NBER WORKING PAPER SERIES

ENDOGENOUS SUDDEN STOPS IN A BUSINESS CYCLE MODEL WITH COLLATERAL CONSTRAINTS: A FISHERIAN DEFLATION OF TOBIN'S Q

Enrique G. Mendoza

Working Paper 12564
http://www.nber.org/papers/w12564

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
October 2006

I am grateful to Guillermo Calvo, Dave Cook, Mick Devereux, Gita Gopinath, Tim Kehoe, Nobuhiro Kiyotaki, Narayana Kocherlakota, Juan Pablo Nicolini, Marcelo Oviedo, Helene Rey, Vincenzo Quadrini, Alvaro Riascos, Lars Svensson, Linda Tesar and Martin Uribe for helpful comments. I am also grateful for comments by participants at the 2006 Texas Monetary Conference, 2005 Meeting of the Society for Economic Dynamics, the Fall 2004 IFM Program Meeting of the NBER, and seminars at the ECB, BIS, Bank of Portugal, Cornell, Duke, Georgetown, IDB, IMF, Harvard, Hong Kong Institute for Monetary Research, Hong Kong University of Science and Technology, Johns Hopkins, Michigan, Oregon, Princeton, UCLA, USC, Western Ontario and Yale. The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

Endogenous Sudden Stops in a Business Cycle Model with Collateral Constraints:A Fisherian Deflation of Tobin's Q
Enrique G. Mendoza
NBER Working Paper No. 12564
October 2006
JEL No. D52,E44,F32,F41

ABSTRACT

The current account reversals, large recessions, and price collapses that define Sudden Stops contradict the predictions of a large class of models in which the current account is a vehicle for consumption smoothing and investment financing. This paper shows that the quantitative predictions of a business cycle model with collateral constraints are consistent with the key features of Sudden Stops. Standard shocks to imported input prices, the world interest rate, and productivity trigger collateral constraints on debt and working capital when borrowing levels are high relative to asset values, and these high-leverage states are endogenous outcomes. In these situations, Irving Fisher's debt-deflation mechanism causes Sudden Stops as the deflation of Tobin's Q leads to a spiraling decline in the prices and holdings of collateral assets. This has immediate effects on output and factor demands because collapsing collateral values cut access to working capital. In contrast with previous findings, collateral constraints induce significant amplification in the responses of macroaggregates to shocks. Because of precautionary saving, Sudden Stops are infrequent events nested within normal cycles in the long run, but they remain a positive probability event.

Enrique G. Mendoza
Research Department
International Monetary Fund
700 19th St Nw
Washington Dc 20431
and NBER
mendozae@econ.umd.edu

1. Introduction

The Great Depression showed that market economies can experience deep recessions that differ markedly from typical business cycle downturns. The recessions that hit emerging economies in the aftermath of the financial crises of the last twelve years illustrated the same fact. In contrast with the Great Depression, however, the loss of access to world capital markets played a key role in emerging markets crises. That is, the crises of emerging economies featured a phenomenon referred to as a "Sudden Stop."

Three striking stylized facts characterize Sudden Stops: (1) large reversals in the current account, (2) deep recessions, and (3) collapses in real asset prices and the price of nontradable goods relative to tradables. The recessions were in most cases the largest downturns experienced by the affected countries since the Great Depression, with GDP, consumption and investment falling well below two standard deviations from their corresponding trends (see Calvo, Izquierdo and Loo-Kung (2006), Mendoza (2002)).

Sudden Stops are a puzzle for a large class of macroeconomic models in which the current account is an efficient vehicle for consumption smoothing and investment financing, and countries enjoy uninterrupted access to credit markets. Models in this class include frictionless real business cycle (RBC) models of small open economies (SOE) as well as models with nominal rigidities. In contrast, the recent literature on Sudden Stops emphasizes the role of frictions in world capital markets. Several studies propose models that predict sudden adjustments in production, absorption and the current account as a result of the adverse effects of financial frictions (see, for example, Calvo (1998), Gopinath (2003), Cook and Choi (2003), Cook and Devereux (2006a, 2006b), Martin and Rey (2006), and Paasche (2001)).

Empirical evidence from Sudden Stops suggests that, in addition to the "demand-side" effects of credit frictions widely studied in the Sudden Stops literature, there are important "supply-side" effects. Growth accounting shows that changes in capital and labor account for a small fraction of the *initial* output collapse (see Bergoeing et. al. (2002) and Section 2 of this paper). This could indicate that a large exogenous drop in total factor productivity (TFP) caused the fall in output. Section 2 of this paper shows, however, that large declines in imported inputs and capacity utilization accounted for an important share of the output collapse in Mexico's 1995 Sudden Stop: Of the 8.5 percent decline in output per worker between the third quarter of 94 and the second quarter of 95, 3.1 percentage points were due to the drop in imported inputs and at least 2.5 percentage points were due to reduced capacity utilization.

The stylized facts of Sudden Stops suggest that an equilibrium business cycle framework aiming to account for this phenomenon should have three desirable features: First, it should produce a stochastic stationary equilibrium in which infrequent Sudden Stops are nested together with normal business cycles. Second, in a Sudden Stop episode, typical realizations of the same underlying exogenous shocks that drive business cycles in non-Sudden Stop periods should result in a sharp reversal in the current account, a deep recession, and a fall in asset prices. Third, endogenous declines in imported inputs and capacity utilization should play a key role in a Sudden Stop's initial output collapse. This paper proposes a model with these features, and explores whether it can deliver quantitative predictions consistent with those of actual Sudden Stops. The model introduces financial frictions similar to those studied in the Sudden Stops literature into an RBC-SOE setup with incomplete markets, imported inputs and capacity utilization.

The standard RBC-SOE model cannot produce Sudden Stops even if imported inputs and endogenous utilization are added. Agents in this model still have unrestricted access to a perfect international credit market. Negative shocks to TFP, the world interest rate, or the world price of imported inputs induce the standard consumption-smoothing and investment-reducing effects of the RBC-SOE model. Large TFP shocks could trigger large output collapses driven by cuts in imported inputs and capacity utilization, but this would still fail to explain the current account reversal and the collapse in consumption (since households would borrow from abroad to smooth consumption). Adding large shocks to the world interest rate or access to external financing could alter these results, but such a theory of Sudden Stops would hinge entirely on unexplained "large and unexpected" shocks. The shocks would need to be large, because by definition they need to induce recessions larger than the normal non-Sudden-Stop recessions, and they would need to be unexpected (i.e. outside the set realizations of shocks agents consider possible), because otherwise agents would self-insure to undo their real effects.

Despite the above shortcomings, large, unexpected shocks often drive current account reversals in the Sudden Stops literature. In contrast, the quantitative findings of this paper show that credit constraints can provide an explanation for Sudden Stops that does not hinge on large, unexpected shocks. The model features two collateral constraints: First, a limit on debt not to exceed a fraction of the liquidation value of collateral assets. Second, a constraint that limits working capital financing not to exceed a fraction of the value of the firms' assets. The emphasis is on studying the quantitative significance of these constraints for the business cycle transmission mechanism, along the lines of the recent Macroeconomics literature on this subject, including Kiyotaki and Moore (1997), Bernanke, Gertler and Gilchrist (1998), Aiyagari and Gertler (1999), Kocherlakota (2000), Cooley, Miramon and Quadrini (2004) and Jermann and Quadrini (2005).

The collateral constraint on debt is similar to the margin constraint used by Mendoza and Smith (2006) in their extension of the Aiyagari-Gertler setup to an environment of global asset trading. This paper incorporates this idea into a quantitative business cycle setup with endogenous capital accumulation and dividends that vary in response to the collateral constraint. The constraint shares some of the features of the collateral constraints studied by Kiyotaki and Moore (1997) and Kocherlakota (2000) and others examined in theoretical studies of Sudden Stops (e.g. Auenhaimer and Garcia (2000), Izquierdo (2000), Paasche (2001)).

The collateral constraint on working capital is a modification of the working capital constraint typical of limited participation models, in which firms borrow to finance factor costs. Neumeyer and Perri (2005), Uribe and Yue (2006) and Oviedo (2004) examine the quantitative implications of world interest rate disturbances in RBC-SOE models with working capital. The model of this paper differs in that firms are required to provide collateral for working capital loans, and the role of this constraint is to amplify recessions during Sudden Stops (rather than generate larger business cycles in general).

The transmission mechanism created by the collateral constraints has three elements that are key for the model's ability to produce Sudden Stops nested within normal cycles:

(1) The constraints only bind when the economy's leverage ratios (the ratios of debt and/or working capital to asset values) are sufficiently high. Otherwise, adverse shocks of standard magnitude do not alter the economy's access to world credit markets, and thus yield the same responses as in a typical RBC-SOE model.

(2) The loss of credit market access is an endogenous result. In particular, the high leverage ratios at which the collateral constraints bind are an endogenous feature of the model. These high-leverage states are reached with positive long-run probability as the outcome of the dynamics driven by optimal plans for debt and capital accumulation under particular sequences of "typical" realizations of exogenous shocks. In contrast with endowment economy models, in which agents use debt to smooth consumption in response to adverse shocks, the economy of this paper accumulates debt also to finance investment and increase consumption in response to persistent favorable shocks. Hence, high-leverage states can be preceded by economic expansions, as observed in many emerging economies. Households self insure to reduce the likelihood of large consumption drops when the collateral constraints bind, but this precautionary saving does not rule out Sudden Stops completely in the long run.

(3) Sudden Stops are driven by three "credit channel" effects that induce amplification, asymmetry and persistence in the macroeconomic effects of exogenous shocks. Two of these credit channels are endogenous financing premia (one on debt and one on working capital) that arise because the effective cost of borrowing rises when the collateral constraints bind. The third is Fisher's (1933) classic debt-deflation mechanism: When the collateral constraint on debt binds, agents liquidate capital in order to meet "margin calls." This fire-sale of assets reduces the price of capital and hence tightens further the constraint, setting off a spiraling collapse of investment and asset prices. This debt-deflation spiral has important real effects: The current account and domestic absorption suffer immediate reversals. Future levels of capital, output, and factor demands fall in response to the initial investment decline. Moreover, the collapse in the value of collateral assets can trigger the collateral constraint on working capital and thus induce immediate declines in production, factor demands and capacity utilization.

The quantitative analysis of the model calibrated to Mexican data shows that economies with and without collateral constraints exhibit largely the same long-run business cycle co-movements. In contrast, these economies yield sharply different responses to one-standard-deviation shocks conditional on a positive-probability initial state with high leverage ratios. The economy with the collateral constraints on debt and working capital yields Sudden Stops with initial collapses in consumption and investment similar to those observed in Mexico's 1995 crash. The current account reversal is larger and the drops in output and asset prices are smaller. Still, the amplification effects induced by the collateral constraints on the responses of *all* macroeconomic aggregates are significantly larger than previous estimates. These effects are weaker if the collateral constraints are replaced with exogenous credit limits, if working capital or imported inputs are removed, and if shocks to the world interest rate or imported input prices are ignored.

The rest of the paper is organized as follows. Section 2 reviews the stylized facts of Mexico's 1994-95 Sudden Stop. Section 3 describes the model economy and characterizes its competitive equilibrium. Section 4 provides a simple example that illustrates the quantitative potential of the debt-deflation mechanism. Section 5 conducts the numerical analysis of the model based on the calibration to Mexican data. Section 6 concludes.

2. Empirical Regularities of Sudden Stops: The Mexican Case

A growing number of empirical studies document the stylized facts of Sudden Stops (e.g. Calvo and Reinhart (1999), Calvo, Izquierdo and Loo-Kung (2006), and Milesi-Ferretti and Razin (2000)). This paper focuses instead on a detailed analysis of Mexico's Suddent Stop following the December, 1994 devaluation. The objectives are to quantify the stylized facts of this Sudden Stop, to identify potentially relevant features of the business cycle transmission mechanism that was at work at that time, and to provide information for the calibration of the model used in the quantitative analysis of Section 5.

Figure 1 plots the levels and the Hodrick-Prescott trend of annual per-capita GDP for the period 1900-2004. The Figure shows two striking facts. First, the Sudden Stop of 1995 produced the largest recession since the Great Depression. Second, the trend of Mexico's GDP per capita fell sharply around the time of the first post-war currency crises in 1976, and its recovery since then has been prevented first by the protracted recession that followed the 1982 Debt Crisis, and later by the 1995 Sudden Stop. Since the Debt Crisis can also be regarded as a Sudden Stop, this observation suggests that vulnerability to Sudden Stops has played a central role in Mexico's growth slowdown (although studying this issue is beyond the scope of this paper).

Figure 2 uses quarterly data for the annualized current-account GDP ratio to illustrate the magnitude of the cutback in external financing during the 1982 debt crisis and the 1995 crash. In both instances, current account deficits of about 8 percent of GDP, which were built up gradually in the years before the Sudden Stops, were fully reversed. In the 1982 episode, the reversal was more gradual but it was also larger and more persistent (the current account shifted into surpluses that averaged about 3 percent of GDP for nearly six years). The 1995 current account reversal was nearly immediate but the country returned to current account deficits in late 1996. In line with these observations, Figure 1 suggests that the recession after 1982 was milder but more prolonged than the one associated with the 1995 Sudden Stop.

Figure 3 illustrates the evolution of three key relative prices: real equity prices (in units of the GDP deflator), the price of imported intermediate goods relative to export prices, and the price of nontradable goods relative to tradables (taken from Mendoza (2002)). The plot starts in 1993 because quarterly data for imported input prices are not available before this year. The Figure shows a boom in equity prices in the two years before the 1995 Sudden Stop, followed by a collapse in 1995. The real price of imported inputs rose by about 15 percent in 1995 and remained high for almost six years, before declining and then rising sharply again since 2002. The relative price of nontradables goods fell by a little more than the increase in imported input prices, but since 1996 the nontradables relative price has followed an increasing trend.

One important aspect of these price movements is the "liability dollarization" problem identified in the Sudden Stops literature. Since foreign debt obligations of agents in emerging economies are generally denominated in units of world tradable goods (i.e.

hard currencies) and this debt is leveraged on incomes and assets denominated in different units (nontradable goods or domestically-produced tradable goods), a sudden drop in the relative price of nontradables, or a sudden surge in the relative price of imported inputs, impairs the ability of domestic agents to service debts. It is also possible that these sharp price movements reflect in part price rigidities. For instance, Cook and Devereux (2006a) report a sharp rise in import prices relative to manufacturing export prices, similar to the one shown here for Mexican data, for South East Asian countries in the 1997-1998 crises. They associate this to a combination of slow adjustment in dollar prices of Asian exports with quick pass-through of exchange rate changes to import prices. The Mexican data show, however, that the relative price of imported inputs in terms of exports can rise sharply even when the exchange rate is not moving much, as has been the case since 2002.

Table 1 summarizes key features of Mexico's business cycles and the Sudden Stop of 1995 using quarterly data. The Table provides standard measures of business cycle variability, co-movement and persistence of macroeconomic time series (GDP, private consumption, fixed investment, the current account-GDP ratio, and equity prices) using the Hodrick-Prescott filter to isolate cyclical components of the data. The Table also reports business cycle moments for the price of imported inputs relative to exports and for Uribe and Yue's (2005) measure of Mexico's real interest rate in world capital markets.

The moments reported in Table 1 are in line with well-known business cycle facts. Investment is more variable than GDP, nondurables consumption is less variable than GDP, all variables exhibit positive first-order autocorrelations, consumption and investment are positively correlated with GDP and the current account-GDP ratio is negatively correlated with GDP. Equity prices are very volatile and procyclical. Intermediate goods prices are slightly more variable than GDP and they are countercyclical. Mexico's real interest rate in world markets is about 70 percent as variable as output and is also countercyclical (by contrast, the standard measure of the world real interest rate, the rate on three-month U.S. tbills deflated by the U.S. CPI is about 60 percent as variable as output but is nearly uncorrelated with Mexico's GDP).

Table 1 also reports measures of the magnitude of the Sudden Stop in each variable (defined as the lowest deviation from trend during the sample period, or the largest one in the case of the current account-GDP ratio and the price of intermediate goods) and the ratio of this measure of Sudden Stop to the standard deviation of the same variable. The latter is an indicator of the extent to which the movements observed during the Sudden Stop exceeded those of Mexico's "typical" business cycles. The magnitude of the recessions in GDP, consumption and investment, the size of the reversal in the current account, and the collapse in asset prices are significantly larger than those of typical Mexican recessions. Except for the asset price collapse and the increase in intermediate goods prices (which measured just below 2 standard deviations), the Sudden Stops in all macro aggregates are well in excess of the two-standard deviation threshold.

Mexican National Accounts data provide further insights into the nature of the 1995 output collapse. In particular, changes in measured capital and labor played a relatively minor role in the fall in output, compared to changes in TFP, imported inputs and capacity utilization. The rationale for considering the latter two is the standard problem biasing Solow residuals as measures of "true" TFP in the presence of large swings in capacity utilization and prices of intermediate goods (see Finn (1995)).

Consider first the contributions of the capital stock (k_t), labor usage (L_t) and the Solow residual measure of TFP (A_t^S) to the 1995 output collapse implied by a Cobb-Douglas technology, $A_t^S k_t^\beta L_t^\alpha$. The first step to measure these contributions is to construct estimates of factor income shares. Mexican data shows that the average labor share on GDP for the period 1988-2002 is $\alpha = 0.35$, but this figure is subject of debate because of problems in measurement of proprietor's income and other forms of labor income. Recently, Garcia (2005) constructed estimates of Mexico's factor shares using household survey data and obtained $\alpha = 0.66$, much higher than the average from national accounts. Using Garcia's estimates of factor shares, a standard capital stock series constructed using the method of perpetual inventories (see Section 4 for details), and labor data from Mexico's National Accounts, the near 7 percent decline in GDP per worker observed between the third quarter of 1994 and the second quarter of 1995 breaks down into a contribution of -11 percentage points of the Solow residual, and a weighted contribution of 4 percentage points due to the capital-GDP ratio (weighted by the share of capital in GDP). Thus, standard Solow residual analysis would suggest that the output collapse was fully driven by a change in TFP that had to be large enough to overcome the growth in the capital-GDP ratio (which grew because investment was still positive in 1994).

The results are very different using a production function for gross output of the form $A_t(m_t k_t)^\beta L_t^\alpha v_t^\eta$, where A_t is "true" TFP, v_t are imported inputs and m_t is the utilization rate of capital. The estimate of η implied by the 1993:1-2005:2 average share of imported inputs in gross output at current prices (measuring gross output as GDP plus imported inputs) is 10.2 percent. Given Garcia's (2005) factors shares and this estimate of η, the implied factor shares of capital and labor in gross output are $\alpha = 0.59$ and $\beta = 0.31$.

Changes in utilization are difficult to gauge because capacity utilization is ignored in National Accounts, and existing indicators based on utilization surveys of limited coverage or indirect proxies, like electricity demand, are unreliable.[1] There are, however, methodologies that can be used to infer utilization rates from National Accounts. This paper uses a methodology that follows from the capacity utilization framework of Calvo (1975), in which the utilization rate determines the rate of depreciation of physical capital according to an isoelastic cost function. Optimal utilization under competitive factor pricing yields a mapping for the utilization rate that is a positive, isoelastic function of the gross output-capital ratio, with an elasticity parameter determined by the ratio of β divided by the long-run investment rate (see Section 4 for details). Combining the estimates of utilization produced by this mapping with data on labor and imported inputs, and using the capital stock estimates and factor shares in gross output mentioned earlier, the contribution of TFP to the 8.5 decline in gross output per worker observed between 1994:3 and 1995:2 falls to 7.7 percentage points (instead of 11 percentage points with the Solow residual). The contribution of the ratio of imported inputs to gross output is -3 percentage points, and the contribution of capacity utilization is -2.5 percentage points.

It is interesting that, because of the large swing in the relative price of imported inputs, and hence in the demand for these inputs, a factor of production that is just 10 percent of gross output can contribute 3 percentage points of the 8.5 percent drop in gross output per worker.[2] Moreover, the contribution of TFP can still be overstated to the

[1] Utilization surveys in Mexico started in November, 1996 and cover only the manufacturing sector.
[2] Finn (1995) made a similar argument for the case of energy in the United States.

extent that there are other key inputs that experienced large price changes. Taking these inputs into account would reduce more the contribution of TFP and increase that of the key inputs. This exercise focuses only on imported inputs because their price relative to export prices can be treated as shocks exogenous to the Mexican economy.

The findings of this TFP accounting exercise indicate that if, as the Sudden Stops literature argues, financial frictions are at the core of the transmission mechanism that triggers Sudden Stops, the effects of those frictions on investment and employment are of little relevance for explaining the *initial* output collapse. Sudden Stops models that focus only on the investment or labor effects of financial frictions could mimic the observed falls in capital and labor and yet explain little of the fall in output. Changes in imported inputs and capacity utilization play a bigger role, and hence Sudden Stops models should include mechanisms that link them with financial frictions.

Interpreting the TFP results as suggesting that financial factors are irrelevant would be misleading because the sudden, sharp reversal of Mexico's current account in 1995 provides independent evidence showing that during Sudden Stops access to external financing is severely restricted. There is also evidence from firm-level data showing that (a) corporate leverage ratios rise in the buildup phase to a Sudden Stop, and (b) when the Sudden Stop hits, leverage ratios collapse. Chapter II of IMF (2002) reports country and regional aggregates of leverage ratios for listed corporations generated with firm-level data on a fiscal year basis from the *Worldscope* database, including ratios of debt to assets and debt to market value of equity. The median debt-to-assets ratio of Mexican corporations rose by 5 percentage points in the two years before the Sudden Stop, and fell by more than 5 percentage points in the fiscal year 1995-1996. For the aggregate of emerging markets in Asia, the median ratio of debt to market value of equity rose from 0.4 to 1.2 between 1996 and 1998, and then fell by 20 percentage points in 1998-1999.

3. A Model of Sudden Stops and Business Cycles with Collateral Constraints

The model economy is a variation of the standard RBC-SOE model with incomplete insurance markets and capital adjustment costs proposed in Mendoza (1991). Two important modifications are introduced here. First, the assumption of perfect credit markets is relaxed to introduce endogenous collateral constraints. Second, the supply-side of the model is modified to introduce imported inputs and endogenous capacity utilization.

3.1 Households

The small open economy is inhabited by a large set of identical, infinitely lived households. The preferences of the representative household are defined over stochastic sequences of consumption c_t and labor supply L_t, for $t=0,\dots,\infty$. Preferences are modeled using Epstein's (1983) Stationary Cardinal Utility (SCU) function, which features an endogenous rate of time preference, so as to obtain a unique, invariant limiting distribution of foreign assets.[3]

[3] Since agents face non-insurable income shocks and the world interest rate is exogenous, precautionary saving leads foreign assets to diverge to infinity with the standard assumption of a constant rate of time preference equal to the interest rate.

The preference specification is:[4]

$$E_0\left[\sum_{t=0}^{\infty}\exp\left\{-\sum_{\tau=0}^{t-1}\rho\left(c_\tau - N(L_\tau)\right)\right\}u\left(c_t - N(L_t)\right)\right] \tag{1}$$

In this expression, *u(.)* is a standard twice-continuously-differentiable and concave period utility function and *ρ(.)* is an increasing, concave and twice-continuously-differentiable time preference function. Following Greenwood et al. (1988), utility is defined in terms of the excess of consumption relative to the disutility of labor, with the latter given by the twice-continuously-differentiable, convex function *N(.)*. This assumption eliminates the wealth effect on labor supply by making the marginal rate of substitution between consumption and labor independent of consumption.

There are other methods that yield well-defined stochastic stationary equilibria in SOE models (see Arellano and Mendoza (2003) and Schmitt-Grohe and Uribe (2002) for details). In the context of models with credit constraints, SCU has the advantage that it can support stationary equilibria in which these constraints can bind permanently. This is because a binding credit constraint drives a wedge between the intertemporal marginal rate of substitution in consumption and the rate of interest. In a stationary state with a binding credit constraint, the rate of time preference adjusts endogenously to accommodate this wedge. In contrast, in models with an exogenous discount factor, credit constraints never bind in the long run (if the rate of time preference is greater or equal to the world interest rate) or always bind at steady state (if the rate of time preference is fixed below the interest rate).

Households choose sequences of consumption, labor supply, investment in domestic capital, k_{t+1}, and foreign borrowing or lending in one-period international bonds, b_{t+1}, so as to maximize SCU subject to the following period budget constraint:

$$c_t = (d_t + q_t)k_t - q_t k_{t+1} + w_t L_t - b_{t+1} + b_t R\exp(\varepsilon_t^R) \tag{2}$$

Households take as given the dividend rate on capital holdings, d_t, the market price of capital, q_t, the wage rate, w_t, and the stochastic gross world real interest rate on foreign assets, $R exp(\varepsilon_t^R)$. ε_t^R is an interest rate shock that follows a Markov process joint with the other shocks defined later in this section.

The world credit market is imperfect. In particular, lenders require households to guarantee their debt by offering domestic assets as collateral. The collateral constraint takes the form of the margin requirement proposed by Aiyagari and Gertler (1999):

$$b_{t+1} \geq -\kappa q_t k_{t+1} \tag{3}$$

[4] Epstein showed that SCU requires weaker preference axioms than those behind the standard utility function with exogenous discounting. Exogenous discounting requires preferences over stochastic future allocations to be risk-independent from past allocations and past allocations to be risk-independent from future allocations, while SCU preferences only require the latter. Epstein proved that a preference order consistent with the weaker axioms has a von Neumann-Morgenstern, time-recursive representation *if and only if* it takes the form of the SCU function.

Thus, households can borrow up to a fraction κ of the market value of their capital. This constraint resembles a debt contract with a margin clause. Margin clauses typically require borrowers to surrender the control of collateral assets when the debt contract is entered and give creditors the right to sell the assets when their market value falls below the contract value. There are also other arrangements in financial markets that operate in a similar way as a margin constraint without explicit margin clauses. These include value-at-risk strategies of portfolio risk management used by investment banks, and capital requirements imposed by regulators on financial institutions and institutional investors. For example, if an aggregate shock hits emerging markets, value-at-risk estimates increase and lead investment banks to reduce their exposure in these markets, but since the shock is aggregate, the resulting sale of assets increases price volatility and leads value-at-risk models to require further portfolio adjustments. Dunbar (2000) provides a detailed account of the central role that these mechanisms played in propagating the Russian crisis of 1998 across the financial markets of emerging and industrial economies.

The margin constraint is not derived here from an optimal credit contract. Instead, the constraint is imposed directly as in the models with endogenous credit constraints examined by Kiyotaki and Moore (1997), Aiyagari and Gertler (1999), and Kocherlakota (2000). Still, a credit relationship with a constraint like (3) could result, for example, from an environment in which limited enforcement prevents lenders to collect more than a fraction κ of the value of a defaulting debtor's assets. In states of nature in which (3) binds, the model produces an endogenous premium over the world interest rate at which borrowers would agree to contracts in which b_{t+1} satisfies (3).[5]

3.2 Firms

Firms are owned by households and discount future profits taking as given the households' stochastic discount factors (i.e., the intertemporal marginal rates of substitution in consumption, the reciprocal of which are denoted by $\tilde{R}_{t+1}^{t}$, for $t=0,\ldots,\infty$ with $\tilde{R}_0^{-1}=1$). Firms produce a tradable good that sells at a world-determined price (normalized to unity without loss of generality). They make plans for factor demands and investment. Capital depreciates at a rate that varies with capacity utilization, as determined by the continuously-differentiable, increasing function $\delta(m_t)$. Thus, as in Calvo (1975), the cost of higher utilization is faster depreciation. Net investment, $z_t = k_{t+1} - k_t$, incurs unitary investment costs determined by the function $\Psi(z_t/k_t)$, which is linearly homogeneous in z_t and k_t.[6] Firms need working capital to pay for a fraction ϕ of their purchases of imported inputs, wage payments, and capacity utilization costs in advance of sales. Firms are required to guarantee working capital loans so working capital financing cannot exceed the fraction κ^f of the value of firms' assets. Imported inputs are purchased

---

[5] Arellano (2005), Cook and Devereux (2006a), Neumeyer and Perri (2005) and Uribe and Yue (2006) study the quantitative implications of country risk for business cycles in small open economies. Arellano endogenizes country risk in a setup of strategic default. Neumeyer and Perri, and Uribe and Yue study the effects of introducing exogenous risk premia of the magnitude observed in the data in models with working capital, while Cook and Devereux conduct a similar experiment in a model with sticky dollar pricing of exports and intra-regional trade.

[6] Separating the utilization-depreciation cost from investment adjustment costs yields a more tractable recursive formulation of the competitive equilibrium that preserves Hayashi's (1982) results regarding the conditions that equate marginal and average Tobin Q.

at an exogenous, stochastic relative price $p\exp(\varepsilon_t^P)$, where $\exp(\varepsilon_t^P)$ represents a shock to the world price of imported inputs. TFP is also subject to random shocks $\exp(\varepsilon_t^A)$.

The firms' problem is to choose labor demand, investment, imported inputs, and the rate of capacity utilization so as to maximize their value:

$$E_0\left[\sum_{t=0}^{\infty}\left(\prod_{j=0}^{t}\left(\tilde{R}_j^{j-1}\right)^{-1}\right)\left(\begin{array}{l}\exp(\varepsilon_t^A)F(m_t k_t, L_t, v_t)\\ \qquad -(1+\phi r_t)\left(w_t L_t + p\exp(\varepsilon_t^P)v_t + \delta(m_t)k_t\right) - z_t\left[1+\Psi\left(\frac{z_t}{k_t}\right)\right]\end{array}\right)\right], \quad (4)$$

where $r_t \equiv R\exp(\varepsilon_t^R)-1$, subject to the law of motion for capital,

$$z_t = k_{t+1} - k_t, \quad (5)$$

and the collateral constraint on working capital financing:

$$R\exp(\varepsilon_t^R)\phi\left(w_t L_t + p\exp(\varepsilon_t^P)v_t + \delta(m_t)k_t\right) \leq \kappa^f q_t k_{t+1}. \quad (6)$$

Working capital is a within-period loan contracted at the beginning of each period and paid off after the current output is sold at the end of each period. Hence, lenders set the limit on working capital considering that the market value of the assets offered as collateral must cover interest and principal on working capital loans.

3.3 Competitive Equilibrium & Credit Channel Effects

A competitive equilibrium for the small open economy is defined by stochastic sequences of allocations $\left[c_t, L_t, k_{t+1}, b_{t+1}, m_t, v_t, z_t\right]_0^{\infty}$ and prices $\left[q_t, d_t, w_t, \tilde{R}_{t+1}\right]_0^{\infty}$ such that: (a) households maximize SCU subject to (2) and (3), taking as given dividends, wages, equity prices, the world interest rate, and the initial conditions (k_0, b_0), (b) firms maximize their value subject to (5) and (6), taking as given wages, the price of imported inputs, the world interest rate, the household discount factors and the initial condition k_o, and (c) the capital, labor and goods markets clear.

In the absence of credit constraints, the competitive equilibrium is the same as in a standard RBC-SOE model. The credit constraints distort this equilibrium by introducing three credit-channel effects. Two of them are external financing premia affecting the cost of borrowing for households and firms and the third is the Fisherian debt-deflation process. These credit-channel effects can be analyzed using the optimality conditions of the competitive equilibrium.

The household's optimality conditions yield the following Euler equation for b_{t+1}:

$$0 < 1 - \frac{\mu_t}{\lambda_t} = E_t\left[\frac{\lambda_{t+1}}{\lambda_t} R\exp(\varepsilon_{t+1}^R)\right] \leq 1 \quad (7)$$

where λ_t is the non-negative Lagrange multiplier on the date-t budget constraint (2), which equals also the lifetime marginal utility of c_t, and μ_t is the non-negative Lagrange multiplier on the collateral constraint (3). It follows from (7) that, when the collateral constraint binds, households face an endogenous external financing premium on the effective real interest rate at which they borrow (R^h_{t+1}) relative to the world interest rate. This expected premium is given by:

$$E_t\left[R^h_{t+1} - R\exp(\varepsilon^R_{t+1})\right] = \frac{\mu_t + \text{cov}(\lambda_{t+1}, \varepsilon^R_{t+1})}{E_t\left[\lambda_{t+1}\right]}, \quad R^h_{t+1} \equiv \frac{\lambda_t}{E_t\left[\lambda_{t+1}\right]} \tag{8}$$

This external financing premium can be viewed as the premium at which domestic agents would choose debt contracts with loan amounts that satisfy the collateral constraint with equality in a credit market in which the constraint is not imposed directly.

In the canonical RBC-SOE model, international bonds are a risk-free asset and μ_t=0 for all t, so there is no premium. If the world interest rate is stochastic, the premium can be positive or negative depending on the sign of the covariance term in (8). If the collateral constraint binds, there is a direct effect by which the multiplier μ_t increases the households' external financing premium. In addition, there is an indirect effect that pushes in the same direction because a binding credit constraint makes it harder to smooth consumption, and hence the covariance between marginal utility and the world interest rate is likely to increase.

The effects of the external financing premium on the asset price valuation of households can be derived from their Euler equation for capital. Solving forward this equation yields the following asset-pricing condition:

$$q_t = E_t\left[\sum_{j=0}^{\infty}\left(\prod_{i=0}^{j}\left(\frac{1}{\tilde{R}^{t+i}_{t+i+1}}\right)\right)d_{t+1+j}\right], \quad \tilde{R}^{t+i}_{t+i+1} \equiv \frac{\lambda_{t+i} - \kappa\mu_{t+i}}{\lambda_{t+i+1}} \tag{9}$$

where $\tilde{R}^{t+i}_{t+i+1}$ is again the reciprocal of the households' stochastic discount factor. Given the financing premium in (8), it can be shown that a collateral constraint binding at t or expected to bind at any future date, increases the rate at which future dividends are discounted and thus lowers the date-t price of equity. In particular, the Euler equations for bonds and equity yield the following expression for the equity premium (the excess return on equity, $R^q_{t+1} \equiv (d_{t+1} + q_{t+1})/q_t$, relative to the gross world interest rate):

$$\begin{aligned} E_t\left[R^q_{t+1} - R\exp(\varepsilon^R_{t+1})\right] &= \frac{\mu_t(1-\kappa) + COV_t(\lambda_{t+1}, \varepsilon^R_{t+1}) - COV_t(\lambda_{t+1}, R^q_{t+1})}{E_t[\lambda_{t+1}]} \\ &= E_t\left[R^h_{t+1} - R\exp(\varepsilon^R_{t+1})\right] - \frac{\mu_t\kappa + COV_t(\lambda_{t+1}, R^q_{t+1})}{E_t[\lambda_{t+1}]} \end{aligned} \tag{10}$$

The above expression collapses to the standard equity premium expression if the collateral constraint does not bind and the world interest rate is deterministic. As Mendoza and Smith (2006) explained, when the collateral constraint binds it induces direct and indirect effects on the equity premium similar to those affecting the external financing premium. The direct effect of the binding collateral constraint is reduced by the term $\frac{\kappa\mu_t}{E_t\left[\lambda_{t+1}\right]}$, which measures the marginal benefit of being able to borrow more by holding an additional unit of capital. There is also a new element in the indirect effect that is not present in the external financing premium and is implicit in the covariance between λ_{t+1} and R^q_{t+1}: since a binding collateral constraint makes it harder for households to smooth consumption and self-insure, this covariance term is likely to become more negative when the constraint binds, thereby increasing the equity premium.

Given the sequence of expected equity returns from (10), the forward solution for the households' valuation of equity can be re-written as:

$$q_t = E_t\left(\sum_{j=0}^{\infty}\left[\prod_{i=0}^{j}\left(\frac{1}{E_t\left[R^q_{t+1+i}\right]}\right)\right]d_{t+1+i}\right) \tag{11}$$

It follows then from (10) and (11) that, as Aiyagari and Gertler (1999) showed, higher expected returns when the collateral constraint binds, or is expected to bind in the future, increase the discount rate of future dividends and lower equity prices in the present.

In general equilibrium, the equity market clears and asset prices adjust so that the households' investment plans are consistent with those formulated by firms. On the firms' side, the optimality conditions for k_{t+1} and z_t are:

$$\left(1+\Psi\left(\frac{z_t}{k_t}\right)+\left[\frac{z_t}{k_t}\right]\Psi'\left(\frac{z_t}{k_t}\right)\right)=\zeta_t \tag{12}$$

$$E_t\left[\left(\tilde{R}^t_{t+1}\right)^{-1}\left(d_{t+1}+\zeta_{t+1}\right)\right]=\zeta_t$$

$$d_{t+1}\equiv \exp(\varepsilon^A_{t+1})m_{t+1}F_1(m_{t+1}k_{t+1},L_{t+1},v_{t+1})+\left[\frac{z_{t+1}}{k_{t+1}}\right]^2\Psi'\left(\frac{z_{t+1}}{k_{t+1}}\right) \tag{13}$$

$$-\,\delta(m_{t+1})\left(1+\phi\left(r_{t+1}+\chi_{t+1}R\exp(\varepsilon^R_{t+1})\right)\right)+\chi_t\kappa^f q_t\tilde{R}^t_{t+1}$$

where ζ and χ are the Lagrange multipliers on the investment equation (5) and the working capital constraint (6) respectively.

Notice that, since firms discount at the households' stochastic discount factors, the forward solution of (13) yields asset prices consistent with those from the households' forward solution (9) when $\zeta_t=q_t$. The optimal choice for z_{t+1} (given k_t and q_t) implied by the firm's optimal investment condition (12) represents the firms' demand for investment resources (i.e., its equity supply function). Since (12) is a standard Tobin Q relationship, the fact that $\Psi(.)$ is increasing and convex implies that there is a positive relationship between investment demand and the equity price, or that the firms' equity supply function is upward sloping. This is because adjustment costs prevent firms from instantaneously adjusting the stock of capital to its long-run level.[7] Hence, when the collateral constraint causes a negative shock to the households' equity demand, firms reduce gradually the capital stock and thus the fall in equity demand is accommodated partly with a cut in firm investment and partly with a fall in the price of equity.

The second external financing premium present in the model arises because of the collateral constraint on working capital financing. Firms observe the date-t realizations of the shocks and, since the date-t capital stock is predetermined, they set factor demands and capacity utilization according to these marginal productivity rules:

$$\exp(\varepsilon^A_t)F_2(m_tk_t,L_t,v_t)=w_t\left[1+\phi(r_t+\chi_tR\exp(\varepsilon^R_t))\right] \tag{14}$$

[7] With the standard functional form $\Psi(z/k)=(a/2)[z/k]$, the elasticity of investment with respect to q is $(kq)/(az)$ so without adjustment costs investment demand would be infinitely elastic.

$$\exp(\varepsilon_t^A)F_1(m_t k_t, L_t, v_t) = \delta'(m_t)\big[1 + \phi(r_t + \chi_t R \exp(\varepsilon_t^R))\big] \tag{15}$$

$$\exp(\varepsilon_t^A)F_3(m_t k_t, L_t, v_t) = p\exp(\varepsilon_t^P)\big[1 + \phi(r_t + \chi_t R \exp(\varepsilon_t^R))\big] \tag{16}$$

In the right-hand-side of each of these conditions, the term $\chi_t Rexp(\varepsilon_t^R)$ reflects the increase in the effective marginal financing cost of working capital caused by the collateral constraint. This external financing premium represents the excess over r_t at which firms in a competitive market of working capital loans would find it optimal to agree to loan contracts that satisfy constraint (6) voluntarily.

The third credit channel present in the model, Fisher's debt-deflation mechanism, is harder to illustrate than the above external financing premia because of the lack of closed-form solutions for equity prices and investment. In light of this, Section 4 provides a simple quantitative example based on a simplified version of the model. Still, the debt-deflation mechanism can be described intuitively: When the households' collateral constraint binds, they respond to "margin calls" from lenders by fire-selling equity. However, when they do this they meet with firms that feature an upward-sloping supply of equity because of Tobin's Q. These firms thus find it optimal to lower investment given the reduced demand for equity and higher discounting of future dividends, and hence equilibrium equity prices fall. But if the households' collateral constraint was binding at the initial equity prices and equity holdings, it must be more binding at the reduced prices and investment levels, so another round of margin calls takes place and Fisher's debt-deflation mechanism is set in motion. Moreover, as the value of collateral assets falls, the collateral constraint on working capital may become binding, and if it does, the Fisherian deflation will also cause a sudden increase in marginal financing costs for firms, resulting in reduced levels of factor demands and capacity utilization.

4. Amplification with Fisherian Deflation: A Simple Example

This Section of the paper uses a simple version of the model to show the potential of the debt-deflation mechanism to produce large amplification effects. This exercise is similar to the one conducted by Kocherlakota (2000) to quantify the amplification induced by credit constraints in the responses of output and asset prices to unanticipated linear income shocks in a simple deterministic model. The two models are very similar, except that his model assumes that a fixed supply of land serves as collateral and agents can borrow up to 100 percent of the value of land (i.e., κ=1).

The model of Section 3 is simplified by adopting these assumptions: (1) perfect foresight, (2) a standard intertemporal utility function that depends only on consumption and has a constant discount factor, (3) zero depreciation of capital, (4) a production technology with capital as the only input, (5) standard functional forms for preferences, technology and adjustment costs (analogous to those used in the quantitative analysis of the full model in Section 5), and (6) a rate of time preference equal to the world real interest rate. The equilibrium conditions of this model, for t=0,…,∞, are the following:

$$u'(c_t) - \mu_t = u'(c_{t+1})\,, \tag{17}$$

$$\left(u'(c_t) - \mu_t \kappa\right) q_t = \frac{u'(c_{t+1})}{R}\left(\beta k_{t+1}^{\beta-1} + \frac{a}{2}\left(\frac{k_{t+2} - k_{t+1}}{k_{t+1}}\right)^2 + q_{t+1}\right), \tag{18}$$

$$c_t = k_t^{\beta} - (k_{t+1} - k_t)\left[1 + \left(\frac{a}{2}\right)\frac{k_{t+1} - k_t}{k_t}\right] - b_{t+1} + Rb_t + \Delta_t\,, \tag{19}$$

and the credit constraint $b_{t+1} \geq -\kappa q_t k_{t+1}$. Condition (17) is the consumption Euler equation (considering that the rate of time preference equals the interest rate), condition (18) is the investment Euler equation (where a is the adjustment cost coefficient and $q_t = 1 + a(k_{t+1} - k_t)/k_t$ is Tobin's Q) and (19) is the resource constraint. The deterministic sequence of linear income shocks $[\Delta_t]_0^{\infty}$ satisfies two conditions: (a) the sequence can be split into an initial shock Δ_0 and a time-invariant shock $\Delta_t \equiv \Delta_{-0}$ for all t>0, and (b) Δ_0 and Δ_{-0} are such that the present discounted value of $[\Delta_t]_0^{\infty}$ is zero. Thus, Δ_0 is a "wealth-neutral" shock to date-0 income, which can be expressed for simplicity as a fraction s of date-0 output, $\Delta_0 = s k_0^{\beta}$ (recall that date-0 output is pre-determined by k_0).

It is straightforward to show that if credit markets are perfect, wealth-neutral shocks do not affect the equilibrium of this economy (except for debt dynamics). Given the initial conditions (k_0, b_0), the model with frictionless credit yields standard textbook solutions:

$$k_{t+1} - k_t = (\Lambda - 1)(k_t - k_{ss}) \tag{20}$$

$$y_t = k_t^{\beta} - (\Lambda - 1)(k_t - k_{ss})\left[1 + \frac{a}{2}\left(\frac{(\Lambda - 1)(k_t - k_{ss})}{k_t}\right)\right] \tag{21}$$

$$c_t = \overline{c} = (1 - R^{-1})\left[\sum_{t=0}^{\infty} R^{-t}\,(y_t + \Delta_t) + b_0 R\right] \tag{22}$$

where y_t is output net of investment and adjustments costs, k_{ss} is the steady state capital stock (which satisfies $\beta k_{ss}^{\beta-1} + 1 = R$), and $0 \leq \Lambda \leq 1$ is the partial-adjustment coefficient of the capital stock that depends on R, β and a. These results imply that consumption is perfectly smooth at a constant fraction of the economy's wealth, while investment, the capital stock, and net income display standard dynamics that converge monotonically to steady state. Wealth-neutral shocks do not alter these outcomes because the present value of $[\Delta_t]_0^{\infty}$ vanishes from the measure of wealth in eq. (22). The shocks alter only bond dynamics, with agents borrowing more (less) at date 0 when Δ_0 is negative (positive).

Consider now the effects of wealth-neutral shocks in the economy with the collateral constraint. To simplify transitional dynamics, κ takes two values known with full certainty: a low value κ^L at t=0 and a high value κ^H for all t>0, with $\kappa^L \leq \kappa^H$ and κ^H high enough so that the collateral constraint does not bind for all t>0. Given κ^L, if s is small enough agents can borrow as needed to afford $\overline{c}$ at date 0 without violating the credit constraint, and hence the results of the perfect-credit-markets case still hold. Since this implies that $\mu_0 = 0$, eq. (18) predicts that in this case the return on k_{t+1} is equal to R. There is, however, a large enough shock $\hat{s}$ that makes the constraint bind at date 0. This is the value of s for which the equilibrium of the perfect-credit-markets case satisfies jointly the resource constraint and the collateral constraint with equality:

$$\overline{c} = y_0 - \hat{s}k_0^{\beta} + Rb_0 + \kappa^L q_0 \left(\Lambda k_0 + (1-\Lambda)k_{ss}\right) \tag{23}$$

The equilibrium of the economy with a binding collateral constraint at date 0 can be solved in two stages. First, the solutions from date 1 forward are given by the solutions from the case with nonbinding credit constraints (eqs. (20)-(22)), adjusted for the initial conditions (k_1,b_1) and incorporating the sequence of time-invariant income shocks that complements the date-0 shock ($\Delta_t \equiv \Delta_{-0} = (R/(1\text{-}R^{-1}))sk_0^{\beta}$ for all t>0). Second, the values of c_0, k_1, b_1 and μ_0 solve the following non-linear equation system:

$$u'(c_0) - \mu_0 = u'\left(\overline{c}(k_1, b_1, \Delta_{-0})\right), \tag{24}$$

$$\begin{aligned}&\left(u'(c_0) - \mu_0\kappa^L\right)\left(1 + a\left(\frac{k_1 - k_0}{k_0}\right)\right) = \\ &\frac{\left(u'(c_0) - \mu_0\right)}{R}\left(\beta k_1^{\beta-1} + \frac{a}{2}\left(\frac{(\Lambda-1)(k_1 - k_{ss})}{k_1}\right)^2 + 1 + a\left(\frac{(\Lambda-1)(k_1 - k_{ss})}{k_1}\right)\right),\end{aligned} \tag{25}$$

$$c_0 = (1-s)k_0^{\beta} - (k_1 - k_0)\left[1 + \left(\frac{a}{2}\right)\left(\frac{k_1 - k_0}{k_0}\right)\right] - b_1 + Rb_0, \tag{26}$$

$$b_1 = -\kappa^L\left[1 + a\left(\frac{k_1 - k_0}{k_0}\right)\right]k_1, \tag{27}$$

where $\overline{c}(k_1, b_1, \Delta_{-0})$ denotes the stationary consumption level that solves eqs. (20)-(22) for initial conditions (k_1,b_1) and $\Delta_{-0} \equiv (R/(1\text{-}R^{-1}))sk_0^{\beta}$.

The intuition behind the debt-deflation mechanism implicit in eqs. (24)-(27) can be explained as follows: When the shock first hits and triggers the constraint (i.e., before considering its endogenous effect on capital and Tobin's Q), k_1 falls as agents seek to offset the impact of their reduced borrowing ability by cutting investment. The cut in investment lowers q_0 and date-1 dividends and rises q_1, so the return on k_1 raises above R. If the credit constraint was set as an exogenous fixed amount, these would be the main adjustments. But the lower k_1 and q_0 tighten further the endogenous collateral constraint, forcing extra adjustments in bond holdings, consumption, investment, and asset prices.

The value of κ^L plays an important role in the debt-deflation process. If κ^L=1, it follows from (25) that the return on k_1 remains equal to R even if the constraint binds, which implies that investment and Tobin's Q remain the same as with perfect credit markets, and hence there is no debt-deflation. Consumption and debt still adjust, but they do so as they would with an exogenous credit constraint. At the other extreme, if κ^L=0 the collateral constraint induces a decline in investment (as well as adjustments in consumption and debt) and raises the return on k_1 above R. But again there is no debt deflation because now capital cannot be used as collateral. Hence, for the debt-deflation mechanism to operate, credit markets must allow borrowers to leverage their assets but only to some degree.

The size of the amplification effects produced by this simple model is illustrated with a set of numerical experiments. These experiments use the parameter values β=0.34,

R=1.086, and a=2.65, which correspond to the calibration to Mexican data discussed in the next section. The value of b_0 is set at 40 percent of GDP, also as in Mexican data, and k_0 is set at 90 percent of k_{ss}. The baseline shock is s=0.027 percent, equivalent to 1 standard deviation of Mexico's GDP in Table 1. Given these parameters, the initial leverage ratio is $b_0/q_{-1}k_0$ =-10.2 percent, hence κ^L=0.102. Under perfect credit markets, the leverage ratio would fall to -12 percent at date 0 (i.e., b_1/q_0k_1 =-0.12) and would continue to fall monotonically to reach -20 percent at steady state. Thus, $\kappa^H \geq 0.2$ guarantees that the credit constraint never binds for any t>0.[8]

Table 2 shows amplification effects for output, Tobin's Q, consumption and the GDP shares of investment and the current account. These effects are measured as differences between responses to the shock with and without collateral constraints in percent of the latter. The Table also includes Kocherlakota's (2000) measure of amplification: the difference between the responses to the shock with the credit constraint and the pre-shock values of the variables in percent of the size of the shock. Five panels of results are reported: Panel A reports the baseline results. Panel B changes the endogenous borrowing constraint into an exogenous constraint set to an amount equal to 10.2 percent of the date-0 value of capital in the economy with perfect credit markets. Panel C sets k_0 1/3[rd] below the steady state capital stock. Panel D raises the adjustment cost coefficient by a factor of 40. Panel E lowers the same coefficient by a factor of 1/40. Panel F shows results for shocks ranging from 2 to 6 percent. The parameter changes in Panels C, D and E, change the initial leverage ratio, so the value of κ^L is adjusted accordingly.

The baseline results show that the credit constraint has important amplification effects for a shock of just 1 standard deviation of the observed output variability. The amplification effects (in absolute value) are 0.6 percent for GDP, 4.5 percent for Tobin's Q, 2.6 percent for consumption, 6.6 percentage points of GDP for investment, and over 9 percentage points of GDP for the current account. The amplification coefficients are 22 percent for GDP and nearly 86 percent for Tobin's Q.

The changes in the results in panels B-F relative to Panel A illustrate the role of key factors affecting the size of amplification effects: (1) the effects are weaker without the endogenous feedback between investment and Tobin's Q that drives the debt-deflation process (Panel B with the exogenous borrowing constraint shows significantly smaller effects than Panel A); (2) the effects are larger if the leverage needs of the economy are stronger (Panel C with initial capital 1/3[rd] below steady state requires higher leverage ratios because the incentives to borrow are stronger the smaller initial capital, and hence income, are relative to long-run values); (3) higher (lower) capital adjustment costs in Panels D and E enlarge (reduce) the effects on asset prices and consumption but reduce (enlarge) the effects on output, investment and the current account – this result reflects the tradeoff of price and quantity effects typical of Tobin Q models when adjustment costs increase; (4) larger shocks produce larger amplification effects (in Panel F, tripling the size of the shock from 2 to 6 percent, which is a little over twice the standard deviation of Mexico's GDP, increases the amplification effects by about 70 percent). These findings extend to the stochastic simulations of Section 5, where binding credit constraints with

[8]Note that the constraint can bind for $0.102 \leq \kappa^L \leq 0.12$ ($\kappa^L \geq 0.102$ because the initial leverage ratio must satisfy the credit constraint and $\kappa^L \leq 0.12$ because when κ^L=0.12, $s = \hat{s}$ =0.027).

large amplification are found in the "high-leverage" region of the state space, and amplification effects are significantly smaller with an exogenous credit constraint.

The results in Table 2 also show that the model can produce larger amplification coefficients for output and asset prices than those computed by Kocherlakota (2000). He found that, varying the share of capital from 0.1 to 0.3, the amplification coefficients were small, ranging from 15 to 35 percent for output and 0.4 to 0.8 percent for the price of land. The amplification coefficient for GDP in Panel A of Table 2 is in line with these findings, but the coefficient for asset prices is much larger. In addition, the results in Panels C and E suggest that higher borrowing needs and/or lower adjustment costs can yield amplification coefficients that exceed Kocherlakota's by large margins. The larger effects obtained here are due in part to the fact that land used as collateral in his model is in fixed supply, which breaks the feedback between asset prices and asset accumulation operating via the debt-deflation process. He noted, however, that using a credit constraint formulated in terms of capital seemed to produce even weaker amplification coefficients. This is consistent with the model examined here because, as explained earlier, the model predicts zero amplification on output and asset prices when κ=1. Thus, the assumption in Kocherlakota's model that agents can use the full value of their assets as collateral also plays a role in explaining the larger effects shown in Table 2.

5. Stochastic Simulation Analysis

5.1 Functional Forms and Numerical Solution Method

This section studies the quantitative implications of the complete model by analyzing the results of numerical simulations calibrated to Mexican data. The functional forms of preferences and technology are the following:

$$u(c_t - N(L_t)) = \frac{\left[c_t - \frac{L_t^{\omega}}{\omega}\right]^{1-\sigma} - 1}{1-\sigma}, \qquad \sigma, \omega > 1, \tag{28}$$

$$v(c_t - N(L_t)) = \gamma\left[Ln\left(1 + c_t - \frac{L_t^{\omega}}{\omega}\right)\right], \qquad 0 < \gamma \leq \sigma, \tag{29}$$

$$F(m_t k_t, L_t, v_t) = A(m_t k_t)^{\beta} L_t^{\alpha} v_t^{\eta}, \qquad 0 \leq \alpha, \beta, \eta \leq 1, \quad \alpha + \beta + \eta = 1, \quad A > 0, \tag{30}$$

$$\Psi\left(\frac{z_t}{k_t}\right) = \frac{a}{2}\left(\frac{z_t}{k_t}\right), \qquad a \geq 0 \tag{31}$$

$$\delta(m_t) = \frac{h m_t^{\theta}}{\theta}, \qquad \theta \geq 1, h > 0. \tag{32}$$

The utility and time preference functions in (28) and (29) are standard from RBC-SOE models. The parameter σ is the coefficient of relative risk aversion, ω determines the wage elasticity of labor supply, which is given by *1/(ω -1)*, and γ is the semi-elasticity of the rate of time preference with respect to composite good *c-N(L)*. The restriction $\gamma \leq \sigma$ is a condition required to ensure that the SCU function supports a unique, invariant limiting distribution of assets (see Epstein (1983)). The Cobb-Douglas production function (30) is the same used in the TFP accounting exercise of Section 2. Equation (31) is the net investment adjustment cost function. Following Hayashi (1982), the production function and the adjustment cost function are linearly homogeneous in their arguments. The isoelastic capacity utilization cost (or depreciation rate) function in (32) is standard from the utilization literature (e.g. Finn (1995), Greenwood et al. (1988)).

The model is solved numerically by representing the competitive equilibrium in recursive form (see Mendoza and Smith (2006) and Arellano and Mendoza (2003) for details on algorithms for solving SOE models with collateral constraints). The endogenous state variables of the problem are k and b. These are chosen from discrete grids of NK non-negative values of the capital stock, $K=\{k_1<k_2<\ldots<k_{NK}\}$, and NB values of bond positions, $B=\{b_1<b_2<\ldots<b_{NB}\}$. The exogenous states are the realizations of shocks in the triple $e=(\varepsilon^A,\varepsilon^R,\varepsilon^P)$. The shocks follow a joint Markov process, which defines the set E of all triples of possible realizations of the shocks and their one-step transition probability matrix π. Hence, the state space of the problem is defined by all triples (k,b,e) in the set $K\times B\times E$. An Appendix with the details on the formulation of the recursive representation of the competitive equilibrium is available from the author on request.

5.2 Calibration to Mexican Data

The calibration exercise assigns values to the model's parameters so that the deterministic stationary equilibrium matches key averages from quarterly Mexican data. This exercise adopts two assumptions to make the calibration easier to compare with typical RBC calibrations: (1) the working capital coefficient is set to $\phi=0$ (otherwise working capital payments distort factor shares) and (2) the collateral coefficients κ and κ^f are high enough so that the constraints do not bind at the deterministic steady state.

In the data, the average ratio of GDP to gross output (gdp/y), with gross output defined as GDP plus imported inputs, is 0.896, and the ratio of imported inputs to GDP (pv/gdp) is 0.114. The average share of imported inputs in gross output is 0.102, hence $\eta=0.102$. This factor share, combined with the 0.66 labor share on GDP from Garcia (2005) implies the following factor shares for the production function of gross output: $\alpha=\left(\frac{0.66}{1+(pv/gdp)}\right)=0.592$ and $\beta=1-\alpha-\eta=0.305$.

National Accounts data for investment and GDP at 1993 prices are used to construct a quarterly time series for k that matches the capital-output ratios estimated by Garcia (2005) using annual data. He used data for the period 1970-2000 and applied the standard perpetual inventories method to estimate capital stocks. The average capital-output ratio of his estimates is 1.88 and his 1980 point estimate is 1.56. The quarterly capital stock series constructed here is based also on the perpetual inventories method, and is targeted to produce quarterly estimates of the capital-output ratio that replicate Garcia's annual estimates for 1980 and the 1970-2000 average. The initial condition applies to 1980 because existing quarterly National Accounts data at constant prices begin in that year. The quarterly capital stock estimates match the targets taken from Garcia's series by setting the 1980:01 capital-output ratio to 1.45 and the depreciation rate to the quarterly equivalent of 8.8 percent per year.

The capital stock estimates imply that the 1980:01-2005:02 average ratio of capital to *gross* output is 1.758. Combined with the 0.088 depreciation rate, this capital-gross output ratio yields an average ratio of investment to gross output (i/y) of 15.5 percent. The optimality condition for capacity utilization can then be used together with this estimate of the i/y ratio to obtain an implied value for the elasticity parameter of the utilization cost function: $\theta = \beta/[(i/y)] = 1.97$. This is possible because at steady state the i/y ratio is also equal to the share of utilization costs in gross output (since at steady state $i=\delta(m)k$)). Given the value of θ , and setting the long-run utilization rate at 0.9, the endogenous depreciation rate $\delta(m)$ matches the value of 0.088 obtained in the computation of the capital stock series by setting h=0.214. Thus, the parameters of the depreciation rate function, which are crucial for determining the marginal cost of utilization and the utilization rate, are inferred from National Accounts data and the optimality conditions of firms in a manner similar to that used to set the Cobb-Douglas factor shares.

In the deterministic stationary state, imported input prices and the real interest rate take their mean values. The mean price of imported inputs is set so as to match the price obtained by dividing the mean ratios of imported intermediate goods to gross output at current and constant prices, which is 1.028. The mean value of the annual-equivalent of the gross real interest rate is derived by imposing the values set for β, θ, (i/y), and the depreciation rate on the Euler equation for capital (eq. (13)) evaluated at steady state and solving for R. The resulting expression yields $R=1+[\delta\beta(1-\theta^{-1})]/(i/y))=1.086$. A real interest rate of 8.6 percent may seem relatively high, but in this calibration it represents the *implied* real interest rate that, given the values of δ, β, and θ, supports Mexico's average investment-gross output ratio as a feature of the deterministic steady state of a standard small open economy model with perfect credit markets.

The households' optimality condition for labor supply equates the marginal disutility of labor with the real wage, which at equilibrium is equal to the marginal product of labor. This equilibrium condition reduces to: $L_t^{\omega} = \alpha \exp(\varepsilon_t^A)F(\cdot)$. Using the logarithm of this expression and Mexican data on gross output and employment growth, the implied value of the exponent of labor supply in utility is $\omega = 1.8461$.

Since aggregate demand in the data includes government expenditures, the model needs an adjustment to consider these purchases in order for the deterministic steady state to match the average private consumption-GDP ratio (0.65). This adjustment is done by setting the deterministic steady state to match the average ratio of government purchases to GDP (0.11), assuming that these government purchases are unproductive and paid out of a time-invariant, ad-valorem consumption tax. The tax is equal to the ratio of the GDP shares of government and private consumption, 0.11/0.65=0.168, which is very close to the statutory value-added tax rate in Mexico. Since this tax is time invariant, it does not distort the households' intertemporal decision margins and any distortion on the consumption-leisure margin does not vary over the business cycle.

Given the preference and technology parameters set in the previous paragraphs, the optimality conditions for L, m, and v, and the firms' steady-state Euler equation for capital accumulation are solved as a nonlinear simultaneous equation system in the steady state levels of k, L, v and m. Given these, the levels of gross output and GDP are computed using the production function and the definition of GDP, and the level of consumption is determined by multiplying GDP times the average consumption-GDP ratio in the data. The value of γ follows then from the steady-state consumption Euler equation: $\gamma = \frac{\ln(R)}{\ln(1+c-\omega^{-1}L^{\omega})} = 0.0166$. As is typical in calibration exercises for RBC-SOE models with SCU preferences (see Mendoza (1991)), the value of the time preference coefficient is very low, suggesting that the "impatience effects" introduced by the endogenous rate of time preference have negligible quantitative implications on business cycle dynamics. Finally, the steady-state foreign asset position follows from the household budget constraint (eq. (2)) evaluated at steady state. This implies a ratio of external debt to GDP of about 79 percent.

The only two parameter values that were not determined in this calibration exercise were those for the adjustment cost coefficient a and the working capital coefficient ϕ. The value of a will be set below so that in the stochastic simulations the model without credit constraints matches the observed business cycle variability of investment. This parameter does not affect the deterministic steady state because total and marginal adjustment costs are zero at steady state. The value of ϕ is set to 0.25 in the stochastic simulations, which implies a stock of working capital loans of about 19 percent of GDP in the deterministic steady state. Data on working capital financing for Mexican firms are not available but the 1994:01-2005:01 average of *total* financing to private nonfinancial firms as a share of GDP is 24.4 percent. Note, however, that this measure includes financing at all maturities and for all uses (including long-term investment financing) so it is likely to overestimate actual working capital financing. Still, ϕ=0.25 is significantly lower than the working capital coefficients used in the studies of RBC-SOE models for emerging markets by Neumeyer and Perri (2005) and Uribe and Yue (2006).

5.3 Markov Process of Exogenous Shocks

Price, TFP, and interest rate shocks follow a joint Markov process. Shocks to the real interest rate and the relative price of imported inputs are taken directly from the data as described in Section 2. TFP shocks can be approximated in two ways. One approach is to use the "true" TFP estimates constructed also in Section 2. However, since this measure of TFP can still be inaccurate because of measurement problems affecting factor shares, input prices and implied utilization rates, the simulations follow a second approach akin to an "identification procedure." This approach calibrates the variability and persistence of TFP shocks so that the model matches the variability and persistence of Mexico's GDP.[9] The procedures starts by using the interest rate, price and TFP shocks taken from the data to estimate an unrestricted, first-order VAR, and the VAR is then approximated as a discrete Markov process using Tauchen's (1991) quadrature method. Next, in the identification stage, the elements of the VAR that pertain to TFP are adjusted so that the model matches the business cycle moments of GDP in the data.

Given that the cyclical components have zero mean, the VAR has the following form:

[9] This is similar to an approach used often in RBC analysis to set the moments of productivity shocks so that the models mimic the cyclical moments of GDP (see Greenwood et al. (1988)).

$$e_{t+1} = RHO \cdot e_t + u_{t,}$$

$$e_t \equiv \begin{bmatrix} \varepsilon_t^R \\ \varepsilon_t^P \\ \varepsilon_t^A \end{bmatrix}, \quad RHO \equiv \begin{bmatrix} \rho_R & \rho_{R,P} & \rho_{R,A} \\ \rho_{P,R} & \rho_P & \rho_{P,A} \\ \rho_{A,R} & \rho_{A,P} & \rho_A \end{bmatrix}, \quad u \equiv \begin{bmatrix} u_t^R \\ u_t^P \\ u_t^A \end{bmatrix} \tag{33}$$

The innovations in u are independently and identically distributed over time, with zero mean and a known stationary variance-covariance matrix *cova(u)*. Tauchen's algorithm takes as input the estimates of *RHO* and *cova(u)* and the desired number of elements of the Markov realizations of the shocks, and it returns as output the vectors of realizations and the transition probability matrix π of moving across states.

The VAR analysis cannot reject the hypothesis that the three shocks follow independent first-order autoregressive processes.[10] The resulting estimates of the autocorrelation and covariance matrices are:

$$RHO \equiv \begin{bmatrix} .571861 & 0 & 0 \\ 0 & .737109 & 0 \\ 0 & 0 & .390259 \end{bmatrix}, \quad \text{cova}(u) \equiv \begin{bmatrix} .000215 & 0 & 0 \\ 0 & .000533 & 0 \\ 0 & 0 & .000091 \end{bmatrix} \tag{34}$$

The standard deviations of the shocks implied by these results are: $\sigma(\varepsilon^R)=0.019$, $\sigma(\varepsilon^P)=0.034$ and $\sigma(\varepsilon^A)=0.01$.

Solving the model without collateral constraints and using the Markov representation of the above VAR yields an output process with a standard deviation of 2.94 percent and a first-order autocorrelation coefficient of 0.545 (using a=2.6 so as to match the observed standard deviation of investment).[11] The corresponding statistics in the data are 2.72 percent and 0.749 respectively (see Table 1). Hence, with the TFP series constructed in Section 2, the model is close to matching the variability of output but it underestimates its persistence. The identification procedure shows that the model can match the variability and persistence of GDP by setting $\sigma(\varepsilon^A)=0.009$ and $\rho(\varepsilon^A)=0.65$ (with a=2.65 to match the observed variability of investment). Thus, the model can mimic the variability and persistence of Mexico's GDP by reducing the standard deviation of TFP shocks by 1/10 of a percentage point and by increasing their autocorrelation from 0.39 to 0.65, relative to the moments of the TFP series constructed with actual data.

5.4 Baseline Results

The baseline experiment compares the results of a simulation in which the collateral constraints never bind (the "frictionless model") with those from three scenarios in which the constraints bind in some states of nature: one with both collateral constraints, one with the constraint on foreign debt only, and one with the constraint on working capital

---

[10] In this case, a simpler way to obtain the Markov approximation of the shocks is by using the "simple persistence" rule, which uses two symmetric points for each Markov process set at ±1 standard deviation, and transition probabilities set to yield the observed autocorrelation of each shock. See Mendoza (1995) for details.
[11] The realizations and transition probabilities of the Markov chains are available from the author.

only. All the simulations use a K grid with 50 nodes and a B grid with 72 nodes, both evenly spaced. There are 2 realizations for each Markov shock, and hence 8 triples of realizations of shocks. Hence, the discrete state space of the model has 52×100×8 coordinates that represent all possible combinations of k, b and e.[12]

Panel I of Table 3 lists the statistical moments that characterize the stochastic steady state of the frictionless economy, computed with the corresponding limiting distribution of k, b, and e. These moments are generally consistent with standard results from RBC-SOE models. As indicated earlier, however, the frictionless model cannot produce Sudden Stops because it assumes that credit markets are perfect. In particular, as shown below, this economy responds to adverse shocks with a relatively smooth adjustment of the current account and "normal" recessions, regardless of the size of debt and working capital financing or the leverage ratios.

Panels II-IV of Table 3 list business cycle moments for the economies with collateral constraints. Panel II shows moments for the economy with the two collateral constraints, based on a simulation with $\kappa = 0.248$ and $\kappa^f = 0.139$. These collateral coefficients imply that the debt leverage ratio ($-b_{t+1}/q_t k_{t+1}$) cannot exceed 24.8 percent and the working capital leverage ratio ($wcap_t/q_t k_{t+1}$, where $wcap_t = \phi(w_t L_t + p_t v_t + \delta(m_t) k_t)$ cannot exceed 13.9 percent. These values of the collateral coefficients are 1½ standard deviations below the mean of the corresponding leverage ratio in the frictionless economy. With these values, the joint probability of hitting both collateral constraints predicted by the model matches the 1.96 percent frequency of Sudden Stops observed in the 1980:1-2005:2 sample of quarterly Mexican data (counting the 1982 Debt Crisis and the 1995 crash as Sudden Stops). The collateral constraint on debt binds with 2 percent probability, and that on working capital binds with 14 percent probability.

The high leverage states at which the economy is vulnerable to Sudden Stops are reached after sequences of realizations of the shocks lead the leverage ratios to approach their ceilings. Because of the curvature of the constant-relative-risk-aversion period utility function, households accumulate precautionary savings to self insure against the risk of large consumption collapses, and hence the long-run probability of states of nature where Sudden Stops are possible is low. As a result, the long-run business cycle indicators in Panel II do not differ much from those of the frictionless economy in Panel I. This result also holds for Panels III and IV for economies that use only one collateral constraint. Hence, the long-run features of business cycles are largely the same with and without the collateral constraints.

The next task is to show that in the economies with collateral constraints Sudden Stops coexist with the normal business cycles summarized in Table 2. To show that this is the case, Figures 4.a and 4.b plot conditional forecast functions of the equilibrium Markov processes of macroeconomic aggregates in response to combined one-standard-deviation shocks to the world interest rate, imported input prices and TFP at t=1. These forecast functions are Markovian impulse response functions that preserve the non-linearity of the decision rules that characterize the equilibrium of the model. The forecast functions are

[12] A simulation with 100 nodes in the K grid and 144 nodes in the B grid showed negligible differences in the first and second moments of the endogenous variables, but CPU time increased by a factor of 7 (using parallelized code and dual Pentium Xeon 3.6Mhz processors).

conditional on initial conditions of high leverage in debt and working capital inside the "Sudden Stop region" of the state space, defined as the region in which adverse shocks trigger a Sudden Stop with positive long-run probability. Since initial conditions in this class are distant from the long-run averages of k and b, the low-frequency dynamics driving these variables back to long-run averages has been removed by plotting the difference between the forecast functions of each economy with collateral constraints and those of the frictionless model, in percent of the means in the latter. The plots show data only for the first 36 quarters after the shocks hit. The initial conditions feature a debt ratio of 51 percent of GDP and a debt leverage ratio of 26.8 percent. The economy with both collateral constraints hits this particular initial state with 0.0012 probability in the long run.[13]

In the economy with both collateral constraints, Figures 4a-4b show that the Sudden Stop impact effects at are: -3 percent in GDP, -5.5 percent in working capital, -3 percent in labor, -5 percent in imported inputs, -2.5 percentage points in capacity utilization, -2.9 percent in "effective capital" (mk), -8 percent in consumption, -32 percent in investment, -7 percent in asset prices, and a current account reversal of 10 percentage points of GDP. Output, factor demands, utilization and working capital recover somewhat in the next period, but the Sudden Stop effects persist because these variables converge to their values in the frictionless economy only by the 24^{th}–30^{th} quarter. Note also that, while the impact effect on capacity utilization is negative, after that firms substitute away from factor demands into more intensive use of existing capital, and this effect is also persistent. Thus, beyond its impact effect contributing to enlarge the initial output collapse, endogenous utilization actually works to weaken Sudden Stop effects. Figure 4b shows that the Sudden Stop effects on consumption, investment, the Tobin Q and the current account dissipate by the 10^{th} quarter. This Figure also shows that the Sudden Stop induces the expected tilt in the time profile of consumption, with lower consumption than in the frictionless economy for the first 15 periods and converging thereafter to a slightly higher consumption level.

A comparison of Sudden Stop dynamics across the three economies with collateral constraints shows that the constraints on debt and working capital need to be combined in order to obtain realistic Sudden Stops. The collateral constraint on debt alone cannot generate declines in output, factor demands or working capital on impact because the capital stock as of this date is predetermined and there is no feedback via working capital. Hence, the "supply side" of the economy that includes only the debt constraint responds on impact to the shocks in the exact same way as the frictionless case. There are Sudden Stop impact effects on consumption, investment, the Tobin Q and the current account, and these are of similar magnitude as those of the economy with the two constraints. After the impact effects, the initial investment and asset price collapses affect supply-side dynamics, and these effects are qualitatively similar but quantitatively weaker than those of the economy with the two collateral constraints.

---

[13] Since economies with the collateral constraint on working capital feature slightly lower long-run averages of consumption, output, factor demands, capacity utilization, and working capital than the frictionless economy, the plots for these economies do not converge to zero (i.e., the variables do not converge to the means of the frictionless economy).

Figures 4a-4b indicate that the economy with the collateral constraint on working capital alone does not produce Sudden Stops. The responses of this economy differ only slightly from those of the frictionless economy. This is because the working capital constraint on its own does not bind when the shocks hit. Hence, the amplification and persistence obtained in the Sudden Stop effects on the supply side of the economy with the two constraints are caused by the feedback from the debt collateral constraint to the working capital collateral constraint: Collapsing asset prices and investment trigger the collateral constraint on working capital and induce a substantial downward adjustment in the supply side of the economy. Moreover, this result shows that the role of working capital in this model is very different from that in the models of Neumeyer and Perri (2005) and Uribe and Yue (2006). The key feature here is that access to working capital is suddenly reduced by feedback effects between endogenous collateral constraints. Working capital per se, or even a stand-alone collateral constraint on working capital, do not produce responses to shocks that differ markedly from those of the frictionless economy.

Kocherlakota (2000) argued that the analysis of business cycle effects of credit constraints should focus on amplification, asymmetry and persistence in the responses to exogenous shocks. The amplification and persistence of Sudden Stops effects in the model of this paper are illustrated in Figures 4a-4b. In addition, it is possible to compute amplification coefficients comparable with Kocherlakota's measure of amplification described in Section 4. Given the stochastic nature of the model, amplification coefficients are computed here as the difference between the values of the variables in the frictionless economy and those pertaining to the economies with collateral constraints, in percent of the standard deviation of each variable in the frictionless economy. Hence, these coefficients measure how much larger are the recessions of a Sudden Stop in units of the standard deviations that measure normal business cycles. Figures 5-7 plot these amplification coefficients for GDP, consumption, investment, the Tobin Q, working capital and capacity utilization for the first 36 quarters after the same shocks considered in Figures 4a-4b hit the economy (using the same initial conditions).

The amplification effects are significantly larger than Kocherlakota's. In the economy with the two collateral constraints, the initial amplification effects are near 100 and 300 percent for GDP and consumption respectively (Figure 5), over 300 percent for investment and the Tobin Q (Figure 6), and about 175 percent for working capital and utilization (Figure 7). These estimates imply that GDP (the price of capital) is lower in the economy with collateral constraints than in the frictionless economy by a deviation from the mean that is 1.1 (3) times larger than the standard deviation of output (the price of capital) in the frictionless economy. Moreover, amplification occurs also beyond the impact effect because of the persistence of the Sudden Stop effects.

The asymmetry of the effects of the collateral constraints on GDP is illustrated in Figure 8, which plots the histogram of the differences in the GDP deviations from means in the economy with the two collateral constraints relative to the frictionless economy using data from a stochastic time-series simulation with 10,000 periods. The histogram shows that there is a bias to the left of the mean. The skewness coefficient is -1.88, indicating that the distribution of GDP has a long left tail.

Figures 9-10 show surface plots of impact effects (i.e., deviations from long-run means on the initial date that the shocks hit the economy) for the entire state space of (k,b)

pairs, instead of the particular pair used in Figures 4a-4b to illustrate a Sudden Stop scenario. Figure 9 shows consumption impact effects. The plot on the left is for the economy with perfect credit markets and the plot on the right is for the economy with collateral constraints. Figure 10 shows similar plots for the impact effects on Tobin's Q.[14]

Figure 9 shows large drops in consumption in response to adverse shocks of standard size in the high-leverage area of (k,b) pairs where the collateral constraints bind. These consumption collapses are much larger than the mild recessions that these shocks cause in the same economy outside of this area, or in the same area but in the economy with perfect credit markets. The impact effects on consumption are nearly identical in the two economies for high values of b that are outside the region in which collateral constraints bind. Note that, because of the precautionary saving effect, the area of very large consumption collapses in the economy with collateral constraints includes many states that have zero probability in the long run. These states can be interpreted as representing outcomes that the model predicts to be the result of "large, unexpected" shocks (which can be viewed as shocks that move the economy to states with leverage ratios that otherwise would have zero probability in the long run). In response to shocks of this type, the model can predict massive consumption collapses of up to 60 percent!

An alternative interpretation of the Sudden Stop scenarios ruled out in the long run by precautionary saving is that they represent scenarios that economies could face in the short run if financial liberalization is adopted when leverage ratios are high. In this situation, a country that has yet to build up a large enough stock of precautionary savings is more vulnerable to suffer a Sudden Stop. Interestingly, the majority of emerging economies have built up large stocks of foreign exchange reserves in recent years that reflect in part their efforts to self insure against future Sudden Stops.

Figure 10 shows the effects of the Fisherian deflation on asset prices. Adverse shocks of typical size cause small declines in the price of capital when credit markets are perfect, or with collateral constraints but outside the area of the state space where they bind. On the other hand, when leverage ratios are high enough for the constraints to bind, the shocks trigger the collateral constraint and this results in agents fire-selling capital. The price of capital sinks below the value that would have prevailed in the economy with perfect credit markets, and this tightens further the collateral constraint, inducing agents to reduce further their capital holdings and cause further price declines. The plot in the right-hand-side of Figure 10 shows the end result of this process. The downward spiral in the price of capital is hampered by the negative wealth effect induced by the adverse effect of reduced investment on future output. Note also that, as before, the states of nature with the largest equity price and investment collapses in Figure 10 are ruled out in the long run by precautionary saving (since they lead to states with very low consumption).

In summary, the economy with the two collateral constraints accounts for several features of Sudden Stops. Unusually large and infrequent recessions can take place in response to shocks of standard magnitude when the economy is highly leveraged, and

[14] The region with the lowest b values in the plots of the economy with collateral constraints cannot support equilibrium allocations because, for the corresponding (k_t,b_t) pairs, there is no (k_{t+1},b_{t+1}) pair in $K\times B$ that can keep c_t positive and at the same time satisfy the collateral constraint.

these Sudden Stops are nested within normal business cycles. The economy arrives at these high-leverage states with positive long-run probability, and in these states binding collateral constraints cause significant amplification, asymmetry and persistence in macroeconomic responses to shocks. One weakness, however, is that the decline in asset prices is about $1/3^{rd}$ the size of the asset price collapse observed in Mexico. Still, the Sudden Stop effects reported in this Section are in response to one-standard-deviation shocks, whereas the price and interest rate shocks observed at the beginning of the 1995 Sudden Stop in Mexico were significantly larger.

5.5 Sensitivity Analysis

The remainder of this Section reports the results of a sensitivity analysis that highlights the key determinants of the model's Sudden Stop effects. This analysis focuses on three issues: the role of endogenous v. exogenous credit constraints, the implications of separate shocks to imported input prices, the world interest rate and TFP, instead of joint shocks, and the implications of removing from the model (one at a time) working capital, endogenous utilization and imported inputs.

Table 4 compares Sudden Stop effects across economies with endogenous and exogenous credit constraints. The Table shows responses in macroeconomic aggregates (as defined to construct Figures 4a-4b) to joint negative one-standard-deviation shocks in the first two periods for three economies: one with both debt and working capital constraints, one with the debt constraint alone, and one with exogenous credit limits. To make the experiments comparable, the exogenous limits are set as ad-hoc credit limits equal to the lowest b and the highest $wcap$ with positive probability in the limiting distribution of the economy with the two endogenous collateral constraints. The results show that the debt-deflation mechanism of the endogenous collateral constraints is crucial for the model's performance. Sudden Stop effects on impact and one period later are several orders of magnitude smaller when the endogenous collateral constraints are replaced by ad-hoc credit limits. Thus, exogenous credit limits produce significantly weaker amplification and persistence than the endogenous collateral constraints. The ad-hoc limits eliminate both the effects of the Fisherian deflation on the debt constraint *and* the feedback from this constraint to the working capital constraint. The former can be seen in that the date-0 responses of i, q and ca/y are similar in the economies with collateral constraints but much smaller in the economy with ad-hoc credit constraints. The fact that ad-hoc credit limits eliminate also the feedback from the debt constraint to the working capital constraint is illustrated in that the impact effects on output, factor demands, utilization and working capital vanish. The ad-hoc limit on working capital is not triggered by the ad-hoc debt limit, and hence the economy with ad-hoc credit limits cannot produce contemporaneous supply side effects.

Table 5 shows date-0 and date-1 impact effects of separate one-standard-deviation shocks to the world interest rate, imported input prices and TFP. Naturally, the effects are weaker than in the baseline case which considers the combined impact of the three shocks. The interesting implication of the results in the Table is that interest rate and price shocks generate larger Sudden Stop effects than TFP shocks. Considering output, for example, the Sudden Stop effects due to an interest rate (price) shock exceed those due to a TFP shock by a factor of 2 (1.4). Interest rate shocks generate the largest effects on consumption and supply side variables, while price shocks generate the largest effects on investment and asset prices.

Table 6 compares the Sudden Stop effects of the baseline exercise (Panel A) with those produced by alternative versions of the model that exclude working capital (Panel B), capacity utilization (Panel C) or imported inputs (Panel D). The Table also lists the long-run probabilities of hitting each of the collateral constraints. Since the collateral constraint on working capital does not bind in the simulations that use this constraint alone, Table 6 reports results only for cases with the two collateral constraints or the debt constraint alone (or just the debt constraint for the economy without working capital).

A comparison of Panels A and B reaffirms the important role that working capital, and the feedback from the debt constraint to the working capital constraint, play in the model's ability to generate large amplification effects during Sudden Stops. This is true not just for the supply side variables that show zero amplification when working capital is removed from the model (Panel B) but also for investment, Tobin's Q and the current-account output ratio, which show larger date-0 Sudden Stop effects when the two constraints are at play (Panel A) than in the economy without working capital.

Panels A and C show that the main contribution of endogenous utilization is that it enlarges the date-0 Sudden Stop effect on output because of the reductions in utilization and effective capital allowed in the baseline case. The resulting contribution to the output collapse is about 2/3rds of a percentage point, which is equivalent to enlarging the date-0 output effect in the economy with constant utilization by 25 percent. This contribution could be larger if the coefficients of the utilization cost function are altered to increase the endogenous response of utilization to the shocks.

The results in Panels A and D indicate that imported intermediate goods are crucial for the baseline results. Removing imported inputs has two important effects: it changes the production technology and it removes the shock on imported input prices that has direct effects on factor costs and working capital. As a result, the binding debt constraint in the economy without imported inputs (Panel D) can no longer trigger the collateral constraint on working capital at the same initial conditions that apply to the baseline case (Panel A). Since the constraint on working capital does not bind, all the date-0 amplification effects on supply side variables are lost, and the effects on the rest of the variables are weakened.

6. Conclusions

This paper shows that the quantitative predictions of an equilibrium business cycle model with collateral constraints are consistent with key features of the Sudden Stop phenomenon. The model features two collateral constraints: A constraint that limits debt not to exceed a fraction of the value of the collateral assets, and a constraint that limits working capital financing not to exceed a fraction of the value of the firms' assets. These constraints only bind in states of nature in which the ratios of debt and/or working capital to asset values are sufficiently high. In turn, these high-leverage states are an endogenous outcome of the model's stochastic competitive equilibrium and, despite strong precautionary saving effects, they remain a positive probability event even in the long run.

The motivation for introducing collateral constraints on both debt and working capital originates in observations from Mexico's Sudden Stop of 1995 suggesting that explanations of Sudden Stops need to connect two sets of facts: First, the large reversal of the current account, the loss of access to credit, and the collapse of equity prices are strong indications that financial frictions are a central feature of Sudden Stops. Second, growth accounting indicates that declines in imported inputs, capacity utilization, and TFP account for the bulk of the large output collapse in its early stages, while capital and labor play a relatively small role. In the model of this paper, the collateral constraint on debt introduces Fisher's debt-deflation mechanism, and the constraint on working capital links credit frictions to factor demands and capacity utilization. The combined effects of these constraints produce quantitatively significant amplification, asymmetry and persistence in the responses of macroeconomic aggregates to shocks of the same magnitude that drive normal business cycles. Because of precautionary saving, Sudden Stops are infrequent events nested within normal business cycles in the stochastic stationary equilibrium of the model.

In simulations calibrated to Mexican data, the long-run business cycle moments of economies with and without credit constraints differ marginally, while the responses to one-standard-deviation shocks conditional on a positive-probability initial state with high leverage ratios differ sharply across the two economies. In particular, the economy with the two collateral constraints produces Sudden Stops with initial declines in consumption and investment similar to those observed in Mexico, a larger reversal in the current account and smaller collapses in output and asset prices. Still, credit-driven amplification effects on the responses of *all* macroeconomic aggregates are significantly larger than those reported in previous studies. Thus, this paper shows that explanations of Sudden Stops need not rely on large, unexpected shocks, and that credit constraints can be used to integrate a theory of business cycles with a theory of Sudden Stops within the same dynamic stochastic general equilibrium framework.

The collateral constraints introduce three credit channel effects. Two of these effects are in the form of endogenous external financing premia that emerge when the constraints bind. These premia reflect the effective real interest rates that lead households and firms to choose levels of debt and working capital that satisfy their credit constraints. The third credit channel is the debt-deflation mechanism. This mechanism plays a key role in the ability of the model to generate large Sudden Stop effects. In a high-leverage state of the economy, adverse shocks of standard magnitudes that would result in RBC-like responses under perfect credit markets trigger the collateral constraint on debt. This causes a fall in physical investment and equity prices which tightens further the constraint and leads to a spiraling collapse of credit, asset prices and investment. Accordingly, Sudden Stop effects in experiments that replace the collateral constraints with exogenous credit limits are much weaker that in the experiments in which the debt-deflation mechanism operates. The debt-deflation process also induces significant feedback from the collateral constraint on debt to the one on working capital, as the decline in the value of collateral assets triggers the limit on access to working capital forcing firms to cut factor demands and reduce capacity utilization. In contrast, the collateral constraint on working capital is of little consequence when introduced without the debt constraint.

Further research on this subject can go in several directions. One interesting extension would be to study the "liability dollarization effect," caused by the fact that the foreign debt of emerging economies is denominated in hard currencies (i.e. tradables goods) but largely leveraged on assets and incomes in domestic currencies and generated by non-tradables industries. In this case, the debt-deflation mechanism can operate through a fall in the relative price of nontradables and lead to a relative price collapse that causes a financial crash. However, since the key aspects of the "liability dollarization" effect are (1) the difference in the units in which debt contracts and incomes and assets securing those contracts are denominated, and (2) sharp movements in the relative price of those two units, the "liability dollarization" effect could also operate via changes in prices of imported inputs if the firms' working capital loans are denominated in units of a world tradable good. In this case, a sharp increase in the price of imported inputs relative to this numeraire increases not just the direct cost of inputs but also the real financing costs faced by firms.

The findings of this paper suggest that the key to reducing the probability of Sudden Stops in emerging economies is in promoting the attainment of levels of financial and institutional development that weaken the contractual frictions behind collateral constraints. In contrast, taking as given the underlying uncertainty in the form of aggregate shocks to TFP, world interest rates and relative prices, tighter capital requirements or "value-at-risk" targets, designed to manage exposure to idiosyncratic risk, can be counterproductive and rise the probability of observing Sudden Stops.

Other policy conclusions derived from this analysis relate to financial contagion and the desirability of holding large stocks of foreign reserves. In the setup of this paper, an emerging economy can have solid domestic policies and competitive, open markets, and still reach a point of high leverage at which a Sudden Stop is caused by a relatively modest increase in the world interest rate, or in the relative price of imported inputs, due to developments elsewhere in the world. If waiting for financial development to eliminate this problem seems naïve, and since tighter credit limits can make things worse, self insurance in the form of a sufficiently large stock of foreign reserves can be a useful way of lowering the probability of facing Sudden Stops that the model favors. Another alternative to explore is a mechanism that could be set at international financial organizations to help emerging economies maintain access to credit by preventing asset prices from collapsing during situations of financial contagion (see, for example, Calvo's (2002) proposal).

References

Aiyagari, S. Rao and Mark Gertler (1999), "Overreaction of Asset Prices in General Equilibrium," *Review of Economic Dynamics*.

Arellano, Cristina (2005), "Default Risk and Aggregate Fluctuations in Emerging Economies," mimeo, University of Minnesota.

Auenhaimer, Leonardo and Roberto Garcia-Saltos (2000), "International Debt and the Price of Domestic Assets, " IMF Working Paper 00/177.

Bernanke, Ben, Mark Gertler, and Simon Gilchrist (1999), "The Financial Accelerator in a Quantitative Business Cycle Framework," in J. Taylor and M. Woodford eds. *Handbook of Macroeconomics*, Volume 1C, ed. by, North-Holland.

Calvo, Guillermo A., (1975), "Efficient and Optimal Utilization of Capital Services," *American Economic Review*, v. 65, no.1, March, 181-186.

______, (1998), "Capital Flows and Capital-Market Crises: The Simple Economics of Sudden Stops," *Journal of Applied Economics*, v.1, pp 35-54.

______, (2002), "Globalization Hazard and Delayed Reform in Emerging Markets," *Economia*, Spring, v. 2, no. 2.

______, and Carmen M. Reinhart (1999), "When Capital Inflows come to a Sudden Stop: Consequences and Policy Options," mimeo, University of Maryland.

______, Alejandro Izquierdo, and Rudy Loo-Kung (2006), "Relative Price Volatility under Sudden Stops: The Relevance of Balance-Sheet Effects,"*Journal of International Economics*, v. 69, pp. 231-254.

Choi, Woon Gyu and David Cook (2003), "Liability Dollarization and the Bank Balance Sheet Channel," *Journal of International Economics*.

Cooley, Thomas, Ramon Marimon and Vincenzo Quadrini (2004), "Aggregate Consequences of Limited Contract Enforceability," *Journal of Pol. Economy*, 111(4).

Cook, David and Devereux, Michael B., (2006a), "External Currency Pricing and the East Asian Crisis," *Journal of International Economics*, v. 69, pp. 37-63.

Cook, David and Devereux, Michael B., (2006b), "Accounting for the East Asian Crisis: A Quantitative Model of Capital Outflows in a Small Open Economies," *Journal of Money, Credit and Banking*, forthcoming.

Dunbar, Nicholas, (2000) *Inventing Money: The Story of Long-Term Capital Management and the Legends Behind it,* John Wiley & Sons Ltd., West Sussex: England.

Epstein Larry G. (1983), "Stationary Cardinal Utility and Optimal Growth under Uncertainty," *Journal of Economic Theory*, 31, 133-152.

Fisher, Irving (1933), "The Debt-Deflation Theory of Great Depressions," *Econometrica* 1, 337-57.

Finn, Mary G. (1995), "Variance Properties of Solow's Productivity Residual and their Cyclical Implications," *Journal of Economic Dynamics and Control*, 19, 1249-1281.

Garcia-Verdu, Rodrigo (2005), "Factor Shares from Household Survey Data," mimeo, Research Department, Banco de Mexico.

Gopinath, Gita (2003), "Lending Booms, Sharp Reversals and Real Exchange Rate Dynamics," *Journal of International Economics*.

Greenwood, Jeremy, Zvi Hercowitz and Gregory W. Huffman (1988), "Investment, Capacity Utilization & the Real Business Cycle", *American Economic Review*, June.

Izquierdo, Alejandro (2000) "Credit Constraints, and the Asymmetric Behavior of Asset Prices and Output under External Shocks," mimeo, The World Bank. June.

Jermann, Urban and Vincenzo Quadrini (2005), "Financial Innovations and Macroeconomic Volatility," mimeo, Wharton School of Business.

Hayashi, Fumio (1982), “Tobin’s Marginal q and Average q: A Neoclassical Interpretation,” *Econometrica*, v. 50, no. 1, 213-224.

International Monetary Fund (2002), *World Economic Outlook*, September.

Kiyotaki, Nobuhiro and John Moore (1997), “Credit Cycles,” *Journal of Political Economy*, v.105, 211-248.

Kocherlakota, Narayana (2000) “Creating Business Cycles Through Credit Constraints,” *Federal Reserve Bank of Minneapolis Quarterly Review*, v. 24, no. 3, Summer, 2-10.

Lane, Philip R. and Gian Maria Milesi-Ferretti, (2001), “The External Wealth of Nations: Measures of Foreign Assets and Liabilities for Industrial and Developing Countries,” *Journal of International Economics*, 55, 263-294.

Martin, Phillippe and Helene Rey, (2006), “Globalization and Emerging Markets: With or Without Crash?,” *American Economic Review*, forthcoming.

Mendoza, Enrique G. (2002), “Credit, Prices, and Crashes: Business Cycles with a Sudden Stop.” In *Preventing Currency Crises in Emerging Markets*, Frankel, Jeffrey and Sebastian Edwards eds. Chicago: University of Chicago Press.

______, (1995), “The Terms of Trade, The Real Exchange Rate and Economic Fluctuations,” *International Economic Review*, v. 36, 101-137, February.

______, (1991), “Real Business Cycles in a Small Open Economy,” *American Economic Review*, v. 81, 797-818, September.

_____, and Katherine A. Smith (2006), “Quantitative Implications of a Debt-Deflation Theory of Sudden Stops & Asset Prices,” *Journal of Int. Econ.,* forthcoming.

Milesi-Ferretti, Gian Maria, and Assaf Razin (2000), “Current Account Reversals and Currency Crises: Empirical Regularities.” In *Currency Crises*, Paul Krugram ed. Chicago: University of Chicago Press.

Neumeyer, P. Andres and Fabrizio Perri (2005), “Business Cycles in Emerging Economies: The Role of Interest Rates,” *Journal of Monetary Economics*, forthcoming.

Oviedo, P. Marcelo (2004), “Intermediation of Capital Inflows: The Macroeconomic Implications of Neoclassical Banks &Working Capital,” mimeo, Iowa State University.

Paasche, Bernhard (2001), “Credit Constraints and International Financial Crises,” *Journal of Monetary Economics*, v.28, 623-650.

Schmitt-Grohe, Stephanie and Martin Uribe (2002), “Closing Small Open Economy Models,” *Journal of International Economics*.

Tauchen, George, (1991), “Quadrature-based Methods for Obtaining Approximate Solutions to Nonlinear Asset Pricing Models,” *Econometrica*, v. 59, no. 2, 371-396.

Uribe, Martin and Zhanwei Vivian Yue, (2006), “Country Spreads and Emerging Countries: Who Drives Whom?,” *Journal of International Economics*, v. 69, 6-36.

Table 1. Mexico: Business Cycle Statistics and the Sudden Stop of 1995

variable	standard deviation	standard dev. relative to GDP	correlation with GDP	first-order autocorrelation	Sudden Stop (date in brackets)	Sudden Stop relative to standard dev.
GDP	2.723	1.000	1.000	0.749	-8.315 (1995:2)	3.053
intermediate goods imports	7.850	2.882	0.905	0.759	-27.229 (1995:2)	3.469
private consumption						
total	3.397	1.247	0.895	0.701	-8.175 (1995:3)	2.407
non durables & services	2.490	0.914	0.893	0.676	-5.649 (1995:2)	2.269
investment	9.767	3.586	0.944	0.816	-30.074 (1995:3)	3.079
current account-GDP ratio	1.560	0.573	-0.754	0.720	3.838 (1995:2)	2.460
equity prices	14.648	5.379	0.570	0.640	-27.397 (1995:2)	1.870
intermediate goods prices	3.345	1.228	-0.377	0.720	5.915 (1995:1)	1.768
world real interest rate	1.958	0.719	-0.590	0.572	6.752 (1995:2)	3.448

Note: The data were expressed in per capita terms, logged and detrended with the Hodrick-Prescott filter. Equity prices are in units of the GDP deflator. Intermediate goods prices are defined as the ratio of the deflator of imported intermediated goods divided by the exports deflator. "Sudden Stop" corresponds to the lowest deviation from trend observed in the corresponding variable (for the current account-GDP ratio it is the largest change in percentage points observed in two consecutive quarters). The world real real interest rate is the sum of the return on 3-month U.S. T bills plus the EMBI+ spread for Mexican sovereign debt minus a measure of expected U.S. CPI inflation (see Uribe and Yue (2005) for details). The data are for the period 1993:1-2005:2, except the Uribe-Yue real interest rate, which is for the period 1994:1-2004:1.

Table 2: Amplification Effects of the Debt-Deflation Mechanism

	Output	Tobin Q	Consumption	Investment GDP Ratio	Current Account GDP Ratio	Amplification coefficients (Kocherlakota's measure) Output	Amplification coefficients (Kocherlakota's measure) Tobin Q
				A. Baseline case			
$\kappa^L = 0.102$	-0.58	-4.45	-2.59	-6.57	9.18	-21.53	-85.66
				B. Exogenous borrowing constraint set at $\kappa^L \times 7.55$ [1]			
$\kappa^L = 0.102$	-0.44	-3.34	-1.75	-4.94	6.73	-16.15	-64.33
				C. Initial capital 1/3rd below steady state capital			
$\kappa^L = 0.112$	-1.76	-12.60	-7.70	-16.52	25.08	-65.72	-293.63
				D. High Adjustment Costs $(a = 100)$			
$\kappa^L = 0.099$	-0.06	-17.30	-9.26	-0.70	9.89	-2.30	-345.28
				E. Low Adjustment Costs $(a = 0.066)$			
$\kappa^L = 0.104$	-2.27	-0.46	-0.56	-26.37	26.98	-85.10	-8.56
$s =$				*F. Changing the size of the shocks (baseline with* κ *=0.102)*			
0.02	-0.52	-3.95	-2.30	-5.79	8.11	-26.02	-103.59
0.04	-0.70	-5.33	-3.11	-7.98	11.12	-17.58	-69.87
0.06	-0.89	-6.72	-3.93	-10.27	14.26	-14.81	-58.73

Note: Amplification effects are measured as differences between economies with and without binding collateral constraint in percent of the value of each variable in the economy without binding collateral constraint. The effects for investment and the current account are changes in percentage points of GDP. Kocherlakota coefficients are differences between economies with and without binding collateral constraints in percent of the size of the shock (sk_0^{β}). All the effects are for date 0 except for output effects that correspond to date 1.

[1] 7.55 is the date-0 value of capital in the economy with perfect credit markets.

Table 3. Long-Run Business Cycle Moments in the Baseline Simulations

variable	standard deviation (in percent)	standard deviation relative to GDP	correlation with GDP	first-order autocorrelation
I. Frictionless Economy				
GDP	2.71%	1.000	1.000	0.744
consumption	2.88%	1.061	0.763	0.842
investment	9.80%	3.611	0.502	0.486
net exports-GDP ratio	2.38%	0.878	0.015	0.585
capital stock	2.97%	1.095	0.502	0.970
foreign assets-GDP ratio	13.28%	4.894		
equity prices	1.93%	0.711	0.244	0.448
debt-value of capital ratio	6.88%	2.535		
intermediate goods	5.08%	1.873	0.779	0.741
capacity utilization	1.48%	0.544	0.482	0.682
working capital	2.94%	1.085	0.985	0.741
working cap.-asset ratio	0.54%	0.200	0.031	0.637
Savings-investment correlation		0.642		
GDP-world interest rate correlation		-0.338		
GDP-int. goods price correlation		-0.314		
II. Economy with Collateral Constraints on Debt (24.8%) & Working Capital (13.9%)				
GDP	2.73%	1.000	1.000	0.759
consumption	2.85%	1.045	0.789	0.836
investment	10.39%	3.808	0.478	0.487
net exports-GDP ratio	2.45%	0.898	0.001	0.567
capital stock	3.29%	1.205	0.549	0.972
foreign assets-GDP ratio	12.52%	4.591		
equity prices	2.06%	0.756	0.229	0.455
debt-value of capital ratio	6.50%	2.384		
intermediate goods	5.05%	1.853	0.776	0.746
capacity utilization	1.48%	0.541	0.388	0.677
working capital	3.01%	1.105	0.976	0.754
working cap.-asset ratio	0.55%	0.200	-0.057	0.643
Savings-investment correlation		0.589		
GDP-world interest rate correlation		-0.386		
GDP-int. goods price correlation		-0.291		
prob. of binding debt constraint		2.000%		
prob. of binding working cap. constraint		16.100%		
III. Economy with Collateral Constraint on Debt (24.8%)				
GDP	2.73%	1.000	1.000	0.747
consumption	2.82%	1.035	0.791	0.835
investment	9.81%	3.601	0.508	0.486
net exports-GDP ratio	2.31%	0.849	0.003	0.562
capital stock	3.03%	1.111	0.509	0.971
foreign assets-GDP ratio	11.89%	4.363		
equity prices	1.93%	0.707	0.251	0.445
debt-value of capital ratio	6.16%	2.261		
intermediate goods	5.09%	1.868	0.780	0.742
capacity utilization	1.48%	0.545	0.467	0.685
working capital	2.95%	1.084	0.985	0.743
working cap.-asset ratio	0.54%	0.199	0.018	0.637
Savings-investment correlation		0.643		
GDP-world interest rate correlation		-0.337		
GDP-int. goods price correlation		-0.314		
prob. of binding debt constraint		2.16%		
IV. Economy with Collateral Constraint on Working Capital (13.9%)				
GDP	2.71%	1.000	1.000	0.756
consumption	2.90%	1.070	0.761	0.842
investment	10.36%	3.821	0.471	0.488
net exports-GDP ratio	2.52%	0.928	0.014	0.588
capital stock	3.23%	1.192	0.542	0.971
foreign assets-GDP ratio	13.91%	5.129		
equity prices	2.06%	0.760	0.221	0.457
debt-value of capital ratio	7.22%	2.662		
intermediate goods	5.04%	1.858	0.775	0.745
capacity utilization	1.47%	0.542	0.403	0.675
working capital	3.00%	1.105	0.976	0.752
working cap.-asset ratio	0.55%	0.202	-0.043	0.643
Savings-investment correlation		0.587		
GDP-world interest rate correlation		-0.387		
GDP-int. goods price correlation		-0.290		
prob. of binding working cap. constraint		15.76%		

Note: Standard deviations are percentages of the corresponding mean, except for variables defined as ratios.

Table 4. Sudden Stop Effects in Response to Joint One-Standard-Deviation Shocks
(differences relative to frictionless economy in percent of frictionless averages)

	Collateral Constraints 1/				Ad-hoc debt limits 2/	
	both constraints		debt only			
	date 0	date 1	date 0	date 1	date 0	date 1
gdp	-2.937	-1.093	0.000	-0.893	0.000	-0.190
c	-8.504	-0.486	-6.103	-0.323	-0.989	-0.552
i	-33.045	-3.833	-27.628	-2.676	-4.605	-2.820
q	-6.906	-0.646	-6.906	-0.444	-1.151	-0.649
ca/gdp	10.154	0.917	9.888	0.778	1.648	0.928
tb/gdp	10.261	0.010	9.888	-0.134	1.648	0.778
L	-3.067	-0.701	0.000	-0.495	0.000	-0.125
v	-5.233	-1.236	0.000	-0.868	0.000	-0.223
m	-2.682	0.602	0.000	0.784	0.000	0.091
mk	-2.779	-1.843	0.000	-1.659	0.000	-0.315
$wcap$	-5.417	-1.263	0.000	-0.890	0.000	-0.226

Note: The Sudden Stop state is defined by initial conditions K=690.7, B=-185.2, which imply a debt/GDP ratio of 51 percent and a debt-to-value-of-capital ratio of 26.8 percent.

1/ Results for the economy with working capital constraint are ommitted because impact effects are zero because the constraint does not bind at the Sudden Stop state in this economy.

2/ Debt and working capital limits set to highest debt and highest level of working capital with positive long-run probability in the economy with both collateral constraints (the limits are 191.20 for debt and 96.03 for working capital)

Table5. Sudden Stop Effects in Response to Separate One-Standard-Deviation Shocks in the Economy with Collateral Constraints on Debt and Working Capital
(differences relative to frictionless economy in percent of frictionless averages)

	interest rate shock		price shock		TFP shock	
	date 0	date 1	date 0	date 1	date 0	date 1
gdp	-1.478	-0.985	-0.997	-0.783	-0.734	-0.528
c	-6.420	-0.528	-5.511	-0.697	-5.798	-0.358
i	-13.096	-5.090	-14.506	-6.430	-11.715	-5.245
q	-2.590	-0.888	-3.165	-1.316	-2.590	-1.128
ca/gdp	6.140	0.924	6.127	1.448	6.176	1.224
tb/gdp	6.189	0.370	6.161	0.944	6.203	0.709
L	-1.525	-0.847	-1.036	-0.608	-0.767	-0.384
v	-2.703	-1.544	-1.781	-1.077	-1.308	-0.689
m	-1.332	-0.284	-0.906	0.024	-0.671	0.118
mk	-1.380	-1.220	-0.938	-1.108	-0.695	-0.802
$wcap$	-2.735	-1.538	-1.844	-1.102	-1.354	-0.694

Note: The Sudden Stop state is defined by initial conditions K=690.7, B=-185.2, which imply a debt-GDP ratio of 0.51 and a debt-to-value-of-capital ratio of 0.268.

Table 6. Sensitivity Analysis of Sudden Stop Effects in Response to One-Standard-Deviation Shocks
(differences relative to frictionless economy in percent of frictionless averages)

	(A) Baseline model				(B) No working capital with debt constraint		(C) Constant utilization rate				(D) No intermediate goods			
	both constraints		debt only				both constraints		debt only		both constraints		debt only	
	date 0	date 1	date 0	date 1	date 0	date 1	date 0	date 1	date 0	date 1	date 0	date 1	date 0	date 1
gdp	-2.937	-1.093	0.000	-0.893	0.000	-0.748	-2.391	-1.314	0.000	-1.314	0.000	-0.751	0.000	-0.751
c	-8.504	-0.486	-6.103	-0.323	-5.119	-0.745	-8.957	-0.789	-6.087	-0.789	-5.088	-0.119	-5.088	-0.119
i	-33.045	-3.833	-27.628	-2.676	-22.835	-3.970	-27.730	0.047	-27.730	0.047	-23.032	5.045	-23.032	5.045
q	-6.906	-0.646	-6.906	-0.444	-5.755	-0.814	-6.906	0.660	-6.906	0.660	-5.755	1.492	-5.755	1.492
ca/gdp	10.154	0.917	9.888	0.778	8.321	1.369	10.099	0.231	9.883	0.231	8.167	-0.786	8.167	-0.786
tb/gdp	10.261	0.010	9.888	-0.134	8.321	0.600	10.162	-0.669	9.883	-0.669	8.167	-1.543	8.167	-1.543
L	-3.067	-0.701	0.000	-0.495	0.000	-0.415	-3.662	-0.731	0.000	-0.731	0.000	-0.415	0.000	-0.415
v	-5.233	-1.236	0.000	-0.868	0.000	-0.729	-6.224	-1.277	0.000	-1.277	na	na	na	na
m	-2.682	0.602	0.000	0.784	0.000	0.660	na	na	na	na	0.000	0.658	0.000	0.658
mk	-2.779	-1.843	0.000	-1.659	0.000	-1.387	0.000	-2.422	0.000	-2.422	0.000	-1.390	0.000	-1.390
$wcap$	-5.417	-1.263	0.000	-0.890	na	na	-5.276	-1.511	0.000	-1.511	0.000	-0.749	0.000	-0.749
$\mathrm{P}[\mu > 0]$	0.020		0.022		0.037		0.028		0.028		0.003		0.003	
$\mathrm{P}[\chi > 0]$	0.161		na		na		0.016		na		0.000		na	

Note: The Sudden Stop state is defined by initial conditions K=690.7, B=-185.2, which imply a debt/GDP ratio of 51 percent and a debt-to-value-of-capital ratio of 26.8 percent. Results for economies with working capital constraint only are ommitted because impact effects are zero as the constraint does not bind at the Sudden Stop state.

Figure 1. Mexico: Real GDP Per Capita 1900-2004
(1980 prices)

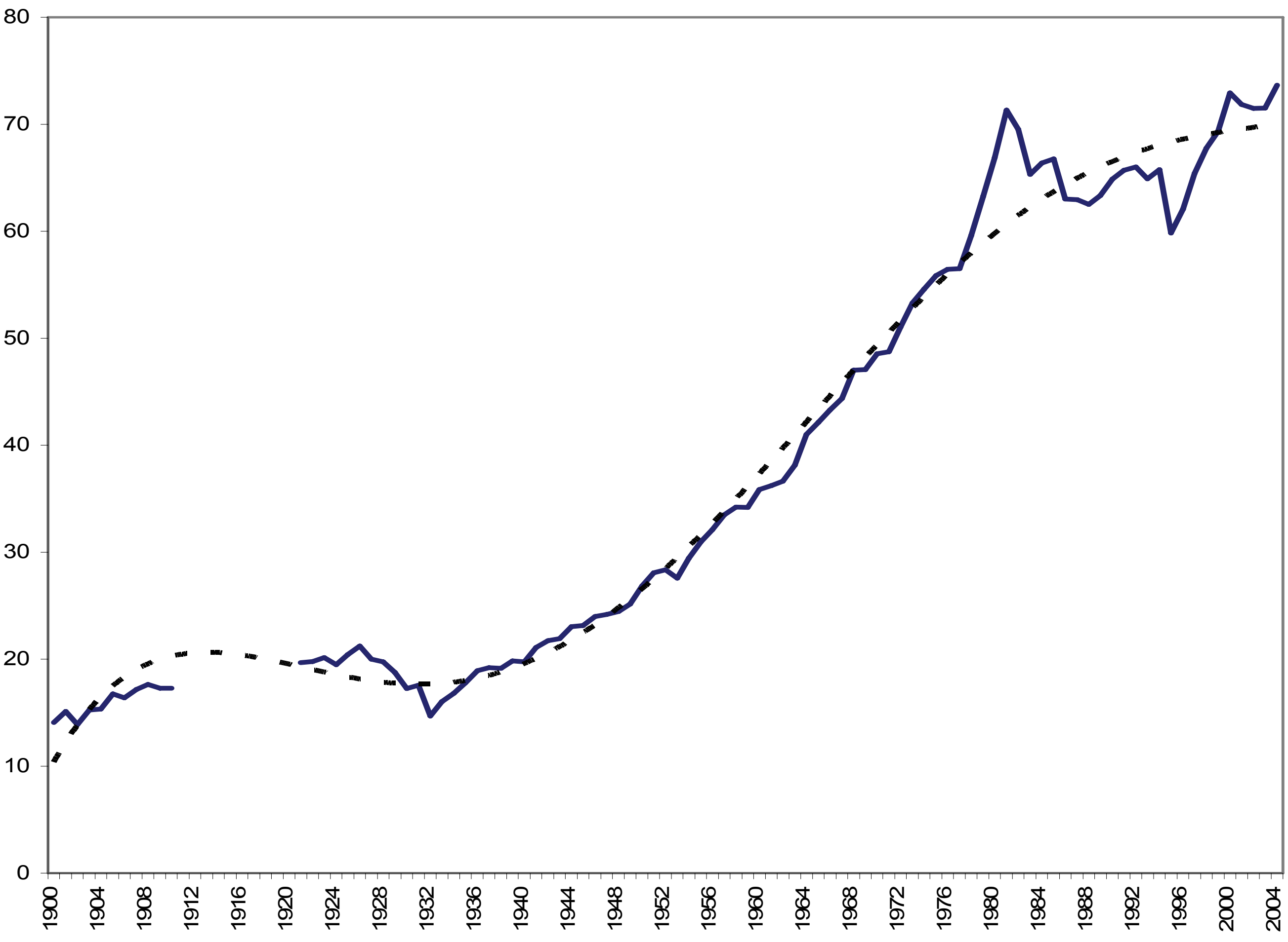


Figure 2. Mexico: Current Account-GDP Ratio
(annualized quarterly data)

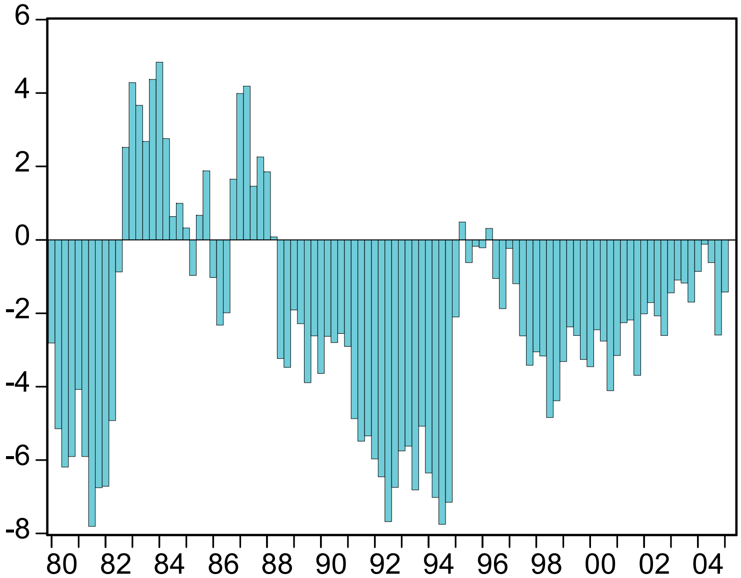
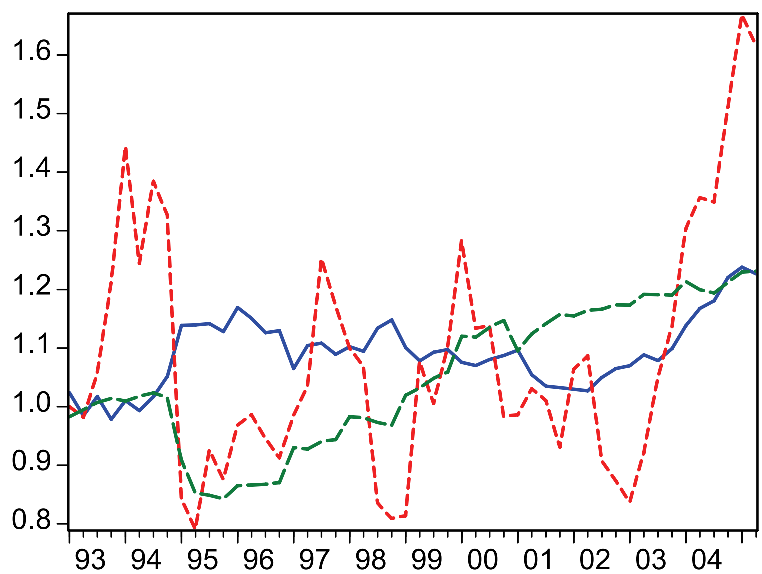
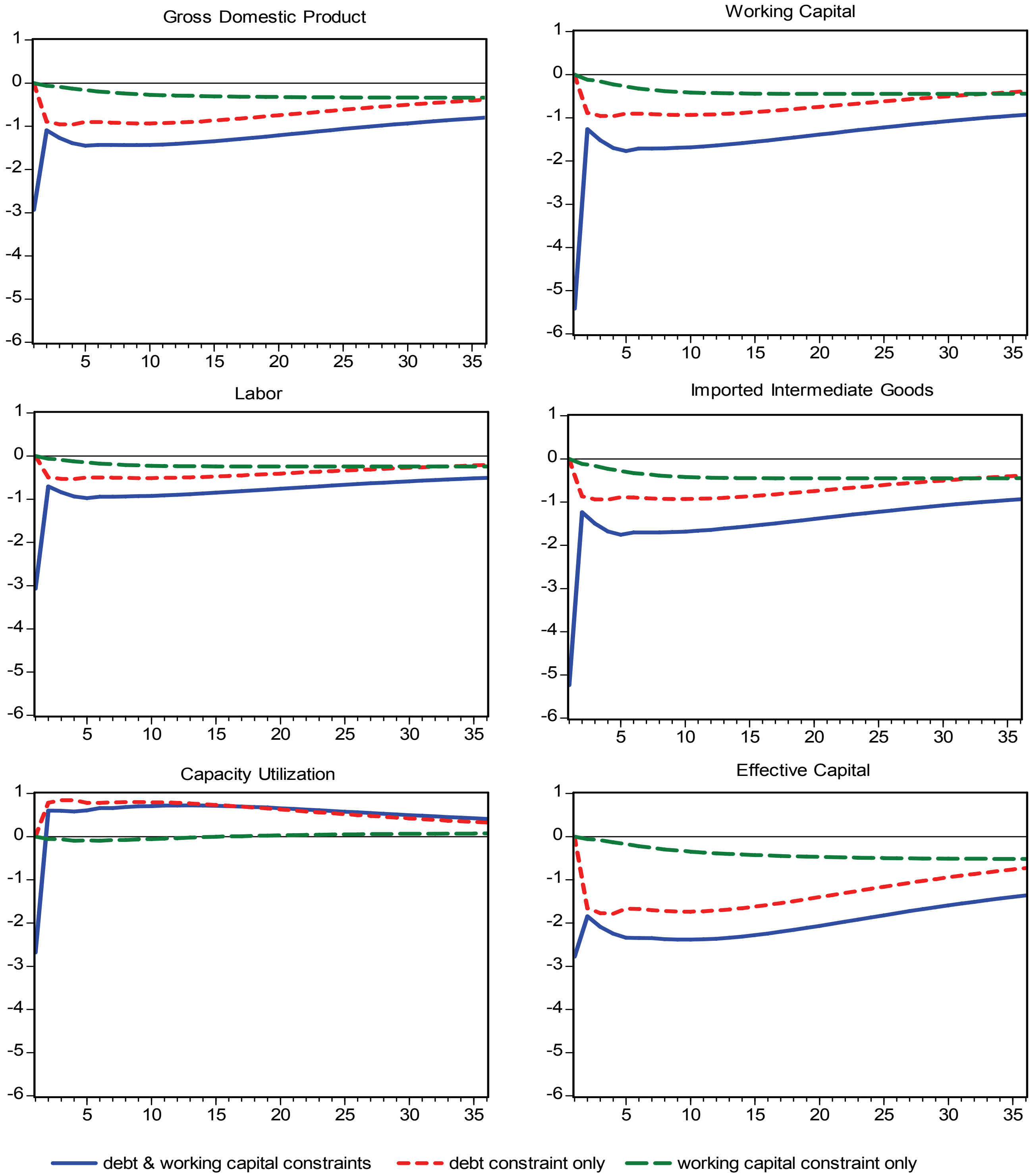


Figure 3. Mexico: Relative Prices of Imported Inputs, Equity & Nontradable Goods



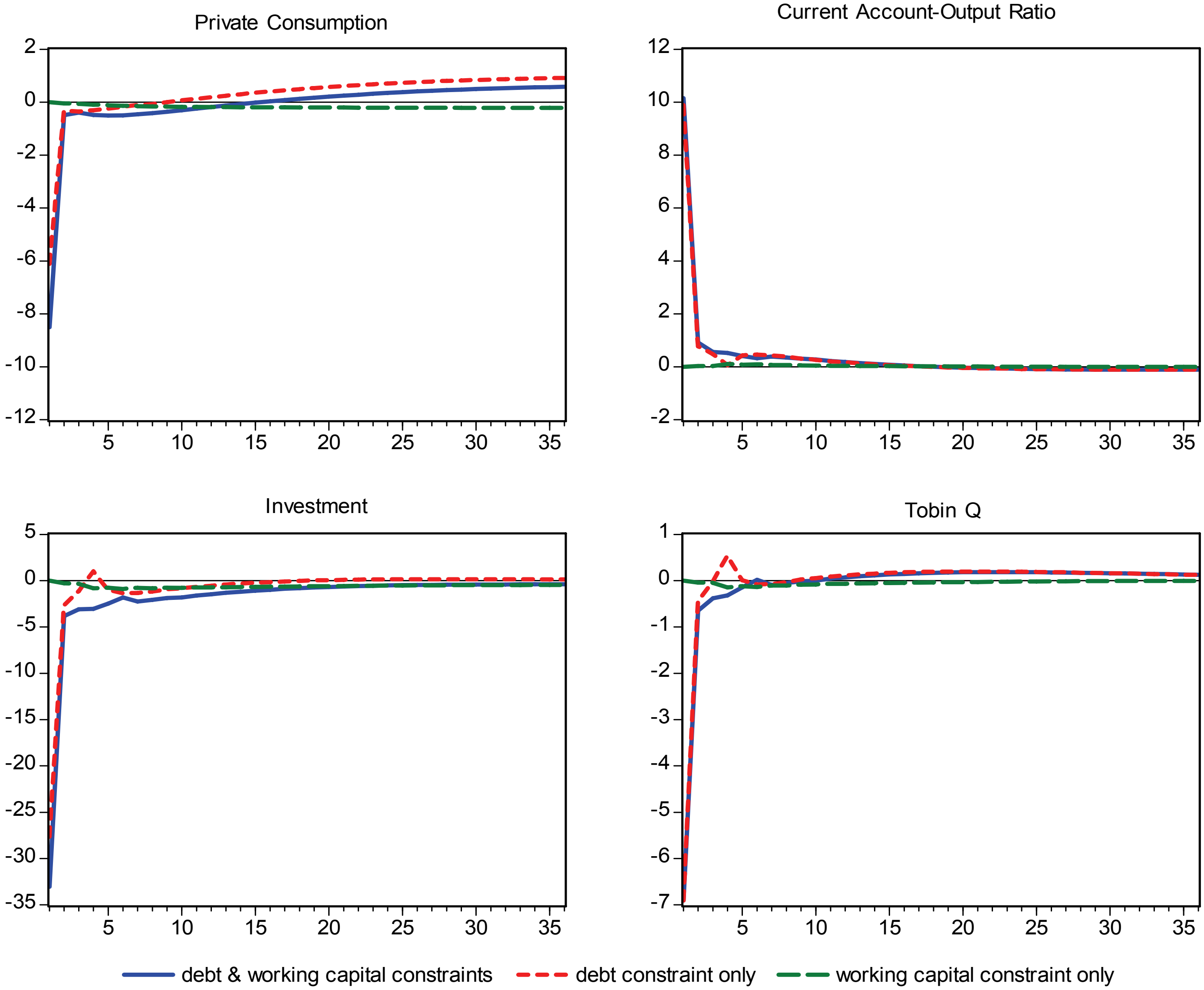
Note: Price of imported inputs relative to exports, equity prices relative to GDP deflator, and price of nontradables (services) to tradables (durables).

Figure 4a. Sudden Stop Dynamics in Response to Adverse One-Standard-Deviation Shocks
(percent deviations from long-run average in the frictionless economy)



Note: Sudden Stop dynamics are measured as differences in Markov forecast functions between economies with collateral constraints and the economy with perfect credit markets, in percent of averages in the latter. The forecast functions are conditional on the initial conditions K=690.7, B=-185.2, which imply a debt/GDP ratio of 51 percent and a debt-to-value-of-capital ratio of 26.8 percent.

Figure 4b. Sudden Stop Dynamics in Response to Adverse One-Standard-Deviation Shocks
(percent deviations from long-run average in the frictionless economy)



Note: Sudden Stop dynamics are measured as differences in Markov forecast functions between economies with collateral constraints and the economy with perfect credit markets, in percent of averages in the latter. The forecast functions are conditional on the initial conditions K=690.7, B=-185.2, which imply a debt/GDP ratio of 51 percent and a debt-to-value-of-capital ratio of 26.8 percent.

Figure 5. Output & Consumption Amplification Effects of Collateral Constraints
in Response to One-Standard-Deviation Shocks
(in units of the corresponding standard deviation in the economy with perfect credit markets)

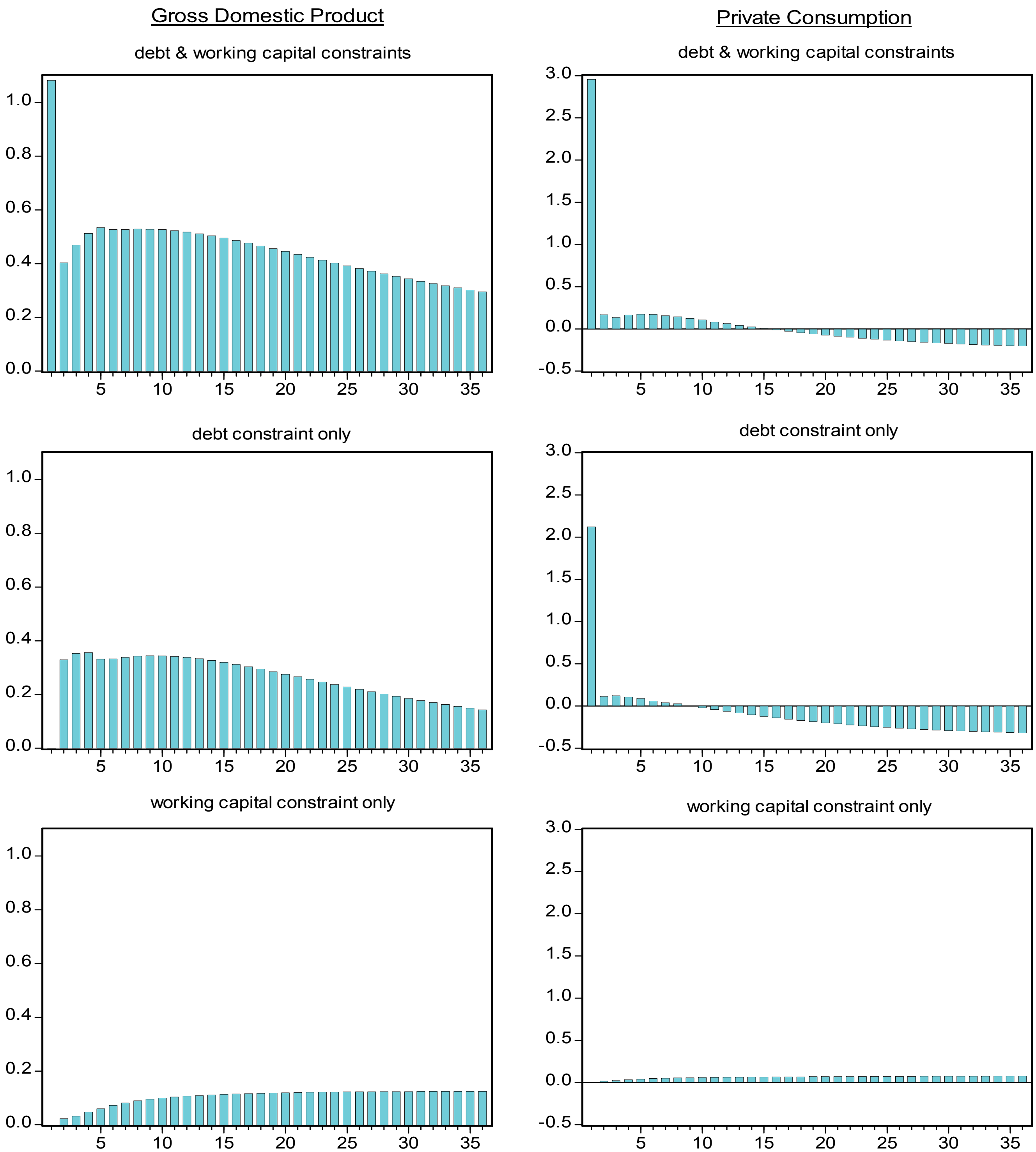


Figure 6. Investment & Tobin Q Amplification Effects of Collateral Constraints
in Response to One-Standard-Deviation Shocks
(in units of the corresponding standard deviation in the economy with perfect credit markets)

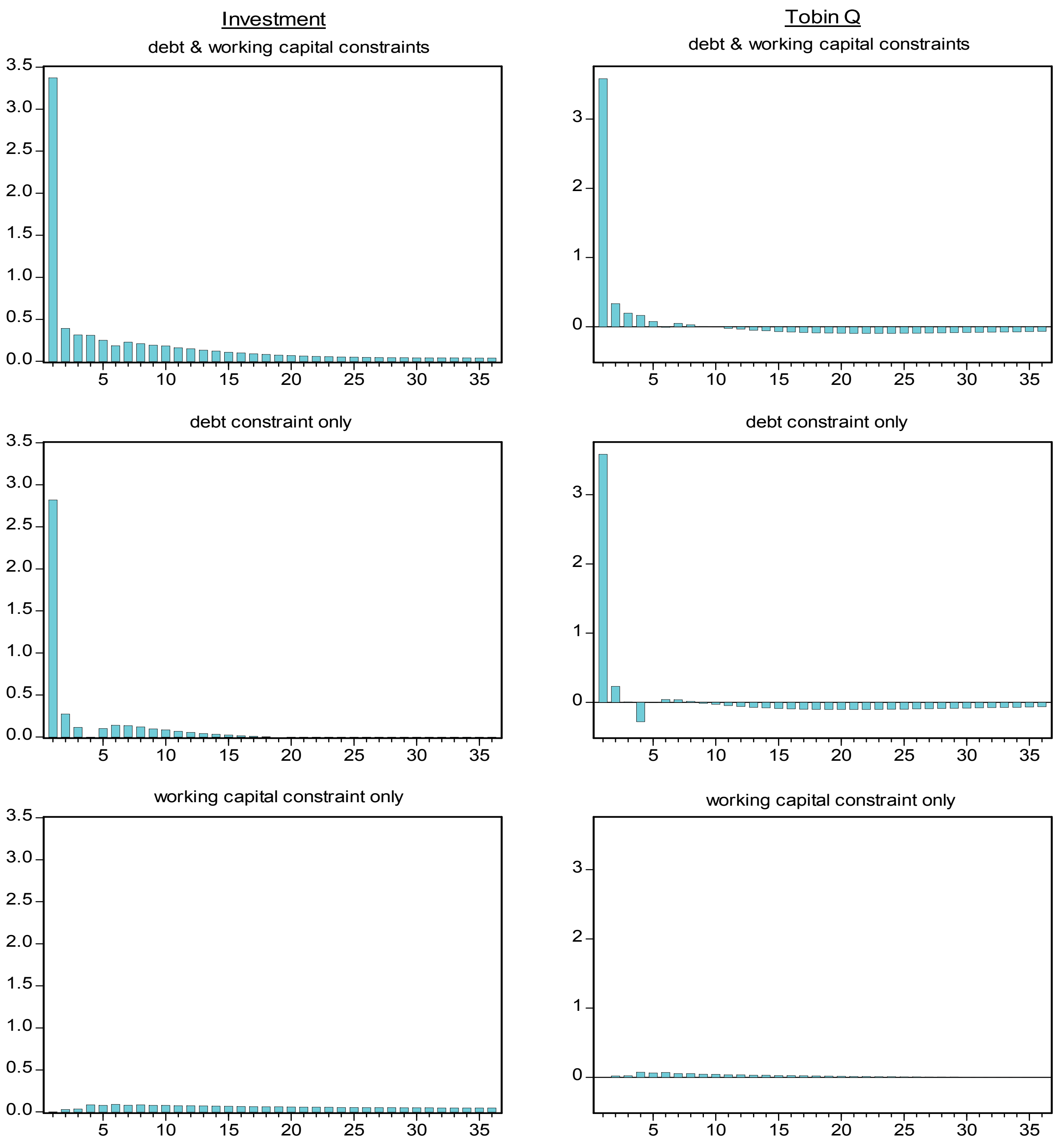


Figure 7. Working Capital & Utilization Amplification Effects of Collateral Constraints in Response to Adverse One-Standard-Deviation Shocks
(in units of the corresponding standard deviation in the economy with perfect credit markets)

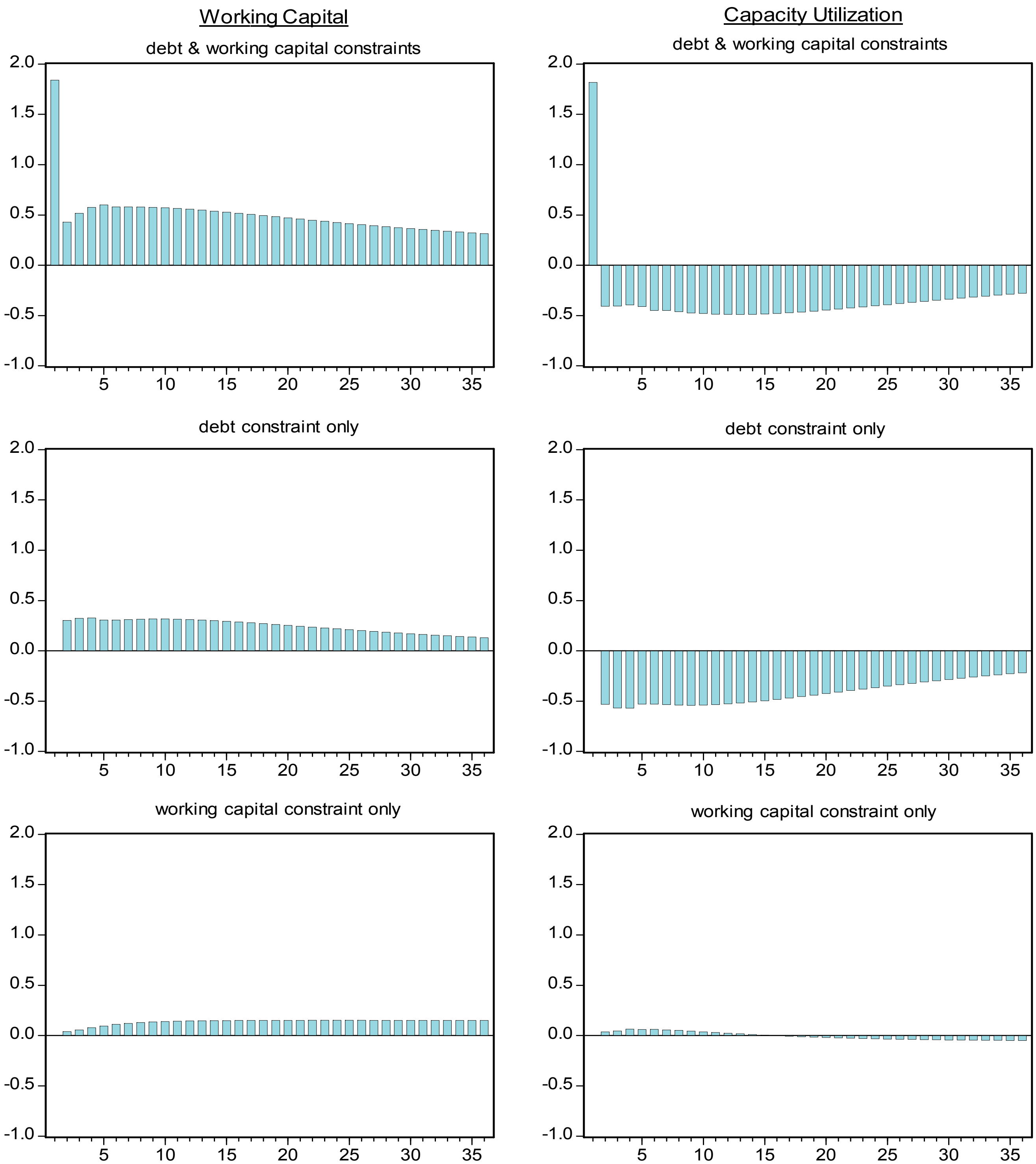
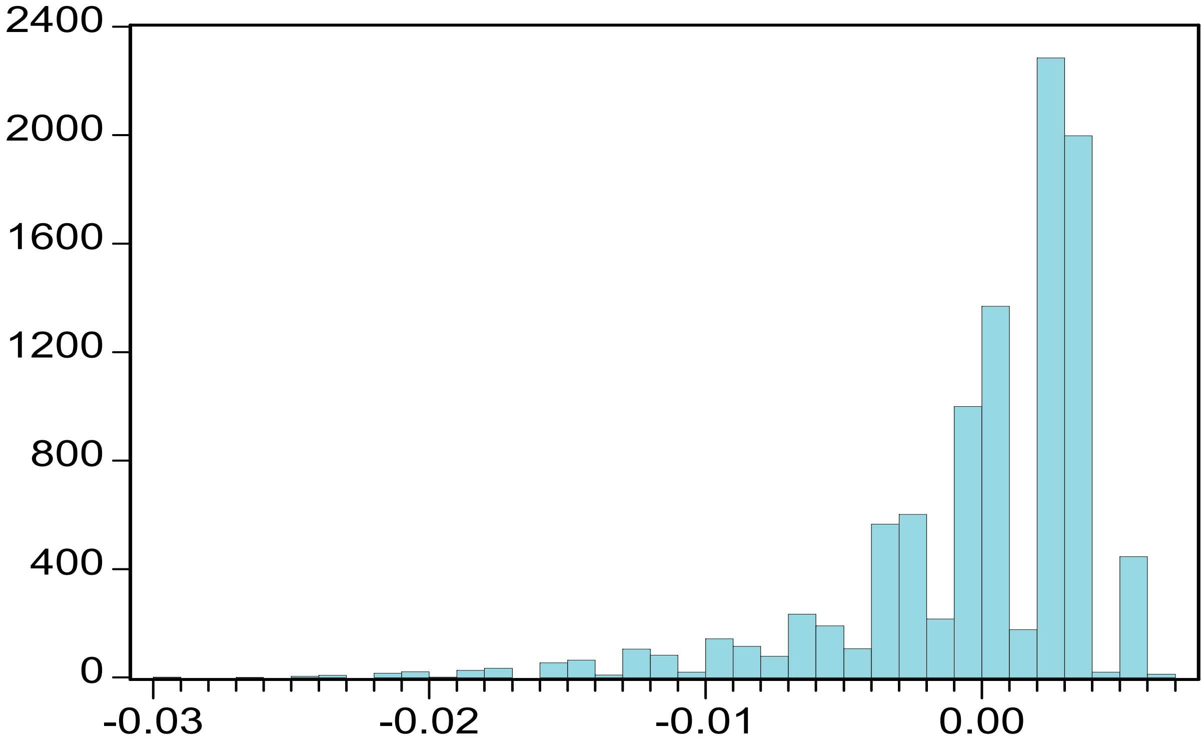


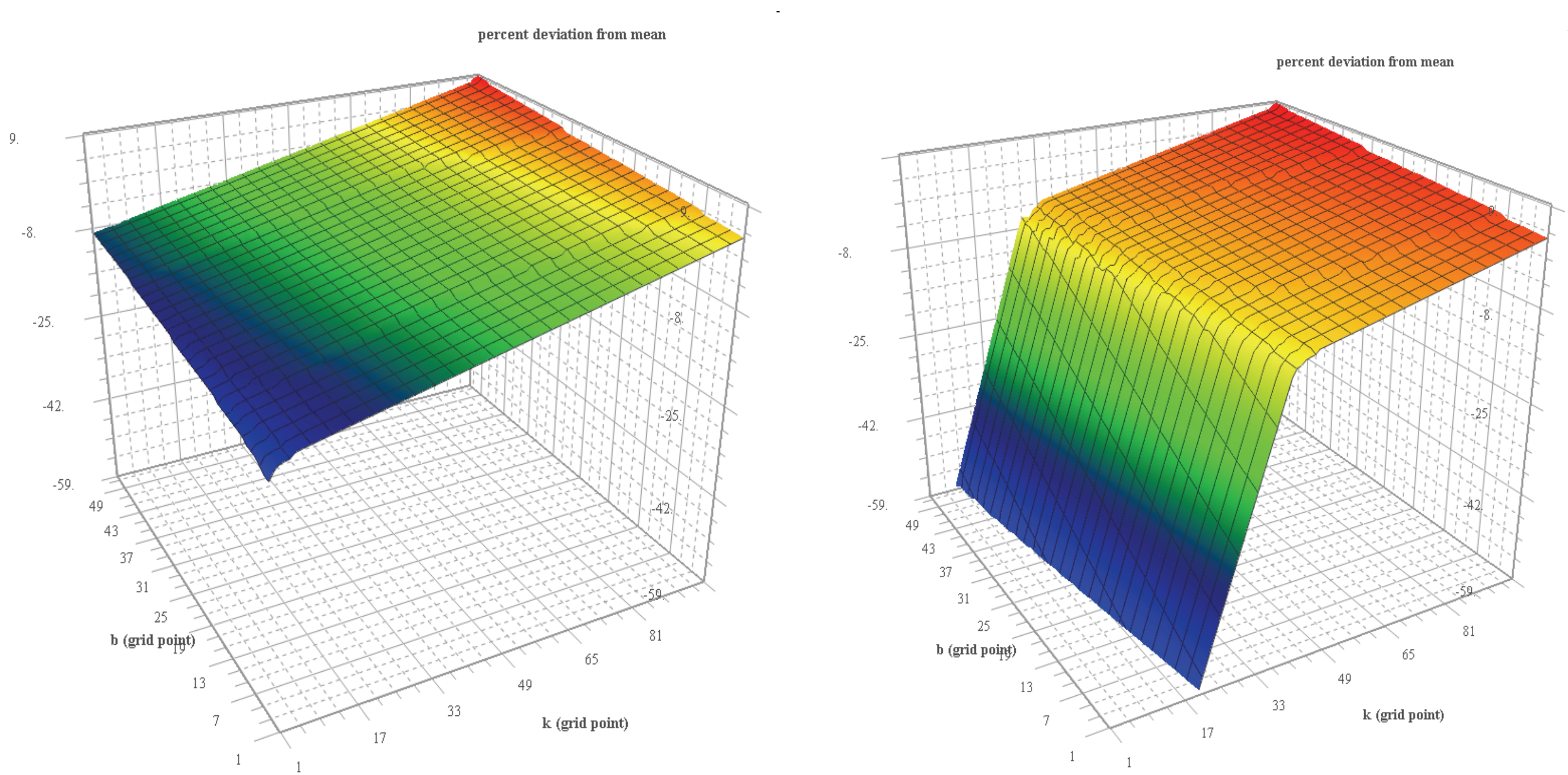
Figure 8. Histogram of GDP Fluctuations in the Economy with the Two Collateral Constraints



Note: Stochastic simulation with 10,000 quarters, differences in deviations from the mean relative to economy with perfect credit markets.

Figure 9. Impact Effects on Consumption with Perfect Credit Markets and with Collateral Constraints

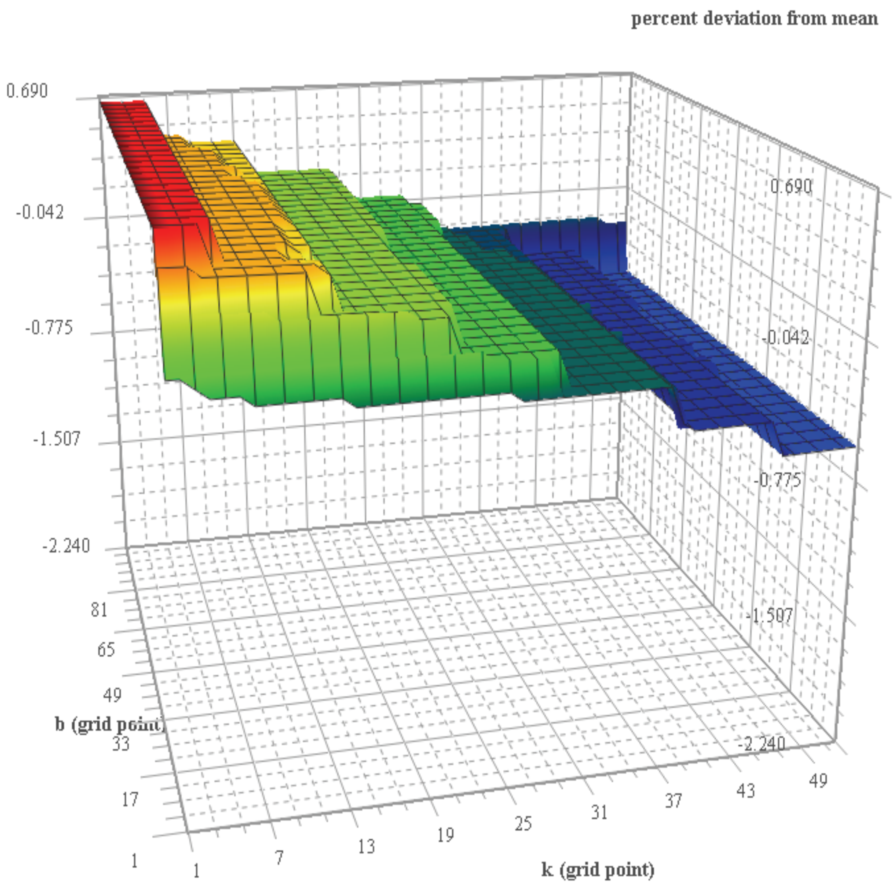
(as a percent of long-run averages, in response to one-standard-deviation shocks to εA, εR, and εP)



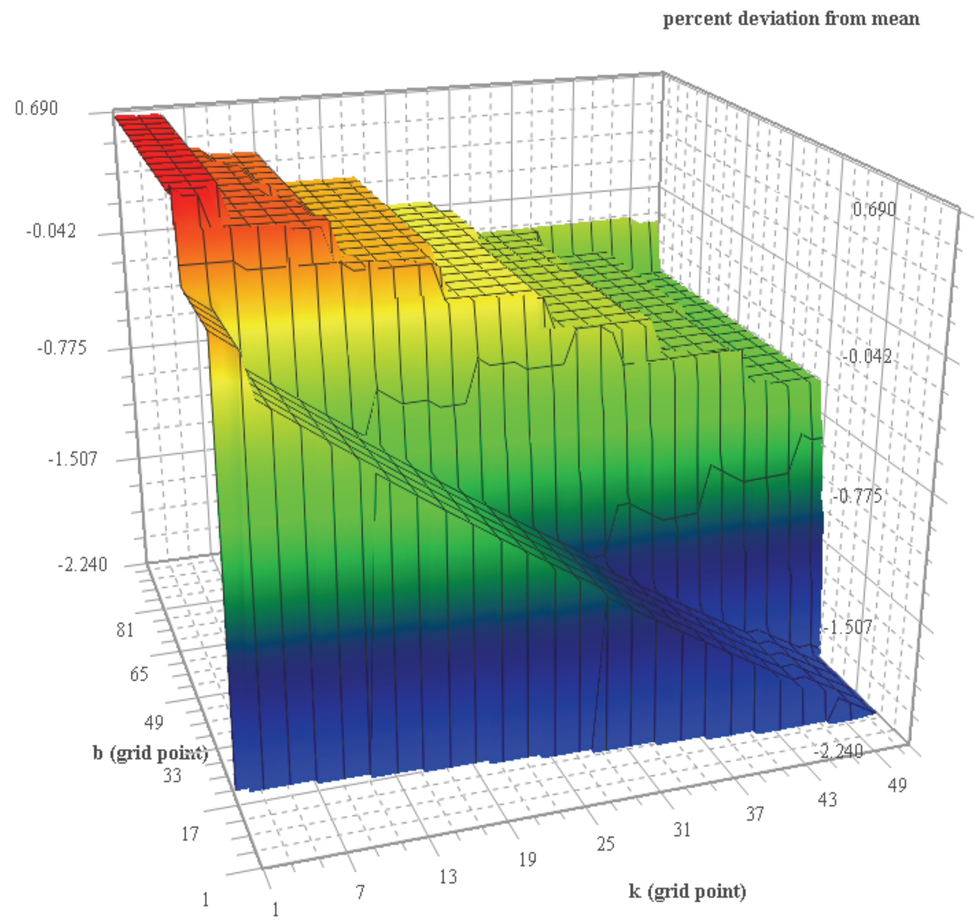
a. economy with perfect credit markets

b. economy with collateral constraints

Figure 10. Impact Effects on Tobin's Q with Perfect Credit Markets and with Collateral Constraints
(as a percent of long-run averages, in response to one-standard-deviation shocks to εA, εR, and εP)



a. economy with perfect credit markets



b. economy with collateral constraints