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SOME ISSUES CONCERNING INTEREST RATE PEGGING, PRICE LEVEL DETERMINACY, AND THE REAL BILLS DOCTRINE

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ABSTRACT

In a recent paper, Canzoneri, Henderson, and Rogoff have shown that it is possible for the monetary authority to peg the nominal interest rate without creating price level indeterminacy in a simplified version of the 1975 Sargent-Wallace model. The present paper begins by reviewing that result, which involves a limiting case of a money supply rule that depicts the authority as responding to current values of the interest rate. Then it shows that there exists an alternative rule that will peg the nominal rate without creating indeterminacy, but that this rule induces a different pattern of price level fluctuations. Next the paper considers whether indeterminacy will prevail if the authority tries to effect a peg in a third way: by simply standing ready to buy and sell securities at the desired rate. Finally, the implication of the foregoing results are drawn for arguments concerning the real bills doctrine and some critical comments are directed at the recent attempted rehabilitation of that doctrine by Sargent and Wallace.

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I. Introduction

In one of the more famous papers of the past decade, Sargent and Wallace (1975) argued that the price level and other nominal magnitudes will be formally indeterminate[1/] in a flexible-price economy free of money illusion and expectational irrationality if the monetary authority tries to use an interest rate as its policy instrument, that is, adopts a policy feedback rule that sets each period's interest rate as a fixed function of previous realizations of relevant variables. A few years later Parkin (1978) and McCallum (1981) showed that an interest rate feedback rule would not lead to nominal indeterminacy in such economies if the rule was designed to have some specified effect on the quantity of money (or the price level) in an upcoming period. Indeterminacy would be avoided if the monetary authority's objective function involved some nominal magnitude, a conclusion reminiscent of Patinkin's (1965, p. 309) dictum that "a necessary condition for the determinacy of the absolute price level ... is that the central bank concern itself with some money value--and in this sense be willing to suffer from money illusion." This contention would appear to be correct for any economy in which private agents' behavior is concerned only with real magnitudes, for without "money illusion" on the part of the monetary authority no nominal variable will enter the system in any way.

Given these arguments, a result recently developed by Canzoneri, Henderson, and Rogoff (1983) is of considerable interest, for it indicates that the interest rate can be virtually pegged at an arbitrary value in an economy of the Sargent-Wallace type.[2/] This result is considerably stronger than that in the McCallum (1981) demonstration, it should be noted, as the latter does not pertain to cases in which parameters of the interest rate feedback rule are autonomous (i.e., unrelated to behavioral characteristics of the private economy).

Furthermore, a pegged interest rate would appear to violate Patinkin's dictum, despite the latter's apparent reasonableness. But because their main concern is with other matters, Canzoneri, Henderson, and Rogoff--henceforth, CHR--do not devote much space or emphasis to their result. Nor do they contrast their own reasoning with that of previous writers. The first object of the present paper, accordingly, is to provide an alternative development of the CHR result, one that facilitates comparison with other analyses.

Furthermore, CHR do not investigate the possibility that the interest rate can be pegged by policy schemes other than the monetary rule used in their discussion. The second object of the present paper is then to consider some alternative schemes. Particular attention is accorded the question of whether nominal indeterminacy obtains--or, in what sense it obtains--if the monetary authority attempts to peg the interest rate not by an extreme version of a money supply rule, as in the CHR setup, but by simply standing ready to buy or sell securities at the rate in question. In the course of addressing this issue, it will become necessary to consider what empirical interpretation should be given to a situation involving nominal indeterminacy.

Finally, a third objective of the paper is to consider the implications of the pegging results for two topics of great importance in the development of monetary thought--Wicksell's cumulative process and the so-called real bills doctrine. Particular attention is devoted, in this portion of the paper, to the recent reconsideration of the real bills doctrine provided by Sargent and Wallace (1982).

Before beginning the analysis, one possible source of confusion should be addressed. In considering the various issues of concern in this paper, it is important to distinguish clearly between price level indeterminacy and nonuniqueness (or multiplicity) of price level solutions. The former, discussed by Patinkin (1961), Sargent-Wallace (1975), and McCallum (1981), involves situations in which the model economy (including specifications of policy behavior) does not determine the value of any nominal magnitude. By constrast, the latter--discussed by Sargent and Wallace (1973), Taylor (1977), McCallum (1983a), and many others--involves situations in which many price level solution paths satisfy the model for each given path of the money stock. Thus indeterminacy pertains to all nominal values but no real values, while nonuniqueness involves multiple paths of real money balances.[3/]

II. Basic Result

In order to facilitate comparison with previous literature and provide a maximum of simplicity, we shall begin the discussion in the context of the two-equation, full-employment IS-LM model used by McCallum (1981). Letting r_t denote the nominal rate of interest with m_t and p_t logs of the money stock and price level, respectively, this model can be written as:

$$(1) \quad r_t = b_0 + E_t p_{t+1} - p_t + v_t \qquad b_0 > 0$$

$$(2) \quad m_t - p_t = c_0 + c_1 r_t + \eta_t \qquad c_0 > 0, c_1 < 0.$$

Here v_t and η_t are independent white noise disturbances while $E_t p_{t+1} = E(p_{t+1} | r_t, \Omega_{t-1})$ with Ω_{t-1} denoting realizations of all variables in periods t-1, t-2,.... Equations (1) and (2) can be thought of as IS and LM functions, respectively; i.e., relationships describing saving/investment and portfolio balance behavior by the private sector.[4/]

In this setting, the basic idea of the CHR result can be exhibited quite simply. To do so, let us initially suppose that the monetary authority adopts a money supply rule of the following form:

$$(3) \quad m_t = \mu_0 + \mu_1 t + \lambda(r_t - \bar{r}) \qquad \lambda > 0$$

From this specification it is clear that it is being assumed that the monetary authority can observe the interest rate for a given period when setting the value of the money supply for that period. It is also clear that the type of policy behavior in question involves a growth rate of money that equals μ_1 on average but differs from that value if the current interest rate departs from its desired value $\bar{r}$. The greater the concern with the closeness of r_t to $\bar{r}$, the larger will be the magnitude of λ.

In obtaining a solution to the model, we wish to exclude "bubble" or "bootstrap" components--i.e., components that exist only because they are arbitrarily expected to exist. One possible reason for excluding bubbles is belief in the substantive hypothesis that they typically do not exist as an empirical matter. Another reason, sufficient for the purposes of this paper, is that the admission of bubble components would lead to a multiplicity of solutions. 5/ As stated above, we wish clearly to distinguish such multiplicities from indeterminacies. Furthermore, such multiplicities make difficult the comparison of outcomes under alternative policies. Consequently, we shall utilize the minimal-state-variable approach, described in McCallum (1983a), which leads to the exclusion of bubble components.

Inspection of the system (1),(2),(3) suggests that the essential relevant state variables are t, v_t, and η_t. 6/ Consequently, reduced-form "solution" equations for p_t and r_t will be of the form

$$(4) \quad p_t = \pi_{10} + \pi_{11}t + \pi_{12}v_t + \pi_{13}\eta_t$$

$$(5) \quad r_t = \pi_{20} + \pi_{21}t + \pi_{22}v_t + \pi_{23}\eta_t.$$

Now the first of these implies that expectations are generated according to

$$(6) \quad E_t p_{t+1} = \pi_{10} + \pi_{11}(t+1).$$

Consequently, substitution of (4), (5), and (6) into (1) yields

$$(7) \quad \pi_{20} + \pi_{21}t + \pi_{22}v_t + \pi_{23}\eta_t = b_0 + \pi_{10} + \pi_{11}(t+1)$$
$$-(\pi_{10} + \pi_{11}t + \pi_{12}v_t + \pi_{13}\eta_t) + v_t,$$

which implies the following relationships between the undetermined coefficients π_{ij} and the basic parameters of the model:

$$(8)\quad \pi_{20} = b_0 + \pi_{11}$$
$$\pi_{21} = 0$$
$$\pi_{22} = -\pi_{12} + 1$$
$$\pi_{23} = -\pi_{13}.$$

Likewise, substitution into (2) yields

$$(9)\quad \mu_0 + \mu_1 t + \lambda(\pi_{20} + \pi_{21}t + \pi_{22}v_t + \pi_{23}\eta_t - \bar{r})$$
$$= \pi_{10} + \pi_{11}t + \pi_{12}v_t + \pi_{13}\eta_t + c_0 + c_1(\pi_{20} + \pi_{21}t + \pi_{22}v_t + \pi_{23}\eta_t) + \eta_t$$

which implies

$$(10)\quad \mu_0 + \lambda(\pi_{20} - \bar{r}) = c_0 + \pi_{10} + c_1\pi_{20}$$
$$\mu_1 + \lambda\pi_{21} = \pi_{11} + c_1\pi_{21}$$
$$\lambda\pi_{22} = \pi_{12} + c_1\pi_{22}$$
$$\lambda\pi_{23} = \pi_{13} + c_1\pi_{23} + 1.$$

Finally, solving equations (8) and (10) we obtain

$$(11)\quad \pi_{10} = \mu_0 - c_0 + \lambda(b_0 + \mu_1 - \bar{r}) - c_1(b_0 + \mu_1) \qquad \pi_{20} = b_0 + \mu_1$$
$$\pi_{11} = \mu_1 \qquad \pi_{21} = 0$$
$$\pi_{12} = (\lambda - c_1)/(1 + \lambda - c_1) \qquad \pi_{22} = 1/(1 + \lambda - c_1)$$
$$\pi_{13} = -1/(1 + \lambda - c_1) \qquad \pi_{23} = 1/(1 + \lambda - c_1)$$

From the expressions for π_{20} and π_{21} we then conclude that--as is obvious for this classical model--a choice of $\mu_1 = \bar{r} - b_0$ is necessary to make the mean value of r_t equal the target value $\bar{r}$. If that choice is made, however, r_t will fluctuate randomly around $\bar{r}$. And by increasing the magnitude of λ, the values of π_{22} and π_{23} can be driven arbitrarily close to zero so the behavior of r_t can be made arbitrarily close to $r_t = \bar{r}$, i.e., to the desired peg. Furthermore, the price level parameters are well defined, even as $\lambda \to \infty$. In particular, in the limit we have $\pi_{12} \to 1$ and $\pi_{13} \to 0$ so that

(12) $p_t = \mu_0 - c_0 - c_1\bar{r} + \mu_1 t + v_t$.

Thus there is no indeterminacy of p_t even in this extreme case. This is, in the present model, the result obtained by CHR.[7/]

III. An Alternative Policy Rule

In a non-stochastic setting, a constant money growth rate of value μ_1 can, of course, be expressed by $m_t = m_{t-1} + \mu_1$ as well as by $m_t = \mu_0 + \mu_1 t$. That leads naturally to the question of whether the following money supply rule will have the same effects on r_t and p_t as (3):

$$(13)\quad m_t = m_{t-1} + \mu_1 + \lambda(r_t - \bar{r}).$$

Now, in the system (1),(2),(13) the relevant set of state variables includes m_{t-1} instead of t so we posit solutions of the form

$$(14)\quad p_t = \pi_{10} + \pi_{11}m_{t-1} + \pi_{12}v_t + \pi_{13}\eta_t$$

$$(15)\quad r_t = \pi_{20} + \pi_{21}m_{t-1} + \pi_{22}v_t + \pi_{23}\eta_t.$$

The implied expectation in this case is

$$(16)\quad E_t p_{t+1} = \pi_{10} + \pi_{11}[m_{t-1} + \mu_1 + \lambda(\pi_{20} + \pi_{21}m_{t-1} + \pi_{22}v_t + \pi_{23}\eta_t - \bar{r})].$$

Substitution into (1) then yields an equation analogous to (7) which implies relationships (analogous to (8)) as follows:

$$(17)\quad \begin{aligned} (1 - \lambda\pi_{11})\pi_{20} &= b_0 - \pi_{11}\lambda\bar{r} + \pi_{11}\mu_1 \\ (1 - \lambda\pi_{11})\pi_{21} &= 0 \\ (1 - \lambda\pi_{11})\pi_{22} &= -\pi_{12} + 1 \\ (1 - \lambda\pi_{11})\pi_{23} &= -\pi_{13}. \end{aligned}$$

In addition, substitution into (2) implies

$$(18)\quad \begin{aligned} \mu_1 + (\lambda - c_1)\pi_{20} - \lambda\bar{r} &= \pi_{10} + c_0 \\ 1 + (\lambda - c_1)\pi_{21} &= \pi_{11} \\ (\lambda - c_1)\pi_{22} &= \pi_{12} \\ (\lambda - c_1)\pi_{23} &= \pi_{13} + 1. \end{aligned}$$

In this case there are two sets of solutions, one in which the second of equations (17)-(18) are satisfied by $\pi_{11} = 1/\lambda$ and the other in which $\pi_{21} = 0$ and $\pi_{11} = 1$. By means of the procedure described in McCallum (1983a, pp.146-7) it can be determined that the second set provides the minimal-state-variable, bubble-free solution--see Appendix B. Thus, after setting $\mu_1 = \bar{r} - b_0$, we have

(19) $\pi_{10} = (1 - c_1)\bar{r} - b_0 - c_0$ $\qquad$ $\pi_{20} = \bar{r}$

$\pi_{11} = 1$ $\qquad$ $\pi_{21} = 0$

$\pi_{12} = (\lambda - c_1)/(1 - c_1)$ $\qquad$ $\pi_{22} = 1/(1 - c_1)$

$\pi_{13} = -(1 - \lambda)/(1 - c_1)$ $\qquad$ $\pi_{23} = 1/(1 - c_1)$

From these, we see that r_t fluctuates randomly about $\bar{r}$. But we also see that the coefficients π_{22} and π_{23} do not approach zero as $\lambda \rightarrow \infty$. On the other hand, π_{12} and π_{13} approach ∞ as $\lambda \rightarrow \infty$. So with the non-stationary version of the money supply rule, responding more strongly to current deviations of r_t from $\bar{r}$ serves to increase (without bound) the variance of p_t and has no effect on the variance of r_t.[8/] [9/]

These last conclusions should not, however, be given much weight for they are not robust to specificational adjustments in the model. Suppose, in particular, that output is not strictly constant but instead departs from a (constant) capacity value in response to a price level surprise term, $p_t - Ep_t|\Omega_{t-1}$, as it does in the CHR (1983) specification. Then the appropriate modification of equations (1) and (2) would be as follows:

(1') $r_t = b_0 + E_t p_{t+1} - p_t + b_1(p_t - Ep_t|\Omega_{t-1}) + v_t$

(2') $m_t - p_t = c_0 + c_1 r_t + c_2(p_t - Ep_t|\Omega_{t-1}) + \eta_t$.

Here $b_1 < 0$ and $c_2 > 0$.

Re-solving the model with these changes leads to no alteration in the solution values for π_{10}, π_{11}, π_{20}, or π_{21}, but for the remaining coefficients we have

$$(20)\quad \pi_{12} = (\lambda - c_1)/\psi_1 \qquad \pi_{22} = (1 + c_2)/\psi_1$$

$$\pi_{13} = (\lambda - 1)/\psi_1 \qquad \pi_{23} = (1 - b_1)/\psi_1$$

where $\psi_1 = (1 - \lambda)(1 + c_2) + (\lambda - c_1)(1 - b_1)$. Consequently, we see that as $\lambda \longrightarrow \infty$, we obtain $\pi_{22} \longrightarrow 0$ and $\pi_{23} \longrightarrow 0$ unless by chance $c_2 = -b_1$. Furthermore, with the same proviso, we see that $\pi_{12} \longrightarrow -1/(b_1 + c_2)$ and $\pi_{13} \longrightarrow -1/(b_1 + c_2)$, so that the variance of p_t does not increase without bound. The behavior of the system is in these two ways the same as in Section II, with the money supply rule (3).[10/]

An important difference can be noted, however, when we reconsider the present model in light of the money supply rule of Section II--i.e., when we consider the system (1'), (2'), (3). Again the values of π_{10}, π_{11}, π_{20}, and π_{21} are unchanged but for the other coefficients we obtain

$$(21)\quad \pi_{12} = (\lambda - c_1)/\psi_2 \qquad \pi_{22} = (1 + c_2)/\psi_2$$

$$\pi_{13} = -1/\psi_2 \qquad \pi_{23} = (1 - b_1)/\psi_2$$

where $\psi_2 = (1 + c_2) + (\lambda - c_1)(1 - b_1)$. Continuing now with or without the presumption that $c_2 \neq -b_1$, we again find that $\pi_{22} \longrightarrow 0$ and $\pi_{23} \longrightarrow 0$ as $\lambda \longrightarrow \infty$. But instead of the limiting values in the previous paragraph we now have $\pi_{12} \longrightarrow 1/(1 - b_1)$ and $\pi_{13} \longrightarrow 0$. Thus with the extended model (1'), (2'), we see that the limiting behavior of the price level is described by

$$(22)\quad p_t = (\mu_0 - c_0 - c_1\bar{r}) + (\bar{r} - b_0)t + \frac{1}{1-b_1} v_t$$

when the money supply rule is (3), and by

$$(23) \quad p_t = \bar{r}(1 - c_1) - b_0 - c_0 + m_{t-1} - \frac{v_t+\eta_t}{b_1+c_2}$$

when the rule is (13). Thus, even with a complete peg of r_t enforced by $\lambda \rightarrow \infty$, the stochastic behavior of p_t (as well as m_t) is different under the two rules. It is then evidently not a complete description of monetary policy to say that the interest rate will be pegged at $r_t = \bar{r}$! 11/

IV. A "Pure" Interest Rate Peg?

Given the conclusion of the last paragraph, it then becomes natural to ask whether it is possible for the monetary authority to peg r_t at the value $\bar{r}$, not by means of a money supply rule, but by simply standing ready to buy or sell securities at the chosen value. The distinction between this "pure" type of pegging operation and the ones previously considered is that, while the others involve money supply behavior, in the present case the money stock is entirely demand determined at the pegged value of the interest rate. The monetary authority is committed, under this type of policy, to maintaining a constant interest rate regardless of the path of m_t that materializes.

Analytically, this type of pure peg is expressed by the simple condition

$$r_t = \bar{r}. \tag{24}$$

Consequently, the system under discussion becomes (1'), (2'), (24), and the variables to be solved for are p_t and m_t, rather than p_t and r_t.[12/] Inspection of equations (1'), (2'), and (24) shows that no lagged variables or trends appear explicitly, so it appears that the relevant set of state variables includes only v_t and η_t.[13/] This tentative assumption also reflects the notion that nothing from the past is relevant to the determination of p_t in a flexible-price model of the type at hand when the monetary authority does not provide a connection between successive periods by its adopted policy.[14/]

We provisionally assume, then, that solution equations will be of the form

$$p_t = \pi_{10} + \pi_{12}v_t + \pi_{13}\eta_t \tag{25}$$

$$m_t = \pi_{30} + \pi_{32}v_t + \pi_{33}\eta_t. \tag{26}$$

Continuing as before, we next note that $E_t p_{t+1} = \pi_{10}$, so substitution of that condition plus (24), (25), and (26) into (1') yields

$$\bar{r} = b_0 + \pi_{10} - (\pi_{10} + \pi_{12} v_t + \pi_{13} \eta_t) + b_1(\pi_{12} v_t + \pi_{13} \eta_t) + v_t \tag{27}$$

which implies the relationships

$$\begin{aligned} \bar{r} &= b_0 \\ 0 &= (b_1 - 1)\pi_{12} + 1 \\ 0 &= (b_1 - 1)\pi_{13}. \end{aligned} \tag{28}$$

Likewise, substitution into (2') yields

$$\begin{aligned} \pi_{30} + \pi_{32} v_t + \pi_{33} \eta_t &= \pi_{10} + \pi_{12} v_t + \pi_{13} \eta_t + c_0 \\ &+ c_1 \bar{r} + c_2(\pi_{12} v_t + \pi_{13} \eta_t) + \eta_t \end{aligned} \tag{29}$$

implying

$$\begin{aligned} \pi_{30} &= \pi_{10} + c_0 + c_1 \bar{r} \\ \pi_{32} &= \pi_{12}(1 + c_2) \\ \pi_{33} &= \pi_{13}(1 + c_2) + 1. \end{aligned} \tag{30}$$

Now from (28) and (30) we readily deduce that $\pi_{12} = 1/(1 - b_1)$, $\pi_{13} = 0$, $\pi_{32} = (1 + c_2)/(1 - b_1)$, and $\pi_{33} = 1$. But these equations fail to determine values for either π_{10} or π_{30}. Furthermore, they require--for the avoidance of inconsistency--that the peg value $\bar{r}$ be chosen to equal the "natural rate" value b_0.[15/] Consequently, in this case we find an apparent nominal indeterminacy with precisely the sort of symptoms as described in McCallum (1981, pp. 323-4).

Reflection upon the last-mentioned symptom leads one, however, to consider the possibility that the previous analysis has been attempted with a sub-minimal set of state variables. The idea is that rational agents, being aware of the classical structure of the economy, will understand that for the policy objective $r_t = \bar{r}$ to be achieved in all future periods it will be necessary for the inflation rate to equal $\bar{r} - b_0$ on average. Furthermore, these rational agents will also know that this magnitude of inflation will necessitate an average money stock growth rate of $\bar{r} - b_0$. But money stock growth rates involve comparisons of successive values of m_t. Thus a relevant determinant of any m_t value must be m_{t-1}. In other words, m_{t-1} appears to be a relevant state variable when policy is specified as in (24).[16/]

To investigate this conjecture, let us then consider solutions to (1'), (2'), (24) of the form

(31) $$p_t = \pi_{10} + \pi_{11} m_{t-1} + \pi_{12} v_t + \pi_{13} \eta_t$$

(32) $$m_t = \pi_{30} + \pi_{31} m_{t-1} + \pi_{32} v_t + \pi_{33} \eta_t .$$

To solve the system in this case, we first note that with $r_t = \bar{r}$ there is no information concerning v_t or η_t available to private agents during period t. Thus the expectational variable in (1') becomes

(33) $$E_t p_{t+1} = \pi_{10} + \pi_{11}(\pi_{30} + \pi_{31} m_{t-1}).$$

Substitution into (1') then yields

(34) $$\bar{r} = b_0 + \pi_{10} + \pi_{11}(\pi_{30} + \pi_{31} m_{t-1}) - (\pi_{10} + \pi_{11} m_{t-1} + \pi_{12} v_t + \pi_{13} \eta_t) + b_1(\pi_{12} v_t + \pi_{13} \eta_t) + v_t$$

which implies

(35) $$\bar{r} = b_0 + \pi_{11}\pi_{30}$$

$$0 = \pi_{11}\pi_{31} - \pi_{11}$$

$$0 = (b_1 - 1)\,\pi_{12} + 1$$

$$0 = (b_1 - 1)\,\pi_{13}.$$

Also, substitution into (2) gives

$$(36) \qquad \pi_{30} + \pi_{31}m_{t-1} + \pi_{32}v_t + \pi_{33}\eta_t = \pi_{10} + \pi_{11}m_{t-1} + \pi_{12}v_t + \pi_{13}\eta_t$$

$$+ c_0 + c_1\bar{r} + c_2(\pi_{12}v_t + \pi_{13}\eta_t) + \eta_t$$

implying

$$(37) \qquad \pi_{30} = \pi_{10} + c_0 + c_1\bar{r}$$

$$\pi_{31} = \pi_{11}$$

$$\pi_{32} = (1 + c_2)\ \pi_{12}$$

$$\pi_{33} = (1 + c_2)\ \pi_{13} + 1.$$

From the expressions in (35) and (37) we quickly see that again $\pi_{12} = 1/(1 - b_1)$, $\pi_{13} = 0$, $\pi_{32} = (1 + c_2)/(1 - b_1)$, and $\pi_{33} = 1$. Also, it is apparent that π_{11} and π_{31} both equal 0 or 1. With the former value, the system would be identical to the one based on (25) and (26), so we take the values $\pi_{11} = \pi_{31} = 1$. Then from the first of equations (35) we find that $\pi_{30} = \bar{r} - b_0$ and from the first of equations (37) that $\pi_{10} = \bar{r}(1 - c_1) - b_0 - c_0$. Thus the solution with m_{t-1} included as a state variable is well behaved; it involves no inconsistency or inability to solve for particular parameters.

This result seems to suggest that a pure pegging policy is feasible, a conclusion that is more drastic than the CHR result that money supply rules can be designed to maintain a constant interest rate. Before accepting this conclusion, however, we need to consider whether a different solution would be obtained if the private agents in our economy behaved as if t, rather than m_{t-1}, were the relevant state variable missing from (25) and (26). That this possibility needs to be examined is indicated by the contrasting results obtained in Section III under the alternative money stock rules (3) and (13).

Accordingly, we now seek solutions to the model (1'), (2'), (24) of the form

(38) $p_t = \pi_{10} + \pi_{11}t + \pi_{12}v_t + \pi_{13}\eta_t$

(39) $m_t = \pi_{30} + \pi_{31}t + \pi_{32}v_t + \pi_{33}\eta_t$,

where the trend variable t appears rather than m_{t-1}. In this case, $E_t p_{t+1} = \pi_{10} + \pi_{11}(t + 1)$ and proceeding as before we find that the relations implied by substitution into (1') are

(40)
$$\bar{r} = b_0 + \pi_{11}$$
$$0 = \pi_{11} - \pi_{11}$$
$$0 = (b_1 - 1)\ \pi_{12} + 1$$
$$0 = (b_1 - 1)\ \pi_{13}.$$

Also, substitution into (2') leads to

(41)
$$\pi_{30} = \pi_{10} + c_0 + c_1\bar{r}$$
$$\pi_{31} = \pi_{11}$$
$$\pi_{32} = (1 + c_2)\ \pi_{12}$$
$$\pi_{33} = (1 + c_2)\ \pi_{13} + 1$$

From these we see that π_{12}, π_{13}, π_{22}, and π_{23} are the same as in the previous example. We also see, however, that $\pi_{11} = \pi_{31} = \bar{r} - b_0$ and that the system fails to determine π_{10} or π_{30}.

At a superficial level, one might be inclined to interpret the non-determination of π_{10} and π_{30} as an indication that this solution is not viable, leaving the solution with the m_{t-1} state variable as the only well-behaved candidate. But that conclusion would be unjustified. No parameters are undetermined in the m_{t-1} solution precisely because the previous period's value of m_t enters the solution equation. But some value of m_{t-1} must be used as an initial value to start up the process, and that value is just as undetermined as is π_{30} in the solution with the time trend. In each case, one initial condition is needed to put the system in operation. Thus the solution with reduced-form equations of the form (38)-(39) is just as worthy a candidate for selection as "the solution" as is the one with reduced-form equations of the form (31)-(32). The conjecture that m_{t-1} must be a relevant state variable is incorrect; the trend variable t would serve as well.

But the solutions with m_{t-1} and t included are different in their implied time series properties for m_t and p_t. In particular, one of these solutions describes the behavior of m_t as a random walk with drift $\bar{r} - b_0$, while the other implies that m_t is generated by a process that departs in a white-noise fashion from a linear trend path with slope $r - b_0$.17/ So while it is true that rational agents should be able to infer that $r_t = \bar{r}$ requires an average money stock growth rate of $\bar{r} - b_0$, there is no way for them to infer whether this average rate is generated in a random-walk or trend-stationary fashion. And of course these different processes for m_t correspond to different processes with the same qualitative characteristics for p_t--and, consequently, to different expectations of future values of p_t.

The main implication of this finding is that a commitment by the monetary authority to peg r_t at the value $\bar{r}$ is not a satisfactory description of policy behavior. In particular, it does not indicate whether or not the authority will permit "base drift" and is therefore not complete enough to enable private agents to form expectations--themselves crucial for asset demand behavior--in a rational manner. A "pure interest rate peg" does not, in other words, constitute a well-formulated monetary policy. While the monetary authority can come arbitrarily close to effecting a peg of the form $r_t = \bar{r}$, it must do so by adopting a money supply rule such as (3) or (13) and by making λ large in magnitude--not by attempting a pure peg.[18/] This conclusion expresses in a new way the old idea that attempts to peg interest rates may be misguided.

It should also be noted, before moving to other matters, that the solutions in this section provide something akin to counterexamples to the indeterminacy proposition of Sargent and Wallace (1975). An even clearer case can be developed, moreover, by use of an interest rate rule of the form $r_t = \alpha_0 + \alpha_1 m_{t-1}$. The resulting solution would provide a counterexample in the sense that p_t would be determinate with a fixed and autonomous feedback rule for r_t provided that an initial condition is available for m_{t-1}. But without such an initial condition one component of the model--the policy rule--would not be well-specified. Thus the Sargent-Wallace (1975, p. 250) discussion goes astray by considering only terminal conditions, as opposed to initial conditions, as possible ways of pinning down nominal magnitudes.

In addition, the foregoing discussion suggests a more detailed interpretation of indeterminacy findings in static, non-stochastic models such as those used by Gurley and Shaw (1960), Patinkin (1961) (1965, pp. 308-9), Sargent (1979, pp. 92-5), and Fischer (1983). In particular, one can see that to restrict the analysis to the fully static case with zero inflation is tantamont to adoption of a solution form like (25) (26). But that seems overly restrictive; admission of non-zero inflation rates would seem desirable even in a non-stochastic framework and such an admission would eliminate certain inconsistencies. There would be a remaining indeterminacy corresponding to the need for an initial condition to go with our equations (35) (37) or

(40) (41). But this hardly seems to be a matter of great consequence since the choice of an initial value for m_t has no effect on any real magnitude, not even real money balances. Thus traditional findings of nominal indeterminacy seem to point out inadequacies of the assumptions utilized, rather than the policies investigated.

Our main result indicates, however, that there is a valid criticism to be made of interest rate pegging policies of the pure type. This criticism is based on the different time series properties of solutions for nominal variables that are consistent with the model when stochastic elements are recognized. These differing solutions indicate that pure pegging--a standing offer to buy and sell securities at a specified interest rate--does not constitute a well-formulated monetary policy.

V. Indeterminacy and the Real Bills Doctrine

An interesting and important interpretation of a pegged interest rate is as a manifestation of policy behavior of the type recommended by the ancient and infamous "real bills doctrine."[19/] This interpretation has been expressed by a number of writers, including Patinkin (1965, p. 309), Humphrey (1982), Sargent (1979, pp. 92-5), and Sargent and Wallace (1975).[20/] Price level indeterminacy is viewed, under this interpretation, as illustrating in an extreme fashion the undesirability of a policy regime of the real-bills type.

In a provocative turnabout, Sargent and Wallace (1982) have recently attempted to provide "something of a rehabilitation of the real bills doctrine" (1982, p. 1214). In developing this new position, these authors (henceforth, S-W) construct an overlapping generations model in which Pareto optimality obtains under a policy regime of the real-bills type, but does not obtain under a regime of the "quantity theory" type.[21/] A subsidiary but crucial strand of the argument focuses on the issue of price level determinacy. In this regard, S-W indicate that the price level is determinate in their 1982 model under the real bills regime even though the latter features an interest rate pegged at the value zero.

Elsewhere, Sargent (1982) has contrasted this finding with the indeterminacy result in the earlier (1975) S-W paper. Indeed, Sargent has (1982, p. 387) conjectured that "the difference in these two analyses stems sensitively from the fact that the older one took the demand function for money and a particular definition of money as primitive objects, while the later paper goes deeper and has primitive objects in the form of preferences, opportunities, endowments, and explicit restrictions on financial intermediation."

One purpose of the present section is to investigate this conjecture; another aim is to comment more generally on the S-W rehabilitation of the real bills doctrine.

We begin by reviewing aspects of the S-W (1982) analysis, adopting notation related (but not identical) to theirs. Under all of the policy regimes considered, the price level P(t) is required to satisfy the following condition in each period, t = 1,2,...:

$$(42) \quad \sum_{h=1}^{N}[w_t^h(t) - w_t^h(t+1)/(1 + r^h(t))]/2 = H(t)/P(t).$$

Here $w_s^h(t)$ is the (exogenous) endowment[22] of member h of generation s in period t while $r^h(t)$ is the real rate of return available between t and t+1 to agent h (h=1,...,N). Also, H(t) is the aggregate nominal stock of currency held by the current old in period t after transfers from, and loan repayments to, the government.[23] The term in square brackets is, given the symmetric Cobb-Douglas utility function used by S-W, the utility-maximizing quantity of real savings in period t by member h of generation t.[24] Thus (42) equates the aggregate saving of each period's young to the aggregate dissaving of its old, with the latter supplying their money balances (currency) inelastically.[25]

After rationalizing the foregoing type of saving behavior and assuming that currency and private loans are the only available stores of value, S-W go on to discuss monetary equilibria[26] under three different policy regimes--arrangements that involve "laissez faire" (LF), "quantity theory" (QT), and "real-bills discount window" (DW) modes of behavior by the monetary authority. Under the LF regime, all individuals face the same intertemporal terms of trade so $r^h(t) = r(t)$ for all h. Also, equilization of returns on currency and private securities[27] requires that

$$(43) \quad r(t) = [P(t)/P(t+1)] - 1.$$

In this regime, moreover, there are no loans to the young and the monetary authority holds constant the stock of currency:

(44) $H(t) = H.$

Together, as S-W show, equations (42), (43), and (44) determine solution sequences for the variables H(t), P(t), and r(t).

Turning now to the real-bills DW regime, conditions (42) and (43) continue to hold.[28/] The monetary authority's behavior, however, is in this case designed to peg the interest rate on private securities at zero: "the government stands ready at every date to grant safe one-period loans in the form of (newly printed) government currency at a zero nominal rate of interest" (S-W, 1982, p. 1225). But that mode of behavior is expressed analytically as

(45) $$r(t) + \frac{P(t+1) - P(t)}{P(t+1)} = 0,$$

which then replaces (44) as the formal representation of policy. And (45) is algebraically equivalent to (43), so the DW system includes only two independent equations, a number that is inadequate to determine values for the three variables H(t), P(t), and r(t). Only r(t) and the ratio H(t)/P(t) can be found from analysis of the model.

The contrary conclusion that there is no indeterminacy problem is reached by S-W (1982, p. 1226) because they treat H(t) as a constant for all t = 1,2... and take that constant value as exogenously given. But since in their model H(t) is the currency stock held by the old in t after transfers, should the government choose to make any, the value of H(t) in the initial period t=1 is not independent of the choice of the monetary authority. The objective of a zero magnitude for r(t) + 1 - P(t)/P(t+1) is inadequate, given private agents' concern for real magnitudes, to determine what constant value of H(t) will prevail in period 1 and thereafter. What the S-W (1982) example shows, then, is that it is feasible in their (non-stochastic) model for the monetary authority both to peg the interest rate at zero and to hold the money stock constant at an arbitrarily chosen value. Given the latter value the price level is determined; without such a value it will not be.[29/]

But this essentially duplicates in a non-stochastic setting the result described in previous sections as pertaining to the 1975 S-W model, in which it is possible to maintain a constant interest rate and a money stock that is constant except for random (white noise) fluctuations, provided that $\bar{r} = b_0$. The random fluctuations in the money stock are not present in the S-W (1982) model because it is non-stochastic in specification, and a constant money stock is compatible with a zero nominal rate of interest because that rate is required by (43) as a necessary condition of a monetary equilibrium.

Thus we see that there is no significant difference, with respect to the indeterminacy status of the two S-W models, to attribute to the fact that the earlier analysis took demand functions as primitive analytical objects. As Patinkin's dictum would suggest, the relevant distinction as far as potential indeterminacy is concerned is whether the demand functions (primitive or derived) are free of money illusion.

A more interesting issue, of course, is whether the 1982 S-W analysis in fact provides any support for the real bills doctrine. In that regard there are two points that need to be made here,[30/] both of which suggest that such support is not provided by the analysis in question. First, as we have seen, the S-W (1982) model is one in which price level determinacy requires that the monetary authority specify a value for H(1), the initial money stock. But the need for any such quantitative specification is precisely what is denied by the real bills doctrine in its claim that a restriction of loans to discounts of real bills is sufficient to guarantee desirable monetary behavior.[31/]

Second, and of greater import, is the fact that the S-W (1982) model is one in which the asset termed "currency" serves as a store of value but not as a medium of exchange.[32/] This fact is crucial in evaluating the main theme of the S-W argument, namely, that Pareto-optimality obtains under the real bills DW regime even though the resulting equilibrium features a relatively large extent of price level variability. With respect to that theme, Laidler (1984) has suggested that the S-W analysis is inconsistent with the views of the doctrine's 18th and 19th century advocates, who "... regarded the maintenance of price level stability as a vital principle of social organization, and would not have entertained a defense of their doctrine which showed that it failed to ensure such stability" (Laidler, 1984, p.). Now, while this suggestion of Laidler's would certainly appear to be correct, the substantive issue of whether a real bills regime is socially desirable does not hinge on what its original proponents believed. If the S-W model were one that satisfactorily depicted the essential features of a contemporary economy, then their results would be of importance regardless of the views of Adam Smith, Thomas Tooke, John Fullerton, et. al.

As it happens, however, the S-W (1982) model does not provide an adequate vehicle for analysis of a contemporary monetary economy. Laidler mentions the model's neglect of uncertainty and production, but not its most fundamental weakness: the asset called "currency" does not serve as a medium of exchange.[33/] That being the case, there is no reason to think of this asset as money--it would be better thought of as government bonds--and thus no reason to think of the reciprocal of its price in terms of goods as "the price level."[34] Nor is there any reason to think of the model's financial intermediaries as "banks"--i.e., as intermediaries with liabilities that serve as money--or to think of the government's loan office as a central bank. What S-W call the price level is just the price of goods relative to the price of government bonds, and the fact that its value fluctuates in the LF and DW regimes is merely the reflection of assumptions regarding tastes and endowments that directly necessitate fluctuations in the real rate of interest.[35]

Thus the S-W (1982) model is, as a consequence of its design, one that is incapable of addressing issues concerning the real bills doctrine. As I have argued elsewhere (1983b)(1983c), the specific feature that makes this model inapplicable to various monetary issues is not its life-cycle structure, but its adherence to a particular version of the "principle of finance theory that assets are valued according to the streams of returns that back them" (S-W, 1982, p. 1214). There is nothing wrong with that principle as stated, except for some ambiguity involving the term "back them," but S-W have for some reason interpreted it to mean that only pecuniary returns should be counted. In particular, the transactions-facilitating services of money are not to be counted, a proviso that seems analogous to requiring that the pleasures of owning a painting by (say) Vermeer must not be taken into account in the analysis of that asset's value. As a result of that proviso, every asset in the model must have the same monetary status--all serve as media of exchange or none do. Thus it becomes impossible to carry out any analysis that involves a distinction between money and non-monetary paper assets, a distinction that is central to the very notion of the "discounting of real bills."

VI. The Wicksellian Dynamic Process

A notable feature of the analysis in Sections II-IV is its evident inconsistency with the Wicksellian notion that policy designed to yield a low nominal rate of interest will tend to induce inflation.[36/] In the analysis of those sections, by contrast, the average rate of inflation moves (across policy regimes) together with the target value of the nominal rate. Indeed, extremely low values of $\bar{r}$--values below the rate b_0--are associated with _falling_ prices, deflations of magnitude $b_0 - r$. We need to consider, then, whether the Wicksellian notion is itself incorrect as a matter of neoclassical theory or if, alternatively, there is some deficiency in the model (1')-(2') as a vehicle for expressing that theory.

The most prominent way in which our analysis differs from that of previous Wicksellian discussions--e.g., Wicksell (1898)(1905), Friedman (1968), Laidler (1972)--is in its assumption that expectations are formed rationally. But given the attractiveness of that assumption, recognition of this difference does not suggest a deficiency in our framework. It leads, rather, only to a re-phrasing of the question, which now becomes: Is the Wicksellian notion incorrect for an economy with rational expectations?

The first thing to be said in this regard is that it does appear incorrect to suggest--as Friedman (1968, p. 5) comes very close to doing--that the monetary authority cannot permanently keep the nominal interest rate low or high, as it chooses, by means of low or high rates of money creation. Ironically, Friedman's hypothesis concerning the natural rate of _unemployment_ seems more convincing today than the interest rate counterpart that he used as a relatively uncontroversial means of introduction.

The second point to be mentioned concerns pegging of the real rate, $r_t - (E_t p_{t+1} - p_t)$. In this regard, reflection indicates that the real rate cannot--in the model (1'),(2')--be maintained at any value differing from the natural rate value b_0; any attempt to do so would founder on an inconsistency similar to the one noted in Section IV (just below equations (30)). But that conclusion is not robust to plausible specificational adjustments. In particular, the inclusion of a real balance effect in (1') would, as is well known, make the steady-state real rate dependent upon the inflation rate. So in an economy in which the real balance effect is operative, the monetary authority can permanently influence the real rate by its money creation activity. It cannot literally peg the real rate under the informational assumptions employed in Sections II-IV, because it cannot observe without error the contemporaneous value of private agents' expected inflation rate. But by adopting a policy rule of the form

$$(46) \qquad m_t = m_{t-1} + \mu_1 + \lambda[r_t - E_t(p_{t+1} - p_t) - \bar{\rho}]$$

and setting μ_1 appropriately, the monetary authority can induce the real rate to fluctuate randomly around the chosen value $\bar{\rho}$. In this case, lower values of $\bar{\rho}$ will require larger values of μ_1 and will be associated with higher inflation rates. This association of high inflation rates and low real rates of interest is reminiscent of the Wicksellian notion, and indeed might be regarded as a reformulated version of the latter. It should be recognized, however, that the phenomena responsible for this association--in particular, the real balance effect--are quite different from those involved in the original version of the Wicksellian notion.

Returning now to the matter of nominal-rate pegging, let us conclude with a recognition that there is one type of potential malfunction that has not been explicitly considered in the foregoing discussion--namely, dynamic instability. It is certainly arguable that dynamic instability would represent Wicksell's hypothesis more accurately than the situation of nominal indeterminacy.[37] For instability to obtain, it is clear that the model at hand would have to be modified in ways that introduce a more interesting dynamic structure. In this respect there are two reasonably attractive possibilities. The first of these would involve replacement of the classical supply function utilized in Sections II-IV with one that makes prices somewhat sticky, i.e., temporarily rigid. To maintain consistency with neoclassical theory the adopted supply function should, however, respect the natural rate hypothesis as applied to employment and output--something that naive Phillips Curve relationships and NAIRU specifications fail to do.[38] The second type of dynamic modification that seems of potential interest is one that reflects some sort of lag in the reaction of policy to r_t observations. In this case the policy rule (13) might, for example, be replaced by

$$(47) \qquad m_t = m_{t-1} + \mu_1 + \lambda(r_{t-1} - \bar{r}).$$

Investigation of such dynamic elaborations is, however, a large task that is beyond the scope of the present paper.

VII. Conclusions

The conclusions of our various investigations can be stated very briefly. First, it is possible--as Canzoneri, Henderson, and Rogoff (1983) have shown--for the monetary authority to peg the nominal interest rate without generating price level indeterminacy in an economy with rational expectations and no private-sector money illusion. But, second, this can be accomplished by adhering to either of two (or more) alternative policy rules, and these have differing implications for the stochastic behavior of the price level. Third, the feasibility of effecting a pure peg is more problematical; merely standing ready to buy and sell securities at the desired rate does not constitute a well-specified policy and so leaves private agents unable to formulate expectations rationally. Fourth, the Sargent-Wallace (1982) analysis of the real bills doctrine is conducted in a model that features indeterminacy to the same extent as their 1975 model. Fifth, the more recent Sargent-Wallace model provides an unsatisfactory framework for consideration of the real bills doctrine because it neglects the medium-of-exchange role of money, thereby negating the possibility of distinguishing between monetary and non-monetary paper assets. Finally, the combination of flexible prices and rational expectations is not favorable to the Wicksellian idea that low nominal interest rates tend to induce inflation.

Appendix A

The object here is to show by means of a familiar example that the inclusion of extraneous state variables, i.e., ones not included in the minimal set, can lead to an infinite multiplicity of solutions. For this purpose, consider the model (1)(2) with a constant money stock, $m_t = m$. Substitution of (1) into (2) then yields

$$(A\text{-}1) \qquad m - p_t = c_3 + c_1(E_t p_{t+1} - p_t) + u_t$$

where $c_3 = c_0 + c_1 b_0$ and $u_t = \eta_t + c_1 v_t$. The minimal-state-variable solution is of the form

$$(A\text{-}2) \qquad p_t = \pi_0 + \pi_1 u_t$$

and straightforward calculations show that $\pi_0 = m - c_3$ and $\pi_1 = -1/(1-c_1)$.

Suppose, however, that one seeks a solution of the form

$$(A\text{-}3) \qquad p_t = \pi_0 + \pi_1 u_t + \pi_2 p_{t-1} + \pi_3 u_{t-1},$$

in which the extraneous state variables p_{t-1} and u_{t-1} are admitted. Then we have

$$(A\text{-}4) \qquad E_t p_{t+1} = \pi_0 + \pi_2(\pi_0 + \pi_1 u_t + \pi_2 p_{t-1} + \pi_3 u_{t-1}) + \pi_3 u_t$$

and substitution into (A-1) gives rise to the implications

$$(A\text{-}5) \qquad \begin{aligned} m &= c_3 + c_1\pi_2\pi_0 + \pi_0 \\ 0 &= c_1\pi_2\pi_1 + c_1\pi_3 + (1-c_1)\pi_1 + 1 \\ 0 &= c_1\pi_2^2 + (1-c_1)\pi_2 \\ 0 &= c_1\pi_2\pi_3 + (1-c_1)\pi_3. \end{aligned}$$

The third of these implies that π_2 equals zero or $(c_1-1)/c_1$. If the former value is chosen, the minimal-state-variable solution is obtained. But if the value $\pi_2 = (c_1-1)/c_1$ is used, then the equations are consistent with any value for π_1. So the analyst concludes, unless side conditions are brought in, that there is an infinity of solution paths for p_t.

Appendix B

To show that the minimal-state-variable solution to the system (1)(2)(13) is as asserted in Section III, consider a more general version of (13) in which the coefficient on m_{t-1} is not necessarily 1.0:

$$m_t = \gamma m_{t-1} + \mu_1 + \lambda(r_t - \bar{r}). \tag{B-1}$$

With this modification, the second of equations (17) and (18) become

$$(1-\lambda\pi_{11})\pi_{21} = \pi_{11}(\gamma-1) \tag{B-2}$$

$$\gamma + (\lambda - c_1)\,\pi_{21} = \pi_{11}. \tag{B-3}$$

Consequently, π_{11} is determined as

$$\pi_{11} = \frac{\delta \pm \sqrt{\delta^2 - 4\lambda\gamma}}{2\lambda} \tag{B-4}$$

where $\delta = 1 + \gamma\lambda - (\lambda - c_1)(\gamma - 1)$. But in the case with $\gamma = 0$ the minimal-state-variable solution will have $\pi_{11} = 0$, so we see that the negative square root in (B-4) gives the appropriate value to π_{11}. Then we consider our case, in which $\gamma = 1$, and obtain

$$\pi_{11} = \frac{1 + \lambda - \sqrt{(1+\lambda)^2 - 4\lambda}}{2\lambda} = \frac{(1+\lambda) - (1-\lambda)}{2\lambda} = 1. \tag{B-5}$$

Appendix C

The purpose here is to provide some bibliographic evidence in support of the interpretation of the real bills doctrine offered in footnote 19, namely, as asserting that desirable behavior of the money supply will result if banks (including the central bank) restrict their loans to the discounting of non-speculative "real bills." Each of the following paragraphs will consist of a brief passage quoted from the indicated source.

... the error which it is the object of the present Chapter to expose; namely, that of imagining that a proper limitation of bank notes may be sufficiently secured by attending merely to the nature of the security for which they are given (Thornton, 1802 [1978], p. 244).

We can only consider their [i.e., the banking school's] view of the influence of bank credit, and more especially of note issues, on prices. This school ... denies any such influence so long as the banks only grant credit to the public in the form of <u>loans</u> on absolutely sound security. Even if the banks are not compelled to redeem their notes in gold they cannot, says Tooke, <u>under such conditions</u> either increase or diminish the total amount of credit instruments in circulation (Wicksell, 1905 [1935], p. 173).

The anti-bullionists ... claimed that as long as currency was issued only by banks, and was issued by them only in the discount of genuine and sound short-term commercial paper, it could not be issued in excess of the needs of business, since no one would borrow at interest funds which he did not need (Viner, 1937, p. 148).

... if only "real" bills are discounted, the expansion of bank money will be in proportion to ... the 'needs of trade,' and ..., when trade contracts, bank loans will be correspondingly paid off. Closely associated with this point of view is the doctrine that, if only commercial loans are made, the currency will have a desirable elasticity and the banks will at all times be in a liquid condition. I shall designate these ideas as the "real-bills doctrine" (Mints, 1945, p. 9).

... any continuing excess of note-issues above a safe figure was automatically prevented ... [provided that] notes were issued only against sound commercial discounts.... The discounting of real trade bills could not lead to an over-issue of paper (Horsefield, 1953, pp. 16, 26).

It was alleged [by the real bills doctrine] that the quantity of money would automatically be properly regulated if the monetary authorities ensured that banks always had enough reserves to meet the demand for loans intended to finance "real" (as opposed to "speculative") investments at an interest rate set "with a view of accommodating commerce and business" (Sargent, 1979, p. 92).

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Footnotes

1. The distinction between "indeterminacy" and "non-uniqueness" will be discussed below.

2. A related line of argument was put forth earlier by Liviatan (1981) and more recently by Dotsey and King (1983) and Goodfriend (1983). Also relevant is the analysis in Sargent-Wallace (1982). Recent papers concerning indeterminacy but bearing less directly on the issues here considered include Begg and Hague (1983), Calvo (1981), and Carlson (1983).

3. The terminology proposed in this paragraph is not currently used by all writers, but is consistent with the usage of Sargent and Wallace (who have contributed leading papers on both topics).

4. Actually, this specification differs in one way from that in McCallum (1981): agents base their expectations of the future price level p_{t+1} on r_t as well as Ω_{t-1}. That change is not crucial for the issues under discussion, but should be kept in mind in obtaining solutions to the various models. For additional discussion of the model, see McCallum (1981).

5. For an illustration of this phenomenon, see Appendix A.

6. Here and in all that follows it is taken for granted that the constant state "variable" 1.0 is also included.

7. It will be noted that the values of π_{22} and π_{23} in (11) would also be small for large negative values of λ and would approach zero for $\lambda \rightarrow -\infty$. Our specification does not, however, permit negative values of λ. One reason for this restriction is that $\lambda > c_1$ is necessary to rule out "process inconsistency", in the language of Flood and Garber (1980), of the type discussed in McCallum (1983a, pp. 159-160).

8. Suppose one were to adopt the other set of solutions to (17) and (18), the $\pi_1 = 1/\lambda$ set that (according to Appendix B) includes bubble components. In this event it would be found that the coefficients

π_{12}, π_{13}, π_{22}, and π_{23} equal (for all λ) the values obtained as $\lambda \to \infty$ in the model with policy rule (3). By itself, this finding might lead one to believe that the procedure is picking the "wrong" solution. But it is also the case that the system requires that $\mu_1 = \lambda(\bar{r} - b_0)$ to avoid an inconsistency, thereby restricting λ to the value 1.0 if $\mu_1 = \bar{r} - b_0$. Also, π_{10} is not determinate. So this solution is very poorly behaved.

9. Compare equations (20) in Goodfriend (1983).

10. Another change from the original model that will yield these qualitative results is the inclusion of a real-balance term in the IS function, provided that this term is written--as consideration of budget equations suggests it should be--as $m_{t-1} - p_t$ (not $m_t - p_t$).

11. This conclusion is also obtained by Goodfriend (1983) and by Dotsey and King (1983).

12. We continue with the modified equations (1') and (2'), rather than (1) and (2), for generality.

13. This is the set indicated by the procedural rule suggested in McCallum (1983a) for cases with white-noise disturbances and no lagged expectations, i.e., include only disturbances and predetermined variables that explicitly appear in the equations of the model.

14. At an earlier stage, it was my belief that this notion was suggested by the analysis of Barro and Gordon (1983, p. 595). That this is not the case will become apparent shortly.

15. That one is bound to meet some problem with $r \neq b_0$ is obvious from the specification of (26).

16. It should be emphasized that this conclusion does not involve a departure from the strategy of limiting our attention to solutions with a minimal set of state variables (i.e., bubble-free solutions). Instead, it reflects a conjecture that in the case at hand the minimal set of state variables includes m_{t-1} as well as v_t and η_t. (Alternatively, p_{t-1} could be used in place of m_{t-1}. The resulting solution would be the same; i.e., the same paths would be generated for the endogenous variables.)

17. In the language of Nelson and Plosser (1982), one of these processes is, and the other is not, trend stationary.

18. The line of argument developed in this section can be applied to the analysis on pp. 323-4 of McCallum (1981).

19. It is not an entirely straightforward task to determine what the real bills doctrine, or commercial loan theory of credit, is. My impression--formed on the basis of discussions by Blaug (1968), Humphrey (1982), Horsefield (1953), Mints (1945), Thornton (1802), Viner (1937), and Wicksell (1905)--is that the doctrine claims that desirable behavior of the money supply will be assured if banks, including the central bank, restrict their loans to the discounting of "real bills," i.e., to the financing of non-speculative investments. (For some supportive quotes, see Appendix C.) This version implies that no quantitative restrictions on the stock of money need to be imposed as a matter of policy, a situation represented by a policy rule that focuses on interest rates rather than any monetary aggregate. In this paper, as in part I of S-W (1982), the discussion presumes a fiat monetary standard. Some writers, including

Adam Smith, have espoused the real bills doctrine under the proviso that the economy be on a commodity money standard, in which case no issue of price level determinacy can arise. (For a discussion of Smith's position, see Laidler (1981).)

20. Actually, Sargent and Wallace (1975) did not mention the real bills doctrine. But Sargent (1982, p. 387) and Sargent-Wallace (1982, p. 1213) have indicated that the 1975 results concerning an interest rate policy rule should be interpreted as a criticism of the real-bills doctrine.

21. Whether the S-W representation of quantity-theory recommendations is accurate is debatable, for reasons to be discussed below.

22. There is but one good, which is perishable.

23. In fact, the magnitude of monetary transfers to the old is zero for all $t = 1,2,\ldots$ in the two S-W regimes that are of concern here. It is nevertheless appropriate to write $H(t)$ as a variable, thereby reflecting the possibility of other regimes and the possibility that loan repayments will fluctuate over time.

24. Here there is no need to spell out the fluctuating endowment patterns that are crucial for other aspects of the S-W discussion.

25. This heuristic interpretation of the equilibrium condition (42) applies under the laissez faire regime. For an interpretation pertaining to the real bills regime, see S-W (1982, p. 1226).

26. That is, equilibria in which currency has a positive value in terms of goods.

27. Requiring this equalization of returns is implicitly to deny that currency serves as a medium of exchange--or, more precisely, does so to a greater extent than private securities--in the modelled economy. It will be seen below that this condition necessitates some re-interpretation of the present discussion.

28. In this regime there are legal restrictions on private security issues. Relation (43) nevertheless obtains in any monetary equilibrium since all asset holders are free to hold government currency (S-W, 1982, p. 1225).

29. This statement is also applicable, it should be noted, to the LF regime.

30. Laidler (1984) has presented other arguments suggesting that the attempted rehabilitation is unsuccessful.

31. Recall fn. 19.

32. This conclusion is based on arguments developed in McCallum (1983b), especially pp. 23-28 and 33-34, and McCallum (1983c).

33. Neglect of uncertainty and production is less fundamental in the sense that much interesting monetary analysis can be, and has been, conducted in non-stochastic models of exchange economies.

34. That this asset is said to carry a zero nominal interest rate is no reason to think of it as currency. In fact, all this condition implies is that the "nominal interest rate" has been computed from its real rate by addition of the inflation rate with the price level based on <u>this</u> asset as numeraire. But the own rate of interest on any paper asset is always zero!

35. This conclusion obviously implies that the indeterminacy analysis in the first part of this section must be reinterpreted as involving a relative price.

36. It is not universally agreed how Wicksell's (1898) (1905) ideas should be expressed in terms of 1980's-style analysis. Thus Sargent and Wallace refer to price level indeterminacy as "Wicksellian" (1982, p. 1213) or as "Wicksell's indeterminacy" (1975, p. 215) while Laidler (1984) contends that this is a misrepresentation, with Wicksellian phenomena requiring some form of "disequilibrium." My own impression is that the main aspect of Wicksell's analysis resides in the hypothesized tendency for low nominal rates to lead to continuing inflation.

37. For the representation to be satisfactory, however, the price level explosion would need to be in an upward direction when $\bar{r}$ is low and in a downward direction when $\bar{r}$ is high.

38. On this point, see McCallum (1982).