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UNDERSTANDING THE EVOLUTION OF THE U.S. WAGE DISTRIBUTION: A THEORETICAL ANALYSIS

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ABSTRACT

In this paper we present an analytically tractable overlapping generations model of human capital accumulation, and study its implications for the evolution of the U.S. wage distribution from 1970 to 2000. The key feature of the model, and the only source of heterogeneity, is that individuals differ in their ability to accumulate human capital. Therefore, wage inequality results only from differences in human capital accumulation. We examine the response of this model to skill-biased technical change (SBTC) theoretically. We show that in response to SBTC, the model generates behavior consistent with several features of the U.S. data including (i) a rise in overall wage inequality both in the short run and long run, (ii) an initial fall in the education premium followed by a strong recovery, leading to a higher premium in the long run, (iii) the fact that most of this fall and rise takes place among younger workers, (iv) a rise in within-group inequality, (v) stagnation in median wage growth (and a slowdown in aggregate labor productivity), and (vi) a rise in consumption inequality that is much smaller than the rise in wage inequality. These results suggest that the heterogeneity in the ability to accumulate human capital is an important feature for understanding the effects of SBTC, and interpreting the transformation that the U.S. economy has gone through since the 1970's.

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1 Introduction

The Ben-Porath (1967) model of human capital accumulation has been one of the workhorses in labor economics in the last 40 years. It has been extensively used to understand such issues as educational attainment, on-the-job training, and wage growth over the life cycle, among others. It is then perhaps surprising that, with few exceptions, this model has not been applied to study the significant changes observed in the U.S. labor markets since the early 1970s. The goal of this paper is to close this gap. In particular, this paper develops and studies the theoretical implications of a particular human capital model (which builds on Ben-Porath (1967)) for the evolution of the wage distribution during this period. The particular model studied is an analytically tractable overlapping-generations model, where the key novel feature is that individuals differ significantly in their ability to accumulate human capital. This is the only source of heterogeneity in this model, and this assumption is motivated by some recent empirical evidence discussed below.

Specifically, this paper focuses on the following three dimensions of the changes in the wage distribution from about 1970 to 2000:[1]

1. The stagnation of median wages (and the slowdown in labor productivity) from about 1973 to 1995 (i.e., changes in the *first* moment of the wage distribution).
2. The substantial changes in overall, between-group and within-group wage inequality during this period (i.e., changes in the *second* moment of the wage distribution).
3. The relatively small rise in consumption inequality despite the large rise in wage inequality (i.e., changes in *lifetime* wage income distribution).

Among the trends mentioned above, perhaps the most puzzling has been the joint behavior of overall wage inequality and between-group inequality (i.e., the education premium), and in particular, their movement in *opposite* directions during the 1970s. Juhn, Murphy, and Pierce (1993) have documented these patterns and stated: "The rise in within-group inequality preceded the increase in returns to observables by over a decade. On the basis of this difference in timing, it seems clear to us that there are at least two unique dimensions of skill (education and skill differences within an education group) that receive unique prices in the labor market" (p. 429). They then added: "Our conclusion is that the general rise in inequality and the rise in education premium are actually distinct economic phenomena" (p. 412). This widely accepted conclusion has then led most of the subsequent literature to search for separate driving forces and mechanisms to explain each of these phenomena.[2] Instead, as we elaborate below, this paper proposes a mechanism that simultaneously generates a monotonic rise in overall inequality and a non-monotonic change in the education premium, despite the fact that the model has one type of skill.

[1] For extensive documentation of these trends, see Bound and Johnson (1992), Katz and Murphy (1992), Murphy and Welch (1992), Juhn, Murphy, and Pierce (1993), Card and Lemieux (2001), Acemoglu (2002), Krueger and Perri (2006), Attanasio, Battistin, and Ichimura (2004), and Autor, Katz, and Kearney (2005a,b).

[2] Some notable exceptions are discussed in the literature review.

Here are the basic features of the model. Individuals begin life with a fixed endowment of "raw labor" (i.e., strength, health, etc.) and are able to accumulate "human capital" (skills, knowledge, etc.) over the life cycle. Raw labor and human capital earn separate wages in the labor market and each individual supplies both of these factors of production at competitively determined prices (wages). Investment in human capital takes place on-the-job unless it equals 100 percent of an individual's time, in which case it is interpreted as "schooling." Individuals who invest full-time for a specified number of years are defined as college graduates. We assume that skills are general (i.e., not firm-specific) and labor markets are competitive. As a result, the cost of human capital investment will be completely borne by the workers, and firms will adjust the hourly wage rate downward by the fraction of time invested on the job (Becker (1965)). Thus, the cost of human capital investment is the forgone earnings while individuals are learning new skills.[3]

We introduce two features into this framework. *First*, we assume that individuals differ in their ability to accumulate human capital. As a result, individuals differ systematically in the amount of investment they undertake, and consequently, in the growth rate of their wages over the life cycle. This assumption is consistent with the recent empirical evidence from panel data on individual wages; see, for example, Lillard and Weiss (1979), Baker (1997), Guvenen (2005, 2006), and Huggett, Ventura, and Yaron (2005). Thus, wage inequality in the model results entirely from the systematic fanning out of wage profiles over the life cycle.

The demand side of the model consists of a linear production technology that takes raw labor and human capital as inputs. The *second* element in the model, and the driving force behind the non-stationary changes during this period, is skill-biased technical change (SBTC) that occurs starting in the early 1970s (for empirical evidence on SBTC, see, for example, Berman, Bound, and Griliches (1994), Autor, Katz, and Krueger (1998), and Machin and Van Reenen (1998)). A key difference of our model is that we do not equate "skill" to education as has often been done in previous studies. Instead, we interpret skill more broadly as human capital and view SBTC as a change that raises the productivity of human capital relative to raw labor. This seemingly small difference in perspective has important consequences. To see this, note that in this model all workers have some amount of human capital (which varies by ability and age) and raw labor (which is the same for all). Therefore, SBTC does not only change wages *between* education groups (because of differences in average human capital levels), but it also affects wages *within* each group differently, depending on the ability and age of each individual. In this framework education is not a separate skill with its unique price, but is merely a noisy indicator of an individual's ability to

[3] The main difference of this model from the standard Ben-Porath framework comes from our introduction of raw labor as a second factor of production (in addition to human capital). The main purpose of this modification is to introduce the concept of "returns to skill" into the Ben-Porath model, which perhaps surprisingly does not exist in that model, but which is clearly crucial for studying skill-biased technical change. An advantage of the particular specification we propose in this paper is that it retains the analytical tractability of the Ben-Porath framework allowing us to solve the model in closed form. We elaborate on these points in the next section.

learn, which in turn is an indicator of his human capital level and of how strongly he responds to SBTC. Therefore, another feature of this model is that it allows us to study both between-group and within-group inequality simultaneously.

The linear production function allows us to solve the model in closed form, derive explicit expressions for the moments of the wage and consumption distributions, and establish our results theoretically. In addition to this analytical convenience, however, the linear form (i.e., perfect substitutability) plays another important role. With *imperfect* substitution, the college premium would be negatively related to the relative supply of college graduates. Several authors have emphasized this link to argue that the fall in the college premium during the 1970s resulted from the rapid increase in the supply of college-educated workers (cf., Katz and Murphy (1992), Juhn, Murphy, and Pierce (1993)). When the production function is linear, however, this link is broken. Therefore, we use this linear technology to highlight a different mechanism, and to show that our results—and especially the non-monotonic behavior of the college premium—are not driven by the relative supply channel emphasized in earlier work. Of course, in reality both channels are probably operational and are complementary to each other.

We first examine the behavior of average wages in response to SBTC. Under a fairly mild assumption, the model generates stagnation in average wages in the short run after SBTC and a rise in the long run. The mechanism can be explained as follows. Because SBTC raises the returns to human capital at all future dates, it leads to a permanent increase in investment rates, since individuals are forward-looking. While this higher investment results in an immediate increase in costs (in the form of forgone earnings), its benefits are realized gradually as the total stock of human capital slowly increases. As a result, observed wages fall in the short run, due to increased investment on the job, and inherit the sluggish growth of the human capital stock thereafter.

Second, a closely related mechanism generates the non-monotonic behavior of the college premium during SBTC. Basically, because college graduates have higher learning ability than those with lower education, their investment increases more in response to SBTC. This *differential* increase in immediate costs (i.e., forgone earnings) results in a fall in their relative wages in the short run. In the long run, however, this higher investment yields a larger increase in their human capital stock, leading to a higher college premium. Third, it is also easy to see that the described mechanism will affect younger workers—who have a longer horizon and thus expect larger benefits from investing—more than older ones, resulting in a more pronounced decline in the college premium among younger workers, consistent with empirical evidence (Katz and Murphy (1992), Card and Lemieux (2001)). Fourth, despite the fall in the college premium in the short run, it can be shown that overall wage inequality *rises* in the model during the same time (proposition 5). Therefore, taken together the second and fourth results show that this model is consistent with the joint behavior of overall wage inequality and between-group inequality observed in the U.S. data mentioned above.

Fifth, and finally, the rise in lifetime income inequality in the model is significantly smaller than the rise in wage inequality. In the model, a high price of human capital generates larger cross-sectional wage inequality because of a *fanning out* of wage profiles (see figure 1). However, note that those individuals who experience a large increase in their wages later in life are exactly those who make larger investments and accept lower wages early on. Because future gains are discounted compared to the early losses in calculating lifetime income, the rise in lifetime inequality remains small. Therefore, the model offers a new mechanism that rationalizes a small change in lifetime inequality with the large increase in wage inequality.

The model presented in the current paper is too stylized to be taken to data directly. In a companion paper (Guvenen and Kuruscu (2006)), we relax several assumptions made in the current paper for analytical tractability, and introduce some important missing features—such as replacing the perfect foresight assumption about future skill prices with Bayesian learning, allowing for imperfect substitution in the production function, and introducing uncertainty, among others. The detailed quantitative analysis in that paper shows that the main mechanisms highlighted in this paper continue to play a central role, and the main conclusions of this paper carry over to that more general case.

There is a vast literature on the empirical trends that motivate this paper. A short list of these papers is mentioned in footnote 1; for excellent surveys of the literature see Katz and Autor (1999) and Acemoglu (2002). An important precursor to our paper is Heckman, Lochner, and Taber (1998), who quantitatively examine the implications of an overlapping-generations model of human capital accumulation for some of the trends mentioned above. To our knowledge, their paper was the first one to emphasize that with human capital accumulation skill prices and observed wages differ, and that this could be important for understanding the recent evolution of the U.S. wage distribution. Our paper however also differs from theirs in several important respects. First, the framework studied here extends the Ben-Porath model to allow for returns to skill, but is still analytically very tractable. As a result, we are able to solve the model in closed form and establish all our results theoretically. Second, a central thesis of our paper is that individuals differ *significantly* in their ability to accumulate human capital and this feature is behind most of the results derived in this paper, whereas heterogeneity in ability is quantitatively very small in Heckman et al (1998, see figure 3). Finally, in the long run after SBTC all measures of inequality increase in the present paper, whereas many of them fall in that paper.

Several studies before us have emphasized the role of a rapid increase in skill demand for rising wage inequality. Important examples include Hornstein and Krusell (1996), Greenwood and Yorukoglu (1997), Galor and Tsiddon (1998), Caselli (1999), Aghion, Howitt, and Violante (2002), and Violante (2002). Greenwood and Yorukoglu emphasize the role of skill in facilitating the adoption of new technologies. They argue that the advent of computer technologies in the 1970s presented such a change, which increased the wages of skilled workers and resulted in a productivity slowdown due to the time it takes to utilize the new technologies effectively. Hornstein and Krusell also make a similar observation, but add that the acceleration in quality improvements during this period has exacerbated measurement problems, further reducing measured productivity growth. Caselli (1999) studies a model where differences in innate ability and newer technologies that are more costly to learn than existing ones result in a rising skill premium. Violante (2002) develops a model of within-group inequality, in which vintage-specific skills, embodied technological acceleration, and labor market frictions combine to generate rising inequality. These papers share the feature that technical change is "embodied" in new machines; instead in our paper, it is "disembodied." It is possible to argue that both types of technological changes have been taking place during this period, so the mechanisms emphasized in these papers are complementary to ours.[4]

Several papers have proposed explanations for the (non-monotonic) behavior of the college premium during this period. In an influential paper, Katz and Murphy (1992) show that a simple supply-demand framework provides a very good fit to the observed behavior of the college premium. Acemoglu (1998) goes one step further and proposes a model to endogenize the demand for skill: essentially, a large rise in the supply of college workers causes firms to direct their innovations to take advantage of this supply, creating an endogenous skill-bias in technological progress. He also shows that an extension of this model with two skills is potentially consistent with the joint behavior of college premium and total wage inequality. Similarly, Krusell et al. (2000) show that an increased demand for skills can result if capital and skills are complementary in the production function, and technical change is investment-specific. Galor and Moav (2000) discuss an extension of their baseline model that can also explain the joint behavior of college premium and total wage inequality.[5] As noted above, a common feature in these papers (as well as in Heckman et al (1998)) is the central role played by the relative supply of skill, which arises from a production function with imperfect substitution between workers with different skill levels. Instead, here we eliminate the effect of relative supply on skill prices, and emphasize a different channel that works through the differential human capital investment response of different groups to SBTC. In addition, the joint behavior of between-group and overall inequality in this model results from a single driving force.

The paper is organized as follows. The next section describes the model. Section 3 presents the theoretical analysis and establishes the results described above. Section 4 discusses some possible extensions, and Section 5 concludes.

[4] In section 4 we consider a version of our baseline model with embodied technical change, and argue that such an extension may be important for explaining the behavior of college enrollment in the short run after SBTC (which was stagnant in the 1970's in the U.S data). However, this version of the model has implications similar to the baseline case for the evolution of the wage distribution, which is the main focus of this paper, and is not as tractable as the baseline model. So we do not analyze it in more detail.

[5] Gould, Moav, and Weinberg (2001) develop a model that can also generate the different behaviors of between- and within-group inequality during this period. However, their explanation relies on the existence of two separate sources of inequality growth.

2 A Baseline Model

2.1 Human Capital Accumulation Decision

The economy consists of overlapping generations of individuals who live for S years. Individuals begin life with an endowment of "raw labor" (i.e., strength, health, etc.), which is the same across individuals and constant over the life cycle, and are able to accumulate "human capital" (skills, knowledge, etc.) over the life cycle, which is the only skill that can be accumulated in this economy. There is a continuum of individuals in every cohort, indexed by $j \in [0,1]$, who differ in their ability to accumulate human capital, denoted by $\widetilde{A}_j$ (also referred to as their "type"). This is the only source of heterogeneity in the model.

Each individual has one unit of time endowment in each period that can be allocated between producing output and accumulating human capital. Let l denote raw labor and $h_{j,s}$ denote the human capital of an s-year-old individual of type j. We assume that raw labor and human capital earn separate wages in the labor market, and each individual supplies both of these factors of production at competitively determined wage rates. Therefore, the "potential income" of an individual—that is, the income he would earn if he spent all his time producing for his employer—is given by $P_L l + P_H h_{j,s}$, where P_L and P_H are the rental prices of raw labor and human capital, respectively. (For clarity of notation, we suppress the dependence of variables on time, except when we specifically want to emphasize time variation.)

Following the standard interpretation of the Ben-Porath (1967) model, we assume that investment in human capital takes place on-the-job unless it equals 100 percent of an individual's time, in which case it is interpreted as "schooling." We assume that skills are general (i.e., not firm-specific) and labor markets are competitive. As a result, the cost of human capital investment will be completely borne by workers, and firms will adjust the hourly wage rate downward by the fraction of time invested on the job (Becker (1965)). Then, the observed wage income of an individual is given by

$$w_{j,s} = [P_L l + P_H h_{j,s}](1 - i_{j,s}) = \underbrace{[P_L l + P_H h_{j,s}]}_{\text{Potential earnings}} - \underbrace{[P_L l + P_H h_{j,s}] \times i_{j,s}}_{\text{Cost of investment}}, \tag{1}$$

where $i_{j,s}$ is the fraction of time spent on human capital investment, henceforth referred to as "investment time." Thus, wage income can be written as the potential earnings minus the "cost of investment," which is simply the forgone earnings while individuals are learning new skills. Since labor supply is inelastic (i.e., conditional on working, all workers supply one unit of time per period), $w_{j,s}$ is also the observed (hourly) wage rate.

Individuals begin their life with zero human capital, $h_{j,0} = 0$, and accumulate human capital according to the following technology:

$$h_{j,s+1} = h_{j,s} + Q_{j,s}, \tag{2}$$

where $Q_{j,s}$ is the newly produced human capital, which will be referred to simply as "investment" in the rest of the paper, and should not be confused with investment time $(i_{j,s})$. New human capital is produced by combining the existing stocks of raw labor and human capital with the available investment time according to

$$Q_{j,s} = \widetilde{A}_j((\theta_L l + \theta_H h_{j,s})i_{j,s})^{\alpha}. \tag{3}$$

The key parameter in this specification is $\widetilde{A}_j$, which determines the productivity of learning. Due to the heterogeneity in $\widetilde{A}_j$, individuals will differ systematically in the amount of investment they undertake, and consequently, in the growth rate of their wages over the life cycle. Another important parameter is $\alpha \in [0,1]$, which determines the degree of diminishing marginal returns in the human capital production function. A low value of α implies higher diminishing returns, in which case it is optimal to spread out investment over time. In contrast, when α is high, the marginal return on investment does not fall quickly, and investment becomes bunched over time. In the extreme case when $\alpha = 1$, individuals will spend either all their time on investment $(i_{j,s} = 1)$ or none at all in a given period. Finally, the parameters θ_L and θ_H determine the relative contributions of each factor to human capital accumulation, and could be time-varying as well.

2.2 Individual's Dynamic Problem

We assume that individuals can borrow and lend at a constant interest rate (denoted by r), which implies that markets are complete. As is well-known, in this case the consumption-savings and income maximization decisions can be disentangled from each other. Therefore, for the purposes of analyzing human capital investment, we concentrate on the lifetime income maximization problem. Individuals solve

$$\max_{\{i_{j,s}\}_{s=1}^{S}} \left[\sum_{s=1}^{S} \left(\frac{1}{1+r} \right)^{s-1} [P_L l + P_H h_{j,s}] (1 - i_{j,s}) \right]$$

subject to (2), (3), and $h_{j,0} = 0$. It should be stressed that this formulation does not rest on the assumption of risk-neutrality, but only requires markets to be complete.

2.3 Aggregate Production Technology

The aggregate factors used in production at a point in time are defined as

$$\begin{aligned} L^{net} &= \sum_{s=1}^{S} \mu(s) \int_j l\,(1 - i_{j,s})\, dj, \quad \text{and} \\ H^{net} &= \sum_{s=1}^{S} \mu(s) \int_j h_{j,s}\,(1 - i_{j,s})\, dj, \end{aligned}$$

where $\mu(s)$ is the (discrete) measure of s-year-old individuals, and the sums are thus taken over the distribution of individuals of all types and ages.[6] The superscript "*net*" indicates that these variables measure the actual amounts of each factor *used in production* (that is, net of the time allocated to human capital investment) to distinguish them from the "total stocks" of these factors available in the economy, which are defined later below. The aggregate firm uses these two inputs to produce a single good, denoted by Y according to

$$Y = Z\left(\theta_L L^{net} + \theta_H H^{net}\right), \tag{4}$$

where Z is the total factor productivity (TFP). For simplicity we assume that capital is not used in production. Note that raw labor and human capital enter the aggregate production function and human capital production in a symmetric manner and with the same productivity parameters (compare (4) to (3)).[7] This assumption allows us to solve the model in closed form.

The firm solves a static problem by hiring factors from households every period to maximize its profit: $Y - P_L L^{net} - P_H H^{net}$. As a result, factor prices are given by the marginal products

$$P_H = \partial Y/\partial H^{net} = \theta_H \qquad \text{and} \qquad P_L = \partial Y/\partial L^{net} = \theta_L.$$

It is useful to compare this production structure to that assumed in some of the previous literature. In these papers, the production technology is typically taken to be a CES function: $Z\left[(\theta_L L)^{\rho} + (\theta_H H)^{\rho}\right]^{1/\rho}$, where now H and L denote the total *work hours of college and high-*

[6] For the population structure assumed so far, $\mu(s) = 1/S$.

[7] When studying the quantitative implications of this model (Guvenen and Kuruscu (2006)), we have experimented with three other specifications of the human capital accumulation function that also seemed a priori plausible. In particular, we wrote the human capital function as $\widetilde{A}_j((\lambda_{L,t} l + \lambda_{H,t} h_{j,s}) i_{j,s})^{\alpha}$ and considered (i) weights that remain constant through SBTC: $\lambda_{L,t} = \overline{\lambda}_L$ and $\lambda_{H,t} = \overline{\lambda}_H$; (ii) an aggregate production function that features imperfect substitution between raw labor and human capital, and weights that are proportional to wages: $\lambda_{L,t} = P_{L,t}$ and $\lambda_{H,t} = P_{H,t}$; and (iii) no role for raw labor in human capital production: $\lambda_{L,t} = 0$ and $\lambda_{H,t} = 1$. The first two cases had implications qualitatively similar to the baseline model described here, while the third displayed some implausible behavior even in steady state (that is, without SBTC). Overall, and perhaps surprisingly, the simplest specification we adopt here for analytical convenience (given in equation (3)) also turned out to have the most plausible quantitative implications.

school workers, respectively. Therefore, in these models, a change in θ_H/θ_L due to SBTC has the same effect on all individuals *within* a given education group. Instead, here we interpret skill more broadly as human capital and do not equate it to education. Since all workers in this model have some amount of human capital (which varies by ability and age) and raw labor (which is the same for all), SBTC not only changes wages *between* education groups (because of differences in average human capital levels), but also affects wages *within* each group differently, depending on the ability and age of each individual. In this sense, this model allows us to study both between-group and within-group inequality simultaneously.

A second implication of the production function assumed in earlier studies is that the education premium is given by $P_H/P_L = (\theta_H/\theta_L)^\rho\,(H/L)^{\rho-1}$ and is therefore decreasing in the relative supply of college graduates. Several authors have emphasized this link to argue that the fall in the college premium during the 1970s resulted from the rapid increase in the supply of college-educated workers (cf., Katz and Murphy (1992), Juhn, Murphy, and Pierce (1993)). However, notice that when the production function is linear, this link is broken: $P_H/P_L = \theta_H/\theta_L$. Thus, we use this linear production technology to show that the non-monotonic behavior of the college premium in this paper is not driven by the relative supply channel emphasized in earlier work.

2.4 Analyzing the Individual's Problem

We now rewrite the problem to simplify the exposition. Using equation (3), the opportunity cost of investing an amount $Q_{j,s}$ can be written as

$$C_j(Q_{j,s}) \equiv (\theta_L l + \theta_H h_{j,s}) i_{j,s} = \left(\frac{Q_{j,s}}{\widetilde{A}_j}\right)^{1/\alpha}.$$

With this transformation, the problem of an individual can be written as

$$\max_{\{Q_{j,s}\}_{s=1}^{S}} \left[\sum_{s=1}^{S}\left(\frac{1}{1+r}\right)^{s-1}(\theta_L l + \theta_H h_{j,s} - C_j(Q_{j,s}))\right]$$

subject to

$$h_{j,s+1} = h_{j,s} + Q_{j,s}, \quad \text{with} \;\; h_{j,0} = 0.$$

The optimality condition which determines the amount of investment at time t is

$$C_j'(Q_{j,s}) = \frac{1}{1+r}\left\{\theta_H(t+1) + \frac{\theta_H(t+2)}{1+r} + ... + \frac{\theta_H(t+S-s-1)}{(1+r)^{S-s-2}}\right\}, \tag{5}$$

where we make explicit the dependence of future prices of human capital on time. The left-hand side of this equation is the marginal cost, and the right-hand side is the marginal benefit (MB)

of investment. The latter is the present discounted value of the future stream of wages that is earned by an additional unit of human capital. An important implication of (5) is that an expected increase in the future prices of skill (the sequence $\theta_H(t)$) will have an immediate impact on current investment decisions because of the forward-looking nature of this equation.

No Returns to Skill in the Ben-Porath Model—The optimality condition (5) highlights an important difference between the current framework and the Ben-Porath model. In the latter, the optimality condition is given by

$$W(t)\, C_j'(Q_{j,s}) = \frac{1}{1+r}\left\{ W(t+1) + \frac{W(t+2)}{1+r} + ... + \frac{W(t+S-s-1)}{(1+r)^{S-s-2}} \right\},$$

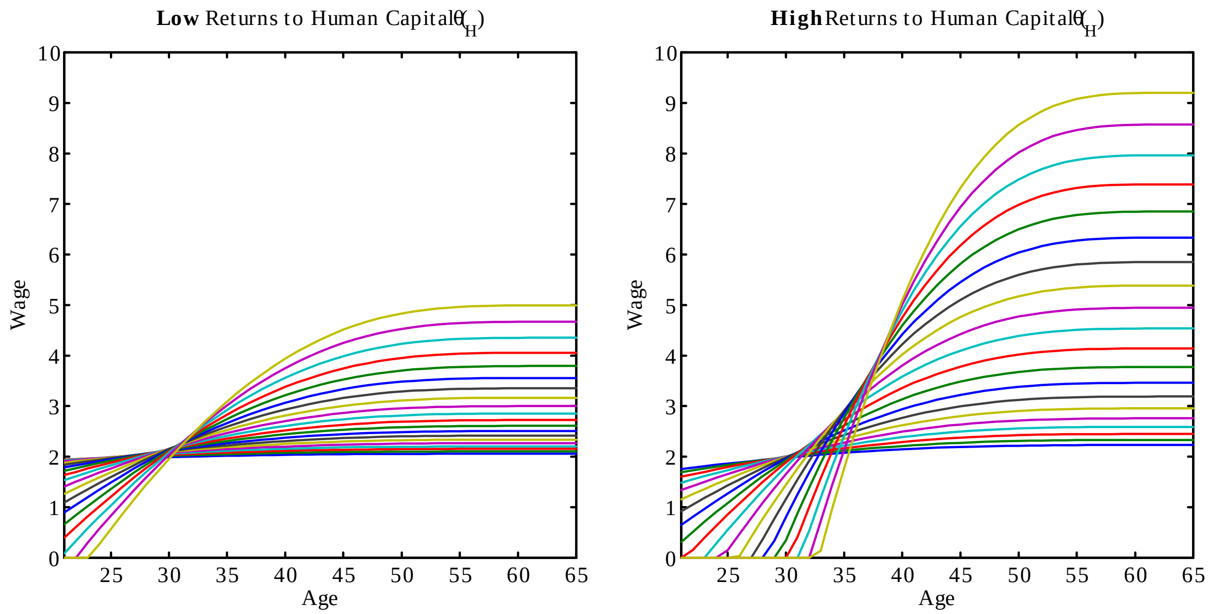
where $W(t)$ is the wage rate in period t, which in turn equals the price of human capital since it is the only factor of production in that model. Now, suppose that the economy is in steady state with $W(t) = \overline{W}$ for all t, and consider the effect of a one-time permanent increase in the wage rate. This change will only have an effect on investment in the first period, but not after that. This is because a permanently higher W will increase both the cost and the benefit of investment by the same amount, after the first period. Therefore, the price of human capital in this standard model does not capture what we think of as a "return on human capital investment."[8] In contrast, in the present framework SBTC takes the form of a permanent increase in θ_H *relative to* θ_L. This increases the benefit of human capital investment (right-hand side of (5)) relative to the cost of investment (left-hand side), and therefore increases the incentives to invest in human capital permanently, without necessarily implying anything about TFP growth.

We now return to analyzing the individual's problem. To illustrate how the model works, consider two economies that differ only in the price of human capital, θ_H and θ_H' with $\theta_H' > \theta_H$. Figure 1 compares the wage profiles of individuals with different ability levels in these two cases. First, note the features common to both cases: workers with high ability invest more than others, accepting lower wages early on in return for higher wages later in life. As a result, wage inequality increases over the life cycle due to the systematic fanning out of the wage profiles. Workers with ability level above a certain threshold invest full time early in life (i.e., they attend college).

A comparison of these two economies reveals a number of important points that are key to understanding the results of this paper about the long-run effects of SBTC. First, a higher price of human capital induces more investment, where the strength of this response increases with ability. As a result, cross-sectional wage inequality increases due to the fanning out of wage profiles. Notice, however, that lifetime income inequality will not rise as much as cross-sectional wage inequality because those with high wages later on are exactly those who invest more, and therefore have low

[8] Although it is possible to generate a persistent increase in investment rates by increasing the *growth rate* of W, there is no evidence of increased TFP growth after the 1970s; in fact there is ample evidence to the contrary.

Figure 1: The Effect of the Returns to Human Capital on Life-Cycle Wage Profiles



wages early in the life cycle. Moreover, since lifetime income is a discounted average of wages over the life cycle, later gains are discounted compared to early losses, so the rise in the lifetime income of high ability individuals remains modest.

3 Theoretical Analysis

In this section we consider a simplified demographic structure that allows us to establish our main results theoretically. In particular we specialize to the "perpetual youth" version of the overlapping-generations model as in Blanchard (1985): individuals can potentially live forever ($S = \infty$) but face a constant probability of death $(1 - \delta)$ every period. Under this assumption, s is no longer a state variable in the human capital problem, simplifying the analysis substantially. We normalize the population size to one, and assume that each period a cohort of measure $(1 - \delta)$ is born to replace the individuals who die. Therefore, the measure of an s-year-old cohort is given by $\mu(s) = (1 - \delta)\delta^{s-1}$. In the rest of the analysis, we restrict our attention to an interior solution; hence we assume that $w_{j,s} \geq 0$ for all j, s. This provides analytical tractability.[9] Finally, we assume that $r = 1/(\delta\beta) - 1$, where β is the pure time discount rate. This assumption implies that individuals choose a constant

[9] This assumption is not as restrictive as it might seem, since it can be satisfied by rescaling l to a larger value. The theoretical results that follow do not depend on the particular value of l.

consumption path over the life cycle.

Characterizing the Steady State Before SBTC

To examine the effects of SBTC, we assume that the economy is in steady state in the period preceding the shock, and characterize how investment, wages, and consumption are determined. In this initial steady state, let $\theta_H(t) = \theta_H$ and $\theta_L(t) = \theta_L$ for all t.

The assumption of constant survival probability simplifies the structure of the model in many ways. First, the optimality condition for investment choice (5) reduces to

$$C_j'(Q_j) = \frac{\theta_H \beta \delta}{1 - \beta\delta},$$

where the marginal benefit of investment is now constant, since the expected life span is now independent of age. Using the functional form for the cost function we get

$$Q_j = A_j \left(\frac{\alpha\delta\beta}{1-\beta\delta} \theta_H \right)^{\alpha/(1-\alpha)}, \tag{6}$$

where $A_j \equiv \widetilde{A}_j^{1/1-\alpha}$. The fact that Q_j is independent of age implies that the human capital stock at age s is simply $h_{j,s} = Q_j(s-1)$. Furthermore, this optimal investment choice satisfies the following equalities, which will be useful in our derivations below:

$$C_j(Q_j) = \alpha C_j'(Q_j) Q_j = \left(\frac{\alpha\delta\beta}{1-\delta\beta} \theta_H \right) Q_j.$$

This expression makes clear that the cost of investment evaluated at the optimal investment level depends on j only through Q_j, implying that the subscript j can be dropped from the cost function: $C_j(Q_j) = C(Q_j)$. The optimal amount of investment time, $i_{j,s}$, is given by the total cost of investment divided by potential earnings:

$$i_{j,s} = \frac{C(Q_j)}{\theta_L l + \theta_H h_{j,s}} = \frac{\alpha\delta\beta}{1-\delta\beta} \times \left[\frac{\theta_L l}{\theta_H Q_j} + (s-1) \right]^{-1}. \tag{7}$$

A few intuitive results can be seen from these expressions. First, equation (6) implies that individuals with higher ability make larger investments: $dQ_j/dA_j > 0$. Second, even though individuals increase their human capital stock by a constant amount Q_j every period, investment *time* falls with age: $di_{j,s}/ds < 0$. Third, equations (6) and (7) can be combined to show that $di_{j,s}/dA_j > 0$: conditional on age, individuals with higher ability also devote a larger fraction of their time investing in human capital. Finally, and most importantly, the increase in investment time in response to SBTC is larger for individuals with higher ability: $d^2 i_{j,s}/d\theta_H dA_j > 0$. These results play a central role for the results that we prove below.

We are now ready to derive an expression for the *average* wage rate in the economy. In order to express the average wage in an easily interpretable form, it is convenient to define some new variables. Define the "average investment" in the economy:

$$\overline{Q} \equiv \sum_{s=1}^{\infty} \mu(s) \int_j Q_j dj = \left(\frac{\alpha\delta\beta}{1-\delta\beta} \theta_H \right)^{\alpha/(1-\alpha)} E(A_j), \tag{8}$$

the corresponding "average cost of investment":

$$C(\overline{Q}) \equiv \sum_{s=1}^{\infty} \mu(s) \int_j C(Q_j) dj = \frac{\alpha\delta\beta}{1-\delta\beta} \theta_H \overline{Q}, \tag{9}$$

the "average human capital stock":

$$H(\overline{Q}) \equiv \sum_{s=1}^{\infty} \mu(s) \int_j h_{j,s} dj = \sum_{s=1}^{\infty} \mu(s)(s-1) \times \int_j Q_j dj = \frac{\delta}{1-\delta} \overline{Q}, \tag{10}$$

and, finally, the "average raw labor endowment" in the economy:

$$L \equiv \sum_{s=1}^{\infty} \mu(s) \int_j l dj = l. \tag{11}$$

Notice that $H(\overline{Q})$ and L measure the aggregate human capital stock and raw labor, *inclusive* of on-the-job investment activities, which should not be confused with H^{net} and L^{net} defined earlier.

At a given point in time, $\overline{Q}$ and $C(\overline{Q})$ only depend on the *future* values of θ_H, whereas the *stock* of human capital only depends on *past* levels of investment, which in turn is determined by the history of θ_H's.[10] Therefore, the former variables will adjust immediately in response to a permanent change in θ_H such as SBTC (making them "jump variables"), whereas $H(\overline{Q})$ will adjust only gradually (making it a "stock variable"). This distinction will play a crucial role in the analysis below.

Now, using the definition of an individual's wage in equation (1), the *average wage rate* in the economy can be calculated as

$$\overline{w} = \sum_{s=1}^{\infty} \mu(s) \int_j w_{j,s} dj = \left[\theta_L l + \theta_H H(\overline{Q})\right] - C(\overline{Q}).$$

[10] As a result, the definition of $H(\overline{Q})$ in equation (10) is only valid in steady state when all past returns were constant. Similarly, the definitions of $\overline{Q}$ and $C(\overline{Q})$ are valid when all future returns are constant.

By substituting the expressions for $H\left(\overline{Q}\right)$ and $C(\overline{Q})$ we obtain

$$\overline{w} = \theta_L l + \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)\theta_H \overline{Q}. \tag{12}$$

Optimal consumption choice.—Given the interest rate $r = 1/(\delta\beta) - 1$, the optimal consumption path is constant over the life cycle, and is given by the fraction $(1-\delta\beta)$ of individuals' lifetime income. Then the *average consumption* in the economy is

$$\overline{c} = \left[\theta_L l + \frac{\delta\beta}{1-\delta\beta}\theta_H \overline{Q}\right] - C(\overline{Q}) = \theta_L l + \left((1-\alpha)\frac{\delta\beta}{1-\delta\beta}\right)\theta_H \overline{Q}. \tag{13}$$

Comparing the last two formulas, it is easy to see that average consumption is less than average wage ($\overline{c} < \overline{w}$) whenever $\beta < 1$. The reason can be explained as follows. Given that the interest rate equals the reciprocal of the effective discount rate $(\delta\beta)$, individuals would like to maintain a constant consumption over their lifetime. But because all individuals have upward-sloping wage profiles, they need to borrow against their future income to maintain a constant consumption path. With a positive interest rate, part of aggregate labor income goes toward paying the interest that accrues on borrowed funds (which, for simplicity, is assumed to be borrowed from the rest of the world). Consequently, average consumption is less than the average wage in the economy.

Characterizing the Behavior after SBTC

In the following subsections, we consider a one-time permanent increase, at time t^*, in the price of human capital from θ_H to $\boldsymbol{\theta}'_H$, while the price of raw labor, θ_L, remains constant.[11] We analyze the behavior of average wages (and labor productivity), the college premium, and overall wage inequality both in the short run and in the long run. For the short-run analysis, we focus our attention to the period immediately after the shock. Analyzing the economy in this time period captures the fact that, in the short run, the human capital stock does not fully adjust yet, but investment jumps to its new level immediately.[12]

3.1 Stagnation of Average Wages (and the Slowdown in Labor Productivity)

Labor economists and macroeconomists have documented two closely related trends during this period: the stagnation of median wage growth and the slowdown in labor productivity growth,

[11] Since θ_L remains unchanged and θ_H increases, SBTC entails a true improvement in aggregate productivity in these experiments. An alternative way of modeling SBTC would be to assume that the rise in θ_H is matched by a symmetric fall in θ_L. Almost all the results studied in the next section carry over to this case, and some of them become easier to prove. For example, the decline in average wage in the short run would be larger in this case. Similarly, consumption inequality would increase even less compared to wage inequality after SBTC. To show that these results do not follow trivially from the decline in θ_L, we assume that θ_L is fixed and θ_H increases.

[12] In Guvenen and Kuruscu (2006) we model SBTC as a gradual change in the price of human capital that takes place over several years, and we show that the main conclusions drawn here remain valid.

which both started with a sharp fall in 1973 and persisted until about 1995. For example, the median real wage has increased by 2.2 percent per year between 1963 and 1973, but actually *fell* by 0.3 percent per year between 1973 and 1989 (Juhn, Murphy, and Pierce (1993)). Similarly, labor productivity (measured as the non-farm business output per hour) has grown by 2.6 percent per year from 1955 to 1973, but only by 1.45 percent per year from 1973 to 1995.[13]

To develop the implications of the model, first consider the average wage in the economy in the initial steady state:

$$\overline{w}_I \equiv \overline{w}|_{t<t^*} = \left[\theta_L l + \theta_H \times H\left(\overline{Q}\right)\right] - C(\overline{Q}). \tag{14}$$

As noted before, θ_H and $C(\overline{Q})$ increase immediately after SBTC, whereas $H\left(\overline{Q}\right)$ adjusts only gradually. Therefore, the average wage immediately after SBTC (in the short run) is given by

$$\overline{w}_{SR} \equiv \overline{w}|_{t=t^*+\varepsilon} = \left[\theta_L l + \boldsymbol{\theta}'_H \times H\left(\overline{Q}\right)\right] - C(\overline{\mathbf{Q}}'), \tag{15}$$

where bold letters with a prime denote the values of variables in the final steady state. The stock of human capital, $H\left(\overline{Q}\right)$, gradually increases to its new steady state value $H\left(\overline{\mathbf{Q}}'\right)$, and the average wage in the new steady state is given by

$$\overline{w}_{LR} \equiv \overline{w}|_{t\to\infty} = \left[\theta_L l + \boldsymbol{\theta}'_H \times H\left(\overline{\mathbf{Q}}'\right)\right] - C(\overline{\mathbf{Q}}'). \tag{16}$$

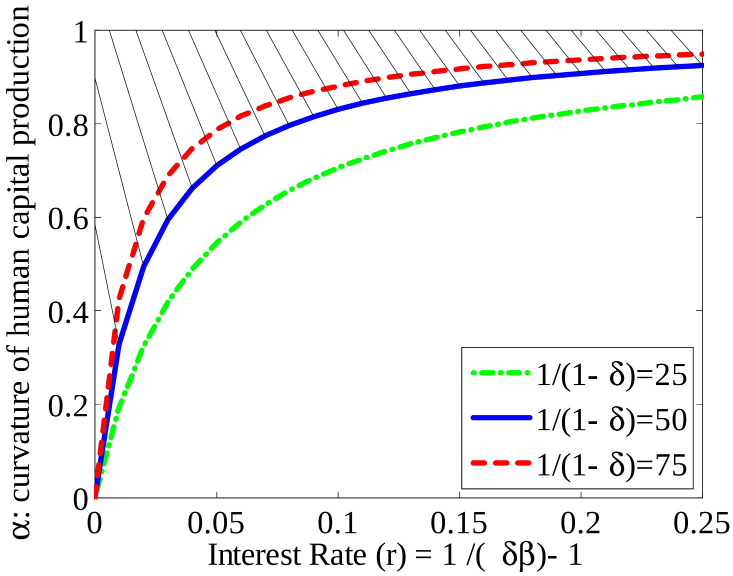
Price, Investment and Quantity Effects.—It is instructive to decompose the changes in the average wage after SBTC into three components. First, using equations (14) and (15), the short-run response of the average wage can be written as

$$\overline{w}_{SR} - \overline{w}_I = \underbrace{\left[\left(\boldsymbol{\theta}'_H - \theta_H\right) \times H\left(\overline{Q}\right)\right]}_{\text{Price Effect } (>0)} + \underbrace{\left[C\left(\overline{Q}\right) - C(\overline{\mathbf{Q}}')\right]}_{\text{Investment effect } (<0)}. \tag{17}$$

First, for a fixed stock of human capital, an increase in θ_H *increases* the wage rate. We call this the "price effect." Second, a higher θ_H also induces more investment, which *reduces* the wage rate by increasing the forgone earnings. We refer to this as the "investment effect." Therefore, the short-run response of the average wage is entirely determined by the relative strengths of these counteracting forces. In other words, whether or not the average wage falls in the short run depends on whether the investment effect dominates the price effect. Below we examine the conditions under which this outcome obtains.

[13] Authors' calculation from the Bureau of Labor Statistics data.

Figure 2: Combinations of (α, r) That Satisfy Condition 1 (Shaded Region)



Similarly, using equations (15) and (16) we have

$$\overline{w}_{LR} - \overline{w}_{SR} = \underbrace{\boldsymbol{\theta}'_H \times \left(H\left(\overline{\mathbf{Q}}'\right) - H\left(\overline{Q}\right)\right)}_{\text{Quantity Effect } (>0)}, \tag{18}$$

which shows that the only change between the short run and the long run is the (slow) adjustment of the human capital stock. We call this long-run channel the "quantity effect." Finally, adding up equations (17) and (18) shows that the total effect of SBTC on the average wage $(\overline{w}_{LR} - \overline{w}_I)$ can simply be written as the sum of the price, investment, and quantity effects. We will use analogous decompositions to examine the effect of SBTC on other variables below, where the main idea will be the same as here.

Before we can state the main result of this section, we need the following condition, which characterizes the parameter values under which the investment effect dominates the price effect.

Condition 1 $\frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)} > \frac{\delta}{1-\delta}$.

Figure 2 illustrates the parameter combinations that satisfy condition 1. From equation (17), it is clear that the investment effect would dominate the price effect if either the initial stock of human capital is low so that the price effect is small, or the response of investment is high. The latter is, in turn, mainly determined by two parameters. First, the response of investment to SBTC is larger when α is high. This is because, as noted earlier, a higher α implies less diminishing marginal returns in human capital production. Consequently, there is little benefit from spreading investment over time (as would be the case if α were low.) Second, for a given (α, δ), a higher β makes the present discounted value of future wages larger, implying a higher benefit from a given increase in the price of human capital. Thus, the response of investment to SBTC increases with β (and the corresponding low interest rate). At the same time, the stock of human capital is increasing in the survival probability, so the price effect is more likely to dominate the investment effect when δ is large.[14] The combination of these three forces gives rise to the the region of admissible parameters shown in Figure 2. This region contains a fairly wide range of plausible parameter combinations. For example, assuming an expected working life of 50 years and an interest rate of 5 percent, any curvature value above 0.71 satisfies condition 1. Estimates of this parameter are typically around 0.8 and higher (see, for example, Heckman (1976), and more recently, Heckman, Lochner, and Taber (1998) and Kuruscu (2006)). The following proposition characterizes the behavior of average wages.

Proposition 1 (Stagnation of Average Wages) *In response to SBTC, for all $\boldsymbol{\theta}'_H > \theta_H$, the average wage (alternatively, labor productivity)*

1. *increases in the long run, i.e., $\overline{w}_{LR} > \overline{w}_I$.*
2. *falls in the short run, i.e., $\overline{w}_{SR} < \overline{w}_I$, if condition 1 holds.*

Proof. *See Appendix A for all the omitted proofs and derivations.* ■

It should be emphasized that for a *marginal* increase in θ_H, condition 1 is not only sufficient but also necessary for the average wage to decline in the short run. However, if the increase in the price of human capital is larger, the average wage would decline under a weaker condition, making condition 1 sufficient but not necessary in general.

The transition path of the average wage after SBTC is easy to characterize: since the stock of human capital increases monotonically over time, it can be seen (by comparing equations (15) and (16)) that after the initial decline the average wage also increases monotonically over time. Furthermore, different measures of the length of this transition can be computed explicitly (proofs in the appendix).

Corollary 1 *After the initial decline, the average wage increases monotonically to its new steady state level. The time it takes for the average wage to complete half of the transition toward the new steady state, $w_{t_1} = w_{SR} + (w_{LR} - w_{SR})/2$, is given by $t_1 = -\log 2/\log \delta$.*

[14] It should be stressed that all three parameters affect both the price and investment effects (which can be seen from equations (8,9,10). So the discussion here, for example, of the impact of α on the cost of investment only highlights the stronger of the two effects it has (even though it affects both the price and investment effects).

See the appendix for the derivation of t_1. Notice that the half-life of the transition depends only on the survival probability, which determines the average age in the economy. In particular, when the population is young on average (δ is small) the transition is faster. Loosely speaking, in this case individuals born before SBTC (who invested at a lower rate for years) are replaced more quickly with younger generations, which increases the average human capital stock more quickly.

For plausible parameter values the transition could be quite slow: for $\delta = 0.98$ (average working life of 50 years), $t_1 = 35$ years, and for $\delta = 0.96$ (average working life of 25 years), $t_1 = 17$ years. However, note that these numbers do not necessarily imply that the average wage will stay below its initial steady state value for this long. An alternative measure of convergence speed is the time it takes for the wage to return to its level before the shock (formula reported in Appendix A; see equation (29)). For $\beta = \delta = 0.98$ and assuming an increase in θ_H of 50 percent implies that the average wage stays below its initial level for 21 years. If $\beta = \delta = 0.96$, the corresponding duration is 11 years. Overall, these back-of-the-envelope calculations suggest that the average wage can stagnate below its initial level for a decade or perhaps longer, and the convergence to the final steady state is likely to be even slower.

Finally, there would be no stagnation in average wages without the endogenous response of investment to SBTC. Thus, the human capital response is essential for these results. The following corollary states this.

Corollary 2 *If individuals' investment behavior did not respond to SBTC (i.e., $\mathbf{Q}'_{j,s} = Q_{j,s}$ for all j, s) the average wage would immediately jump after SBTC from its initial steady state value to final steady state value.*

3.2 Between-Group Wage Inequality (College Premium)

The wages of college graduates declined throughout the 1970s relative to the wages of less-educated individuals. Starting in the early 1980s this trend reversed course, and the college premium increased sharply in the subsequent two decades. For example, the average male college graduate earned 52 percent more than a high-school graduate in 1970; this premium fell to 41 percent in 1979 and increased back to 84 percent by 2000 (Autor, et al (2005a)). In this section, we characterize the behavior of the college premium in the model. We show that under condition 1, SBTC leads to a non-monotonic change in the college premium similar to that observed in the data.

Consistent with the standard interpretation of the Ben-Porath model, the perspective adopted in this paper is that educational labels merely represent some threshold level for the human capital investment completed. Thus, a "college graduate" is defined as an individual who has invested above a certain threshold in a specified number of periods.[15] Since there is a one-to-one relationship between investment time and ability at every age, there is a corresponding threshold ability level above which all individuals ($A_j > A_{j^*}$) become college graduates. However, this threshold depends on the price of human capital and will therefore change in response to SBTC. In the following, we abstract from these changes in A_{j^*}. This is because allowing for changes in A_{j^*} would affect the ability composition of each education group over time, thereby potentially confounding the effects of changes in the premium on education with changes in the returns to ability. Hence, we fix the ability distribution of each group and analyze how their wages change in response to SBTC.

Let $\overline{Q}_c$ and $E_c[A]$ denote the average investment and average ability of college graduates, respectively. We define $\overline{Q}_n$, and $E_n[A]$ in an analogous fashion for individuals without a college degree. From the discussion above, it is clear that $E_c[A] > E_n[A]$, which also implies $\overline{Q_c} > \overline{Q}_n$ from equation (8). Finally, let $\overline{w}_c$ (alternatively $\overline{w}_n$) be the average wage of college (high school) graduates. Then, the college premium before SBTC is

$$\omega_I^* \equiv \left.\frac{\overline{w}_c}{\overline{w}_n}\right|_{t<t^*} = \frac{\theta_L l + \theta_H H\left(\overline{Q}_c\right) - C(\overline{Q}_c)}{\theta_L l + \theta_H H\left(\overline{Q}_n\right) - C(\overline{Q}_n)}.$$

The college premium in the *short run* (that is, immediately after SBTC) is given by

$$\omega_{SR}^* \equiv \left.\frac{\overline{w}_c}{\overline{w}_n}\right|_{t=t^*+\varepsilon} = \frac{\left[\theta_L l + \boldsymbol{\theta}'_H H\left(\overline{Q}_c\right)\right] - C(\overline{\mathbf{Q}}'_c)}{\left[\theta_L l + \boldsymbol{\theta}'_H H\left(\overline{Q}_n\right)\right] - C(\overline{\mathbf{Q}}'_n)}. \quad (19)$$

In the long run, the premium is given by

$$\omega_{LR}^* \equiv \left.\frac{\overline{w}_c}{\overline{w}_n}\right|_{t\to\infty} = \frac{\left[\theta_L l + \boldsymbol{\theta}'_H H\left(\overline{\mathbf{Q}}'_c\right)\right] - C(\overline{\mathbf{Q}}'_c)}{\left[\theta_L l + \boldsymbol{\theta}'_H H\left(\overline{\mathbf{Q}}'_n\right)\right] - C(\overline{\mathbf{Q}}'_n)}. \quad (20)$$

The following proposition characterizes the behavior of the college premium.

Proposition 2 (Behavior of College Premium) *In response to SBTC, for all $\boldsymbol{\theta}'_H > \theta_H$, the college premium*

(i) rises in the long run, i.e., $\omega_{LR}^* > \omega_I^*$,

(ii) falls in the short run, i.e., $\omega_{SR}^* < \omega_I^*$, if condition 1 holds.

[15] More formally, the condition can be stated as $\sum_{s=1}^{\widetilde{s}} 1\{i_{j,s} > i^*\} \geq S_c$, where i^* is the investment time threshold (which is equal to 1 in the standard Ben-Porath model), $\widetilde{s}$ is the individual's current age, and S_c is the number of years of schooling required to qualify as a college graduate.

Despite the similarities between the statements of propositions 1 and 2, there is an important difference between the two. While the stagnation of average wages only requires the endogenous response of human capital investment to SBTC (i.e., that $C(Q)$ increases after the shock), the fall in the college premium requires, in addition, that this response be different across education groups. In other words, if heterogeneity in ability was eliminated from the model, average wages would still stagnate after SBTC, but the college premium would not fall in the short run.[16]

Since college graduates accumulate skills faster than high school graduates, the college premium increases monotonically toward the new steady state value after the initial fall. Moreover, it can be easily shown that the response of the college premium is proportional to the ability differential between college and high school graduates.

Corollary 3 *In response to SBTC, the decline (increase) in the college premium in the short run (long run) is larger, when the ability differential between college graduates and high school graduates,* $E_c[A]/E_n[A]$, *is larger.*

Decomposing the College Premium.—To understand the behavior of the college premium further, an intuitive discussion is helpful. For the sake of this discussion, assume that there are no differences in ability within each education group, and the investment levels are denoted by Q_c and Q_n for college and non-college groups, respectively. However, investment *time* will vary within each education group due to differences in age, and hence, in potential earnings. Using the expression for investment time in (7), we can rewrite the college premium as

$$\omega^* = \frac{\theta_L l + \theta_H H(Q_c)}{\theta_L l + \theta_H H(Q_n)} \times \frac{1-\overline{i_c}}{1-\overline{i_n}} = \underbrace{\frac{l + (\theta_H/\theta_L) H(Q_c)}{l + (\theta_H/\theta_L) H(Q_n)}}_{G_1} \times \underbrace{\frac{1-\overline{i_c}}{1-\overline{i_n}}}_{G_2}, \tag{21}$$

where all the variables that appear in this expression are defined as before, but the averages are now taken with respect to the group indicated by the subscript.[17]

The first term in the decomposition, G_1, captures the price and quantity effects of changes in θ_H. Both of these effects are larger for college graduates because they have a larger human capital stock, and moreover, their human capital stock increases more after SBTC (though the latter happens only gradually). The key point to note is that there is no reason for G_1 to behave in any way other than increase monotonically after SBTC. If there was no investment response in

[16] It should be clear from this discussion that one can prove a result analogous to corollary 2 for the college premium: if individuals' investment did not respond to SBTC, the college premium would immediately jump from its initial steady state to its final steady state value.

[17] $\overline{i}_c$ is the average investment time of college graduates, which is calculated as the ratio of each individual's potential earnings $(\theta_L l + \theta_H Q_c(s-1))$ to average potential earnings of that group $(\theta_L l + \theta_H \frac{\delta}{1-\delta}\overline{Q}_c)$ as weights. $\overline{i}_n$ is defined analogously.

the model, G_2 would be constant over time and the college premium would be proportional to G_1 and would also increase monotonically.

The *differential* investment response captured by G_2 is thus crucial for the initial decline in the college premium. There are two reasons for the initial decline in G_2. First, after SBTC college graduates increase their investment time more than high school graduates. In the long run, this follows from the fact that $d^2 i_{j,s}/d\theta_H dA_j > 0$ mentioned above. The same can be shown to be true in the short run.[18] A second and reinforcing effect responsible for the fall in G_2 is that the initial *level* of investment time is larger for college graduates. As a result, even the same *amount* of increase in investment time would cause a decline in $\left(1-\bar{i}_c\right)/\left(1-\bar{i}_n\right)$. Overall, then, the college premium falls initially because G_2 (which depends on the jump variables, $\bar{i}_c$ and $\bar{i}_n$) falls quickly, but then recovers as G_1 gradually increases over time.

3.2.1 College Premium Within Age Groups

A well-documented fact is that the behavior of the college premium in the United States during this period has been different for different experience groups (see Katz and Murphy (1992) and Murphy and Welch (1992)). In particular, these authors show that the fall and rise in the overall college premium were largely attributable to this behavior among individuals with less experience. In contrast, the fall and rise in the premium among more experienced individuals have been very much muted. Similarly, Card and Lemieux (2001) focus on age groups (rather than experience), and examine data from the United Kingdom and Canada in addition to the United States. They find the same pattern to emerge in these countries as well.

To examine this issue, we now look at the college premium among s-year-old individuals, which is given in the first steady state by

$$\omega_I^*(s) = \frac{\theta_L l + \theta_H \overline{Q}_c(s-1) - C(\overline{Q}_c)}{\theta_L l + \theta_H \overline{Q}_n(s-1) - C(\overline{Q}_n)}. \tag{22}$$

Similarly, the premium in the short run and in the long run after SBTC is defined analogously to equations (19) and (20).

Proposition 3 (Behavior of College Premium Within Age Groups) *Define* $\underline{s} = 1 + \frac{\alpha\delta\beta}{1-\delta\beta}$ *and* $\overline{s} = 1 + \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)}$. *Then in response to SBTC, the college premium among s-year-old individuals*

[18] More formally, we evaluate how the increase in investment time changes with A: we calculate $d^2 i/dAd\boldsymbol{\theta}'_H$, which equals a positive constant times $\theta_L l + (2\alpha-1)\frac{\delta}{1-\delta}\boldsymbol{\theta}'_H\overline{Q}$. It is clear that $\alpha > 0.5$ is a sufficient condition for this cross-partial to be positive. When $\alpha < 0.5$, this will still hold true if $\theta_L l/(\frac{\delta}{1-\delta}\boldsymbol{\theta}'_H\overline{Q})$ is large enough.

Table 1: Evolution of College Premium Within Age Groups After SBTC

	College premium within s-year-old individuals:		
If **s** satisfies:	$\mathbf{s} \leq \underline{s}$	$\underline{s} < \mathbf{s} < \overline{s}$	$\overline{s} \leq \mathbf{s}$
Short run:	*Declines*	*Declines*	Increases
$\log(\omega^*_{SR}(s)/\omega^*_I(s))$	(< 0)	(< 0)	(> 0)
Long run:	*Declines*	Increases	Increases
$\log(\omega^*_{LR}(s)/\omega^*_I(s))$	(< 0)	(> 0)	(> 0)

Notes: See proposition 3 for the definitions of $\underline{s}$ and $\overline{s}$. The table displays the behavior of the college premium for a *marginal* increase in θ_H.

(i) *falls in the short run,* $\omega^*_{SR}(s) < \omega^*_I(s)$*, if and only if* $s < \overline{s}$*,*

(ii) *rises in the long run,* $\omega^*_{LR}(s) > \omega^*_I(s)$*, if and only if* $s > \underline{s}$*.*

An important difference of this proposition from the previous one is that here the decline in the college premium for young individuals does not require condition 1, and therefore, holds under more general conditions than proposition 2. Furthermore, from proposition 3, it is also easy to conjecture that whether the *average* college premium falls in the short run will depend on whether there are sufficiently many young individuals in the population. In fact, this is exactly what condition 1 in proposition 2 ensures: the condition that the average age in the population be less than $\overline{s}$ (that is, $1/(1-\delta) < \overline{s}$) is exactly the same as condition 1.

Proposition 3 partitions the population into three age groups,[19] where the college premium displays distinct behaviors (shown in Table 1). To explain the intuition for these results, it is convenient to take a first-order Taylor series approximation to the college premium for s-year-old individuals (in the initial steady state, short run, and long run), which yields

$$\log\left(\frac{\omega^*_{SR}(s)}{\omega^*_I(s)}\right) \approx \underbrace{\left[(\boldsymbol{\theta}'_H - \theta_H) \times (\overline{Q}_c - \overline{Q}_n)(s-1)\right]}_{\text{Differential Price Effect (>0)}} + \underbrace{\left[\left(C(\overline{Q}_c) - C(\overline{\mathbf{Q}}'_c)\right) - \left(C(\overline{Q}_n) - C(\overline{\mathbf{Q}}'_n)\right)\right]}_{\text{Differential Investment Effect (<0)}},$$

and

$$\log\left(\frac{\omega^*_{LR}(s)}{\omega^*_{SR}(s)}\right) \approx \underbrace{\boldsymbol{\theta}'_H \times \left[\left(\overline{\mathbf{Q}}'_c - \overline{Q}_c\right) - \left(\overline{\mathbf{Q}}'_n - \overline{Q}_n\right)\right](s-1)}_{\text{Differential Quantity Effect (>0)}},$$

[19] The necessity part of the proposition (the "only if" part) applies for a marginal increase in θ_H. For larger increases, $s < \overline{s}$ is a sufficient condition for the college premium to decline, but not a necessary condition. The same comment applies to part (ii) of the proposition. The reason we consider this stronger form of the proposition is because it allows us to divide the age range into three distinct groups in table 1.

(up to a constant scaling factor).[20] Adding up these two equations shows that the total change in log education premium, $\log\left(\omega^*_{LR}(s)/\omega^*_I(s)\right)$, is simply given by the sum of the price, investment, and quantity effects. Comparing these last two equations to their counterparts derived earlier for the average wage (equations (17) and (18)), we note that the only change here is the appearance of "double differences." For example, the price effect here results from the *differential* impact of the increase in θ_H on the human capital stocks of college and non-college workers. The same goes for investment and quantity effects. However, inspecting these two equations also shows that the three effects have the same signs on the college premium as they had on the average wage. This is because college workers have higher ability on average, and therefore (i) they have a larger human capital stock before the shock (resulting in a positive price effect), (ii) they increase their investment by more after the shock (negative investment effect), and (iii), in the long run they experience a larger increase in their human capital stock (positive quantity effect).

To understand the behavior of the college premium among different age groups, two points should be noted. First, the price and quantity effects on the college premium increase with age—notice the multiplicative $(s-1)$ terms that appear in these two effects—whereas the investment effect does not vary with age. Therefore, in the short run the constant investment effect dominates the price effect for younger individuals, but not for older ones who experience a larger price effect ($\mathbf{s} \geq \overline{s}$). The formal proof in the appendix establishes that $\overline{s} > 1$, so that the college premium does fall among a group of young individuals in the short run, but not for the old. Second, as before, the only difference of long run is the additional quantity effect. As a result, some of the relatively younger individuals ($\underline{s} < \mathbf{s} < \overline{s}$) also experience a rise in the premium, and only the very young see a decline in the long run.

3.3 Within-Group Wage Inequality

A well-known empirical fact, first documented by Juhn, Murphy, and Pierce (1993, figure 3), is that the wage growth in a given percentile of the wage distribution during SBTC has been monotonically related to the ranking of that percentile before SBTC. In particular, wages in the higher percentiles in 1963 also experienced high growth from 1963 to 1989, while the opposite happened at lower percentiles. Consequently, the rise in wage inequality happened by the stretching out of the entire wage distribution. The following proposition states that the same outcome happens in the present model.

Proposition 4 (Within-Group Inequality) *Let $w_I(\Omega)$ be the average wage at the Ω^{th} percentile*

[20] To the extent that the change in the college premium is large in response to SBTC, these approximations would not be accurate for quantitative purposes. However, they are useful for explaining the intuition of the results, and we employ them here for that purpose. They are not however used in any proof or derivation.

of the wage distribution before SBTC and $w_{LR}(\Omega)$ be the average wage at the Ω^{th} percentile of the wage distribution in the new steady state after SBTC. Then, $w_{LR}(\Omega)/w_I(\Omega)$ is increasing in Ω.

Juhn, Murphy, and Pierce (1993, figure 5) also show that the same fanning out of the wage distribution is obtained when one conditions on a given age group. This is also true in the present model. The intuition is simple and can already be seen from figure 1, which shows that a higher price of human capital stretches out the wage distribution at every age (above a threshold) without a change in the relative ranking of individuals. Therefore, individuals who earn high wages before SBTC also experience a larger increase in their wages after SBTC (except for very young individuals). The next corollary states this result, and the proof follows.

Corollary 4 *Let $w_I(\Omega|s)$ and $w_{LR}(\Omega|s)$ be the average wage at the Ω^{th} percentile of the wage distribution conditional on age before SBTC and in the new steady state after SBTC, respectively. Then, $w_{LR}(\Omega|s)/w_I(\Omega|s)$ is increasing in Ω when $s > \underline{s}$.*

3.4 Overall Wage Inequality

As mentioned earlier, the movement of overall inequality and between-group inequality in opposite directions in the 1970s has received much attention in the literature. In this section we analyze the behavior of overall wage inequality in response to SBTC both in the short run and in the long run. We show that under a simple sufficient condition, overall inequality *rises* in the short run. Together with the fact that the college premium *falls* in the short run under condition 1, this shows that the present model is consistent with the joint behavior of college premium and overall wage inequality.

To this end, first, the cross-sectional variance of wages can be shown to be

$$Var(w) = \left(n_1 Var(A) + n_2 E[A]^2\right) \times \theta_h^{2/(1-\alpha)}, \tag{23}$$

where the coefficients n_i's are all positive in the rest of the text (the exact expressions are provided in Appendix A). In order to eliminate the effect of the levels of variables on measures of inequality, we focus on the coefficient of variation of wages, which we denote by $CV(w)$. It is given by

$$CV(w) \equiv \frac{Std(w)}{\overline{w}} = \frac{\left[n_1 Var(A) + n_2 E[A]^2\right]^{1/2}}{\frac{\theta_L l}{\theta_H^{1/(1-\alpha)}} + n_4 E[A]}. \tag{24}$$

This expression shows that wage inequality is driven by two sources. First, heterogeneity in learning ability (captured by $Var(A)$) creates wage differences within every age group, and therefore increases wage inequality. Second, a higher average learning ability ($E[A]$) generates more wage growth over the life cycle—and hence wage differences across age groups—thereby increasing the variance in the numerator; but it also increases the average wage, thereby increasing the denominator. Consequently, the effect of average ability on inequality is ambiguous, whereas the effect of ability heterogeneity on inequality is always positive.

The expression in (24) shows that the coefficient of variation is increasing in θ_H, implying that, compared to the initial steady state, wage inequality will be higher in the new steady state after SBTC. In fact, this result could be anticipated from figure 1, which shows both the widening of the wage distribution within age groups, and the steepening of profiles across age groups when θ_H is higher.

Finally, it is also possible to derive a (complicated) expression for the coefficient of variation of wages immediately after SBTC (i.e., in the short run) and prove the following result:

Proposition 5 (Rise in Overall Wage Inequality) *In response to SBTC, for all $\boldsymbol{\theta}'_H > \theta_H$, wage inequality (as measured by the coefficient of variation):*

1. *increases in the long run, i.e., $CV_{LR}(w) > CV_I(w)$,*
2. *also increases in the short run, i.e., $CV_{SR}(w) > CV_I(w)$, if $\beta = 1$.*

The proof is in Appendix A. While $\beta = 1$ is sufficient for an increase in wage inequality in the short run, it is far from being necessary. When $\beta < 1$, there is still a wide range of parameters resulting in an increase in wage inequality in the short run, though we have not been able to find a simple sufficient condition in that case.

When taken together, propositions 2 and 5 show that overall wage inequality and between-group inequality (college premium) move in opposite directions in the short run after SBTC. As noted earlier, this result is consistent with evidence from the U.S. data, and the present framework delivers this outcome despite being a one-skill model.[21]

[21] An important reason for the rise in overall inequality—despite a falling college premium—is that in the short run, average wage profiles shift downward and become *steeper*, increasing the wage differences between the young and the old. To see this, consider the average wage of individuals at age s immediately after SBTC:

$$\overline{w}_{SR}(s) = \left[\theta_L l + \boldsymbol{\theta}'_H \left(\overline{Q}(s-1)\right)\right] - C(\overline{\mathbf{Q}}').$$

Notice that $\overline{Q}(s-1)$ is the *stock* of human capital of s-year-old individuals and is thus relatively fixed in the short run. As mentioned earlier, the price effect is larger for older individuals who have a higher human capital stock compared to younger individuals. This differential price effect steepens the wage profiles immediately after SBTC. Second, the investment effect, $C(\overline{\mathbf{Q}}')$, is independent of age, so it reduces the level of wages for all age groups by the same amount, which shifts the experience profile of wages downward, and therefore, increases *percentage* differences between the young and the old. In turn, this further increases inequality (as measured by the coefficient of variation) across different age groups. As a result, overall wage inequality will increase—despite a fall in college premium—because of a steepening age profile of wages. The steepening of wage profiles after SBTC is consistent with the U.S. data: Katz and Murphy (1992, Table 1) report that the wages of workers with 1–5 years of experience fell by 10.2 percent compared to workers with 26–35 years of experience between 1971 and 1987.

3.4.1 Wage Inequality and Consumption Inequality

The measures of wage inequality discussed so far (i.e., overall, between-group and within-group) relate to the distribution of wages at a point in time. In that sense, they provide some "snapshot" measures of inequality. For many purposes, however, it is of interest to know whether the observed changes in these snapshots imply a parallel change in lifetime income inequality.

A somewhat surprising relevant empirical finding is that the rise in consumption inequality has been muted compared to the rise in wage inequality during this period. Although there remains some disagreement about the exact magnitude of the rise in consumption inequality (mainly due to data problems), several authors report findings broadly supporting this conclusion (see, for example, Krueger and Perri (2006) and Attanasio, Battistin, and Ichimura (2004)). Moreover, the change between the 90th and 50th percentiles of the consumption distribution has not tracked the large rise in the 90-50 percentile wage differences. Autor, Katz, and Kearney (2004) document this fact and find it puzzling.

We now examine the behavior of lifetime wage income inequality in response to SBTC. Note that under the assumptions made so far, individuals choose a constant consumption path over their life cycle. As a result, consumption inequality equals lifetime wage inequality and we use the two interchangeably. First it can be shown (see Appendix A) that the variance of consumption equals

$$Var(c) = n_3 Var(A)\theta_h^{2/(1-\alpha)}. \tag{25}$$

This expression differs from the one for the variance of wages (equation (23)) in two ways. First, as noted earlier, part of the variance of wages is due to the differences in wages across age groups (when $(E(A) > 0)$). This effect is not present in the variance of consumption, because individuals with the same ability will consume the same amount regardless of their age, since they have the same lifetime income. Therefore, the variance of consumption is driven by heterogeneity in learning ability, which is the only source of permanent differences in lifetime incomes. Second, a given heterogeneity in learning ability ($var(A)$) results in smaller consumption inequality than wage inequality (i.e., $n_3 < n_1$, which can be shown easily). The intuition for this result is that those individuals who have high wages later in life are exactly those who make larger investments and accept lower wages early on. Therefore, consumption inequality is always lower than cross-sectional wage inequality in steady state.

We now use these results to examine how wage and consumption inequality *change* relative to each other in response to SBTC. In particular, when the subjective time discount rate is zero, we can show that $CV(w)^2$ will always increase more than $CV(c)^2$ in response to SBTC, regardless of other parameter values. The next proposition states this result.

Proposition 6 (Rise in Wage and Consumption Inequality) *Assume that $\beta = 1$. In response to SBTC, for all $\boldsymbol{\theta}'_H > \theta_H$, wage inequality rises more than consumption inequality in the long run, i.e., $CV_{LR}(w)^2 - CV_I(w)^2 > CV_{LR}(c)^2 - CV_I(c)^2$.*

To prove this proposition, note that when $\beta = 1$, we have $\bar{c} = \overline{w}$ (using equations (12), and (13)). Then combining equations (12), (23) and (25), we have

$$CV(w)^2 - CV(c)^2 = \frac{n_2\left(Var(A) + E\left[A\right]^2\right)}{\left\{\frac{\theta_L l}{\theta_H^{1/(1-\alpha)}} + n_4 E\left[A\right]\right\}^2},$$

in a given steady state. It is easy to see that this expression is increasing in θ_H. Therefore, it is higher in steady state after SBTC than in the initial steady state, which completes the proof.

Note that the difference between wage and consumption inequality increases more with an increase in θ_H when $var(A)$ is larger. Furthermore, if SBTC is modeled as involving a simultaneous fall in θ_L, then the difference between wage and consumption inequality would increase even further after SBTC. Although we have not been able to extend this result to the more general case with $\beta < 1$, in the quantitative analysis (Guvenen and Kuruscu (2006)) we have found wage inequality to increase (substantially) more than consumption inequality for a wide range of parameter values.

4 Extensions and Discussion

4.1 College Enrollment versus On-the-Job Training

Although the main focus of the present paper is on the evolution of the wage distribution, the model also makes predictions about the behavior of college enrollment, and in particular, implies that it will increase in response to SBTC. To show this, we first define an individual to be currently *enrolled in college* if his investment time exceeds a threshold level i^* (consistent with the standard interpretation of the Ben-Porath model). Now, let Π_m for $m = I, SR$, and LR denote the fraction of population enrolled in college in the initial steady state, the short run and the long run respectively. We have the following result (proof in the appendix).

Proposition 7 (College Enrollment) $\Pi_{SR} > \Pi_{LR} > \Pi_I$ *for all s. Therefore, in response to SBTC the fraction of population enrolled in college increases in the long-run, but increases even more in the short-run.*

The reason enrollment is highest in the short run follows from the fact that the opportunity cost of investing—which is determined by current potential earnings—does not change immediately after SBTC while the potential future benefits (determined by $\boldsymbol{\theta}'_H$) increases. Over time, as the price of human capital rises, investment becomes more costly and college enrollment falls to its final steady state level which is still higher than the initial level.

While college enrollment overall increased in the United States from 1970 to 2000, consistent with the proposition's prediction for the long-run behavior, the enrollment rate was stagnant in the 1970's which is at odds with the model's prediction for the short run (cf., Card and Lemieux (2001)). The latter, counterfactual, implication follows from our assumption that SBTC happens in a completely *disembodied* fashion: that is, the productivity of all human capital rises at the same rate regardless of when it is acquired. As a result, the incentive to invest is strongest immediately after SBTC begins, and strongest among young individuals which takes the form of increased college enrollment.

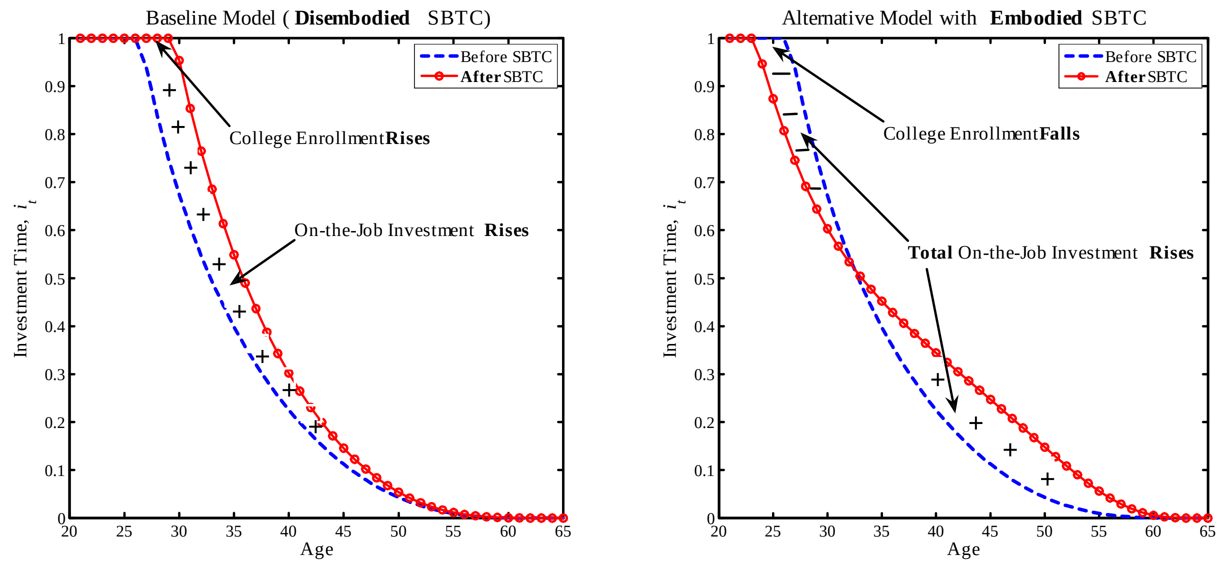
Although the assumption of disembodied SBTC proved to be analytically very convenient, in reality some types of technical change *are* embodied in new types of human capital. With embodied SBTC however, it is easy to show that college enrollment does not necessarily rise, and in fact may fall, in the short run. At the same time on-the-job investment still rises, which causes the college premium to fall in the short run (and rise in the long run) as in the baseline case analyzed earlier. Therefore, it seems that if part of SBTC takes place in an embodied fashion, the counterfactual implications about college enrollment could be overturned while keeping the plausible implications of the model for the evolution of the wage distribution. The drawback of such a model—with embodied SBTC—is that it is not nearly as analytically tractable as our baseline framework. Therefore, below we provide an intuitive derivation of the results just described and relegate the details of the model and proofs to Appendix B. A full investigation of this model is left for future work. We now turn to this variation of the model.

4.1.1 *Embodied* Skill-Biased Technical Change (*E*SBTC)

Consider a modification to the baseline model where technical change is embodied. Specifically, each period a new vintage of human capital becomes available, whose productivity is $(1+\gamma)$ times higher than the preceding vintage. At the same time, the arrival of these more productive vintages makes all existing vintages of human capital partially obsolete, and their productivity falls by $(1-a\gamma)$ each period after their arrival, where $a > 0$ controls the rate of obsolescence. For example, the productivity of vintage-t^* human capital at time t^*+5 will be $\theta_H(1+\gamma)(1-a\gamma)^4$, whereas the productivity of a newer one, say vintage-(t^*+2), will be $\theta_H(1+\gamma)^3(1-a\gamma)^2$ in the same period.[22]

[22] More generally, at time $t+k$ the value of vintage-t human capital ($t \geq t^*$) is given by $\theta_H(1+\gamma)^{t+1-t^*}(1-a\gamma)^{k-1}$.

Figure 3: College Enrollment and On-the-Job Training in the Short Run: "Disembodied" versus "Embodied" SBTC



Optimal investment of an $s-$year-old individual during ESBTC ($t \geq t^*$) is determined by

$$C_j'(Q_t^j(s)) = \underbrace{\beta\theta_H(1+\gamma)^{t+1-t^*}}_{MB_1^*} \times \underbrace{\left[1+\beta(1-a\gamma)+\beta^2(1-a\gamma)^2+...+\beta^{S-s-1}(1-a\gamma)^{S-s-1}\right]}_{MB_2^*} \tag{26}$$

Investment decision before t^* can be obtained by setting $\gamma \equiv 0$.[23] Note that at time t^* individuals realize that investment *later* in life has become more attractive because more productive vintages of human capital will become available by that time (captured by $(1+\gamma)^{t+1-t^*}$). At the same time, human capital becomes obsolete more quickly, reducing the marginal benefit from investing: $\partial MB_2^*/\partial\gamma < 0$. But crucially, this obsolescence reduces the marginal benefit more for *younger* individuals who have a longer horizon than for older individuals: $\partial^2 MB_2^*/\partial\gamma\partial s > 0$. Therefore, the effect of ESBTC is to spread investment more evenly over the life-cycle. In fact, depending on the parameter values, investment at young ages can fall below, while investment at older ages exceed, their levels before t^*. An example of this is shown in the right panel of figure 3, which plots the optimal investment of individuals at different ages (for a given ability level). For the parameter

[23] It is clear from this optimality condition that because future productivity improvements will come *embodied* in future vintages of human capital, they do not increase current incentives to invest—only the productivity of the current vintage, $\theta_H(1+\gamma)^{t+1-t^*}$, enters the optimality condition. (Contrast this with optimality condition (5) in the baseline model where future rises in productivity do increase current investment.)

choices in this example, in the first steady state individuals up to age 26 would choose to enroll in college (dashed line). Immediately after the start of ESBTC (at time t^*) the cross-sectional profile of investment *flattens*: young individuals reduce their investment—those who are 24 to 26 years-old decide not to attend college anymore—whereas older individuals increase their on-the-job training (line with circles).[24] In contrast, the left panel plots the results from the baseline model where both college enrollment and on-the-job investment increases in the short-run for the reasons discussed earlier.

Although the main mechanism discussed above holds for any S, the algebra becomes tedious very quickly as the time horizon expands. Therefore, we prove the main result in Appendix B for $S = 3$, which is the shortest life span necessary to establish our results. Specifically, we show that (under some parameter conditions) ESBTC reduces college enrollment in the short-run, while increasing investment later in life (i.e., on-the-job investment). Because on-the-job training increases, the behavior of the college premium and average wages in response to ESBTC is qualitatively the same as in the baseline model: that is, they fall in the short-run but rise in the long run. Appendix B contains the formal statements and proofs of these results. Therefore, we conclude that the short-run response of college enrollment to SBTC is sensitive to whether technical change is embodied or disembodied, whereas the implications about the evolution of the wage distribution as well as the implications for long-run behavior seem quite robust to that distinction.

4.2 Bringing the Model to the Data

To isolate and highlight the central mechanisms discussed in this paper, the present model abstracts from several important features and makes some stark assumptions, such as assuming a perpetual youth demographic structure, assuming perfect foresight about future skill prices after the arrival of SBTC, assuming perfect substitutability in the production function, assuming that SBTC happens as a one-time jump in the level of human capital price, among others. As a result, the present model is too stylized to be taken directly to the data. In a companion paper (Guvenen and Kuruscu (2006)), we relax these assumptions and provide a detailed quantitative assessment of the model's ability to explain the evolution of the U.S. wage distribution since the 1970's.

An important finding from that paper is that even with few modifications the baseline model proposed in this paper is quite successful in quantitatively explaining the evolution of the U.S. wage distribution.[25] Basically the model generates a fall in the college premium throughout the 70's and a strong recovery thereafter; stagnation in median wages that last for several decades; a rise in

[24] Of course, as time passes MB_2^* remains constant whereas MB_1^*continues to rise, which implies higher investment at all ages compared to the pre-shock levels. As a result, the investment of all individuals rise in the long run.

[25] Essentially the simplest model we consider in that paper relaxes the perpetual youth assumption made here and assumes SBTC takes place over several decades rather than as a one-time jump, but keeps the remaining structure of the baseline model in this paper intact.

within-group inequality that matches closely the empirical counterpart; as well as a very small rise in consumption inequality despite a large rise in wage inequality.

As an extension, we also relax the perfect foresight assumption made here and allow individuals to learn about SBTC in a Bayesian fashion (and again solve the model numerically). Under some plausible scenarios, this extension does not overturn the conclusions of the present paper, but new and interesting channels emerge from the interaction of learning and human capital accumulation. For example, the uncertainty brought about by Bayesian learning does *not* dampen investment incentives. We show in that paper exactly why this is the case. As a result, this extension in fact *strengthens* some of the results obtained here (such as the decline in the college premium in the short-run). In some cases, even if individuals initially underestimate (significantly) the rise in the average growth rate of skill prices due to SBTC, the college premium continues to fall significantly in the short run. Finally we also show that allowing for imperfect substitution in production does not change any substantive conclusions reached without it. Overall, that analysis suggests that the mechanisms and channels that we study theoretically in this paper also play a *quantitatively* important role for understanding the effects of SBTC and the transformation that the U.S. economy has gone through since the 1970's.

5 Conclusions

In this paper we have studied the implications of a tractable overlapping-generations model of human capital accumulation for several trends about the evolution of the wage distribution since the early 1970's. The key element in the model is the interaction between skill-biased technical change—which is interpreted broadly as a rise in the price of human capital—and heterogeneity in the ability to accumulate human capital. Because of the latter heterogeneity, the response of different individuals to SBTC is systematically different from each other. As a result, the model generates rich behavior in the relative wages of individuals depending on their age and ability, thereby creating interesting dynamics in the evolution of the wage distribution. The model is consistent with some prominent trends observed in the U.S. data, such as the joint behavior of the college premium (which fell first and then rose strongly) and overall inequality (which rose throughout this period) despite the fact that the model has one type of skill; the rise in within-group inequality; the stagnation of average wages for an extended period of time; and the small increase in lifetime (consumption) inequality despite the large rise in wage inequality. The baseline model implies that college enrollment should increase in the short run after SBTC which is at odds with the U.S. experience in the 1970's (at least for males). We discuss an extension with embodied SBTC which is consistent with a falling or stagnating college enrollment in the short run while keeping the main implications of the baseline model about the wage distribution intact.

An area for future research is to study whether the mechanism proposed in this paper can also help understand the different experiences of several developed economies during this period (cf., Katz and Autor 1999). To study this question, the current model can be extended to allow for differences in labor market institutions—such as in the progressivity of income taxes—across countries and over time. Notice that progressivity makes the wage structure more compressed (as does unionization, which could also be thought of as another variation across countries and over time), which in turn will hamper the incentives to accumulate human capital. For example, an economy which responds to SBTC by compressing the wage structure will not experience a large increase in inequality, but will also not be able to accumulate the requisite human capital, and therefore experience the growth surge that happens several decades after the onset of SBTC. In future work, we intend to explore the predictions of such a model systematically, and apply them to understand cross-country differences in inequality trends.

A Appendix: Derivations and Proofs of Propositions

Proof of Proposition 1

Substituting the optimal investment level leads to the following expressions for the average wage before the shock and in the short run.

$$\overline{w}_{SR} = \theta_L l + \left(\frac{\delta}{1-\delta}\theta'_H \left(\frac{\alpha\delta\beta}{1-\beta\delta}\theta_H\right)^{\alpha/(1-\alpha)} - \frac{\alpha\delta\beta}{1-\delta\beta}\theta'_H \left(\frac{\alpha\delta\beta}{1-\beta\delta}\theta'_H\right)^{\alpha/(1-\alpha)} \right) E\left[A\right].$$

$$\overline{w}_{I} = \theta_L l + \left(\frac{\delta}{1-\delta}\theta_H \left(\frac{\alpha\delta\beta}{1-\beta\delta}\theta_H\right)^{\alpha/(1-\alpha)} - \frac{\alpha\delta\beta}{1-\delta\beta}\theta_H \left(\frac{\alpha\delta\beta}{1-\beta\delta}\theta_H\right)^{\alpha/(1-\alpha)} \right) E\left[A\right].$$

Then $w_{SR} < w_I$ iff

$$\frac{\theta'_H}{\theta_H}\left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\left(\frac{\theta'_H}{\theta_H}\right)^{\alpha/(1-\alpha)}\right) < \frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}.$$

To see under what conditions this inequality is satisfied, consider the function

$$f(x) = x\left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}x^{\alpha/(1-\alpha)}\right).$$

Notice that a skill-biased technical change is equivalent to increasing $x = \frac{\theta'_H}{\theta_H}$ above 1. Therefore, if $f'(1) < 0$, $f'(x) < 0$ for $x > 1$, then the inequality above is satisfied and $w_{SR} < w_I$.

$$f'(1) = \frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)},$$

therefore $f'(1) < 0$ and $f'(x) < 0$ for $x > 1$, iff $\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)} < 0$.■

Corollary 2: Calculating the Transition time for Average Wages

1. *How long does it take for the average wage to complete half of the transition* $\left(\left(\overline{w}_{LR} - \overline{w}_{SR}\right)/2\right)$*?*

Average wage t periods after the shock is given by

$$\overline{w}_t = \theta_L l - C(\overline{\mathbf{Q}}') + \boldsymbol{\theta}'_H \frac{\delta}{1-\delta}\overline{\mathbf{Q}}' + \left(\overline{Q} - \overline{\mathbf{Q}}'\right)\boldsymbol{\theta}'_H \frac{\delta^{t+1}}{1-\delta}.$$

Define the half-life of the transition to be the smallest integer, t_1, that satisfies

$$\overline{w}_{t_1} - \overline{w}_{SR} > \left(\overline{w}_{LR} - \overline{w}_{SR}\right)/2 = \frac{1}{2}\frac{\delta}{1-\delta}\boldsymbol{\theta}'_H\left(\overline{\mathbf{Q}}' - \overline{Q}\right), \tag{27}$$

where we substituted the definitions of $\overline{w}_{LR}$ and $\overline{w}_{SR}$ from equations (15) and (16) to obtain the second equality. The left-hand side of this last expression is

$$\overline{w}_{t_1} - \overline{w}_{SR} = \overline{w}_{t_1} - \overline{w}_0 = \frac{\delta}{1-\delta}\boldsymbol{\theta}'_H\left(\overline{\mathbf{Q}}' - \overline{Q}\right)\left(1 - \delta^{t_1}\right) \tag{28}$$

by using the definition of $\overline{w}_{t_1}$ above, and equation ((15)). Now, combining equations (27) and (28):

$$\delta^{t_1} < \frac{1}{2} \Leftrightarrow t_1 > -\frac{\log 2}{\log \delta},$$

which can be solved for the minimum integer t_1. It is easy to see that the transition time t_1 is decreasing in δ. So, the speed of convergence is higher when δ is smaller.

2. *How long does it take for the average wage to reach its initial steady state level?*

Normalize the period of shock to zero: $t^* = 0$. Then for all $t > 0$ we have

$$\overline{w}_t - \overline{w}_I = -\boldsymbol{\theta}'_H \overline{\mathbf{Q}}' \left\{ \underbrace{\left[\left(\frac{\theta_H}{\boldsymbol{\theta}'_H} \right)^{1/(1-\alpha)} - 1 \right] \left[\frac{\delta}{1-\delta} - \frac{\alpha\beta\delta}{1-\delta\beta} \right]}_{<0} + \underbrace{\left[1 - \left(\frac{\theta_H}{\boldsymbol{\theta}'_H} \right)^{\alpha/(1-\alpha)} \right]}_{>0} \frac{\delta^{t+1}}{1-\delta} \right\}.$$

Under condition 1, the average wage falls immediately after the shock: $\overline{w}_1 - \overline{w}_I < 0$. Then the expression above is going to increase monotonically and cross zero after some point. There exists a t_2 such that $\overline{w}_I - \overline{w}_t > 0$ for $t \geq t_2$, which is the nearest integer larger than the t^+ that solves

$$\delta^{t^+} = \frac{1 - \left(\frac{\theta_H}{\boldsymbol{\theta}'_H} \right)^{1/(1-\alpha)}}{1 - \left(\frac{\theta_H}{\boldsymbol{\theta}'_H} \right)^{\alpha/(1-\alpha)}} \left[\frac{1}{1-\delta} - \frac{\alpha\beta}{1-\delta\beta} \right] (1-\delta). \tag{29}$$

Proof of Proposition 2

Let

$$\phi = \frac{\overline{Q_c}}{\overline{Q}_n} = \frac{\overline{\mathbf{Q}_c}'}{\overline{\mathbf{Q}}'_n} = \frac{E_c(A)}{E_n(A)}.$$

Substitute $\overline{Q_c} = \phi \overline{Q}_n$ and $C(\overline{Q}_n) = \frac{\alpha\delta\beta}{1-\delta\beta} \theta_H \overline{Q}_n$ in the college premium to get

$$\omega_I^* = \frac{\theta_L l + \phi \theta_H \overline{Q}_n \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta} \right)}{\theta_L l + \theta_H \overline{Q}_n \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta} \right)}$$

and

$$\omega_{LR}^* = \frac{\theta_L l + \phi \boldsymbol{\theta}'_H \overline{\mathbf{Q}}'_n \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta} \right)}{\theta_L l + \boldsymbol{\theta}'_H \overline{\mathbf{Q}}'_n \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta} \right)}.$$

Since $\boldsymbol{\theta}'_H \overline{\mathbf{Q}}'_n > \theta_H \overline{Q}_n$, if the function $g(x) = \frac{\theta_L l + \phi x}{\theta_L l + x}$ is increasing in x, then $\omega_{LR}^* > \omega_I^*$.

$$g'(x) = \frac{\phi \theta_L l - \theta_L l}{(\theta_L l + x)^2}$$

$g'(x)$ is positive iff $\phi > 1$. Then $\omega_{LR}^* > \omega_I^*$ iff $E_c(A) > E_n(A)$.

The premium in the short run can be written as

$$\omega_{SR}^* = \frac{\theta_L l + \phi \left(\frac{\delta}{1-\delta} \boldsymbol{\theta}'_H \overline{Q}_n - \frac{\alpha\delta\beta}{1-\beta\delta} \boldsymbol{\theta}'_H \overline{\mathbf{Q}}'_n \right)}{\theta_L l + \frac{\delta}{1-\delta} \boldsymbol{\theta}'_H \overline{Q}_n - \frac{\alpha\delta\beta}{1-\beta\delta} \boldsymbol{\theta}'_H \overline{\mathbf{Q}}'_n} = \frac{\theta_L l + \phi x_{SR}}{\theta_L l + x_{SR}},$$

where

$$x_{SR} = \frac{\delta}{1-\delta} \boldsymbol{\theta}'_H \overline{Q}_n - \frac{\alpha\delta\beta}{1-\beta\delta} \boldsymbol{\theta}'_H \overline{\mathbf{Q}}'_n.$$

Let

$$x_I = \frac{\delta}{1-\delta}\theta_H \overline{Q}_n - \frac{\alpha\delta\beta}{1-\beta\delta}\theta_H \overline{Q}_n.$$

If $x_{SR} < x_I$, then $\omega^*_{SR} < \omega^*_I$. Therefore we will characterize the condition under which $x_{SR} < x_I$. Plugging optimal investment choices, we can show that $x_{SR} < x_I$ iff

$$\frac{\boldsymbol{\theta}'_H}{\theta_H}\left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\left(\frac{\boldsymbol{\theta}'_H}{\theta_H}\right)^{\alpha/(1-\alpha)}\right) < \frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}.$$

This is the same condition as in proposition 1, therefore $\omega^*_{SR} < \omega^*_I$ for all $\boldsymbol{\theta}'_H > \theta_H$ if $\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)} < 0$.■

Proof of Proposition 3

The proof is very similar to the proof of proposition **2.** Let

$$\phi = \frac{\overline{Q_c}}{\overline{Q}_n} = \frac{\overline{\mathbf{Q}_c}'}{\overline{\mathbf{Q}}'_n} = \frac{E_c(A)}{E_n(A)}.$$

The premium in the short run can be written as

$$\omega^*_{SR}(s) = \frac{\theta_L l + \phi\left(\boldsymbol{\theta}'_H \overline{Q}_n(s-1) - \frac{\alpha\delta\beta}{1-\beta\delta}\boldsymbol{\theta}'_H \overline{\mathbf{Q}}'_n\right)}{\theta_L l + \boldsymbol{\theta}'_H \overline{Q}_n(s-1) - \frac{\alpha\delta\beta}{1-\beta\delta}\boldsymbol{\theta}'_H \overline{\mathbf{Q}}'_n} = \frac{\theta_L l + \phi x_{SR}}{\theta_L l + x_{SR}},$$

where $x_{SR} = \boldsymbol{\theta}'_H \overline{Q}_n(s-1) - \frac{\alpha\delta\beta}{1-\beta\delta}\boldsymbol{\theta}'_H \overline{\mathbf{Q}}'_n$. Let $x_I = \theta_H \overline{Q}_n(s-1) - \frac{\alpha\delta\beta}{1-\beta\delta}\theta_H \overline{Q}_n$.

The education premium declines in the short run iff $x_{SR} < x_I$.

$$x_{SR} < x_I \iff \frac{\boldsymbol{\theta}'_H}{\theta_H}\left(s - 1 - \frac{\alpha\delta\beta}{1-\delta\beta}\left(\frac{\boldsymbol{\theta}'_H}{\theta_H}\right)^{\alpha/(1-\alpha)}\right) < s - 1 - \frac{\alpha\delta\beta}{1-\delta\beta}.$$

Define the function

$$f_s(x) = x\left(s - 1 - \frac{\alpha\delta\beta}{1-\delta\beta}x^{\alpha/(1-\alpha)}\right).$$

Notice that a skill-biased technical change is equivalent to increasing $x = \frac{\theta'_H}{\theta_H}$ above 1. Therefore, if $f'_s(1) < 0$ then $\omega^*_{SR}(s) < \omega^*_I(s)$.

$$f'_s(1) < 0 \iff s < 1 + \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)}.$$

Therefore, $\omega^*_{SR}(s) < \omega^*_I(s)$ if $s < 1 + \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)}$.

The college premium in the long run is given by

$$\omega^*_{LR}(s) = \frac{\theta_L l + \phi\left(s - 1 - \frac{\alpha\delta\beta}{1-\beta\delta}\right)\boldsymbol{\theta}'_H \overline{\mathbf{Q}}'_n}{\theta_L l + \left(s - 1 - \frac{\alpha\delta\beta}{1-\beta\delta}\right)\boldsymbol{\theta}'_H \overline{\mathbf{Q}}'_n}.$$

Since $\boldsymbol{\theta}'_H \overline{\mathbf{Q}}'_n > \theta_H \overline{Q}_n$ and $\phi > 1$, the college premium would increase in the long run if $s - 1 - \frac{\alpha\delta\beta}{1-\beta\delta} > 0$.■

Derivation of the Variances of Wages and Consumption

The wage of an s-year-old individual of type j who is $w_{j,s} = \theta_L l + \theta_H Q_j(s-1) - C_j(Q_j)$. We rewrite it as $w_{j,s} = m_j + p_j(s-1)$, where $m_j = \theta_L l - C_j(Q_j)$ and $p_j = \theta_H Q_j$.

The average wage is given by

$$\overline{w} = \sum_{s=1}^{\infty}(1-\delta)\delta^{s-1}\int_j w_{j,s} = \sum_{s=1}^{\infty}(1-\delta)\delta^{s-1}\int_j \left[m_j + p_j(s-1)\right].$$

With some algebra we get

$$\overline{w} = \overline{m} + \frac{\delta}{1-\delta}\overline{p} = \theta_L l - C(\overline{Q}) + \frac{\delta}{1-\delta}\theta_H\overline{Q} = \theta_L l + \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)\theta_H\overline{Q},$$

where

$$\overline{m} = \theta_L l - \int_j C_j(Q_j) = \theta_L l - C(\overline{Q}) = \theta_L l - \frac{\alpha\delta\beta}{1-\delta\beta}\theta_H\overline{Q}$$

and $\bar{p} = \theta_H \int_j Q_j = \theta_H\overline{Q}$. Using the expression for Q_j, we get

$$\overline{Q} = \left(\frac{\alpha\delta\beta}{1-\beta\delta}\theta_H\right)^{\alpha/(1-\alpha)} E\left[A\right].$$

The consumption of type j individual is $c_j = m_j + \frac{\beta\delta}{1-\beta\delta}p_j$. Then the average consumption is

$$\bar{c} = \overline{m} + \frac{\delta\beta}{1-\delta\beta}\bar{p} = \theta_L l + \left(\frac{\delta\beta}{1-\delta\beta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)\theta_H\overline{Q}$$

Variance of wages is given by

$$\begin{aligned} Var(w) &= \int_j\sum_{s=1}^{\infty}(1-\delta)\delta^{s-1}\left[m_j + p_j(s-1) - \overline{m} - \frac{\delta}{1-\delta}\bar{p}\right]^2 \\ &= Var(m) + \frac{2\delta}{1-\delta}Cov(m,p) + \frac{\delta(1+\delta)}{(1-\delta)^2}Var(p) + \frac{\delta}{(1-\delta)^2}\bar{p}^2 \end{aligned}$$

and variance of consumption is

$$\begin{aligned} Var(c) &= \int_j\left[m_j + \frac{\beta\delta}{1-\beta\delta}p_j - \overline{m} - \frac{\beta\delta}{1-\beta\delta}\overline{p}\right]^2 \\ &= Var(m) + \frac{2\delta\beta}{1-\delta\beta}Cov(m,p) + \frac{(\delta\beta)^2}{(1-\delta\beta)^2}Var(p). \end{aligned}$$

$$\begin{aligned} Var(m) &= \left(\frac{\alpha\delta\beta}{1-\delta\beta}\right)^2\theta_H^2 Var(Q) \\ Var(p) &= \theta_H^2 Var(Q) \end{aligned}$$

$$
\begin{aligned}
Cov(m,p) &= -\frac{\alpha\delta\beta}{1-\delta\beta}\theta_H^2 Var(Q) \\
Var(Q) &= \left(\frac{\alpha\delta\beta\theta_H}{1-\delta\beta}\right)^{2\alpha/(1-\alpha)} Var(A)
\end{aligned}
$$

Plugging in the expressions above, we get

$$
Var(w) = \left(\frac{\delta(1+\delta)}{(1-\delta)^2} - \frac{2(\alpha\delta\beta)\delta}{(1-\delta\beta)(1-\delta)} + \frac{(\alpha\delta\beta)^2}{(1-\delta\beta)^2}\right)\theta_h^2 Var(Q) + \frac{\delta}{(1-\delta)^2}\theta_h^2\overline{Q}^2
$$

$$
\begin{aligned}
&= \left(\frac{\delta(1+\delta)}{(1-\delta)^2} - \frac{2(\alpha\delta\beta)\delta}{(1-\delta\beta)(1-\delta)} + \frac{(\alpha\delta\beta)^2}{(1-\delta\beta)^2}\right)\left(\frac{\alpha\delta\beta}{1-\delta\beta}\right)^{2\alpha/(1-\alpha)} \theta_h^{2/(1-\alpha)} Var(A) \\
&\quad + \frac{\delta}{(1-\delta)^2}\left(\frac{\alpha\delta\beta}{1-\delta\beta}\right)^{2\alpha/(1-\alpha)} \theta_H^{2/(1-\alpha)} E\left[A\right]^2
\end{aligned}
$$

and

$$
\begin{aligned}
Var(c) &= \left[\frac{(1-\alpha)\delta\beta}{1-\delta\beta}\right]^2 \theta_h^2 Var(Q) \\
&= \left[\frac{(1-\alpha)\delta\beta}{1-\delta\beta}\right]^2 \left(\frac{\alpha\delta\beta}{1-\delta\beta}\right)^{2\alpha/(1-\alpha)} \theta_h^{2/(1-\alpha)} Var(A).
\end{aligned}
$$

Therefore in the formulas in the text:

$$
\begin{aligned}
n_1 &\equiv \left(\frac{\delta(1+\delta)}{(1-\delta)^2} - \frac{2(\alpha\delta\beta)\delta}{(1-\delta\beta)(1-\delta)} + \frac{(\alpha\delta\beta)^2}{(1-\delta\beta)^2}\right)\left(\frac{\alpha\delta\beta}{1-\delta\beta}\right)^{2\alpha/(1-\alpha)} > 0, \\
n_2 &\equiv \frac{\delta}{(1-\delta)^2}\left(\frac{\alpha\delta\beta}{1-\delta\beta}\right)^{2\alpha/(1-\alpha)} > 0, \qquad n_3 \equiv \left[\frac{(1-\alpha)\delta\beta}{1-\delta\beta}\right]^2 \left(\frac{\alpha\delta\beta}{1-\delta\beta}\right)^{2\alpha/(1-\alpha)} > 0, \\
n_4 &\equiv \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)\left(\frac{\alpha\delta\beta}{1-\beta\delta}\right)^{\alpha/(1-\alpha)} > 0.
\end{aligned}
$$

Proof of Proposition 4

Remember that $w_{s,j} = \theta_L l + \theta_H Q_j(s-1) - C(Q_j)$. Plugging in the optimal investment, we can write $w_{s,j} = \theta_L l + n_5 \theta_H^{1/(1-\alpha)} y$, where $n_5 = \left(\frac{\alpha\delta\beta}{1-\beta\delta}\right)^{\alpha/(1-\alpha)}$ and $y = \left(s - 1 - \frac{\alpha\delta\beta}{1-\beta\delta}\right)A$. It is clear that $w_{s,j}$ is increasing in y. Hence, one's relative position Ω in the wage distribution is positively related to y.

The wage of an agent with y before the shock is given by $w_I(y) = \theta_L l + n_5 \theta_H^{1/(1-\alpha)} y$. The corresponding wage in the long run is $w_{LR}(y) = \theta_L l + n_5 \boldsymbol{\theta}_H'^{1/(1-\alpha)} y$. It is then easy to show that $w_{LR}(y)/w_I(y)$ is increasing in y.■

Proof of Proposition 5

a. **Long run:**

$$\begin{aligned} & CV(w)^2\Big|_{\theta'_H} - CV(w)^2\Big|_{\theta_H} \\ = & \frac{n_1 Var(A) + n_2 E\left[A\right]^2}{\overline{w}'^2}\theta_h'^{2/(1-\alpha)} - \frac{n_1 Var(A) + n_2 E\left[A\right]^2}{\overline{w}^2}\theta_h^{2/(1-\alpha)} \\ = & \left(n_1 Var(A) + n_2 E\left[A\right]^2\right)\left(\frac{\theta_h'^{2/(1-\alpha)}}{\overline{w}'^2} - \frac{\theta_h^{2/(1-\alpha)}}{\overline{w}^2}\right) \end{aligned}$$

where $\overline{w}$ and $\overline{w}'$ are the average wages in the old and new steady. Plugging in

$$\overline{w} = \theta_L l + \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)\left(\frac{\alpha\delta\beta}{1-\beta\delta}\right)^{\alpha/(1-\alpha)} \theta_H^{1/(1-\alpha)} E\left[A\right]$$

and

$$\overline{w}' = \theta_L l + \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta}\right)\left(\frac{\alpha\delta\beta}{1-\beta\delta}\right)^{\alpha/(1-\alpha)} \theta_H'^{1/(1-\alpha)} E\left[A\right]$$

we get

$$\begin{aligned} & CV(w)^2\Big|_{\theta'_H} - CV(w)^2\Big|_{\theta_H} \\ = & \theta_L l\left\{n_1 Var(A) + n_2 E\left[A\right]^2\right\}\left[\theta_H'^{1/(1-\alpha)}\overline{w} + \theta_H^{1/(1-\alpha)}\overline{w}'\right] \times \frac{\theta_H'^{1/(1-\alpha)} - \theta_H^{1/(1-\alpha)}}{\overline{w}^2\overline{w}'^2}. \end{aligned}$$

Since n_1 and n_2 are positive, and $\theta'_H > \theta_H$,

$$CV(w)^2\Big|_{\theta'_H} - CV(w)^2\Big|_{\theta_H} > 0$$

An alternative way is to look at the derivative of $CV(w)^2\big|_{\theta_H}$ with respect to θ_H. It is easy to see that in fact $CV(w)^2\big|_{\theta_H}$ increases with θ_H.

b. **Short run:** Remember that the wage in the short run is

$$w_{j,s}^{SR} = \underbrace{\theta_L l - C_j(\mathbf{Q}'_j)}_{m'_j} + \underbrace{\boldsymbol{\theta}'_H Q_j}_{p'_j}(s-1).$$

Notice that the difference between $w_{j,s}$ and $w_{j,s}^{SR}$ is that we have replaced m_j and p_j with m'_j and p'_j. Hence the average wage in the short run is $\overline{w}_{SR} = \overline{m}' + \frac{\delta}{1-\delta}\overline{p}'$ and the variance of wages in the short run is

$$Var_{SR}(w) = Var(m') + \frac{2\delta}{1-\delta}Cov(m', p') + \frac{\delta(1+\delta)}{(1-\delta)^2}Var(p') + \frac{\delta}{(1-\delta)^2}\overline{p}'^2.$$

$$\begin{aligned} Var(m') &= \left(\frac{\alpha\delta\beta}{1-\delta\beta}\boldsymbol{\theta}'_H\right)^2 \times \underbrace{Var(\mathbf{Q}')} \\ &= \left(\frac{\alpha\delta\beta}{1-\delta\beta}\boldsymbol{\theta}'_H\right)^2 \left(\frac{\boldsymbol{\theta}'_H}{\theta_H}\right)^{2\alpha/(1-\alpha)} Var(Q) \end{aligned}$$

$$
\begin{aligned}
Var(p') &= \theta_H'^2 Var(Q) \\
Cov(m', p') &= -\frac{\alpha\delta\beta}{1-\delta\beta}\theta_H'^2 Cov(Q, \mathbf{Q}') \\
Cov(Q, \mathbf{Q}') &= \left(\frac{\boldsymbol{\theta}'_H}{\theta_H}\right)^{\alpha/(1-\alpha)} Var(Q).
\end{aligned}
$$

Then we can write the variance in the short run as

$$
Var_{SR}(w) = \left\{ \left(\frac{(\alpha\delta\beta)^2}{(1-\delta\beta)^2} x^{2/(1-\alpha)} - \frac{2\alpha\delta^2\beta}{(1-\delta\beta)(1-\delta)} x^{(2-\alpha)/(1-\alpha)} + \frac{\delta(1+\delta)}{(1-\delta)^2} x^2 \right) Var(Q) + \frac{\delta}{(1-\delta)^2} \overline{Q}^2 x^2 \right\} \theta_h^2,
$$

where $x = \boldsymbol{\theta}'_H/\theta_H$. Similarly the average wage in the short run can be written as

$$
\overline{w}_{SR} = \theta_L l + \left(\frac{\delta}{1-\delta} x - \frac{\alpha\delta\beta}{1-\delta\beta} x^{1/(1-\alpha)} \right) \theta_H \overline{Q}.
$$

We look at what happens to $CV_{SR}^2(w)$ if x is increased marginally above one, or equivalently $\boldsymbol{\theta}'_H$ is increased marginally above θ_H. Hence we compute

$$
\frac{d}{dx} CV_{SR}^2(w) = 2 \frac{\frac{\overline{w}_{SR}}{2} \frac{d}{dx} Var_{SR}(w) - Var_{SR}(w) \frac{d}{dx} \overline{w}_{SR}}{\overline{w}_{SR}^3}.
$$

If $\frac{d}{dx} CV_{SR}^2(w) > 0$ then we conclude that inequality increases in the short run with an increase in price of human capital. Since wage is positive, $\frac{d}{dx} CV_{SR}^2(w)$ would be positive if $\frac{\overline{w}_{SR}}{2} \frac{d}{dx} Var_{SR}(w) - Var_{SR}(w) \frac{d}{dx} \overline{w}_{SR} > 0$. Then we look at this expression at $x = 1$, which is equal to

$$
\begin{aligned}
&\frac{\overline{w}_{SR}}{2} \frac{d}{dx} Var_{SR}(w) - Var_{SR}(w) \frac{d}{dx} \overline{w}_{SR} \\
&= \left\{ \left[\frac{1}{1-\alpha} \frac{(\alpha\delta\beta)^2}{(1-\delta\beta)^2} - \frac{2-\alpha}{1-\alpha} \frac{2\alpha\delta^2\beta}{(1-\delta\beta)(1-\delta)} + \frac{\delta(1+\delta)}{(1-\delta)^2} \right] Var(Q) + \frac{\delta}{(1-\delta)^2} \overline{Q}^2 \right\} \\
&\quad \times \left\{ \theta_L l + \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{1-\delta\beta} \right) \theta_H \overline{Q} \right\} \\
&\quad - \theta_H \overline{Q} \left(\frac{\delta}{1-\delta} - \frac{\alpha\delta\beta}{(1-\delta\beta)(1-\alpha)} \right) \\
&\quad \times \left\{ \left(\frac{(\alpha\delta\beta)^2}{(1-\delta\beta)^2} - \frac{2\alpha\delta^2\beta}{(1-\delta\beta)(1-\delta)} + \frac{\delta(1+\delta)}{(1-\delta)^2} \right) Var(Q) + \frac{\delta}{(1-\delta)^2} \overline{Q}^2 \right\}.
\end{aligned}
$$

Evaluating this expression at $\beta = 1$ gives

$$
\frac{\delta}{(1-\delta)^2} \left\{ (1+\delta-2\alpha\delta) Var(Q) \theta_L l + \theta_L l \overline{Q}^2 + \frac{\alpha\delta^2}{1-\delta} \theta_H \overline{Q}^3 + (1-\alpha^2+\alpha^2\delta) \frac{\alpha\delta^2}{(1-\delta)(1-\alpha)} \theta_H \overline{Q} Var(Q) \right\}.
$$

Notice: $1+\delta-2\alpha\delta > 0$ and $1-\alpha^2+\alpha^2\delta > 0$ for all $\alpha \leq 1$. Therefore, $\frac{d}{dx} CV_{SR}^2(w)\big|_{x=1} > 0$ when $\beta = 1$.■

Proof of Proposition 7:

Let $A_I^*(s)$ denote the ability threshold in the initial steady state above which all individuals enroll in college (that is, $i^j(s) \geq i^*$ for $A_j > A_I^*(s)$). Using equation (7) we have:

$$A_I^*(s) = \frac{\eta_0}{\theta_H^{1/(1-\alpha)}} \left[\frac{\theta_L l}{\eta_1 / i^* - \eta_2 (s-1)} \right]$$

where η_0, η_1 and η_2 are positive constants. As could be expected, the threshold is increasing with s implying that college enrollment falls with age. Replacing θ_H in this expression with $\boldsymbol{\theta}'_H$ yields the threshold in the long-run after SBTC, which is denoted by $A_{LR}^*(s)$. Since $\boldsymbol{\theta}'_H > \theta_H$ it follows that $A_{LR}^*(s) < A_I^*(s)$. Similarly, the threshold ability level in the short-run after SBTC can be shown to be:

$$A_{SR}^*(s) = \frac{\eta_0}{\boldsymbol{\theta}'^{1/(1-\alpha)}_H} \left[\frac{\theta_L l}{\eta_1 / i^* - \eta_2 (s-1) \left(\theta_H / \boldsymbol{\theta}'_H \right)^{1/(1-\alpha)}} \right],$$

again, using equation (7). It is easily noted that $A_{SR}^*(s) < A_{LR}^*(s) < A_I^*(s)$.

Total college enrollment rate can be written as: $\Pi_m = \frac{1}{S} \sum_{s=1}^{S} 1 \left\{ A^j > A_m^*(s) \right\}$ for $m = I, SR$ and LR. Then, the fact that $A_{SR}^*(s) < A_{LR}^*(s) < A_I^*(s)$ implies $\Pi_{SR} > \Pi_{LR} > \Pi_I$.■

B Appendix: A Model with Embodied SBTC

An embodied SBTC is defined as a one-time jump at time t^* in γ from 0 to γ^*. Notice that if ESBTC continues forever all individuals will be college graduates *in the long run*. Although this does not affect any of our conclusions, it may seem like an extreme outcome. Alternatively, we can assume that ESBTC continues for T ($> S$) years after which time γ reverts back to zero. For a given choice of parameters, we can choose T such that the new steady state features an enrollment rate less than 1.

Let $i_m(s)$ denote the average investment time of individuals of age s in state m (short-run, long-run, etc.) More generally, at time $t+k$ the value of a vintage of skill acquired at time $t \geq t^*$ is given by $\theta_H(1+\gamma)^{t+1-t^*}(1-a\gamma)^{k-1}$. Let Q^τ denote vintage-τ human capital. The following table shows the timing of investment and the stream of wages that result from that investment.

t^*	t^*+1	t^*+2	$\cdots$	t^*+k
acquire Q^{t^*}	$(1+\gamma)Q^{t^*}$	$(1+\gamma)(1-a\gamma)Q^{t^*}$	$\cdots$	$(1+\gamma)(1-a\gamma)^{k-1}Q^{t^*}$
	acquire Q^{t^*+1}	$(1+\gamma)^2 Q^{t^*+1}$	$\cdots$	$(1+\gamma)^2(1-a\gamma)^{k-2}Q^{t^*+1}$
		$\vdots$		$\vdots$
				$(1+\gamma)^k Q^{t^*+k-1}$

We first state a condition on the parameters of the model, and then present the main result of this section.

Condition 2 $\gamma(1+\beta-\beta a(1+\gamma)) < 0.$

Proposition 8 *In response to* ***embodied*** *SBTC, in the short run, the average investment time of*

1. *old individuals increases, i.e.,* $i_{SR}(2) > i_I(2)$,
2. *young individuals falls, i.e.,* $i_{SR}(1) < i_I(1)$, *if condition 2 holds.*

Proof. The marginal benefit of investment for old individuals is $\beta\theta_H$ before ESBTC and $\beta\theta_H(1+\gamma)$ at time t^*. Hence, investment of all old individuals increases in the short-run since the marginal benefit of investment is larger. Note that potential earnings does not change, hence investment time also increases. Similarly, the marginal benefit of investment for young individuals is given by $\beta\theta_H(1+\beta)$ before ESBTC and by $\beta\theta_H(1+\gamma)(1+\beta(1-a\gamma))$ at time t^*. Hence, investment of young agents fall in the short-run if $\beta\theta_H(1+\gamma)(1+\beta(1-a\gamma)) < \beta\theta_H(1+\beta)$ which is equivalent to condition 2.■

This result formalizes the intuition discussed above that an embodied technical change *flattens* the life-cycle profile of human capital investment: that is, individuals invest less early in life—therefore college enrollment falls—and more later in life—on-the-job training rises.[26]

Next we turn our attention to what happens to college premium and average wages. We need the following condition to characterize the behavior of college premium.

Condition 3 $[(1+\gamma)(1+\beta(1-a\gamma))]^{1/(1-\alpha)} + [1+\gamma]^{1/(1-\alpha)} > [1+\beta]^{1/(1-\alpha)} + 1.$

Proposition 9 *In response to ESBTC, the college premium*

1. *rises in the long-run, i.e.,* $\omega^*_{LR} > \omega^*_I$,
2. *falls in the short run, i.e.,* $\omega^*_{SR} < \omega^*_I$ *if conditions 2 and 3 hold.*

Proof. It is straightforward to show that, the college premium falls if average wage falls. Since the stock of human capital and its price do not change in the short-run the average wage would fall if on-the-job training increases in the short-run. Since investment early in life (college enrollment) falls in the short-run under condition 2, on-the-job training would be larger if total cost of human capital investment is larger which would occur under condition 3.■

Because on-the-job training increases in the short run, the behavior of the college premium in response to ESBTC is qualitatively the same as in the baseline model.[27] We want to note that the intersection of condition 2 and 3 is not an empty set. Looking at these conditions, one can see that condition 2 holds if depreciation parameter a is larger. The opposite is true for condition 3. Hence, both conditions hold for intermediate values of a.

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[27] The details of these results are available from the authors upon request.

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