NBER WORKING PAPER SERIES

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Working Paper No. 1300

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
March 1984

Financial support was provided by the National Science Foundation. The research reported here is part of the NBER's research program in Economic Fluctuations. Any opinions expressed are those of the authors and not those of the National Bureau of Economic Research.

NBER Working Paper #1300
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ABSTRACT

This paper describes a theoretical and empirical study of the possibility of rational bubbles in the relative price of gold. The critical implication of the theoretical analysis is that, if rational bubbles exist, the time series of the relative price of gold, as well as any time series obtained by differencing a finite number of times, is nonstationary. The empirical evidence relating to this nonstationarity property involves diagnostic checks for stationarity carried out in both the time domain and the frequency domain. This evidence strongly suggests that the process generating the first difference of the log of the relative price of gold is stationary, a finding that is inconsistent with the existence of rational bubbles. More broadly, the empirical analysis finds a close correspondence between the time series properties of the relative price of gold and the time series properties of real interest rates, which the theory relates to the time series properties of the fundamental component of the relative price of gold. In sum, the evidence is consistent with the combined conclusion that the relative price of gold corresponds to market fundamentals, that the process generating first differences of market fundamentals is stationary, and that actual price movements do not involve rational bubbles.

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In recent years the price of gold has fluctuated widely relative to the prices of other goods and services and the realized return from holding gold for short periods has fluctuated widely relative to realized real interest rates on short-term assets. For example, from January 1975 through March 1983, the monthly inflation rate, measured by the U.S. CPI for All Items Less Mortgage Interest Costs, averaged .61%, with a standard deviation of .29%, a minimum of .04% and a maximum of 1.25%, while the monthly rate of change of the price of gold, measured by the average of daily values of the London P.M. price fix, averaged 1.19%, with a standard deviation of 8.22%, a minimum of -16.80%, and a maximum of 48.39%. Moreover, during the same period, the real monthly return from holding commercial paper averaged .17%, with a standard deviation of .39%, a minimum of -.66%, and a maximum of 1.18%, while the real monthly return from holding gold averaged .58%, with a standard deviation of 8.13%, a minimum of -17.77%, and a maximum of 46.68%.

Do these relatively wide fluctuations in gold's price and real return reflect rational behavior? If so, are they attributable solely to changes in market fundamentals, or do they involve the existence of rational bubbles?

This paper describes a theoretical and empirical study of the possibility of rational bubbles in the relative price of gold. The critical implication of the theoretical analysis is that, if rational bubbles exist, the time series of the relative price of gold, as well as any time series obtained by differencing a finite number of times, is nonstationary. This implication suggests a strategy for obtaining evidence against the existence of rational bubbles that, in contrast to other empirical strategies, does not depend on accepting untested hypotheses about market fundamentals or about the factors generating bubbles.

The empirical analysis involves diagnostic checks for stationarity using data for the relative price of gold from January 1975 through March 1983. The diagnostic checks are carried out in both the time domain and the frequency domain. The results strongly suggest that the fluctuations in the relative price of gold during this period did not involve rational bubbles.

1. A Model of the Relative Price of Gold

The theoretical model consists of a single equation that specifies that the relative price of gold satisfies a condition of equality between the value of the existing stock of gold and the portfolio demand for gold.

$$(1) \quad S_t + p_t = \beta(E_t p_{t+1} - p_t) - \gamma E_t r_{t+1} + u_t$$

where S_t is the logarithm of the stock of gold at date t,

p_t is the logarithm of the price of gold at date t relative to a general index of prices of goods and services,

r_{t+1} is an index of real rates of return on other assets (the real interest rate) from date t to date t+1,

E_t is an operater that denotes a rational expectation, i.e. an expectation consistent with the true model, conditional on information available at date t,

u_t represents the effects of all factors other than expected rates of return on the demand for gold at date t,

and β and γ are positive constants.

Equation (1) says that the value in terms of goods and services of the portfolio demand for gold depends positively on the expected real rate of return from holding gold, which is simply the expected rate of change in the relative price of gold, depends negatively on expected real rates of return on other asets, and depends on other factors incorporated in u_t. This model is a simplified version of the model of the market for a storable commodity developed in Diba and Grossman (1983) and is similar to the model of inflation that is discussed extensively together with related literature in that paper.

The analysis assumes that S_t, r_t, and u_t are exogenously determined random variables and that the sequences $\{E_t S_{t+j}\}$, $\{E_t r_{t+j}\}$, and $\{E_t u_{t+j}\}$ do not grow exponentially with j, for any t. The assumed exogeneity of S_t means that the model abstracts from the distinction between different states of gold, such as unmined ore, bullion, or jewelry. The theoretical analysis in Diba and Grossman (1983) considers the market for refined gold separately and explicitly allows for the effect of the expected relative price of gold on the rate of extraction and refining of ore. These complications , however, do not change the qualitative implications of the existence of rational bubbles on which the present analysis focuses. In any event, because no reliable data exist for the stock of gold, these alternative formulations are not distinguishable empirically, and the econometric analysis must treat S_t, like u_t, as an unobserved variable.

The assumed exogeneity of r_t means that the model abstracts from any dependence of the real interest rate on events in the gold market. Specifically, the analysis presumes that, even if the relative price of gold exhibited rational bubbles, these bubbles would not be reflected in real interest rates. A possible interpretation of this assumption of exogeneity is that the real interest rate is determined by close correspondence with either the physical rate of return on capital or the rate of time preference.

An implicit assumption that is critical for the possible existence of rational bubbles in the price of gold is that the cumulative flow of services generated by the stock of gold, aggregated over an infinite time horizon, is not finite. To see this point, contrast the case of a truly exhaustible resource with given finite initial reserves, of which oil might be an example, with the case of an inexhaustible resource, of which some types of land might be an example. We can analyze the time path of the price of oil as follows: First, profit maximization by owners of the resource and/or portfolio balance determines the time path of the rate of price increase relative to the rate of interest. Second, substituting for price in the consumers' demand functions and aggregating over time gives cumulative consumption as a function of the initial price alone. Finally, equating cumulative consumption to the initial reserves determines the initial price, and hence the entire price path, uniquely.

In the contrasting case of land, the fundamental component of the price equals the present value of the flow of rental income. However, because the cumulative value of the flow of services from land is not finite, we cannot determine uniquely the time path of the price of land in the same way as the price of oil. The present analysis treats the stock of gold to be inexhaustible like the stock of land. The rationale is that, even if gold is refined into bullion or manufactured into jewelry, it continues to generate an unending flow of services in the form of security, visual satisfaction, or whatever.

2. Solution of the Model

A solution for equation (1) requires a solution for a partial difference equation that governs the formation of expectations at date t. To obtain this equation, lead (1) j periods, $j > 1$, rearrange terms, operate on both sides with E_t, and apply the law of iterated expectations, $E_t(E_{t+j}x_{t+j+1}) = E_t x_{t+j+1}$, to get,

$$(2) \quad E_t p_{t+j+1} = (1+\beta^{-1})E_t p_{t+j} - \beta^{-1}E_t(u_{t+j} - \gamma r_{t+1+j} - S_{t+j}).$$

Equation (2) is a partial, rather than ordinary, difference equation because $E_t p_{t+j}$ depends on both t and j. To solve for $E_t p_{t+j}$, fix t and treat (2) as an ordinary difference equation in j. Because the eigenvalue $1+\beta^{-1}$ is greater than unity and $E_t(S_{t+j} - u_{t+j} + \gamma r_{t+j+1})$ does not grow exponentially, the forward-looking particular solution involves a convergent sum. This solution--see Sargent (1979, pp. 171-177)--is

$$(3)\quad E_t p^*_{t+j} = (1+\beta)^{-1} \sum_{k=0}^{\infty} (1+\beta^{-1})^{-k} E_t(u_{t+j+k} - \gamma r_{t+1+j+k} - S_{t+j+k}),$$

$$j \geq 1.$$

To obtain a forward-looking particular solution for p_t, set $j = 1$ in (3) and substitute for $E_t p_{t+1}$ in (1), to get

$$(4)\quad p^*_t = (1+\beta)^{-1} \sum_{i=0}^{\infty} (1+\beta^{-1})^{-i} E_t(u_{t+i} - \gamma r_{t+1+i} - S_{t+i}).$$

We refer to the particular solution, p^*_t, as the fundamental component of the relative price of gold (FC).

To obtain the general solution for p_t, define $\zeta_t \equiv p_t - p^*_t$. Then, by (1) and (4), ζ_t must satisfy the homogeneous equation

$$(5)\quad E_t \zeta_{t+1} - (1+\beta^{-1})\zeta_t = 0.$$

The solutions to equation (5) depend crucially on assumptions about formation of expectations. For example, under perfect foresight, ζ_{t+1} is known at date t and the general solution to equation (5) would be

$$\zeta_t = c(1+\beta^{-1})^t,$$

where c is a constant to be determined by an initial condition.

More generally, under rational expectations, we can also satisfy equation (5) with solutions to the stochastic difference equation

$$(6)\quad \zeta_{t+1} - (1+\beta^{-1})\zeta_t = z_{t+1},$$

where z_i is a random variable, representing new information available at date i, that satisfies

$$(7) \quad E_j z_i = \begin{cases} z_i & \text{for} \quad i \leq j \\ 0 & \text{for} \quad i > j. \end{cases}$$

The key to the relevance of equation (6) for the general solution is that equation (5) relates ς_t to $E_t \varsigma_{t+1}$, rather than to ς_{t+1} itself.

The solution to equation (6) is

$$(8) \quad \varsigma_t = c(1+\beta^{-1})^t + \sum_{i=1}^{t} (1+\beta^{-1})^{t-i} z_i .$$

Equation (8) involves the sum of two elements. We refer to $c(1+\beta^{-1})^t$ as the deterministic bubble component of price (DBC) and to $\sum_{i=1}^{t} (1+\beta^{-1})^{t-i} z_i$ as the stochastic bubble component of price (SBC). Adding together the expressions in equations (4) and (8) gives the general solution for the time path of the relative price of gold,

$$(9) \quad p_t = (1+\beta)^{-1} \sum_{i=0}^{\infty} (1+\beta^{-1})^{-i} E_t(u_{t+i} - \gamma r_{t+1+i} - S_{t+i})$$
$$+ c(1+\beta^{-1})^t + \sum_{i=1}^{t} (1+\beta^{-1})^{t-i} z_i .$$

3. Components of the Solution

The solution for the relative price of gold given by equation (9) contains three components--FC, DBC, and SBC. The fundamental component (FC) involves the current values of the expected real interest rate, the other demand variables, and the stock of gold together with a sum, with exponentially declining weights, of expected future values of these variables.

The deterministic bubble component (DBC) equals the product of the eigenvalue raised to the power t and a constant that depends on an initial condition. In some cases, the history of an asset price might include an initial condition that precludes the existence of DBC. Referring to the solution given by equation (9), note that, given an initial price p_o, the constant c equals $p_o - p_o^*$. Thus, if at date $t = 0$ the price equals its FC, p_o^*, then the constant, c, equals zero, and, hence, without an unanticipated change in the structure of the model, DBC equals zero for all dates $t > 0$.

A reasonable interpretation of the history of the market for gold, however, would be that, because of changes in the policy of official intervention, its current structure has existed only since about 1975--see Kettell (1982). This period seems too short to presume that it has recorded a price of gold equal to its FC. Thus, in the present case, it does not seem relevant to appeal to initial conditions to rule out DBC a priori.

The final element in the solution, the stochastic bubble component (SBC), involves an average of new information, represented by the random variable z_i, that became available from date 1 through date t, weighted by powers of the eigenvalue that decrease as i approaches t. The restrictions on z, given by equation (7), imply that market participants know current and past values of z and that z is serially independent with zero mean. Moreover, z_t is not correlated with any variable in the information set of date $t-1$.

Any information on a new or newly observed phenomenon that satisfies, either itself or through its innovations, the restrictions on z given by equation (7) can affect p_t in the way prescribed by SBC as long as, beginning at date 1, individuals held expectations consistent with the existence of SBC. Specifically, the existence of SBC can involve a reaction by market participants to new information about an intrinsically irrelevant variable, i.e., a variable that is not a member of the set of exogenous variables present in FC, or it can involve over-reaction to new information about a truly relevant variable. Moreover, the random variable z_t need not have a stationary distribution. For example, Blanchard and Watson (1982) specify a model in which, although the variable driving SBC satisfies equation (7), its distribution changes over time in such a way as to create a constant probability that the stochastic bubble will burst.

The fact that market participants observe z does not imply that the identity of z is inferable from any economic theory or that there exist readily available data that provide a history of z. The econometric analysis must treat z as a potential set of unobservable random variables that satisfy equation (7) and enter the solution for the relative price of gold in the way given by equations (8) and (9).

The intuitive distinction between FC and the bubble components is that, if the market collectively misunderstands FC, individuals can gain by contradicting the market, whereas if the market does not expect a price bubble, individuals who act on the basis of price forecasts incorporating a bubble will lose. The bubble components are rational in the sense that, as long as the market collectively uses them in forecasting prices, individuals (with small market shares) also have an incentive to do so, even if they understand that price incorporates bubble components.

4. Evidence of Rational Bubbles

The objective of the present study is to determine whether actual movements in the relative price of gold are to any extent attributable to rational bubbles. The existence of rational bubbles would imply both that expectations are rational and that price does not conform to the fundamental component (FC) of its solution. The main problem for empirical analysis of rational bubbles is that we cannot directly observe them separately from

FC. As a result, conclusions about the existence of rational bubbles typically depend on accepting untested hypotheses either about the form of FC--for example, by specifying a distribution for the unobserved variables in FC--or about the nature of rational bubbles--for example, by focusing on either DBC or SBC or by specifying a distribution for the stochastic variable driving SBC. These objections apply, in particular, to the conclusions drawn by Flood and Garber (1980) and Flood, Garber, and Scott (1982), who attempt direct estimation of rational deterministic bubbles in the value of money, and by Blanchard and Watson (1982), who attempt indirect tests for rational stochastic bubbles in stock prices and the price of gold. Diba and Grossman (1983) contains a fuller critique of these studies.

In general, evidence in favor of rational bubbles is difficult to interpret, because, in the absence of independent knowledge about the form of FC and the nature of the possible rational bubbles, observations that are attributable to rational bubbles can also be attributed to FC. Examination of the theoretical solution obtained above for the relative price of gold, however, reveals a promising possibility for obtaining evidence against the existence of rational bubbles that is unambiguous in the sense that it is not conditional on accepting untested hypotheses about the processes generating FC or about the factors responsible for bubbles.

The theoretical model implies that rational bubble components can enter the solution for the relative price of gold only in the self-confirming way prescribed by equations (8) and (9). Specifically, the existence of rational bubbles at date t, according to the form of the bubble terms in equation (8), implies that the relative price of gold at any date $t+j$, $j > 0$, and its rational expectation formed at date t include these same terms multiplied by the eigenvalue, $1+\beta^{-1}$, raised to the power j. Because this eigenvalue is greater than unity, rational bubbles describe a divergent price path. More precisely, equation (9) implies that, if the time series of the relative price of gold contains a rational bubble, a nonstationary stochastic process generates its nth difference, for any finite n. (A related implication of equation (9) is that, if the rational bubble is stochastic, a nonstationary process also generates the deviations of price from any deterministic trend.)

These implications of rational bubbles for the stochastic process that generates the relative price of gold reflect an essential aspect of the structure of the model. The property that the eigenvalue is greater than unity results from the positive relation between the portfolio demand for gold and the expected real rate of return from holding gold ($\beta > 0$). This property also ensures a meaningful specification of the fundamental component relating current price to the future values of the exogenous variables. Specifically, because the eigenvalue is greater than unity, the forward-looking solution p_t^* in equation (4) converges for all $\{E_t S_{t+i}\}_{i=0}^{\infty}$, $\{E_t r_{t+1+i}\}_{i=0}^{\infty}$, and $\{E_t u_{t+i}\}_{i=0}^{\infty}$ sequences that do not grow exponentially at a rate of β^{-1} or faster.

The present empirical analysis of the existence of rational bubbles, based on these theoretical results, investigates the stationarity properties of the processes generating time series of appropriately differenced observations of the relative price of gold. In practice, drawing conclusions about stationarity from finite samples is difficult. Most basically, there is no sampling theory that provides statistical tests at specific levels of significance for the stationarity of the process generating a time series.

The present analysis follows the standard practice of using diagnostic checks based on sample autocorrelation functions and estimated spectra to make judgments about stationarity. One problem with these procedures is that a finite sample drawn from a time series generated by a stationary stochastic process can exhibit signs of nonstationarity. Another problem, which is especially relevant in the present context, is that sufficient differencing of any finite sample from a time series can create the appearance of stationarity.

This last problem suggests the need for an objective criterion for limiting the number of times that we difference the observations of the relative price of gold before drawing conclusions about stationarity. The conservative strategy adopted here is to difference the observations of prices as many times as it is necessary to difference any observable stochastic variables in FC to obtain time series that appear to be generated by stationary processes. The basis for this criterion is that the model implies that nonstationarity of the stochastic variables in FC would make FC nonstationary. Consequently, differencing fewer times than necessary to make the processes generating the variables in FC stationary would not make the process generating prices stationary even in the absence of rational bubbles.

If, as turns out to be the case, this procedure involves only low-order differencing and the resulting differenced time series of prices seems to be generated by a stationary process, we can reasonably conclude that price fluctuations during the sample period did not involve rational bubbles. More generally, equivalence between the number of times necessary to difference to achieve the appearance of stationarity in the observable variables in FC and in the price of gold would suggest that price corresponds to FC and that the processes generating the unobserved variables in FC, at least after differencing the same number of times, are stationary. (If, alternatively, after differencing this number of times, the process generating the price of gold does not seem to be stationary, the situation is ambiguous. Possible explanations would include either the presence of rational bubbles, or the nonstationarity of the unobserved stochastic variables in FC, or the failure of a small sample to reveal the true stationarity of the generating process, or irrational behavior by market participants.)

A complication in the present case is that none of the stochastic variables in FC--u_{t+i}, $E_t r_{t+1+i}$, and S_{t+i}--are directly observable. Rational expectations, however, relates the process generating the time series of $E_t r_{t+1+i}$ to the process generating the time series of r_t. Consequently, it seems reasonable to set the number of times to difference prices equal to the number of differences of r_t necessary to obtain a time series that appears stationary.

The following example illustrates this strategy and its rationale: Suppose that the process generating r_t is nonstationary, but that the process generating first differences of r_t is stationary. Specifically, assume that $E_t r_{t+1+i}$ is based on a univariate autoregressive process of order k for r_t that has one eigenvalue equal to unity and $k-1$ eigenvalues inside the unit circle. In this case, the term

$\sum_{i=0}^{\infty} (1+\beta^{-1})^{-i} E_t r_{t+1+i}$, which is part of FC, is equal to a

distributed lag function of the last k observations of the real interest rate. Thus, a nonstationary process generates FC, but, if a stationary process generates first differences of the other stochastic variables in FC, a stationary process generates first differences of FC. Accordingly, evidence that a nonstationary process generates p_t and that a stationary process generates first differences of p_t would support the combined conclusions that no rational bubbles are present, that price corresponds to FC, and that a stationary process generates first differences of u_t and S_t.

5. Diagnostic Checks in the Time Domain

The empirical analysis uses monthly data from January 1975 through March 1983 (99 observations). The relative price of gold is the average of daily values of the London P.M. price fix divided by the CPI covering. All Items Less Mortgage Interest Costs for the United States. The real interest rate is the nominal interest rate on one-month U.S. commercial paper minus the percentage change in the same price index.

Although the pattern exhibited by a sample autocorrelation depends on various properties of the process generating a time series, as well as on sample size, two characteristics of sample autocorrelations, considered together, provide a good basis for judgments about stationarity. First, sample autocorrelations of nonstationary time series typically die out slowly as lag length increases--see, for example, Wichern (1973). This property, of course, can also be associated with a stationary process with strong positive serial correlation. Second, the sample autocorrelation at one lag of a first difference of a stationary time series is typically negative. In fact, a rule of thumb in deciding how many times to difference a time series to achieve stationarity is to stop one step before negative autocorrelation at one lag is obtained.

The table reports sample autocorrelations for one through ten lags of the real interest rate and its first and second differences and of the logarithm of the relative price of gold and its first and second differences.

Table of Sample Autocorrelations

Variable \ Number of lags	1	2	3	4	5	6	7	8	9	10
r_t	.96	.89	.82	.78	.75	.72	.71	.71	.70	.66
Δr_t	.29	-.12	-.18	-.17	-.03	-.17	-.16	.20	.29	.06
$\Delta^2 r_t$	-.21	-.24	-.06	-.09	.21	-.12	-.24	.18	.24	-.03
p_t	.98	.95	.92	.90	.87	.83	.80	.76	.72	.68
Δp_t	.26	-.13	-.02	.18	.19	-.03	.03	.09	.06	.02
$\Delta^2 p_t$	-.21	-.34	-.06	.11	.15	-.14	-.03	.07	.00	-.17

The sample autocorrelations for r_t are all positive and drop off slowly. This pattern corresponds closely to the theoretical patterns implied by Wichern's formula for integrated moving average time series.

Such an autocorrelation pattern could also result from a stationary process--for example, the AR(1) process $r_t = \lambda r_{t-1} + \varepsilon_t$, where ε_t is white noise, if the coefficient λ is close to unity. If r_t is stationarity, however, the autocorrelation of Δr_t at one lag, given by $(\lambda-1)/2$ for the AR(1) case, is negative. Thus, a critical piece of evidence in favor of nonstationarity of r_t is that the sample autocorrelation of Δr at one lag, $\hat{\rho}_1$, is significantly greater than zero at the five percent level. (Under the null hypothesis that $\rho_1 = 0$, the statistic

$$\frac{1}{2} \ln \frac{1+\hat{\rho}_1}{1-\hat{\rho}_1}$$

is approximately normally distributed with mean zero and variance $(N-3)^{-1}$, for moderately large sample size, N.)

The table also indicates that the properties of the sample autocorrelations for p_t at all lags and for Δp_t at one lag are the same as for r_t and Δr_t. This finding accords with the theoretical model. Nonstationarity of the process generating r_t implies that the processes generating FC and p_t are also nonstationary.

The analytical strategy requires next consideration of the process generating Δr_t. Although initially positive, the sample autocorrelations for Δr_t drop off rapidly. Moreover, the sample autocorrelation for $\Delta^2 r_t$ at one lag is significantly less than zero at the five percent level. Using the converse of the arguments above, both of these findings suggest that the process generating Δr_t is stationary.

Given that Δr_t seems to be generated by a stationary stochastic process, the critical step, in investigating the existence of rational bubbles is to form a judgment about the stationarity of the process generating Δp_t. The relevant evidence, as it turns out, is strikingly clear. The properties of the sample autocorrelations for Δp_t at all lags and for $\Delta^2 p_t$ at one lag are the same as for Δr_t and $\Delta^2 r_t$. Thus, the process generating Δp_t seems to be stationary. This finding is not consistent with the existence of rational bubbles. Overall, the diagnostic checks in the time domain suggest that price corresponds to FC and that the process generating first differences of FC is stationary.

6. Diagnostic Checks in the Frequency Domain

Although stationarity is a necessary property for describing a stochastic process by its spectrum, in practice estimated spectra are useful for identifying nonstationarity--see, for example, Jenkins and Watts (1968, pp. 7-8). Working with estimated spectra rather than sample autocorrelations avoids the potential problem that high correlations among neighboring values can distort the sample autocorrelation. In contrast, the estimated spectrum isolates the effects of nonstationarity at the low frequencies.

Although estimated spectra of most economic time series show highest densities for the lowest frequency, the concentration of spectral power at the lowest frequencies should be especially pronounced if the process generating the time series is nonstationary. Specifically, the estimated spectra of nonstationarity time series usually suggest the presence of a "spike" at the zero frequency. In other words, the estimated spectrum rises sharply as it approaches the low frequencies and stays flat over a band near the zero frequency.

The figures depict the logarithms of estimated spectra for r_t, Δr_t, p_t, and Δp_t. In these estimates, each series is transformed to deviations from its mean and is "padded" by adding

46 zeros. In other words, the reported estimates use 144 ordinates for 98 observations. To obtain consistent estimates, the periodograms are smoothed by applying a "tent window" of width 11.

The reported estimates do not use a "taper". Tapers are often used in spectral estimation to scale the ends of the unpadded part of the series and to smooth the transition to the zeros in the padded part. Although advantageous for many applications, using a taper can make a spectral peak at some low frequency appear to be a spike at the zero frequency by smoothing out a fall in the spectrum as it goes from a low frequency to zero. In general, tapers cause loss of "resolution", i.e., the ability of the estimators to distinguish fine structure in the spectrum--see Koopmans (1974, pp. 303-308).

The results from analysis in the frequency domain are in every respect consistent with the results from analysis in the time domain. The estimated spectra of r_t and p_t both strongly suggest the presence of spikes at the zero frequency. These findings confirm the conclusion that the processes generating r_t and p_t seem to be nonstationary.

In contrast, the estimated spectra of Δr_t and Δp_t do not suggest the presence of spikes at the zero frequency. For both of these time series, spectral power increases from the zero frequency to a peak at a frequency between $\pi/9$ and $\pi/7$. These findings confirm the conclusion that the processes generating Δr_t and Δp_t seem to be stationary.

Other experiments involved estimating spectra using several combinations and variations of the number of ordinates and windows. Trials included 108 and 288, in addition to 144, ordinates, and flat and tent windows of widths 3, 5, 7, 9, 11, 13. The properties of the spectra estimated in these trials were much the same as those depicted in the figures.

7. Conclusions

The recent literature includes many examples that show that rational bubbles in the prices of infinitely lived assets are theoretically possible. Given that asset markets seem to be efficient in the sense of rational expectations, the wide fluctuations in the prices of certain assets, such as gold, suggest, moreover, the possibility that rational bubbles actually exist. The assumption of rational expectations, however, places strong restrictions not only on the relation between asset prices and market fundamentals, but also on the form that rational bubbles can take. Specifically, in the present analysis, if the time series of the relative price of gold contains a rational bubble, a nonstationary stochastic process generates its nth difference, for any finite n. This implication follows from the positive dependence of the demand for gold on the expected rate of return from holding gold and the requirement that a rational bubble involves expectations that are on average self confirming.

The empirical evidence relating to this nonstationarity property involves diagnostic checks for stationarity carried out in both the time domain and the frequency domain. This evidence strongly suggests that the process generating the first difference of the log of the relative price of gold is stationary, a finding that seems inconsistent with the existence of rational bubbles. More broadly, the empirical analysis finds a close correspondence between the time series properties of the relative price of gold and the time series properties of real interest rates, which the theory relates to the time series properties of the fundamental component of the relative price of gold. In sum, the evidence is consistent with the combined conclusion that the relative price of gold corresponds to market fundamentals, that the process generating first differences of market fundamentals is stationary, and that actual price movements are <u>not</u> attributable to rational bubbles.

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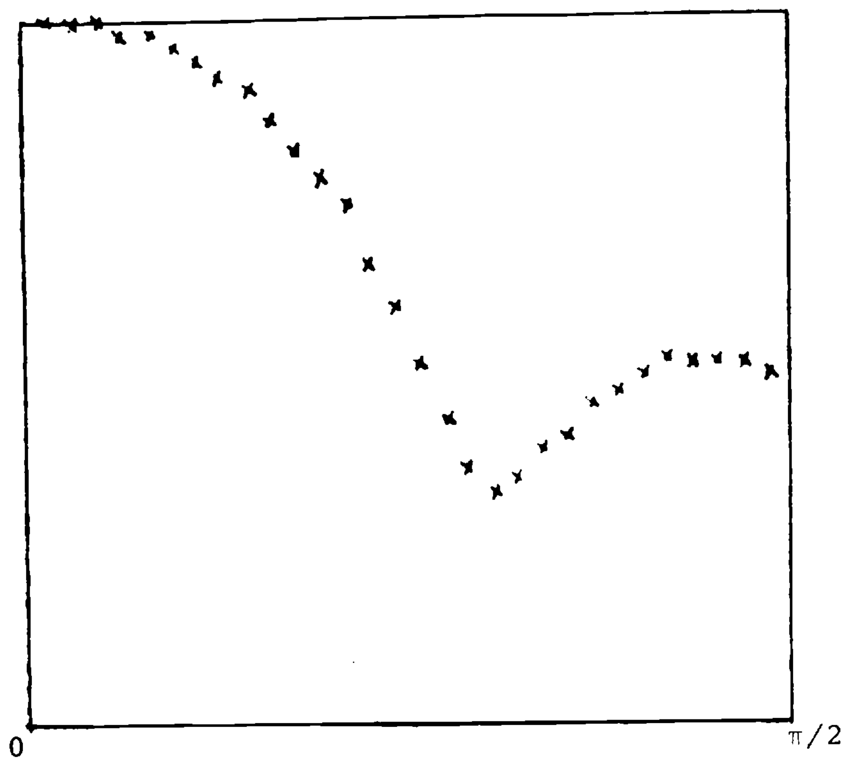
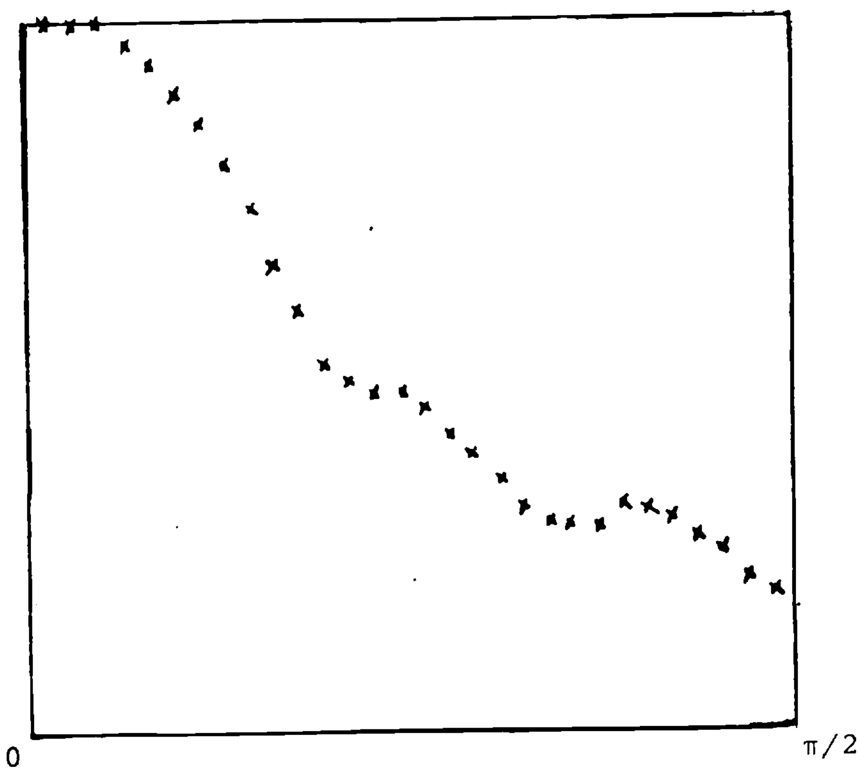
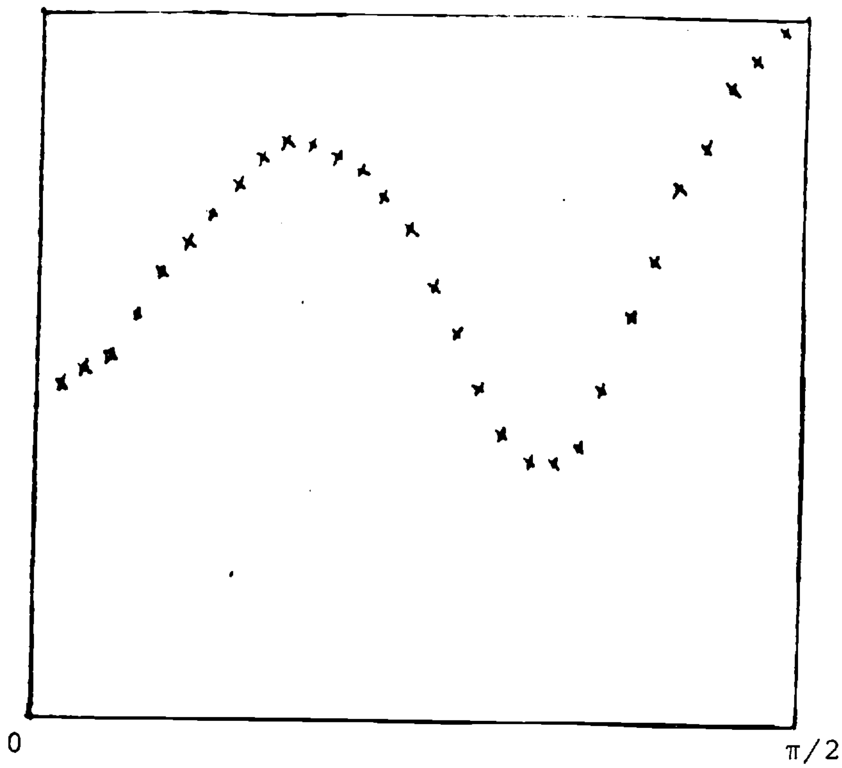


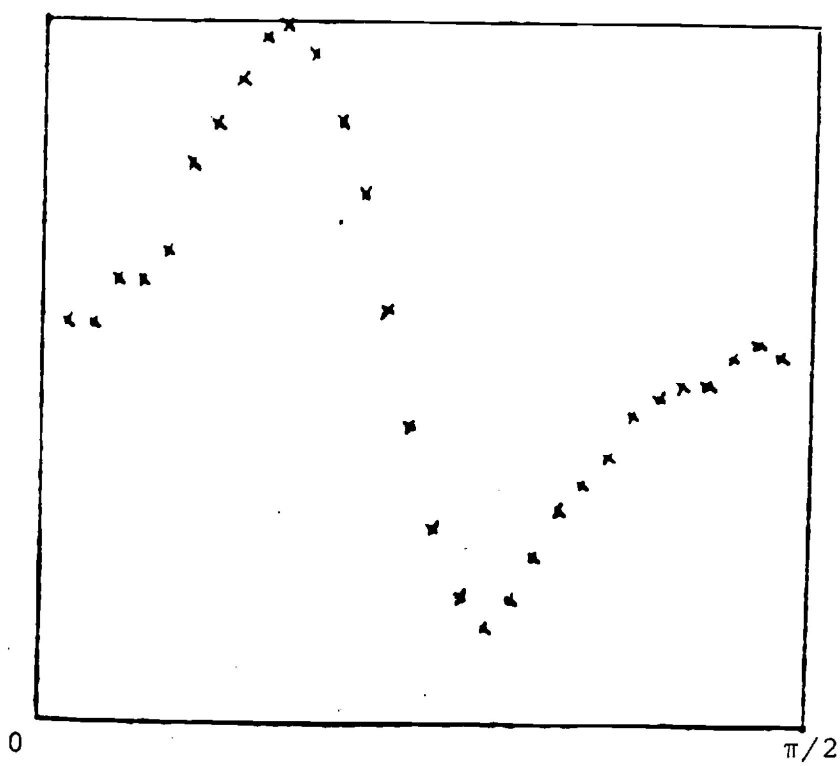
Log of Estimated Spectrum for r_t



Log of Estimated Spectrum for p_t



Log of Estimated Spectrum for Δr_t



Log of Estimated Spectrum for Δp_t