-car services, we have preferred to make no adjustment to the series, as no simple adjustment is available. Finally, W is the automobile industry real wage, in terms of the PCE deflator.

A detailed description of the data and data sources is given in appendix B.

Deterministic components

All equations, those of the reduced form and those describing the (C,W) process have unconstrained seasonal dummies and time trends. Time trends are assumed to be cubic in time. Sales and production display practically no time trend. The cubic term is important only for the price series: Relative prices steadily decline until approximately 1973 and their trend value appears constant since then.

There is one additional component that was not considered in the model but appears in the data: there were four major strikes during the sample period. A complete treatment would formalize when and how they were anticipated; we stop short of doing this and simply allow for dummies for all months of each strike, as well as for the month preceding and the month following each strike. Coefficients on these dummies are of some interest and are reported in appendix B.

Exogeneity tests

We have seen that under the joint hypothesis of the last section, C and W should be exogenous with respect to (S,Y,P). More precisely, the joint hypothesis implies $G(L) = 0$ in:

$$\begin{bmatrix} C \\ \\ W \end{bmatrix} = H(L) \begin{bmatrix} C-1 \\ \\ W-1 \end{bmatrix} + G(L) \begin{bmatrix} S-1 \\ Y-1 \\ P-1 \end{bmatrix} + u$$

We consider lag lengths of 4, 6, 8, 10, 12 and consider two test statistics. The first is the likelihood ratio test statistic $\lambda = T \quad \ln (\ |\hat{\Sigma}_R| \ / \ |\hat{\Sigma}_u| \ )$ and the other is $\lambda' = (T-K) \quad \ln (\ |\hat{\Sigma}_R| \ / \ |\hat{\Sigma}_u| \ )$, where K is the number of coefficients in each equation of the unconstrained model. It has been suggested that if K is large in relation to T, λ' is more reliable (Nelson and Schwert [1983]). We find λ to be significant at all lag lengths, λ' to be insignificant, except marginally at lag length of 4. Relying on λ', we decide to maintain the assumption of exogeneity. Detailed results and a further discussion of the properties of λ and λ' are given in Appendix C. Based also on likelihood ratio tests, we choose a lag length of 4 to characterize the process for (C,W).[6]

<u>Reduced form evidence</u>

Table 1 gives the reduced form regressions for (S,Y,P), each of them estimated by OLS. Given the assumption of a lag length of 4 for (C,W), the reduced form includes current and up to 3 lagged values of C and W. The coefficients on state variables depend only on structural parameters and thus, the model strongly suggests signs for these coefficients[7]. The coefficients on the exogenous variables are functions of both the structural parameters and the parameters of the (C,W) process; the model still suggests likely signs, on the sums of coefficients on C and W for example.

Table 1. Unconstrained reduced form estimation. Original model.

		X_{-1}	I_{-1}	Y_{-1}	C	$\sum_{i=1}^{3} C_i$	W	$\sum_{i=1}^{3} W_i$	R^2	SSR	D.W.
S mean:	7.72	-.03	.00	.25	6.3	-1.2	.00	.00	.89	28.7	1.65
(OLS)		(.01)	(.04)	(.05)	(1.0)		(.02)				
Y mean:	7.74	-.03	-.32	.41	3.0	.4	-.01	.08	.88	64.5	1.98
(OLS)		(.01)	(.06)	(.08)	(1.5)		(.04)				
P mean:	87.48	-.15	-.07	.14	-2.3	.1	.27	.18	.99	110.1	.73
(OLS)		(.02)	(.08)	(.10)	(1.9)		(.06)				(ρ)
(P)		-.12	-.22	.15	-.6	1.3	.10	.20	.99	44.7	.80
(GLS)		(.04)	(.10)	(.07)	(1.17)		(.04)				
Mean		379.2	19.48	7.74	7.44		112.73				

Is the sign consistent with predictions of the model?

	X_{-1}	I_{-1}	Y_{-1}	C	W
S	Yes	---	Yes	Yes	---
Y	Yes	Yes	Yes	Yes	Yes
P	Yes	Yes	No	No	Yes

Period of estimation: 1966-5 to 1979-12 Monthly dummies, strike dummies, t, t^2, t^3.

Consider first the two quantity equations. The signs on the coefficients of the state variables are all consistent with the model. High producers' inventories depress production and increase sales, both tending to reduce the initial high level of inventories. High consumers' inventories depress both purchases and production. High levels of production in the previous month increase production this month, because of costs of changing production; they also lead firms to increase sales. The signs of the sums of coefficients on consumption are also consistent with the model: Higher consumption leads to higher sales and production. Real wages have no noticeable effects on either sales or production.

The price equation is however in substantial contradiction with the model. Consumers' and dealers' inventories decrease the price, an effect consistent with the model. The effect of lagged production, as well as the effect of consumption are inconsistent with the model. The main problem is however the low Durbin-Watson, indicative either of serial correlation unexplained by the model, or of misspecification.

Table 1 presents us with a problem and a puzzle. <u>The problem is the inconsistency of the price equation with the model</u>. <u>The puzzle is the inconsistency of estimated price and quantity equations</u>. How can there be serial correlation in the price but not in the quantity equations? How can high values of lagged production lead to both higher sales and an increase in the price of cars? We can think of three possible explanations, which we consider in turn before turning to structural estimation of the models implied by two of them.

How to reconcile observed price and quantity behavior?

The first possibility is to allow for serial correlation in the structural disturbance terms. Although this would in general lead to serial correlations in all reduced form equations, combinations of structural parameters and serial correlation coefficients may produce serial correlation only in the price equation.[8] We do not find this extension particularly attractive. "Explaining" serial correlation in prices by unexplained serial correlation of disturbances does not appear useful. We do not consider this direction further.

The second is suggested by the consistency of the quantity equations with the model. It is simply that prices are measured with error. This is clearly an easy way out but not a totally convincing one. The price series is carefully constructed. It responds as we expect to lagged consumers' and dealers' inventories, and it responds quite strongly, as appendix A shows, to events such as strikes. We nevertheless consider this direction in the next section.

The third starts from the premise that serial correlation may hide the role of the lagged price as a dependent variable. The lagged price could be a state variable if firms face or perceive costs of adjusting prices. We explore this direction in sections VI and VII.[9]

Section V. Structural Estimation

Efficient estimation implies joint estimation of the process generating (C,W) (equation (5)), and of the constrained reduced form characterizing the process generating (S,Y,P) (equation 6). Since both the parameters in the covariance matrix and the deterministic components are unrestricted we can concentrate the likelihood function in the usual fashion. The concentrated likelihood function depends on the 16 parameters of H and the 8 structural parameters $(a_0, a_1, a_2, b, b_0, b_1, b_2, b_3)$. Joint estimation is still difficult and we use the 2-step method which is simpler but less efficient. We estimate H in equation (5) by OLS and then replace H by the estimated $\hat{H}$ in equation (6). The second step implies therefore maximization over the 8 structural parameters only. One disadvantage of this method is that the reported standard errors would be correct only if H was known exactly, and will therefore understate the true standard errors. Also, likelihood ratio tests of the overidentifying restrictions imposed by the structural model will be biased towards rejecting these restrictions.

In this section, we carry out estimation under the assumption that prices are measured with error.

Estimation without the price equation

As explained in our earlier discussion of identification, dropping the price equation implies that the structural parameters $(a_0, a_1, a_2, b, b_0, b_1, b_2)$ are identified only up to a scale factor. We therefore choose the normalization $b_1 = 1$. The results are reported in table 2.

The upper part reports the constrained and unconstrained reduced forms; the

Table 2. Estimation of structural model using only quantity equations.

Implied reduced form

	X_{-1}	I_{-1}	Y_{-1}	C	$\sum_{i=1}^{3} C_i$	W	$\sum_{i=1}^{3} W_i$
S unconstrained	-.03 (.01)	.00 (.01)	.25 (.05)	6.3 (1.0)	-1.2	.00 (.02)	.00
constrained	-.03	.10	.10	4.4	- .1	.01	.00
Y unconstrained	-.03 (.01)	-.32 (.06)	.41 (.08)	3.0 (1.5)	.4	-.01 (.04)	.08
constrained	-.02	-.24	.43	2.6	- .1	.04	.00

$$\hat{\Sigma}_u = \begin{bmatrix} .175 & .120 \\ .120 & .393 \end{bmatrix} \qquad \hat{\Sigma}_c = \begin{bmatrix} .215 & .124 \\ .124 & .443 \end{bmatrix}$$

$\lambda = -2(L_R - L_u) = 61.80$; $(\lambda - 15)/\sqrt{30} = 8.5$; $(\lambda' - 15)/\sqrt{30} = 5.7$

λ likelihood ratio test statistic

λ' modified likelihood ratio test statistic

Estimated structural parameters

a_0	a_1	a_2	b_0	b_1	b_2	b_3	b
2.01 (.95)	.007 (.003)	2.20 (1.03)	.00 (.00)	1.00 *	.20 (.07)	.13 (.35)	.003 (.078)

* by normalization

unconstrained reduced form is repeated from table 1. The model is formally rejected with high confidence: The likelihood ratio test statistic, and the modified likelihood ratio test statistic are respectively 8 and 5 standard deviations away from their mean under the null hypothesis. Looking at the coefficients however, we see that the structural model is able to replicate them quite accurately: rejection does not come from any single source.[10] Our own conclusion is that the model provides an adequate explanation of the behavior of quantities.

The lower part gives the estimated structural parameters which underlie the constrained reduced form. It is difficult to say just by looking at them whether they are reasonable. We shall study their implications for the dynamic behavior of S and Y in the last section. We may already say that they are reasonable and consistent with existing estimates. The parameters for consumers imply large costs of adjustment but a substantially larger impact of temporary rather than permanent changes in prices. Supply parameters can be compared to those in Blanchard [1983]. Like those, they show no apparent convexity of the cost function but substantial convexity in costs of changing production. The convexity of the cost of being away from target inventory is higher, but the desired marginal target inventory to sales ratio is smaller in the present study.[11]

Estimation with the price equation, allowing for measurement error.

If we assume that the measurement error is uncorrelated with current and lagged values of C and W as well as with lagged disturbances, we can do estimation keeping the price equation but allowing for serial correlation in the price equation. We assume that the measurement error leads to an AR(1)

Table 3. Estimation of structural model with measurement error in the price equation

		X_{-1}	I_{-1}	Y_{-1}	C	$\sum_{i=1}^{3} C_i$	W	$\sum_{i=1}^{3} W_i$	ρ
S	unconstrained	-.03 (.01)	.00 (.01)	.25 (.05)	6.3 (1.0)	-1.2	.00 (.02)	.00	
	constrained	-.03	.10	.09	4.4	- .2	.07	.01	
Y	unconstrained	-.03 (.01)	-.32 (.06)	.41 (.08)	3.0 (1.5)	.4	-.01 (.04)	.08	
	constrained	-.02	-.24	.43	2.7	-.1	.05	.00	
P	unconstrained	-.12 (.04)	-.22 (.10)	.15 (.07)	-.6 (1.17)	1.3	1.0 (.04)	.20	.80
	constrained	.00	.00	.00	.00	.00	.00	.00	.95

$\lambda = -2(L_R - L_u) = 100.32$; $(\lambda-25)/\sqrt{50} = 10.7$; $(\lambda'-25)/\sqrt{50} = 9.7$

$$\hat{\Sigma}_u = \begin{bmatrix} .175 & .120 & .000 \\ .120 & .393 & .010 \\ .000 & .010 & .264 \end{bmatrix}$$

$$\hat{\Sigma}_c = \begin{bmatrix} .216 & .125 & -.006 \\ .125 & .442 & .021 \\ -.006 & .021 & .343 \end{bmatrix}$$

Estimated structural parameters

	a_0	a_1	a_2	b_0	b_1	b_2	b_3	b
10^{-4} x	1.87	.007	2.05	.00	.86	.18	.16	.00

disturbance in the reduced form price equation. The results are reported in table 3.

The estimates of coefficients in the quantity equations of the constrained reduced forms are nearly identical to those given in table 2. Why this is, is clear from the price equation. Prices are entirely "explained" by the serially correlated disturbance and thus give no additional information.

To both fit quantity equations and find no effect of state and exogenous variables on prices, the model must assume "extremely flat supply and demand curves". The structural parameters tell us that this is indeed the case. Their relative values are very similar to those reported in table 2. Their absolute values however are very small and highly implausible.

To summarize, dropping the price equation altogether gives reasonable estimates of relative convexities. Keeping it and allowing for measurement error does not however allow us to explain the behavior of prices.

Section VI. Costs of adjusting prices

The alternative interpretation of the results in table 1 is that serial correlation in the price equation hides the role of lagged prices as state variables.

Three explanations have been suggested in the literature for "price stickiness", or the apparent effect of lagged prices on current prices. The first is that there truly are transaction costs (Mussa [1976]), and thus costs to changing nominal prices. The second is that changes in prices, presumably real prices, are costly because they lead to unfavorable reactions by consumers. The third is that, because of oligopolistic behavior, all firms agree not to change their markups in the face of fluctuations in demand. Each of these three approaches requires a different formalization. The last two have not however been formalized, and indeed appear difficult to formalize. Thus our approach, following Rotemberg [1982], is simply to introduce convex costs of adjusting nominal or real prices, or mark ups and to see whether such costs can explain the data.

Given our introduction of costs of adjusting prices, we have to give up the assumption that the market is competitive. We assume that all firms act as a single monopolist; allowing for monopolistic competition instead would make little difference. This introduces an additional complication: even in the absence of costs of adjusting prices, a monopolist selling a durable good faces a problem of time consistency. (This problem was examined by Bulow [1982]). We assume that the monopolist chooses a time consistent solution, that at any point of time, he chooses a sequence of current and anticipated prices so as to maximize its market value. In turn we assume that consumers understand the rule followed by the monopolist so that the resulting equilibrium is a rational expectation equilibrium.

Characterization of the Maximization Problems

Because the algebra involved is somewhat complicated, it is useful to describe briefly how we derive the reduced form associated with the time consistent solution of the maximization problem.

We first solve for consumers' demand as a function of their lagged stock as well as of current and expected prices. We then solve for the first order conditions of the monopolist who takes as given this demand function. If the monopolist is time consistent, it will each period satisfy the first order conditions for the first period of his current maximization problem.

Thus the set of first period first order conditions of the monopolist, together with the first order condition of the consumer gives us the structural form of the model. This structural form is then reduced to an observable form by the same steps as in Section III. The reader uninterested in technical details can turn directly to the reduced form.

We first derive the demand function. The consumer problem is the same as in Section I and thus the linearized first order condition is still:

$$(a_1 + (1 + \Theta^2\beta)a_2)\, X_t - \Theta a_2\, X_{t-1} - \Theta\beta a_2\, E(X_{t+1} \mid \Omega_t) =$$

$$-P_t + \Theta\beta\, E\,(P_{t+1}, \mid \Omega_t) + a_0 C_t + \xi_t$$

It is convenient to reparametrize it as:

$$(7)\quad (1 + \beta^{-1}\alpha_1^2)\, X_t - \beta^{-1}\alpha_1\, X_{t-1} - \alpha_1\, E(X_{t+1} \mid \Omega_t) =$$

$$\alpha_2\, (-P_t + \Theta\beta E(P_{t+1} \mid \Omega_t) + a_o C_t + \xi_t)$$

where α_1 is the smallest root of:

$$\alpha^2 - ((a_1 + (1 + \Theta^2\beta)a_2) / \Theta a_2)\, \alpha + \beta = 0$$

and $\alpha_2 \equiv (\beta\Theta a_2)^{-1}\, \alpha_1$

This reparametrization allows for an easy factorization which gives the demand function:

$$(8) \quad X_t = \beta^{-1}\alpha_1 X_{t-1} + \alpha_2 \sum_{i=0}^{\infty} \alpha_1^i E(-P_{t+i} + \Theta\beta P_{t+i+1} + a_0 C_{t+i} \mid \Omega_t) + \alpha_2 \xi_t$$

$$(9) \quad S_t = X_t - \Theta X_{t-1}$$

The maximization problem of the monopolist is now:

$$\max \quad E\left(\sum_{i=0}^{\infty} \beta^i \Pi_{t+i} \mid \Omega_t\right), \ \beta < 1 \text{ , where:}$$

$$\Pi_t \equiv P_t(X_t - \Theta X_{t-1}) - W_t B_0(Y_t, \eta_{0t}) - B_1(Y_t - Y_{t-1}, \eta_{1t})$$

$$-B_2(I_{t-1} - b_3 S_t, \eta_{2t}) - B_4(P_t - P_{t-1}, \eta_t)$$

All derivatives are as before and

$$B_{4p} \equiv \delta B_4/\delta(P-P_{-1}) \gtreqless 0 \text{ as } (P-P_{-1}) \gtreqless 0 \ ; \qquad B_{4pp} > 0$$

subject to

$$I_t = I_{t-1} + Y_t - X_t + \Theta X_{t-1} \qquad \text{and}$$

$$X_t = \beta^{-1}\alpha_1 X_{t-1} + \alpha_2 \sum_{i=0}^{\infty} \alpha_1^i E(-P_{t+i} + \Theta\beta P_{t+i+1} + a_0 C_{t+i} \mid \Omega_t) + \alpha_2 \xi_t$$

The objective function is the same as in Section II, except for the convex cost of adjusting real prices. For notational simplicity, we introduce explicitly only costs of adjusting real prices: costs of adjusting nominal prices or mark ups lead to additional cost terms and the extension to these cases is straightforward. The firm faces two constraints, the accumulation equation and the demand function; S_t has been eliminated using (9)

Let λ_{it}, λ_{xt} be the Lagrange multipliers associated with the accumulation equation and the demand equation respectively. The linearized system of first order conditions can be written in the following form (if α_2, b_1, b_2, b_3 are different from zero and if b_3 different from unity, a condition we shall assume to be satisfied):

$$\begin{bmatrix} Y_t \\ I_t \\ P_t \\ X_t \\ \hdashline E(\lambda_{xt+1} \mid \Omega_t) \\ E(\lambda_{it+1} \mid \Omega_t) \\ E(Y_{t+1} \mid \Omega_t) \\ E(P_{t+1} \mid \Omega_t) \\ E(X_{t+1} \mid \Omega_t) \end{bmatrix} = A \begin{bmatrix} Y_{t-1} \\ I_{t-1} \\ P_{t-1} \\ X_{t-1} \\ \hdashline \lambda_{xt} \\ \lambda_{it} \\ Y_t \\ P_t \\ X_t \end{bmatrix} + B \begin{bmatrix} C_t \\ W_t \end{bmatrix} + \text{disturbances}$$

First order conditions and matrices A and B are given in appendix A. The matrices A and B depend on the structural parameters which are now $(\beta, a_0, \alpha_1, \alpha_2)$ for the consumer after our reparametrization and $(\Theta, \beta, b, b_0, b_1, b_2, b_3, b_4)$ for the firm, where $b_4 \equiv B_{4pp}$ is the degree of convexity of the cost of adjusting prices.

This structural form implies that if there exists a unique stationary solution, it is such that $(\lambda_{xt}, \lambda_{it}, Y_t, P_t, X_t)$ depends on the state variables $(Y_{t-1}, I_{t-1}, P_{t-1}, X_{t-1})$, current and expected values of C and W and current disturbances. Such a solution will exist if and only if four roots of A are inside and five roots of A are outside the unit circle; in the present case, we cannot know a priori whether this condition will be satisfied.

This structural form has been derived under the assumption of costs of adjusting real prices. If there are instead costs of adjusting mark ups, that is price-wage ratios, the structural form has as forcing variables not only C_t and W_t but also W_{t-1} and $E(W_{t+1} \mid \Omega_t)$, as in deciding the current mark up firms take into account the lagged mark-up which depends on lagged wages and the expected mark-up next period which depends on the expected wage. If there are costs of adjusting nominal prices, the structural form has as forcing variables C_t, W_t, $\mathbb{P}_{t-1}$, $\mathbb{P}_t$, and $E(\mathbb{P}_{t+1} \mid \Omega_t)$ where $\mathbb{P}_t$ is the price level. The reason is again that nominal prices, which depend directly on $\mathbb{P}_t$, depend also on the lagged and the expected price levels.

Because the price level appears in one of the three cases, we include it in the information set, which includes therefore current and lagged values of C, W and $\mathbb{P}$. Following the same steps as in Section III gives the following reduced form:

$$\text{(10)} \quad \begin{bmatrix} S_t \\ Y_t \\ P_t \end{bmatrix} = \Pi_1 \begin{bmatrix} X_{t-1} \\ I_{t-1} \\ Y_{t-1} \\ P_{t-1} \end{bmatrix} + \Pi_2 \Phi_t + \text{(disturbances)}$$

$$\phi_t' = [C_t, W_t, \mathbb{P}_t], \qquad \Phi_t' \equiv [\Phi_t, \ldots\ldots, \Phi_{t-k+1}]$$

The reduced form includes also two equations for λ_{it} and λ_{xt}. As λ_{it} and λ_{xt} are however unobservable, we do not use these two equations. We have also replaced X_t on the right hand side of (10) by S_t, using $S_t = X_t - \Theta X_{t-1}$, so that the system (10) is more easily compared to the system (6) in Section III. The matrix Π_1 depends only on the structural parameters (β, Θ, a_0, α_1, α_2, b, b_0, b_1, b_2, b_3, b_4,). The matrix Π_2 depends on both structural parameters and on H, the matrix characterizing the joint process of (C, W, $\mathbb{P}$). The reduced form is similar to that of the original model, except for the lagged real price P_{t-1}, which is now a state variable and thus appears in all three equations, and for the presence of the current and lagged price level.

Section VII. Costs of adjusting prices. Empirical Evidence

Consider first the results from unconstrained estimation of the reduced form; the results are reported in the first three lines of Table 4a. The state variables X_{-1}, I_{-1} and Y_{-1} have effects of the expected sign on sales, production and prices; this represents an improvement in comparison to the original model. The state variable P_{-1} has a strong effect on P but no significant effect on the two quantities S and Y. Turning to the exogenous variables, the sign of the sum of coefficients on consumption is as expected in the quantity equations, but not in the price equation. The wage appears to play a role only in the price equation. Finally the price level appears to be significant in the sales and price equations. We cannot tell from this reduced form if this is because it helps predict future consumption and real wages, or because there are costs to adjusting nominal prices. The large negative coefficient in the price equation, which implies that an increase in the price level is associated with a decrease in the real price of cars suggests that the second explanation might be more plausible.

The main characteristic of this estimated reduced form is the significant presence of lagged prices in the price equation but not in the quantity equations. The question is therefore whether there is a set of structural parameters which can generate such a result.

The rest of Table 4a gives the answer to this question. It gives the implied reduced form from structural estimation under the assumption that there are costs of adjusting real prices. The answer is very clearly negative. The modified likelihood ratio test statistic is more than ten standard deviations from the mean under the null hypothesis. The results of structural estimation

Table 4a. Estimation with costs of adjusting real prices. Implied reduced forms.

		X_{-1}	I_{-1}	Y_{-1}	P_{-1}	C	$\sum_{i=1}^{3} C_{-i}$	W	$\sum_{i=1}^{3} W_{-i}$	IP	$\sum^{3} IP_{-i}$	$\hat{\Sigma}$		
Unconstrained Reduced Form	S	-.06	-.01	.27	-.07	6.8	-2.4	.0	.0	.49	-.58	.158	.110	.012
		(.01)	(.04)	(.05)	(.05)	(1.1)		(.0)		(.21)				
	Y	-.04	-.32	.38	.11	4.1	.6	-.04	.04	.44	-.52		.366	.015
		(.02)	(.06)	(.08)	(.07)	(1.7)				(.32)				
	P	-.02	-.05	-.03	.77	1.8	.87	.12	.0	-.55	.54			.267
		(.02)	(.05)	(.07)	(.06)	(1.4)		(.04)		(.28)				
Constrained Reduced Form	S	-.16	.10	.03	-.49	3.4	.35	.05	.01	-.83	.48	.558	.426	.045
	Y	-.09	-.30	.37	-.28	1.4	.23	.03	.00	-.55	.32		.688	.031
	P	-.03	-.19	-.10	.84	.4	.08	.01	.00	-.19	.12			.334

Table 4b. Structural parameter estimates

Consumer:	a_0	α_1	α_2	(a_1)	(a_2)	
	1.76	.96	1.57	.00	.66	
Producer	b	b_0	b_1	b_2	b_3	b_4
	.00	.00	47.2	21.2	.00	209

under the alternative assumptions that there are costs of adjusting nominal prices or mark-ups are nearly identical and are not reported. The models are unable to explain the large effect of lagged prices on current prices without at the same time implying a large negative effect of prices on both sales and production. These results cast substantial doubt on the ability of costs of adjusting prices to explain the joint behavior of prices and quantities.

The structural parameters associated with structural estimation are reported in Table 4b. Given the lack of success of the models in explaining the data, we shall not spend time to discuss them in detail. The parameters characterising consumer behavior are quite similar to those obtained earlier. The parameters characterising producer behavior are quite different. Not surprisingly, given the price equation, they show a very high cost to changing prices.

Section VIII. Summary and Interpretation

Our empirical results allow for two quite different conclusions. If we are willing to believe that the serial correlation in the price equation, as reported in Table 1, is due entirely to measurement error, then it appears that we have found a satisfactory model of the automobile market. If we believe, however, that prices are correctly measured, then we are at a loss to provide an integrated explanation of observed movements in both prices and quantities.

Assuming that prices are badly measured, what do we learn from the structural coefficients reported in Table 2?

Consider first the parameters that characterize the consumer's problem. Holding prices fixed, a permanent increase in consumption, say ΔC, will generate a steady state increase in the desired stock of cars, ΔX, given by

$$\Delta X = \frac{a_2}{a_1 + (1-\theta-\theta\beta+\theta^2\beta)a_2} \Delta C$$

Our parameter estimates imply that a permanent increase in real personal consumption expenditures of $10 billion, holding prices fixed, will lead consumers to increase their desired stock of automobiles by a little under a quarter million cars. This translates into a permanent increase in sales of about 45,000 cars a month. Assuming an average car price of $3,000 (in 1972 dollars), we would estimate that about one sixth of any permanent increase in consumption would be allocated to new car purchases.

The estimated value of a_2 indicates that consumers act as if they faced substantial costs of adjustment. Figure 1 describes the response of new car purchases due to the permanent increase in consumption described above, holding prices fixed. A little less than one half of the difference between initial

Figure 1

PARTIAL EQUILIBRIUM RESPONSE OF SALES TO A PERMANENT INCREASE IN CONSUMPTION

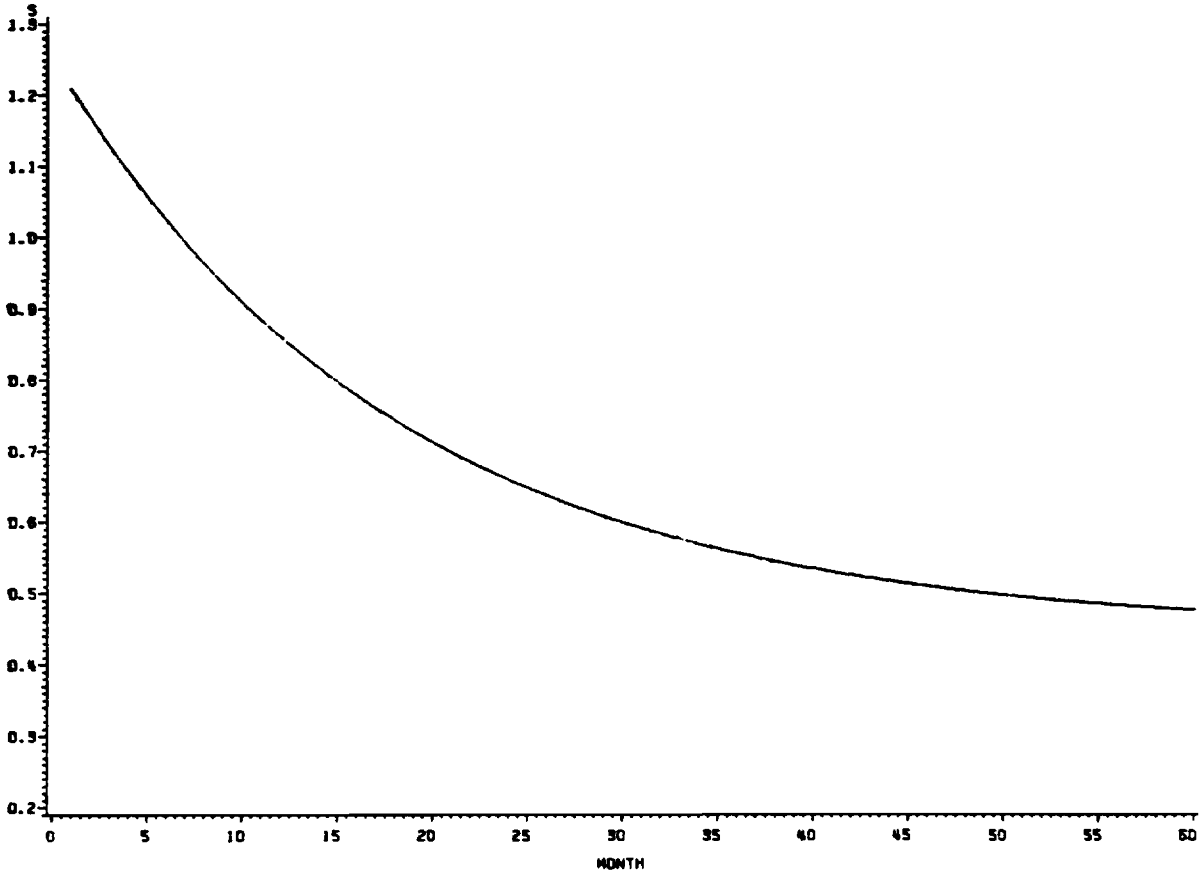
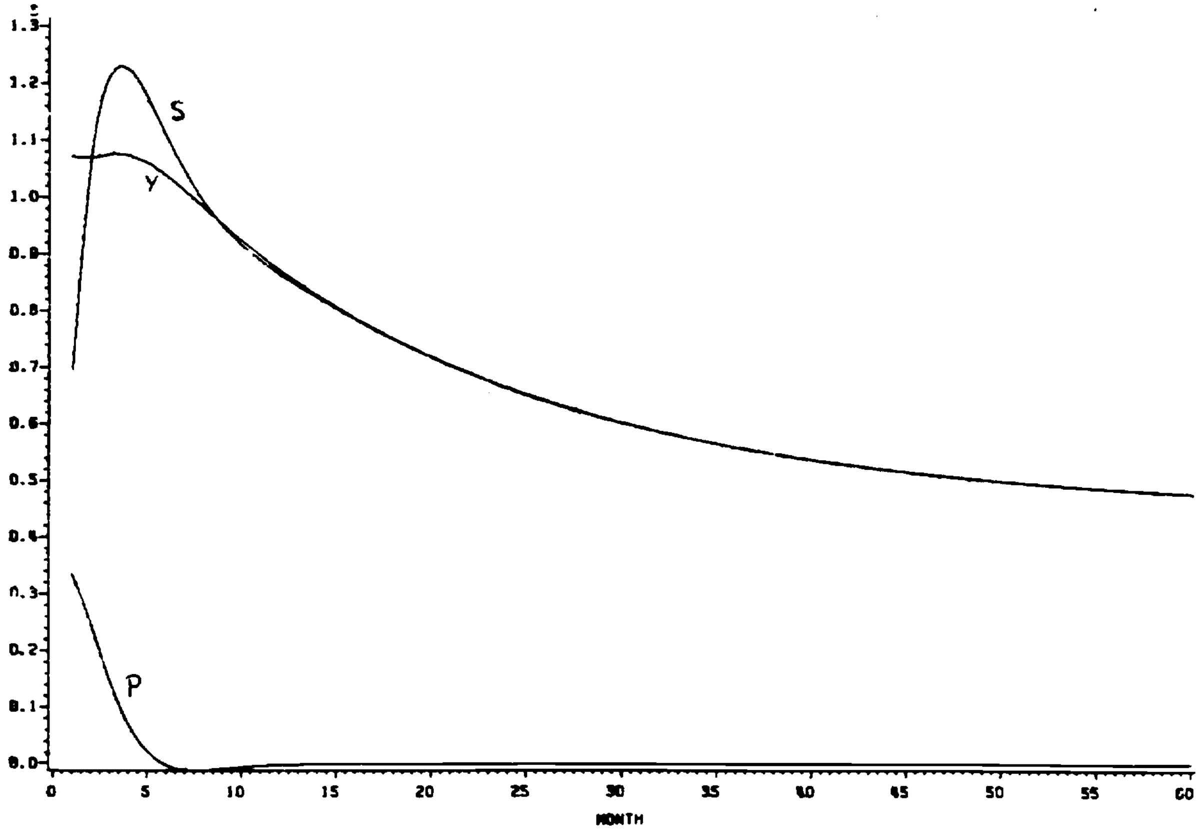


Figure 2

GENERAL EQUILIBRIUM RESPONSE OF (S,Y,P) TO A PERMANENT INCREASE IN CONSUMPTION



stocks and the new steady state value is resolved in the first year. It takes almost five years to complete 95% of the adjustment.

Consider now the parameters which characterize the firms' problem. The estimated cost parameters indicate that although it is costly to change the level of production, the long-run average cost curve is essentially flat. Deviations of inventories from desired levels are costly, and on the margin dealers like to have 113 more cars on their lots in anticipation of a 100 car increase in sales.

Figure 2 describes the general equilibrium response of sales, production and prices to a permanent $10 billion dollar increase in real consumption.

Prices increase temporarily to offset the costs of adjusting production to meet the higher level of sales. As output stabilizes at a new and higher level, these costs of adjustment dissipate and prices fall back to their original level since marginal costs are constant.

Sales jump less than the partial equilibrium model predicts, since consumers anticipate the declining pattern in prices and postpone their purchases somewhat. In the long run, they face essentially constant prices, so the steady state stocks and purchases of automobiles are virtually identical to those predicted by the partial equilibrium model.

Production jumps immediately in response to the higher demand but does not peak until almost one year later. The amplitude of the production response is greater than that of sales, reflecting the need to restore inventories to their new steady state value. In the long run, production increases by just enough to match the steady state increase in sales.

Appendix A

Structural form of the original model:

$$\begin{bmatrix} X_t \\ I_t \\ Y_t \\ E(S_{t+1}\ |\Omega) \\ E(Y_{t+1}\ |\Omega) \\ E(P_{t+1}\ |\Omega) \end{bmatrix} = A \begin{bmatrix} X_{t-1} \\ I_{t-1} \\ Y_{t-1} \\ S_t \\ Y_t \\ P_t \end{bmatrix} + B \begin{bmatrix} C_t \\ W_t \end{bmatrix} + D \begin{bmatrix} \xi_t \\ \mu_t \\ \sigma_t \end{bmatrix}$$

$A = -F^{-1}K$, $B = -F^{-1}G$, $D = -F^{-1}$ where F, K, G are given by:

$$F \equiv \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -(a_1+a_2) & 0 & 0 & \Theta\beta a_2 & 0 & \Theta\beta \\ 0 & \beta b_2(1-b_3) & 0 & \beta b_2 b_3(b_3-1) & 0 & -\beta \\ 0 & 0 & 0 & 0 & -\beta b_1 & 0 \end{bmatrix}$$

$$K \equiv \begin{bmatrix} -\Theta & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ \Theta a_2 & 0 & 0 & 0 & 0 & -1 \\ 0 & b_2 b_3 & 0 & -b_2 b_3^2 & 0 & 1 \\ 0 & -b_2 b_3 & -b_1 & b_2 b_3 & b_1(1+\beta)+b_o & -1 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ a_o & 0 \\ 0 & 0 \\ 0 & b \end{bmatrix}$$

First order conditions for the time consistent monopolist (Section VI):

$$Y_t : \quad W_t B_{oy}(Y_t, \eta_{ot}) + B_{1y}(Y_t - Y_{t-1}, \eta_{1t})$$

$$-\beta E [B_{1y}(Y_{t+1} - Y_t, \eta_{1t+1}) | \Omega_t] = \lambda_{it}$$

$$I_t : \quad \lambda_{it} = -\beta E [B_{2i}(I_t - b_3(X_{t+1} - \Theta X_t), \eta_{2t+1}) | \Omega_t] + \beta E [\lambda_{it+1} | \Omega_t]$$

$$P_t : \quad (X_t - \Theta X_{t-1}) - \alpha_2 \lambda_{xt} - B_{4p} (P_t - P_{t-1}, \eta_t)$$

$$+ \beta E [B_{4p}(P_{t+1} - P_t, \eta_{t+1}) | \Omega] = 0$$

$$X_t : \quad - b_3 B_{2i} (I_{t-1} - b_3(X_t - \Theta X_{t-1}), \eta_{2t})$$

$$+ \beta b_3 E [B_{2i}(I_t - b_3(X_{t+1} - \Theta X_t), \eta_{2t+1}) | \Omega_t]$$

$$= (P_t - \lambda_{it}) - \Theta\beta (E [P_{t+1} | \Omega_t] - E [\lambda_{it+1} | \Omega_t])$$

$$- \lambda_{xt} + \alpha_1 E [\lambda_{xt+1} | \Omega_t]$$

$$\lambda_{it} : \quad I_t - I_{t-1} - Y_t + X_t - \Theta X_{t-1} = 0$$

$$\lambda_{xt} : \quad -\beta^{-1} \alpha_1 X_{t-1} + (1 + \beta^{-1} \alpha_1^2) X_t - \alpha_1 E [X_{t+1} | \Omega_t] =$$

$$\alpha_2 (-P_t + \Theta \beta E [P_{t+1} | \Omega_t] + a_o C_t + \xi_t).$$

Note that to solve for the first order conditions of the firm, we use as a constraint the demand function and not the FOC of consumers. Once the FOC are derived, it is however more convenient to use the FOC of consumers, equation (7), to write and solve the structural model. Linearizing the above FOC, the structural form can be written as:

$$\begin{bmatrix} Y_t \\ I_t \\ P_t \\ X_t \\ E(\lambda_{xt+1}|\Omega_t) \\ E(\lambda_{it+1}|\Omega_t) \\ E(Y_{t+1}|\Omega_t) \\ E(P_{t+1}|\Omega_t) \\ E(X_{t+1}|\Omega\ ) \end{bmatrix} = A \begin{bmatrix} Y_{t-1} \\ I_{t-1} \\ P_{t-1} \\ X_{t-1} \\ \lambda_{xt} \\ \lambda_{it} \\ Y_t \\ P_t \\ X_t \end{bmatrix} + B \begin{bmatrix} C_t \\ W_t \end{bmatrix} + (\text{disturbances})$$

$A = -F^{-1} K$, $B = -F^{-1} G$, where F, K, G are:

$$F = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\Theta\beta\alpha_2 & -\alpha_1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta b_4 & 0 \\ 0 & -\Theta\beta b_3 b_2 & 0 & 0 & \alpha_1 & \Theta\beta & 0 & -\Theta\beta & \beta\Theta b_3^2\ b_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & \beta b_1 & 0 & 0 \\ 0 & -\beta b_2 & 0 & 0 & 0 & \beta & 0 & 0 & \beta b_3 b_2 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$K = \begin{bmatrix} 0 & 0 & 0 & -\beta\alpha_1 & 0 & 0 & 0 & \alpha_2 & (1+\beta^{-1}\alpha_1^2) \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & b_4 & -\Theta & -\alpha_2 & 0 & 0 & -b_4(1+\beta) & 1 \\ 0 & b_3 b_2 & 0 & b_3^2 b_2 \Theta & -1 & -1 & 0 & 1 & -b_3^2 b_2(1+\beta\Theta^2) \\ b_1 & 0 & 0 & 0 & 0 & 1 & -(b_0+(1+\beta)b_1) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 & -\beta b_3 b_2 \Theta \\ 0 & -1 & 0 & -\Theta & 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$

$$G = \begin{bmatrix} -\alpha_2 a_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -b & 0 & 0 \end{bmatrix}$$

Appendix B. Data sources and construction.

Sales and production.

Production data, Y, for U.S. and Canada are obtained by aggregation of divisions (not including Checkers Motors and Volkswagen of America), from Ward's Automotive Reports, weekly, 1965-1979.

Sales data, U.S., from same source

Sales data, Canada, from Statistics Canada CANSIM # D 2369

Sales outside of U.S. and Canada, from Statistics Department, Motor Vehicle Manufacturers Association of the U.S., Inc. Detroit.

Sales, S, are defined as the sum of the three.

Inventories.

Consumers' inventories, X, are computed using the accumulation equation, with $\Theta = .98$. Level chosen so that there is no trend. The choice of $\Theta = .98$ (monthly) implies an annual depreciation rate of .78 which is consistent with the average rate found from studying prices of used cars.

Producers-Dealers inventories, I, computed using the accumulation equation. Benchmark described in Blanchard [1983].

Throughout the paper, quantities of automobiles are expressed in units of 100,000 vehicles.

Prices, Consumption and Wages:

Price, CPI component for new cars, from BLS. (Description in BLS 3400B, 414, 1-3); normalized to equal 100 in 1972: 7.

Wage, Production worker average hourly earnings, in \$, SIC 3711, deflated by PCE implicit deflator, from BLS; normalized to equal 100 in 1972:7.

Consumption, constant dollar personal consumption expenditures, U.S. expressed on an annualized basis in units of 100 billion 1972 dollars.

<u>Coefficients on strike dummies</u> in S, Y, P equations of Table 1.

	S	Y	P		S	Y	P
August 1967	-.17	-1.53	-.76	August 1970	-.73	-.77	.26
Sept.	-.13	-1.56	-.57	Sept.	-.84	-1.47	.44
Oct.	-.65	-2.35*	2.85*	Oct.	-.94	-3.85*	5.10*
Nov.	-.01	- .93	2.73*	Nov.	-1.56*	-2.57*	6.76*
				Dec.	-.81	.83	4.75*
Sept. 1969	1.15*	1.13	-1.78				
Oct.	-.33	- .03	1.55	August 1976	-.22	1.05	.52
Nov.	-.14	- .31	1.65	Sept.	-.55	-1.06	.59
Dec.	.94	.90	1.19	Oct.	-.70	-1.98*	2.30*
				Nov.	-.15	.35	1.62

* : t statistic above 2.

Appendix C. Exogeneity tests.

The test is $G(L) = 0$ in

$$\begin{bmatrix} C \\ W \end{bmatrix} = H(L) \begin{bmatrix} C_{-1} \\ W_{-1} \end{bmatrix} + G(L) \begin{bmatrix} S_{-1} \\ Y_{-1} \\ P_{-1} \end{bmatrix} + u$$

The two test statistics are

$$\lambda = (T \quad) \ell n \; (|\hat{\Sigma}_R|\hat{\Sigma}_u|) \;, \quad \lambda' = ((T-K) \quad) \ell n (|\hat{\Sigma}_R| / |\hat{\Sigma}_u|)$$

where T is the number of observations, K the number of parameters in each equation in the unconstrained case and q the number of restrictions. For large q, $(\lambda - q)/(2q)^{1/2}$ is approximately normal (0, 1). The results of exogeneity tests for various lag lengths are reported in table C1. All tests were conducted over the sample period 1967:1 to 1979:12. If the lag length is ℓ, we have $T = 156$, $K = 32 + 5\ell$ and $q = 6\ell$.

Table C 1. Exogeneity tests of (C, W).

Lag length:	$(\lambda - q)/(2q)^{1/2}$	$(\lambda' - q)/(2q)^{1/2}$
4	5.6	2.6
6	5.7	1.8
8	7.4	1.7
10	7.3	.6
12	8.2	- .2

Since the two asymptotically equivalent test statistics suggested very different conclusions, we investigated the actual small sample distribution of λ and λ'. Taking the least squares estimates (under H_0: $G(L) = 0$) for a lag length of 4 to be true parameters, 500 independent artificial samples of

164 observations were generated. We conditioned in each case on the actual four initial values. The average value of $(\lambda-q)/(2q)^{1/2}$ turned out to be 1.9, and our value of 5.6 was exceeded 8 times, that is 1.6% of the time. The χ^2_1 approximation to the distribution of λ was poor but the approximation to the distribution of λ' was good. These results led us to base our inferences on λ'. It is of some interest to note that the acceptance of exogeneity of (C, W) does not appear to reflect low power of the test procedure. Taking the least squares estimates in the unconstrained case, for a lag length of 4, as the true parameter values, a further 100 independent samples were generated. The statistic $(\lambda-q)/(2q)^{1/2}$ averaged 13.5 and never fell below 8.80.

FOOTNOTES

[1] We have in mind price equations, as summarized for example in the Eckstein [1970] volume, and standard consumption, investment and inventory equations. To define estimated price equations as reduced forms may be unfair to the intent of that research, which usually interprets these as implicit supply schedules.

[2] We prefer to write the initial problem in general rather than quadratic form, and then to linearize the first order conditions. We find the presentation of a decision problem more intuitive if done in this way. An alternative is to start with a quadratic specification. As the estimated model is linear in both cases, the issue is one of presentation, not of substance.

[3] This section borrows heavily from a previous paper (Blanchard [1983]) which studied the supply side in detail. The reader is referred to that paper for a more complete description of the industry and a more complete discussion of the formalization of the decision problem of suppliers.

[4] We need to know the value of Θ to construct the X series. We could in principle estimate β. Numerous recent papers indicate that obtaining accurate estimates of β is difficult. Our choice of .98 for β is arbitrary. Values of β between .95 and 1.00 would affect estimates of other structural parameters very little.

[5] This idiosyncratic allocation of production is due to the Canadian Automobile Agreement of 1965.

[6] The first order condition for consumption (which we do not use) implies that the projection of C on lagged C and W should have zero coefficients on all variables other than C lagged once. We have not imposed this constraint on the bivariate regression.

[7] We cannot however prove that these signs are implied by the model.

[8] In the standard model where demand and supply are functions of the current price and a disturbance, a flat supply curve, a steep demand curve and a serially correlated supply disturbance could for example generate serial correlation in price but not in quantity.

[9] An extension of the initial model which would not help solve the puzzle would be the relaxation of the assumption of a competitive market to allow suppliers to act either as time consistent or time inconsistent monopolists; this would lead to the same specification of the reduced form as in Table 1.

[10] Note that, although an increase in real wages increases cost and thus would tend to decrease sales and production, the effect of W is positive on both S and Y in the constrained reduced form. This is because high values of W imply higher values of C in the future, this leading to higher anticipated demand, to higher sales and production.

[11] We do not know why the results of the two studies differ. They share the same specification of the supply side and the same normalization $b_1 = 1$. The previous

study is partial equilibrium, and does estimation at the division rather than the industry level. Thus the difference could be due either to the use of mediocre instruments in the first study, or to aggregation problems in the second study.

REFERENCES

Bernanke, B. "Permanent Income, Liquidity and Expenditure on Automobiles: Evidence from Panel Data." National Bureau of Economic Research W.P. 756, September 1981.

Blanchard, O.J. "The Production and Inventory Behavior of the American Automobile Industry." Journal of Political Economy, 91-3, (June 1983): 365-400.

-- "Identification in Dynamic Linear Models with Rational Expectations", mimeo, January 1982.

-- and C. Kahn, "The Solution of Linear Difference Models under Rational Expectations", Econometrica, 48-51, (July 1980): 1305-1313.

Bulow, J. "Durable Goods Monopolists" Journal of Political Economy, 90-2, (April 1982): 314-332.

Eckstein, O. (ed.) The Econometrics of Price Determination (Washington, D.C.: Federal Reserve Board, 1972).

Mussa, M. "Sticky Prices and Disequilibrium Adjustment in a Rational Model of the Inflationary Process", mimeo, 1978.

Nelson, C.R. and G.W. Schwert. "Tests for Predictive Relationships Between Time Series Variables: A Monte Carlo Investigation," Journal of the American Statistical Association, 77, (March 1982): 11-18

Rotemberg, J. "Monopolistic Price Adjustment and Aggregate Output", Review of Economic Studies, Vol XLIX (4), no 158: 517-532

Sargent, T.J. "Beyond Demand and Supply Curves", American Economic Review, 72(7), (May 1982): 382-387.

Wykoff, C. "A User Cost Approach to New Automobile Purchases", Review of Economics and Statistics, Vol XL(3) no 123, (July 1973): 377-390.