NBER WORKING PAPER SERIES

OPTIMAL RESERVE MANAGEMENT AND SOVEREIGN DEBT

Laura Alfaro
Fabio Kanczuk

Working Paper 13216
http://www.nber.org/papers/w13216

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
July 2007

We thank Joshua Aizenman, Manuel Amador, Jaewoo Lee, Enrique Mendoza, Julio Rotemberg, two anonymous referees and participants at the Federal Reserve Bank of San Francisco's Annual Pacific Basin Conference and University of São Paulo for valuable comments and suggestions. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

Optimal Reserve Management and Sovereign Debt
Laura Alfaro and Fabio Kanczuk
NBER Working Paper No. 13216
July 2007, Revised October 2008
JEL No. F32,F33,F34,F4

ABSTRACT

To study the joint decision of holding sovereign debt and reserves, we construct a stochastic dynamic equilibrium model that incorporates willingness-to-pay incentive problems. In this setup, debt and assets are not perfect substitutes, as reserves can be used even after a country has defaulted. We calibrate the model to a sample of emerging markets. We obtain that the reserve accumulation does not play a quantitatively important role in this model. In fact, the optimal policy is not to hold reserves at all. This finding is robust to considering interest rate shocks, sudden stops, contingent reserves and reserve dependent output costs.

Laura Alfaro
Harvard Business School
Morgan Hall 263
Soldiers Field
Boston, MA 02163
and NBER
lalfaro@hbs.edu

Fabio Kanczuk
University of São Paulo
R. Dr Alberto Cardoso de Melo Neto 110/131A
Sao Paulo-S.P.-CEP 01455-100
BRAZIL
kanczuk@usp.br

1 Introduction

There is a renewed interest in policy and academic circles about the optimal level of foreign reserves sovereign countries should hold. This recent interest follows the rapid rise in international reserves held by developing countries. In 2006, for example, reserve accumulation amounted to 20% of GDP in low- and middle-income countries; whereas this number was close to 5% in high-income countries. The increase in reserve assets is not exclusive of China or the East Asian countries; it has become widespread phenomenon among emerging markets, including countries that hold a large amount of external debt (see Figure 1). This practice has raised interesting questions in the literature regarding the cost and benefits of reserve accumulation. The cost of holding reserves has been estimated at close to 1% of GDP for all developing countries (Rodrik 2006).[1] Against this cost, an explanation commonly advanced is that countries may have accumulated reserves as an insurance mechanism against the risk of an external crisis—self protection through increased liquidity.[2]

Recently, researchers have undertaken the task of writing analytical models to characterize and quantify the optimal level of reserves and provide policy advice to countries.[3] Most of the formal models used in the current analysis tend to take the level of international debt as given and solve for the optimal liquidity-insurance services that reserves can provide. In other words, the recent literature has set aside the joint decision of holding sovereign debt and reserves. Although this strategy has allowed for a better understanding of the interplay between foreign reserves and a sovereign's access to international markets, there are several concerns with this approach. First, some of the implications of this assumption may not be generalized once one considers the joint decision by a sovereign to hold foreign debt and reserves. For example, in models a la Eaton and Gersovitz (1981), international debt serves an "insurance" role. That is,

---

[1] Rodrik (2006) estimates the cost of reserves as the spread between the private sector's cost of short-term borrowing abroad and the yield the Central Bank earns on its liquid foreign assets.

[2] For policy advice in this direction, see Feldstein (1999) and Caballero (2003). The "Greensan-Guidotti-IMF" rule proposes full coverage of short-term external liabilities to mitigate the risk of currency crises.

[3] See for example Aizenman and Lee (2005), Aizenman and Marion (2003), Caballero and Panageas (2005), Jeanne (2007), and Lee (2004).

the demand for international loans derives from a desire to smooth consumption. Given the sovereign's willingness-to-pay incentive problems, additional reserves in these types of models tend to reduce sustainable debt levels.[4] Second, the strategy of assuming constant debt levels does not address one of the puzzles behind the current accumulation of reserves. Sovereign countries have an alternative way of reducing the probability and negative effects of external crisis: to reduce the level of sovereign debt. That is, even in the case where reserve accumulation has positive liquidity benefits in terms of reducing the probability of suffering financial crises and the output costs associated with it, a similar net asset position can be obtained by reducing instead the level of foreign debt.[5]

In this paper, we address these concerns by incorporating debt sustainability issues into the optimal reserve management analysis. Our main objective is to study the implications of the joint decision of holding sovereign debt and reserves. We construct a stochastic dynamic equilibrium model of a small open economy with non contingent debt and reserve assets. Our starting setup is a model a la Eaton-Gersovitz and similar to Arellano (2008) and Aguiar and Gopinath (2006), but where the sovereign has the choice to hold reserves. In this setup, debt and assets are not perfect substitutes, as reserves can be used even after a country has defaulted. In other words, both defaultable debt and reserves are means to smooth consumption. We assess the quantitative implication of the model by calibrating a sample of emerging markets. We obtain that reserve accumulation does not play a quantitatively important role in the model. In fact, a robust result that emerges from our numerical exercises is that the optimal policy is not to accumulate reserves.[6]

We extend the model in several ways. Most of the recent papers advancing the insurance motive

---

[4] The cost to a sovereign of the denial of foreign credit is that the country must resort to other methods for consumption smoothing (such as building stock piles) or it must accept a greater fluctuation in its consumption. This penalty and hence the sustainable levels of foreign debt are higher the greater the cost to the borrower of exclusion. This cost in turn is higher the more limited domestically available options for smoothing consumption are and the lower the cost of smoothing via the international capital markets is (i.e., the lower the world interest rate); see Eaton et al. (1986).

[5] Rodrik (2006) mentions that countries could choose instead to reduce short-term debt in order to gain liquidity. More generally, countries can reduce foreign debt.

[6] We perform several exercises exploring the robustness of the result; see section 4.

behind reserve accumulation have been motivated by the sudden loss of access to international capital markets and the collapse of domestic production which have characterized the emerging markets crises of the nineties—a phenomenon Calvo (1998) labeled as “sudden stops.”[7] In line with the sudden stops literature, most of the formal analysis studying optimal reserve management model reserve accumulation as a cushion against external shocks and capital flows reversals while the level of international debt is taken as given. That is, in these models, countries engage in reserve accumulation to seek self-insurance against the potential tightening of international financial constraints rather than income fluctuation per se as in more standard debt models. These models have made important progress in providing an integral framework to analyze quantitatively some of the limitations and risks of the integration of developing countries to international financial markets. But by taking the level of debt as given, they have abstracted from willingness-to-pay concerns associated with sovereign debt and the sovereign’s choice over the composition of its assets and liabilities.

In order to analyze the interplay of both effects, we study an economy that is (i) hit by random shocks in the interest rate (which we take to be “contagion shocks”), and (ii) in addition to the interest rate shocks, the economy faces additional output costs associated with abrupt current account reversals (which we take to mean “sudden stops”). However, as mentioned, our model retains willingness-to-pay concerns. Note that since we focus on debt and reserve management problems, in this paper we do not attempt to advance the understanding of the contagion or sudden stops phenomena.[8] Instead we incorporate them into the analysis in a highly stylized form. Our approach, nevertheless, reveals interesting results. Once again the

[7] Additional empirical regularities include the sharp contractions in domestic production and consumption and the collapse of the real exchange rate and asset prices. See Arellano and Mendoza (2003) and Edwards (2004) for overviews of the stylized facts and the literature studying the “sudden stops” phenomena.

[8] See Mendoza (2006) for important work on this direction. As the author explains, the economics behind sudden stops remains to be understood. The standard real business cycle models do not seem to account for the main stylized facts of these events. Durdu, Mendoza and Terrones (2008) quantitatively analyze the role of foreign assets in a model of incomplete asset markets in which precautionary saving affects asset holding via business cycle volatility, financial globalization, and sudden-stop risk.

optimal policy does not involve accumulating reserves. Rather, the government reacts by reducing the amount of outstanding debt. Moreover, these results are robust to the possibility of holding contingent reserves, which, as suggested by Caballero and Panageas (2005), are a more efficient device to insulate the country from sudden stops.

We then study the role of output costs. The sovereign debt literature has found output costs to play an important quantitative role even in models where debt should otherwise be sustainable (see Alfaro and Kanczuk (2005)). Although these output losses are well documented, their micro-foundation remains to be understood.[9] Nevertheless, some empirical evidence suggests that reserves may reduce the output costs associated with sudden stops (see Frankel and Cavallo (2004)). In order to incorporate these stylized facts in our analysis, we consider a case where foreign reserves reduce output costs exogenously. Interestingly, in this case, if reserves reduce output costs, they reduce sustainability. For this reason, we obtain once again that it is optimal not to hold reserves.

The main implication that emerges from our work is that greater attention should be given to the explicit modeling of the sovereign's motivations, incentives, and constraints (political economy rationales) as they seem necessary towards understanding the observed levels of foreign reserve holdings.[10] To be sure, scholars have considered additional motives for holding reserves. In earlier literature, reserve holdings were associated with exchange rate management policies.[11] More recently, it has been argued that the rapid rise in reserves has little to do with self insurance, but instead with the policymaker's desire to prevent the appreciation of the currency and maintain the competitiveness of the exporting sector (the mercantilist view by Dooley et al. (2003)).[12] Yet, another strand of the literature has considered political economy issues. Aizenman and Marion (2003), for example, analyze the role of interest groups, corruption, and opportunistic behavior by future policy makers. In their setup, a policy recommendation to increase international reserve

[9] See Calvo (2000), Dooley (2000) and Mendoza and Yue (2008) for potential explanations.

[10] Amador (2003), for example, studies the role of political economy considerations in a sovereign debt model.

[11] See Frenkel and Jovanovic (1981), Edwards (1983) and Flood and Marion (2002) for a review of the literature.

[12] See Aizenman and Lee (2005) for a test of the importance of precautionary and mercantilist motives in accounting for the hoarding of international reserves; for "fear of floating" see Calvo and Reinhart (2002).

holdings may be welfare reducing because it may increase the chance of a financial crisis. More generally, our work suggests that policy recommendations may differ from the consensus once sovereign debt issues are considered. This, is in line with Rogoff`s (1999) critical remark: "Whereas the debate over why countries repay may seem rather philosophical, it is quite dangerous to think about grand plans to restructure the world financial system without having a concrete view on it."

The rest of the paper is organized as follows. Section 2 presents the benchmark model. Section 3 defines the data and calibration. The results and their robustness are discussed in Section 4, which is complemented by a comment about the role of reserves as collateral. Section 5 concludes.

2 Model

Our economy is populated by a sovereign country that borrows funds from a continuum of international risk-neutral investors. The economy faces uncertainty in output. As preferences are concave in consumption, households prefer a smooth consumption profile. In order to smooth consumption, the benevolent government may choose optimally to default on its international commitments. As in Arellano (2008) and Aguiar and Gopinath (2006), if the government defaults on its debts, it is assumed to be temporarily excluded from borrowing in the international markets. The novelty of our model is that the government has available another device to smooth consumption: in addition to issuing debt, the government can hold international reserves. And even when the government is excluded from borrowing international funds, a reserve buffer can be used to reduce consumption volatility.

In more precise terms, we assume the sovereign's preferences are given by

$$U = E\sum_{t=0}^{\infty}\beta^t u(c_t)$$

with,
$$u(c) = \frac{c^{1-\sigma}-1}{(1-\sigma)} \tag{1}$$

where $\sigma > 0$ measures the curvature of the utility, $\beta \in (0, 1)$ is the discount factor, and c_t denotes household consumption.

If the government chooses to repay its debt, the country's budget constraint is given by

$$c_t = \exp(z_t) - (B_t - q_t^B B_{t+1}) + (R_t - q_t^R R_{t+1}) \tag{2}$$

where B_t denotes the debt level in period t, R_t denotes the reserve level in period t, and z_t is the technology state in this period, which determines the output level. The debt and reserve price functions, $q^B(s_t)$ and $q^R(s_t)$, are endogenously determined in the model, and potentially depend on all the states of the economy, s_t, which will be described in each of the different scenarios we study. In the benchmark version of the model, the state of the economy is completely defined by the ordered set $s_t = (B_t, R_t, z_t)$.

We assume the technology state z_t can take a finite number of values and evolves over time according to a Markov transition matrix with elements $\pi(z_i, z_j)$. That is, the probability that $z_{t+1} = z_j$ given that $z_t = z_i$ is given by the matrix π element of row i and column j.

When the government chooses to default, the economy's constraint is

$$c_t = (1-\delta)\exp(z_t) + (R_t - q_t^R R_{t+1}) \tag{3}$$

where the parameter δ governs the additional loss of output in autarky, a common feature in sovereign debt models (see Alfaro and Kanczuk (2005)). After defaulting, the sovereign is temporarily excluded from issuing debt. In particular, we assume that θ is the probability that it regains full access to the international credit markets.

To grasp some intuition, consider a sovereign's choice to default, and compare expressions (2) and (3). On the one hand, defaulting entails an instantaneous reduction in the costs of rolling debt. This means higher consumption, particularly when the debt service is large. On the other hand, defaulting implies a lower output and a reduction in the possibilities of smoothing consumption in future periods. This is because the sovereign loses access to international credit markets. Reserves should have a role in these circumstances, because they allow for some consumption smoothing even when the sovereign cannot issue debt. In other words, even though this is a model of borrowing, there is still a role for reserves; defaultable debt is not contingent enough. The question we ask is whether there is a quantitative role for reserves in a calibrated version of the model.

International investors are risk neutral and have an opportunity cost of funds given by ρ_t, which denotes the risk-free rate. The investors' actions are to choose the debt price q_t^B, which depends on the perceived likelihood of default, and the reserves price q_t^R.

For investors to be indifferent between the riskless asset and lending to a country, it must be that

$$q_t^B = \frac{(1-\psi_t)}{(1+\rho_t)} \tag{4}$$

and

$$q_t^R = \frac{1}{(1+\rho_t)} \tag{5}$$

where ψ_t is the probability of default, which is endogenously determined and depends on the sovereign incentives to repay debt.

The timing of the decisions is as follows. In the beginning of each period, the sovereign starts with debt level B_t and reserve levels R_t, and receives the endowment $\exp(z_t)$. She faces the reserve price schedule $q_t^R(s_t)$ and the bond price schedule $q_t^B(s_t)$. Taking these two schedules as given, the sovereign simultaneously makes three decisions: (i) she chooses the next level of reserves, R_{t+1}; (ii) she decides whether to default on her debt or not; and (iii) if she decides not to default, she chooses the next level of debt, B_{t+1}.

The model described is a stochastic dynamic game. We focus exclusively on the Markov perfect equilibria. In these equilibria, the sovereign does not have commitment, and players act sequentially and rationally. In order to describe the equilibrium, notice first that the international investors are passive, and their actions can be completely described by equations (4) and (5). In order to write the sovereign problem recursively, let ν^G denote the value function of the sovereign if she decides to maintain a good credit history this period (G stands for good credit history). Similarly let ν^B denote the value function of the sovereign once it defaults (B stands for bad credit history). The value of being in a good credit standing at the start of a period can then be defined as,

$$v = Max\{v^G, v^B\} \tag{6}$$

This indicates that the sovereign defaults if $\nu^G < \nu^B$. The value functions ν^G can be written as,

$$v^G(s_t) = Max\{u(c_t) + \beta E v(s_{t+1})\} \tag{7}$$

subject to (2), and the value function ν^B by

$$v^B(s_t) = Max\{u(c_t) + \beta[\theta E v^G(s_{t+1}) + (1-\theta) E v^B(s_{t+1})]\} \tag{8}$$

subject to (3).

To compute the equilibrium, it is useful to define a default set as the states of the economy in which the sovereign chooses to default. The default set in turn determines the prices q^B and q^R, through expressions (4) and (5). With these prices at hand, one can solve the sovereign problem (6), (7), and (8). The solution for (7) determines a default, which can be used in the next iteration.

3 Calibration

We calibrate our model so that each period corresponded to one year. Our list of 28 emerging economies is composed of those countries classified as such by *The Economist*.[13] We use data available since 1965. To calibrate the technology state z, we estimate an $AR(1)$ process for the (logarithm) of the GDP for each country i, that is,

$$\ln(y_{i,t+1}) = \alpha_i \ln(y_{i,t}) + \varepsilon_{i,t+1}, \text{ where } \varepsilon_{i,t} \approx N(0, \sigma_{\varepsilon i})$$

We obtain that the GDP weighted average (across countries) parameters are α = 0.85 and σ_ε = 0.044, and the dispersion around these values is fairly small. We then assume the technology state can be discretized into three possible values, z^{low}, z^{avg} and z^{high}. They are spaced so that the extreme values are 2.5 standard deviations away from the mean. We use the Quadrature Method (Tauchen, 1986) to calculate the transition probabilities. We also discretized the space state of debt and reserves, assuming they could take values from zero to 60% of GDP. We assumed these grids were large and fine enough not to affect the decision rules.

We set the probability of redemption θ = 0.5, which implies an average stay in autarky of 2 years, in line with the estimates by Gelos et al. (2003).[14] In the benchmark case, the output cost is set equal to δ =

[13] Argentina, Brazil, Chile, China, Colombia, Czech Republic, Egypt, Estonia, Hong Kong, India, Indonesia, Israel, South Korea, Latvia, Lithuania, Malaysia, Mexico, Pakistan, Peru, Philippines, Russia, Saudi Arabia, Singapore, Slovak Republic, Slovenia, South Africa, Turkey, Venezuela.

[14] There are differences in the average stay in autarky among the different default events. For example, as noted by Sturzenegger and Zettelmeyer (2007), from initial declaration of debt services problems to final agreements, the default episodes of the 1980s took longer than the 1990s one (due to the overly optimistic assumptions and the regulatory incentives faced by the banks against declaring a sovereign in default). We experiment with longer average default periods (6 years) obtaining similar results.

10%, but we experiment with other values as well, $\delta = 5\%$ and $\delta = 20\%$ as in Alfaro and Kanczuk (2005).[15] Following the Real Business Cycle literature, we calibrate the parameter of preference curvature $\sigma = 2$ and the risk free rate $\rho = 0.04$, which corresponds to the U.S. interest rate. As Arellano (2008) and Aguiar and Gopinath (2006) note, in the model debt is used to front load consumption and thus high impatience is necessary for generating features of the data such as the high debt levels and reasonable default in equilibrium. Correspondingly, we set the time preference factor equal to $\beta = 0.50$, but we experiment with higher values as well. A summary of the calibration parameters is found in Table 1.

4 Results

4.1 The Model with No Reserves

In order to compare our results with the literature, we first solve for an economy without reserves. That is, we assume that $R_t = 0$ for any t, and solve, given the state, for the optimal default choice and for the optimal level of debt. In this simple case, the state of the economy is completely defined by the asset level and the technology, $s_t = (B_t, z_t)$.

Figures 2a and 2b depict, respectively, the sovereign's default decision and the choice of the next period debt level contingent on the sovereign not defaulting during this period. The curve denoted by "base" refers to the most favorable situation: one where the sovereign is hit by the high technology shock, z^{high}. The curve denoted by "low technology" refers to the case where the sovereign suffers a bad technology shock, z^{low}.

Figure 2a indicates that when the technology shock is high ("base") the sovereign defaults when the debt level is higher than 50.5% of GDP. If the technology shock is low ("low technology"), the maximum amount of sustainable debt is 41.1%. Since, as expected, default is used as a means to smooth consumption, default is more likely the lower the output levels.

Figure 2b shows that the next period debt level is higher the better the technology state and the higher this period debt. The positive relationship between consecutive debt levels was anticipated because, for a given technology shock, the sovereign would attempt to avoid sharp changes in the level of debt as

[15] Note that for lower values of δ, the model can sustain very low debt levels relative to the stylized facts.

these imply higher consumption volatility. The positive relationship between technology and debt is a fairly surprising result, although already discussed by other researchers (see Arellano (2008)). It means that the sovereign does not use debt primarily as a way to smooth consumption, a departure from the "pure" Eaton and Gersovitz (1981) framework. Instead, debt is predominantly used to front-load consumption, given that the discount factor β is lower than the inverse of the risk-free interest rate. As mentioned, consumption smoothing is mostly achieved through default, as in contingent debt service models, such as Grossman and Van Huyck (1988).

Calculating the invariant distribution of the states, we determine that the sovereign is excluded from the market 1.29% of the time, and the average debt is 46.0% of output, as reported in Table 2. These results are broadly consistent with the stylized facts—the average debt holding for the countries of figure 1 is approximately 41%.

4.2 The Role of Reserves

In our benchmark model, we allow the sovereign to hold reserves. The state of the economy is defined by $s_t = (B_t, R_t, z_t)$. We solve again for the equilibrium and obtain the optimal choices of defaulting, holding of debt, and reserves, which are reported in figures 3a to 3d. These figures contain 3 curves each. As before, we report the decision rules for the cases named "base" and "low technology," which refer respectively to situations with good and bad technology shock, and reserves kept equal to zero ($R_t = 0$). But, in addition, these figures report the decision rule for the case named "high reserves," which correspond to a situation with the worst technology state (z^{low}) and the maximum level of reserves in the grid chosen in our computer implementation (R_t = 60 % of GDP).

Figure 3a should be compared with Figure 2a. It shows that the decision to default is the same for the cases "base" and "low technology." The effect of holding reserves ("high reserves") is to reduce the amount of sustainable debt. This is expected because reserve holdings reduce the cost of exclusion from the capital markets. Figure 3b is very similar to Figure 2b. It indicates that holding reserves reduces the next period level of debt.

Figures 3c and 3d show the decision to hold reserves in the next period when the sovereign has and does not have access to the capital markets, respectively. These figures are not particularly interesting, because most of the curves (precisely the "base" and "low technology" ones) coincide with the horizontal axis. The important result one draws from Figures 3c and 3d is that the sovereign chooses to hold low levels of reserves in the next period, whatever the state of the economy in this period. In particular, she decides to deplete her reserve holdings whenever her outstanding debt level is lower than 6.3% of GDP. Furthermore, and more interestingly, by jointly analyzing the "high reserves" curves from Figures 2b and 2c, one can see that the sovereign would tend to use her reserve holdings to repay part of her debt.

Calculating the invariant distribution of the states, we obtain that the sovereign is excluded from the market 1.29% of the time, the average debt is 46.0% of output, and the average reserve level is zero (see Table 2). Notice that the debt level and the frequency of default results are the same that we obtained in the economy without reserves. This is consistent with the fact that reserve holdings are equal to zero.

Indeed, the main lesson from this experiment is that it is optimal not to hold reserves at all. To appreciate the intuition behind this result, consider first the benefits and costs of holding reserves. As discussed before, the benefits of holding reserves are associated with the consumption smoothing they allow in case the country defaults. Reserve holding are also costly, as the sovereign impatience is higher than the reserves remuneration. Or, in other words, in order to build its stock of reserves, an economy has to consume less. But, more importantly, reserve holdings endogenously affect the willingness to default of the sovereign. As a consequence, reserve holdings reduce the amount of "sustainable" debt, or, more precisely, increases debt services for a given level of debt. Thus, reserve holdings make debt more costly to the sovereign.

4.3 Robustness: Parameter Calibration

Holding reserves may seem to be *a priori* suboptimal because the sovereign can, instead, reduce the amount of outstanding debt. That is, both reserve holdings and outstanding debt are alternative ways of achieving consumption smoothing. In bad times, when the economy is hit by a bad output shock, in order to smooth consumption, the sovereign can increase the amount of debt to its maximum sustainable level. But in the case when the sovereign has outstanding debt levels lower than the maximum sustainable level, there is "more room" to borrow before debt becomes unsustainable. (More precisely, since default is less likely when outstanding debt is smaller, borrowing is relatively cheaper).

However, this logic is too simplistic for many reasons. First, reserves are a device to smooth consumption even after the country has defaulted, a circumstance in which the country cannot increase debt. Second, reserves affect the sovereign's willingness-to-pay in complex ways. And, in particular, there is no reason for the amount of sustainable debt to vary linearly and uniformly with the amount of reserve holdings. In other words, there is no guarantee that reducing outstanding debt by, say, X dollars, increases the ability to smooth the same amount that holding X dollars as reserves does. Third, as Grossman and Han (1999) show, smoothing consumption through increasing debt is less effective than smoothing consumption through defaulting. Or, using their typology, "contingent service" generates more consumption smoothing than "contingent debt." And since reserves are useful even after defaulting, they can be even more so when the sovereign opts to pay service contingently.

To further grasp this last point, we develop a simple two-period economy in the appendix. This very stripped-down model shows how the combination of "defaultable" debt and reserves (a risk free bond) completes the market. It underscores that there is indeed a theoretical role for holding reserves in the model and also indicates which parameters are crucial to make reserves positive in equilibrium. In particular, in the two-period economy we show that the equilibrium amount of reserves should increase both with the parameter β (discount factor) and with the parameter σ_ε (the standard deviation of the endowment process).

Taking all these together, one can conclude that there are many reasons why reserves holdings and less debt outstanding are not perfect substitutes. Consequently, it is not possible to say, *a priori*, that the optimal reserve holding is zero. Our result is, therefore, a quantitative one, which depends on the model and calibration. As a direct consequence, it becomes crucial to understand how robust our result is to the parameter calibration, which we tackle next, and to model specification, which we analyze in the following sections.

First, to check if our results are highly dependent on the benchmark calibration, we simulate the model with parameters changed by amounts equal to 10% of their benchmark values. We obtain that zero optimal reserve result survive this tests.

Then, following the analysis in the appendix, we experiment with much higher values for β. We report the results in table 3. Interestingly, a simulation with $\beta = 0.90$ implies debt values equal to 21.4% of

GDP, which is fairly in line with Asian countries. But, again for this case, the optimal level of reserves is equal to zero. As we keep increasing β, we obtain that the country does not default anymore. In such situation, debt and (negative) reserves become perfect substitutes, as they have the same payoff in all states of nature. Indeed, for β higher than 0.945 the amount of reserves in equilibrium becomes positive, but the amount of debt is now equal to zero. Thus, these experiments never result in positive amounts for both reserves and debt.[16] They show the model may imply in positive reserve amounts, but do not illustrate the potential role of reserves in completing the market.

Once more following the two-period analysis, we then experiment with much higher values for σ_ε, the volatility associated with the endowment process. Table 4 reports the results. As we increase σ_ε, we obtain the sovereign tends to default more often and to reduce the amount of debt. This is again the indication that default is the chosen alternative to smooth consumption. For $\sigma_\varepsilon = 0.33$, which is 7.5 times the volatility of the benchmark calibration, the equilibrium amount of reserves becomes positive. This is exactly what we are looking for, a situation in which reserves and debt coexist to complete the market. As we increase σ_ε further, to 10 times the benchmark calibration, reserves are positive even when the economy is being excluded from the international markets. Finally, for $\sigma_\varepsilon = 1.32$, which is 30 times the benchmark calibration, reserve holdings are greater than GDP, and the probability of default surprisingly falls[17].

The implication of this set of experiments is that there is indeed a theoretical role for reserves in the

[16] As a side issue, one should remember that in our model the parameter β is capturing not only impatience but also the fact that emerging countries grow more than developed ones (the lenders), and therefore want to frontload consumption. More formally, one could write a model in which endowment grows at a rate $(1 + \gamma)$, and transform all the variables into their stationary counterparts. The new model would resemble our model with the original β replaced by β', with $\beta' = \beta^*/ (1 + \gamma)^{\sigma - 1}$. Here β^* is the impatience parameter which can be calibrated by the international interest rate, whereas $(1 + \gamma)$ measures the difference in output growth between the emerging and developed countries. Thus, in practice, β' should be lower than $\beta^* = 1/(1 + \rho) = 0.962$.

[17] To better grasp the quantitative implication of these alternative assumptions, note that in the benchmark calibration, with $\sigma_\varepsilon = 0.044$, the welfare costs of fluctuations were $9.7.10^{-4}$ consumption units. In contrast, endowment volatilities of $\sigma_\varepsilon = 0.33$ and $\sigma_\varepsilon = 1.32$ imply, respectively, in welfare costs of $5.5.10^{-2}$ and $8.7.10^{-1}$ units of consumption.

model, but not for reasonable parameter calibration. Reserve holdings are positive only if (i) the endowment process is much more volatile than what we observe in emerging countries[18], or (ii) the impatience parameter is set to values that imply zero debt, which is also inconsistent to what we observe in emerging markets and conflicts with the consumption front-loading that we expect from fast growing economies.[19]

In the next sections we evaluate the robustness of our results to the model specification. We return to the benchmark calibration and increasingly change the features of the model.

4.4 Contagion

As a first robustness test to model specification, we consider the existence of “contagion effects,” or shocks to the international interest rate. The intuition here is that interest rate shocks may be an additional motivation to default on debt; debt services may become too costly to the sovereign. And, in as much as defaulting becomes a good alternative, holding reserves becomes a useful device to smooth consumption in the periods the sovereign is excluded from the international capital markets.

In order to simulate contagion, we modify equations (4) and (5) to the following expressions

$$q_t^B = \frac{(1-\psi_t)}{(1+\rho_t^B)} \tag{4'}$$

and

$$q_t^R = \frac{1}{(1+\rho_t^R)} \tag{5'}$$

where we now denote the risk free rate by both ρ^B and ρ^R. This notation may initially appear awkward, as there should be only one rate. But, as we discuss next, this is a useful way to consider “irrational” contagion effects in the model.

We then assume that the risk-free interest rate associated with the debt can take on of two values: $\rho^B \in \{0.04, 0.20\}$. The low value corresponds to tranquil times and the high value to nervous times. We further

---

[18] The results in terms of volatility are similar to those in Durdu, Mendoza, and Terrones (2008).

[19] Note that one could also increase the persistence of the endowment process, or boost the curvature of the utility function in order to amplify the importance of consumption smoothing. However, this would again imply unrealistic parameter calibrations.

assume that ρ^B evolves over time according to a Markov process. We calibrate the transition matrix that describes this process such that, on average, a contagion occurs every ten years and lasts for two years. This is in line with the stylized facts about sudden stops (see Edwards (2004) and Mendoza (2006).

Whereas the risk free rate associated with the debt oscillates, we assume the risk-free rate associated with reserves is constant and equal to $\rho^R = 0.04$. That is, the emerging markets always get the regular (and thus lower) interest rate remuneration on their reserves, and hence reserve accumulation does not benefit from the higher interest rates associated with contagion.

The idea of this model specification is that in nervous times there is some type of "irrational" contagion, which makes the emerging market face a high international rate, regardless of the risk of default. It is useful to remember that in our model the probability of default is endogenously determined and affects the equilibrium bond price q^B even for a constant ρ^B. The occurrence of contagion is thus something else. It represents a situation in which the emerging market faces a higher international rate even if it does not default.

Notice also that the effect of contagion is akin to (partly) shutting down the international market. That is, in nervous times the emerging economy can still borrow funds abroad, but it has to do so at a higher cost. The calibration of $\rho^B = 0.20$ implies the interest premia that are usually seen during crisis. Raising ρ^B further is equivalently to increasingly closing the market. Noteworthy, this contrasts to the usual modeling in the literature, in which the maximum amount of debt is reduced to an exogenously specified level during contagion. In our model, the credit constraint occurs through prices rather than quantity, and the debt ceiling is endogenously determined and affected by the holdings of reserves.

In this specification, the state of the economy is defined by $s_t = (B_t, R_t, z_t, \rho^B{}_t)$. After solving for the equilibrium, we can again analyze the policy functions. Figures 4a and 4b depict the default and the next period of debt choices, and are analogous to Figures 3a and 3b. The curves "base," "low technology," and "high reserves" are defined as before, and refer to the case in which $\rho^B = 0.04$, that is, tranquil times. The additional curve, "high interest," corresponds to the case with the worst technology shock, with the highest reserve holdings and with the high international interest rate $\rho^B = 0.20$. That is, this curve carries the same hypotheses of the curve labeled "high reserves" in addition to the interest rate shock or contagion assumptions.

Notice that the results for the cases “base,” “low technology,” and “high reserves” are the same as before. The case “high interest” shows that contagion implies that the sovereign opts to default at smaller debt levels. This could imply more defaults, as one would have expected. However, at the same time, contagion implies an abrupt reduction in the amount of debt outstanding. Thus, it is not possible, by looking only at these figures, to know the true impact of contagion on the frequency of defaults.

When we calculate the invariant distribution of the states, we obtain that the sovereign is excluded from the market only 0.52% of the time, the average debt is 25.1% of output, and the average reserve level is zero (see Table 2). Thus, contrary to our initial intuition, the sovereign optimally responds to the existence of contagion by reducing the outstanding debt and defaulting less frequently instead of defaulting more often. As a consequence, reserves again play no role. We chose not to report the decision of reserve holding (the analogs to curves 3c and 3d) because they do not add additional information. As before, it suffices to know that the sovereign opts to deplete her reserve holdings for any relevant state of the economy. We also redo this experiment with many other parameter calibrations, and with much higher contagion premiums (e.g., with $\rho^B = 2.00$), getting similar results.

4.5 Sudden Stops

A pertinent criticism of our contagion experiment is that it fails to capture some of the most crucial characteristics of sudden stops. In particular, although we assumed sudden increases in the lending rate, we have not considered that there might be output costs associated with abrupt reversals in the current account (or, equivalently, sharp reduction of debt holdings). Maybe because of this, our solution pointed out that the sovereign would choose to reduce its debt holdings, instead of defaulting.

This criticism motivates us to modify the model and include exogenous output costs associated with sudden stops. This amounts to changing our equation (2) to

$$c_t = (1-\Delta)\exp(z_t) - (B_t - q_t^B B_{t+1}) + (R_t - q_t^R R_{t+1}) \qquad (2')$$

where Δ are the sudden-stop output costs. We set Δ = 10% if $B_{t+1} < B_t - 0.05$, and $\Delta = 0$ otherwise. By doing so, we are calibrating output costs equal to 10% every time there is a current account reversal greater than

5% of GDP (see Calvo et al. (2006)).

Figures 5a and 5b are the analogs to figures 4a and 4c, once the sudden stop output costs are introduced in the model. Comparing Figure 5a with 4a, one notices that additional output costs associated with sudden stops reduce debt sustainability. The reason is such output costs reduce the benefits associated with the access to the international capital market, and thus make default more appealing. Figure 5b is surprising for the non-monotonicity of the policy functions, for which we do not have good intuition. But it reveals, as expected, that interest rate shocks do not imply in abrupt reductions of debt.

In line with these observations, the invariant distribution of the states (Table 2) reveals that the sudden-stop economy has more debt outstanding than the contagion economy (30.7% against 25.1% of GDP respectively), and is associated with more default episodes (1.54%). Noticeably, there are more defaults and less debt in the sudden-stop economy than in the benchmark economy. However, once again we obtained that reserve holdings are equal to zero.

We experimented with the sudden-stop economy using other parameters (higher output costs and smaller current account reversal tolerance, in addition to the various combinations of β, σ_ε and ρ^B), and obtained that the optimality of holding no reserves seems to be very robust to different calibrations. Our general reading of this experiment is that, in order to respond to sudden-stop shocks, the sovereign does not increase the default frequency much but, instead, reduces the amount of outstanding debt (when compared to the "benchmark" economy outcomes). This, in turn, implies that sudden-stops are not quantitatively important to rationalize building a reserve buffer.

4.6 Contingent Reserves

In a series of papers, Caballero advocates that emerging markets should hold contingent reserves.[20] The intuition is fairly simple. Contingent reserves, which pay higher rates during "sudden stops," should be

[20] See Caballero (2003) and Caballero and Panageas (2005). Caballero and Panageas (2005) have in mind concrete examples of instruments with contingent payoffs. However, more generally one can think that if capital outflows and sudden stops are associated with devaluations, the payoff of reserves (usually denominated in foreign currency) will be negatively correlated with these events.

better than non-contingent reserves since they allow for more consumption smoothing. The authors also describe the conditions for a contingent asset to qualify for this use (it cannot be controlled by the individual country, and its payoff has to be correlated with the occurrence of sudden stops) and quantify the gains from its use. For the purpose of this paper, the possibility of holding contingent reserves should be seen as an additional reason to holding a reserve buffer, as it increases its benefits.

In order to simulate the existence of contingent reserves in our economy, we let the price of reserves take on two values: $\rho^R \in \{\rho_{LOW}, \rho_{HIGH}\}$. We also assume that there is a perfect correlation between ρ^R and ρ^B, that is, $\rho^R = \rho_{LOW}$ when $\rho^B = 0.04$, and $\rho^R = \rho_{HIGH}$ when $\rho^B = 0.20$. We keep the rest of the model identical to the sudden-stop economy.

The fact that international investors are risk neutral implies that there is a restriction on the values of ρ_{LOW} and ρ_{HIGH}. Since the expected value of the contingent reserves should be equal to the risk less interest rate, this pins down ρ_{LOW} for a given value of ρ_{HIGH}. In order to simulate the economy, we experiment for different values for ρ_{HIGH}. In fact, the choice of ρ_{HIGH} corresponds to different leverages, or different blends of contingent and non-contingent reserves.

However, regardless of the choice of ρ_{HIGH}, the result was the same and identical to the case with non-contingent reserves. That is, the invariant distribution results in the economy with contingent reserves are equal to those obtained with the sudden-stop economy in the previous subsection. The policy functions obtained are also identical to those reported in figures 5a and 5b.

As before, experimentation with many combinations of parameters did not lead to new insights. The essential contribution of this experiment is that the benefits from contingent reserves are not quantitatively important enough to reverse the previous result. Once again, the optimal reserve holding is zero.

4.7 Output Costs Reduction

Defaulting countries experience output costs beyond the observed reduction in investment and labor. Alfaro and Kanczuk (2005) find that additional output costs of defaulting are necessary to sustain the debt levels observed in emerging markets even in a model of contingent services.[21] Although these additional output losses are well documented, their micro-foundation remains to be understood. One possibility, pointed out by Dooley (2000), is that the loss in output is caused by the inability of debtors and creditors to quickly renegotiate contracts and the inability to condition the loss of output ex-ante by reasons of nonpayment. This creates a time interval during which residents of the country in default are unable to borrow from locals or foreigners, for example, due to the inability of new credits to be credibly senior to existing credits. If this is factually relevant, then reserves could potentially be used to mitigate the output costs of default. The empirical evidence provided by Frankel and Cavallo (2004) suggest that reserves may reduce output costs following a default.[22]

To investigate the possibility that this could affect our results, we analyze a case where reserves reduce output costs in an exogenous fashion. To implement this in the model, we change equation (3) to

$$c_t = \left(1 - \frac{\delta}{1 + \omega_1 R_t^{\omega 2}}\right)\exp(z_t) + (R_t - q_t^R R_{t+1}) \tag{3'}$$

Note that the output cost δ was substituted by the expression $\delta/(1 + \omega R_t^{\omega 2})$, where ω_1 and ω_2 are positive parameters and R_t is the level of reserves (measured in percentage of output). As our benchmark, we set $\omega_1 = 100$ and $\omega_2 = 1$, which implies that, when reserves amount to 10% of GDP, the costs from defaulting halves. We also experimented with $\omega_2 < 1$, for which the marginal reduction in the output cost of the first unit of reserves tends to infinity.[23]

From the solution of the model, we obtain again the policy functions for the default choice and the next period level of debt, which we depict in Figures 6a and 6b. The most noticeable change is that debt is now less sustainable than before, as one would expect. The invariant distribution points out to lower debt levels and fewer defaults (see Table 2). Once again, we obtain that reserve holdings are equal to zero. This

---

[21] As Grossman and Han (1999) point out, in contingent service models debt is sustainable even if the sovereign can save after defaulting and regardless the existence of additional output costs.

[22] Although there is a view that foreign reserves might reduce the probability of sudden stops, the empirical evidence on this relation is not conclusive, see Edwards (2004), Frankel and Cavallo (2004) and Rodrik and Velasco (1999).

[23] We thank Joshua Aizenman for this suggestion.

was robust to all values of ω_1 and ω_2 we tried, including choices such as $\omega_1 = 10000$ and $\omega_2 = .5$. Paradoxically, as before, adding a new role for reserves implies in fewer defaults and, thus, no need for reserves.

4.8 Discussion: Reserves as Collateral

From the point of view of our framework, the main role of reserves is to act as a buffer stock, useful for consumption smoothing even when the sovereign is excluded from capital markets. As we discussed, and the quantitative exercises show, according to this view, reserve holdings are consistent with less debt sustainability. A completely different view, sometimes advanced in the literature, is that reserves can be used as collateral, thus increasing debt sustainability.

But can reserves really be used as collateral? Our view is that this role, if it exists, is at best very limited. Reserves are legally protected against attachment by creditors. Under the Foreign Sovereign Immunities Act of 1976 of the United States and similar laws in other countries, central bank assets, including international reserves, are usually protected against attachment.[24] Hence, reserves work as collateral only in the case where governments willingly, and more importantly credibly, pledge them as such.[25] Most governments, however, are generally careful to keep their reserve assets untouchable, thus limiting their role as collateral. As Eaton et al. (1986) note, "if the collateral is retained in the borrowing country, there is no mechanism by which the creditor can seize it, and if the collateral is moved outside the country, where the creditor can seize it, the value of the loan is effectively reduced by the value of the collateral. A fully and effectively collateralized loan would then be of no value to the borrower."

[24] This is also the case even when the central bank is the issuer of the debt. The Bank for International Settlements (BIS) in Switzerland is also protected against attachment proceedings; governments and central banks in many cases place assets with the BIS. This, however, is not the case in Germany, where under German law, reserves are open for attachment. See Scott (2005) and Sturzenegger and Zettelmeyer (2006).

[25] Reserves, however, can affect the bargaining position of a country in a negotiation. See Detragiache (1996) for a bargaining model that incorporates reserves along these lines.

5. Conclusions

On the one hand, reserve accumulation is costly because it implies in less current consumption. On the other hand, holding reserves may be beneficial. Reserves can be used to smooth consumption when a country (i) is excluded from capital markets after defaulting, (ii) is hit by "irrationally" high interest rates due to international contagion, or (iii) suffers from a so-called "sudden-stop" phenomenon. In addition, reserves may be particularly useful (iv) if their payoff is contingent on the state of nature, or (v) if they can mitigate the output costs associated with a default.

We study the optimal reserve policy in a stochastic dynamic general equilibrium model which is calibrated to match a typical emerging market economy. The model is general enough to contemplate the potential benefits of reserves mentioned above and, importantly, endogenously determine the optimal debt level. We obtain that the optimal policy is not to hold reserves at all.

To obtain our results, we resorted to many simplifications. In particular we acknowledge the lack of micro foundations related to the sudden stop phenomenon. More generally, why countries hold debt remains to be understood in the literature. We believe that a better understanding of these issues could improve the analysis, but we do not expect that doing so would change our results qualitatively in terms of the desirability of reducing the amount of outstanding debt as opposed to increasing reserves.

Overall, our paper contributes to the existing debate by arguing that current reserve holding do not seem to correspond to the optimal behavior of a sovereign that can both choose the levels of debt and hold reserves. Some form of transaction costs, which were not considered in our model, could rationalize countries holding some small amount of reserves, say, to cover for their very short-term debt. But this cannot account for large reserve stocks.

Our results have normative implications. They suggest that a positive model, that aims to reproduce the stylized facts, would need to focus on alternative motivations for reserve accumulation. Perhaps the recent reserve holdings observed in emerging markets can be better explained by political economy motivations associated with exchange rate management policies. Yet, another issue worth considering is the interaction between the government's and the private sector's holdings of debt and reserves. Exploring these rationales further is an important topic for future research.

6 References

Aguiar, M. and G. Gopinath. 2006. Defaultable Debt, Interest Rates and the Current Account, Journal of International Economics 69, 64--83.

Amador, Manuel. 2003. A Political Economy Model of Sovereign Debt Repayment. Mimeo.

Aizenman, J. and J. Lee 2005. International Reserves: Precautionary Versus Mercantilist Views, Theory and Evidence. IMF Working Paper 01/198.

Aizenman, J. and N. Marion 2003. International Reserve Holdings with Sovereign Risk and Costly Tax Collection. The Economic Journal 114, 569--591.

Alfaro, L. and F. Kanczuk. 2005. Sovereign Debt as a Contingent Claim: A Quantitative Approach. Journal of International Economics 65, 297--314.

Arellano, C. 2008. Default Risk and Income Fluctuations in Emerging Markets. American Economic Review, forthcoming.

Arellano, C. and E. Mendoza. 2003. Credit Frictions and 'Sudden Stops' in Small Open Economies: an Equilibrium Business Cycle Framework for emerging Market Crises, in: Chandha, A. (Ed), Dynamic Macroeconomics Analysis: Theory and Policy in General Equilibrium, Cambridge University Press.

Calvo, G. 1998. Capital Flows and Capital Market Crises: The Simple Economics of Sudden Stops. Journal of Applied Economics 1, 35--34.

Calvo, G. 2000. Balance of Payments Crises in Emerging Markets: Large Capital Inflows and Sovereign Governments, in: Krugman, P. (Ed.), Currency Crises, Chicago: The University of Chicago Press.

Calvo, G., A. Izquierdo and E. Talvi. 2006. Phoenix Miracles in Emerging Markets: Recovering without Credit from Systemic Financial Crises. NBER Working Paper 12101.

Caballero, R. 2003. On the International Financial Architecture: Insuring Emerging Markets. NBER Working Paper 9570.

Caballero, R. and S. Panageas. 2005. Contingent Reserves Management: An Applied Framework. Mimeo.

Calvo, G. and C. Reinhart. 2002. Fear of Floating. The Quarterly Journal of Economics 117, 379--408.

Detragiache, E. 1996. Fiscal Adjustment and Official Reserves in Sovereign Debt Negotiations. Economica

63, 81--95.

Dooley, M. 2000. Can Output Losses Following International Financial Crises be Avoided? NBER Working Paper 7531.

Dooley, M., D. Folkerts-Landau, and P. Garber. 2003. An Essay on the Revived Bretton Woods System. NBER Working Paper 9971.

Durdu, B., E. Mendoza, and M. Terrones. 2008. Precautionary Demand for Foreign Assets in Foreign Stop Economies: An Assessment of the New Mercantilism. Journal of Development Economics, forthcoming.

Eaton, J. and M. Gersovitz. 1981. Debt with Potential Repudiation: Theoretical and Empirical Analysis. The Review of Economic Studies 48, 289--309.

Eaton, J., M. Gersovitz and J. E. Stiglitz. 1986. The Pure Theory of Country Risk. European Economic Review 30, 481--513.

Edwards, S. 1983. The Demand for International Reserves and Exchange Rate Adjustments: The Case of LDCs. 1964-1972. Economica 50, 269--80.

Edwards, S. 2004. Thirty years of Current Account Imbalances, Current Account Reversals and Sudden Stops. IMF Staff Papers 51, 1--49.

Feldstein, M. 1999. A Self-Help Guide for Emerging Markets. Foreign Affairs, March/April 1999.

Flood, R. and P. Marion. 2002. Holding International Reserves in an Era of High Capital Mobility, in: Collins S. and Rodrik, D. (Eds), Brookings Trade Forum 2001, Washington, DC: Brookings Institution Press.

Frankel, J. and E. A. Cavallo. 2004. Does Openness to Trade Make Countries More Vulnerable to Sudden Stops, Or Less? Using Gravity to Establish Causality. NBER Working Paper 10957.

Frenkel, J. and B. Jovanovic. 1981. Optimal International Reserves: A Stochastic Framework. Economic Journal 91, 507--514.

Gelos, G. R., R. Sahay and G. Sandleris. 2003. Sovereign Borrowing by Developing Countries. What Determines Market Access? IMF Working Paper 04/221.

Grossman, H. I. and T. Han. 1999. Sovereign Debt and Consumption Smoothing. Journal of Monetary

Economics 44, 49--58.

Grossman, H. I. and J. B. Van Huyck. 1988. Sovereign Debt as a Contingent Claim: Excusable Default, Repudiation, and Reputation. American Economic Review 78, 1088--1097.

Jeanne, O. 2007. International Reserves in Emerging Market Countries: Too Much of a Good Thing. Brookings Papers on Economic Activity, forthcoming.

Lee, J. 2004. Insurance Value of International Reserves: An Option Pricing Approach. IMF Working Paper 04/175.

Mendoza, E. 2006. Endogenous Sudden Stops in a Business Cycle Model with Collateral Constraints: A Fisherian Deflation of Tobin's Q. NBER Working Paper 12564.

Mendoza, E. and V. Yue, 2008. A Solution to the Default Risk-Business Disconnect. Mimeo.

Rodrik, D. 2006. The Social Cost of Foreign Exchange Reserves. NBER Working Paper 11952.

Rodrik, D. and A. Velasco. 1999. Short-Term Capital Flows. NBER Working Paper 7364.

Rogoff, K. 1999. Institutions for Reducing Global Financial Instability. Journal of Economic Perspectives 13, 21--42.

Scott, H. 2005. International Finance: Transactions, Policy, and Regulation, 12th Edition. New York: Foundation Press.

Tauchen, G. 1986. Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions. Economics Letters 20, 177--181.

Sturzenegger, F. and J. Zettelmeyer. 2007. Debt Defaults and Lessons from a Decade of Crisis. Cambridge: MIT Press.

7 Appendix: A Two-Period Economy

To better grasp the role of reserves in our model, this appendix develops a two-periods stripped down version. As such, it cannot shed light on the "willingness to pay" incentives of the sovereign, which hinge on the costs from the exclusion from the market. But this simpler version underscores how the combination of reserves and "defaultable" debt completes the market. It also suggests which parameters are crucial to obtaining positive reserves in equilibrium.

In this simple economy, the sovereign preferences are given by

$$\mathrm{u}(c_1, c_2) = c_1 + \beta\, E\,[\, u(c_2)\,] \tag{A1}$$

where c_1 and c_2 denote households' consumption in the first and second period, $\beta \in (0, 1)$ is the discount factor, and u is a second differentiable function with $u' > 0$ and $u'' < 0$. Households receive an endowment y in the second period, according to the following stochastic process,

$$y = 1 + \sigma \text{ with probability equal to } \tfrac{1}{2} \text{ (good state of nature), and} \tag{A2}$$

$$y = 1 - \sigma \text{ with probability equal to } \tfrac{1}{2} \text{ (bad state of nature).}$$

Note that we only kept the curvature of the utility function in the second period, as this is enough to create the gains from smoothing consumption across the different states of nature. In turn, this assumption makes the first period endowment irrelevant for the maximization problem. The parameter β has the role of indicating the benefit from consuming earlier rather than latter. The parameter σ equals the standard deviation of the (second period) endowment process.

The sovereign has two instruments to transfer resources across periods: defaultable bonds and reserves. As usual, reserves correspond to riskless bonds, which bear interest rates given by ρ. In contrast, defaultable bonds are contingent claims. The sovereign only repays debt in the good state of nature (high endowment). More explicitly, the country budget constraint can be written by,

$$c_1 = B - R \tag{A3}$$

$$c_2 = 1 + \sigma + R(1 + \rho) - B(1 + \mathrm{r}) \text{ in the good state of nature and}$$

$$c_2 = 1 - \sigma + R(1 + \rho) \text{ in the bad state of nature}$$

where B is the debt level, R is the amount of reserves, and r denotes the "contractual" interest rate on debt. International investors are risk neutral, and must be indifferent between the riskless asset and lending to a country. This implies,

$$1 + \rho = (\tfrac{1}{2})\,(1 + r) + (\tfrac{1}{2})\, 0 \tag{A4}$$

The maximization of the sovereign problem as a function of the level of debt B and reserves R implies the system of equations, which define the equilibrium allocation of the model.

$$u'(1 - \sigma + R(1 + \rho)) = 1/[\beta(1+\rho)] \tag{A5}$$

$$u'(1 + \sigma + R(1 + \rho) - B(1 + r)) = 1/[\beta(1+\rho)] \tag{A6}$$

The first of these equations can be used to show the following comparative statics,

Proposition. *The equilibrium level of reserves increases with the increases with the discount factor and with the volatility of the endowment, i.e.,* $\partial R/\partial\beta \geq 0$ and $\partial R/\partial\sigma \geq 0$ ■

Higher discount factor means that the costs of consuming latter are smaller, and since reserves are a means to postpone consumption, its equilibrium amount should increase with β. Reserves increase with σ because they interact with debt to complete the markets, and this insurance motive becomes more important when endowment volatility is higher.

It should be noted that this simple analysis is implicitly considering that the solution is interior. By doing so, it is allowing reserves to be negative, in which case *R* actually corresponds to riskless debt. In contrast, the full fledged model makes the realistic assumption that reserves are constrained to be non negative. To make this point more explicit, consider the example in which $\rho = 0$ and $u(.)$ is a logarithmic function. With these additional assumptions, we can obtain the explicit solution $R = \beta + \sigma - 1$. Thus, in this particular case, reserves are only positive (and thus really mean reserves) when $\beta + \sigma > 1$. In other words, if we had constrained reserves to be non-negative, they would only exist for $\beta + \sigma > 1$.

A second related issue with this simple analysis is that it assumes that the sovereign always defaults in bad states of natures. This is a useful assumption here because, as mentioned, we cannot incorporate the willingness to pay analysis in a two-period model. However, in the full-fledged model, default is chosen optimally, and the sovereign may choose never to default. In such case, debt *B* and reserves *R* would become perfect substitutes. The experiments in section 4.3 show that this is indeed what happens when we choose high β values.

Table 1: Calibration of Benchmark Model

Technology autocorrelation	$\alpha = 0.85$
Technology standard deviation	$\sigma_{\varepsilon} = 0.044$
Probability of redemption	$\theta = 0.50$
Output costs	$\delta = 0.10$
Risk aversion	$\sigma = 2.00$
Risk free interest rate	$\rho = 0.04$
Discount Factor	$\beta = 0.50$

Table 2: Invariant Distributions for Alternative Models

Model Specification	Exclusion from Market (% time)	Debt if not excluded (% GDP)	Reserves if not excluded (% GDP)	Reserves if excluded (% GDP)
No Reserves	1.29	46.0	0	0
Benchmark	1.29	46.0	0	0
Contagion	0.52	25.1	0	0
Sudden-Stops	1.54	30.7	0	0
Contingent Reserves	1.54	30.7	0	0
Output Reduction	0.54	28.7	0	0

Table 3: Invariant Distributions in Benchmark Model with Higher discount factor

Discount factor (β)	Exclusion from Market (% time)	Debt if not excluded (% GDP)	Reserves if not excluded (% GDP)	Reserves if excluded (% GDP)
.50	1.29	46.0	0	0
.70	1.29	30.8	0	0
.90	1.29	21.4	0	0
.94	0	18.9	0	-
.945	0	0	.0071	-
.95	0	0	49.2	-
.955	0	0	75.8	-
.96	0	0	295.4	-

Table 4: Invariant Distributions in Benchmark Model with Higher Volatility

Volatility (σ_ε)	Exclusion from Market (% time)	Debt if not excluded (% GDP)	Reserves if not excluded (% GDP)	Reserves if excluded (% GDP)
0.044	1.29	46.0	0	0
0.11	2.56	45.9	0	0
0.22	11.2	44.3	0	0
0.33	11.2	42.1	4.1	0
0.44	11.2	39.8	16.6	0.2
0.66	11.2	32.9	81.2	3.1
1.32	8.1	8.7	131.0	0.6

Figure 1: External Debt and Reserve Holdings of Emerging Countries in 2006 (as a % GDP)

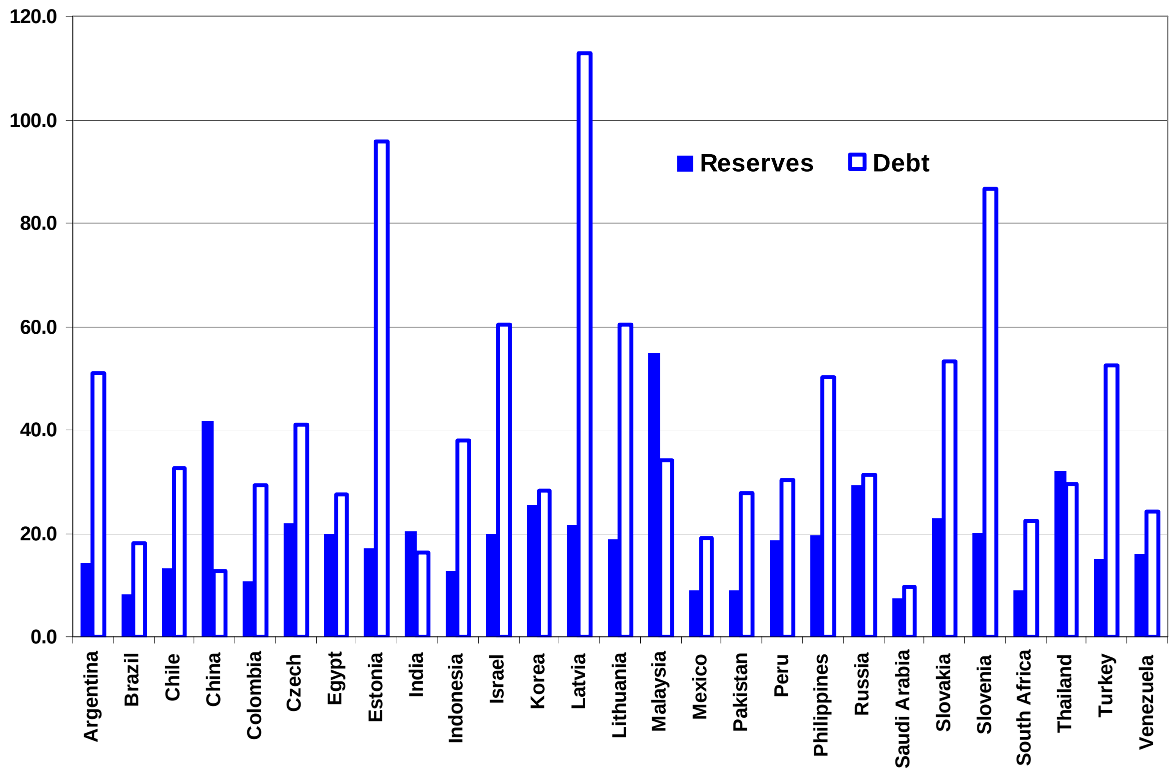

Figure 2a: Default Decision in the "No Reserves" Economy

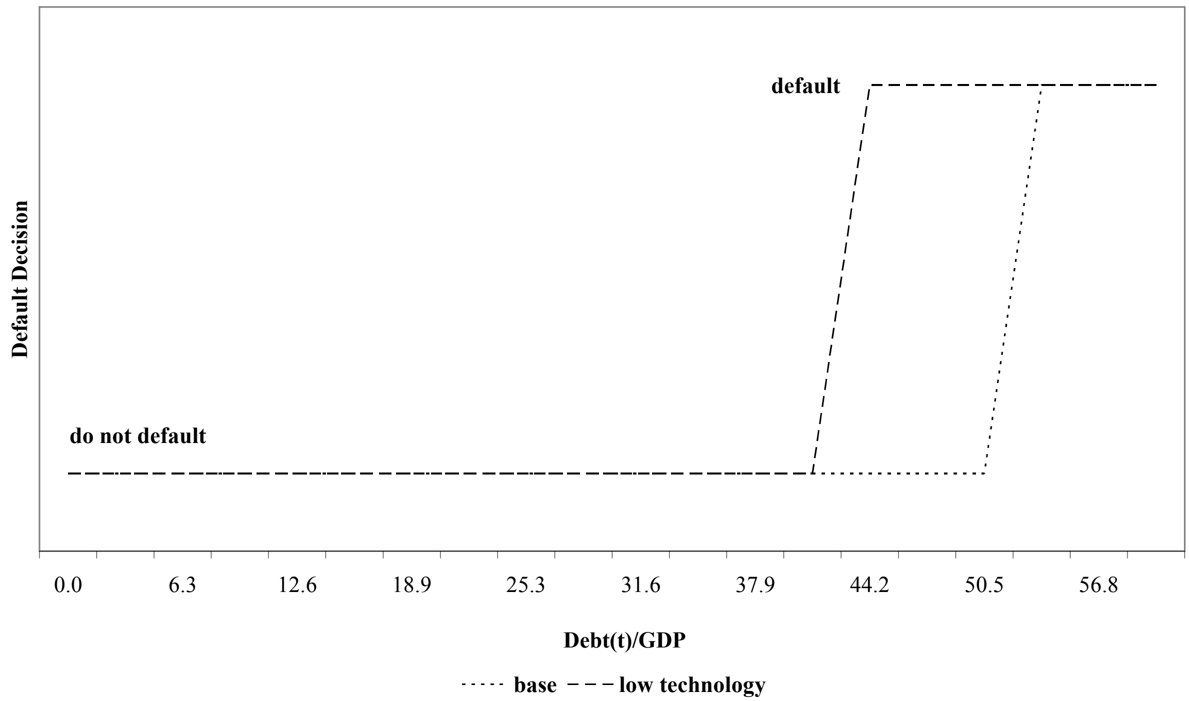

Figure 2b: Debt Decision in the "No Reserves" Economy

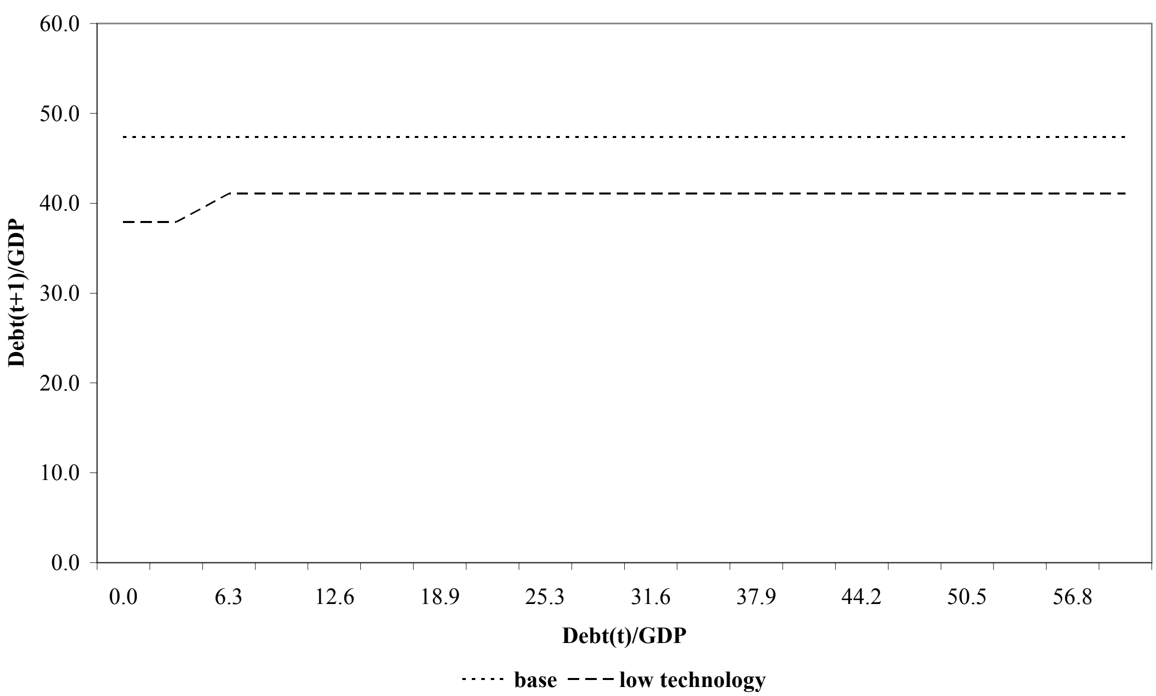

Figure 3a: Default Decision in the "Benchmark" Economy

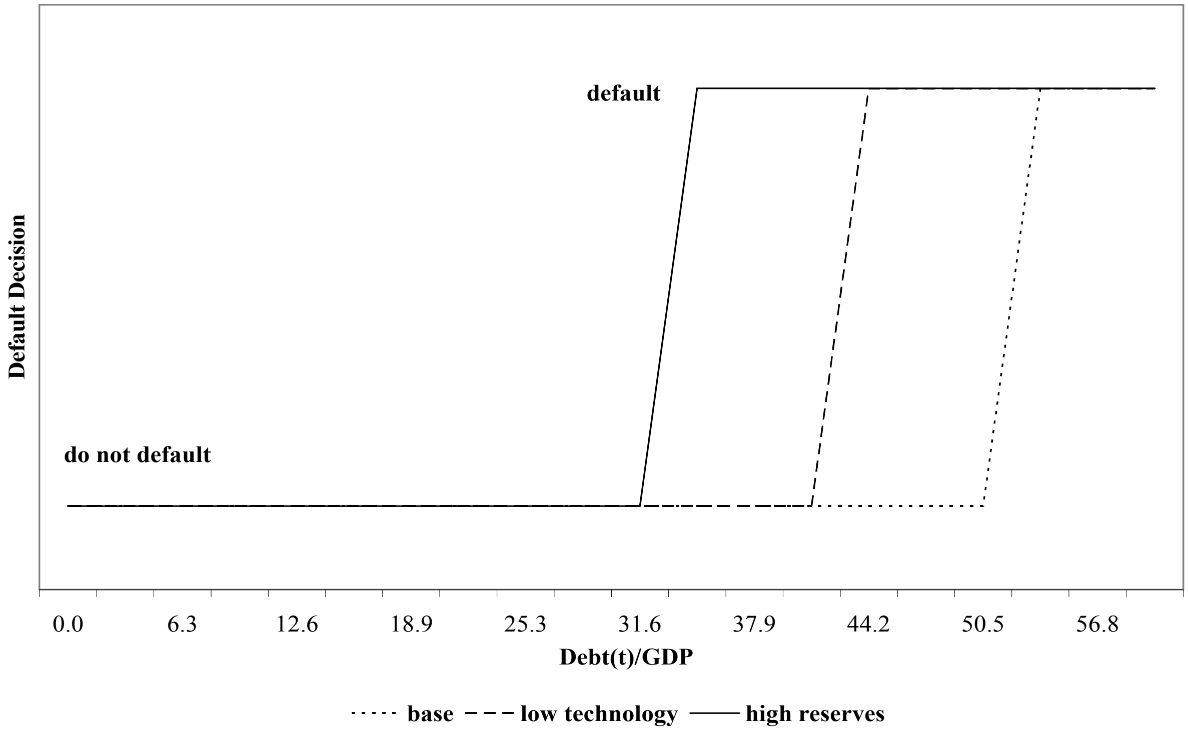

Figure 3b: Debt Decision in the "Benchmark" Economy

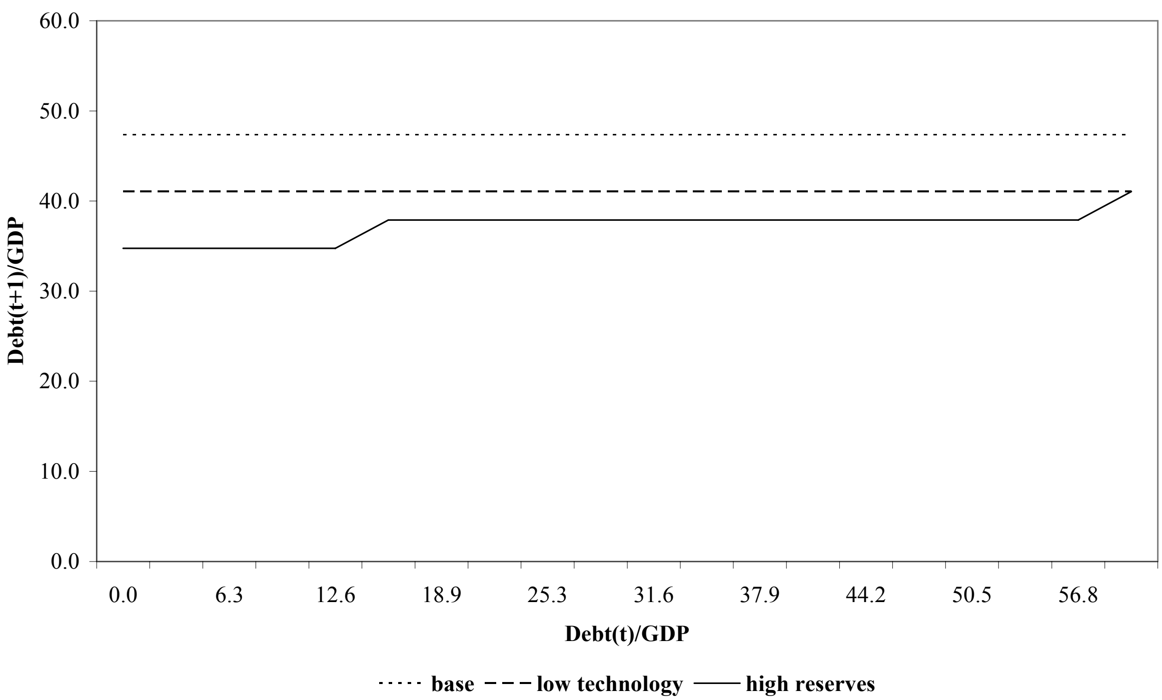

Figure 3c: Reserve Decision in the "Benchmark" Economy, with Access to the International Capital Market

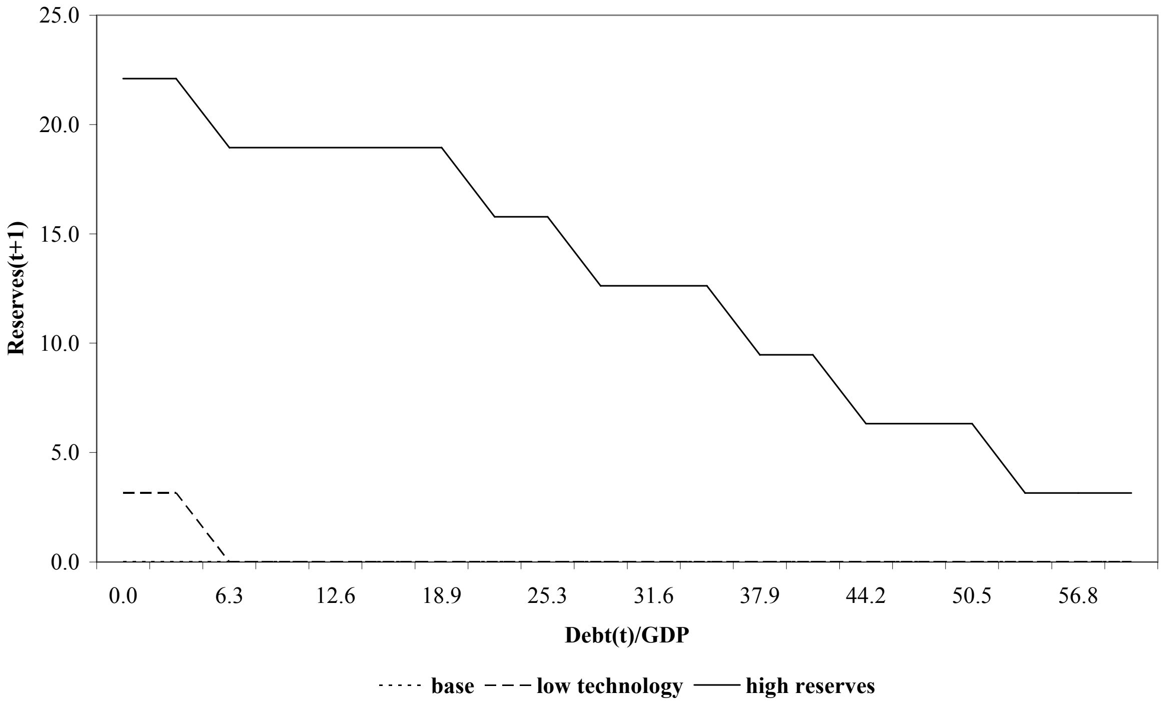

Figure 3d: Reserve Decision in the "Benchmark" Economy, with NO Access to the International Capital Market

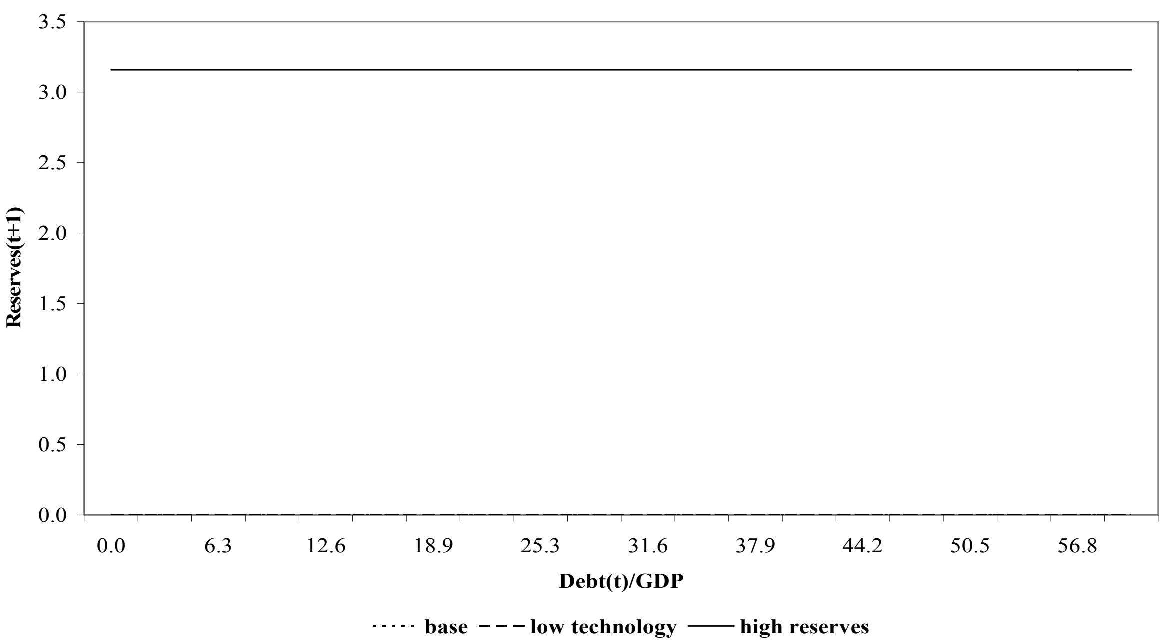

Figure 4a: Default Decision in the "Contagion" Economy

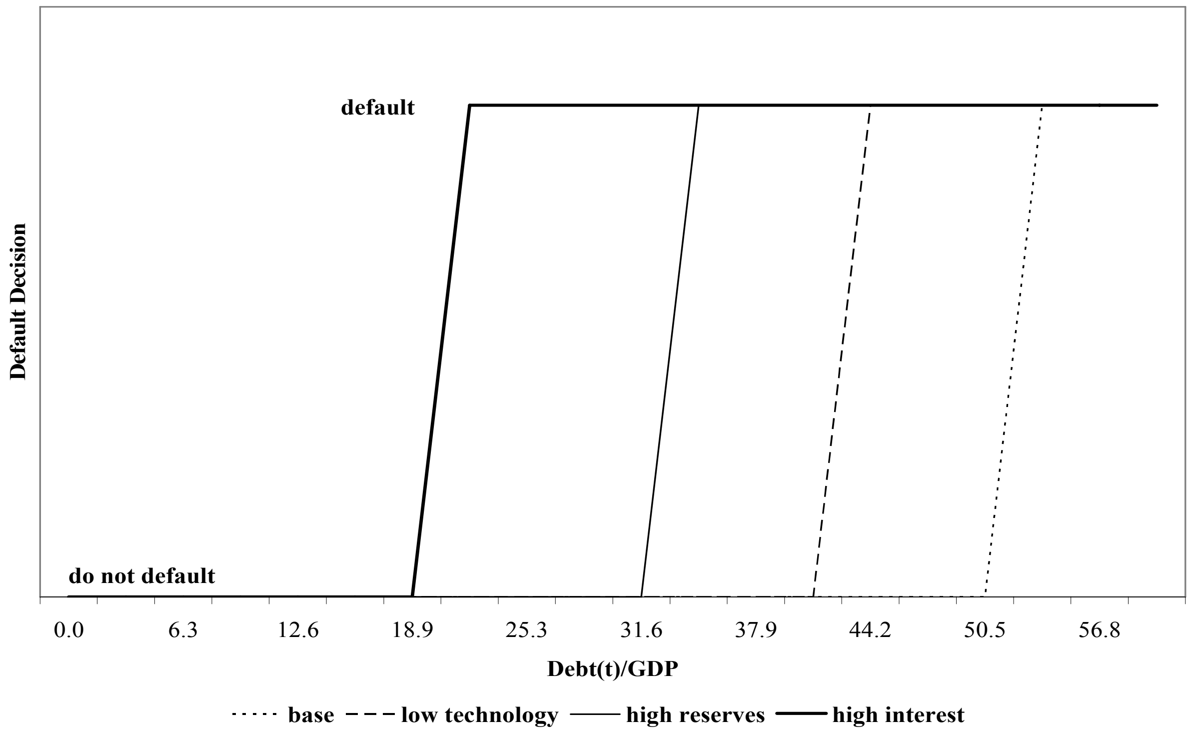

Figure 4b: Debt Decision in the "Contagion" Economy

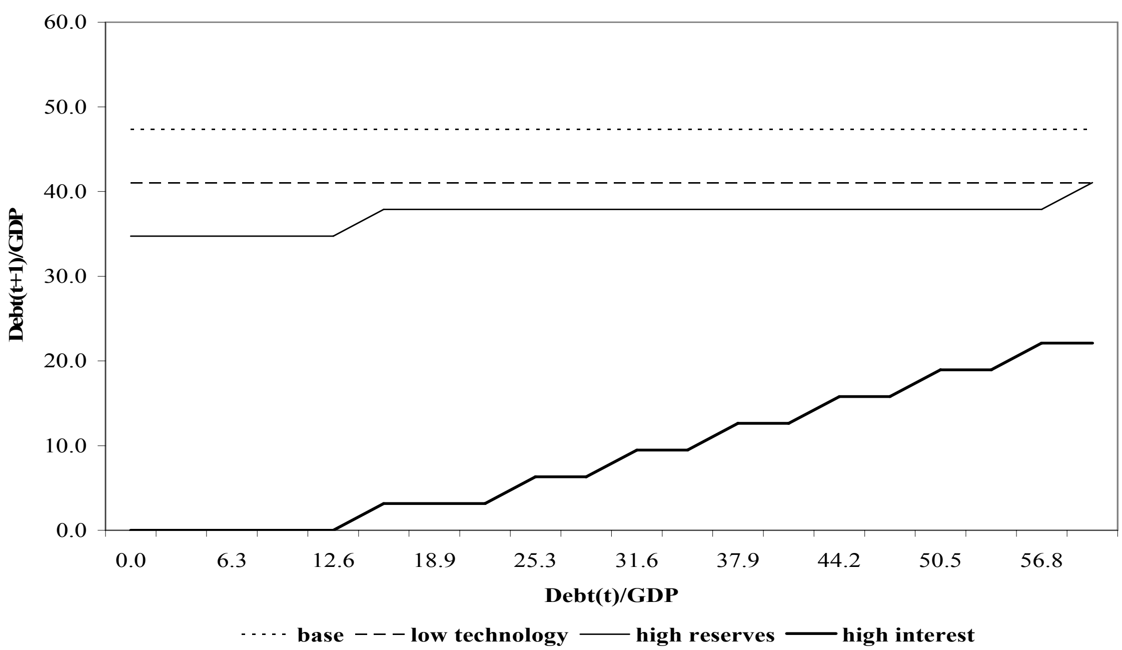

Figure 5a: Default Decision in the "Sudden-Stop" Economy

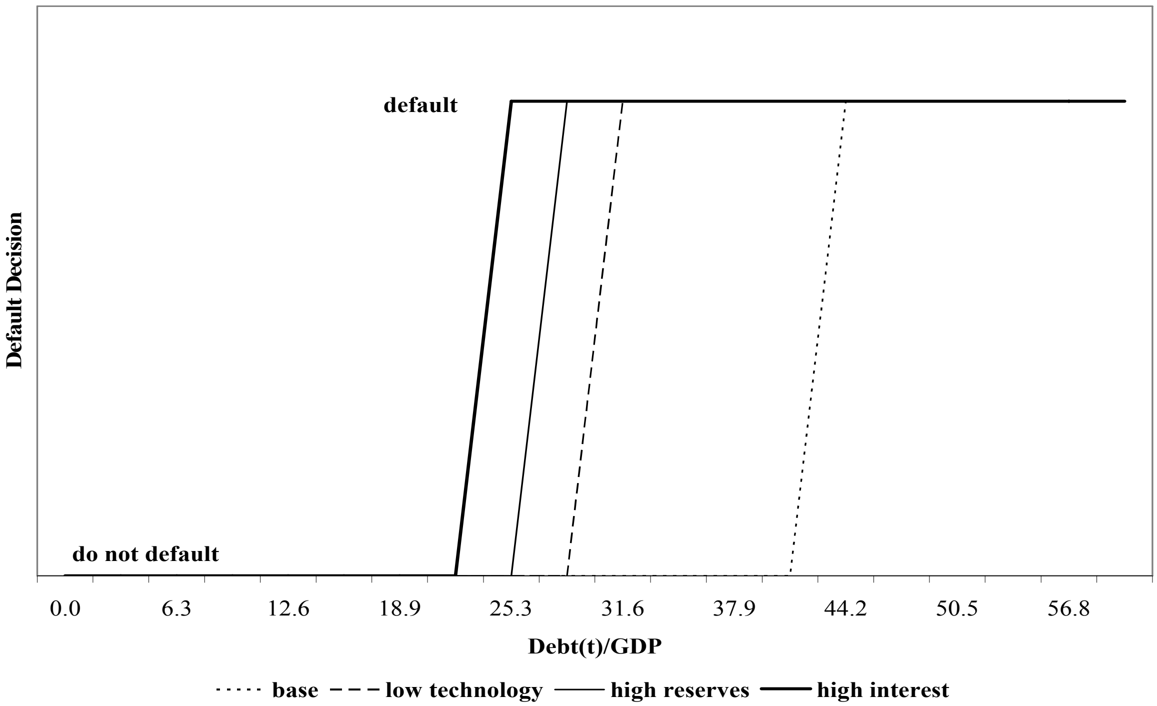

Figure 5b: Debt Decision in the "Sudden-Stop" Economy

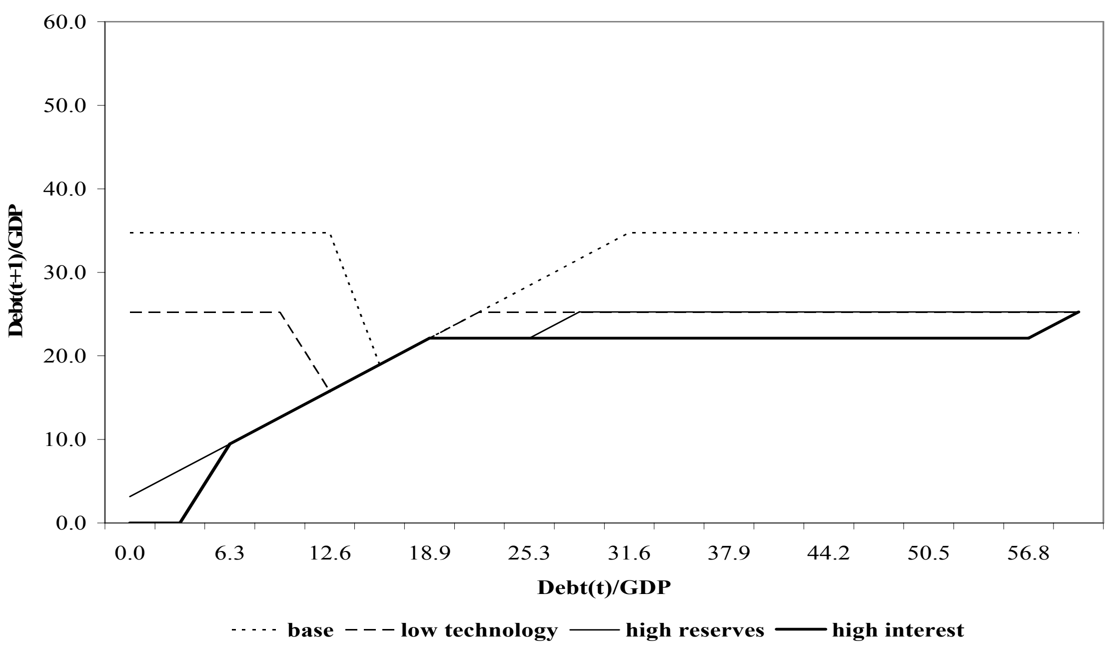

Figure 6a: Default Decision in the "Output Reduction" Economy

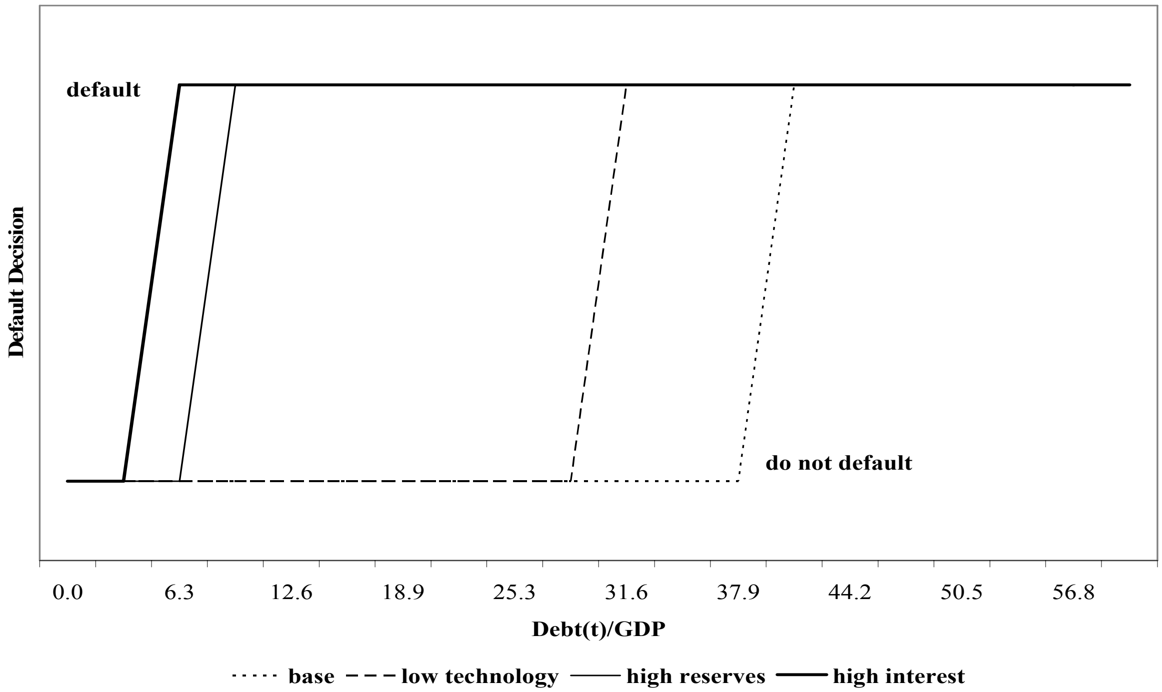

Figure 6b: Debt Decision in the "Output Reduction" Economy