NBER WORKING PAPER SERIES

ARE SHOCKS TO THE TERMS OF TRADE SHOCKS TO PRODUCTIVITY?

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Working Paper 13111
http://www.nber.org/papers/w13111

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
May 2007

ABSTRACT

International trade is frequently thought of as a production technology in which the inputs are exports and the outputs are imports. Exports are transformed into imports at the rate of the price of exports relative to the price of imports: the reciprocal of the terms of trade. Cast this way, a change in the terms of trade acts as a productivity shock. Or does it? In this paper, we show that this line of reasoning cannot work in standard models. Starting with a simple model and then generalizing, we show that changes in the terms of trade have no first-order effect on productivity when output is measured as chain-weighted real gross domestic product. The terms of trade do affect real income and consumption in a country, and we show how measures of real income change with the terms of trade at business cycle frequencies and during financial crises.

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1. Introduction

The terms of trade — the price of imports relative to the price of exports — vary greatly over time and country. This variation makes the terms of trade a natural candidate for explaining country performance. Intuitively, we can think about foreign trade as a production technology: a country's exports are the inputs to the technology, and these inputs are turned into outputs that are recorded as a country's imports. Exports are transformed into imports at the rate that is the ratio of the price of exports to the price of imports, which is just the reciprocal of the terms of trade. Viewed in this way, an increase in the terms of trade acts much like a technology shock: the same amount of exports now produces a smaller amount of imports.

In figures 1 and 2 we plot two well-known examples of terms of trade shocks. Figure 1 shows the contractions in real GDP in the United States that accompanied the sharp increases in the terms of trade, these coming largely from the OPEC oil embargo in 1973 and the Iranian Revolution in 1979. In fact, writing in the early 1980s, Hamilton (1983) points out that all but one of the post–World War II economic downturns in the United States up until that time had been preceded by an upward spike in the price of imported oil. Figure 2 presents the analogous data for Mexico, which in 1983 and 1995 suffered severe debt crises that brought with them sharp increases in the price of imports. These deteriorations of the terms of trade were accompanied by large contractions in real GDP. The correlation coefficient for changes in real GDP and changes in the terms of trade is −0.30 for the United States and −0.73 for Mexico. In these figures, we also plot total factor productivity (TFP), which is even more strongly correlated with the terms of trade for the United States and almost as correlated for Mexico, with correlation coefficients −0.54 and −0.71, respectively. Data like these certainly seem to support the intuition that shocks to the terms of trade affect the economy as shocks to productivity.

In this paper, we show that standard models do not support this line of reasoning. The problem lies in the construction of real gross domestic product (GDP), the most common measure of a country's output. The effect of a shock to the terms of trade on real GDP is not the same as the effect of a productivity shock and is highly dependent upon the method used to construct real GDP. When real GDP is constructed using the chain-weighting method specified in the United Nations System of National Accounts, terms of trade shocks have no first-order effects if inputs of factors are constant. When real GDP is constructed using fixed base year prices, the effect of a terms of trade shock is ambiguous: in some cases a deterioration of the

1

terms of trade can even increase real GDP! In this paper we bring this accounting to bear on the terms of trade and productivity relationship. As productivity is computed using GDP as the measure of output, the terms of trade cannot have a direct effect on a country's TFP. An increase in the terms of trade lowers the purchasing power of the country, which can be very painful in terms of consumption and welfare, but does not impact TFP directly.

The empirical literature on growth is replete with examples of the association of the terms of trade and output growth. Easterly, Kremer, Pritchett, and Summers (1993) study a large panel of countries to uncover the sources of long-run growth and aggregate volatility. They conclude that "shocks, especially to the terms of trade, play a large role in explaining variance in growth." In setting out a framework for studying developing country growth, Easterly, Islam, and Stiglitz (2001) find that the terms of trade volatility are more correlated with output volatility than are the standard deviations of many of the usual suspects: money growth, fiscal balance, and capital flows to name a few. Becker and Mauro (2005) use a large panel of countries to study how output drops are related to various external shocks and, using the likelihood of the shock and the associated output drop, compute the cost of the different shocks. They find that the costliest shocks, particularly for developing countries, are terms of trade shocks. The idea underlying many of these conclusions is succinctly summarized by Easterly, Islam, and Stiglitz (2001), who write, "For small *open* economies, adverse terms of trade shocks can have much the same effect as negative technology shocks, and this is one of the important differences between macroeconomics in these economies and that which underlies some of the traditional closed economy models."

In line with the above reasoning, we show that, in standard models, a shock to the terms of trade has an effect on consumption and welfare that is similar to a TFP shock. The analogy between the terms of trade and productivity breaks down when we calculate their effects on real GDP and productivity. When real GDP is measured at base period prices and domestic factors of production are held fixed, the effect of a terms of trade shock on real GDP is determined by the current terms of trade relative to the base period terms of trade. If the current import price is the same as the base period price, then the shock has no effect. If the current price is higher (lower) than the base period price, the effect is negative (positive). In this case, a change in the terms of trade can have a first-order effect on GDP, but this result follows from an artifact of the deflation method and not from an underlying structural relationship. When we consider real

GDP calculated as a chain-weighted index — as is now the standard for many countries — these artifacts disappear. Changes in the terms of trade do not have a first-order impact on real GDP, and TFP remains unchanged. These ideas are well understood by economists interested in index numbers and national income accounting. See, for example, Diewert and Morrison (1986) and Kohli (1983, 2004).

We expand the simple examples to show that our results easily generalize to richer environments. We show that a shock to the terms of trade can affect the supply of productive factors like labor and that the effects of these shocks, as in the simple examples, also have an ambiguous impact on real GDP. A third set of results shows how the effect of a terms of trade shock on real GDP and consumption varies with the elasticity of substitution between the domestic factors and the imported input. As the elasticity of substitution decreases, changes in the terms of trade have larger impacts on consumption but smaller impacts on real GDP. When the production function uses domestic and imported inputs in fixed proportions, changes in the terms of trade have a large impact on consumption, but no impact on real GDP.

If the terms of trade do not have a clear effect on measures of real GDP and TFP, where are their effects visible? In national accounting measures, the terms of trade affect gross domestic income (GDI). In a closed economy, real GDI and real GDP are the same, but in an open economy they are not. In section 5 we discuss alternative measures of real income, including the concept of command basis GDP used by the U.S. Bureau of Economic Analysis. These measures do respond to changes in the terms of trade and reflect how the purchasing power of an economy changes as foreign prices change.

The problems highlighted in this paper are part of a much larger issue faced by quantitative researchers. Developing good intuition is paramount in understanding how models work, and constructing analogies, such as the one between the terms of trade and productivity, can be very helpful in developing intuition. When evaluating the quantitative properties of a model, however, the statistics taken from the model must be constructed in the same way as they are in the data. As we show below, it is exactly in this dimension that the analogy between the terms of trade and productivity breaks down. In comparing models to data, the researcher is faced with two choices. Either the statistics can be collected from the model as they are by the economists at the statistical agencies, or the data can be reconfigured to mimic the constructs in the model. We take the first approach in sections 2 through 5 and show how the model's GDP

— as it would be constructed by a national income accountant — behaves in unexpected ways. In section 6 we take the second approach and use the data that underlies GDP to construct a measure that corresponds to the variables relevant in the terms of trade and productivity shocks analogy.

This paper identifies a puzzle. As figures 1 and 2 illustrate, deteriorations in the terms of trade are frequently accompanied by declines in productivity. If there is a causal mechanism that links shocks to the terms of trade to movements in productivity, researchers need to identify it.

2. Simple model

We begin by considering a simple model in which the single factor of production, labor, is supplied inelastically, and in which there are no distortions or rigidities. We subsequently show how our results extend to models with variable labor supply and models with distortions. We begin with the case where real GDP is measured in terms of base year prices because the calculations are simpler. We then show how the results can be extended to the case where real GDP is calculated with chain-weighted prices.

2.1. Closed economy

We first consider a closed economy in which labor is supplied inelastically, $\ell = \overline{\ell}$. Here we show that a fall in productivity in the intermediate goods sector produces a fall in GDP and in TFP, a result that does not carry over when we reinterpret the model as that of an open economy in which intermediate goods are imported.

There are two goods produced in this economy at each date t. The first good, the y good, is consumed by consumers and used in the production of the second good, the m good. The y good is produced using labor and intermediate inputs of the m good according to the production function

$$y_t = f(\ell_t, m_t). \tag{1}$$

We assume the production function, f, has constant returns to scale, is concave, and is continuously differentiable. We later analyze the case where f is a fixed proportions production function. The m good is produced using only intermediate inputs of the q good. The production function is

$$m_t = \frac{x_t}{a_t}, \tag{2}$$

where a_t is a unit output requirement that is stochastic. We assume that the m good is sold in competitive markets at price p_t. The m good producer chooses m_t and x_t to minimize costs and to earn 0 profits. The condition that equilibrium profits be 0 is

$$p_t = a_t. \tag{3}$$

The feasibility condition is

$$c_t + x_t = y_t. \tag{4}$$

We normalize the price of the y good to be 1. Expenditure on final goods in the closed economy is only consumption, so on the expenditure side, real GDP, Y_t, is

$$Y_t = c_t = y_t - x_t. \tag{5}$$

On the output side, real GDP is calculated as the base period value of gross output minus the base period value of intermediate inputs:

$$Y_t = \left(y_t + p_0 m_t \right) - \left(p_0 m_t + x_t \right) = y_t - x_t, \tag{6}$$

where $p_0 = a_0$ is the base period price of the m good.

To calculate the impact of an increase in a, a decline in productivity in the m good sector, we note that a competitive economy chooses m_t to solve

$$\max f(\bar{\ell}, m_t) - a_t m_t. \tag{7}$$

The first-order condition for this problem is

$$f_m(\bar{\ell}, m_t) = a_t. \tag{8}$$

Using the implicit function theorem we obtain

$$m'(a_t) = \frac{1}{f_{mm}(\bar{\ell}, m(a_t))} < 0. \tag{9}$$

Suppose that $a_{t+1} > a_t$ increases, that is, that productivity in the intermediate goods sector falls. How does real GDP change? The first-order change is

$$Y(a_{t+1}) - Y(a_t) \approx Y'(a_t)(a_{t+1} - a_t),\tag{10}$$

where

$$Y(a_t) = f(\overline{\ell}, m(a_t)) - a_t m(a_t).\tag{11}$$

Differentiating (11), we use (8) to obtain

$$Y'(a_t) = f_m(\overline{\ell}, m(a_t))m'(a_t) - a_t m'(a_t) - m(a_t) = -m(a_t) < 0.\tag{12}$$

Real GDP and productivity decline.

Equation (12) provides an expression for first-order changes in real GDP when the production function f is continuously differentiable. When f is a fixed proportions function, where

$$y_t = \min[\ell_t, m_t / b],\tag{13}$$

we can obtain exact expressions. In this case, $m(a_t) = b\overline{\ell}$ and real GDP is

$$Y(a_t) = \overline{\ell} - a_t b\overline{\ell},\tag{14}$$

which implies that the first-order expression in (12) is exact.

2.2. Open economy

Now consider an open economy with the same structure as that of the closed economy in which m is an imported intermediate input, x are exports of the y good, and p is the terms of trade. To make the analysis identical to that in the closed economy, we assume balanced trade,

$$p_t m_t = x_t.\tag{15}$$

By comparing (15) to (2), we see how the terms of trade in the open economy, p, and the productivity parameter in the closed economy, a, are similar. Real GDP is now

$$Y_t = c_t + x_t - p_0 m_t = y_t - p_0 m_t = f(\overline{\ell}, m_t) - p_0 m_t,\tag{16}$$

where p_0 is price of imports (relative to exports) in the base year. A competitive economy continues to choose m_t to solve

$$\max f(\overline{\ell}, m_t) - p_t m_t \tag{17}$$

with the corresponding first-order condition defining an implicit function $m(p)$:

$$f_m(\overline{\ell}, m_t) = p_t \tag{18}$$

$$m'(p_t) = \frac{1}{f_{mm}(\overline{\ell}, m(p_t))} < 0 \tag{19}$$

An increase in p — a deterioration in the terms of trade — has the identical impact on consumption and welfare as the decline in productivity in the closed economy. But what happens to real GDP and productivity?

$$Y(p_t) = f(\overline{\ell}, m(p_t)) - p_0 m(p_t) \tag{20}$$

$$Y'(p_t) = f_m(\overline{\ell}, m(p_t))m'(p_t) - p_0 m'(p_t) = (p_t - p_0)m'(p_t). \tag{21}$$

To the extent that the terms of trade in the period before the deterioration takes place, p_t, are close to the terms of trade in the base period, p_0, there is no first-order change in measured real GDP or in productivity. Notice that, if $p_t < p_0$, real GDP may even increase in response to a negative terms of trade shock. When we use chain-weighted real GDP, this sensitivity to the base period price is eliminated.

In the case where f is the fixed proportions function (13), real GDP is

$$Y(p_t) = \overline{\ell} - p_0 b \overline{\ell}, \tag{22}$$

which does not change at all as the terms of trade change. Notice that the fixed proportions case is where consumption,

$$c(p_t) = (1 - p_t b)\overline{\ell}, \tag{23}$$

and therefore welfare, falls the most in response to a deterioration in the terms of trade.

The intuition for our results is simple. A deterioration in the terms of trade causes domestic output to fall, but it also causes imports valued at base period prices to fall. Real GDP

7

is the difference between the two, (20), and the envelope theorem says that the two effects cancel to first order. With fixed proportions production, the two effects are exactly equal.

3. Extensions to the simple model

In this section, we add variable labor supply and distortions to the model. To the extent that shocks to the terms of trade change the labor supply, they can change real GDP, but not productivity. Real GDP can even rise in response to a negative terms of trade shock, although welfare falls. The model with distortions is more complicated. We analyze a model with tariff distortions and show that an increase in tariffs acts like a shock to the terms of trade but has no first-order effects on GDP if initial tariffs are 0.

3.1. Variable labor supply

Suppose that there is a representative consumer who values both consumption and leisure $z = \overline{\ell} - \ell$. The utility function of this consumer is $u(c,z)$, and the consumer solves

$$\max u(c_t, \overline{\ell} - \ell_t) \tag{24}$$

$$\text{s.t. } c_t = w_t \overline{\ell},$$

where $w_t = f_\ell(\ell_t, m_t)$. The first-order condition for this problem is

$$w_t u_c(c_t, \overline{\ell} - \ell_t) = u_z(c_t, \overline{\ell} - \ell_t), \tag{25}$$

which implicitly defines the function $\ell(w)$:

$$w_t u_c(w_t \ell(w_t), \overline{\ell} - \ell(w_t)) = u_z(w_t \ell(w_t), \overline{\ell} - \ell(w_t)) \tag{26}$$

$$\ell'(w_t) = -\frac{u_c(c_t, \overline{\ell} - \ell_t) + u_{cc}(c_t, \overline{\ell} - \ell_t) w_t \ell_t - u_{cz}(c_t, \overline{\ell} - \ell_t) \ell_t}{u_{cc}(c_t, \overline{\ell} - \ell v) w_t^2 - 2 u_{cz}(c_t, \overline{\ell} - \ell_t) w_t + u_{zz}(c_t, \overline{\ell} - \ell_t)}. \tag{27}$$

Consider the constant elasticity of substitution case, where

$$u(c,z) = \begin{cases} \left(c^\rho + \gamma z^\rho - 1 - \gamma\right)/\rho & \text{for } \rho \leq 1, \ \rho \neq 0 \\ \log c + \gamma \log z & \text{for } \rho = 0 \end{cases}. \tag{28}$$

Here

$$\ell'(w) = \frac{\rho c^{\rho-1}}{(1-\rho)\left(w^2 c^{\rho-2} + \gamma(\bar{\ell}-\ell)^{\rho-2}\right)}. \tag{29}$$

Notice that $\ell'(w)$ has the same sign as ρ.

How do w and m vary with p? We can use the first-order conditions for profit maximization to define implicit functions $w(p)$ and $m(p)$

$$f_\ell(\ell(w(p)), m(p)) = w(p) \tag{30}$$

$$f_m(\ell(w(p)), m(p)) = p. \tag{31}$$

Differentiating, we obtain

$$f_{\ell\ell}(\ell, m)\ell'(w)w'(p) + f_{\ell m}(\ell, m)m'(p) = w'(p) \tag{32}$$

$$f_{\ell m}(\ell, m)\ell'(w)w'(p) + f_{mm}(\ell, m)m'(p) = 1 \tag{33}$$

We can solve this system of two equations in the two unknowns $w'(p)$ and $m'(p)$ to obtain

$$w'(p) = \frac{f_{\ell m}(\ell, m)}{f_{mm}(\ell, m) - \left(f_{mm}(\ell, m)f_{\ell\ell}(\ell, m) - f_{\ell m}(\ell, m)^2\right)\ell'(w)} \tag{34}$$

$$m'(p) = \frac{1 - f_{\ell\ell}(\ell, m)\ell'(w)}{f_{mm}(\ell, m) - \left(f_{mm}(\ell, m)f_{\ell\ell}(\ell, m) - f_{\ell m}(\ell, m)^2\right)\ell'(w)} \tag{35}$$

As long as the denominator of these expressions is negative, real wages fall with an increase in p, a deterioration of the terms of trade. If $1 - f_{\ell\ell}(\ell, m)\ell'(w)$ is positive, imports fall. Notice that we can construct perverse examples if $\rho < 0$, which implies that $\ell'(w) < 0$, where deteriorations to the terms of trade force the consumer to work more.

Letting $c(p) = f(\ell(w(p)), m) - pm(p)$, we can use the envelope theorem to show that the change in consumer welfare is

$$\frac{d}{dp}u(c(p_t), \bar{\ell} - \ell(w(p_t))) = -u_c(c_t, \bar{\ell} - \ell_t)m_t < 0. \tag{36}$$

What happens to real GDP and productivity when the terms of trade change? First consider real GDP:

$$Y(p_t) = f(\ell(w(p_t)), m(p_t)) - p_0 m(p_t) \tag{37}$$

$$Y'(p_t) = f_\ell(\ell_t, m_t)\ell'(w_t)w'(p_t) + f_m(\overline{\ell}, m_t)m'(p_t) - p_0 m'(p_t) \tag{38}$$

$$Y'(p_t) = f_\ell(\ell_t, m_t)\ell'(w_t)w'(p_t) + (p_t - p_0)m'(p_t) \tag{39}$$

Notice that real GDP can either rise or fall with an increase in the terms of trade, but, if $\ell'(w_t) > 0$, which implies that $w'(p_t) < 0$, and if $(p_t - p_0)m'(p_t)$ is small, real GDP falls.

Now consider productivity $Y(p_t)/\ell(w(p_t))$:

$$\frac{d}{dp_t}\frac{Y(p_t)}{\ell(w(p_t))} = \frac{\ell(w_t)Y'(p_t) - Y(p_t)\ell'(w_t)w'(p_t)}{\ell(w_t)^2}. \tag{40}$$

Substituting in the expressions $Y(p_{t-1})$ in (37) and for $Y'(p_{t-1})$ in (39), we obtain

$$\frac{d}{dp_t}\frac{Y(p_t)}{\ell(w(p_t))} = \frac{(p_t - p_0)\left(\ell_t m'(p_t) - m_{t-1}\ell'(w_t)w'(p_t)\right)}{\ell_t^2}. \tag{41}$$

Once again, this term is close to 0 if $p_t - p_0$ is close to 0, and this sensitivity to base period prices is eliminated with chain-weighted real GDP.

In the fixed coefficients case, real GDP is

$$Y(p_t) = (1 - p_0 b)\ell(w(p_t)), \tag{42}$$

and productivity does not change as the terms of trade change.

3.2. Tariffs

In this section, we consider a model with tariff distortions. Tariff changes are much like terms of trade shocks, except that tariff revenues are spent domestically. We model tariff revenues as a lump-sum rebate to the representative consumer. In the presence of tariff distortions, changes in the terms of trade can have first-order effects on real GDP and productivity, although these effects are small to the extent that tariffs are small or the production function f is close to fixed coefficients.

Once again, a useful benchmark is provided by the closed economy model. We assume that the government imposes an *ad valorem* tax τ on intermediate inputs. To keep the

discussion simple, assume again that the labor supply is fixed. A competitive economy chooses m_t to solve

$$\max f(\bar{\ell}, m_t) - (1 + \tau_t) a_t m_t . \tag{43}$$

In the case where f is continuously differentiable, the first-order condition is

$$f_m(\bar{\ell}, m_t) = (1 + \tau_t) a_t . \tag{44}$$

We first consider the case where $\tau_t = \tau$ is fixed and a_t fluctuates. The implicit function theorem implies that

$$m'(a_t) = \frac{1 + \tau}{f_{mm}(\bar{\ell}, m(a_t))} < 0 . \tag{45}$$

How do real GDP change and consumption change?

$$Y(a_t) = f(\bar{\ell}, m(a_t)) - a_t m(a_t) \tag{46}$$

$$Y'(a_t) = \left(f_m(\bar{\ell}, m(a_t)) - a_t \right) m'(a_t) - m(a_t) = \tau a_t m'(a_t) - m(a_t) \tag{47}$$

Notice that the tariff distortion introduces an additional term into (12).

We now consider the case where $a_t = a$ is fixed and τ_t fluctuates:

$$m'(\tau_t) = \frac{a}{f_{mm}(\bar{\ell}, m(\tau_t))} < 0 \tag{48}$$

$$Y(\tau_t) = f(\bar{\ell}, m(\tau_t)) - am(\tau_t) \tag{49}$$

$$Y'(\tau_t) = \tau_t a m'(\tau_t) \tag{50}$$

To the extent that the tax before the increase, τ_t, is close to 0, the first-order impact of increasing it is small. In the fixed coefficients case, where $m'(\tau_t) = 0$, real GDP does not change.

In the calculations in the open economy case, where τ is an *ad valorem* tariff on imports, fluctuations in tariffs have the same impact on real GDP as fluctuations are the same:

$$Y'(p_t) = ((1 + \tau) p_t - p_0) p_t m'(p_t) \tag{51}$$

$$Y'(\tau_t) = ((1 + \tau_t) p_t - p_0) p_t m'(\tau_t) \tag{52}$$

11

Notice that the effect on real GDP of an increase in the terms of trade, or of an increase in the tariff, is close to 0 to the extent that either $(1+\tau_t)p_t - p_0$ is close to 0 or to the extent that f is close to a fixed proportions function. In terms of the impact on consumption and welfare, the two cases are very different:

$$c'(p_t) = ((1+\tau)p_t - p_0)p_t m'(p_t) - m(p_t) \tag{53}$$

$$c'(\tau_t) = ((1+\tau_t)p_t - p_0)p_t m'(\tau_t) \tag{54}$$

Consumption falls much more in the case of a deterioration of the terms of trade than it does when tariffs are increased because the revenue generated is rebated to the representative consumer.

4. Chain-weighted real GDP

Currently, the U.S. Bureau of Economic Analysis in its National Income and Product Accounts (NIPA) and the U.N. Statistics Division in its System of National Accounts (SNA) recommend the use of chain-weighted price indices to deflate GDP. In this section we show how the results of the previous two sections carry over to chain-weighted real GDP. Although the calculations are a little more complicated, an advantage of using chain-weighted real GDP is that the annoying terms involving base period prices disappear.

The United States' NIPA accounting uses Fisher chain-weights. So does Statistics Canada. Most countries that follow U.N. SNA national income accounting use Laspeyres chain-weighting, although both Fisher weighting and Paasche weighting are allowed. We start by showing that Fisher chain-weighting eliminates the terms involving $p_0 - p_t$ and then briefly discuss how this result extends to Laspeyres weighing and Paasche weighting.

To keep our discussion simple, we consider the open economy model with fixed labor supply and no tariffs. The extension to the more general model is obvious. Fisher chain-weighted real GDP is

$$Y_t(p_t) = \frac{f(\bar{\ell}, m(p_t)) - p_t m(p_t)}{P_t}, \tag{55}$$

where the Fisher chain-weighted price index is the geometric average of the Paasche and the Laspeyres index between the current period and the previous period:

$$P_{t+1} = \left(\frac{f(\overline{\ell}, m(p_{t+1})) - p_{t+1} m(p_{t+1})}{f(\overline{\ell}, m(p_{t+1})) - p_t m(p_{t+1})}\right)^{\frac{1}{2}} \left(\frac{f(\overline{\ell}, m(p_t)) - p_{t+1} m(p_t)}{f(\overline{\ell}, m(p_t)) - p_t m(p_t)}\right)^{\frac{1}{2}} P_t, \tag{56}$$

where $P_0 = 1$ in the base period.

We can write chain-weighted real GDP as

$$Y(p_{t+1}) = \frac{f(\overline{\ell}, m(p_{t+1})) - p_{t+1} m(p_{t+1})}{\left(\dfrac{f(\overline{\ell}, m(p_{t+1})) - p_{t+1} m(p_{t+1})}{f(\overline{\ell}, m(p_{t+1})) - p_t m(p_{t+1})}\right)^{\frac{1}{2}} \left(\dfrac{f(\overline{\ell}, m(p_t)) - p_{t+1} m(p_t)}{f(\overline{\ell}, m(p_t)) - p_t m(p_t)}\right)^{\frac{1}{2}} P_t}, \tag{57}$$

which yields a Fisher chain-weighted quantity index.

$$Y(p_{t+1}) = \left(\frac{f(\overline{\ell}, m(p_{t+1})) - p_{t+1} m(p_{t+1})}{f(\overline{\ell}, m(p_t)) - p_{t+1} m(p_t)}\right)^{\frac{1}{2}} \left(\frac{f(\overline{\ell}, m(p_{t+1})) - p_t m(p_{t+1})}{f(\overline{\ell}, m(p_t)) - p_t m(p_t)}\right)^{\frac{1}{2}} Y(p_t). \tag{58}$$

The natural logarithm of chain-weighted real GDP is

$$\log Y(p_{t+1}) = \frac{1}{2} \log\left(f(\overline{\ell}, m(p_{t+1})) - p_{t+1} m(p_{t+1})\right) - \frac{1}{2} \log\left(f(\overline{\ell}, m(p_t)) - p_{t+1} m(p_t)\right)$$
$$+ \frac{1}{2} \log\left(f(\overline{\ell}, m(p_{t+1})) - p_t m(p_{t+1})\right) + \frac{1}{2} \log\frac{\left(f(\overline{\ell}, m(p_t)) - p_t m(p_t)\right)}{P_t^2} \tag{59}$$

Differentiating, we obtain

$$\frac{d \log Y(p_{t+1})}{dp_{t+1}} = \frac{f_m(\overline{\ell}, m(p_{t+1})) m'(p_{t+1}) - p_{t+1} m'(p_{t+1}) - m(p_{t+1})}{2\left(f(\overline{\ell}, m(p_{t+1})) - p_{t+1} m(p_{t+1})\right)}$$
$$+ \frac{m(p_t)}{2\left(f(\overline{\ell}, m(p_t)) - p_{t+1} m(p_t)\right)} + \frac{f_m(\overline{\ell}, m(p_{t+1})) m'(p_{t+1}) - p_t m'(p_{t+1})}{2\left(f(\overline{\ell}, m(p_{t+1})) - p_t m(p_{t+1})\right)} \tag{60}$$

Since $f_m(\overline{\ell}, m(p_{t+1})) = p_{t+1}$, this simplifies to

$$\frac{d \log Y(p_{t+1})}{dp_{t+1}} = -\frac{m(p_{t+1})}{2\left(f(\overline{\ell}, m(p_t)) - p_t m(p_t)\right)} + \frac{m(p_t)}{2\left(f(\overline{\ell}, m(p_t)) - p_{t+1} m(p_t)\right)}$$
$$+ \frac{(p_{t+1} - p_t) m'(p_{t+1})}{2\left(f(\overline{\ell}, m(p_{t+1})) - p_t m(p_{t+1})\right)}. \tag{61}$$

Evaluating this expression at $p_{t+1} = p_t$, we obtain

$$\frac{d \log Y(p_t)}{dp_{t+1}} = 0,\tag{62}$$

which implies that any effect of changes in the terms of trade on chain-weighted real GDP is of second order.

Suppose that, instead of Fisher weighting, the national statistics agency uses Laspeyres weighting. The relationship that corresponds to (58) is

$$Y(p_{t+1}) = \left(\frac{f(\overline{\ell}, m(p_{t+1})) - p_t m(p_{t+1})}{f(\overline{\ell}, m(p_t)) - p_t m(p_t)} \right) Y(p_t).\tag{63}$$

Notice that, in this case, the deflator in (55) is

$$P_{t+1} = \left(\frac{f(\overline{\ell}, m(p_{t+1})) - p_{t+1} m(p_{t+1})}{f(\overline{\ell}, m(p_{t+1})) - p_t m(p_{t+1})} \right) P_t,\tag{64}$$

that is, a Paasche price index. In the case where the prices in (63) are those of the second period, p_{t+1} — that is, where the quantity index is Paasche — the deflator in (64) uses the quantity weights in the first period, $f(\overline{\ell}, m(p_t))$ and $m(p_t)$ — that is, the price index is Laspeyres. In either case, a simple argument that follows that in equations (59)–(62), but with less algebra, proves that the first-order effect of a change in the terms of trade on chain-weighted real GDP is 0.

5. Elasticity of substitution

Except for the case where the production function f combines domestic inputs and imported inputs in fixed proportions — where there are analytical formulas for real GDP — we have relied on the implicit function theorem and first-order approximations to determine the impact of terms of trade shocks on real GDP. In this section, we investigate the impacts of large shocks for the case where f is constant elasticity of substitution:

$$f\left(\ell_t, m_t\right) = \left((1-\beta)\ell_t^{\rho} + \beta m_t^{\rho} \right)^{\frac{1}{\rho}},\tag{65}$$

where the parameter β determines the share imports in production. The elasticity of substitution between imported intermediates and labor is $\sigma = 1/(1-\rho)$. This elasticity is frequently referred to as the Armington elasticity. Producers choose inputs ℓ_t and m_t to minimize costs,

$$\min w_t \ell_t + p_t m_t \tag{66}$$

$$\text{s.t.} \ \left((1-\beta)\ell_t^\rho + \beta m_t^\rho\right)^{\frac{1}{\rho}} \geq y,$$

and, in equilibrium, profits are 0,

$$\left((1-\beta)\ell_t^\rho + \beta m_t^\rho\right)^{\frac{1}{\rho}} - w_t \ell_t - p_t m_t = 0. \tag{67}$$

Fixing $\ell_t = \overline{\ell}$, we can use these conditions to obtain the demand for imports

$$m(p_t) = (1-\beta)^{\frac{1}{\rho}} \overline{\ell} \left(p_t^{\frac{\rho}{1-\rho}} \beta^{\frac{-\rho}{1-\rho}} - \beta \right)^{\frac{-1}{\rho}}. \tag{68}$$

This allows us to express real GDP in base period prices as

$$Y(p_t) = \left((1-\beta)\overline{\ell}^\rho + \beta m(p_t)^\rho\right)^{\frac{1}{\rho}} - p_0 m(p_t). \tag{69}$$

Before studying how real GDP changes are related to the elasticity of substitution, we must choose a value for β. The first-order conditions for the producers' problem (66) imply that

$$\frac{m_t}{\left((1-\beta)\overline{\ell}^\rho + \beta m_t^\rho\right)^{\frac{1}{\rho}}} = \left(\frac{p_t}{\beta}\right)^{\frac{1}{\rho-1}}. \tag{70}$$

The left-hand side of the equation is the share of imports in gross output. For each value of ρ we choose the parameter β so that imports make up 8 percent of gross output when the import price is 1. This value is consistent with U.S. data, where imports average 7.8 percent of gross output in the NAICS classified data over the period 1998–2005.

In figure 3, we plot the changes in real GDP that result from changes in the terms of trade for different values of the elasticity of substitution. In this example, we have assumed that the terms of trade in the period prior are the same as those in the base year, so the first-order effect is

15

0 in equation (21). The first-order effect can be seen in the figure, where the change in real GDP from a small change in the terms of trade is negligible. The figure also shows the impact of larger changes in the terms of trade. The elasticity of substitution between domestic and foreign inputs is commonly specified at or around 2.0 in international real business cycle models. The average annual change in absolute value of the terms of trade for OECD countries is 3.5 percent. A 3.5 percent increase in the relative price of imports leads to a 0.0058 percent decrease in real GDP when the elasticity of substitution between domestic and foreign factors is 2.0. When the elasticity of substitution is 6.67, the same deterioration of the terms of trade causes a 0.032 percent decrease in real GDP, and, when the elasticity is 0.33, a 3.5 percent increase in the terms of trade decreases real GDP by 0.0019 percent.

Although an increase in the terms of trade has little effect on real GDP, its effect on consumption can be significant. In figure 4, we plot the change in consumption that results from changing the terms of trade. The less substitutable imports are in production, the more painful are increases in the price of imports. In the fixed proportions case, in which real GDP does not change at all with the terms of trade, the consumption and welfare effects of a change in the terms of trade are the largest.

6. Alternative income measures

If real GDP does not accurately reflect the real purchasing power of an open economy, are there measures that do? In this section we discuss measures of real domestic income that incorporate the terms of trade.

GDP in current prices represents the current value of both production and income in both open and closed economies. Real GDP and real income, though equivalent in a closed economy, are not necessarily equivalent in an open economy. The difference between real GDP and real gross domestic income (GDI) in the open economy arises from the deflation of the trade balance. Real GDP is computed by deflating the current value of the components of GDP by their respective implicit price deflators, P,

$$GDP_t = \frac{C_t}{P_t^C} + \frac{I_t}{P_t^I} + \frac{G_t}{P_t^G} + \frac{X_t}{P_t^X} - \frac{M_t}{P_t^M}, \tag{71}$$

while one method of computing real income is

$$GDI_t = \frac{C_t}{P_t^C} + \frac{I_t}{P_t^I} + \frac{G_t}{P_t^G} + \frac{X_t - M_t}{P_t^M}.$$ (72)

Notice that real GDP, as a measure of production, values exports as an output and imports as an input, while real GDI values the nominal trade balance in terms of the amount of imports that can be purchased. The U.S. Bureau of Economic Analysis (2006) refers to GDI as *command-basis* GDP, rather than real gross domestic income as it is defined in the United Nations' 1993 System of National Accounts (2001). United Nations (2001) also allows for several definitions of real GDI that differ by the index used to deflate $X_t - M_t$, including the export price index or the domestic absorption price index.

The United Nations (2001) allows for various definitions of the GDI because there is no natural way to deflate the proceeds from foreign trade. The debate over the real trade balance in the national income and product accounts is a long one, going back to the early era of national income accounting. Working within the NIPA framework, Nicholson (1960), Bjerke (1968), and others proposed different methods of deflating the income from trade, with most of them arguing for either the import or the export price deflator, though some (Burge and Geary 1957) propose using one deflator when the trade balance is positive and another when the trade balance is negative. As index number theory progressed, prominent researchers in the field developed alternative indices of welfare and productivity (Diewert and Morrison 1986) and real domestic income (Kohli 2004) that accounted for the terms of trade.

Which method should we use? Mahdavy and Silver (1989) compare these methods and find that, for most industrial countries, the choice of deflator is not important. They find the choice of deflator can be important for non-industrial countries. For simplicity, and to be consistent with the methods used by the BEA, we will use the command basis GDP measure (72) in what follows.

Command GDP offers an alternative way of viewing a country's performance. For countries in which either the terms of trade have been stable or trade is not an important factor in output, real GDP and command GDP are similar. The United States is a good example of this case. In figure 5 we plot real GDP and command GDP for the United States, as well as the terms of trade. The terms of trade have stayed fairly steady over the last 20 years, and command GDP and real GDP are almost indistinguishable. In contrast, Switzerland's terms of trade have steadily improved, falling 21.4 percent since 1981, as can be seen in figure 6. The figure also

shows how command GDP has grown significantly faster than real GDP in Switzerland; from 1981Q1–2006Q2 command GDP grew 18.1 percent more than real GDP. Command GDP grew at 2.0 percent per year over this period, compared to the dismal 1.5 percent per year growth in real GDP. Some Swiss economists, notably Kohli (2004), have used measures similar to command GDP to help explain why many do not believe the Swiss economy is doing poorly, despite the lack of growth in real GDP since 1973. For further discussion of Switzerland's economic performance, including the impact of the terms of trade, see Kehoe and Ruhl (2003, 2005).

Recently, Feenstra, Heston, Timmer, and Deng (2004) have proposed adding a new variable, which they call expenditure-side real GDP to the Penn World Tables. This variable is the purchasing power parity equivalent of the SNA concept of GDI where the domestic absorption price index is used to deflate the trade balance. They argue that it is this concept of national income that should be used when researchers are interested in studying welfare, while the traditional concept of real GDP, which they refer to as output-side real GDP, should be used when researchers are interested in studying production. What concept of GDP is currently reported in the Penn World Tables? Feenstra, Heston, Timmer, and Deng (2004) argue that inconsistencies in the GDP calculations make it neither one nor the other and that these inconsistencies need to be eliminated and both variables need to be reported.

6.1. Business cycle frequencies

It makes sense to model some countries as small open economies. These countries are small in the sense that they do not influence world prices, and thus the country's terms of trade are exogenously given. It is easy to imagine one of these small open economies being buffeted by shocks to its terms of trade and this in turn affecting the county's GDP. Although terms of trade shocks cannot have much of an effect on real GDP, particularly given the magnitude of these shocks and the low level of substitutability usually assumed in these models, we can use the command GDP measure to calculate how real income changes over the business cycle.

Figures 7 and 8 plot Hodrick-Prescott filtered log real GDP and log command GDP for the United States and Switzerland. In both cases, the volatility of command GDP is lower than that of real GDP. Command GDP is 20.1 percent less volatile than real GDP in the United States and 24.1 percent less volatile than real GDP in Switzerland. For the United States, real and

command GDP move together; the correlation coefficient is 0.91. For Switzerland, the two series do not move as closely; the correlation coefficient is 0.50.

6.2. Depressions

A crisis in a developing economy may be accompanied by deteriorations of the country's terms of trade. Mexico, for example, has weathered two crises in the last 20 years, the first in 1982–86, and the second in 1994–95. As shown in figure 9, the terms of trade increased by 85 percent from 1981 to 1986 and by 8.6 percent from 1994 to 1995. These periods were also periods of significant declines in output: from 1981 to 1986 real GDP fell by 2.6 percent, and from 1994 to 1995 real GDP fell by 6.2 percent. We have seen in the previous sections that the change in the terms of trade cannot be the cause of the declines in real GDP. How does the situation change when the changing terms of trade are also taken into account?

Figure 9 plots both real GDP and command GDP in Mexico. During the first crisis, real GDP fell by 2.6 percent, but command GDP — real domestic income — fell by 10.0 percent. Command GDP fell by more during the second crisis as well, declining 8.7 percent from 1994 to 1995 compared to the 6.2 percent decline in real GDP over the same period. The output drops associated with financial crises like the ones in Mexico are frequently used as evidence of the painful nature of the withdrawal of credit to a country. The evidence on real domestic income suggests that these "sudden stop" episodes are even more painful than the GDP evidence suggests!

7. Concluding Remarks

In standard models, an adverse shock to the terms of trade acts like an adverse shock to productivity along many dimensions: income and consumption fall. In one crucial dimension, however, a terms of trade shock acts nothing like a productivity shock: real GDP, the most common measure of a country's output, is often unchanged in standard models. In returning to our original question, if we are to use real GDP as a measure of production, then total factor productivity also remains unchanged. Although the terms of trade are shocks to a country's income, they are not shocks to a country's productivity.

So how can we account for the relationships in figures 1 and 2? This paper shows that we cannot expect standard models to do so. One line of promising research argues that there are

other responses to terms of trade shocks. The change in relative prices may induce reallocations across goods and sectors that involve nonproductive activities like retraining, or capital may go idle, both contributing to lower output and measured TFP. The literature on developing country crises is one area in which progress is being made in modeling the frictions that may help account for the relationship between the terms of trade, real GDP, and productivity. Beginning with standard models, Meza and Quintin (2006) introduce labor hoarding and variable capital utilization, Mendoza (2006) introduces financial market frictions, and Kehoe and Ruhl (2006) introduce frictions in reallocating labor across sectors. These papers have had some success in replicating the relationships discussed above, but the exact specification and the quantitative importance of these frictions remains a question for future research.

Figure 10 presents some data that should serve as a caution to researchers. In Switzerland over the period 1970–2000, improvements in the terms of trade have been associated with declines in real GDP and in productivity, with correlation coefficients of 0.53 and 0.58, respectively.

References

Becker T. and P. Mauro (2005), "Output Drops and the Shocks that Matter," IMF Working Paper 06172.

Bjerke, K. (1968), "Some Reflections on the Terms of Trade," *Review of Income and Wealth*, 14, 183–98.

Bureau of Economic Analysis (2006), "A Guide to the National Income and Product Accounts of the United States," http://www.bea.gov/bea/an/nipaguid.pdf.

Burge, R. W. and R. C. Geary (1957), "Balancing of a System of Accounts in Real Terms," *A System of Price and Quantity Indexes for National Accounts*. United Nations.

Diewert, W. E. and C. J. Morrison (1986), "Adjusting Output and Productivity Indexes for Changes in the Terms of Trade," *Economic Journal*, 96, 659–79.

Easterly, W., R. Islam, and J. E. Stiglitz (2001), "Shaken and Stirred: Explaining Growth Volatility," in B. Pleskovic and N. Stern, editors, *Annual World Bank Conference on Development Economics 2000*. The World Bank, 191–211.

Easterly, W., M. Kremer, L. Pritchett, and L. H. Summers (1993), "Good Policy or Good Luck? Country Growth Performance and Temporary Shocks," *Journal of Monetary Economics*, 32, 459–83.

Feenstra, R. C., A. Heston, M. P. Timmer, and H. Deng (2004), "Estimating Real Production and Expenditures across Nations: A Proposal for Improving the Penn World Tables," NBER Working Paper 10866.

Hamilton, J. D. (1983), "Oil and the Macroeconomy since World War II," *Journal of Political Economy*, 91, 228–48.

Kehoe, T. J. and K. J. Ruhl (2003), "Recent Great Depressions: Aggregate Growth in New Zealand and Switzerland, 1973–2000," *New Zealand Economic Papers*, 37, 5–40.

Kehoe, T. J. and K. J. Ruhl (2005), "Is Switzerland in a Great Depression?" *Review of Economic Dynamics*, 8, 759–75.

Kehoe, T. J. and K. J. Ruhl (2006), "Sudden Stops, Sectoral Reallocations, and Real Exchange Rates," University of Minnesota and the University of Texas at Austin.

Kohli, U. (1983), "Technology and the Demand for Imports," *Southern Economic Journal*, 50, 137–150.

Kohli, U. (2004), "Real GDP, Real Domestic Income, and Terms of Trade Changes," *Journal of International Economics*, 62, 83–106.

Mahdavy, K. and M. Silver (1989), The Measurement of a Nation's Terms of Trade Effect and Real National Disposable Income within a National Accounting Framework," *Journal of the Royal Statistical Society*, 152, 87–107.

Mendoza, E. G. (2006), "Endogenous Sudden Stops in a Business Cycle Model with Collateral Constraints: a Fisherian Deflation of Tobin's Q," University of Maryland.

Meza, F and E. Quintin (2006) "Financial Crises and Total Factor Productivity," Universidad Carlos III and Federal Reserve Bank of Dallas.

Nicholson, J. L. (1960), "The Effects of International Trade on the Measurement of Real National Income," *Economic Journal*, 70, 608–12.

United Nations (2001), *System of National Accounts 1993*, Internet Access System, http://unstats.un.org/unsd/sna1993/introduction.asp.

Figure 1
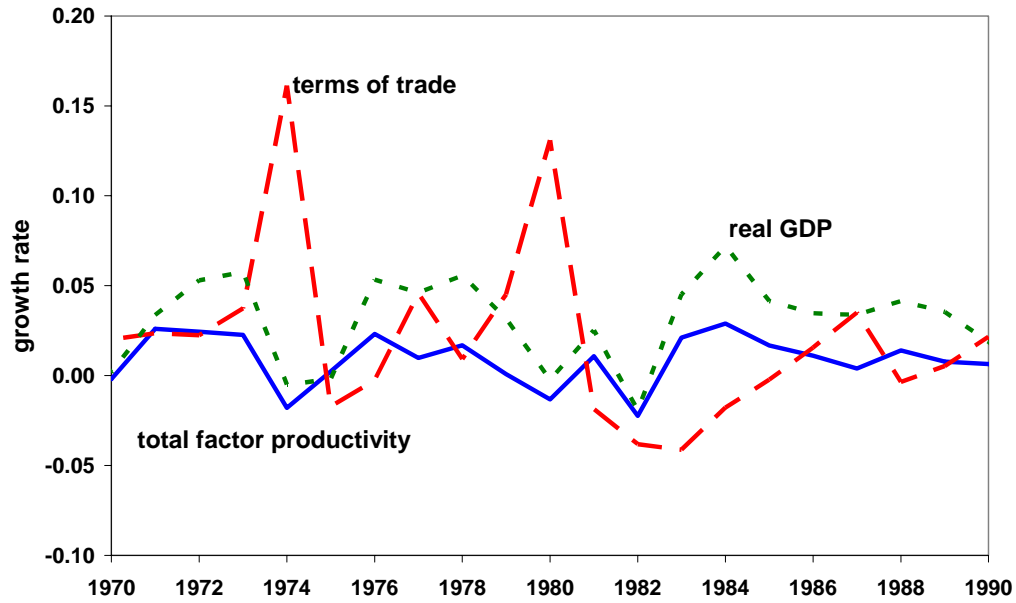
United States

Figure 2

Mexico
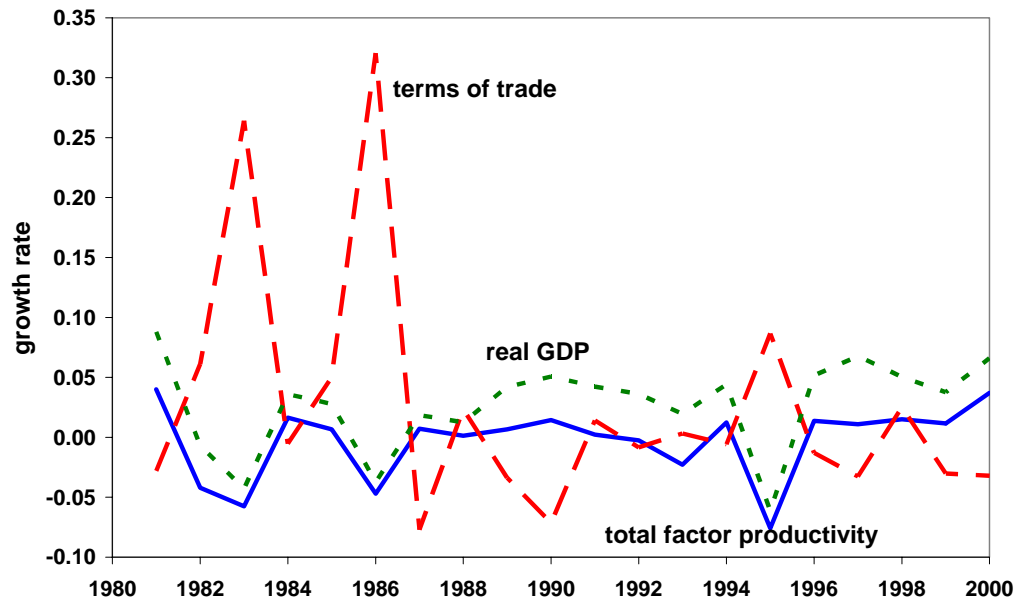
Figure 3

Real GDP and the elasticity of substitution
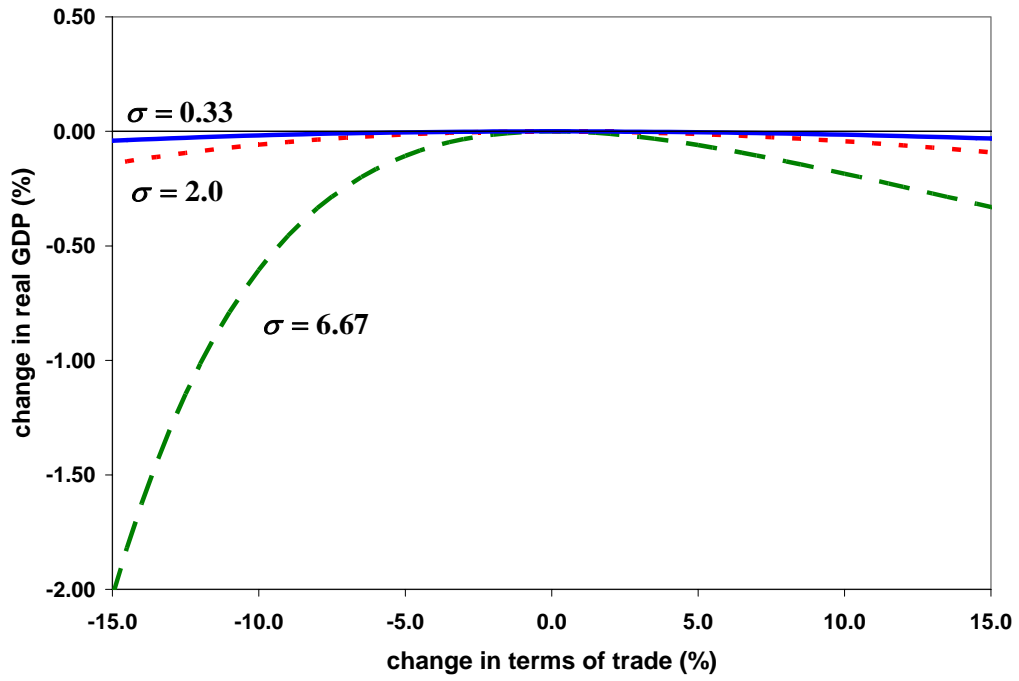
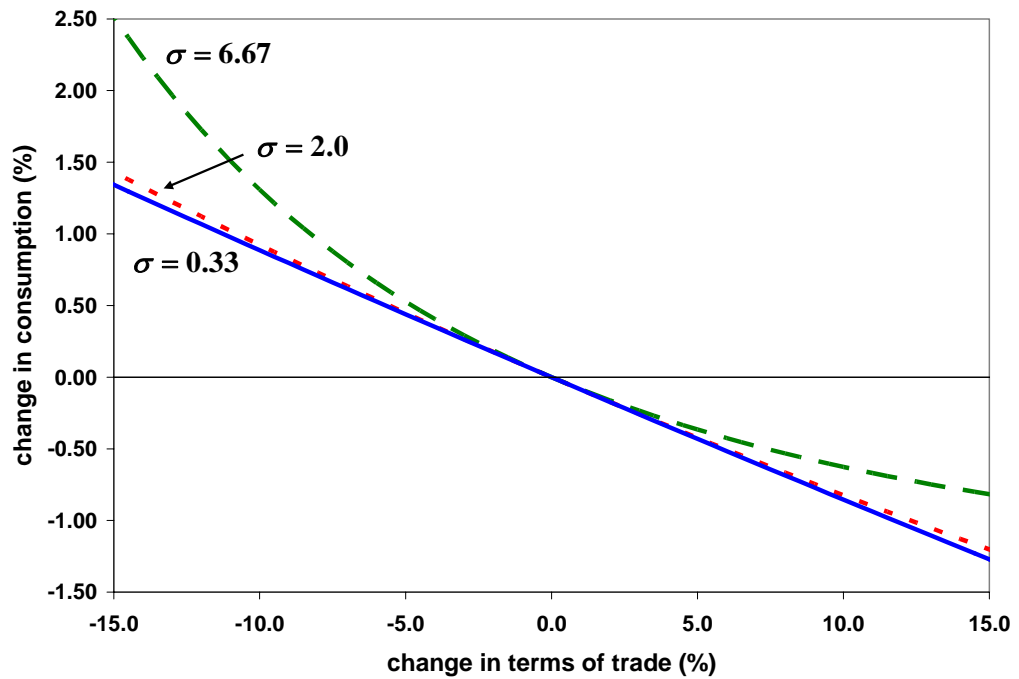## Figure 4

Consumption and the elasticity of substitution
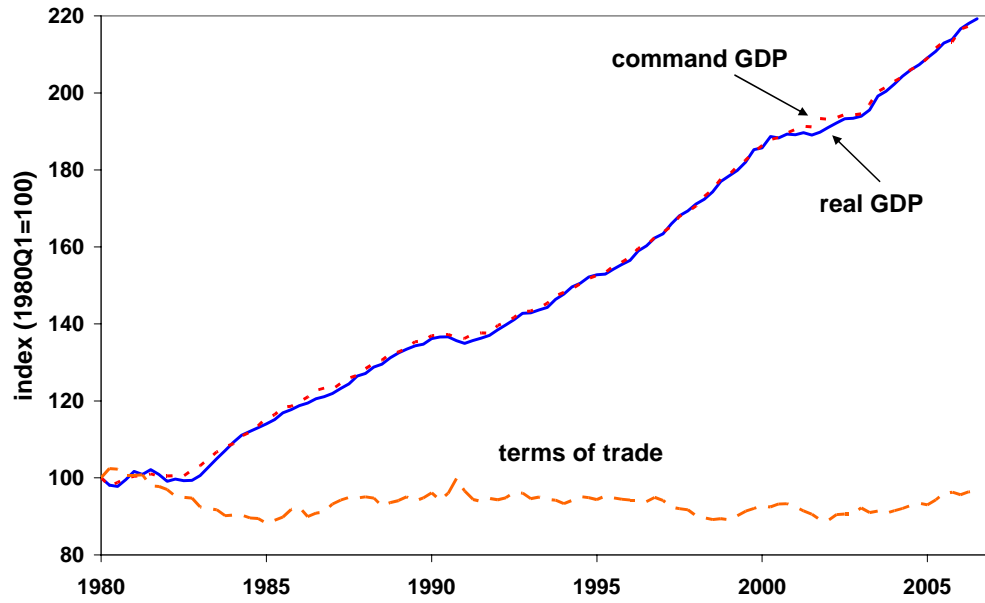

24

Figure 5

United States

Figure 6

Switzerland
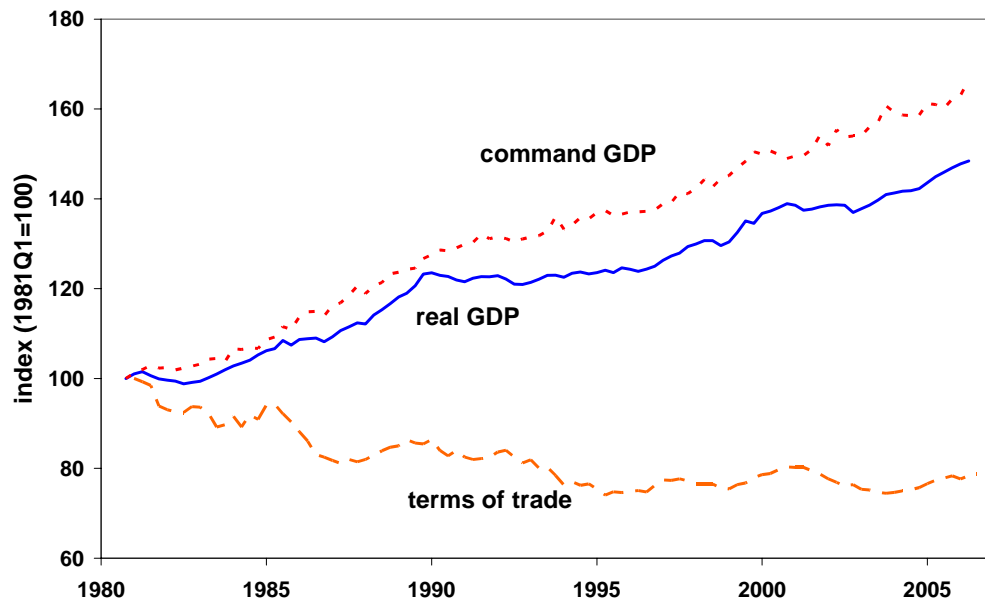
Figure 7

United States
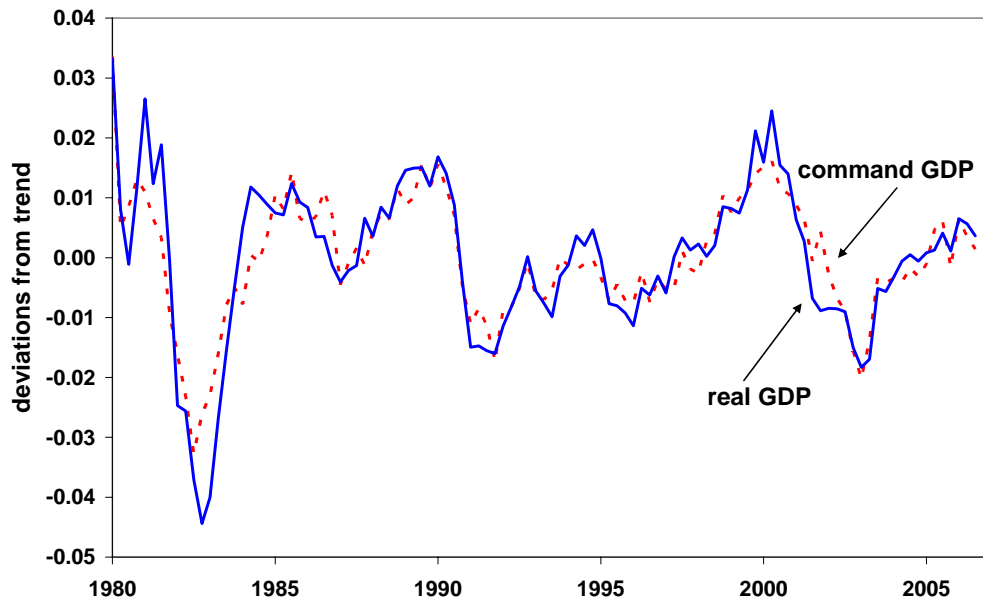
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Switzerland
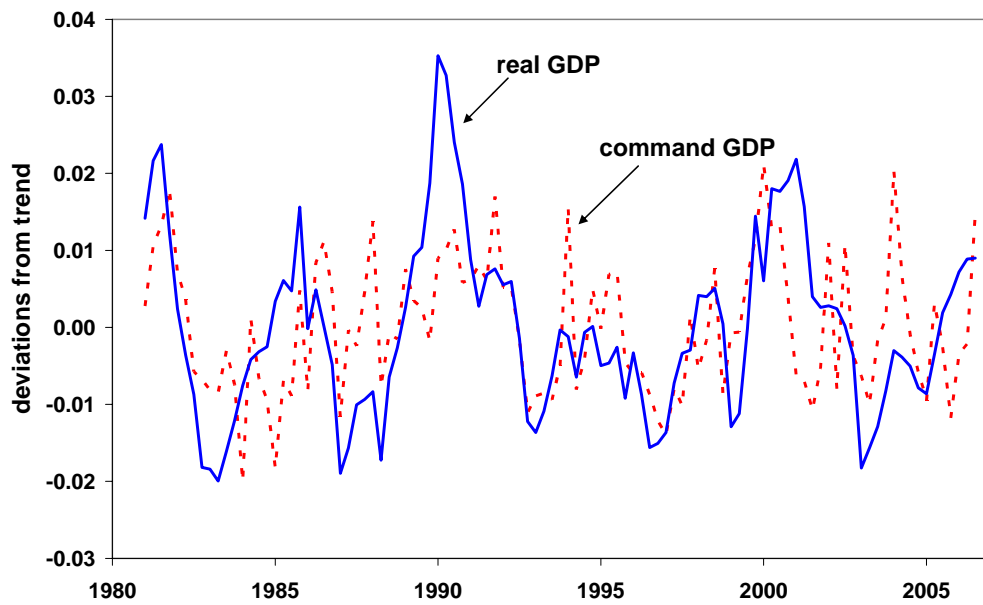
Figure 9

Mexico

Figure 10

Switzerland