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IMMIGRATION AND THE SURVIVAL OF SOCIAL SECURITY:
A POLITICAL ECONOMY MODEL

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ABSTRACT

In the political debate people express the idea that immigrants are good because they can help pay for the old. The paper explores this idea in a dynamic political-economy setup. For this purpose we develop an OLG political economy model of social security and migration. We characterize sub-game perfect Markov equilibria where immigration policy and pay-as-you-go (PAYG) social security system are jointly determined through a majority voting process. The main feature of the model is that immigrants are desirable for the sustainability of the social security system because the political system is able to manipulate the ratio of old to young and thereby the coalition which supports future high social security benefits. We demonstrate that the older is the native born population the more likely is that the immigration policy is liberalized and the social security system survives.

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Economic analysis of the inter-generational and intra-generational aspects of the sustainability of social security has had a revival of sorts in recent time. The analytical underpinnings of a political-economy equilibrium model in which migration and taxes interact focusing on the inter-generational aspect of social security, is yet to be worked out.

The net present value of costs and benefits from a pay-as-you-go social security are negative for the young people and positive for the elderly, in any pay-as-you-go social security system. If people all vote their economic interest, there will be a pivotal age such those who are younger, will favor smaller social security benefits and those who are older will favor larger benefits. Browning (1975) points out that if a single once-and-for-all election were held to determine the level of social security, then a coalition of persons of median age and higher would select an inadequate pension plan that benefits the current elderly at the expense of all future generations. The issue of social security levels that is frequently visited by voters, is however less clear. Cooley and Soares [1999] and Boldrin and Rustichini [2] study the interaction between capital accumulation and social security in general equilibrium models of a closed economy with constant population growth rates where levels of social security payments are determined periodically by majority voting. Prescott and Rios-Rull (2000) argue that a necessary feature for equilibrium is that beliefs about the behavior of other individuals (current and future) are rational. In stationary OLG environments this implies that any future generation in the same situation as the initial generation must do as well as the initial generation did in that situation. They conclude that the existing equilibrium concepts in the literature do not satisfy this condition. They then propose an alternative equilibrium concept, organizational equilibrium, that satisfies this condition. They show that equilibrium exists, it is unique, and it improves over autarky without achieving optimality. Moreover, the equilibrium can be readily found by solving a maximization program. Razin, Sadka and Swagell (2001, 2002b) develop an OLG model where the extent of taxation and redistribution policy is generally determined as a political-economy equilibrium by a balance between those who gain from higher taxes/transfers and those who lose. In a stylized model of migration and human capital formation, they show – somewhat against the conventional wisdom – that low-skill immigration may lead to a lower tax burden and less redistribution than would be the case with no immigration, even though migrants (naturally) join the pro-tax/transfer coalition. The model captures two conflicting effects of migration on taxation and redistribution. On the one hand, migrants who are net beneficiaries of the welfare state will join forces with the low income

native-born voters in favor of higher taxes and transfers. On the other hand, redistribution becomes more costly to the native-born as the migrants share the redistribution benefits with them. But the governments transfers, in this model, accrued uniformly to young and old. Thus the intra-generational transfer is the trigger for the so-called fiscal leakage from the median voter to the net beneficiaries of the welfare system, but not the inter-generational transfer as in a pure social security model.

‘A slower growth of the population implies a smaller labor force. It also slows the growth of national saving rate, because the proportion of the population of individuals who are dissavers rises relative to the proportion of the population of individuals who are savers. If a low birth rate leads to slow population growth, it is easy to see that an exogenous increase in immigration of young will reduce the average age of the population and increase population growth. Immigration does not provide in itself a full-fledged long-term solution to falling birth rates and an ageing population, but it is one of the available tools within a broader policy mix. Feldstein (2006) states: "...the common prescription of increased immigration would do little to reduce the future fiscal burden" of current tax-financed systems of social pensions. Instead of considering migration as determined at the source, and workers entering the "open doors to heaven" (Borjas 1999), the question who is allowed into a country depends on active immigration policy on part of the receiving countries. Countries more often than not enact quotas, point systems, and the like, in order to select those immigrants whom they deem most desirable. This view presupposes that the country under consideration is attractive for potential immigrants. In the US immigration debate the vast majority of citizens favor much tougher immigration rules. This could be due to congestion in public goods and/or dislike of foreigners (mostly mexicans). However, immigration survives unchecked because henceforth it has not been a salient issue in elections and there is a large business lobby keeping it in place. Thus, immigration is an endogenous policy variable in the general equilibrium-political economy system, as much as the level of social security itself. When the population ages, the median voter is likely to be more favorable to immigration. At the same time immigration can bring out political economic forces that would affect the size of the pay-as-you-go social security system.

The main purpose of the present paper is to highlight the interactions between migration policy and social security system, in a political economy setup. To focus on the pure intergenerational aspects of the social security sustainability issue we completely abstract from intra-generational income transfers. There is a pay-as-you-go (PAYG) social security system, which employs payroll taxes

(at a flat rate) on a representative working young in order to finance a uniform benefit to the aged population. We are able to characterize subgame-perfect Markov equilibrium paths for different patterns of population growth among the native-born and the immigrants populations. The tax-migration policy is endogenously determined by the conventional effect on wages, savings, etc. But it is also driven by strategic considerations concerning the effect of the current tax-migration policy on the next period tax-migration policy. The current young would like to influence the old-young composition of next period voters through the current migration policy and taxes. We demonstrate that the older are the native-born population the more likely is that the immigration policy is liberalized and that the social security system will survive.

The model generates different types of equilibrium paths. When the current migration policy is the only state variable, the Markov sub-game perfect equilibrium is characterized by a "demographic switching" strategy, where the young decisive voter admits a limited number of immigrants, in order to change the decisive voter's identity from young to old in the next period. The aim is to maximize his social security benefits when old in the next period. When next period capital per (native-born) worker is an additional state variable, there is another channel of influence of the current period policy variables through savings. Thus, the young decisive voter can have another possible strategy, a "demographic steady" strategy, where she admits the maximum amount of immigrant, and in so doing she renders a majority of young every period. In this case, both "demographic switching" and "demographic steady" strategies are incorporated in the same equilibrium creating other possible equilibria. To understand strategic voting of this kind we begin with a base line model, with no private saving traded assets, in which next period policy variables are influenced only by the current migration policy. The extended model with private savings and capital accumulation includes also an equilibrium path in which both the "demographic switching" and "demographic steady" strategies are at play.

The main prediction of both the base-line and the extended models is that immigrants are desirable for the sustainability of the social security system because the political system is able to manipulate the ratio of old to young and thereby the coalition for future high social security benefits.

The paper is organized as follows. We survey the literature background for our paper in Section 2. Section 3 provides analysis of a base line model, where there is no private savings, the economy does not accumulate capital, and factor prices are exogenous. Section 4 extends the base line model to include private

saving, capital accumulation and the endogenous determination of the factor prices. Section 5 considers the effect of aging in the extended model. Section 6 concludes.

2 Literature Background

Standard theory of the determinants of the size of the government in a direct democracy highlight the relationship between the scope of redistribution, i.e. the extent of the welfare state, and pre-tax income inequality. Two interpretations have been suggested to explain this dependence: Lovell (1975) emphasizes the size of the government as a provider of public goods, while others such as Meltzer and Richard (1981) emphasize the role of the government in redistributing income. See Persson and Tabellini (1999) for a survey. As Razin, Sadka and Swagel (2002a; 2002b) have recently pointed out, a third potential channel is the leakage of welfare benefits to groups such as immigrants or the elderly.

The analysis of the inter-generational and intra-generational aspects of the sustainability of social security, has had a revival of sorts in recent time. There are few economic explanations to the sustainability of the social security system in a political economy equilibrium context. Early papers put an emphasis on dynamic inefficiency, where social security may improve the welfare if the implicit rate of return from social security is larger than the real rate of return from capital accumulation (Samuelson (1958)). Others argue that because the median voters in future generations, treat their past contributions as a sunk cost, they must find it in their interest to preserve the system until their retirement (Browning (1975), Sjoblom (1985)). Another idea is that social security systems are favorable from intergenerational redistribution motives, either because of direct within-cohort redistribution reasons (Persson and Tabellini (2000)) or for intergenerational redistribution reasons due to crowding out effect of social security schemes on capital accumulation, leading to a reduction of real wages and increase in the real returns to capital (Boldrin and Rustichini (2000) and Cooley and Soares (1999)). See also Galasso and Profeta (2002) for a survey.

The ongoing demographic changes have also far-reaching implications on PAYG social security systems. From an empirical investigation the effect of the proportion of elderly people in the population on the size of social security benefit per retiree turn out not to be significant (Mulligan and Sala-i-Martin (1999)) or even negative (Razin, Sadka and Swagell (2002a)). Some explanation has been offered to try and explain this very fact. Cooley and Soares (1999) and Boldrin and Rustichini (2000) show that an increase in the proportion

of elderly in the population raises, on the one hand, the median voter's age, and thus increases the size of the system. But on the other hand, an increase in the proportion of elderly raises the dependency ratio, thereby decreasing per capita benefits. The overall effect on the per capita pension benefit is thus ambiguous, since more resources are shared among more retirees. Boldrin and Rustichini (2000) model PAYG social security systems as the outcome of majority voting within a OLG model with production. When voting, individuals make two choices: pay the elderly their pensions or default, which amount to promise themselves next period. Under general circumstances, there exist equilibria where pensions are voted into existence and maintained. Bergstrom and Hartman (2005) estimate the expected present value of benefits and costs to US voters of each age and sex for a small permanent increase in social security benefits. They reach the conclusion that as the population ages maintaining social security will become more expensive but at the same time, the median age of voters will also rise leading to a majority of selfish voters who favor maintaining current benefit levels. See also Hassler, Mora, Storesletten and Ziliboti (2003).

From the point of view of the sustainability of PAYG social security, immigration may come to the rescue of PAYG social security systems. Razin and Sadka (1999, 2004) make an argument that highlights the importance of migration in enlarging the labor force in OLG models with pay-as-you-go fiscal system, for current and future generations of native-born. They consider an overlapping-generations model, where each generation lives for two periods. In each period a new generation with a continuum of individuals is born. Each individual possesses a one unit of labor-schooling time endowment in the first period, when young. There is a pay-as-you-go (PAYG) pension system, which employs payroll taxes (at a flat rate) on the working young in order to finance a uniform benefit to the aged. in an infinite-horizon, overlapping generations economy, this net burden is perfectly consistent with a net gain to the native-born population. The additional obligation of the fiscal system to pay pension benefits to the incoming migrants, when they retire, could be shifted forward, in effect, indefinitely. If, hypothetically, the world would come to a stop at a certain point of time in the future, the young generation at that point would bear the deferred cost of the present migration. But in an ever-lasting economy, the migrants, by supplying work and helping the financing the pension benefit of period zero to native-born retirees, are a boon to the host country population: old, young, and future generations. Storesletten (2000) and Lee and Miller (2000) calibrate a general equilibrium overlapping generations model to investi-

gates whether a reform of immigration policies could resolve the fiscal problems associated with the aging. Storesletten find that selective immigration policies, involving increased inflow of working-age high and medium-skilled immigrants, can remove the need for a future fiscal reform. Lee and Miller on the other hand reach the conclusion that since immigrants have lower education and higher fertility rates than that of the native-born, a higher amount of immigrant admitted into the economy will ease temporarily the projected fiscal burden of retiring baby boomers in few decades although its overall fiscal consequences would be quite small. Feldstein (2006) analyses ways in which governments can respond to the budget problems implied by aging. In particular, the paper address the question whether the use of increased immigration can offset the slow growth of capital or the decline in the native labor force which reduces the tax base. He argues that the common prescription of increased immigration would do little to reduce the future fiscal burden, and that the only alternative is to shift from a pure tax-financed system to a mixed system that supplements the tax financed benefits with benefits based on increased saving financial investment.

Despite the strong implications of immigration policy and a lively political debate, surprisingly few studies addressed the political economy of immigration policy. The first paper which studied immigration policy in a political economy setup was the paper by Benhabib (1997). He examines the determination of immigration policies that impose capital and skill (human capital) requirements on heterogeneous immigrants through majority voting process. Dolmas and Huffman (2004) and Ortega (2005) add another angle to the political debate and model the joint decisions over immigration quotas and redistributive tax policy. Both address the voting process in a dynamic set-up, where the native-born voters' preferences over immigration are influenced by the prospect that immigrants will be voting over future tax policy. The paper of Dolmas and Huffman refers to the decisions over immigration quotas and redistributive tax policy in subsequent three periods model with different degree of international capital mobility. The latter paper considers a infinite horizon general equilibrium model of immigration and redistribution policies, with a heterogeneously skilled population who chooses an immigration policy by majority vote while anticipating that immigration affects the skill premium and the skill composition of the electorate. Razin, Sadka and Swagell (2002b) show in a stylized model of migration and human capital formation that low-skill immigration may lead to a lower tax burden and less redistribution than would be the case with no immigration, even though migrants (naturally) join the pro-tax/transfer coalition. This is due to two conflicting effects of migration on taxation and redistribution. On the

one hand, migrants who are net beneficiaries of the welfare state will join forces with the low income native-born voters in favor of higher taxes and transfers. On the other hand, redistribution becomes more costly to the native-born as the migrants share the redistribution benefits with them. These models clearly dial with intra-generational transfer, and not the inter-generational transfer as in a pure social security model.

3 A Base Line Model

The economy is populated by overlapping generations of identical individuals. Individuals live for two periods. When young, the representative individual works and makes labor-leisure. Underdeveloped capital markets do not allow any private savings. Social security is therefore the only means of intertemporal transfers. When old, the individual retires, and receives social security benefits. The tax-transfer system is "pay as you go" where in every period the government levies a flat tax on the young's wage income, which fully finances the social security benefits paid to the old. Immigrants enter the economy when young, and gain the right to vote only in the next period, when old. They have the same preferences as those of the native-born, except from having a higher population growth rate. Immigrants are fully integrated into the social security system upon arrival into the country. Offsprings of immigrants are like native-born in all respects (in particular, they have the same rate of population growth).

We assume that the utility of the representative young individual is logarithmic ¹, given by:

$$U^y(w_t, \tau_t, b_{t+1}) = \text{Log}[w_t l_t (1 - \tau_t) - \frac{l_t^{\Psi+1}}{\Psi + 1}] + \beta \text{Log}[b_{t+1}] \quad (1)$$

$$U^o(b_t) = b_t \quad (2)$$

where U^y and U^o are the utility functions of young and old individuals, $\beta \in [0, 1]$ is the discount factor, and $\Psi > 0$ a disutility parameter (also equals to the labor supply elasticity with respect to the wage rate). The transfer payments to the old at period t , b_t , are financed by collecting a flat income tax rate, $\tau_t \in [0, 1]$, from the young individual's wage income at the same period, $w_t l_t$, where l_t denotes the hours worked.

Labor is a single input in the production of a homogenous final good. The production function is linear:

¹Note that this type of utility function implies that there are no income effects on the demand for leisure (Greenwood, Hercowitz, and Huffman (1988)).

$$Y_t = N_t \quad (3)$$

where Y_t and N_t are period t output and labor supply, respectively. Competitive equilibrium wage rate, equals to the marginal productivity of labor, is constant and normalized to unity. A worker can be either native-born or immigrant, perfectly substitutable, and with equal productivities. The immigration quotas is expressed as a certain percentage of the number of young individuals in the native-born population, $\gamma \in [0, 1]$ ². Labor supply is given by:

$$N_t = L_t l_t (1 + \gamma_t) \quad (4)$$

where L_t is the number of young individuals in the native-born population (old people do not work).

Immigrants have the same preferences as the native-born population, but different population growth rates. We assume that the native-born population has a lower population growth rate, $n \in [-1, 1]$, than that of the immigrant population, $m \in [-1, 1]$, so that, $n < m$. We also assume that the immigrant's descendants are completely integrated into the economy and therefore have the same population growth rate as the native-born population does. The number of young native-born individuals at period t can be written as follows,

$$L_t = L_{t-1}(1 + n) + \gamma_{t-1} L_{t-1}(1 + m) \quad (5)$$

In addition, immigrants are also assumed to contribute to, or benefit from, the social security system in the same way as the native-born. Because the social security system redistributes income from the young to the old, the balanced government budget constraint implies:

$$b_{t+1} L_t (1 + \gamma_t) = \tau_{t+1} w_{t+1} l_{t+1} L_{t+1} (1 + \gamma_{t+1}) \quad (6)$$

Re-arranging the expression yields:

$$b_{t+1} = \frac{\tau_{t+1} w_{t+1} l_{t+1} [(1 + n) + \gamma_t (1 + m)] (1 + \gamma_{t+1})}{(1 + \gamma_t)} \quad (7)$$

Labor-leisure decisions of young individuals are derived, as usual, from utility maximization, taking the prices and policy choices as given:

$$l_t^\Psi = w_t (1 - \tau_t) \quad (8)$$

²A ceiling for γ is set equal to one, which means that the number of immigrants cannot surpass the number of native born.

Substituting for b_t , b_{t+1} and l_t in equations (7) and (8) into equation (1), the indirect utility functions of the young individual can be written as:

$$V^y(w_t, \tau_t, \tau_{t+1}, w_{t+1}) = \text{Log}\left[\frac{\Psi}{\Psi+1} w_t l_t (1 - \tau_t)\right] + \beta \text{Log}\left[\frac{\tau_{t+1} w_{t+1} l_{t+1} [1 + n + \gamma_t (1 + m)] (1 + \gamma_{t+1})}{(1 + \gamma_t)}\right] \quad (9)$$

such that,

$$l_t^\Psi = w_t (1 - \tau_t) \quad (10)$$

$$l_{t+1}^\Psi = w_{t+1} (1 - \tau_{t+1}) \quad (11)$$

Substituting for b_t in equations (7) into equation (2), yields the indirect utility functions of the old individual:

$$V^o(b_t) = \frac{\tau_t w_t l_t [(1 + n) + \gamma_{t-1} (1 + m)] (1 + \gamma_t)}{(1 + \gamma_{t-1})} \quad (12)$$

Note that the old individual prefers that the immigration quotas will be as large as possible, because more immigration would raise the total amount of tax collected, and thus the social security benefits she receives. The old preferable tax rate is the "Laffer point" tax rate, where the tax revenues, and therefore the social security benefits, are maximized. The tax rate at that point is equal to $\frac{\Psi}{\Psi+1}$.

The young individual prefers naturally that the current tax rate is as low as possible, namely zero. Concerning immigration quotas, the young preferences are ambiguous. On one hand, a larger quotas increases next period social security benefits per old individual. This is due to the fact that larger quotas increases the number of young in the next generation (some of these are offspring of the current immigrants) more than it increases the number of the old (who happen to be the current young) in the next period. This is due to the assumption that immigrants have a higher population growth rate than that of the native-born ($m > n$). Thus, the number of next period old recipients of social security increases but the total sum of next period social security benefits increases even more. This means that next period social security benefits per old individual (b_{t+1}) are higher the larger is the immigration quotas.

On the other hand, larger immigration quotas also influence the identity of next period decisive voter. A higher current immigration quota would raise the ratio of next period old to young voters. This result from the assumption that

immigrants gain the right to vote in the second period of their life, when old. A larger current immigration quotas increases the number of next period young voters by more than the number of next period old voters (since we assumed $m > n$). Thus, the current young voter which will be old in the next period, will favor the largest possible quotas. This changes next period decisive voter's identity from young to old in the next period in order to lead to a majority of old in the next period.

3.0.1 A Political-Economic Equilibrium

We employ a subgame-perfect Markov equilibrium of perfect foresight, as our equilibrium concept (see Krusell and Rios-Rull (1996)):

Definition 1 *A subgame-perfect Markov equilibrium is defined as a vector of policy decision rules, $\Psi = (T, G)$, where $T : [0, 1] \rightarrow [0, 1]$, is the taxation policy rule, $T(\gamma_{t-1})$, and $G : [0, 1] \rightarrow [0, 1]$, is the immigration quotas policy rule, $G(\gamma_{t-1})$, such that the following functional equation holds:*

1. $\hat{\Psi}(\gamma_{t-1}) = \arg \max_{\pi_t} V^i(\gamma_{t-1}, \pi_t, \pi_{t+1})$ subject to $\pi_{t+1} = \Psi(\gamma_t)$, where $\pi_t = (\tau_t, \gamma_t)$ is defined as the vector of policy platform, and V^i is the indirect utility of the current decisive voter.

2. *The fixed-point condition requires that if next period policy outcome is derived by the vector of policy decision rules- Ψ , the maximization of the indirect utility of the current decisive voter will reproduce the same law of motion, $\hat{\Psi}(\gamma_{t-1}) = \Psi(\gamma_{t-1})$.*

The subgame-perfect Markov equilibrium notion states that the expected policy function, which depends on the current state variables, must be self-fulfilling. The policy variables which are the tax rate, τ_t , and the immigration quotas, γ_t , have to maximize the decisive voter's indirect utility function, while taking into account that next period decision rules depend on the state variable i.e. the current immigration quotas.

The subgame-perfect Markov equilibrium is characterized by a "demographic switching" strategy. The assumption is that immigrants enter the country while young and gain the right to vote only in the next period when they are old, voters take into account the effect of admitting a certain number of immigrants on the composition of voters and their voting preferences in the next period. Moreover, when the number of young exceed the number of old in the population, the young decisive voter admits a limited number of immigrants, in order to change the decisive voter's identity from young to old in the next period and maximize the next period benefits she receives.

The equilibrium path depends on the native-born and immigrant's population growth rates. If the population growth rates of the native-born and immigrants are both positive, there is a steady state with no taxation/social security benefits. If alternatively, the sum of the population growth rates is negative, there is also a steady state, but with a certain level of taxation/social security benefits (the "Laffer point" tax rate) and full openness to immigration. Otherwise, the sum of the population growth rates can be positive and the native-born population's population growth rate negative. In this case, there is a "demographic switching" equilibrium path where some quotas on immigration always prevails while there is an alternate period by period taxation/social security policy, depending on the identity of the decisive voter. In a given period there is a certain amount of taxation/social security benefits (the "Laffer point" tax rate) and a maximum migration quota, while in the next there is no taxation/social security benefits and a more restrictive policy towards immigration.

. Since immigrants gain the right to vote only in the second period of their life in the host economy, the next period ratio of old to young voters in the native-born population, denoted by u_{t+1} , is given by:

$$u_{t+1} = \frac{(1 + \gamma_t)}{(1 + n) + \gamma_t(1 + m)} \quad (13)$$

Assuming that in case of a tie the old will be the decisive, the condition, $u_{t+1} < 1$, assures a majority of young individuals in the next period, while the condition, $u_{t+1} \geq 1$, assures a majority of old individuals. Therefore, the state variable of the economy, affects the next period ratio of young to old voters, u_{t+1} , which sets the profile of the next period decisive voter.

The Markov Perfect political equilibrium of the baseline model and its possible equilibrium paths, which depend on the population growth rates of the native-born and immigrant populations, can be formalized as follows:

Proposition 2 *There exists an equilibrium with the following feature :*

$$T(\gamma_{t-1}) = \begin{cases} \tau_t = 0 & \text{if } u_t(\gamma_{t-1}) < 1 \\ \tau_t = \frac{\Psi}{\Psi+1} & \text{otherwise} \end{cases} \quad (14)$$

$$G(\gamma_{t-1}) = \begin{cases} \gamma_t = -\frac{n}{m} & \text{if } u_t(\gamma_{t-1}) < 1 \\ \gamma_t = 1 & \text{otherwise} \end{cases} \quad (15)$$

where γ_t is restricted to be between zero and one. Under the assumption that the native-born population growth rate is lower than that of the immigrant's, there are three possible equilibrium paths, depending on the population growth

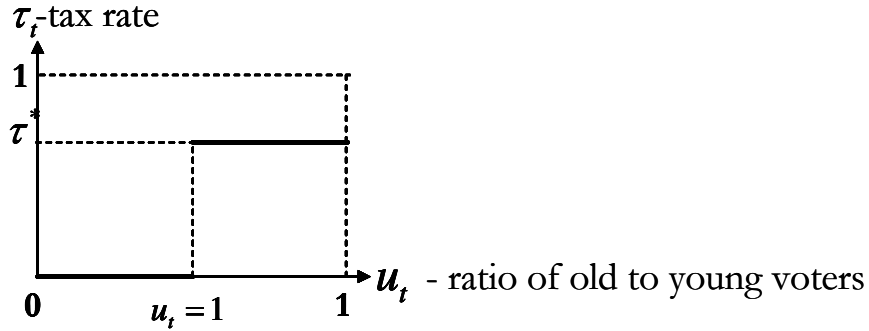


Figure 1:

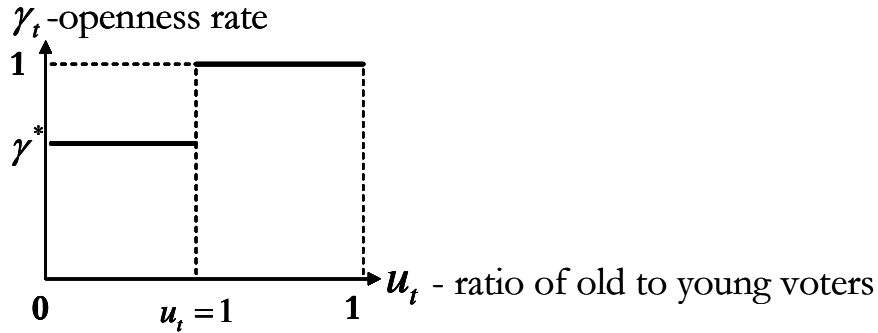


Figure 2:

rates of the native-born and immigrant population, as follows: 1. if $n > 0$, there is no taxation/social security benefits; 2. if $m + n < 0$, migration quota is set at its maximum, and there is a positive level of taxation/social security benefits (the "Laffer point" tax rate). 3. if $n < 0$ and $m + n > 0$, there is a "cycling" equilibrium path, where some positive level of immigration always prevails while there is an alternate taxation/social security policy; in periods where the decisive voter is old, the economy is fully opened to immigration and there is a positive level of taxation (the "Laffer point" tax rate); whereas in periods where the current decisive voter is young, there is no taxation/social security benefits and a more restrictive policy towards immigration.

The proposition is proved in the appendix.

The interpretation of the proposition is as follows.

If the old-young ratio is smaller than one ($u_t < 1$), the decisive voter in the current period is a young voter. The young decisive favors naturally a zero tax rate but has two conflicting considerations regarding the desired immigration quotas. On one hand, if there is full openness towards immigration: there will be more young working people in period $t + 1$, and therefore, the tax revenue that will be collected from a larger work force and needed to support retirement benefits, will increase. The young decisive voter in period t , who will be old in period $t + 1$, would benefit from the more generous social security benefits. On the other hand, if the immigration policy is excessively large, the decisive voter in period $t + 1$ will be a young voter. This voter will want to see the tax rate in period $t + 1$ reduced to zero; hence no social security benefits whatsoever in period $t + 1$. There is a threshold level of immigration quotas, $\gamma_t = -n/m$, which is exactly the level of the immigration policy that would equate the number of old and the number of young in period $t + 1$. Thus, by choosing the immigration quotas at this level, the decisive voter in period t would finely balance the two conflicting forces on period $t + 1$ social security benefits, so as to maximize these benefits. Observe that this young voter's preferable immigration quotas is chosen *strategically*, aimed to influence the identity of the decisive voter in the next period from young voter to old.

If the old-young ratio is higher or equal to one ($u_t \geq 1$), the decisive voter in the next period is an old voter. This voter will naturally vote for the most liberal immigration policy possible, because only the current social security benefits matter to this voter. If $u_t \geq 1$ the immigration quotas is therefore equal to its maximum level (i.e., one). Also the tax revenue is set at the "Laffer point", where the tax rate is equal to $\frac{\Psi}{\Psi+1}$. Because this way the current social security benefits are maximized.

There are three possible equilibrium paths depending on the population growth rates of the native-born and immigrant populations.

The first equilibrium path is the one where the population growth rate of the native-born and the immigrant growth rate are both positive; that is, $n, m > 0$. In this case, the level of social security benefits is zero. This is due to the fact that for every level of immigration, the number of next period young voters exceeds the number of next period old voters. Therefore, the decisive voter in the current and all the following periods is the young voter, and her preferences are for zero labor tax. The young voter is indifferent concerning the level of immigration because it has no influence on her current income, nor on the next period decisive voter's identity. The resulting equilibrium path is one in which there is a majority of young voters, and the social security system is

dismanteled, for ever.

If the sum of the native-born and immigrant population growth rates is negative, $m + n < 0$, the number of next period old voters always exceeds the number of next period young voters. Thus, along the equilibrium path a majority of old will always prevail, which validates a permanent existence for the social security system and a maximum flow of immigrants. The current decisive voter is using the immigration quota as a means to affect the identity of the next period decisive voter through the mix of old and young voters in the next period. This is the second equilibrium type.

The third equilibrium type obtains if the native-born and immigrant populations growth rates are: $n < 0$, and $m + n > 0$. This equilibrium path is characterized by an alternate taxation/social security policy over two consecutive periods. Some positive level of immigration always prevails. This is due to a "demographic switching" strategy of the current and next period young voters. The reason is that when there is a majority of old, their preferable immigration quotas is the at the maximum and the tax rate is at the "Laffer point". Because $m + n > 0$ and the old decisive voter allows as much as possible immigrants, the number of next period young voters exceed the number of next period old voters. Thus, in the next period the decisive voter must be the young. This voter opts for a zero tax rate, and does vote strategically on immigration levels. This means setting immigration at the threshold level, $\gamma_t = -n/m$. The identity of the next period decisive voter will change from young to old. (A possibility of such demographic changes exists because the native-born population growth rate is negative while the immigrant population growth rate is positive.) This creates a cycling effect of an alternate taxation/social security policy, with a certain level of immigration, depending on the identity of the decisive voter.

4 The Extended Model: Private Saving, Capital Accumulation and Endogenous Factor Prices

The base line model assumes zero private savings; hence no capital accumulation at all. In this section, we introduce private saving. This means that intertemporal transfers are both through private savings and through the social security system. The aggregate savings of the current young population generates next period aggregate capital. The latter is used as a factor of production, along with the labor input in the next period. The production function exhibits

constant return to scale. Another feature of the extended model is the wage rate, as well as the rate of interest, are endogenously determined along the equilibrium path. Social security benefits are financed, as before, by a payroll tax in a pay-as-you-go system.

The utility of the representative young individual, as before, is logarithmic.

$$U^y(w_t, \tau_t, s_t, r_{t+1}, b_{t+1}) = \text{Log}(w_t l_t (1 - \tau_t) - s_t - \frac{l_t^{\Psi+1}}{\Psi+1}) + \beta \text{Log}(b_{t+1} + (1 + r_{t+1})s_t) \quad (16)$$

$$U^o(b_t) = b_t + (1 + r_t)s_{t-1} \quad (17)$$

where r_t is the interest rate, and s_t is the savings of the young at period t .

The production function is a Cobb-Douglas production function which is assumed to use both labor and capital as its factors of production:

$$Y_t = N_t^{1-a} K_t^\alpha \quad (18)$$

where K_t is the aggregate amount of capital and N_t is defined as in the previous section. The wage rate and interest rate are determined by the marginal productivity conditions (capital is assumed to depreciate completely at the end of the period):

$$w_t = (1 - a)(1 + \gamma_t)^{-a} l_t^{-a} k_t^\alpha \quad (19)$$

$$r_t = \alpha(1 + \gamma_t)^{1-a} l_t^{1-a} k_t^{\alpha-1} - 1 \quad (20)$$

where k_t is capital per (native-born) worker. The balanced government budget constraint is derived as in the previous section:

$$b_{t+1} = \frac{\tau_{t+1} w_{t+1} l_{t+1} [(1 + n) + \gamma_t (1 + m)] (1 + \gamma_{t+1})}{(1 + \gamma_t)} \quad (21)$$

The saving-consumption decision of young individuals are made by maximizing their utility while taking the prices and policy choices as given, and the labor-leisure decision is given as in the previous section:

$$s_t = \frac{1}{1 + \beta} \left(\beta \frac{\Psi}{\Psi + 1} w_t l_t (1 - \tau_t) - \frac{b_{t+1}}{1 + r_{t+1}} \right) \quad (22)$$

$$l_t^\Psi = w_t (1 - \tau_t) \quad (23)$$

The market clearing condition requires that the net domestic saving generates net domestic investment:

$$s_t = k_{t+1} \left(\frac{1 + n + \gamma_t(1 + m)}{1 + \gamma_t} \right) \quad (24)$$

Solving for b_{t+1} from equations (21) and (22), and substituting b_{t+1} in equations (14), the utility indirect function of the young can be written as follows:

$$\begin{aligned} V^y(w_t, \tau_t, r_{t+1}, \tau_{t+1}) &= \text{Log} \left(\frac{1}{1+\beta} \frac{\Psi}{\Psi+1} w_t l_t (1 - \tau_t) (1 + \beta f(\tau_{t+1})) \right) \\ &+ \beta \text{Log} \left(\frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} w_t l_t (1 - \tau_t) (1 + \beta f(\tau_{t+1})) (1 + r_{t+1}) \right) \end{aligned} \quad (25a)$$

where $f(\tau_{t+1}) = \frac{\frac{1-\alpha}{1-\frac{1-\alpha}{1+\beta}} \frac{1}{1+\beta} \tau_{t+1}}{1 - \frac{1-\alpha}{1+\beta} \frac{1}{1+\beta} \tau_{t+1}}$, such that,

$$k_{t+1} = \frac{\beta}{1 + \beta} \frac{\Psi}{\Psi + 1} \frac{(1 + \gamma_t) w_t l_t (1 - \tau_t) (1 - f(\tau_{t+1}))}{1 + n + \gamma_t(1 + m)} \quad (26)$$

$$l_t^\Psi = w_t (1 - \tau_t) \quad (27)$$

$$l_{t+1}^\Psi = w_{t+1} (1 - \tau_{t+1}) \quad (28)$$

and substituting b_t from equation (21) and k_t from equation (24), in equations (17), the utility indirect function of the old can be written as follows:

$$\begin{aligned} V^o(\gamma_{t-1}, k_t, w_t, r_t, \tau_t) &= \frac{\tau_t w_t l_t [(1+n) + \gamma_{t-1}(1+m)] (1 + \gamma_t)}{(1 + \gamma_{t-1})} + \\ &(1 + r_t) k_t \left(\frac{1+n + \gamma_{t-1}(1+m)}{(1 + \gamma_{t-1})} \right) \end{aligned} \quad (29)$$

such that,

$$l_t^\Psi = w_t (1 - \tau_t) \quad (30)$$

As in the previous analysis, the old individual favors a positive level of tax rate at a "Laffer Point" ($\tau^* = \frac{\Psi}{\Psi+1}$), and the largest immigration quotas.

The preferences of the young, which will be discussed in the next section, differ from the baseline model as they are influenced by capital accumulation and endogenous factor prices effects.

4.0.2 Political-Economic Equilibrium Paths

The Markov sub-game Perfect equilibrium definition for the extended model is as follows:

Definition 3 *A Markov perfect political equilibrium is defined as a vector of policy decision rules, $\Psi = (T, G)$, and private decision rule, S , where $T : [0, 1] \rightarrow [0, 1]$, is the tax policy rule, $\tau_t = T(\gamma_{t-1}, k_t)$, and $G : [0, 1] \rightarrow [0, 1]$,*

is the immigration policy rule, $\gamma_t = G(\gamma_{t-1}, k_t)$, and $S : [0, \infty) \rightarrow [0, \infty)$, is the saving decision rule, $k_{t+1} = S(\pi_t, k_t)$, such that the following functional equations hold:

1. $\hat{\Psi}(\gamma_{t-1}, k_t) = \arg \max_{\pi_t} V^i(\gamma_{t-1}, \pi_t, \pi_{t+1})$ subject to $\pi_{t+1} = \Psi(\gamma_t, S(\pi_t, k_t))$.
2. $S(\pi_t, k_t) = \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{(1+\gamma_t)w_t l_t (1-\tau_t)(1-f(\tau_{t+1}))}{1+n+\gamma_t(1+m)}$, with $\tau_{t+1} = T(\gamma_t, S(\pi_t, k_t))$.
3. The fixed-point condition requires that if next period policy outcome is derived by the vector of policy decision rules- Ψ , the maximization of the indirect utility of the current decisive voter subject to the law of motion of the capital stock, will reproduce the same law of motion, $\hat{\Psi}(\gamma_{t-1}, k_t) = \Psi(\gamma_{t-1}, k_t)$, as in 1.

Policy variables have to maximize the decisive voter's indirect utility function, while taking into account the law of motion of capital and the fact that next period decision rules depend on the state variables i.e. the current period immigration quotas and next period capital per (native-born) worker; as for the definition of equilibrium in the base line model. Equilibrium paths depend on the native-born and immigrant population growth rates (as in the baseline model) and on the initial stock of capital per (native-born) worker.

There are three types of equilibrium paths.

The first type, is characterized by a "demographic switching" strategy, similarly to the base-line model,. When the decisive voter is young, she admits a finely tuned number of immigrants in order to change the decisive voter's identity from young to old in the next period; similarly to the base-line model, The additional effect caused by the existence of savings and the endogeneity of factor price determination, is only quantitative.

The two other equilibria types are however different from the base-line model.

The additional state variable, the stock of capital per (native-born) worker, plays now a crucial role. Rational voters take into account that the current policy variables can affect next period policy variables not only through the composition of old to young voters, but also through the effect on next period capital per (native-born) worker; the additional state variable. There is another possible strategy of the young, where the decisive voter chooses to admit the maximum amount of immigrants. In so doing the voter renders a majority for the young in every period; a "demographic steady" strategy, Equilibria which combine strategies concerning both the old-young composition in the population, and the level of capital, exist for a range of values of the capital per (native-born) worker, for which the "demographic steady" strategy dominates. There are now tow types of equilibrium paths in which the "demographic steady" strategy prevails. Ithe first, the equilibrium tax rate is a *decreasing* function of

the capital per (native-born) worker. In the second, the equilibrium tax rate is an *increasing* function of the capital per (native-born) worker.

In the first type of Markov sub game Perfect equilibrium (referred to by "demographic switching strategy" equilibrium) policy rules do not depend on the capital per (native-born) worker state variable.

Proposition 4 *There exists an equilibrium with the following feature:*

$$T(\gamma_{t-1}) = \begin{cases} \tau_t = 0 & \text{if } u_t(\gamma_{t-1}) < 1 \\ \tau_t = \frac{\Psi}{1+\Psi} & \text{otherwise} \end{cases} \quad (31)$$

$$G(\gamma_{t-1}) = \begin{cases} \gamma_t = \text{Min}[\gamma^*, -\frac{n}{m}] & \text{if } u_t(\gamma_{t-1}) < 1 \\ \gamma_t = 1 & \text{otherwise} \end{cases} \quad (32)$$

$$S(\pi_t, k_t) = \begin{cases} S(\pi_t, k_t, \tau_{t+1} = \frac{\Psi}{1+\Psi}) & \text{if } u_t(\gamma_{t-1}) < 1 \\ S(\pi_t, k_t, \tau_{t+1} = 0) & \text{otherwise} \end{cases} \quad \begin{cases} \text{if } n < 0 \cap \\ m + n > 0 \\ \text{if } m + n < 0 \\ \text{otherwise } n > 0 \end{cases} \quad (33)$$

where γ_t is restricted to be between zero and one, and γ^* is given explicitly in the appendix. The equilibrium paths depends on the population growth rates and on the initial amount of capital per (native-born) worker the economy is endowed with. There are three main types of equilibrium paths which are similar to the previous section: 1. if $n > 0$, there is no taxation and social security benefits and some restrictions on immigration. 2. if $m + n < 0$, there is full openness to immigration and a positive level of taxation (the "Laffer point" tax rate). 3. if $n < 0$ and $m + n > 0$, there is a "cycling" equilibrium path, where some level of immigration always prevails and there is an alternate taxation policy: in periods where the decisive voter is old, the economy is fully opened to immigration and there is a positive level of taxation (the "Laffer point" tax rate); and in periods where the decisive voter is young, there is no taxation and there are some restrictions on immigration.

The intuition is the same as in section 2. Because the decision rules in type one equilibrium do not depend on the capital per (native-born) worker, the Markov sub game Perfect equilibrium of the base line model is essentially a reduced form of this equilibrium. The equilibrium paths depend on the native-born and immigrant's population growth rates as in the baseline model, but naturally are also quantitatively influenced by the amount of initial stock of capital per (native-born) worker. The larger the initial stock of capital, the higher is the amount of capital accumulated every period.

The second and third types of Markov Perfect equilibrium of the extended model (referred to as "combined strategy" equilibria) are specified as follows:

Proposition 5 *Under several conditions on the parameters of the model, which are specified in the appendix, there exist two other equilibria types, $i = 1, 2$, with the following features:*

$$T^i(\gamma_{t-1}, k_t) = \begin{cases} \tau_i(k_t) & \text{if } k_t \in [F(\bar{\tau}), F(\tau_i)] \\ 0 & \text{otherwise} \end{cases} \quad \text{if } u_t(\gamma_{t-1}) < 1$$

$$\frac{\Psi}{1+\Psi} \quad \text{otherwise} \quad (34)$$

$$G^i(\gamma_{t-1}, k_t) = \begin{cases} 1 & \text{if } k_t \in [F(\bar{\tau}), F(\tau_i)] \\ \text{Min}[\gamma^*, -\frac{n}{m}] & \text{otherwise} \end{cases} \quad \text{if } u_t(\gamma_{t-1}) < 1$$

$$1 \quad \text{otherwise} \quad (35)$$

$$S^i(\pi_t, k_t) = \begin{cases} S^i(\pi_t, k_t, \tau_{t+1} = \tau_i(k_{t+1})) & \text{if } k_t \in [F(\bar{\tau}), F(\tau_i)] \\ S^i(\pi_t, k_t, \tau_{t+1} = \frac{\Psi}{1+\Psi}) & \text{otherwise} \end{cases} \quad \text{if } u_t(\gamma_{t-1}) < 1$$

$$\begin{cases} S^i(\pi_t, k_t, \tau_{t+1} = \tau_i(k_{t+1})) & \text{if } k_t \in [g_i(F(\tau_i)), g_i(F(\bar{\tau}))] \\ S^i(\pi_t, k_t, \tau_{t+1} = 0) & \text{otherwise} \end{cases} \quad \text{if } \begin{matrix} m+n > 0 \\ \cap \\ n < 0 \end{matrix} \quad (36)$$

$$\begin{cases} S^i(\pi_t, k_t, \tau_{t+1} = \frac{\Psi}{1+\Psi}) & \text{if } m+n < 0 \\ S^i(\pi_t, k_t, \tau_{t+1} = \tau_i(k_{t+1})) & \text{if } k_t \in [F(\bar{\tau}), F(\tau_i)] \\ S^i(\pi_t, k_t, \tau_{t+1} = 0) & \text{otherwise} \end{cases} \quad \text{if } n > 0$$

where $x = 1 + \frac{(1+\Psi)\alpha\beta}{\Psi+\alpha}$, $\bar{\tau} = \frac{\Psi(1+\beta)+\alpha}{\Psi(1+\beta)+\alpha+\beta}$ and $g_i(F)$ and $F(\tau)$ are functions given in the appendix. The two equilibrium, $i = 1, 2$, differ essentially in the implicit functions of the tax rate denoted by $\tau_i(k_t)$, which are defined in the appendix. In the first type of equilibrium, $\tau_1(k_t)$ is a decreasing function in k_t , while in the second type $\tau_2(k_t)$ is an increasing function in k_t . The equilibrium paths, depends on the native-born and immigrant population growth rates and on the initial capital the economy is endowed with: 1. if $n > 0$ and $k_t \in [F(\bar{\tau}), F(\tau_i)]$, there is a positive tax rate which depends on the capital per (native-born) worker state variable and full openness to immigration. If $k_t \notin [F(\bar{\tau}), F(\tau_i)]$, there are at least few periods in which there is no taxation and some restriction on immigration. 2. if $m+n < 0$, there is fix and positive tax rate (the "Laffer point" tax rate), and a full openness to immigration. 3. if $n < 0$ and $m+n > 0$, there is a range of k_t , for which there is a positive tax rate which depends on the capital per (native-born) worker state variable and full openness to immigration. If k_t is not in this range, there are at least few periods in which there

is a "demographic switching" equilibrium path, where some level of immigration always prevails and there is an alternate taxation policy: in periods where the decisive voter is old, the economy is fully opened to immigration and there is a positive level of taxation (the "Laffer point" tax rate); and in periods where the decisive voter is young, there is no taxation and there are some restrictions on immigration.

The proposition is proved in the appendix.

The additional equilibrium paths in the extended model are characterized by different optimal strategy of the young depending on the values of the capital per (native-born) worker : for a range of values of the capital per (native-born) worker state variable the decision rules of the young decisive voter do not change the next period decisive voter's identity, while for other values of the capital per native-born work force the "demographic switching" strategy is still optimal. When the "demographic steady" strategy is optimal, the tax rate is positive and depends on the amount of capital per (native-born) worker and there are no restrictions on immigration. The reason for the additional strategy result from the additional state variable- the capital per (native-born) worker, which influences next period policy variables. The decisive young voter do not have to engage in a strategy of influencing next policy variables only by changing next period decisive voter's identity through immigration quotas (meaning by admitting a limited amount of immigrants), as both current policy variables, the tax and the migration quota, both influence the amount of capital per (native-born) worker. This, in turn, influences next period policy variables.

The difference between the two "combined strategy " equilibrium tax rates, as a function of capital per (native-born) worker, is due to conflicting forces of the effect of the next period tax rate on next period capital per native-born workers. On one hand, a higher tax rate in the next period raises future social security benefits. Larger benefits, tend to reduce current savings. This would cause a reduction in next period capital per native-born work force ("Effect One"). On the other hand, a higher next period tax rate tend to decrease the amount of hours worked next period which lowers the next period interest rate and social security benefits. The consequent fall in the current young future income induces more savings. This tends to increase next period capital per (native-born) worker ("Effect Two").

In the "combined strategy " equilibrium, where the tax rate is decreasing in the amount of capital per (native-born) worker, "Effect One" is stronger

than "Effect Two"³. There are no restriction on immigration since larger immigration quotas has an additional positive effect on the indirect utility of the young. When "Effect One" is stronger than "Effect Two", larger immigration quotas increases next period tax rate (since it decreases next period capital per (native-born) worker⁴), which raises next period future social security benefits. This additional positive effect of immigration quotas on the indirect utility of the young raises the preferable immigration quotas leading to no restriction on immigration.

In the second equilibrium, where the tax rate is increasing in the amount of capital per (native-born) worker, "Effect One" is weaker than "Effect Two". In that case also, there are no restriction on immigration since larger immigration quotas has an additional positive effect on the indirect utility of the young, but from another reason. When "Effect One" is weaker than "Effect Two", larger immigration quotas decreases next period tax rate (since it decreases next period capital per (native-born) worker) which increase the amount of hours worked next period and raises the young future income. This additional positive effect of immigration quotas on the indirect utility of the young raises the preferable immigration quotas leading to no restriction on immigration.

Equilibrium paths depend on the population growth rates and the amount of capital per (native-born) worker:

1. The population growth rates of the native-born and immigrant populations are positive, $n, m > 0$. In this case, the number of next period young voters exceeds the number of next period old voters, which means that the decisive voter is always young. Therefore, if the capital per (native-born) worker is in the range $[F(\bar{\tau}), F(\tau_i)]$, than the optimal strategy of the young is always to vote for no restrictions on immigration and a positive tax rate which depends on the capital per (native-born) worker. If the initial capital is not in the range $[F(\bar{\tau}), F(\tau_i)]$, zero tax rate and a positive immigration quotas, below the ceiling, are chosen by the young. Capital evolves in a way that it is possible to have a period where the amount of capital per (native-born) worker enters the range $[F(\bar{\tau}), F(\tau_i)]$; if it does, from then on the current young again vote for no restrictions on immigration and a tax rate which depends on the capital per (native-born) worker.

³The equilibrium property of the tax function in the first equilibrium, is already noted by Forni (2005).

⁴A larger immigration quotas increases the number of young who save leading to higher aggregate capital accumulation in the next period. But since we assumed that $m > n$, immigration quotas increases even more the number of next period young. Thus the amount of next period capital per native-born work force decreases.

2. If the sum of the population growth rates is negative, then the number of old voters always exceeds the number of young voters. This means that the decisive voter is always young. In that case the old sets the tax rate at the "Laffer point", and no restrictions on immigration.

3. If the sum of the population growth rates is positive, but the native-born population growth rate is negative, there are two possible equilibrium path types. If the capital per (native-born) worker is in the relevant range⁵, the optimal strategy of the young is to set a tax rate that depends on the capital per (native-born) worker, and to set no restrictions on immigration. If the capital per (native-born) worker is outside this range, there is a "cycling" equilibrium path where some level of immigration always prevails and there is an alternate taxation policy. Capital evolves over time in a way that there could be a period where the capital per (native-born) worker enters the relevant range; Once in this range, the optimal strategy of the young is to set a tax rate that depends on the capital per (native-born) worker, and to set no restrictions on immigration.

5 The Effect of Aging

We are now in position to conduct a comparative dynamics across demographic regimes. We analyze the effect of aging of the native and the immigrant populations on the size of the social security system and on immigration restrictions. Aging of the population is specified by a reduction in the population growth rate of the population (life expectancy is assumed to be exogenously fixed).

Proposition 6 1. *Aging of both populations can move the system to an equilibrium with a certain level of taxation/social security benefits (the "Laffer point" tax rate) and no restrictions on immigration.*

2. *The aging of the native-born (immigrants) population enlarge (reduce) immigration quotas set by the young in the "cycling" equilibrium paths.*

3. *In the "combined strategy" equilibrium types, aging can move the system from a "demographic switching" equilibrium path to an equilibrium path where the tax rate is positive and depends on the capital per (native-born) worker, $\tau_t = \tau(k_t)$, whereas the immigration quotas is at the ceiling level, $\gamma_t = 1$, or vice versa.*

⁵The relevant range for k_t is as follows: If the decisive voter is young, and $k_t \in [F(\bar{\tau}), F(\tau_i)]$, the optimal decision rules are $\pi_t = (\tau_i(k_t), 1)$, from that period and on. If the decisive voter is old and $k_t \in [g_i(F(\tau_i)), g_i(F(\bar{\tau}))]$, then next period decisive voter is young and $k_{t+1} \in [F(\bar{\tau}), F(\tau_i)]$. Thus, in that case also, the optimal decision rules are $\pi_t = (\tau(k), 1)$ from that period and on.

The intuition of the result is as follows. Both the aging of native-born and immigrants populations, can move the system to an equilibrium where the sum of the population growth rates are negative, $n + m < 0$. In this case, the old are in the majority every period. The old set the tax rate at the "Laffer point", and liberalize immigration policy as much as possible.

Aging of the native-born (immigrants) population enlarge (reduce) the immigration quotas, $\gamma_t = \text{Min}[\gamma^*, -\frac{n}{m}]$, in the "cycling" equilibrium path when there is a majority of the young. The effect of aging on the immigration policy results from the fact that aging changes the ratio of young to old individuals in the next period⁶. Larger immigration quotas increases more the number of next period young (some of these are offspring of the current immigrants) than it increases the number of current young which are next period old, since the immigrants population growth rate is higher. Thus, aging of the native-born which decreases the number of next period native-born descendant, will enlarge the preferable immigration quotas. The aging of immigrants will have the opposite result. Larger immigration quotas will increase less the ratio of next period young to old, since it increases less the number of offspring of current immigrants the lower is m . Thus, aging of the immigrant population will reduce the preferable immigration quotas.

Aging affect the capital per (native-born) worker, and thus can move the system from a cycling equilibrium path, where $k_t \notin [F(\bar{\tau}), F(\tau_i)]$, to another equilibrium path where the tax rate is positive and depends on the capital per (native-born) worker, $\tau_t = \tau(k_t)$, and the immigration quotas is maximal $\gamma_t = 1$, or vice versa.

6 Conclusion

In the political debate people express the idea that immigrants are good because they can help pay for the old. We flash out the political economy mechanism whereby immigration strengthens the social security system in the presence of ageing. That is, the older are the native-born and immigrant populations the more likely is that the immigration policy is liberalized and that the social security system will survive. But when there exists some restriction on immi-

⁶In the case where $\gamma_t = -\frac{n}{m}$, the ratio effect the openness rate through the number of young to old voters next period, and in the case where $\gamma_t = \gamma^*$, the ratio effect the openness rate through its capital per native born in the next period (more old individual in the next period, increases the total amount of capital, but more young in the next period, decreases the per capital per native born work force).

gration, aging of the native-born population enlarges immigration quotas, while the aging of immigrants reduces them.

For this purpose we develop an OLG political economy model of social security and migration to explore how immigration policy and a pay-as-you-go (PAYG) social security system are jointly determined. The social security system is a pay-as-you-go, which employs payroll taxes on the working young in order to finance a social-security benefit to the aged. Immigrants enter the economy when young, and gain the right to vote only in the next period, when old. Except from having a higher population growth rate, they have the same preferences and contribute to and benefit from the welfare state in the same way as the native-born. Their offspring are assumed to be completely integrated into the country and have the same population growth as the native-born.

The model is a political economy model where the political decisions regarding labor taxation and immigration quotas are taken simultaneously, through majority voting. Markov sub-game perfect political equilibria of the game feature a dynamic of repeated voting where individuals have a forward looking property, in the sense that they take into account the effect of their current voting on the next period voting decisions. The Markov sub-game perfect types of equilibria depends on the state variables included in the model. When the immigration quotas is the only state variable, voters engage in a "demographic switching" strategy in the sense that under the assumption that immigrants gain the right to vote only in the next period when they are old, voters take into account the effect of admitting a certain number of immigrants on the composition of voters and their voting preferences in the next period. Moreover, when the number of young exceed the number of old in the population, the young decisive voter admits a limited number of immigrants, in order to change the decisive voter's identity from young to old in the next period and maximize her next period benefits. When there is an additional state variable- the stock of capital per (native-born) worker, there is another channel of influence on next period policy variables. Thus, there can be another possible strategy of the young, a "demographic steady" strategy, where she chooses to admit the maximum amount of immigrant, and in so doing she renders a majority of young every period. In this case, both "demographic switching" and "demographic steady" strategies are incorporated creating other possible equilibria.

An interesting extension could be to introduce heterogeneity within the native-born and the immigrants population in terms of labor productivity. This would bring into the current model intra-generational distribution aspects. In addition it would also creates other possible types of representative voters i.e

old native-born/ immigrants and young native-born/ immigrants voters, which can create interesting interaction and different strategies in the voting process.

7 Appendix

7.1 Proposition I:

Proof. We must show that the vector of policy decision rules, $\Psi = (T, G)$, as defined in the proposition, satisfies the equilibrium

conditions:

1. $\widehat{\Psi}(\gamma_{t-1}) = \arg \max_{\pi_t} V^i(\gamma_{t-1}, \pi_t, \pi_{t+1})$ subject to $\pi_{t+1} = \Psi(\gamma_t)$.
2. $\widehat{\Psi}(\gamma_{t-1}) = \Psi(\gamma_{t-1})$.

If $u_t \geq 1$ than the decisive voter is old. Substituting for l_t from equation (8) into (12), the utility of the old can be rewritten as:

$$V^o(\gamma_{t-1}) = \frac{\tau_t(1 - \tau_t)^{\frac{1}{\Psi}} [(1 + n) + \gamma_{t-1}(1 + m)](1 + \gamma_t)}{(1 + \gamma_{t-1})} \quad (37)$$

It is straightforward to show that $V^o(\gamma_{t-1})$ is maximized by setting $\gamma_t = 1$ and $\tau_t = \frac{\Psi}{\Psi+1}$.

If $u_t < 1$ than the decisive voter is young. From equation (9), the utility of the young voter subject to $\pi_{t+1} = \Psi(\gamma_t)$ is given by:

$$V^y(\gamma_{t-1}) = \begin{cases} \text{Log}\left[\frac{\Psi}{\Psi+1}(1 - \tau_t)^{\frac{\Psi+1}{\Psi}}\right] & \text{if } u_{t+1} < 1 \\ \text{Log}\left[\frac{\Psi}{\Psi+1}(1 - \tau_t)^{\frac{\Psi+1}{\Psi}}\right] + \beta \text{Log}\left[\frac{2\frac{\Psi}{\Psi+1}(1 - \frac{\Psi}{\Psi+1})^{\frac{1}{\Psi}} [1+n+\gamma_t(1+m)]}{(1+\gamma_t)}\right] & \text{otherwise} \end{cases} \quad (38)$$

In that case $V^y(\gamma_{t-1})$ is maximized by setting $\gamma_t = -\frac{n}{m}$ and $\tau_t = 0$. It should be noted that in the case where the population growth

rates are positive, than for every immigration quota there is a majority of young in every period, and thus the young decisive

voter in every period will indifferent between all possible immigration quotas.

■

7.2 Proposition II:

Proof. As in the previous proposition, we must show that the vector of policy decision rules, $\Psi = (T, G)$, as defined in the proposition,

satisfies the equilibrium conditions:

1. $\widehat{\Psi}(\gamma_{t-1}) = \arg \max_{\pi_t} V^i(\gamma_{t-1}, \pi_t, \pi_{t+1})$ subject to $\pi_{t+1} = \Psi(\gamma_t)$.
2. $\widehat{\Psi}(\gamma_{t-1}) = \Psi(\gamma_{t-1})$.

$$3. S(\pi_t, k_t) = \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{(1+\gamma_t)w_t l_t (1-\tau_t)(1-f(\tau_{t+1}))}{1+n+\gamma_t(1+m)}, \text{ with } \tau_{t+1} = T(\gamma_t).$$

Consider first the case where there is a majority of old in period t , i.e. $u_t \geq 1$.

Using the fact that,

$$w_t l_t = \left((1-\alpha)k_t^\alpha (1+\gamma_t)^{-\alpha} \right)^{\frac{1+\Psi}{\Psi+\alpha}} (1-\tau_t)^{\frac{1-\alpha}{\Psi+\alpha}} \quad (39)$$

$$1+r_t = \alpha \left((1-\alpha)k_t^{-\Psi} (1+\gamma_t)^\Psi (1-\tau_t) \right)^{\frac{1-\alpha}{\Psi+\alpha}} \quad (40)$$

the utility of the old voter can be rewritten as:

$$V^o(\gamma_{t-1}, k_t) = \frac{\tau_t ((1-\alpha)(1+\gamma_t)^{-\alpha} k_t^\alpha)^{\frac{1+\Psi}{\Psi+\alpha}} (1-\tau_t)^{\frac{1-\alpha}{\Psi+\alpha}} [(1+n)+\gamma_{t-1}(1+m)](1+\gamma_t)}{(1+\gamma_{t-1})} \quad (41)$$

$$+\alpha \left((1-\alpha)k_t^{-\Psi} (1+\gamma_t)^\Psi (1-\tau_t) \right)^{\frac{1-\alpha}{\Psi+\alpha}} k_t \left(\frac{1+n+\gamma_{t-1}(1+m)}{(1+\gamma_{t-1})} \right)$$

It can be proved that $V^o(\gamma_{t-1}, k_t)$ is maximized by setting $\gamma_t = 1$ and $\tau_t = \frac{\Psi}{\Psi+1}$.

Consider next the case where there is a majority of young in period t , i.e. $u_t < 1$. Substituting for $w_t l_t (1-\tau_t)$ and $1+r_{t+1}$ from

equations (39) and (40), the utility of the young voter subject to $\pi_{t+1} = \Psi(\gamma_t)$, can be written in the Lagrangian form, in the following way:

$$L(\gamma_t, \tau_t, k_t) = \begin{cases} L(k_t) \text{ with } \tau_{t+1} = 0, \text{ and } \gamma_{t+1} = \text{Min}[\gamma^*, -\frac{n}{m}] & \text{if } u_{t+1} < 1 \\ L(k_t) \text{ with } \tau_{t+1} = \frac{\Psi}{\Psi+1}, \text{ and } \gamma_{t+1} = 1 & \text{otherwise} \end{cases} \quad (42)$$

where $A = (1+\beta) \text{Log} \left(\frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} (1-\alpha) \right) + \beta \text{Log} \left(\alpha \left((1-\alpha) \right)^{\frac{1-\alpha}{\Psi+\alpha}} \right)$, λ_1 is the Lagrangian multiplier, and L is defined as

follows:

$$L = \begin{cases} A + (1+\beta) \text{Log} \left[\left(k_t^\alpha (1+\gamma_t)^{-\alpha} (1-\tau_t) \right)^{\frac{1+\Psi}{\Psi+\alpha}} (1+\beta f(\tau_{t+1})) \right. \\ \quad \left. + \beta \text{Log} \left(k_{t+1}^{-\Psi} (1+\gamma_{t+1})^\Psi (1-\tau_{t+1}) \right)^{\frac{1-\alpha}{\Psi+\alpha}} \right] \\ - \lambda_1 \left(k_{t+1} - \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{(1+\gamma_t) \left((1-\alpha) k_t^\alpha (1+\gamma_t)^{-\alpha} (1-\tau_t) \right)^{\frac{1+\Psi}{\Psi+\alpha}} (1-f(\tau_{t+1}))}{1+n+\gamma_t(1+m)} \right) \end{cases} \quad (43)$$

As a first step it is easy to prove that the indirect utility of the young subject to constant next period policy variables, is

maximized by setting $\gamma_t = \gamma^*$ and $\tau_t = 0$, where $\gamma^* \in [0, 1]$ and is defined as follows:

$$\gamma^* = \frac{\beta(1-\alpha)\Psi(n-m) + \alpha(1+\Psi)(1+n)x}{-\alpha(1+\Psi)(1+m)x} \quad (44)$$

We will prove that $V^y(\gamma_{t-1}, k_t)$ is maximized by $\gamma_t = \text{Min}[\gamma^*, -\frac{n}{m}]$ ⁷.

⁷If the population growth rates are both positive, than it is straightforward to see that $V^y(\gamma_{t-1}, k_t)$ is maximized by $\gamma_t = \text{Max}[\gamma^*, -\frac{n}{m}]$, $\tau_t = 0$.

If $\gamma^* \leq -\frac{n}{m}$ than it is sufficient to prove that the indirect utility of the young is higher by setting $\gamma_{t+1} = 1$, $\tau_{t+1} = \frac{\Psi}{\Psi+1}$ than by setting

$\gamma_{t+1} = \gamma^*$, $\tau_{t+1} = 0$. It is easy to see that the higher is next period immigration quotas the higher is the indirect utility of the young

as it increases next period interest rate. Regarding next period tax rate, it is sufficient to prove that:

$$0 = \text{Log}[(1 + \beta f(0))^{1+\beta}] \leq \text{Log}\left[\left(1 + \beta f\left(\frac{\Psi}{\Psi+1}\right)\right)^{1+\beta} \left(1 - \frac{\Psi}{\Psi+1}\right)^{\beta \frac{1-\alpha}{\Psi+\alpha}}\right] \quad (45)$$

since,

$$0 = \text{Log}[(1 - f(0))^{-\Psi \beta \frac{1-\alpha}{\Psi+\alpha}}] \leq \left(\text{Log}\left[\left(1 - f\left(\frac{\Psi}{\Psi+1}\right)\right)^{-\Psi \beta \frac{1-\alpha}{\Psi+\alpha}}\right]\right) \quad (46)$$

Define the function: $D(\Psi) = \text{Log}\left[\left(1 + \beta f\left(\frac{\Psi}{\Psi+1}\right)\right)^{1+\beta} \left(1 - \frac{\Psi}{\Psi+1}\right)^{\beta \frac{1-\alpha}{\Psi+\alpha}}\right]$. The derivative of $D(\Psi)$ is the following expression:

$$\left(\frac{1}{\Psi+1}\right)^{\frac{\Psi+\alpha+\beta-\alpha\beta}{\Psi+\alpha}} \left(\frac{(\alpha-1)\beta(1+\beta)^2 \left(\frac{(\Psi+\alpha)(1+\beta)}{\Psi+\alpha+\beta+\alpha\beta\Psi}\right)^{\beta-1}}{\frac{(\Psi(1-\alpha)(\Psi+\alpha)+(\Psi+1)(\Psi+\alpha+\beta+\alpha\beta+\alpha\beta\Psi)\text{Log}\left(\frac{1}{\Psi+1}\right))}{(\Psi+\alpha+\beta+\alpha\beta\Psi)^2}}\right) \quad (47)$$

Since the derivative of $D(\Psi)$ is positive for every $\Psi > 0$, and $D(\Psi = 0) = 0$, than $D(\Psi)$ is positive for every $\Psi > 0$.

Otherwise, if $\gamma^* > -\frac{n}{m}$, we must prove that the following holds,

$$\begin{aligned} & \text{Log}\left((1 + \gamma^*)^{-\alpha}\right)^{\frac{1+\Psi}{\Psi+\alpha}(1+\beta)} + \text{Log}\left(\left(\frac{(1+\gamma^*)((1+\gamma^*)^{-\alpha})^{\frac{1+\Psi}{\Psi+\alpha}}}{1+n+\gamma^*(1+m)}\right)^{-\Psi} \left(1 - \frac{n}{m}\right)^{\Psi}\right)^{\beta \frac{1-\alpha}{\Psi+\alpha}} \\ & \leq (1 + \beta) \text{Log}\left[\left(1 - \frac{n}{m}\right)^{-\alpha \frac{1+\Psi}{\Psi+\alpha}} \left(1 + \beta f\left(\frac{\Psi}{\Psi+1}\right)\right)\right] + \\ & \text{Log}\left(\left(\frac{(1-\frac{n}{m})\left((1-\frac{n}{m})^{-\alpha}\right)^{\frac{1+\Psi}{\Psi+\alpha}} \left(1 - f\left(\frac{\Psi}{\Psi+1}\right)\right)}{1+n-\frac{n}{m}(1+m)}\right)^{-\Psi} 2^{\Psi} \left(1 - \frac{\Psi}{\Psi+1}\right)\right)^{\beta \frac{1-\alpha}{\Psi+\alpha}} \end{aligned} \quad (48)$$

As $D(\Psi)$ is positive for every $\Psi > 0$, and for $\gamma^* > -\frac{n}{m}$ the following holds,

$$(1 + \beta) \text{Log}\left[\left(\frac{1 + \gamma^*}{1 - \frac{n}{m}}\right)^{-\alpha}\right]^{\frac{1+\Psi}{\Psi+\alpha}} + \beta \text{Log}\left(\left(\frac{1 + \gamma^*}{1 - \frac{n}{m}}\right)^{-\Psi(1-\alpha) \frac{1+\Psi}{\Psi+\alpha}} \left(\frac{1 - \frac{n}{m}}{2}\right)^{\Psi}\right)^{\frac{1-\alpha}{\Psi+\alpha}} \leq 0 \quad (49)$$

it is sufficient to prove that,

$$\beta \text{Log}\left(\frac{1 + n - \frac{n}{m}(1+m)}{1 + n + \gamma^*(1+m)}\right)^{-\Psi \frac{1-\alpha}{\Psi+\alpha}} \leq \beta \text{Log}\left(1 - f\left(\frac{\Psi}{\Psi+1}\right)\right)^{-\Psi \frac{1-\alpha}{\Psi+\alpha}} \quad (50)$$

Substituting γ^* from equation (44) into equation (49), we can rewrite the inequality in the following way,

$$\left(1 + \frac{1-\alpha}{\alpha} \frac{1}{1+\beta} \frac{\Psi}{\Psi+1}\right) \geq \frac{\beta(1-\alpha)\Psi}{\alpha(1+\Psi)x\frac{1}{m}} \quad (51)$$

Since this expression is positive, it completes the proof that $V^y(\gamma_{t-1}, k_t)$ is maximized by setting $\gamma_t = \text{Min}[\gamma^*, -\frac{n}{m}]$ and

$\tau_t = 0$. ■

7.3 Proposition III:

Proof. The proof will consist of two parts. The first part will prove that when there is a majority of young voters the policy decisions for

the tax rate and immigration quotas stated maximizes the young indirect utility function, under the assumption that next period

decisive voter is young. The second part will complete the proof and show that under certain conditions on the models

parameters, the vector of policy decision rules as defined in the proposition, satisfies the equilibrium conditions (for equilibrium $i = 1, 2$).

The first part of the proof:

We follow the proof of Forni (2004) to obtain the policy variables. The policy variables are obtained by using as a constrain the

first derivative with respect to the policy variables of the logarithm of the capital accumulation equation. The policy variables

are the following:

$$\left(1 + \frac{1-\alpha}{\alpha} \tau_t(k_t)\right)^{1+\beta} (1 - \tau_t(k_t))^{\frac{\beta(1-\alpha)}{\Psi+\alpha}} = k_t^{-x} c \quad (52)$$

$$\gamma_t = 1 \quad (53)$$

where $x = 1 + \frac{(1+\Psi)\alpha\beta}{\Psi+\alpha}$, and c is a positive constant of integration. The policy decision rule of the immigration quotas is at its

maximal value, and the policy decision rule of the tax rate is implicitly given in equation (52). Define this implicit function by

$F(\tau) = \left(1 + \frac{1-\alpha}{\alpha} \tau\right)^{1+\beta} (1 - \tau)^{\frac{\beta(1-\alpha)}{\alpha+\Psi} \frac{1}{c}}^{-\frac{1}{x}}$. The function $F(\tau)$ is decreasing in τ for $\tau \in [0, \bar{\tau}]$, where $\bar{\tau} = \frac{\Psi(1+\beta)+\alpha}{\Psi(1+\beta)+\alpha+\beta}$, and

increasing in τ for $\tau \in [\bar{\tau}, 1]$. Thus, according to equation (52), for every value of capital per (native-born) worker, k_t , there are

two solutions for $\tau_t(k_t)$ in the range $[0, 1]$. The first solution, denoted by $\tau_1(k_t)$ which is the policy taxation rule in the first

equilibrium of the proposition $i = 1$, is decreasing in k_t where $k_t \in [F(\bar{\tau}), F(0)]$; while the second solution, denoted by

$\tau_2(k_t) \in [0, \bar{\tau}]$, which is the policy taxation rule in the first equilibrium of the proposition $i = 1$, is increasing in k_t where $k_t \in [F(\bar{\tau}), F(1)]$.

The solution for the policy variables given in equations (52) and (53), will be proved to satisfy the first order conditions of the

problem. Substituting for $w_t l_t (1 - \tau_t)$ and $1 + r_{t+1}$ from equations (39) and (40), the young voter's indirect utility function under

the assumption that next period decisive voter is young which set the next period policy decision rules for the tax rate and

immigration quotas to be $\tau_{t+1} = \tau_{t+1}(k_{t+1})$, and $\gamma_{t+1} = 1$ respectively, can be written in its Lagrangian form as follows:

$$\begin{aligned}
L = & A + (1 + \beta) \text{Log} \left(k_t^\alpha (1 + \gamma_t)^{-\alpha} (1 - \tau_t) \right)^{\frac{1+\Psi}{\Psi+\alpha}} + (1 + \beta) \text{Log} [(1 + \beta f(\tau_{t+1}(k_{t+1}))) \\
& + \beta \text{Log} (k_{t+1}^{-\Psi} 2^\Psi (1 - \tau_{t+1}(k_{t+1})))^{\frac{1-\alpha}{\Psi+\alpha}} \\
& - \lambda_1 (k_{t+1} - \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{(1+\gamma_t)((1-\alpha)k_t^\alpha (1+\gamma_t)^{-\alpha} (1-\tau_t))^{\frac{1+\Psi}{\Psi+\alpha}} (1-f(\tau_{t+1}(k_{t+1})))}{1+n+\gamma_t(1+m)}) \\
& - \lambda_2 (\tau_t - 1) - \lambda_3 (-\tau_t) - \lambda_4 (\gamma_t - 1) - \lambda_5 (\gamma_t)
\end{aligned} \tag{54}$$

The Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial \tau_t} = -\frac{1 + \Psi}{\Psi + \alpha} \frac{1 + \beta}{1 - \tau_t} - \lambda_1 \frac{1 + \Psi}{\Psi + \alpha} \frac{k_{t+1}}{1 - \tau_t} - \lambda_2 + \lambda_3 = 0 \tag{55}$$

$$\frac{\partial L}{\partial \gamma_t} = -\alpha \frac{1 + \Psi}{\Psi + \alpha} \frac{1 + \beta}{1 + \gamma_t} + \lambda_1 \frac{k_{t+1}}{1 + \gamma_t} \left(\frac{n - m}{1 + n + \gamma_t(1 + m)} - \alpha \frac{1 + \Psi}{\Psi + \alpha} \right) - \lambda_4 + \lambda_5 = 0 \tag{56}$$

$$\begin{aligned}
\frac{\partial L}{\partial k_{t+1}} = & \left(\frac{\beta(1 + \beta)}{1 + \beta f(\tau_{t+1}(k_{t+1}))} - \frac{\lambda_1 k_{t+1}}{1 - f(\tau_{t+1}(k_{t+1}))} \right) \frac{\partial f(\tau_{t+1})}{\partial \tau_{t+1}} \frac{\partial \tau_{t+1}(k_{t+1})}{\partial k_{t+1}} \\
& - \frac{\beta(1 - \alpha)}{\Psi + \alpha} \frac{1}{1 - \tau_{t+1}(k_{t+1})} \frac{\partial \tau_{t+1}(k_{t+1})}{\partial k_{t+1}} + \frac{1}{k_{t+1}} \left(-\beta \frac{\Psi(1 - \alpha)}{\Psi + \alpha} \right) - \lambda_1 \tag{57}
\end{aligned}$$

$$k_{t+1} = \frac{\beta}{1 + \beta} \frac{\Psi}{\Psi + 1} \frac{(1 + \gamma_t) w_t l_t (1 - \tau_t) (1 - f(\tau_{t+1}(k_{t+1})))}{1 + n + \gamma_t(1 + m)} \tag{58}$$

$$\tau_t - 1 \leq 0, \lambda_2 \geq 0 \text{ and } \lambda_2 (\tau_t - 1) = 0 \tag{59}$$

$$-\tau_t \leq 0, \lambda_3 \geq 0 \text{ and } \lambda_3 (-\tau_t) = 0 \tag{60}$$

$$\gamma_t - 1 \leq 0, \lambda_4 \geq 0 \text{ and } \lambda_4 (\gamma_t - 1) = 0 \tag{61}$$

$$-\gamma_t \leq 0, \lambda_5 \geq 0 \text{ and } \lambda_5(\gamma_t) = 0 \quad (62)$$

Substituting for λ_1 in equation (56) in equations (54) and (55), they can be rewritten as:

$$\frac{\partial L}{\partial \tau_t} = -\lambda_2 + \lambda_3 = 0 \quad (63)$$

$$\frac{\partial L}{\partial \gamma_t} = \frac{(1+\beta)}{1+\gamma_t} \left(\frac{-n+m}{1+n+\gamma_t(1+m)} \right) - \lambda_4 + \lambda_5 = 0 \quad (64)$$

Since we have assumed that $m > n$ from equation (64) we derive that γ_t has to be a corner solution where $\gamma_t = 1$. The other

constraint regarding τ_t , may be bounding or not, meaning that $\tau_t = \tau_t(k_t) \in [0, 1]$.

The optimal solutions should also satisfy the second order sufficient conditions, meaning that the bordered Hessian of the

Lagrangian should be negatively defined. Since the solution of the immigration quotas is a corner solution where the largest

immigration quotas maximizes the young voter's indirect utility function, the bordered Hessian of the Lagrangian is equal to:

$$-g_\tau \left(g_\tau \frac{\partial^2 L}{\partial^2 k_{t+1}} - g_k \frac{\partial^2 L}{\partial k_{t+1} \partial \tau_t} \right) + g_k \left(g_\tau \frac{\partial^2 L}{\partial \tau_t \partial k_{t+1}} - g_k \frac{\partial^2 L}{\partial^2 \tau_t} \right) \quad (65)$$

where g_τ and g_k are the derivatives of the capital per (native-born) worker constraint, from equation (57), with respect to τ_t

and k_{t+1} respectively. The bordered Hessian can be rewritten in the following way:

$$-(1+\beta)^2 + \frac{2x^2(1 + \frac{1-\alpha}{\alpha}\tau_t)^2(1-\tau_t)^2 \left(\frac{1-\alpha}{\alpha}\right)^2}{\left((1+\beta)\frac{1-\alpha}{\alpha}(1-\tau_t) - \frac{\beta(1-\alpha)}{\Psi+\alpha}(1 + \frac{1-\alpha}{\alpha}\tau_t) \right)^2 \left(1 + \frac{1-\alpha}{\alpha}\frac{1}{1+\beta}\tau_t\right)^2} \quad (66)$$

This expression is positive for values of the tax rate, τ_t , which satisfies the following inequalities:

$$0 \leq -(1+\beta) + \frac{\sqrt{2}x(1 + \frac{1-\alpha}{\alpha}\tau_t)(1-\tau_t) \left(\frac{1-\alpha}{\alpha}\right)}{\left((1+\beta)\frac{1-\alpha}{\alpha}(1-\tau_t) - \frac{\beta(1-\alpha)}{\Psi+\alpha}(1 + \frac{1-\alpha}{\alpha}\tau_t) \right) \left(1 + \frac{1-\alpha}{\alpha}\frac{1}{1+\beta}\tau_t\right)} \quad (67)$$

$$0 \geq (1+\beta) + \frac{\sqrt{2}x(1 + \frac{1-\alpha}{\alpha}\tau_t)(1-\tau_t) \left(\frac{1-\alpha}{\alpha}\right)}{\left((1+\beta)\frac{1-\alpha}{\alpha}(1-\tau_t) - \frac{\beta(1-\alpha)}{\Psi+\alpha}(1 + \frac{1-\alpha}{\alpha}\tau_t) \right) \left(1 + \frac{1-\alpha}{\alpha}\frac{1}{1+\beta}\tau_t\right)} \quad (68)$$

Thus, $\tau_t \in [\tau_1, \tau_2]$ is the range for which the bordered Hessian of the Lagrangian is negatively defined, where τ_1 and τ_2 denotes

the solutions of these equations respectively⁸. The solution of the tax rate in the first equilibrium, $\tau_1(k_t)$, is optimal in the range $k_t \in [F(\bar{\tau}), F(\tau_1)]$, and the solution of the tax rate in the second equilibrium, $\tau_2(k_t)$, is optimal in the range $k_t \in [F(\bar{\tau}), F(\tau_2)]$.

The second part of the proof:

As in the previous proposition, we must show that the vector of policy decision rules, $\Psi = (T, G)$, as defined in the proposition, satisfies the equilibrium conditions:

1. $\hat{\Psi}(\gamma_{t-1}, k_t) = \arg \max_{\pi_t} V^i(\gamma_{t-1}, k_t, \pi_t, \pi_{t+1})$ subject to $\pi_{t+1} = \Psi(\gamma_t, k_t)$.
2. $\hat{\Psi}(\gamma_{t-1}, k_t) = \Psi(\gamma_{t-1}, k_t)$.
3. $S(\pi_t, k_t) = \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{(1+\gamma_t)w_t l_t (1-\tau_t)(1-f(\tau_{t+1}))}{1+n+\gamma_t(1+m)}$, with $\tau_{t+1} = T(\gamma_t, k_t)$.

Proposition III refers to the both equilibrium $i = 1, 2$. The first equilibrium $i = 1$, is the case where $F(\tau)$ is decreasing in τ for

$\tau \in [\tau_1, \bar{\tau}]$. Thus, the solution in this case which is denoted by $\tau_1(k_t)$, is decreasing in $k_t \in [F(\bar{\tau}), F(\tau_1)]$. The second equilibrium

$i = 2$, is the case where $F(\tau)$ is increasing in τ for $\tau \in [\tau_1, \bar{\tau}]$. Thus, the solution in this case which is denoted by $\tau_2(k_t)$, is decreasing in $k_t \in [F(\bar{\tau}), F(\tau_2)]$.

Consider first the case where there is a majority of old in period t , i.e. $u_t \geq 1$. The utility of the old voter is the same as in the

previous proposition and thus $V^o(\gamma_{t-1}, k_t)$ is maximized by setting $\gamma_t = 1$ and $\tau_t = \frac{\Psi}{\Psi+1}$. But unlike the previous proposition the saving of the young in period t also set next period policy variables.

In the case of a majority of old, the policy decision rules are set by $\gamma_t = 1$ and $\tau_t = \frac{\Psi}{\Psi+1}$. In order for the next period policy

decision rules to be set according to the decision rules: $\gamma_{t+1} = 1$ and $\tau_{t+1} = \tau_i(k_{t+1})$, the next period capital per (native-born)

worker, k_{t+1} , should be in the range: $[F(\bar{\tau}), F(\tau_i)]$ where k_{t+1} is defined by the following equation:

$$k_{t+1} = \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{2}{2+n+m} \left((1-\alpha)k_t^\alpha 2^{-\alpha} \left(1 - \frac{\Psi}{\Psi+1}\right) \right)^{\frac{1+\Psi}{\Psi+\alpha}} (1 - f(\tau_i(k_{t+1}))) \quad (69)$$

The derivative of k_{t+1} by k_t when the policy decision rules are set by $\gamma_t = 1$ and $\tau_t = \frac{\Psi}{\Psi+1}$ and next period tax rate is set

⁸Since these equations are second order polynomials, $[\tau^1, \bar{\tau}]$ and $[\bar{\tau}, \tau^2]$, τ^1 is the minimal solution according to the first polynomial equation and τ^2 is the maximal solution according to the second polynomial equation.

according to $\tau_i(k_{t+1})$, is negative in the first case for $\tau \in [\tau_1, \bar{\tau}]$ and positive in the second for $\tau \in [\bar{\tau}, \tau_2]$. Thus, the range of k_t

for which $k_{t+1} \in [F(\bar{\tau}), F(\tau_i)]$ is $k_t \in [g_1(F(\tau_1)), g_1(F(\bar{\tau}))]$ in the first case and $k_t \in [g_2(F(\bar{\tau}), g_2(F(\tau_2))]$ in the second, where

$g_i(y)$ is defined by⁹:

$$g_i(y) = \left(\left(\frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{2}{2+n+m} (1-f(\tau_i(y))) \left((1-\alpha)2^{-\alpha} \left(1 - \frac{\Psi}{\Psi+1}\right) \right)^{\frac{1+\Psi}{\Psi+\alpha}} \right)^{-1} y \right)^{\frac{\Psi+\alpha}{\alpha(1+\Psi)}} \quad (71)$$

Otherwise, if $k_t \notin [g_1(F(\tau_1)), g_1(F(\bar{\tau}))]$ in the first case or $k_t \notin [g_2(F(\bar{\tau}), g_2(F(\tau_2))]$ in the second, then in order to obtain the

condition $k_{t+1} \notin [F(\bar{\tau}), F(\tau_1)]$, also for k_{t+1} as defined by the following equation:

$$k_{t+1} = \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{2}{2+n+m} \left((1-\alpha)k_t^\alpha 2^{-\alpha} \left(1 - \frac{\Psi}{\Psi+1}\right) \right)^{\frac{1+\Psi}{\Psi+\alpha}} (1-f(0)) \quad (72)$$

we have to require for the first equilibrium that if $k_t > g_1(F(\tau_1)) \implies k_{t+1} > F(\bar{\tau})$ and if $k_t < g_1(F(\bar{\tau})) \implies k_{t+1} < F(\tau_1)$, while for

the other equilibrium that if $k_t > g_2(F(\bar{\tau})) \implies k_{t+1} > F(\bar{\tau})$ and if $k_t < g_2(F(\tau_2)) \implies k_{t+1} < F(\tau_2)$.

Consider next the case where there is a majority of young in period t , i.e. $u_t < 1$.

If $k_t \in [F(\bar{\tau}), F(\tau_i)]$, we must prove that the indirect utility of the young voter subject to $\pi_{t+1} = \Psi(\gamma_t, k_t)$, is maximized by

$\gamma_t = 1$ and $\tau_t = \tau_i(k_t)$. Substituting for $w_t l_t (1 - \tau_t)$ and $1 + r_{t+1}$ from equations (39) and (40), the young voter's indirect utility

function subject to $\pi_{t+1} = \Psi(\gamma_t, k_t)$, can be written in its Lagrangian form as follows:

$$L^i(\pi_t, k_t, \gamma_{t-1}) = \begin{cases} L^i(k_t) \text{ with } \pi_{t+1} = \Psi(\tau_i(k_{t+1}), 1) & \text{if } k_{t+1} \in [F(\bar{\tau}), F(\tau_i)] \\ L^i(k_t) \text{ with } \pi_{t+1} = \Psi(0, \text{Min}[\gamma^*, -\frac{n}{m}]) & \text{otherwise} \\ L^i(k_t) \text{ with } \pi_{t+1} = \Psi(\frac{\Psi}{\Psi+1}, 1) & \text{otherwise} \end{cases} \quad \text{if } u_{t+1} < 1 \quad (73)$$

⁹If the sum of the fertility rates are negative, $n+m < 0$, than the identity of the decisive voter is always old, which means that next period capital per native born labor force always equal:

$$k_{t+1} = \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{2}{2+n+m} \left((1-\alpha)k_t^\alpha 2^{-\alpha} \left(1 - \frac{\Psi}{\Psi+1}\right) \right)^{\frac{1+\Psi}{\Psi+\alpha}} \left(1 - f\left(\frac{\Psi}{\Psi+1}\right)\right) \quad (70)$$

where L^i is as defined in equation (54) after substituting for $\tau_{t+1} = \tau_i(k_{t+1})$. The previous part of the proposition, proved that if next period decision rules are set by $\tau_{t+1} = \tau_i(k_{t+1})$, and $\gamma_{t+1} = 1$, the optimal solution for the young is to set $\tau_t = \tau_i(k_t)$, and $\gamma_t = 1$.

In addition, we have shown that under the assumption that next period policy decision rule are given according to equations

(31) and (32), the young voter's indirect utility function is maximized by $\gamma_t = \text{Min}[\gamma^*, -\frac{n}{m}]$ and $\tau_t = 0$. Therefore we must show

that if $k_t \in [F(\bar{\tau}), F(\tau_i)]$, the value of the young voter's indirect utility function is higher under the following decision rules: $\tau_t = \tau_i(k_t)$,

and $\gamma_t = 1$. Since the value of the young voter's indirect utility function under the first decision rules is constant in k_t , and the

value of the young voter's indirect utility function under the second decision rules is increasing in k_t , the first condition on the

parameters of the model is to require that both values of the indirect utility of the young should equate at $k_t = F(\tau_i)$:¹⁰

$$\begin{aligned} & \text{Log} \left(\left(\frac{2}{2+n+m} \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \right)^{-(1+\beta)} (2^\Psi (1-\alpha))^{\frac{\beta(1-\alpha)}{\Psi+\alpha}} \alpha^\beta c \right) = \\ & (1+\beta) \text{Log} \left(\left((1-\alpha) F(\tau_i)^\alpha (1+\gamma_t)^{-\alpha} \right)^{\frac{1+\Psi}{\Psi+\alpha}} (1+\beta f(\frac{\Psi}{\Psi+1})) \right) + \\ & \text{Log} \left(\alpha \left((1-\alpha) \left(1 - \frac{\Psi}{\Psi+1} \right) 2^\Psi \left(\frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{(1+\gamma_t) \left((1-\alpha) F(\tau_i)^\alpha (1+\gamma_t)^{-\alpha} \right)^{\frac{1+\Psi}{\Psi+\alpha}} (1-f(\frac{\Psi}{\Psi+1}))}{1+n+\gamma_t(1+m)} \right)^{-\Psi} \right)^{\frac{1-\alpha}{\Psi+\alpha}} \right)^\beta \end{aligned} \quad (74)$$

In addition, we must require that if $k_t \in [F(\bar{\tau}), F(\tau_i)]$, than also k_{t+1} which is equal to the following expression:

$$k_{t+1} = \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{2}{2+n+m} \left((1-\alpha) k_t^\alpha 2^{-\alpha} (1-\tau_i(k_t)) \right)^{\frac{1+\Psi}{\Psi+\alpha}} (1-f(\tau_i(k_{t+1}))) \quad (75)$$

will be in the relevant range, i.e. $k_{t+1} \in [F(\bar{\tau}), F(\tau_i)]$ ¹¹. Since the deriva-

¹⁰If the fertility rates are both positive than next period policy variables are: $\gamma_t = \text{Max}[\gamma^*, -\frac{n}{m}]$ and $\tau_t = 0$, since next period decisive voter is young. Thus the condition in this case is simply: $\text{Log} \left[\left(\frac{2}{2+n+m} \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \right)^{-(1+\beta)} (2^\Psi (1-\alpha))^{\frac{\beta(1-\alpha)}{\Psi+\alpha}} \alpha^\beta c \right] =$

$$(1+\beta) \text{Log} \left(F(\tau_i)^\alpha (1+\gamma_t)^{-\alpha} \right)^{\frac{1+\Psi}{\Psi+\alpha}} + \beta \text{Log} \left(\left(\frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{(1+\gamma_t) \left((1-\alpha) F(\tau_i)^\alpha (1+\gamma_t)^{-\alpha} (1-\tau_t) \right)^{\frac{1+\Psi}{\Psi+\alpha}}}{1+n+\gamma_t(1+m)} \right)^{-\Psi} (1+\gamma^*)^\Psi \right)^{\frac{1-\alpha}{\Psi+\alpha}}$$

¹¹It should be noted that if $k_{t+1} = \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{(1+\gamma_t)}{1+n+\gamma_t(1+m)} \left((1-\alpha) (1+\gamma_t)^{-\alpha} k_t^\alpha \right)^{\frac{1+\Psi}{\Psi+\alpha}} (1-f(\frac{\Psi}{\Psi+1}))$ is also in the relevant range, under the previous condition, the value of the young voter's indirect utility function is higher under the first decision rules. Thus the optimal solution is $\tau_t = \tau_i(k_t)$, and $\gamma_t = 1$.

tive of k_{t+1} by k_t is negative in both equilibrium

$i = 1, 2$, if $k_t \in [F(\bar{\tau}), F(\tau_i)]$, than $k_{t+1} \in [\underline{k}_i, \bar{k}_i]$, where \underline{k}_i and \bar{k}_i are defined respectively by the following equations:

$$\bar{k}_i = \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{2}{2+n+m} ((1-\alpha)(F(\bar{\tau}))^\alpha 2^{-\alpha})^{\frac{1+\Psi}{\Psi+\alpha}} (1-f(\tau_i(\bar{k}_i))) \quad (76)$$

$$\underline{k}_i = \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{2}{2+n+m} ((1-\alpha)(F(\tau_i))^\alpha 2^{-\alpha}(1-\bar{\tau}))^{\frac{1+\Psi}{\Psi+\alpha}} (1-f(\tau_i(\underline{k}_i))) \quad (77)$$

Therefore the required condition is that $[\underline{k}_i, \bar{k}_i] \subseteq [F(\bar{\tau}), F(\tau_i)]$.

Otherwise, if $k_t \notin [F(\bar{\tau}), F(\tau_i)]$, we must prove that the indirect utility of the young voter subject to $\pi_{t+1} = \Psi(\gamma_t, k_t)$, is

maximized by $\gamma_t = \text{Min}[\gamma^*, -\frac{n}{m}]$ and $\tau_t = 0$.

For $k_t > F(\tau_i)$ according to the first condition the value of the young voter's indirect utility function is for $\gamma_t = \text{Min}[\gamma^*, -\frac{n}{m}]$ and

$\tau_t = 0$. It should be noted that since this optimal solution changes next period decisive voter from young to old for all values of

k_{t+1} , there are no additional conditions on k_{t+1} ¹².

For $k_t < F(\bar{\tau})$ if $k_{t+1} > F(\tau_i)$, where k_{t+1} is defined as in equation (74), then as was proved the young voter's indirect utility

function is maximized by $\gamma_t = \text{Min}[\gamma^*, -\frac{n}{m}]$ and $\tau_t = 0$. In order to derive the condition that $k_{t+1} > F(\tau_i)$ for all $k_t < F(\bar{\tau})$, we must

require that $\bar{k}_i \geq F(\tau_i)$. Combining the previous condition with the latter, we derive the conditions: $\underline{k}_i \geq F(\bar{\tau})$ and $\bar{k}_i = F(\tau_i)$.

An additional condition is necessary in order to insure that $\tau_i(k_t)$ is the only solution in the range $[F(\bar{\tau}), F(\tau_i)]$ ¹³.

The condition requires that for $\tau_j(k_t)$ where $i \neq j$, if $k_t \in [F(\bar{\tau}), F(\tau_i)]$ and next period decision rules are given according to

$\gamma_{t+1} = 1$ and $\tau_{t+1} = \tau_i(k_{t+1})$ than $k_{t+1} \notin [F(\bar{\tau}), F(\tau_i)]$, where k_{t+1} is defined by:

$$k_{t+1} = \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{2}{2+n+m} ((1-\alpha)k_t^\alpha 2^{-\alpha}(1-\tau_j(k_t)))^{\frac{1+\Psi}{\Psi+\alpha}} (1-f(\tau_i(k_{t+1}))) \quad (78)$$

Since $\tau_2(k_t)$ is increasing in k_t , $\tau_1(k_t)$ is decreasing in k_t , the derivative of k_{t+1} by k_t are positive in both cases. Therefore the

¹²In the case where fertility rates are positive than we should also require that if $k_t > F(\tau_i) \Rightarrow k_{t+1} > F(\tau_i)$, where: $k_{t+1} = \frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{2}{2+n+m} ((1-\alpha)F(\tau_i)^\alpha (1+\gamma^*)^{-\alpha} (1-0))^{\frac{1+\Psi}{\Psi+\alpha}} (1-f(0))$

¹³This is due to the fact that as was proved in the first part of the proof, both solution for the tax rate $\tau_1(k_t)$ and $\tau_2(k_t)$ maximizes the young voter's indirect utility function.

required conditions, should be that for $k_t \in [F(\bar{\tau}), F(\tau_i)]$, either $k_{t+1} < F(\bar{\tau})$ or $k_{t+1} > F(\tau_i)$. Defined $\bar{\underline{k}}_i$ and $\underline{\underline{k}}_i$ by the following equations:

$$\frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{2}{2+n+m} \left((1-\alpha)(F(\tau_i))^\alpha 2^{-\alpha} (1-\tau_j(F(\tau_i))) \right)^{\frac{1+\Psi}{\Psi+\alpha}} (1-f(\tau_i(\bar{\underline{k}}_i))) = \bar{\underline{k}}_i \quad (79)$$

$$\frac{\beta}{1+\beta} \frac{\Psi}{\Psi+1} \frac{2}{2+n+m} \left((1-\alpha)(F(\bar{\tau}))^\alpha 2^{-\alpha} (1-\tau_j(F(\bar{\tau}))) \right)^{\frac{1+\Psi}{\Psi+\alpha}} (1-f(\tau_i(\underline{\underline{k}}_i))) = \underline{\underline{k}}_i \quad (80)$$

Namely, the sufficient conditions are either $\underline{\underline{k}}_i < F(\bar{\tau})$ or $\bar{\underline{k}}_i > F(\tau_i)$. ■

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