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Guido Lorenzoni

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ABSTRACT

This paper presents a model of business cycles driven by shocks to consumer expectations regarding aggregate productivity. Agents are hit by heterogeneous productivity shocks, they observe their own productivity and a noisy public signal regarding aggregate productivity. The shock to this public signal, or "news shock," has the features of an aggregate demand shock: it increases output, employment and inflation in the short run and has no effects in the long run. The dynamics of the economy following an aggregate productivity shock are also affected by the presence of imperfect information: after a productivity shock output adjusts gradually to its higher long-run level, and there is a temporary negative effect on inflation and employment. A calibrated version of the model is able to generate realistic amounts of short-run volatility due to demand shocks, in line with existing time-series evidence. The paper also develops a simple method to solve forward-looking models with dispersed information.

Guido Lorenzoni
MIT Department of Economics
E52-251C
50 Memorial Drive
Cambridge, MA 02142
and NBER
glorenzo@mit.edu

1 Introduction

A common idea, both in the business community and in policy circles, is that changes in consumer and investor expectations have a causal effect on cyclical fluctuations. In an old-fashioned keynesian model a shift in consumer expectations would be captured by a "demand shock," simply added as an exogenous disturbance to the IS equation. Existing empirical work in this tradition shows that a sizeable fraction of output volatility can be attributed to demand shocks, and, in informal discussions, traces these shocks to changes in private sector expectations.[1] However, two questions remain open: is it possible to build a consistent general equilibrium model that features these types of demand shocks, and, if so, what restrictions does the theory impose on the economy's response to them?

In this paper I address these questions by building a model where the fundamental source of uncertainty are long-run shifts in aggregate productivity, which are not directly observed by the agents in the economy. Consumers and firms form expectations based on noisy public sources of information, summarized by a public signal. The noise component in this signal, or "news shock," causes aggregate mistakes in agents' expectations about productivity. These mistakes lead to deviations of output from its natural level, which have the typical features of aggregate demand shocks. They have a temporary nature and they generate positive comovement between output, inflation and employment. On the other hand, actual productivity shocks, in this environment, have the features of aggregate supply shocks. They generate negative comovement both between output and inflation, and between output and employment.

Next, I turn to the restrictions that this theory imposes on equilibrium behavior. If demand shocks are caused by aggregate mistakes about productivity, then there must be a relation between the volatility due to actual productivity shocks and the volatility due to temporary mistakes. As a matter of fact, for each level of volatility in productivity shocks, the theory places an upper bound on the amount of short-run volatility that demand shocks can generate. This leads to the question: can the model generate a realistic fraction of demand-driven output volatility? To address this issue, I calibrate the model, look at its implications for the variance-decomposition of output at various horizons, and compare it with existing identified VAR studies. This exercise shows that the model can produce time series with around 30% of

[1] See Blanchard and Watson (1987), Shapiro and Watson (1988), Blanchard and Quah (1989), Gali (1992). Note that most studies identify this type of demand shocks separately from monetary shocks and shocks to government expenditure.

short-term volatility due to demand shocks. This is in line with existing evidence, based either on long-run restrictions or on sign restrictions on output and price responses. The crucial parameter that determines the relevance of demand shocks is the precision of the public signal. When the public signal is either too precise or too imprecise, demand shocks play only a small role. In the first case, the economy converges immediately to the full information equilibrium, in the second case, agents tend to disregard public signals. The empirical success of the model depends on choosing an intermediate level for the signal precision.

The model introduces heterogeneous productivity shocks in an island economy *à la* Lucas (1972). Agents observe productivity in their own island and a noisy public signal of the aggregate level of productivity. The model features households of consumers-producers and monopolistic competition. Each period, the household sets the price of his own good and the consumer travels to the other islands to buy the goods produced there. As households accumulate price and quantity signals they learn the aggregate productivity level, and, absent further shocks, the economy converges towards its full information equilibrium.

The "supply side" of the model is familiar. As in the classic papers by Phelps (1969) and Lucas (1972), agents confound aggregate and relative price movements and this explains why increases in nominal spending have non-neutral effects. The novel element of the model is the "demand side," that is, the determination of nominal spending by optimizing consumers in an environment with imperfect information. Each consumer believes, correctly, that equilibrium output will gravitate towards the full information equilibrium, determined by average productivity. This will determine the spending of other consumers, and, thus, affect his expected income. However, the latter also depends on the productivity of the sector where the consumer works. If this sector is relatively less productive than the rest of the economy, the relative price of its output would be higher and sales will be lower. Due to these two forces, the consumer's income expectations end up being an average between his expectation about aggregate productivity and his observation of local productivity. Therefore, equilibrium output will be a weighted average of perceived and realized productivity. The analysis in the paper shows how the model parameters determine the relative weights of perceived and realized productivity, and, hence, the effects of news shocks on output.

The main obstacle to studying news shocks in a model with imperfect information is the analysis of forward-looking consumer behavior. I approach the problem by studying a log-linear approximation of the optimal consumption policy. This does not eliminate the problem of infinite regress that arises in models where agents "forecast the forecasts of others," identified in Townsend (1983). However, it reduces it to a problem of characterizing an infinite sequence of average first moments of agents' beliefs. Therefore, the paper also develops a tractable method to study business cycle models with imperfect information.

The paper is related to two recent strands of literature. First, Woodford (2002), Mankiw and Reis (2002), and Sims (2003), have renewed attention to imperfect information and limited information processing as causes of sluggish adjustment in prices and other macroeconomic variables.[2] On the other hand, a rich literature, starting with Morris and Shin (2002), has emphasized that, in environments with strategic complementarities and imperfect information, public sources of information can cause persistent deviations of economic variables from their "fundamental" value.[3] This paper puts together ideas from these two literatures to build a theory of demand shocks. On the one hand, imperfect information causes sluggish price adjustment and allows for demand shocks to have non-neutral effects. On the other hand, the presence of a public signal on productivity introduces a source of non-fundamental demand shocks.

An alternative take on the idea of cycles driven by expectational mistakes, is to focus on mistakes by the monetary authority. This idea has been developed in models with sticky prices, assuming that the central bank has imperfect information about the economy's fundamentals.[4] This paper takes a different but complementary approach, by focusing on the private sector mistakes and making stark simplifying assumptions about monetary policy. The integration of optimal monetary policy in an environment with news-driven demand shocks is pursued in a companion paper.[5]

There is a growing empirical and theoretical literature that studies the effect of news on macroeconomic fluctuations, including Cochrane (1994), Danthine, Donaldson, and Johnsen (1998), Beaudry and Portier (2004 and 2006) and Jaimovich and Rebelo (2005). This literature has focused on news about the future. This is, in part, due to the fact that, in representative agent models, there is always perfect information about current productivity. The theoretical work in this area has shown that it is not easy, in a neoclassical environment, to obtain standard

[2] See also Hellwig (2005), Milani (2005), Nimark (2005), Adam (2006), Bacchetta and Van Wincoop (2006), Maćkowiak and Wiederholt (2006).

[3] See Hellwig (2002), Angeletos and Pavan (2004), Amato, Morris and Shin (2005), Bacchetta and Van Wincoop (2005), Allen, Morris and Shin (2006).

[4] See Aoki (2003), Orphanides (2003), Reis (2003), Svensson and Woodford (2003, 2005), Tambalotti (2003).

[5] Lorenzoni (2006).

comovements in consumption, investment, and hours, following a news shock. The problem, which was early recognized by Barro and King (1984), is that if the current technological frontier is unchanged, high expected future TFP will lead to reduced investment and to a reduction in hours worked.[6] This paper takes a different course, and explores the idea that the current aggregate state of the economy is uncertain. The relation between these two approaches is further discussed in the conclusions.

There is a vast literature that studies cycles due to shifts in expectations in models with increasing returns and multiple equilibria.[7] There are some common elements and some differences between that literature and the approach in this paper. Both stress the role of complementarities in consumption decisions. In models with increasing returns this complementarity is purely technological. In my approach, the complementarity is due to the permanent-income behavior of consumers: an increase in aggregate spending increases the expected income of all consumers. This complementarity explains why expected productivity, and not just realized productivity, determine aggregate consumption.[8] On the other hand, the approach based on multiple equilibria is silent about the sources of expectations-driven fluctuations, relying on pure shifts in beliefs. The approach followed here, instead, focuses on deriving theory-based restrictions on the behavior of demand shocks, exploiting the fact these are related to uncertainty about long-run changes in productivity.

The paper is organized as follows. Section 2 presents the model. In Section 3 I derive the main qualitative predictions of the model. Section 4 contains the calibration and variance-decomposition exercise. Section 5 concludes.

2 The model

The setup is a model of monopolistic competition with heterogeneous productivity shocks and an island structure *à la* Phelps-Lucas. Individual productivity is given by the sum of an aggregate productivity shock and an idiosyncratic shock. Agents only observe individual productivity and a noisy public signal of aggregate productivity.

The economy is populated by a continuum of households indexed by $i \in [0, 1]$. Each

[6] Beaudry and Portier (2004) and Jaimovich and Rebelo (2005) show possible resolutions of this problem, the first based on a model of economies of scope, the second based on adjustment costs in the investment rate, and preferences that exhibit a weak short-run wealth effect on labor supply.

[7] See references in Benhabib and Farmer (1999).

[8] The latter type of complementarity, sometimes called an "aggregate demand externality," only matters when price adjustment is sluggish and changes in nominal spending are non-neutral.

household is located on a different island and is made of two agents: a consumer and a producer. There are two type of commodities. A durable good, H, in fixed supply, and a continuum of perishable specialized goods, each produced in a different island. The durable good produces a flow of services each period and can be transported across islands. It will be used as commodity money, and will be called "money" from now on.

Each period t a subset $J_{it} \subset [0,1]$ of specialized goods is selected randomly for each consumer i. The subset J_{it} represents his consumption basket. The consumer travels to the islands producing the goods in his basket and exchanges money for the specialized goods produced there. The random assignment of consumers to islands is necessary to generate noisy price signals. The details are described in Appendix A.

The essential ingredients of the model are: (i) unobservable aggregate productivity shocks, (ii) partially revealing prices and quantities, (iii) variable velocity of circulation of money. The island structure of the model delivers the first two features. The latter feature is necessary in order to allow for demand shocks with a fixed money stock.[9] The presence of a single asset, money, that gives a flow of services is a way of obtaining variable velocity, while keeping the simplest possible financial structure. Note that the model is formally equivalent to a model with fiat money and real balances in the utility function. However, a setting with commodity money allows for a more natural description of the trading protocol.

Preferences and technology. Household preferences are represented by the utility function

$$E\left[\sum_{t=0}^{\infty}\beta^t u\left(C_{it}, X_{it}, N_{it}\right)\right],$$

where

$$u\left(C_{it}, X_{it}, N_{it}\right) = \log C_{it} + \alpha \log X_{it} + \frac{1}{1+\eta}N_{it}^{1+\eta},$$

C_{it} is a composite good defined below, X_{it} is consumption of services of the durable good, and N_{it} are hours worked in the production of good i.

The composite consumption good is a standard CES aggregate including all the varieties produced in the islands visited in period t:

$$C_{it} = m^{-\frac{\sigma}{\sigma-1}}\left(\int_{J_{it}} C_{ijt}^{\frac{\sigma-1}{\sigma}}\,dj\right)^{\frac{\sigma}{\sigma-1}},$$

[9] In the Lucas (1972) model, as in the recent versions of Woodford (2002) and Hellwig (2005), the cash-in-advance constraint is always binding. Therefore, the only type of shocks to nominal spending that are allowed, are exogenous shocks to money supply.

with $\sigma > 1$, where C_{ijt} is the consumption of good j by consumer i, and m is the measure of J_{it}, which is constant.

The production function on island i is linear and is given by

$$Y_{it} = A_{it} N_{it}.$$

The productivity parameters A_{it} are different across islands and are the fundamental source of uncertainty in this economy.

There is a fixed stock $\bar{H}$ of money in the economy. Each period the household starts with the initial stock H_{it}. Consumers carry H_{it} to the islands they visit and transfer $H_{it} - X_{it}$ to the local producers. The rest, X_{it}, produces services which generate the utility flow $\alpha \ln X_{it}$. The money used in transactions, $H_{it} - X_{it}$, does not produce any service at date t.[10] Money is stored one for one from period t to period $t + 1$.

Uncertainty. Let a_{it} denote the log of A_{it}. The variable a_{it} has an aggregate component, a_t, and an idiosyncratic one, ϵ_{it},

$$a_{it} = a_t + \epsilon_{it}.$$

The cross sectional distribution of ϵ_{it} satisfies $\int_0^1 \epsilon_{it} di$. The aggregate component a_t follows the random walk:

$$a_t = a_{t-1} + u_t.$$

The aggregate productivity shock u_t is the fundamental aggregate shock in this economy. It would be straightforward to add a deterministic component to productivity growth. To save on notation, I normalize it to zero.

At the beginning of each period, agents observe a public signal

$$s_t = a_t + e_t.$$

The noise component, e_t, is the "news shock" which will be at the center of the analysis. The aggregate shocks u_t and e_t are independent, serially uncorrelated, and normally distributed with zero mean and variances $\left(\sigma_u^2, \sigma_e^2\right)$. For each agent i, the idiosyncratic shock, ϵ_{it}, is also normal, with zero mean and variance σ_ϵ^2, serially uncorrelated, and independent of the aggregate shock.

[10] The only role of this assumption is to ensure that the cash-in-advance constraint is never binding, so that linearization methods can be employed.

Finally, there is a "sampling shock," ζ_{it}, which determines a bias in the composition of the consumption basket of consumer i. Namely, average productivity in the islands in J_{it} is equal to $a_t + \zeta_{it}$. The sampling shock is i.i.d., normally distributed with variance σ_ζ^2.

Trading and information. The only trades allowed in this economy are spot trades of goods for money. At the beginning of period t, producer i observes a_{it} and s_t, sets the price of his good, P_{it}, and stands ready to deliver any quantity of the good at that price.[11] This is the "pricing stage," or stage I.

After prices are set, consumer i travels to the islands in J_{it} and observes the prices of the goods produced there. Then, he chooses the consumption vector $\{C_{ijt}\}_{j \in J_{it}}$, trades, and enjoys the services of the money not used for transactions, X_{it}, where

$$X_{it} = H_{it} - \int_{J_{it}} P_{jt} C_{ijt} dj.$$

This is the "trading stage," or stage II. Consumers do not communicate with producers during the trading stage, so consumers do not know the quantity traded in the home island when they are making their spending decisions.

At the end of the period, the consumer returns to his island and observes the quantity sold by the producer, Y_{it}. The stock of money at the end of the period is given by the money in the hand of the consumer, plus the money accumulated by the producer. Thus, the household budget constraint at date t is:

$$H_{it+1} = H_{it} - \int_{J_{it}} P_{jt} C_{ijt} dj + P_{it} Y_{it}.$$

The pattern of trade across islands is represented in Figure 1.[12]

Equilibrium. Consider the vector z_t, which summarizes the history of past aggregate shocks:

$$z_t = \langle a_t, s_t, a_{t-1}, s_{t-1}, ...a_0, s_0 \rangle .$$

Let $\mathcal{Z}$ be the set of possible histories z_t. Let μ_{it} be a probability measure on $\mathbb{R}^{t-1}$. It represents the beliefs of consumer i on the vector of realized productivity levels $\langle a_{t-1}, ...a_0 \rangle$, at the beginning of period t, before the current shocks are realized. Three objects will be used to

[11] As usual in models of price setting, I assume that the size of the shocks is small, so that it is always optimal to produce the quantity demanded.

[12] The price index $\overline{P}_{it}$, used in the figure, is defined below.

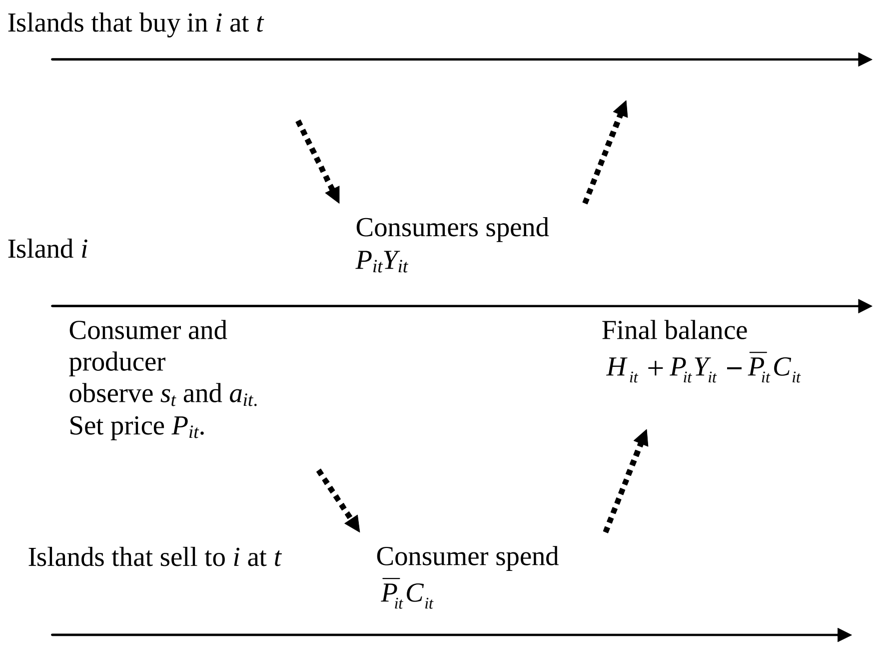

Figure 1: Timeline.

define a symmetric rational expectations equilibrium. First, an individual pricing policy rule, which takes the form $P_{it} = \mathcal{P}(A_{it}, H_{it}, \mu_{it})$. Second, a consumption policy rule, which takes the form $C_{it} = \mathcal{C}\left(A_{it}, H_{it}, \overline{P}_{it}, \mu_{it}\right)$, where the price index $\overline{P}_{it}$ is defined as:

$$\overline{P}_{it} = \left(\int_{J_{it}} P_{jt}^{1-\sigma} dj\right)^{\frac{1}{1-\sigma}}. \tag{1}$$

Third, a sequence of cross-sectional distributions for money holdings and beliefs, described by the CDF $G(H_{it}, \mu_{it}; z_t)$ for each $z_t \in \mathcal{Z}$.

A *symmetric rational expectations equilibrium* is given by a pair $\mathcal{P}$ and $\mathcal{C}$, and a sequence of distributions G, that satisfy two conditions: optimality and consistency. Optimality requires that the policy rules are optimal for the individual household, taking as given the cross sectional distributions G and the fact that all other households follow $\mathcal{P}$ and $\mathcal{C}$. Consistency requires that, for each state z_{t+1} that follows z_t, the cross sectional distribution $G(.; z_{t+1})$ is derived from the distribution $G(.; z_t)$ using the individual law of motion for money holdings implied by $\mathcal{P}$ and $\mathcal{C}$, and using rational updating of agents' beliefs. The details of the equilibrium construction are in Appendix A.

3 News shocks, output and prices

In this section I analyze the equilibrium determination of output, prices and employment, first in the case of full information and then in the case of imperfect information.

3.1 Full information

First, consider the simplest case, with no heterogeneity. Productivity is equal across islands, $\sigma_\epsilon^2 = 0$, and, in a symmetric equilibrium, prices are identical across islands. In this case the economy boils down to an economy with a representative consumer and full information about current productivity. Given the assumption of log preferences and the absence of capital, employment is constant in equilibrium and output is proportional to aggregate productivity. All the proofs for this section are in Appendix B.

Proposition 1 *In the representative agent case equilibrium output, employment and prices are:*

$$Y_t = A_t \left(\frac{\sigma - 1}{\sigma} \right)^{\frac{1}{1+\eta}}, N_t = \left(\frac{\sigma - 1}{\sigma} \right)^{\frac{1}{1+\eta}},$$
$$P_t = \frac{1-\beta}{1-\beta+\alpha} \frac{\bar{H}}{A_t}.$$

The equilibrium price level P_t is inversely related to the productivity level A_t. Since the supply of money is fixed, when the supply of specialized goods increases their relative price falls. Given the assumptions made about preferences, P_t adjusts so as to keep a constant velocity of circulation, $P_t Y_t / \bar{H}$.[13]

Consider now the richer case where idiosyncratic uncertainty is present, i.e. σ_ϵ^2 is positive, but maintain the assumption of full information about current shocks. Under a log-linear approximation, the following proposition shows that the aggregate behavior of the economy is the same as in the representative agent case. From now on, I will use lowercase variables to denote the logarithm of the corresponding uppercase variable. Also, to simplify notation, I will omit constant terms when there is no risk of confusion.

Proposition 2 *Under full information about current shocks aggregate output and prices, in log-linear approximation, are:*

$$\begin{aligned} y_t &= a_t, \\ p_t &= -a_t. \end{aligned}$$

The proof of this result provides a useful introduction to the case of imperfect information. Under full information, output is fully determined by the optimal pricing condition. The first order condition for price setting can be written, in log-linear terms, as:

$$p_{it} - \overline{p}_{it} = -a_{it} + c_{it} + \eta \left(y_{it} - a_{it} \right). \tag{2}$$

For an individual agent, $p_{it} - \overline{p}_{it}$ reflects the relative price of his output in terms of his consumption basket. This equation can be interpreted as follows: the household sets his target relative price $p_{it} - \overline{p}_{it}$ proportional to his marginal cost, measured in terms of consumption. His

[13] The presence of a stochastic trend in P_t, inversely related to aggregate productivity growth, is a minor nuisance associated to the assumption of constant money supply. The predictions regarding surprise inflation are robust to different scenarios for money growth. Section 4 shows that these predictions survive in a model with a stationary price level. The case of constant money supply, on the other hand, has clear advantages in terms of exposition.

marginal cost depends negatively both on his productivity, a_{it}, and on his expected marginal rate of substitution between consumption and leisure, captured by $-(c_{it}+\eta(y_{it}-a_{it}))$.

Aggregating both sides of (2), one obtains:

$$0=(1+\eta)(y_t-a_t) \tag{3}$$

and the first result follows. In the full information equilibrium, when agents set their price, p_{it}, they perfectly forecast the price of their consumption basket. Moreover, the average relative price across all agents in the economy must be zero (in log-linear approximation). This, given the structure of the model, implies that output responds one for one to productivity.[14]

Summing up, when shocks are fully observed, output is determined by the supply side of the model and is completely independent of demand dynamics. The demand side only determines equilibrium prices. The consumer Euler equation can be written as:

$$c_{it}=E_t\left[c_{it+1}\right]+\xi\left(h_{it}-\overline{p}_{it}-c_{it}\right)+E_t\left[\overline{p}_{it+1}\right]-\overline{p}_{it}. \tag{4}$$

where ξ is a positive constant, which depends on β and α.[15] The term $\xi\left(h_{it}-\overline{p}_{it}-c_{it}\right)$ on the right-hand side captures the service flow of money, as a fraction of nominal spending. The term $E_t\left[\overline{p}_{it+1}\right]-\overline{p}_{it}$ captures the expected real interest rate, which is equal to the inverse of expected inflation. Note that the relevant rate of inflation is individual specific, given the differences in consumption baskets across agents.

Aggregating both sides of (4) gives:

$$a_t=E_t\left[a_{t+1}\right]-\xi\left(p_t+a_t\right)+E_t\left[p_{t+1}\right]-p_t, \tag{5}$$

where I use the result that output is equal to productivity and the assumption of constant money supply. Substituting the candidate equilibrium prices on the right-hand side, the last three terms cancel out. The equilibrium price path has the property that: (i) the service flow of money is a constant fraction of nominal spending, (ii) the average expected real rate is constant.[16] Given that output is a random walk, the real rate is equal, up to a constant factor, to the "natural interest rate" for this economy.

[14] The same result holds if agents are restricted to observe only the aggregate shock a_t at the pricing stage, but not the productivity shocks in the islands they will visit. In this case, agents can still perfectly forecast the average price and the average consumption, and aggregating across agents still gives (3).

[15] The expression for ξ is:

$$\xi=\frac{1-\beta}{\beta}\frac{\alpha+1-\beta}{\alpha}.$$

[16] In fact, this is the only stationary price path consistent with (5).

3.2 Imperfect information

Turning to imperfect information, I now focus on a limit case that illustrates well the central mechanism of the model. Consider the case where idiosyncratic uncertainty is large with respect to the volatility of the innovations in aggregate productivity. That is, let the ratios $\sigma_\epsilon^2/\sigma_u^2$ and $\sigma_\zeta^2/\sigma_u^2$ approach infinity. At the same time, keep the ratio σ_e^2/σ_u^2 constant, that is, consider a fixed precision of the public signal s_t. Then, the relative precision of the private signals observed by each agent —individual productivity levels and prices— becomes very small. Therefore, agents put a very small weight on their private information when forecasting aggregate variables. In the limit, their forecasts of aggregate variables are identical and are solely based on the public signal s_t.[17]

In equilibrium, at the end of each period t, agents learn the aggregate productivity shock by observing their current sales. This happens because, by observing s_t and y_{it}, the household is able to infer exactly the shocks u_t and e_t.[18] Due to this result, a_{t-1} is common knowledge across agents at t, and agents only need to form expectations about the current shock u_t. Denote "public expectations" at time t as $E_t^P[.] \equiv E[.|s_t, a_{t-1}]$. The following proposition characterizes the equilibrium.

Proposition 3 *Let $\sigma_\epsilon^2/\sigma_u^2 = \sigma_\zeta^2/\sigma_u^2 = \infty$ and $\sigma_e^2/\sigma_u^2 < \infty$. Then, equilibrium output and prices satisfy:*

$$y_t = a_t + \lambda\left(E_t^P[a_t] - a_t\right) - \left(p_t - E_t^P[p_t]\right) \tag{6}$$

$$p_t - E_t^P[p_t] = \frac{\lambda + \eta}{1 + \eta\sigma}\left(E_t^P[a_t] - a_t\right) \tag{7}$$

where $\lambda \in (0,1)$. The public expectation of aggregate output and prices in the current and future periods are:

$$\begin{aligned} E_t^P[y_{t+j}] &= E_t^P[a_t] \text{ for } j = 0, 1, \ldots \quad, \\ E_t^P[p_{t+j}] &= -E_t^P[a_t] \text{ for } j = 0, 1, \ldots \quad. \end{aligned}$$

The constant λ is a function of all the model parameters, α, β, η and σ.[19] In the remainder of this section I will discuss various properties of λ.

[17] Furthermore, given the high dispersion of price signals, expectations are, in the limit, the same at the pricing and at the trading stage. Note that private shocks still matter for pricing and consumption decisions at the individual level, and agents do not have common information regarding individual variables, like a_{it} or p_{it}. Agents only agree on their expectations about aggregates.

[18] This is a general result which does not depend on linearization nor on the assumption of large values for $\sigma_\epsilon^2/\sigma_u^2$ and $\sigma_\zeta^2/\sigma_u^2$. See Appendix A, p. 28.

[19] See the proof of Proposition 3, in Appendix B.

In this simple case, the responses of output, prices and employment are fully determined by two variables: realized productivity, a_t, and the error in the public forecast of aggregate productivity, $a_t - E_t^P[a_t]$. The latter determines both the price surprise $p_t - E_t^P[p_t]$, and the output gap $y_t - a_t$.

3.2.1 Price and output responses

The expression for the public expectation of aggregate productivity is:

$$E_t^P[a_t] = a_{t-1} + \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2}(u_t + e_t). \tag{8}$$

Substituting (8) in (6) and (7) gives the equilibrium responses of output and prices to the underlying shocks, u_t and e_t. The next proposition gives a characterization of these responses.

Proposition 4 *Under the assumptions of Proposition 3, in equilibrium:*
(i) a positive news shock, $e_t > 0$, increases output and employment, leads to a positive price surprise and to a negative output surprise;
(ii) a positive productivity shock, $u_t > 0$, increases output, reduces employment, leads to a negative price surprise and to a positive output surprise.

To gain some intuition for these results, it is useful to go back to the optimality conditions at the pricing and at the consumption stage. Consider the effects of a positive news shock e_t. The public expectation of productivity increases, while actual productivity is unchanged. Under imperfect information, the Euler equation becomes:

$$c_{it} = E_{it}^{II}[c_{it+1}] + \xi(h_{it} - \overline{p}_{it} - c_{it}) + E_{it}^{II}[\overline{p}_{it+1}] - \overline{p}_{it}, \tag{9}$$

where $E_{it}^{II}[.]$ denotes the expectation of agent i at the trading stage. For the moment, focus on future expected consumption and leave aside the last three terms on the right-hand side. Expectations about current productivity affect expected future productivity, which will determine future income. Moreover, they affect the location of the demand curve faced by producer at date t, which determines current income and, thus, individual money balances at the beginning of period $t+1$. Therefore, high expected productivity tends to increase expected future consumption.

Consider now the optimal pricing condition in the case of imperfect information:

$$p_{it} - E_{it}^I[\overline{p}_{it}] = -a_{it} + E_{it}^I[c_{it}] + \eta\left(E_{it}^I[y_{it}] - a_{it}\right), \tag{10}$$

where $E_{it}^{I}[.]$ denotes the expectation at the pricing stage.[20] When expected productivity is higher than actual productivity, for producer i, he tries to increase the relative price of good i. There are two reasons for this: high expected output shifts up the demand curve faced by producer i, and high expected output leads to higher consumption by household i and, thus, to higher marginal costs in terms of consumption. As all agents are trying to increase their relative price, they end up increasing the absolute price level above their common target. Their common mistake results in a positive surprise in the price level. This mechanism is closely related to the mechanism in Phelps (1969) and Lucas (1972), where agents confound absolute and relative price changes.

Going back to the demand side, note that a positive price surprise tends to dampen the response of aggregate consumption through its effects on the expected real rate, as shown by equation (9).[21] This happens because a positive price surprise is associated to an increase in the real rate perceived by consumers. This follows from three observations: (i) agents' expectations about the current and the future aggregate price level are the same, (ii) at the pricing stage, they observe exactly their own price, and, (iii) at the consumption stage, they observe exactly the price of their consumption basket. The total response of output to a news shock depends both on the effect on expected income, discussed above, and on the effect on the expected real rate. Proposition 4 shows that the first effect always dominates.

This decomposition of the effects of a news shock has a formal counterpart in the expression for the coefficient ψ_e, which represent the equilibrium response of output to e_t:

$$\psi_e \equiv \left(\lambda - \frac{\lambda+\eta}{1+\sigma\eta}\right)\frac{\sigma_u^2}{\sigma_u^2+\sigma_e^2}. \tag{11}$$

The term λ captures the effect of e_t on expected income, while the term $-(\lambda+\eta)/(1+\sigma\eta)$ captures the effect of e_t on the expected real interest rate. In the next sub-section, I will discuss the effect of the various model parameters on ψ_e.

Consider now the effects of a productivity shock u_t. This shock increases both actual productivity, a_t, and the public expectation about it, $E_t^P[a_t]$. However, due to imperfect information, the effect of u_t on $E_t^P[a_t]$ is smaller than the effect on a_t.[22] Therefore, $E_t^P[a_t]-a_t$ is negative when $u_t>0$, and gives a negative price surprise. This happens because the

[20] Both (9) and (10) are derived in the general case, and do not use the assumption of large idiosyncratic shocks.

[21] The term $\xi(h_{it}-\overline{p}_{it}-c_{it})$ does not add insight to the intuition. In fact, when $\beta\to 1$ this term is negligible.

[22] See (8) and note that $\frac{\sigma_u^2}{\sigma_u^2+\sigma_e^2}<1$.

expected upward shift in the demand curve faced by each producer is smaller than the realized productivity increase, and, thus, producers try to reduce their relative prices. On the quantity side, consumption increases given that both expected income increases and the expected real rate decreases. To derive the response of employment I need to determine whether the total effect on output is larger or smaller than one. Proposition 4 shows that the effect on output is smaller than one and employment decreases temporarily after a positive productivity shock.[23]

3.2.2 The effect of news shocks

I turn now to the determinants of ψ_e, defined in (11), which represents the response of output to news shocks. Consider first the effect of increasing the discount factor β. The next proposition shows that as β approaches 1, the response of output to news increases and reaches its maximum value for $\beta = 1$.

Proposition 5 *The response of output to the news shock is bounded above by:*

$$\bar{\psi}_e \equiv \eta \frac{\sigma - 1}{1 + \sigma\eta} \frac{\sigma_u^2}{\sigma_u^2 + \sigma_e^2}, \tag{12}$$

and $\lim_{\beta \to 1} \psi_e = \bar{\psi}_e$.

As consumers become more patient, their spending becomes more responsive to productivity expectations. Consider the average household with $\epsilon_{it} = 0$. After a positive news shock, expected current income increases less than expected future income. This is because household i, observing his lower productivity today, increases the price of good i, and expects to sell less than other households. In the future, household i expects his productivity to catch up with the average level and the own price effect goes away. As agents become more patient they tend to respond more to future income, and, thus, they respond more to expected productivity.

Consider now the effect of the parameters σ and η on the response of output to news. Remember that σ is the elasticity of demand and η is the inverse of the Frisch elasticity of labor supply. It is useful to decompose the effects of these parameters in two parts. First, keeping λ constant, the parameters σ and η, affect the response of the price surprise. Second, the parameters affect the parameter λ.

Consider again the average household, with $\epsilon_{it} = 0$, after a positive news shock. Producer i thinks his productivity, a_{it}, is lower than expected aggregate productivity, $E_t [a_t]$, and tries

[23] The result that an increase in productivity leads to a temporary reduction in employment in an environment with imperfect information has been independently derived by Kawamoto (2004).

to increase his relative price. However, the optimal price response to a perceived productivity differential will be smaller if the elasticity of demand, σ, is higher or if the elasticity of labor supply, $1/\eta$, is lower. At the aggregate level this implies that an aggregate productivity mistake will have smaller effects on the price surprise. A smaller price surprise also means a smaller effect on the expected real rate, and thus a bigger response of consumption to the positive increase in expected income. In conclusion, for a fixed λ, an increase in σ and η reduces the price surprise, and increases the effect on output.

This is related to the notion that η and σ affect the degree of strategic complementarity in pricing. Through this channel, a higher degree of strategic complementarity in pricing increases the output response to demand disturbances, as it is common in the neo-keynesian literature.[24]

However, changes in σ and η also affect the parameter λ, which determines the endogenous response of consumption. Consider the case of σ. Under realistic assumptions about parameters, λ is a decreasing function of σ.[25] The intuition for this is the following. Consider again the average household, after a positive news shock. Producer i is trying to increase his relative price. A higher elasticity σ implies that his total sales will respond more to the relative price increase.[26] Therefore, the agent expected income responds less to the news shock. This means that a higher σ induces consumers, in the aggregate, to respond less to news shocks. That is, this mechanism tends to decrease λ.

Therefore, in general, the relation between the parameters σ and η and the coefficient ψ_e is ambiguous. In numerical examples with realistic parameter values, the first effect tends to dominate, and an increase in σ and η tends to increase ψ_e. This is further illustrated in the dynamic simulations below.

4 Equilibrium dynamics

In this section I use numerical simulations to analyze the dynamic responses of output, employment and prices, and to address a basic quantitative question: what fraction of output volatility can be explained by news shocks? In particular, I look at the ability of the model to replicate the variance decompositions obtained in existing empirical studies.

In order to perform this quantitative exercise, I modify the model assumptions to obtain a

[24] See the discussion in Chapter 3 (§1.3) of Woodford (2003).

[25] In general, depending on parameters, λ can be either decreasing or increasing in σ, the same is true for η.

[26] In part, this effect is undone by the endogenous change in the optimal price: with higher elasticity the agent will increase his price less. For realistic parameter values the direct effect dominates.

more realistic behavior for nominal prices and to allow for interesting learning dynamics. The details of the model, together with a description of the computational method used, are in Appendix C. Here, I will give a brief overview of both.

On the model side, it is useful to eliminate the mechanical relation between productivity and the price level, which is due to the assumption of constant money supply. To do that, I assume that the stock of money in the hands of each household grows over time, and that the growth rate of the aggregate money stock is the same as that of aggregate productivity. I also introduce a random component in individual money growth, so that agents cannot infer aggregate productivity growth from observing it. Furthermore, the model, as it is, has no learning dynamics and, thus, no propagation mechanism. The observation of output at the end of date t, y_{it}, together with the observation of s_t at the beginning of the period, fully reveals the past shocks u_t and e_t to each household. To introduce learning dynamics in the model, I assume that each producer is also hit by a local demand shock, n_{it}, which determines the measure of consumers purchasing good i. The shock n_{it} adds noise to the agents' signals and implies that households learn gradually about the underlying aggregate shocks. Once more, the details can be found in Appendix C.

To compute the equilibrium I need to find the coefficients of the relation between aggregate quantities and prices and the state z_t. To solve for optimal individual behavior, I apply a method of undetermined coefficients, together with a Kalman filter to solve the agents' inference problem. The state space is approximated using the truncated state vector $z_t^{(T)} = \{a_t, s_t, ..., a_{t-T}, s_{t-T}\}$. For T sufficiently large the choice of T does not affect the results.[27]

The parameter β is set equal to 0.99, so the time period can be interpreted as a quarter. The value of η is set to 0.33, corresponding to a Frisch labor elasticity of 3, and the value of σ is set to 7.5, which implies a mark-up of around 15%. Both values are in the range of those used in existing DSGE studies with monopolistic competition and price rigidities. The parameter α is set equal to 0.01 in the benchmark parametrization. This preference parameter, together with $\beta = 0.99$, implies that the money-to-output ratio in steady state is equal to 2. Considering that the durable good is the only form of financial wealth in the economy, this does not seem an unreasonable parametrization.[28]

It remains to choose values for the variance of the shocks. For a variance decomposition exercise, in a linearized model, the choice of σ_u is merely a normalization, and I set $\sigma_u = 0.1$.

[27] Typically 35 periods are sufficient for the parameterizations presented.

[28] Moreover, as β is close to 1, the results presented are not sensitive to the choice of α.

For the noise in the public signal I choose a benchmark value of $\sigma_e = 0.62$. The precision of the public signal, $1/\sigma_e^2$, is the crucial variable that will determine the importance of demand shocks. Therefore, in the following, I will consider a range of values for σ_e and look at their implications for short-run output volatility. The main role of the idiosyncratic shocks in the model is to prevent full information revelation. Therefore, I calibrate their variance in order to obtain a realistic speed of learning in the economy. I measure the speed of learning in terms of the number of periods agents take to learn about a permanent productivity shock. In particular, let $a_{t|t} \equiv \int_0^1 E_{it}[a_t]\, di$, denote the average expectation about current productivity. The average speed of learning is measured by the number of periods it takes for $a_t - a_{t|t}$ to fall by a factor of $1/2$, after a permanent productivity shock u_t, i.e., by the half-life of the average expectational error. I choose a speed of learning of 4 periods (one year).[29] The historical experience with major shifts in productivity growth is that they have typically been recognized by professional economists and central bankers with a lag of at least two years. On this basis, this parametrization seems a reasonable starting point.

4.1 Dynamic responses

Figure 2 shows the responses of output, employment and prices to the fundamental shock, u_t, and to the news shock, e_t.[30] The last panel illustrates the dynamics of the average expectational error regarding aggregate output, $y_t - y_{t|t}$, where $y_{t|t} \equiv \int_0^1 E_{it}[y_t]\, di$.

The qualitative responses are analogous to the ones obtained in Proposition 4 for the model with no learning dynamics. The qualitative behavior of output and inflation is consistent with the evidence from identified VARs, e.g. Shapiro and Watson (1988) and Gali (1992, Table III). Namely, following the news shock both output and inflation increase, while following a productivity shock output increases, while inflation decreases. Furthermore, the negative response of employment to the permanent technology shock is consistent with the evidence presented in Gali (1999) and Francis and Ramey (2003). Recently, there has been substantial controversy regarding this empirical finding and more generally regarding the use of VAR evidence with semi-structural identification assumptions.[31] This controversy has highlighted

[29] In the benchmark calibration this corresponds to:

$$\sigma_\epsilon = \sigma_\zeta = \sigma_n = \sigma_v = 5.$$

I have experimented with different sets of values for these four parameters. For a given speed of learning, aggregate behavior does not seem to depend on the specific values chosen for the four parameters.

[30] For both, I report the response to a one-standard-deviation shock.

[31] See Christiano, Eichenbaum and Viguffson (2003), Chari, Kehoe and McGrattan (2004), Gali and Rabanal

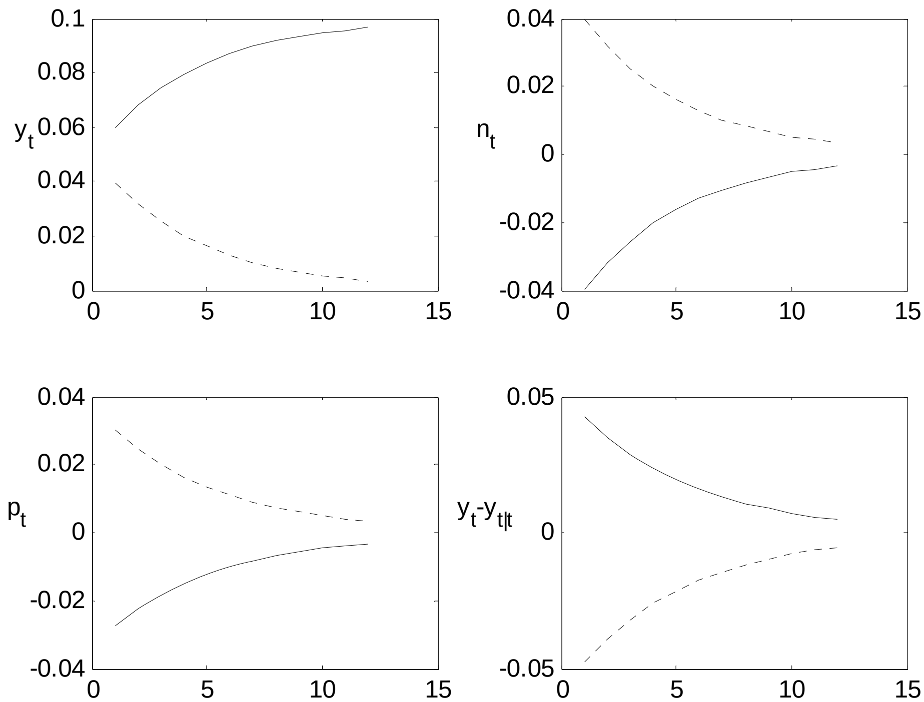

Figure 2: Impulse responses of prices, output, employment and the output surprise. Solid line: response to productivity shock. Dashed line: response to news shock.

the need of a theory-based rationale for identification assumptions. The present model has the advantage of being consistent with the identification in a large body of empirical work.[32]

Finally, news shocks are associated to negative output surprises, while fundamental shocks to positive output surprises. This is an additional testable restriction of the model, which can potentially be taken to the data, using survey data to estimate $y_{t|t}$.

4.2 Comparative dynamics

The analysis in Section 3 shows that the model parameters affect the economy response to the two shocks by determining: (i) the response of spending to expected productivity changes, (ii) the degree of strategic complementarity in pricing. Here, I show that these observations extend to the model considered here. Figure 3 illustrates the effect of changing β from 0.99 to 0.95. The top two panels report the benchmark impulse responses, for comparison. As discussed in sub-section 3.2.2 the parameter β is determinant for the response of consumption to expected productivity. A reduction in β, by shortening the horizon of the consumers, reduces the response of spending to expected changes in productivity. As a consequence, news shocks have a smaller effect, both in terms of prices and in terms of quantities.

Figure 4 shows the responses of prices and output for different levels of σ. In the top two panels I report the results for the benchmark level $\sigma = 7.5$ and in the bottom two panels those for $\sigma = 15$. A higher level of elasticity of substitution increases the degree of strategic complementarity in pricing, it reduces the response of prices to both shocks, it increases the response of output to a news shock and reduces the response of output to a fundamental shock.

4.3 How much short-run volatility can news shocks generate?

The structure of the model imposes a bound on the fraction of output volatility that can be explained by the news shock. If the public signal is very noisy agents would put little weight on it, while if the signal is very precise the economy would converge very fast to the full information equilibrium. In both cases, the news shock would only explain a small fraction of output volatility. Therefore, the question I address here is whether intermediate levels of signal precision can generate realistic values for the fraction of output volatility explained by the demand shocks.

(2004).

[32] In particular, the model is consistent both with long-run restrictions *à la* Blanchard and Quah (1989), and with sign restrictions, used e.g. in Blanchard (1989) and Canova and De Nicolo (2002).

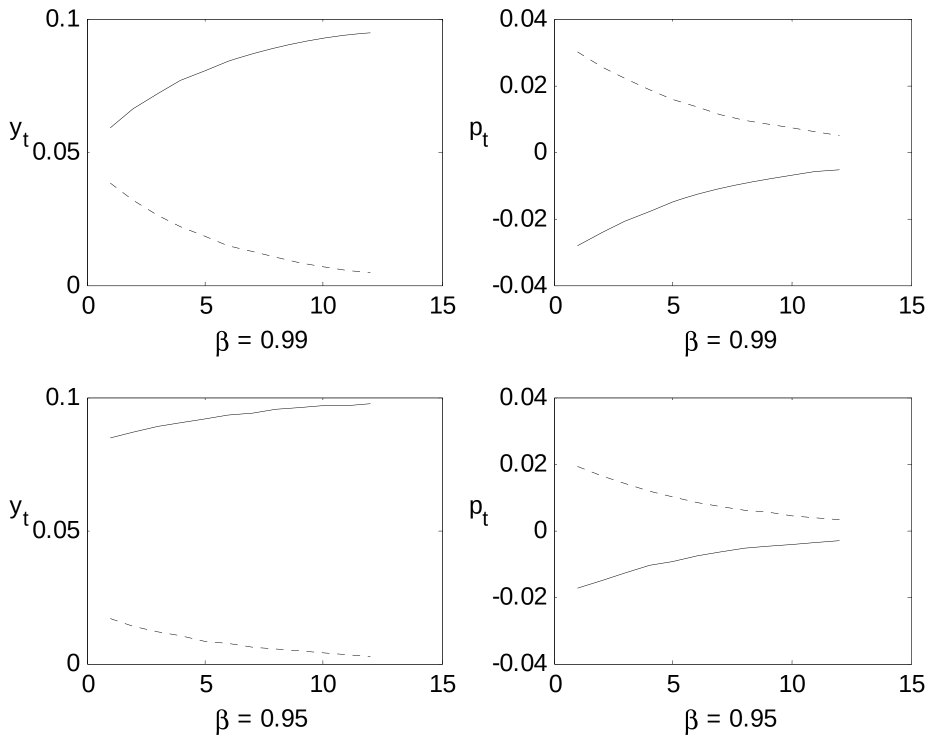

Figure 3: Changing the discount factor.

Solid line: response to productivity shock. Dashed line: response to news shock. Shocks as in Figure 3.

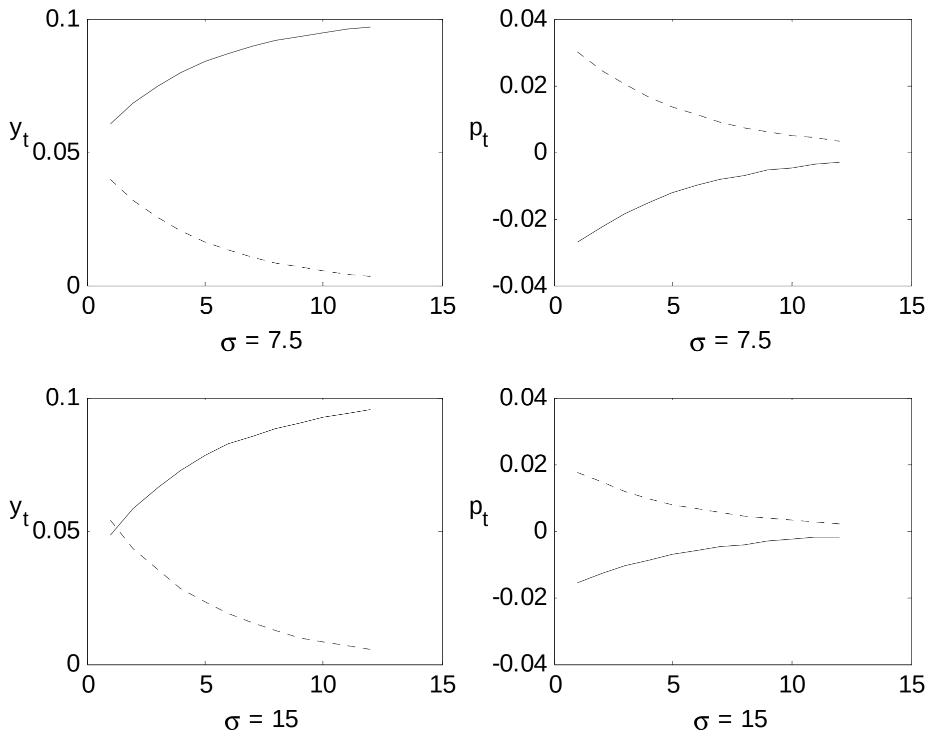

Figure 4: Changing the elasticity of substitution σ.
Solid line: response to productivity shock. Dashed line: response to news shock.

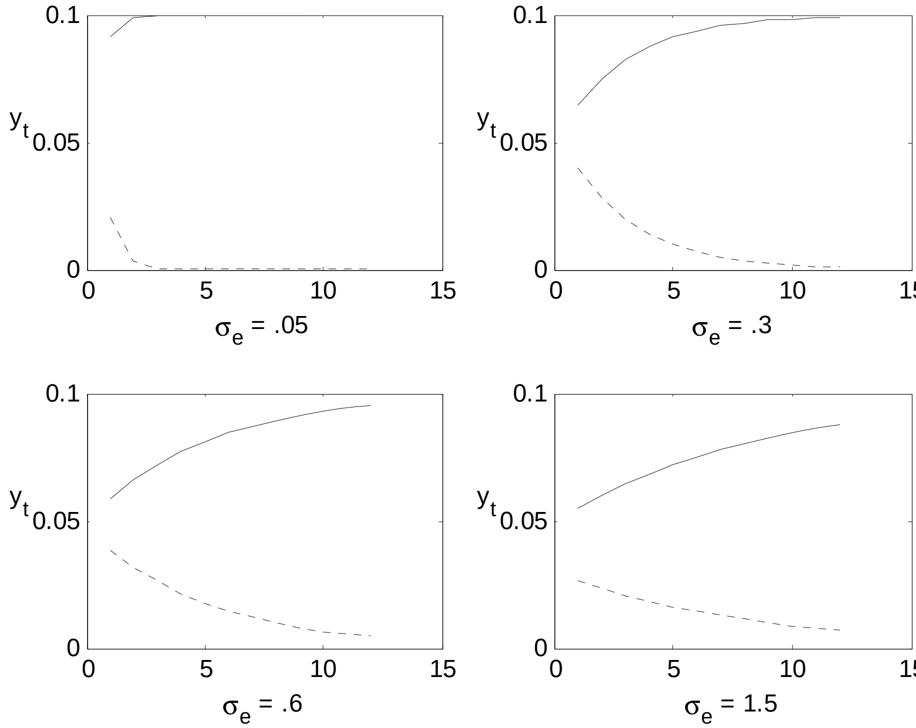

Figure 5: Signal precision and output responses.
Solid line: response to productivity shock. Dashed line: response to news shock.

Figure 5 illustrates the effects of changing the precision of the signal (i.e. changing σ_e) on the dynamic response of output to the two shocks.[33] The first panel shows that if the public signal is very precise, the news shock has very small and temporary effects. Moving to the second and third panel, the effect of the news shock increases. However, in the fourth panel the noise becomes so large that the effect on impact of the news shock tends to decrease. On the other hand, as the quality of the signal deteriorates agents take a longer time to learn the long-run equilibrium, and the effect of the news shock becomes more persistent.

Table 1 summarizes the model implications in terms of the fraction of output volatility explained by the news shock at different horizons. For comparison, the last column reports the values obtained by Gali (1992) (Table IV), where the empirical counterpart to the news shock and the productivity shock are, respectively, the "IS shock" and the "supply shock."

The first column reports the variance decomposition for the benchmark parameters. The

[33] Note that each panel reports the response to a one-standard-deviation shock.

model appears able to generate demand-driven short-run volatility in line with empirical observations, both for output and for prices. The two main results from this table are: (a) it is possible to choose σ_e so that the news shock generates 30% of output volatility on impact; (b) if one chooses σ_e to generate this amount of short-run volatility, then the news shock explains 15% of output volatility at a one-year horizon, i.e., around 3/4 of the observed output volatility due to demand shocks at the same horizon (19%). Result (a) is not obvious given the non-monotonicity of the relation between σ_e and short-run volatility discussed above. This is illustrated in columns (ii) and (iii) which show that, if I move away from an intermediate range for σ_e, I obtain lower demand-driven volatility. Result (b) clearly depends on the choice of the learning speed. This is illustrated in column (iv). In the benchmark calibration I set σ_ϵ equal to 5, in order to obtain a learning-time of 4 quarters. If I reduce σ_ϵ to 1, column (iv) shows that the learning time falls to 3.4 quarters and the effect of the news shock drops considerably.[34] Summing up, with a learning time of one year, the pure effect of learning dynamics is able to generate demand shocks that are both sizeable and persistent.

Notice that, in order to obtain a learning time of 4 quarters, I have to assume a very high volatility for idiosyncratic shocks, namely shocks 50 times larger than aggregate productivity shocks. This extreme assumption is needed for two reasons. First, to keep the model tractable, I have assumed that all shocks are i.i.d., so that every agent collects a large sample of independent observations in a short amount of time. Allowing for a more realistic autocorrelation structure for the shocks would slow down learning and introduce an additional source of persistence. Second, the model structure is very simple, and, in particular, prices are a very good signal of aggregate expectational mistakes. Introducing monetary shocks uncorrelated to productivity, or other sources of nominal volatility, would confound the inference problem of the agents. This would allow me to obtain the same learning speed with more realistic levels of idiosyncratic volatility. At this stage, large idiosyncratic shocks are a stand-in for all the unmodelled sources of uncertainty that complicate the learning problem of actual consumers. Further work is needed to address more explicitly these sources of uncertainty and assess their relevance. One possible avenue to consider, is to model the idiosyncratic noise as due to costly information processing, rather than to actual noisy observations, as in the limited attention approach of Sims (2003).

[34] Here I am changing the value of σ_ϵ, keeping the other idiosyncratic variances constant.

Table 1. Variance decomposition

	i	ii	iii	iv	Gali (1992)
	σ_e= .62 σ_ϵ= 5	σ_e= .3 σ_ϵ= 5	σ_e= 1.5 σ_ϵ= 5	σ_e= .62 σ_ϵ= 1	
Output					
1 quarter	0.30	0.27	0.19	0.22	0.31
2 quarters	0.24	0.19	0.15	0.17	—
5 quarters	0.14	0.09	0.10	0.09	0.19
10 quarters	0.07	0.03	0.05	0.04	0.10
Prices					
1 quarter	0.53	0.58	0.34	0.52	0.37
2 quarters	0.53	0.58	0.33	0.52	—
5 quarters	0.52	0.58	0.32	0.52	0.52
10 quarters	0.52	0.58	0.30	0.52	0.51
Learning time	4.0	2.1	6.9	3.4	

Note: The table reports the fraction of forecast volatility explained by the e_t shock. Last column from Table IV in Gali (1992).

5 Concluding remarks

In this paper I have derived the basic implications of a model of demand shocks based on misperceptions about aggregate productivity. In particular, I have focused on the fraction of output volatility that is explained by demand shocks. Further restrictions can be derived and tested. One could be more specific about the information in the public signal s_t, and test the model implications in terms of the economy's response to this signal. For example, one can focus on the information contained in publicly released macroeconomic statistics. Rodriguez Mora and Schulstald (2006) present evidence showing that aggregate consumption responds more to public announcements regarding GNP, than to actual movements in GNP.[35] This evidence is clearly consistent with the model presented here. Moreover, recent work by Beaudry and Portier (2006) shows that the stock market can be used as a measure of aggregate expectations about future productivity. The model in this paper does not fit well with the identification strategy in their paper, given that it only allows for permanent technology

[35] The authors look at the effect on aggregate consumption of changes in public statistics regarding GNP, including both the series representing the initial data release and the series for the revised data, which are published later but which are more precisely measured. In this way, they can identify a positive effect due to the noise included in the initial data release.

shocks.[36] Adding to the model news about the future, would make it possible to compare its implications with their evidence. This extension could also be useful to assess the relation between the identification approach in Beaudry and Portier (2006) and the approach based on aggregate demand/aggregate supply shocks.[37] Finally, one can exploit the fact that consumers' average expectations will reflect the underlying observed signals, and test the theory using information from consumer surveys data. Recent work by Barsky and Sims (2006) pursues this approach, using data from the Michigan Survey of Consumers.[38]

The results of this paper raise a number of questions about what determines the information collected by consumers, and what determines the overall precision of the information they acquire. Veldkamp (2006) and Hellwig and Veldkamp (2006) consider models where the decision to acquire information is treated explicitly. They show that, in environments where there are gains from coordination, agents tend to acquire similar pieces of information, and thus the potential for coordinated mistakes is bigger. Therefore, the role for demand shocks may actually be magnified when the information structure is endogenous. On the other hand, Maćkowiak and Wiederholt (2006), focus on endogenous information processing, in a price setting with limited attention. They show that the level of aggregate volatility affects the amount of attention agents devote to the shocks in their local markets and to aggregate shocks. Here I have assumed that agents observe their local shock with perfect precision, while they observe the aggregate shock with noise. The results in Maćkowiak and Wiederholt (2006) offer a microfoundation for this assumption. At the same time, they show that this assumption may be sensitive to changes in macroeconomic uncertainty.

[36] Beaudry and Portier (2006) focus on shocks to expectations about future TFP, which are orthogonal to current TFP shocks. Clearly, such shocks are not present if productivity is a random-walk.

[37] Conceptually, the two approaches simply differ in terms of the orthogonal decomposition of the shocks they adopt. Here I focus on the decomposition (u_t, e_t), which leads to a standard keynesian identification. An alternative decomposition is $\left(u_t + e_t, u_t - \frac{\sigma_u^2}{\sigma_e^2} e_t\right)$, which separates the "information shock" $u_t + e_t$, captured by the innovation in s_t. This second decomposition is closer to the identification in Beaudry and Portier (2006).

[38] Namely, they try to identify both an information shock —corresponding to $u_t + e_t$ in the current model—, and an animal spirit shock — corresponding to the news shock e_t in the current model. When unemployment is not included in the VAR, they find a significant relation with consumer confidence measures for the first type of shock, but not for the latter. When unemployment is included, instead, their evidence is also consistent with a significant relation between the news shock and consumer confidence.

Appendix

A. Definitions and linearization

This appendix contains the details of the model which are not in the main text, a formal definition of rational expectations equilibrium, and a definition of the notion of log-linear equilibrium that is used as an approximation.

Consumption baskets

The consumption basket J_{it} is selected so that the distribution $\{\epsilon_{jt} : j \in J_{it}\}$ is $N\left(\zeta_{it}, \sigma^2_{\epsilon|\zeta}\right)$. The sampling shock for consumer i, ζ_{it}, is drawn from a distribution $N\left(0, \sigma^2_{\zeta}\right)$. The sampling shocks satisfies $\int_0^1 \zeta_{it} di = 0$. The variances $\sigma^2_\epsilon, \sigma^2_\zeta$ and $\sigma^2_{\epsilon|\zeta}$ satisfy

$$\sigma^2_\epsilon = \sigma^2_\zeta + \sigma^2_{\epsilon|\zeta}.$$

This ensures that the cross sectional distribution of productivity shocks observed by consumers is identical to the distribution across producers.

Equilibrium

Price and demand indexes. To define a rational expectations equilibrium it is useful to define two functions, which give the price and demand indexes associated to the triple $\langle \mathcal{P}, \mathcal{C}, \{G\}\rangle$.

The price index function $\overline{\mathcal{P}}\left(\zeta_{it}, z_t\right)$ gives the price index for a consumer with sampling shock ζ_{it}. This price index is derived substituting the individual pricing rule $P\left(e^{a_{it}}, H_{it}, \mu_{it}\right)$ in expression (1) and integrating using the distribution $G\left(H_{it}, \mu_{it}; z_t\right)$ for H_{it} and μ_{it}, and the distribution $N\left(a_t + \zeta_{it}, \sigma^2_{\epsilon|\zeta}\right)$ for the productivity shocks a_{it}.

The demand index function $D\left(z_t\right)$ gives the constant in the demand curve faced by the producer. The demand for good i by consumer j is:

$$C_{jit} = P_{it}^{-\sigma} C_{jt} \overline{P}^{\sigma}_{jt}.$$

Aggregating across consumers gives $Y_{it} = P_{it}^{-\sigma} D$, where D is the demand index

$$D = \int C_{it} \overline{P}^{\sigma}_{it} di.$$

Substitute C_{it} and $\overline{P}_{it}$ on the right-hand side, using the individual consumption rule $C\left(e^{a_{it}}, H_{it}, \overline{\mathcal{P}}\left(\zeta_{it}; z_t\right), \mu_{it}\right)$ and the price index function derived above. Integrate using the distribution $G\left(H_{it}, \mu_{it}; z_t\right)$ for H_{it} and μ_{it}, the distribution $N\left(0, \sigma^2_\zeta\right)$ for the sampling shock ζ_{it}, and $N\left(0, \sigma^2_\epsilon\right)$ for the productivity shock a_{it}. This gives the function $D\left(z_t\right)$.

Optimality and consistency. The household problem is to set prices and consumption to maximize expected utility, subject to the constraints:

$$\begin{aligned}
Y_{it} &= P_{it}^{-\sigma} \mathcal{D}\left(z_t\right), \\
Y_{it} &= A_{it} N_{it}, \\
X_{it} &= H_{it} - \overline{P}_{it} C_{it}, \\
H_{it+1} &= H_{it} - \overline{P}_{it} C_{it} + P_{it} Y_{it}, \\
\overline{P}_{it} &= \overline{\mathcal{P}}\left(\zeta_{it}, z_t\right),
\end{aligned}$$

and to the measurability constraints reflecting the information available to consumers at each stage. Optimality requires that P and C are a solution to this optimization problem.

The beliefs of household i at the pricing stage, μ_{it}^{I}, are given by the bayesian update of μ_{it} based on the observation of A_{it}. The beliefs at the trading stage, μ_{it}^{II}, are the update of μ_{it}^{I} based on $\overline{P}_{it} = \overline{\mathcal{P}}\left(\zeta_{it}, z_t\right)$. The end-of-period beliefs μ_{it+1} of household i at the end of the period are the update of μ_{it}^{II} based on the observation of $D_t = D\left(z_t\right)$.

Incidentally, the fact that D_t is a function of z_t can be used to show that, in equilibrium, quantity observations fully reveal the aggregate productivity shock. In Section 4, instead, I introduce local demand shocks n_{it}, and the demand for producer i will be $e^{n_{it}}D\left(z_t\right)$. This assumption prevents full revelation in equilibrium.

The consumption and pricing rule, together with the bayesian updating rules described above define an individual law of motion that, for each pair H_{it} and μ_{it}, and each realization of the shocks u_t, e_t, ϵ_{it} and ζ_{it}, gives us H_{it+1} and μ_{it+1}. The consistency requirement asks that $G\left(H_{it+1}, \mu_{it+1}; z_{t+1}\right)$ is derived from $G\left(H_{it}, \mu_{it}; z_t\right)$ using this law of motion and the exogenous distributions of ϵ_{it} and ζ_{it}.

Log-Linearization

In order to study the equilibrium I resort to log-linearization. The choice of a linear approximation is dictated by the presence of imperfect information. In the original non-linear form the inference problem of the agents is daunting. In linear form, the Kalman filter can be used and recursive methods can be applied to characterize the equilibrium.

In a linear equilibrium aggregate output and prices, y_t and p_t, are:

$$p_t = \phi z_t, \tag{13}$$

$$y_t = \psi z_t, \tag{14}$$

where ϕ and ψ are two vectors of coefficients. The solution of the model requires solving a fixed point problem to find the coefficients ϕ and ψ that are consistent with optimality, belief rationality and consistency.

To use linearization methods in presence of imperfect information, requires an extra assumption. Namely, I assume that the agents in the model also use a linear model when drawing inferences from prices and quantity observations. In particular they use the model:

$$\begin{aligned} \overline{p}_{it} &= \phi z_t + \phi_\zeta \zeta_{it}, \\ y_{it} + \sigma p_{it} &= \left(\psi + \sigma\phi\right) z_t + n_{it}, \end{aligned}$$

where $\overline{p}_{it}$ is the log of the price index for agent i, and $y_{it} + \sigma p_{it}$ is the log of the demand index for agent i, and ϕ_ζ is a coefficient to be determined in equilibrium. Here, to be general, I allow for local demand shocks, n_{it}, as in the model of Section 4 (see Appendix C). Note that, apart from using a linear approximation, consumers use the correct model of the economy, i.e. they use values of ϕ and ψ and ϕ_ζ that are consistent with equilibrium behavior.

When agents use a linear model, their beliefs about the state z_t are normally distributed and their dynamics can be characterized using the Kalman filter. Then, the state vector for the agent decision problem is reduced to: the money balances and the first moment of his beliefs about z_t. This simplification of the state vector allows me to linearize the individual pricing and consumption rules $\mathcal{P}$ and $\mathcal{C}$. This rules can then be aggregated and the equilibrium is found as the solution of a fixed point problem. Linearity helps in three dimensions: it simplifies the inference problem faced by each individual, it simplifies the state space for individual decision rules, and it allows for aggregation of individual decision rules.

In the baseline model, studies in Section 3 agents learn the state z_t at the end of period t. This allows me to derive the equilibrium in closed form, these derivations are in Appendix B. In the extended model of Section 4, instead, I have to keep track of the agents expectations regarding all past shocks.

In Appendix C I describe how to compute the linear individual decision rule in this case, and I derive the steady-state cross sectional distributions of h_{it} and $E_{it}[z_t]$.

Finally, a brief remark on an issue that arises when taking linear approximations in a non-stationary model with imperfect information. The random walk for a_t makes the model non-stationary. Therefore, when taking a log-linear approximation I need to normalize variables by A_t, e.g. set $\hat{c}_{it} = \ln(C_{it}/A_t)$. However, with imperfect information agents do not observe A_t, This means that, for example, in the consumer first order conditions I will have $E_{it}[\hat{c}_{it}]$ and $E_{it}[\hat{c}_{it}] \neq \hat{c}_{it}$. In order to recover agents' decision rules it is convenient to add and subtract a_t whenever an expression like $E_{it}[\hat{c}_{it}]$ appears, e.g.

$$E_{it}[\hat{c}_{it}] = c_{it} - E_{it}[a_t] + E_{it}[a_t].$$

In this way, I derive the first order conditions for consumption and prices, (9) and (10).

B. Proofs for Section 3

Proof of Proposition 1

The first order conditions for pricing and consumption are:

$$\frac{1}{\overline{P}_{it}C_{it}}Y_{it} - \frac{\sigma}{\sigma-1}\frac{1}{A_{it}}\left(\frac{Y_{it}}{A_{it}}\right)^{\eta}\frac{Y_{it}}{P_{it}} = 0,$$

and

$$\frac{1}{\overline{P}_{it}C_{it}} = \alpha\frac{1}{\bar{H} - \overline{P}_{it}C_{it}} + \beta E_t\left[\frac{1}{\overline{P}_{it+1}C_{it+1}}\right].$$

Substituting the conjectured equilibrium prices and quantities it is immediate to check that household behavior is optimal.

Remark

A preliminary remark on the limit results in sub-section 3.2: analytical results can also be derived for the case of finite values of idiosyncratic variances. Moreover, it is possible to show, by continuity, that the behavior of the economy in the case of infinite idiosyncratic variances is indeed the limit of the finite variances case. The details are available from the author.

Proof of Proposition 3

Begin with the following guess for aggregate prices and output:

$$\begin{aligned} p_t &= -a_{t-1} + \phi_u u_t + \phi_e e_t, \\ y_t &= a_{t-1} + \psi_u u_t + \psi_e e_t, \end{aligned}$$

and guess that ϕ and ψ are such that

$$\begin{aligned} E_t^P[p_t] &= -E_t^P[a_t], \\ E_t^P[y_t] &= E_t^P[a_t]. \end{aligned}$$

Using this guess, the expected demand curve faced by producer i is:

$$E_{it}[y_{it}] = E_t^P[a_t] - \sigma p_{it}.$$

The optimal pricing condition is (10) in the text. Substituting the expected demand derived above gives

$$p_{it} = \frac{1}{1+\sigma\eta}(\overline{p}_{it} + c_{it}) - \frac{1+\eta}{1+\sigma\eta}a_{it} + \frac{\eta}{1+\sigma\eta}E_{it}[y_t + \sigma p_t].$$

For the individual policy rule for consumption use the conjecture

$$c_{it} + \overline{p}_{it} = b_h h_{it} + b_a \left(a_{it} - E_{it} \left[a_t \right] \right). \tag{15}$$

The Euler equation (9), in the text, can be rewritten as

$$c_{it} + \overline{p}_{it} = (1 - \delta) h_{it} + \delta E_t^{II} \left[c_{it+1} + \overline{p}_{it+1} \right],$$

where $\delta \equiv \frac{\alpha\beta}{\alpha + (1-\beta)^2} \in (0,1)$, while the agent budget constraint is, in log-linear terms:

$$h_{it+1} = h_{it} + \theta \left(p_{it} + y_{it} - \overline{p}_{it} - c_{it} \right), \tag{16}$$

where $\theta \equiv \frac{1-\beta}{\alpha+1-\beta}$.

Using the individual policy (15) the Euler equation becomes:

$$\overline{p}_{it} + c_{it} = (1 - \delta) h_{it} + \delta b_h E_{it} \left[h_{it+1} \right],$$

and, using the initial guesses, the budget constraint gives:

$$\begin{aligned} E_{it} \left[h_{it+1} \right] &= h_{it} - \theta\sigma \frac{1+\eta}{1+\eta\sigma} \left(\overline{p}_{it} + c_{it} \right) + \theta \frac{(1+\eta)(\sigma-1)}{1+\eta\sigma} a_{it} + \\ &\quad + \theta \frac{1+\eta}{1+\eta\sigma} E_{it} \left[y_t + \sigma p_t \right]. \end{aligned}$$

Putting together the last two equations gives the following two equations, that determine b_h and b_a:

$$\left(1 + \delta\theta\sigma \frac{1+\eta}{1+\eta\sigma} b_h \right) b_h = 1 - \delta + \delta b_h, \tag{17}$$

$$\left(1 + \delta\theta\sigma \frac{1+\eta}{1+\eta\sigma} b_h \right) b_a = \delta\theta \frac{(1+\eta)(\sigma-1)}{1+\eta\sigma} b_h. \tag{18}$$

The first equation can be rewritten as

$$\theta \frac{\delta}{1-\delta} \frac{\sigma(1+\eta)}{1+\eta\sigma} b_h^2 = 1 - b_h. \tag{19}$$

This equation has a unique positive solution in $(0,1)$. The second equation is a linear equation that gives b_a.

Aggregating the consumption equation across consumers and using the assumption of large idiosyncratic shocks, gives:

$$y_t = b_a \left(a_t - E_t^P \left[a_t \right] \right) - p_t. \tag{20}$$

Aggregating the optimal pricing condition gives

$$\begin{aligned} p_t &= \frac{1}{1+\sigma\eta} b_a \left(a_t - E_t^P \left[a_t \right] \right) - \frac{1+\eta}{1+\sigma\eta} a_t - \frac{\eta(\sigma-1)}{1+\sigma\eta} E_t^P \left[a_t \right] = \\ &= -E_t^P \left[a_t \right] + \frac{1 - b_a + \eta}{1+\sigma\eta} \left(E_t^P \left[a_t \right] - a_t \right). \end{aligned}$$

With the definition

$$\lambda \equiv 1 - b_a,$$

this gives (7). Substituting in (20) gives (6).

To prove that $\lambda \in (0,1)$ notice that, given that $b_h > 0$ one obtains

$$\delta\theta b_h \frac{(1+\eta)(\sigma-1)}{1+\eta\sigma} < 1 + \delta\theta b_h \frac{(1+\eta)\sigma}{1+\eta\sigma}$$

which implies $b_a \in (0,1)$. Therefore, as long as b_h and b_a satisfy (17) and (18) the initial guesses are verified and consumer behavior is optimal.

Proof of Proposition 4

The coefficient ψ_e was derived in the text, see (11). This coefficient is positive as long as:

$$\lambda\sigma > 1. \tag{21}$$

Using (17) and (18) λ can be written as a function of b_h.

$$\lambda = 1 - \frac{\sigma - 1}{\sigma} \frac{(1-\delta)(1-b_h)}{1-\delta(1-b_h)}.$$

Substitute this expression in (21). After some algebra one obtains that (21) is satisfied if and only if $(\sigma - 1) b_h > 0$ which is satisfied as long as $b_h > 0$.

Proof of Proposition 5

Consider equation (19), which determines b_h. It is straightforward to show that $\lim_{\delta\to 1} b_h = 0$ which, given (18) implies $\lim_{\delta \to 1} b_a = 0$. Moreover, the definition of δ gives $\lim_{\beta\to 1}\delta = 1$. Putting these results together gives

$$\lim_{\beta\to 1}\lambda = 1,$$

which proves the statement.

C. Assumptions and computational method for Section 4

Additional assumptions

The durable good in the hands of household i grows at the rate R_{it}. The household budget constraint becomes:

$$H_{it+1} = R_{it+1}\left(H_{it} - \int_{J_{it}} P_{jt}C_{ijt}dj + P_{it}Y_{it}\right).$$

Let $R_{it} = R_0 \exp(u_t + v_{it})$, where u_t is the aggregate productivity shock, and v_{it} is an i.i.d. shock to the individual rate of return, with distribution $N\left(0, \sigma_v^2\right)$. As usual, assume that the idiosyncratic shocks satisfy $\int_0^1 v_{it} = 0$.

Furthermore, assume that producer i is visited by a subset of measure of consumers equal to $\exp(n_{it})$. The variable n_{it} is i.i.d., normal with variance σ_n^2, and satisfies $\int_0^1 e^{n_{it}}di = 1$.

The equilibrium definition in Appendix A is easily adapted to this case. Note that the results in Section 3 can be extended to the case of money growth, if one maintains $\sigma_n^2 = 0$ and the assumption of large idiosyncratic shocks. Furthermore, one can show that, as $\beta \to 1$, the behavior of output is the same in the two cases. Analytical results for this case are available from the author.

Computation

Consider the linear equilibrium:

$$\begin{aligned} p_t &= \phi z_t, \\ y_t &= \psi z_t. \end{aligned}$$

Given a vector of parameters (ϕ, ψ) I derive optimal decision rules. Using bayesian updating, I also find the agents' first-order expectations regarding z_t. The optimal decision rules can then be aggregated to obtain a new vector of parameters $\left(\phi', \psi'\right)$. I iterate this procedure until I find a fixed point.

Optimal decision rules. Write the individual decision rules in the following form:

$$p_{it} = q_h h_{it} + q_a a_{it} + q_z^I E_{it}^I [z_t], \tag{22}$$
$$c_{it} = b_h h_{it} + b_a a_{it} + b_p \overline{p}_{it} + b_z^I E_{it}^I [z_t] + b_z^{II} E_{it}^{II} [z_t]. \tag{23}$$

Let me write here the first order conditions, the budget constraint, and the demand for good i:

$$\begin{aligned} p_{it} - E_{it}^I [\overline{p}_{it}] &= -a_{it} + E_{it}^I [c_{it}] + \eta \left(E_{it}^I [y_{it}] - a_{it} \right), \\ c_{it} + \overline{p}_{it} &= (1-\delta) h_{it} + \delta E_t^{II} \left[c_{it+1} + \overline{p}_{it+1} \right], \\ h_{it+1} &= h_{it} + \theta (p_{it} + y_{it} - \overline{p}_{it} - c_{it}) + r_{it+1}, \\ y_{it} &= y_t + \sigma (p_t - p_{it}) + n_{it}. \end{aligned}$$

Substituting the individual decisions rules and using the law of iterated expectations these four equations become:

$$\begin{aligned} p_{it} = & \frac{1}{1+\eta\sigma} \{ \left[(1 + b_p + \eta\sigma) \phi + b_z^I + b_z^{II} + \eta\psi \right] E_{it}^I [z_t] + \\ & b_h h_{it} + (b_a - 1 - \eta) a_{it} \}, \end{aligned}$$

$$\begin{aligned} \overline{p}_{it} + c_{it} = & (1-\delta) h_{it} + \delta (1 + b_p) \phi A E_{it}^{II} [z_t] + \delta b_h E_{it}^{II} [h_{it+1}] + \\ & \delta b_a e_1 A E_{it}^{II} [z_t] + \delta \left(b_z^I + b_z^{II} \right) A E_{it}^{II} [z_t], \end{aligned}$$

$$\begin{aligned} E_{it}^{II} [h_{it+1}] &= h_{it} + \theta \left(p_{it} + E_{it}^{II} [y_{it}] - \overline{p}_{it} - c_{it} \right), \\ E_{it}^{II} [y_{it}] &= (\psi + \sigma\phi) E_{it}^{II} [z_t] - \sigma p_{it}, \end{aligned}$$

where e_1 is the vector $[1, 0, 0...]$.

Substituting the linear rules on the left-hand side and matching coefficients gives:

$$\begin{aligned} q_h &= \frac{1}{1+\eta\sigma} b_h, \\ q_a &= \frac{1}{1+\eta\sigma} (b_a - 1 - \eta), \\ q_z &= \frac{1}{1+\eta\sigma} \left[(1 + b_p + \eta\sigma) \phi + b_z^I + b_z^{II} + \eta\psi \right], \end{aligned}$$

and

$$\begin{aligned} b_h &= \frac{1}{1+\delta b_h \theta} \left[((1-\delta) + \delta b_h) + \delta b_h \theta (1-\sigma) q_x \right], \\ b_a &= \frac{1}{1+\delta b_h \theta} \delta b_h \theta (1-\sigma) q_a, \\ b_p &= -1, \\ b_z^I &= \frac{1}{1+\delta b_h \theta} \delta b_h \theta (1-\sigma) q_z, \\ b_z^{II} &= \frac{1}{1+\delta b_h \theta} \left[\delta (1 + b_p) \phi A + \delta b_h \theta (\psi + \sigma\phi) + \delta b_a e_1 A + \delta \left(b_z^{II} + b_z^I \right) A \right]. \end{aligned}$$

For a given pair ψ and ϕ these equations can be solved for q and b. Notice that the parameters q_h, q_a, b_h, b_a, b_p can be determined separately, without knowledge of ψ and ϕ.

Bayesian updating. The learning problem of the individual agent can be solved recursively using the Kalman filter. It is convenient to divide the filtering problem in three stages. The first two stages correspond to the pricing and trading stages I and II. The last stage correspond to the time when consumers return to their island of origin and observe the quantity sold at the end of period t. Denote end of period expectations as $E_{it}[.]$.

The law of motion for the exogenous state z_t is:

$$z_{t+1} = Az_t + B \begin{pmatrix} u_t \\ e_t \end{pmatrix}.$$

For computational purposes we will consider the truncated version of z_t, $z_t^{[T]} = \{a_t, s_t, ...a_{t-T}, s_{t-T}\}$. In this case, A and B are:

$$A = \begin{bmatrix} 1 & \mathbf{0}_{1\times 2T-1} \\ 1 & \mathbf{0}_{1\times 2T-1} \\ \mathbf{I}_{2(T-1)} & \mathbf{0}_{2(T-1),2} \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ \mathbf{0} & \mathbf{0} \end{bmatrix}.$$

Given ϕ and ψ I can derive the expressions for the Kalman filter for agent i. Agent i first observes the vector of exogenous signals S_{it}, where

$$S_{it} = \begin{pmatrix} a_{it} & r_{it} & s_t \end{pmatrix}',$$

then observes $\overline{p}_{it}$, and, finally, observes the quantity signal $y_{it} + \sigma p_{it}$.

The relation between the signals $(S_{it}, \overline{p}_{it}, y_{it} + \sigma p_{it})$ and the aggregate state z_t is given by:

$$\begin{aligned} S_{it} &= Gz_t + F(\epsilon_{it}, v_{it})', \\ \overline{p}_{it} &= \phi z_t + \phi_\zeta \zeta_{it}, \\ y_{it} + \sigma p_{it} &= (\psi + \sigma\phi) z_t + n_{it}, \end{aligned}$$

where

$$G = \begin{bmatrix} 1 & 0 & 0 & \mathbf{0} \\ 1 & 0 & -1 & \mathbf{0} \\ 0 & 1 & 0 & \mathbf{0} \end{bmatrix}, F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix},$$

and where ϕ_ζ is the elasticity of the price index to the sampling shock ζ_{it}, to be determined later.

Let

$$\begin{aligned} \Sigma_V &= \begin{bmatrix} \sigma_\epsilon^2 & 0 \\ 0 & \sigma_v^2 \end{bmatrix}, \\ Q &= \phi, \ R = \psi + \sigma\phi. \end{aligned}$$

The agents expectations are given by

$$\begin{aligned} E_{it}^I[z_t] &= E_{it-1}[z_t] + C(S_{it} - E_{it-1}[S_{it}]), & \text{(24a)} \\ E_{it}^{II}[z_t] &= E_{it}^I[z_t] + L\left(\overline{p}_{it} - E_{it}^I[\overline{p}_{it}]\right), & \text{(24b)} \\ E_{it}[z_t] &= E_{it}^{II}[z_t] + M\left(q_{it} - E_{it}^{II}[q_{it}]\right). & \text{(24c)} \end{aligned}$$

To derive the Kalman gains $C, L,$ and M use the following definitions:

$$\Omega = Var_{it-1}[z_t], \ \Omega_I = Var_{it}^I[z_t], \ \Omega^{II} = Var_{it}^{II}[z_t].$$

Then the Kalman gains are:

$$\begin{aligned} C' &= (G\Omega G' + F\Sigma_V F')^{-1} G\Omega, & \text{(25a)} \\ L' &= \left(Q\Omega_I Q' + \phi_\zeta^2 \sigma_\zeta^2\right)^{-1} Q\Omega_I, & \text{(25b)} \\ M' &= \left(R\Omega_{II} R' + \sigma_n^2\right)^{-1} R\Omega_{II}. & \text{(25c)} \end{aligned}$$

The matrices Ω need to satisfy the equations:

$$\Omega_I = \Omega - \Omega G' \left(G\Omega G' + F\Sigma_V F'\right)^{-1} G\Omega, \tag{26a}$$

$$\Omega_{II} = \Omega_I - \Omega_I Q' \left(Q\Omega_I Q' + \phi_\zeta^2 \sigma_\zeta^2\right)^{-1} Q\Omega_I, \tag{26b}$$

$$\hat{\Omega} = \Omega_{II} - \Omega_{II} R' \left(R\Omega_{II} R' + \sigma_n^2\right)^{-1} R\Omega_{II}. \tag{26c}$$

Using the law of motion of z_t we obtain the steady state condition:

$$Var_t\left[z_{t+1}\right] = A\hat{\Omega}A' + B\Sigma B' = \Omega,$$

where

$$\Sigma = \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & \sigma_e^2 \end{bmatrix}.$$

To solve for the matrices Ω I use iterations on the (26). Then the Kalman gains C, L and M are derived from (25).

Fixed point. The average first order expectations can be expressed in terms of the current state as

$$z_{t|t^I} = \Xi_I z_t, \quad z_{t|t^{II}} = \Xi_{II} z_t, \quad z_{t|t} = \Xi z_t.$$

Using the updating equations and aggregating across consumers gives:

$$\begin{aligned} z_{t|t} &= (I - MR)(I - LQ)(I - CG) A z_{t-1|t-1} + \\ &\quad ((I - MR)((I - LQ) CG + LQ) + MR) z_t, \end{aligned}$$

which gives the following expression for Ξ,

$$\begin{aligned} \Xi &= (I - MR)(I - LQ)(I - CG) A\Xi + \\ &\quad ((I - MR)((I - LQ) CG + LQ) + MR), \end{aligned}$$

and similar expressions for Ξ_I and Ξ_{II}. These matrices are infinite dimensional. When using the truncated vector $z_t^{[T]}$ one finds finite dimensional matrices $\Xi^{[T]}$ that approximate Ξ (more on this approximation below). This is the only step where the use of the truncated vector $z_t^{[T]}$ requires an approximation.

Having expressions for $z_{t|t^I}$ and $z_{t|t^{II}}$ in terms of the current state variable I can use the equilibrium relations to obtain:

$$\begin{aligned} \phi' &= (q_h + q_a) e_1 + q_z \Xi_I, \\ \psi' &= (b_h + b_a) e_1 + b_p \phi + b_z^I \Xi_I + b_z^{II} \Xi_{II}. \end{aligned}$$

Moreover, consistency of the pricing rules with the price indexes requires $\phi_\zeta = b_a$.

In order to evaluate the accuracy of the approximation due to the truncation of the state space, I evaluate the distance between the vectors $q_z^{[T]}\Xi_I^{[T]}, b_z^{I[T]}\Xi_I^{[T]}, b_z^{II[T]}\Xi_{II}^{[T]}$ and $q_z^{[T+k]}\Xi_I^{[T+k]}, b_z^{I[T+k]}\Xi_I^{[T+k]}, b_z^{II[T]}\Xi_{II}^{[T+k]}$, for a given k.

Cross-sectional dispersion. It is also possible to derive the equilibrium joint distribution of money holdings and beliefs. Define the idiosyncratic component of agents' expectations as

$$J_{it} = E_{it}\left[z_t\right] - z_{t|t},$$

and define J^I_{it} and J^{II}_{it} in a similar way. Use the relations (24) to obtain the following recursive expression for the individual forecast errors:

$$\begin{aligned} J^I_{it} &= (I - CG)\, AJ_{it-1} + CFV_{it}, \\ J^{II}_{it} &= (I - LQ)\, J^I_{it} + L\zeta_{it}, \\ J_{it} &= (I - MR)\, J^{II}_{it} + Mn_{it}. \end{aligned}$$

This gives the law of motion for the individual forecast errors:

$$\begin{aligned} J_{it} = \; & (I - MR)\,(I - LQ)\,(I - CG)\, AJ_{it-1} + \\ & + (I - MR)\,(I - LQ)\, CFV_{it} + (I - MR)\, L\zeta_{it} + Mn_{it}. \end{aligned}$$

Define $x_{it-1} \equiv h_{it} - r_{it}$. The wealth dynamics are given by

$$\begin{aligned} x_{it} = \; & \left[\theta\left((1-\sigma)\, q_z - b^I_z\right)(I - CG)\, A + b^{II}_z\,(I - LQ)\,(I - CG)\, A\right] J_{it-1} + \\ & + \left[1 + \theta\,(1-\sigma)\, q_x - \theta b_x\right] x_{it-1} + \theta b^{II}_z L\zeta_{it} + \theta n_{it} \\ & + \left[\theta\left((1-\sigma)\, q_z - b^I_z\right) CF + \theta b^{II}_z\,(I - LQ)\, CF\right] V_{it} + \\ & + \theta\left((1-\sigma)\, q_a - b_a\right)\epsilon_{it} + \left[1 + \theta\left((1-\sigma)\, q_x - b_x\right)\right] v_{it}. \end{aligned}$$

Using the relations just derived one can write the joint dynamics of individual wealth and expectations in matrix form as:

$$\begin{pmatrix} J_{it} \\ x_{it} \end{pmatrix} = W_1 \begin{pmatrix} J_{it-1} \\ x_{it-1} \end{pmatrix} + W_2 \begin{pmatrix} \epsilon_{it} \\ v_{it} \\ n_{it} \\ \zeta_{it} \end{pmatrix}.$$

The steady state distribution of J_{it} and x_{it} is normal with variance-covariance matrix $\Sigma_{J,x} = W_1 \Sigma_{J,x} W_1' + W_2 \Sigma_{id} W_2'$.

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