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AGENCY-BASED ASSET PRICING

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ABSTRACT

We analyze the interaction between managerial decisions and firm value/asset prices by embedding the standard agency model of the firm into an otherwise standard asset pricing model. When the manager-agent's compensation depends on the firm's stock price performance, stock prices are set to induce the creation of future cash flows, instead of representing the discounted value of exogenous cash flows, as in the standard model. In our case, stock prices are formed via trading in the market to induce the managers to hold the number of shares consistent with the optimal effort level desired by the outside investors. We compare two price formation mechanisms, corresponding to two firm ownership structures. In the first, stock prices are formed competitively among a continuum of dispersed investors. In the second, stock prices are set by a single block shareholder, as a bargaining solution. Under both mechanisms there are persistent, dynamic, patterns of asst prices, The level of the equity premium and the return volatility depend on the risk aversion of the agents in the economy and the ownership structure of firms.

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1 Introduction

Financial Economics is somewhat schizophrenic. On the one hand, Corporate Finance views managerial decisions as being made by managers acting as the agents of outside investors. These managers are imperfectly controlled; they must be induced to make efforts with compensation contracts linked to the firm's performance. The firms cash flows are endogenous as they depend on managerial effort. On the other hand, Asset Pricing largely views corporate cash flows as exogenous and focuses on the proper way to discount these assumed cash flows, with a stochastic discount factor or pricing kernel. In this paper we provide an integrated model in which the cash flows of the firm are endogenized via an agency model and prices are formed in the stock market.

When there is no trade between the principal/investor and the agent/manager in the secondary stock markets, a long-term contract between the principal and the agent can be designed so that the manager's and the investor's marginal rates of substitution between present and future payoffs are equal as in, for example, Spear and Srivastava (1987). In our model, besides the shareholdings of the manager, there is no other explicit contract between the inside equity holder (the manager) and the outside equity holders (the investors). The manager's incentive to make an effort comes purely from his holding of equity shares.[1] The initial shareholdings distribution represents the initial contract between the manager and the investors, and this initial split determines how the manager and the investors will share the output of the firm, on average, in the future.

In an optimal long term contract, in order to generate incentives for the manager to make efforts, the managerial compensation arrangement needs to be reset in response to the realization of the firm's output.[2] In our model, indirect renegotiation between the manager and the investors occurs only through stock market trading, which changes the allocation of shareholdings between the manager and the investors and, consequently, changes the future sharing of the output of the firm, as well as the incentive for the manager to expend efforts. In general, the path dependency of the equilibrium outcome does not allow corporate decisions to be separated from asset pricing. However, we focus on some type of Markovian strategies, which allows us to use the shareholdings of the manager as a state variable for the economy. The interaction between corporate compensation and asset prices play a key role in determining the dynamics of the output, and it is crucial to include a "corporate finance" state variable for a standard asset pricing model to replicate the real economy.

We explain trade in the stock market and stock prices as part of the dynamics of corporate

[1] We could include some form of (price sensitive) contract in the model, but for simplicity we do not do that. Also, for simplicity we do not allow the manager to trade securities other than his own company's stock. Because the manager is risk averse he clearly would like to diversify and, to that extent, prices of other stocks would also be affected in equilibrium.

[2] See, for example, Rogerson (1985) for a discussion on the memory of the optimal dynamic contract.

governance. This view of asset pricing contrasts with most asset pricing research, which takes the firms' cash flows as exogenous and studies the effects on equity price of various types of preferences in pure exchange economies. We show, however, that stock prices do not simply reflect discounted future cash flows. Rather stock prices are formed, in part, to entice the manager to hold a certain fraction of the outstanding equity shares, a fraction that provides an incentive to generate future cash flows. Stock prices and the ownership structure of equity holdings of the manager and the outside shareholders fluctuate over time, influenced both by the incentive effects on future output due to the manager's holdings, but also due to risk sharing considerations. Risk sharing considerations results in persistence of ownership shares, and hence of prices, as investors are reluctant to have large price moves to induce large changes in the stock-holding of the manager. We also show that the level of the equity premium and the return volatility depend on the risk aversion of the agents in the economy and the ownership structure of firms. Rich patterns of stock price dynamics occur, depending on the endogenous trading of stock between the manager and outside investors, although output is i.i.d. conditional on the manager's effort choice. We display these dynamics with numerical examples.

In our setting since asset pricing is linked to corporate governance, we study two price formation mechanisms, corresponding to the two canonical corporate ownership structures. The first is where ownership is dispersed. This is the classic view of the separation of ownership and control that is at the root of Corporate Finance. Stock prices are formed competitively among a continuum of small dispersed investors. The other extreme is the case of a single outside blockholder, who sets the stock price as a bargaining solution with the manager. Blockholders are widely viewed as being able to exert closer control and, indeed, here the ability to bargain directly with the manager will have that effect. In our model, this increased "control" is due to how the stock price is formed.

We show that the ownership structure matters for price formation and return dynamics. When outside investors are dispersed they behave competitively and symmetrically. They coordinate their beliefs about trading behavior and the effort choice of the manager. The equity price consistently reflects these beliefs and induces the manager to behave as expected. We model this as a game played between a controlling manager and a continuum of outside investors. When there is a single blockholder, this outside investor sets the price to induce the manager to behave in his best interests. In this case, as distinct from the case with a continuum of investors, the equity price is not required to be consistent with the expectation of future dividends. This allows the block investor to negotiate with the manager more effectively. The ownership structure matters. Generally, the model generates equity premia and equity return volatility, depending on risk aversion, as well as the ownership structure. But, in particular, with single blockholder there is a higher equity premium and higher equity return volatility.

There are a few papers that do integrate corporate governance considerations into asset pricing. These include Dow, Gorton and Krishnamurthy (2005), Danthine and Donaldson

(2003), and Philippon (2003). Dow, Gorton and Krishnamurthy (2005) incorporate Jensen's free cash flow theory into a dynamic equilibrium model and study the effect of imperfect corporate control; they first solve the optimal auditing rule, and then study the effect of auditing cost (agency cost) on equilibrium term structure pattern. Danthine and Donaldson (2003) analyze the optimal contracting problem in a stochastic growth model when shareholders hire a self-interested manager; they show the difference between the optimal contract and the standard practice of corporate governance, and thus demonstrate the discrepancy between the delegated management economy and the standard representative agent business cycle model. While Dow, Gorton and Krishnamurthy (2005) and Danthine and Donaldson (2003) focus on the impact of corporate governance on firm outputs and/or asset prices, Philippon (2003) adopts an opposite view by studying the impact of economic conditions on corporate governance. None of these studies analyze the interaction between corporate governance and asset prices, which is captured in our study.

We proceed as follows. In Section II, we first set-up the model, then, to build some understanding of the effects of the agency problem on asset pricing, we study a benchmark case, in which the manager's shareholdings are fixed. In Section III, we study the cases with trading by the manager when there are a continuum of investors or a single blockholder, respectively, followed by some discussion on the model implications and related empirical literature. In Section IV, we provide some numerical examples and discuss the model's implications for asset prices. We conclude in Section V.

2 Model Set-up and A Benchmark Case

2.1 The Model Set-up

There are two types of agents in the economy, an inside equity holder and some outside equity holders. The inside equity holder is the manager who owns some shares of the company and whose effort choice affects the output of the firm. The outside equity holders are investors in the market. They do not manage the firm directly, but their trading affects the market price of the equity. We consider two situations: a continuum of small homogeneous outside equity holders and a second case with a single outside equity holder, a blockholder. For the case with a continuum of outside investors, we only consider symmetric (among investors) equilibria, and we will refer to a "representative investor" or just "investor" in that case.

At $t = 0$, the manager and the investor sign a contract to start the firm. The investor invests some capital in the firm, and receives a fraction $1 - \alpha_0$ of the firm's shares. The inside equity holder (the manager) then owns α_0 portion of the total equity. At the beginning of each period t, the manager chooses an effort level e_t, which is unobservable by the investor. Conditional on this effort level, the output each period y_t, which is perishable, has a distribution with density function $f(y_t|e_t)$, which is defined on a support $\mathcal{Y} = [\underline{y}, \overline{y}] \subseteq \mathcal{R}^+$. The support $\mathcal{Y}$ is invariant

to the effort level e_t. Therefore, the output y_t is an imperfect signal of the effort choice in period t. In each period, after y_t is realized, it is distributed as a dividend, and the manager and the investor start trading in the stock market. The fraction of shares that the manager owns before trade is α_{mt}. The manager chooses to trade $\Delta\alpha_{mt}$. Therefore, the manager owns $\alpha_{mt+1} = \alpha_{mt} + \Delta\alpha_{mt}$ fraction at the beginning of period $t+1$. The fraction of equity that the investor owns before trade is α_{it}, and the investor chooses to trade $\Delta\alpha_{it}$ at the price p_t. Therefore, the investor owns $\alpha_{it+1} = \alpha_{it} + \Delta\alpha_{it}$ shares at the beginning of period $t+1$. After trading in the stock market, the next period starts. To summarize, the timing in each period t is as follows:

1. The manager, with a share of $\alpha_{mt} \in [0,1]$, chooses an effort level e_t;
2. Nature chooses output y_t according to the density function $f(y_t|e_t)$;
3. All the output is distributed as dividends; the manager gets $\alpha_{mt}y_t$, and the investor gets $\alpha_{it}y_t$;
4. Stock trading starts. After trading, the manager ends up with α_{mt+1} shares of the equity, and the investor ends up with α_{it+1} shares of the equity;
5. Consumption occurs and next period starts.

At time t, the consumption of the manager is given by:

$$c_{mt} = \alpha_{mt}y_t + (\alpha_{mt+1} - \alpha_{mt})p_t,$$

and the consumption of the investor is given by:

$$c_{it} = \alpha_{it}y_t + (\alpha_{it+1} - \alpha_{it})p_t.$$

Thus, an allocation can be represented as $\{c_{mt}, c_{it}, \alpha_{mt+1}, \alpha_{it+1}\}_{t=0}^{\infty}$, and in equilibrium, $\alpha_{mt+1} + \alpha_{it+1} = 1$ and $c_{mt} + c_{it} = y_t$. In the rest of the paper, we will use α_t to denote fraction of equity held by the manager at the beginning of time t.

The manager is risk averse, with preferences represented by a utility function $-g(e) + u(c)$ for the consumption and effort level each period. Assume that $u_c > 0$, $u_{cc} < 0$, and $g_e > 0$. The manager's lifetime utility is:

$$\sum_{t=0}^{\infty} \delta^t[-g(e_t) + u(c_{mt})],$$

where $\delta \in (0,1)$ is the discount factor.

The (representative) investor is also risk averse, with the preferences represented by a utility function $v(.)$ for consumption each period. His lifetime utility is:

$$\sum_{t=0}^{\infty} \delta^t v(c_{it}).$$

We impose short sale constraints on both the manager and investor. Therefore, $\alpha_t \in [0,1]$ for any $t \geq 0$.

Let $\Phi(\alpha_0)$ denote the dynamic game between the manager and investor when the representative investor starts the game holding $1-\alpha_0$ shares of the firm. At the beginning of period t the manager possesses an information set, which may be written as $h_m^t = \{\alpha_0\} \cup \{\alpha_{\tau+1}, e_\tau, p_\tau, y_\tau\}_{\tau=0}^{t-1} \in H_m^t$ for $t \geq 1$, and $h_m^0 = \{\alpha_0\}$. A (pure) strategy for the manager associates a schedule $\sigma_{mt}(h_m^t)$ with each $t = 0, 1, 2...$, and $\sigma_{mt} : H_m^t \to S_m$, where S_m is the stage strategy space with element s_{mt}.

The investor does not observe the effort choice of the manager. At the beginning of period t the investor possesses an information set, which may be written as $h_i^t = \{\alpha_0\} \cup \{\alpha_{\tau+1}, p_\tau, y_\tau\}_{\tau=0}^{t-1} \in H_i^t$ for $t \geq 1$, and $h_i^0 = \{\alpha_0\}$. A (pure) strategy for the investor associates a schedule $\sigma_{it}(h_i^t)$ with each $t = 0, 1,...,$ and $\sigma_{it} : H_i^t \to S_i$, where S_i is the stage strategy space with element s_{it}.

In this paper, we will study sequential equilibria, in which the strategies of both the manager and the investor depend only on public information. This type of equilibrium is called *Perfect Public Equilibrium (PPE)* (see Fudenberg, Levine, and Maskin (1994)). The public history is $h^t = \{y_\tau\}_{\tau=0}^{t-1} \in H^t$ for $t \geq 1$ with $h^0 = \varnothing$.[3] A strategy profile for $\Phi(\alpha_0)$ is a pair of strategies $\sigma = (\sigma_m, \sigma_i)$. Let $\Sigma = \Sigma_m \times \Sigma_i$ denote the set of all strategy profiles for $\Phi(\alpha_0)$.

2.2 A Benchmark Case of No Trading

To start to understand the model we analyze a benchmark case in which the manager is not allowed to trade his shares in the stock market. The manager makes an effort choice each period, and this effort choice will be influenced by the fraction of the equity that he owns. Equity prices are decided in a competitive market with a continuum of outside investors. By studying this case, we can show the comparative static that a change of the fraction of equity held by the manager initially can change the incentive for him to make effort. Later on we will also compare the results when the manager trades to this benchmark case.

For a given α, the manager's life time utility is $\sum_{t=0}^{\infty} \delta^t [-g(e_t) + u(\alpha y_t)]$. The manager chooses an effort level to maximize his lifetime utility:

$$\begin{aligned} U_0(\alpha) &= \max_{\{e_t\}} \sum_{t=0}^{\infty} \delta^t E[-g(e_t) + u(\alpha y_t)|e_t] \\ &= \max_{\widetilde{e}} \int_{\mathcal{Y}} [-g(\widetilde{e}) + u(\alpha y)] f(y|\widetilde{e}) dy + \delta U_0(\alpha). \end{aligned} \tag{1}$$

The optimal effort choice, e, satisfies the first order condition:

$$-g_e(e) + \int_{\mathcal{Y}} u(\alpha y) f_e(y|e)] dy = 0. \tag{2}$$

[3] We do not include α_{t+1} or p_t in the public history at time t, because in a PPE, it is a function of $h^t = \{y_\tau\}_{\tau=0}^{t-1}$.

We assume that the second order condition holds:

$$-g_{ee}(e) + \int_{\mathcal{Y}} u(\alpha y) f_{ee}(y|e)]dy < 0.$$

The lemma below demonstrates how the manager's incentive to make efforts is affected by his shareholdings.

Lemma 1 *If (1) the manager's utility function satisfies the condition that $yu_c(\alpha y)$ is increasing (decreasing) in y, and (2) $f(y|e)$ satisfies the Monotone Likelihood Ratio Condition (MLRC), then the optimal effort level, e, is increasing (decreasing) in the fraction of equity held by the manager, α.*[4]

Proof. We need to prove that $e_\alpha = \frac{\partial e}{\partial \alpha} \geq 0$. Taking the derivative of equation (2) with respect to α gives:

$$0 = e_\alpha[-g_{ee}(e) + \int_{\mathcal{Y}} u(\alpha y) f_{ee}(y|e)dy] + \int_{\mathcal{Y}} yu_c(\alpha y) f_e(y|e)]dy.$$

We can see $e_\alpha > 0$ iff $\int_{\mathcal{Y}} yu_c(\alpha y) f_e(y|e)]dy > 0$, since $-g_{ee}(e) + \int_{\mathcal{Y}} u(\alpha y) f_{ee}(y|e)dy < 0$.

Write:

$$\int_{\mathcal{Y}} yu_c(\alpha y) f_e(y|e)dy = \int_{\mathcal{Y}} yu_c(\alpha y) \frac{f_e(y|e)}{f(y|e)} f(y|e)dy.$$

If $yu_c(\alpha y)$ is increasing in y, we can conclude:

$$\int_{\mathcal{Y}} yu_c(\alpha y) \frac{f_e(y|e)}{f(y|e)} f(y|e)dy > 0,$$

since $\int_{\mathcal{Y}} \frac{f_e(y|e)}{f(y|e)} f(y|e)dy = 0$ and $\frac{f_e(y|e)}{f(y|e)}$ is increasing in y (implied by the Monotone Likelihood Ratio Condition (MLRC)).

Similarly if $yu_c(\alpha y)$ is decreasing in y, we can conclude:

$$\int_{\mathcal{Y}} yu_c(\alpha y) \frac{f_e(y|e)}{f(y|e)} f(y|e)dy < 0.$$

■

Therefore, in the benchmark case where we do not allow the manager to trade his own shares, when the MLRC holds, the optimal effort level is increasing or decreasing with the fraction of equity held by the manager depending on the risk aversion of the manager. For a manager with constant relative risk aversion (γ_m), $\gamma_m > 1$ implies $yu_c(\alpha y)$ is decreasing in y and the optimal effort choice of the manager is decreasing in α, and when $\gamma_m < 1$, the manager expends more effort with more shares. Intuitively, for a very risk averse manager, the marginal

[4] The Monotone Likelihood Ratio Condition (MLRC) holds if the ratio $f_e(y|e)/f(y|e)$ is an increasing function of y. In words, it says that a high output realization is more likely to be due to a high effort choice.

gain from extra consumption decreases quickly as y goes up, and that sabotages the incentive for the manager to makes greater effort.

Before we proceed to the case in which the manager is allowed to trade, we study the incentive for the manager to (deviate to) trade in the benchmark case.

For the lemma below, we assume that the Arrow-Pratt measure of relative risk aversion of the investors and the manager are constant. Then: $-\frac{c_i v_{cc}(c_i)}{v_c(c_i)} = \gamma_i$, and $-\frac{c_m u_{cc}(c_m)}{u_c(c_m)} = \gamma_m$.

Lemma 2 *If the market price of equity is formed based on the beliefs that the manager will never trade any shares in the stock market, then generically the manager has an incentive to trade as long as* $\gamma_i \neq \gamma_m$.

Proof. The expected utility of the manager is given by:

$$U(\alpha) = \sum_{t=0}^{\infty} E[-g(e) + u(\alpha y_t)|e],$$

where e is the optimal effort level in the equilibrium. Without loss of generality, we assume $\alpha \in (0, 1)$.

Without loss of generality, assume $y = \overline{y}$ is realized, and consider a one shot deviation. After $\overline{y}$ is realized, the manager is facing the following optimization problem:

$$\begin{aligned} & \max_{\alpha'} u(c) + \delta E[U(\alpha')] \\ s.t.\ c &= \alpha\overline{y} + (\alpha - \alpha')p(\overline{y}) \end{aligned}$$

The first order condition with respect to α' is:

$$-u_c(c)p(\overline{y}) + \delta U_\alpha(\alpha') = 0.$$

The Envelope Theorem tells us that:

$$U_\alpha(\alpha') = \sum_{t=0}^{\infty} \delta^t \int y_t u_c(\alpha' y_t) f(y_t|e(\alpha')) dy_t.$$

Given α, the investors' trading determines the equity price:

$$p(\overline{y}) = \frac{1}{v_c((1-\alpha)\overline{y})} E[\sum_{t=1}^{\infty} \delta^t y_t v_c((1-\alpha)y_t)|e(\alpha)].$$

It is easy to check that at $\alpha' = \alpha$, the first order derivative with respect to α' is:

$$\begin{aligned} & -u_c(c)p(\overline{y}) + \delta U_\alpha(\alpha) \\ = & \sum_{t=1}^{\infty} \delta^t \int_{\mathcal{Y}} y_t [u_c(\alpha y_t) - u_c(\alpha\overline{y}) \frac{v_c((1-\alpha)y_t)}{v_c((1-\alpha)\overline{y})}] f(y_t|e(\alpha)) dy_t, \end{aligned}$$

which is $\begin{matrix} > \\ < \end{matrix} 0$ if $\gamma_u \begin{matrix} > \\ < \end{matrix} \gamma_v$. Therefore, we can see that the manager in general will have an incentive to trade in the market. ■

Intuitively, when the manager is more risk averse than the investor, the market price will be such that the manager wants to trade to smooth his consumption over time (sell in bad times (low output) and buy in good times). When the manager is less risk averse, the manager will trade to take the advantage of the price (buy in bad times and sell in good times).

The equity price can be written as:

$$p(\alpha, y) = \int_{\mathcal{Y}} \frac{\delta v_c(c_i')}{v_c(c_i)}[y' + p(\alpha, y')]f(y'|e),$$

where $c_i = (1-\alpha)y$ and $c_i' = (1-\alpha)y'$.

We do not explicitly have a bond in the economy, but that is equivalent to assuming that bonds are in zero net supply and thus no one trades bonds in a symmetric (among investors) equilibrium. We can write the price of a risk free bond as follows:

$$b(\alpha, y) \equiv \int_{\mathcal{Y}} \frac{\delta v_c(c_i')}{v_c(c_i)}[y' + p(\alpha, y')]f(y'|e).$$

Lemma 3 *Assume the investor has a constant Arrow-Pratt measure of relative risk aversion and that $f(y|e)$ satisfies the Monotone Likelihood Ratio Condition, Then:*

(i) if both $yv_c(y)$ and $yu_c(\alpha y)$ are monotonic in y and $\frac{\partial yv_c((1-\alpha)y)}{\partial y}\frac{\partial yu_c(\alpha y)}{\partial y} > 0$ (< 0), then the equity price is increasing (decreasing) in α;

(ii) if the manager's utility function satisfies the condition that $yu_c(\alpha y)$ is increasing (decreasing) in y, then the price of the risk free bond is decreasing (increasing) in α and the risk free interest rate is increasing (decreasing) in α.

Proof. For (i), we can write:

$$p(\alpha, y) = \frac{\delta}{1-\delta}\int_{\mathcal{Y}} y' \frac{v_c(c_i')}{v_c(c_i)} f(y'|e)dy'.$$

Differentiating the equity price with respect to α gives us:

$$\begin{aligned} p_\alpha(\alpha, y) &= \frac{1}{v_c(c_i)}\int_{\mathcal{Y}} \delta y' v_c(c_i')e_\alpha f_e(y'|e)dy' \\ &\quad + \frac{1}{v_c^2(c_i)}\{v_c(c_i)\int_{\mathcal{Y}} -\delta y'^2 v_{cc}(c_i')f(y'|e)dy' + yv_{cc}(c_i)\int_{\mathcal{Y}} \delta y' v_c(c_i')f(y'|e)dy'\}. \end{aligned}$$

With a constant Arrow-Pratt measure of relative risk aversion for the investor, i.e. $\frac{-c_i v_{cc}(c_i)}{v_c(c_i)} = \gamma_v$, the last two terms sum to zero and we have:

$$p_\alpha(\alpha, y) = \frac{e_\alpha}{v_c(c_i)}\int_{\mathcal{Y}} \delta y' v_c(c_i')\frac{f_e(y'|e^*(\alpha))}{f(y'|e^*(\alpha))}f(y'|e)dy'.$$

By Lemma 1, the result is immediate.

For (ii), similarly, we can show:

$$b_\alpha(\alpha, y) = \frac{e_\alpha}{v_c(c_i)}\int_{\mathcal{Y}} \delta v_c(c_i')\frac{f_e(y'|e^*(\alpha))}{f(y'|e^*(\alpha))}f(y'|e)dy'.$$

The result is immediate. ■

Lemma 3 shows the impact of the level of equity-linked compensation on security prices, in a static setting. The bond price is mainly affected by the manager's preference towards risk, and so is the risk free interest rate. As we have discussed, when the manager is very risk averse, his effort choice is actually decreasing with α, and the risk free interest rate (conditional on y) is decreasing with α. So, roughly speaking, when output is high, if the manager with high risk aversion increases his shareholdings to smooth his consumption (investors are less averse), then the interest rate will actually go down. The interest rate will go up with high output if the investors are more risk averse (in which case the manager sells shares when output is high).

The dependence of the equity price on α demonstrates how the interaction of the investor's preferences with the manager's preferences affects the equity price. When the manager is very risk averse, a lower effort choice by the manager reduces output on average. However, if the investor is also very risk averse, the equity price (conditional on y) is increasing with α.

Next, we study the impact of α on the equilibrium equity risk premium. The conditional equity premium can be written as:

$$\pi(\alpha, y) = \int_{\mathcal{Y}} [\frac{y' + p(\alpha, y')}{p(\alpha, y)} - \frac{1}{b(\alpha, y)}] f(y'|e) dy'.$$

We also define the realized equity premium as follows:

$$\Pi(\alpha, y, y') = \frac{y' + p(\alpha, y')}{p(\alpha, y)} - \frac{1}{b(\alpha, y)}.$$

Lemma 4 *Define* $g(\alpha, y, y') = \Pi_\alpha(\alpha, y, y') + \Pi(\alpha, y, y') e_\alpha \frac{f_e(y'|e)}{f(y'|e)}$. *If (i) the investor has a constant Arrow-Pratt measure of relative risk aversion, and (ii)* $f(y|e)$ *satisfies the Monotone Likelihood Ratio Condition, and (iii)* $g(\alpha, y, y')$ *is increasing (decreasing) with* y'*, then the conditional equity premium* $\pi(\alpha, y)$ *is increasing (decreasing) with* α *.*

Proof. We know that:

$$\frac{\delta}{v_c(c_i)} E[v_c(c_i') \Pi(\alpha, y, y')|e] = 0.$$

With CRRA preferences for the investor, differentiating the above equation with respect to α gives:

$$E[v_c(c_i') g(\alpha, y, y')|e] = 0.$$

It is easy to check that $\pi_\alpha(\alpha, y) = E[g(\alpha, y, y')|e]$. We know:

$$\begin{aligned} COV[v_c(c_i'), g(\alpha, y, y')|e] &= E[v_c(c_i') g(\alpha, y, y')|e] - E[v_c(c_i')|e] E[g(\alpha, y, y')|e] \\ &= -E[v_c(c_i')|e] E[g(\alpha, y, y')|e]. \end{aligned}$$

We know $E[v_c(c_i')|e] > 0$, then $COV[v_c(c_i'), g(\alpha, y, y')|e]$ and $E[g(\alpha, y, y')|e]$ have different signs. Since $v_c(c_i')$ is decreasing with y', when $g(\alpha, y, y')$ is increasing (decreasing) with y', we know $COV[v_c(y'), g(\alpha, y, y')|e] < 0$ (> 0) and $E[g(\alpha, y, y')|e] > 0$ (< 0). ■

In the above lemma, the sign of $g_{y'}(\alpha, y, y')$ for condition (iii) is uncertain. In the expression for $g(\alpha, y, y')$, MLRC implies $\frac{f_e(y'|e)}{f(y'|e)}$ is increasing with y', and we know $\Pi(\alpha, y, y')$ is increasing with y'. However, the sign of $\frac{\partial \Pi_\alpha(\alpha, y, y')}{\partial y'}$ is uncertain. To see this we have:

$$\frac{\partial \Pi_\alpha(\alpha, y, y')}{\partial y'} = \frac{\partial \Pi_{y'}(\alpha, y, y')}{\partial \alpha} = \frac{p(\alpha, y)p_{\alpha y'}(\alpha, y') - [1 + p_{y'}(\alpha, y')]p_\alpha(\alpha, y)}{p^2(\alpha, y)}.$$

Therefore, the level of the manager's shareholdings are not crucial to determine the level of the equity premium. Instead, as we will discuss in the next section, it is the fluctuations of the manager's shareholdings that generates a high equity premium.

3 Models with Trading by the Manager

3.1 Trading with A Continuum of Outside Investors

In this subsection, we first define Perfect Public Equilibrium (PPE). To do so we need to first define an auxiliary model in which the manager's strategy is fixed in advance. In this case, the equilibrium is a standard competitive equilibrium, with the fraction of equity outstanding, $\{1 - \alpha_t\}_{t=1}^\infty$, and an exogenous effort choice, $\{e_t\}_{t=0}^\infty$, following a process that is determined by the manager's strategy. Based on this auxiliary model, we can state the definition of PPE by adding the incentive constraints of the manager, as will be seen.

3.1.1 Competitive Equilibria – An Auxiliary Model

In this subsection, we will characterize the competitive equilibria of the dynamic economy in which the strategy of the manager is exogenously given. In the game, the manager cannot commit to an effort level or a trading amount. However, in our later characterization of the sequential equilibria, along the equilibrium path the investor acts as if the manager commits to the effort level and trading amount. We denote this exogenous strategy as $\sigma_m = \{e_t(h^t), \alpha_{t+1}(h^t, y_t)\}_{t=0}^\infty$. Denote this economy by $\Phi(\alpha_0|\sigma_m)$.

As in Lucas (1978), consider an economy with a representative investor, who is maximizing the quantity:

$$E\left[\sum_{t=0}^{\infty} \delta^t v(c_{it})\right]$$
$$s.t.\ c_{it} \leq \alpha_{it}(y_t + p_t) - \alpha_{it+1}p_t,$$

where consumption c_{it} is consists of the dividends paid on the equity shares held by the investor as well as any shares sold.

Output, y_t, follows a Markov process, with the density function defined as:

$$\widehat{f}(y_t|y^{t-1}) = f(y_t|e_t(y^{t-1})).$$

The competitive equilibrium of this economy, $\Phi(\alpha_0|\sigma_m)$, is a sequence of stock prices $p = \{p_t\}_{t=0}^{\infty}$ and a sequence of consumption and share allocations $q = \{c_{it}(h^t, y_t), \alpha_{it+1}(h^t, y_t)\}_{t=0}^{\infty}$, such that:

1. Given $\{p_t\}_{t=0}^{\infty}$, $\{\alpha_{it+1}\}_{t=0}^{\infty}$ maximizes $E\{\sum_{t=0}^{\infty} \delta^t v(c_{it})\}$.
2. $\alpha_{it+1} = 1 - \alpha_{t+1}$, for any t.

Define:

$$M_t \equiv E[(y_t + p_t v_c(c_{it}))|e_t]. \tag{3}$$

This quantity represents the increase in an investor's utility, in period t, had he started with an additional fraction of equity and sold all the additional equity for consumption. The role of M_t will become clear in the next subsection.

Consider a one-period economy where the investors have initial shareholdings of $\alpha_{it} = 1-\alpha_t$. The manager chooses to hold α_{t+1} at the end of the period, and the investors' augmented utility function over the consumption and the end-of-period shareholdings is $v(c_{it})+\delta M_{t+1}\alpha_{it+1}$, where M_{t+1} is an exogenous parameter. The competitive equilibrium of this static economy is a price p_t and a pair (α_{it+1}, c_{it}) such that:

1. Given p_t, α_{it+1} maximizes $v(c_{it}) + \delta M_{t+1}\alpha_{it+1}$.
2. $\alpha_{it+1} = 1 - \alpha_{t+1}$.

- Assumption: $v_c(c) < \overline{v}_c < \infty$.

Lemma 5 *M_t is bounded from above in a competitive equilibrium of the economy $\Phi(\alpha_0|\sigma_m)$.*

Proof. See the Appendix. ■

We will need the boundedness of M_t to show that the transversality condition holds. Let $CE(\alpha_{it}, M_{t+1})$ denote the set of competitive equilibrium allocations (c_{it}, α_{it+1}) of this economy. We have the following lemma.

Lemma 6 *For $q^* = \{c^*_{it}, \alpha^*_{it+1}\}_{t=0}^{\infty}$ to be a competitive allocation of the economy $\Phi(\alpha_0|\sigma_m)$, a necessary and sufficient condition is that, for all t and h^t,*

$$(c^*_{it}, \alpha^*_{it+1}) \in CE(\alpha^*_{it}, M^*_{t+1}).$$

Proof. See the Appendix. ■

Basically we show in the above lemma that the transversality condition is satisfied; that is necessary to write an infinite horizon dynamic problem recursively.

3.1.2 Perfect Public Equilibria

A strategy profile $\sigma = \{\sigma_m, \sigma_i\}$ for $\Phi(\alpha_0)$, together with the realization of nature's randomness with respect to the output, generates a unique random outcome path $\{e_t, \alpha_{t+1}, p_t, y_t\}_{t=0}^{\infty}$ and a corresponding consumption process $c = \{c_{mt}, c_{it}\}_{t=0}^{\infty}$. The stage strategies of the manager and the investor can be written as:

$$\begin{aligned} \sigma_{mt}(h^t) &= (e_t(h^t), \alpha_{t+1}(h^t, y_t)), \\ \sigma_{it}(h^t) &= \alpha_{it+1}(h^t, y_t, p_t). \end{aligned}$$

The value of $\sigma \in \Sigma$ is a vector with two elements: expected lifetime utility of the manager and expected lifetime utility of the investor:

$$\text{Manager:} \qquad U(\alpha_0, \sigma) = E\left[\sum_{t=0}^{\infty} \delta^t u(c_{mt}, e_t)\right],$$

$$\text{Investor:} \qquad V(\alpha_0, \sigma) = E\left[\sum_{t=0}^{\infty} \delta^t v(c_{it})\right],$$

where $c_{mt} = \alpha_t y_t - (\alpha_{t+1} - \alpha_t)p_t$ and $c_{it} = (1 - \alpha_t)y_t + (\alpha_{t+1} - \alpha_t)p_t$.

A strategy profile $\sigma \in \Sigma$ induces, after any history $h^t \in H^t$, a strategy profile $\sigma_t = \sigma|_{h^t} \in \Sigma$. For $s \geq 0$ and $h^s \in H^s$, we have:

$$\sigma(s)|_{h^t}(h^s) = \sigma(t + s)(h^t, h^s).$$

Consequently, we can define sequential equilibria with public strategies as follows:

Definition 1 *A strategy profile σ is a PPE for $\Phi(\alpha_0)$ if for any $t \geq 0$, history $h^t \in H^t$ with corresponding current α_t, the following conditions are satisfied:*

1. *The manager has no incentive to deviate, i.e., for any $\sigma'_m \neq \sigma_m$, $U(\sigma_t) \geq U(\sigma'_{mt}, \sigma_{it})$;*
2. *For the effort choice $e = \{e_s\}_{s=t}^{\infty}$ and $\alpha = \{\alpha_s\}_{s=t+1}^{\infty}$ induced by σ_{mt}, with the random realization of the outputs conditional on the the effort choice, $\{1 - \alpha_{s+1}, c_{is}\}_{s=t}^{\infty}$ is a competitive equilibrium of $\Phi(\alpha_t|\sigma_{mt})$.*

These two conditions require that the manager's and the investor's continuation strategies be best responses after any history h^t. The investor will be worse off by deviating, according to the optimality of the investor's decision in a competitive equilibrium.

Different from Atkeson (1991), the recursive formalization of PPE involves not only the payoffs to the manager and the investor but also the marginal value of shares for the investors. Similar to the definition in (3), for any $\sigma \in \Sigma$, we define the marginal value of shares for the investors as:

$$M(\sigma) = E[(y_0 + p_0)v_c(c_{i0})|e_0].$$

In our environment, as in Phelan and Stacchetti (2001), the state variable is a distribution across a continuum of agents. Here it is the distribution of shareholdings across investors. When the distribution of shareholdings is degenerate and almost all investors hold the same fraction of equity, a given sequential equilibrium delivers a lifetime utility U to the manager holding α shares of the equity and a lifetime utility V to each of the measure one of investors holding $1-\alpha$ shares of the equity. It also delivers a lifetime utility $\widehat{V}(\widehat{\alpha}_i|\alpha)$ for each investor holding an off-equilibrium path fraction of equity $\widehat{\alpha}_i \neq 1-\alpha$. As argued by Phelan and Stacchetti, for the symmetric (among investors) sequential equilibrium, it is *sufficient* to work with equilibrium value correspondences, $U(\alpha)$ and $V(\alpha)$, and with the derivative $M(\alpha)$ at $\widehat{\alpha} = \alpha$ of the function $\widehat{V}(\widehat{\alpha}_i|\alpha)$.

Define $\Gamma(\alpha)$ to be the set of values which the manager and the investor can obtain and the marginal value of shares for the investor from a symmetric sequential equilibrium, for each initial share holding of the manager $\alpha \in [0,1]$. So:

$$\Gamma(\alpha) = \{(U(\sigma_m,\sigma_i), V(\sigma_m,\sigma_i), M(\sigma_m,\sigma_i))|(\sigma_m,\sigma_i) \text{ is a PPE and } \alpha_0 = \alpha\}.$$

In the Appendix, we briefly discuss the factorization of the $\Gamma(\alpha)$ into a stage payoff and a continuation payoff. This step is necessary to write the equilibrium outcomes recursively. See Abreu, Pearce, and Stacchetti (1990).

In our set up, the only public payoff relevant information at the beginning of each period is the current share position of the manager, α. The past influences current play only though its effect on α. It seems natural to restrict our attention to strategies where the manager and the investor's decisions only depend on α. These strategies, which condition on the realization of a *state variable*, are commonly known as *Markovian strategies*. A *Markov Perfect Public Equilibrium* (MPPE) is a Perfect Public Equilibrium in which both the manager and the investor play time-invariant Markovian strategies. Following the definition of *Markov Perfect Equilibrium* (MPE) as in Fudenberg and Tirole (1991), we define MPPE below.

Definition 2 *A Markov Perfect Public Equilibrium is a profile of state-dependent strategies that yields a PPE in every truncated continuation game.*

However, in our setting, there can be many MPPEs, depending how the off-equilibrium beliefs are constructed. For example, given α, after a certain output y is realized, on the equilibrium path the manager will trade to hold α' for the next period, and this can be implemented by specifying a low-payoff-to-the-manager PPE in an off-equilibrium continuation game starting with $\alpha'' \neq \alpha'$. The off-equilibrium continuation PPE can be any PPE. At the same time, if, after an output value $y' \neq y$ is realized, α'' is the on the equilibrium path, the continuation

PPE can be different from the one that is off the equilibrium path even though they all start with the same α''.

To fix this arbitrariness of off-equilibrium threats, we consider a special class of MPPE, so called *Strong Markov Perfect Public Equilibrium* (SMPPE) as defined below.

Definition 3 *A Strong Markov Perfect Public Equilibrium is a profile of state-dependent strategies that yields the same MPPE in every truncated continuation game in which the manager starts with the same equity share position.*

It is easy to check that an SMPPE is a set of functions, $\{U(\alpha), V(\alpha), M(\alpha), e(\alpha), p(\alpha, \alpha', y), \alpha'(\alpha, y)\}$, satisfying the following conditions:

1. Competitive price formation among investors:

$$\begin{aligned} 0 &= -p(\alpha, \widetilde{\alpha}', y)v_c(\widetilde{c}_i) + \delta M(\widetilde{\alpha}') \qquad \text{for any } \widetilde{\alpha}' \in [0,1] \\ \text{where } \widetilde{c}_i &= (1-\alpha)y + (\widetilde{\alpha}' - \alpha)p(\alpha, \widetilde{\alpha}', y) \end{aligned} \tag{4}$$

2. Optimal portfolio choice by the manager:

$$\begin{aligned} \alpha'(\alpha, y) &= \arg\max_{\widetilde{\alpha}' \in [0,1]} u(\widetilde{c}_m) + \delta U(\widetilde{\alpha}') \\ \text{where } \widetilde{c}_m &= \alpha y - (\widetilde{\alpha}' - \alpha)p(\alpha, \widetilde{\alpha}', y). \end{aligned} \tag{5}$$

3. Optimal effort choice by the manager:

$$\begin{aligned} e(\alpha) &= \arg\max_{\widetilde{e}} \left\{ -g(\widetilde{e}) + \int_{\mathcal{Y}} \left[u(c_m) + \delta U(\alpha'(\alpha, y)) \right] f(y|\widetilde{e}) dy \right\} \\ \text{where } c_m &= \alpha y - (\alpha'(\alpha, y) - \alpha)p(\alpha, \alpha'(\alpha, y), y). \end{aligned} \tag{6}$$

4. Consistency of marginal value of shares for the investor:

$$\begin{aligned} M(\alpha) &= \int_{\mathcal{Y}} [p(\alpha, \alpha'(\alpha, y), y) + y] v_c(c_i) f(y|e) dy \\ \text{where } c_i &= (1-\alpha)y + (\alpha'(\alpha, y) - \alpha)p(\alpha, \alpha'(\alpha, y), y). \end{aligned} \tag{7}$$

In equilibrium, we also have the following Bellman equations:

$$U(\alpha) = -g(e(\alpha)) + \int_{\mathcal{Y}} \left[u(c_m(\alpha, y)) + \delta U(\alpha'(\alpha, y)) \right] f(y|e(\alpha)) dy \tag{8}$$

$$V(\alpha) = \int_{\mathcal{Y}} \left[v(c_i) + \delta V(\alpha'(\alpha, y)) \right] f(y|e(\alpha)) dy \tag{9}$$

$$M(\alpha) = \int_{\mathcal{Y}} \frac{y + p(\alpha, \alpha'(y), y)}{p(\alpha, \alpha'(y), y)} \delta M(\alpha'(\alpha, y)) f(y|e(\alpha)) dy. \tag{10}$$

The competitive price formation condition in (4) imposes an off-equilibrium price consistency condition, i.e., every point (p, α') on the demand function for the representative investor is consistent with the SMPPE starting with α'. Similarly, the term $U(\widetilde{\alpha}')$ in the optimal portfolio choice problem of the manager in (5) reflects that the future payoff will follow the same functional form $U(.)$ no matter whether it is on or off the equilibrium path, which is the critical condition for an SMPPE.

- Assumption: $g(e)$ and $f(y|e)$ are continuous, and for the continuous functions $U(\alpha')$ and $p(\alpha, \alpha', y)$, there exists a continuous solution $\{e(\alpha), \alpha'(\alpha, y)\}$ for the following programming problem:

$$\begin{aligned} & \max_{\widetilde{e}\in[\underline{e},\overline{e}]} -g(\widetilde{e}) + \int_{\mathcal{Y}} \max_{\widetilde{\alpha}'\in[0,1]} [u(\widetilde{c}_m) + \delta U(\widetilde{\alpha}')] f(y|e(\alpha))dy \\ s.t.\ \widetilde{c}_m &= \alpha y - (\widetilde{\alpha}' - \alpha) p(\alpha, \widetilde{\alpha}', y). \end{aligned}$$

We will use this assumption for the proof of the proposition below.

Proposition 1 *There exists a Strong Markov Perfect Public Equilibrium.*

Proof. Given a continuous price function $p(\alpha, \alpha', y)$, we have:

$$U(\alpha) = T_U(U)(\alpha) = \max_{\widetilde{e}\in[\underline{e},\overline{e}]} -g(\widetilde{e}) + \int_{\mathcal{Y}} \max_{\widetilde{\alpha}'\in[0,1]} [u(\widetilde{c}_m) + \delta U(\widetilde{\alpha}')] f(y|e(\alpha))dy. \tag{11}$$

It is easy to show that $T_U(.)$ is a contraction mapping, and there exists a unique continuous function $U(.)$ such that $U = T_U(U)$.

Given $p(\alpha, \alpha', y)$, let $e(\alpha)$ and $\alpha'(\alpha, y)$ be a continuous solution to the above programming in (11). Given $e(\alpha)$ and $\alpha'(\alpha, y)$, the investors are in a pure exchange economy, as in Lucas (1978), and the pricing function $\widehat{p}(\alpha, \alpha', y)$ for this economy satisfies the following condition:

$$v_c(\widehat{c}_i)\widehat{p}(\alpha, \alpha', y) = T_p(v_c\widehat{p}) = \int_{\mathcal{Y}} [\delta v_c(\widehat{c}_i')y' + \delta v_c(\widehat{c}_i')\widehat{p}(\alpha', \alpha'', y')] f(y'|e(\alpha')dy'.$$

We can show that $T_p(.)$ is a contraction mapping, and that there exists a unique continuous function $v_c\widehat{p}(.)$ such that $v_c\widehat{p} = T_p(v_c\widehat{p})$.

Thus, we have demonstrated a mapping from $p(\alpha, \alpha', y)$ to $(e(\alpha),\ \alpha'(\alpha, y))$, and then from $(e(\alpha),\ \alpha'(\alpha, y))$ to $\widehat{p}(\alpha, \alpha', y)$. Let $C([0,1]^2 \times [\underline{y}, \overline{y}])$ be the set of continuous functions defined on $[0,1]^2 \times [\underline{y}, \overline{y}]$. By the Brouwer-Schauder-Tychonoff Fixed Point Theorem,[5] we know there exists a fixed point $p(\alpha, \alpha', y) \in C([0,1]^2 \times [\underline{y}, \overline{y}])$ such that $\widehat{p}(\alpha, \alpha', y) = p(\alpha, \alpha', y)$. ■

[5] See, for example, Aliprantis and Border (1999).

3.1.3 Analysis of the Equilibria

The trading behavior of the manager affects his equilibrium effort choice over time. The manager's trading depends on the stock price. Investors' beliefs affect the asset price and consequently affect the manager's stock position and his effort choice. This chain is the link between the stock price and corporate governance. However, competitive trading among the continuum of outside investors imposes a constraint on their ability to affect the manager's incentive to make efforts. Any overpricing or underpricing of equity, which would generate a greater incentive for the manager to make an effort, can not survive in a competitive environment. We now formalize this description.

We start with the manager's trading behavior. The next proposition gives a necessary condition under which the manager does not trade.

Proposition 2 *In an SMPPE, for some $\alpha \in (0,1)$, the manager does not trade for any $y \in \mathcal{Y}$ only if the ratio of the marginal utilities of the investor and the manager, $\frac{v_c((1-\alpha)y)}{u_c(\alpha y)}$, does not vary with the output, y.*

Proof. When there is no trade at α, the condition in (4) implies:

$$-p(\alpha,\alpha,y)v_c((1-\alpha)y) + \delta M(\alpha) = 0 \qquad \text{for any } y \in \mathcal{Y}.$$

The first order condition for (5) with respect to α' gives us (setting $\alpha' = \alpha$):

$$-p(\alpha,\alpha,y)u_c(\alpha y) + \delta U_\alpha(\alpha) = 0.$$

Combining two expressions together we have:

$$\frac{v_c((1-\alpha)y)}{u_c(\alpha y)} = \frac{M(\alpha)}{U_\alpha(\alpha)} \text{ for any } y \in \mathcal{Y},$$

which gives our result. ■

If the condition in the above proposition does not hold, then the manager will trade. It is easy to check that, if the manager and the investor both have CRRA preferences and they have different risk aversion coefficients, then the manager is going to trade, for any α.

Before we show some further results on the manager's trading behavior, let us prove the following lemma.

Lemma 7 *For two continuous function $x_1(y) > 0$ and $x_2(y)$, if $X_1 = \int_{\mathcal{Y}} x_1(y)dy$ and $X_2 = \int_{\mathcal{Y}} x_2(y)dy$, then there exists $y^* \in \mathcal{Y}$ such that $\frac{X_2}{X_1} = \frac{x_2(y^*)}{x_1(y^*)}$.*

Proof. Assume $\frac{x_2(y)}{x_1(y)} > \frac{X_2}{X_1}$ for all $y \in \mathcal{Y}$, then we have $X_2 = \int_{\mathcal{Y}} x_2(y)dy > \int_{\mathcal{Y}} x_1(y)\frac{X_2}{X_1}dy = X_2$, a contradiction. Similarly, it is not possible $\frac{x_2(y)}{x_1(y)} < \frac{X_2}{X_1}$ for all $y \in \mathcal{Y}$. By the Mean Value Theorem, we know that there exists some y^* such that $\frac{X_2}{X_1} = \frac{x_2(y^*)}{x_1(y^*)}$. ■

With Lemma 7, we can show the following result.

Proposition 3 *In an SMPPE, for any $\alpha \in (0,1)$, there exists a $y^*(\alpha) \in \mathcal{Y}$ such that $\alpha'(\alpha, y^*(\alpha)) = \alpha$.*

Proof. Taking the derivative with respect to α for (8), and using the Envelope Theorem, we have:

$$U_\alpha(\alpha) = \int_{\mathcal{Y}} u_c(c_m)[y + p - (\alpha' - \alpha)p_\alpha]f(y|e)dy$$

Combine with (7) and use Lemma 7, we know that there exists a $y^*(\alpha) \in \mathcal{Y}$, such that:

$$\frac{U_\alpha(\alpha)}{M(\alpha)} = \frac{u_c(c_m^*)[y^* + p^* - (\alpha'^* - \alpha)p_\alpha^*]}{v_c(c_i^*)(y^* + p^*)}.$$

>From Proposition 4, we know:

$$\frac{U_\alpha(\alpha')}{M(\alpha')} = \frac{u_c(c_m)[p + (\alpha' - \alpha)p_{\alpha'}] - \lambda_0 + \lambda_1}{v_c(c_i)p}.$$

At y^*, we have:

$$\frac{U_\alpha(\alpha)}{M(\alpha)} = \frac{U_\alpha(\alpha'^*)}{M(\alpha'^*)} \times \frac{u_c(c_m^*)[y^* + p^* - (\alpha'^* - \alpha)p_\alpha^*]p^*}{(y^* + p^*)\{u_c(c_m^*)[p^* + (\alpha'^* - \alpha)p_{\alpha'}^*] - \lambda_0^* + \lambda_1^*\}},$$

which is true for any $\alpha \in (0,1)$. Therefore, $\alpha'^* = \alpha$ for any $\alpha \in (0,1)$. ■

Proposition 3 imposes some restriction on the manager's trading behavior. For any $\alpha \in (0,1)$, there exists some level of output at which the manager will not trade. In general, the manager will sell shares at some levels of output and buy share at some others. In the long run, the distribution of the manager's shareholdings converge to a steady state, and the manager's shareholdings fluctuate around a certain level. As we can imagine, the trading behavior of the manager generates dynamic patterns in his shareholdings, outputs and asset prices, as we will demonstrate in the section with numerical examples.

The trading activities between the manager and the investor change the share distribution between them, and thus the incentives for the manager to make effort fluctuate over time in response to output shocks. A natural question is: can trading replicate a long term contract? Or in a broader picture: can the market replace central planning? The next proposition answers these questions.

Proposition 4 *In an SMPPE, for any $\alpha'(\alpha, y) \in (0,1)$,*

$$\frac{M(\alpha')}{U_\alpha(\alpha')} = \frac{v_c(c_i)}{u_c(c_m)} \frac{p}{p + (\alpha' - \alpha)p_{\alpha'}}.$$

Proof. We know that:

$$M(\alpha') = \frac{1}{\delta} v_c(c_i)p.$$

With Lagrange multipliers $\lambda_0(\alpha, y) \geq 0$ and $\lambda_1(\alpha, y) \geq 0$ for the constraints $\alpha' \geq 0$ and $1 - \alpha' \geq 0$, the first order condition for (5) implies:

$$U_\alpha(\alpha') = \frac{1}{\delta}\{u_c(c_m)[p + (\alpha' - \alpha))p_{\alpha'}] - \lambda_0 + \lambda_1\}.$$

For $\alpha'(\alpha, y) \in (0, 1)$, $\lambda_0 = \lambda_1 = 0$. Combining the above equation with the condition in (4), we get our results. ■

Recall that $M(\alpha) = \frac{\partial \widehat{V}(\widehat{\alpha}_i|\alpha)}{\partial \widehat{\alpha}_i}$ at $\widehat{\alpha}_i = 1 - \alpha$, where $\widehat{V}(\widehat{\alpha}_i|\alpha)$ is the lifetime utility for an investor holding an off-equilibrium path fraction of equity $\widehat{\alpha}_i$. In general, $\frac{\partial \widehat{V}(\widehat{\alpha}|\alpha)}{\partial \widehat{\alpha}}|_{\widehat{\alpha}=\alpha}$ is not the same as $-V_\alpha(\alpha)$, where $V(\alpha)$ is the equilibrium value function for the representative investor in an SMPPE. In the standard long-term contracting problem, as discussed in the Introduction, the optimal contract requires that the principal's and the agent's marginal rates of substitution between present and future payoffs be equal, i.e., $\frac{V_\alpha(\alpha')}{U_\alpha(\alpha')} = \frac{-v_c}{u_c}$(see Spear and Srivastava (1987)). In our case, we have a continuum of investor-principals, and the corresponding expression is $\frac{-M(\alpha')}{U_\alpha(\alpha')}$. It would be equal to $\frac{-v_c}{u_c}$ if $(\alpha' - \alpha)p_{\alpha'} = 0$. Note that there are two sources for the distortion. The first one comes from the trading between the manager and the investors, $\alpha' - \alpha$. The second one comes from the change in the equity price when the manager trades (i.e., the manager is not an infinitesimal trader), $p_{\alpha'}$.

Therefore, the answers to the questions raised earlier are "No." In other words, the payoff dependency on output under an optimal long term contract can not be replicated through trading in the stock market. The equity price is formed competitively, and this restricts the effectiveness of the market in disciplining the manager. Also, the manager will take advantage of the equity price by trading with investors, and this prevents the manager from making efforts as investors would like.

The next proposition tells us how the manager's effort choice depends on his shareholdings.

Proposition 5 *In an SMPPE, if (1) for some* α, $\alpha' \in (0, 1)$ *for all* y, *(2)* $u_c(c_m)[y + p - (\alpha' - \alpha)p_\alpha]$ *is increasing (decreasing) in* y, *and (3)* $f(y|e)$ *satisfies the Monotone Likelihood Ratio Condition, then the manager's equilibrium effort choice,* e, *is increasing (decreasing) in the fraction of equity held by the manager,* α.

Proof. Differentiate the first order condition for (6) with respect to α, giving:

$$0 = \frac{\partial e}{\partial \alpha}[-g_{ee}(e) + \int_{\mathcal{Y}} u(c_m) f_{ee}(y|e)dy] + \int_{\mathcal{Y}} [y + p - (\alpha' - \alpha)p_\alpha]u_c(c_m)f_e(y|e)]dy.$$

Notice that $\alpha' \in (0, 1)$ guarantees $\partial \frac{u(c_m) + \delta U(\alpha')}{\partial \alpha'} = 0$ from the first order condition for (5), and that eliminates the $\frac{\partial \alpha'}{\partial \alpha}$ term from the above expression.

We can see that $\frac{\partial e}{\partial \alpha} > 0$ iff $\int_{\mathcal{Y}}[y + p - (\alpha' - \alpha)p_\alpha]u_c(c_m)f_e(y|e)]dy > 0$, since $-g_{ee}(e) + \int_{\mathcal{Y}} u(\alpha y) f_{ee}(y|e)dy < 0$ due to the optimality of e. The result is implied by the Monotone Likelihood Ratio Condition. ■

When the manager is allowed to trade with the outside investors, the marginal gain to him from holding an additional fraction of the equity comes from the output, y, the equity price, p, and the trading benefit (or cost) when he trades, $(\alpha' - \alpha)p_\alpha$. Assume $p_\alpha < 0$ (this could happen when $e_\alpha < 0$), imagine that in equilibrium that the manager is smoothing his consumption through trading, i.e., selling when output is low and buying when output is high. Then c_m will be less volatile than αy, the no trading case, which implies that $u_c(c_m)$ is decreasing with y to a lesser extent. At the same time, the trading increases the dependence of $y + p - (\alpha' - \alpha)p_\alpha$ on y. The trading behavior of the manager makes the investor's consumption c_i more volatile than in the no-trading case, and that makes the price more volatile. Also, higher (lower) outputs leads the manager to buy, making $y + p - (\alpha' - \alpha)p_\alpha > y + p$, and vice versa. Therefore, $u_c(c_m)[y + p - (\alpha' - \alpha)p_\alpha]$ is decreasing with y to a lesser extent because of trading. Thus, e is decreasing with α to a lesser extent. Similarly, when $p_\alpha > 0$, and the manager buys at a low output realization and sells at a high output realization, then e is increasing with α to a greater extent.

The next lemma gives a condition for the sign of p_α.

Lemma 8 *For a manager with constant relative risk aversion, γ_i, if in equilibrium $1 - \frac{\gamma_i(\alpha' - \alpha)p}{c_i} > 0$ (< 0), then $p_\alpha < 0$ (> 0).*

Proof. Differentiate the condition in (4) with respect to α, and we have:

$$p_\alpha v_c + p v_{cc}[-y - p + (\alpha' - \alpha)p_\alpha] = 0.$$

Replace v_{cc} with $-\frac{\gamma_i v_c}{c_i}$, and the result is immediate. ■

In general, a higher α means a lower consumption for the investors, thus a higher marginal utility of consumption today, which drives down the equity price. When the manager sells shares ($\alpha' - \alpha < 0$), $p_\alpha < 0$ always holds. However, when the manager buys shares, a higher price will reduce consumption further. Therefore, it is possible that the price has to go down to balance the equation (4), and this is more likely when γ_i is large.

3.2 Trading with A Single Outside Blockholder

Now we will study the case with a single outside investor, a blockholder. In this case, the outside investor can freely manipulate the equity price to generate the incentive for the manager to make effort. The time line of each period is as follows:

1. The manager chooses an effort level;

2. After the output is realized, the blockholder makes a take-it-or-leave-it offer to the manager, (α', p) –this is the stock trading;

3. The manager chooses to accept or reject this offer;

4. Consumption occurs and then the next period starts.

The above bargaining structure is equivalent to a structure in which the investor posts his demand function (by submitting limit orders), and the manager pick a volume to trade.

The investor's strategy σ_i can be written as $\{\alpha_{t+1}(h^t, y_t),\ p_t(h^t, y_t)\}_{t=0}^{\infty}$; the manager's strategy σ_m can be written as $\{e_t(h^t),\ I_t(h^t, y_t, \alpha_{t+1}, p_t)\}_{t=0}^{\infty}$, where $I_t \in \{accept,\ reject\}$.

Define $\Gamma(\alpha)$ as the set of payoffs to the manager and the investor in a PPE:

$$\Gamma(\alpha) = \{(U(\sigma_m, \sigma_i), V(\sigma_m, \sigma_i) | (\sigma_m, \sigma_i) \text{ is a PPE and } \alpha_0 = \alpha\}.$$

Following Atkeson (1991), we can factor the payoffs of the manager and the investor in a PPE, $(U, V) \in \Gamma(\alpha)$, into a stage payoff plus a continuation payoff:

$$\begin{aligned} U &= E[u(c_m) - g(e) + \delta U' | e] \\ V &= E[v(c_i) + \delta V' | e] \end{aligned}$$

where $c_m = \alpha y - I(\alpha', p, y)(\alpha'(y) - \alpha)p(y)$, $c_i = (1-\alpha)y + I(\alpha', p, y)(\alpha'(y) - \alpha)p(y)$, and $(U', V') \in \Gamma(\alpha')$.

Again, the only public payoff relevant information at the beginning of each period is the current share position of the manager. As before, we consider SMPPE. In this setting, a MPPE might involve threats, and SMPPE requires that threats have to be from the same class of SMPPE. To see this consider the following example. On the equilibrium path of an MPPE, at time t, given α, the manager's strategy is to "reject any offer other than (α', p')," while the investor's strategy is to "offer (α', p')." However, it might be the case that there exists an offer $(\alpha'', p'') \neq (\alpha', p')$ such that a SMPPE starting with (α'', p'') will yield a better expected payoff for the manager than rejecting the offer, then the manager's rejection of any offer other than (α', p') constitutes an incredible threat.

It is easy to check that an SMPPE is a set of functions, $\{U(\alpha),\ V(\alpha),\ e(\alpha),\ p(\alpha, y),\ \alpha'(\alpha, y)\}$, solving the following optimization problem:

$$\begin{aligned} \text{[P1]} \quad &: \quad \{e, \alpha', p\} = \arg\max_{\widetilde{e}, \widetilde{\alpha}', \widetilde{p}} \int_{\mathcal{Y}} v(\widetilde{c}_i) + \delta V(\widetilde{\alpha}'(y)) f(y|\widetilde{e}) dy \\ \text{s.t. } 0 &= -g_e(\widetilde{e}) + \int_{\mathcal{Y}} [u(\widetilde{c}_m(y)) + \delta U(\widetilde{\alpha}'(y))] f_e(y|\widetilde{e}) dy \qquad (12) \\ 0 &\leq \left[u(\widetilde{c}_m(y)) + \delta U(\widetilde{\alpha}'(y))\right] - [u(\alpha y) + \delta U(\alpha)] \qquad (13) \\ 0 &\leq \widetilde{\alpha}'(y) \leq 1, \qquad (14) \end{aligned}$$

where $\widetilde{c}(y) = \alpha y - (\widetilde{\alpha}' - \alpha)\widetilde{p}$ and $\widetilde{c}_i = (1-\alpha)y + (\widetilde{\alpha}' - \alpha)\widetilde{p}$.

In equilibrium, with the optimal α' and p, we also have the Bellman equations for the value functions for the manager and the investor:

$$U(\alpha) = -g(e) + \int_{\mathcal{Y}} [u(c_m) + \delta U(\alpha')] f(y|e) dy \tag{15}$$

$$V(\alpha) = \int_{\mathcal{Y}} v(c_i) + \delta V(\alpha') f(y|e) dy. \tag{16}$$

The constraint in (13) implies that the manager will accept any offer that makes him better off given that he will do the same in the continuation game.

Lemma 9 *The solution to the program [P1] always has constraint (13) binding.*

Proof. It is trivial if $\alpha'(y) = \alpha$. For the output y such that $\alpha'(y) \neq \alpha$, if the constraint (13) is not binding, then the investor can always manipulate p (but keep α' unchanged) to reduce c_m (and c_i is increased) to make the constraint (13) binding. ■

The above lemma tells us that the investor will always make an offer such that the manager is indifferent between the choices of *accept* and *reject*. Before we show the existence of an SMPPE, we prove the following proposition.

Proposition 6 *(i) In an SMPPE, given α, the manager will expend the same effort as in the benchmark case with no trading. Thus if $yu_c(\alpha y)$ is increasing (decreasing) in y, and $f(y|e)$ satisfies the Monotone Likelihood Ratio Condition (MLRC), then the manager's equilibrium effort choice, e, is increasing (decreasing) in the fraction of equity held by the manager, α.*

(ii) In an SMPPE, $U(\alpha) = U_0(\alpha)$, where $U_0(\alpha)$ is the value function for the manager in the benchmark case with no trading, as defined in (1).

Proof. For part (i), constraint (12) and the binding of the constraint (13) imply:

$$\begin{aligned} 0 &= -g_e(e) + \int_{\mathcal{Y}} \left[u(c_m(y)) + \delta U(\alpha'(y))\right] f_e(y|e) dy \\ &= -g_e(e) + \int_{\mathcal{Y}} \left[u(\alpha y) + \delta U(\alpha)\right] f_e(y|e) dy. \end{aligned}$$

The rest of the proof follows the proof for Lemma 1, and is thus omitted.

For part (ii), we know:

$$\begin{aligned} U(\alpha) &= -g(e) + \int_{\mathcal{Y}} [u(c_m(y)) + \delta U(\alpha'(y))] f(y|e) dy \\ &= -g(e) + \int_{\mathcal{Y}} [u(\alpha y) + \delta U(\alpha)] f(y|e) dy, \end{aligned}$$

which implies:

$$U(\alpha) = \frac{1}{1-\delta}[-g(e) + \int_{\mathcal{Y}} u(\alpha y) f(y|e) dy] = U_0(\alpha),$$

since the effort choice is the same as in benchmark case according to part (i). ■

In an SMPPE equilibrium, the manager is always indifferent between trading or not trading. As a result, the manager he will expend the same effort as in the benchmark case with no trading, and his expected lifetime utility will be the same as well.

Now we are ready to show the existence of an SMPPE.

Proposition 7 *There exists a unique Strong Markov Perfect Public Equilibrium solving [P1].*

Proof. Given $U(.) = U_0(.)$, it is easy to show that $T(V) = \max_{\widetilde{e},\widetilde{\alpha}'(y),\widetilde{p}(y)} \int_{\mathcal{Y}} v(\widetilde{c}_i(y)) + \delta V(\widetilde{\alpha}'(y)) f(y|\widetilde{e}) dy$ subject to constraints (12), (13), and (14) satisfies Blackwell's sufficient conditions (monotonicity and discounting) for being a contraction mapping for any bounded function $V(.)$ defined on $[0,1]$.

Monotonicity: if $V_1(\alpha) \leq V_2(\alpha)$, for all $\alpha \in [0,1]$, implies $T(V_1)(\alpha) \leq T(V_2)(\alpha)$ for all $\alpha \in [0,1]$.

Discounting: $T(V+c)(x) \leq T(V)(x) + \delta c$, for any $c \geq 0$. ■

The next proposition shows the equilibrium condition that is satisfied by $U(\alpha)$ and $V(\alpha)$.

Proposition 8 *In an SMPPE,* $\frac{V_\alpha(\alpha'(y))}{U_\alpha(\alpha'(y))} = -\frac{v_c(c_i(y))}{u_c(c_m(y))}$.

Proof. Pointwise variations in α' and p for equations (16) and (15) give:

$$\begin{aligned} 0 &= v_c(c_i(y))(\alpha'(y) - \alpha) + v_c(c_i(y))p(y)\frac{\Delta\alpha'}{\Delta p} + \delta V_\alpha(\alpha'(y))\frac{\Delta\alpha'}{\Delta p} \\ 0 &= -u_c(c_m(y))(\alpha'(y) - \alpha) - u_c(c_m(y))p(y)\frac{\Delta\alpha'}{\Delta p} + \delta U_\alpha(\alpha'(y))\frac{\Delta\alpha'}{\Delta p}. \end{aligned}$$

The result is immediate. ■

The result in Proposition 8sharply contrasts with the one with dispersed investors. In this case the manager's and the investor's marginal rates of substitution between present and future payoff are equal. This condition is a necessary condition for an optimal long term contract, as in Spear and Srivastava (1987). However, the bargaining structure imposes some restrictions on the reservation value for the manager. In our case, the manager has a time varying reservation value, which is his lifetime utility from holding a certain amount of shares without trading. In Spear and Srivastava (1987), the long term contract guarantees a certain payoff at time zero, but there is no lower bound (or a constant lower bound) on the manager's payoff thereafter. In Section 4, we will use some numerical examples to compare the effort choice in this case to that in the case with continuum investors.

Lemma 10 *In an SMPPE equilibrium, $U(\alpha)$ is increasing with α and $V(\alpha)$ is decreasing with α.*

Proof. Constraint (12) and the binding of the constraint (13) imply:

$$\begin{aligned} U_\alpha(\alpha) &= \int_{\mathcal{Y}} \frac{\partial\left[u(c_m)+\delta U(\alpha')\right]}{\partial\alpha} f(y|e)dy \\ &= \int_{\mathcal{Y}} \frac{\partial\left[u(\alpha y)+\delta U(\alpha)\right]}{\partial\alpha} f(y|e)dy, \end{aligned}$$

which can be rearranged as:

$$(1-\delta)U_\alpha(\alpha) = \int_{\mathcal{Y}} y u_c(\alpha y) f(y|e)dy > 0.$$

By Proposition 8, $U_\alpha(\alpha) > 0$ implies $V_\alpha(\alpha) < 0$. ■

Below we show some properties of the asset prices, consumption levels, shareholdings of the manager, and the lifetime utilities of the manager and the investor. These will be useful when we study the numerical examples later.

Proposition 9 *In an SMPPE, $p > 0$ when $\alpha' \neq \alpha$.*

Proof. >From Lemma 9, we know that the constraint (13) is binding, i.e., $u(c_m)+\delta U(\alpha') = u(\alpha y) + \delta U(\alpha)$. Part (ii) of Proposition 6 implies that U is increasing in α in equilibrium. Therefore, when $\alpha' > \alpha$, we must have $c_m(y) < \alpha y$, i.e. $\alpha y - (\alpha' - \alpha)p < \alpha y$, which implies $p > 0$. Similarly for the case when $\alpha' < \alpha$. ■

This proposition rules out the possibility that the investor will make an offer that awards more (less) shares and provides more (less) consumption to the manager at the same time.

Lemma 11 *In an SMPPE, if $U(\alpha)$ is concave, then $V(\alpha)$ is concave.*

Proof. With multipliers μ_1, $\mu_2(y) \leq 0$, $\mu_3(y) \leq 0$, and $\mu_4(y) \leq 0$, the Lagrangean for [P1] can be written as:

$$\begin{aligned} \mathcal{L}(\alpha) &= \int_{\mathcal{Y}} v(c_i) + \delta V(\alpha') f(y|e)dy + \mu_1 \left[-g_e(e) + \int_{\mathcal{Y}} [u(c_m)+\delta U(\alpha')] f_e(y|e)dy\right] \\ &\quad + \int_{\mathcal{Y}} \mu_2 \{\left[u(c_m)+\delta U(\alpha')\right] - [u(\alpha y)+\delta U(\alpha)]\}dy + \int_{\mathcal{Y}} \mu_3 \alpha' dy + \int_{\mathcal{Y}} \mu_4 (1-\alpha')dy. \end{aligned} \tag{17}$$

The first order condition with respect to p gives:

$$0 = v_c(c_i) f(y|e) - \mu_1 u_c(c_m) f_e(y|e) - \mu_2 u_c(c_m).$$

Differentiating (17) with respect to α, gives:

$$\begin{aligned} V_\alpha(\alpha) &= \mathcal{L}_\alpha(\alpha) = \int_{\mathcal{Y}} -(y+p)v_c(c_i)f(y|e)dy + \mu_1\left[\int_{\mathcal{Y}}(y+p)u_c(c_m)f_e(y|e)dy\right] \\ &\quad + \int_{\mathcal{Y}} \mu_2\{(y+p)u_c(c_m) - [yu_c(\alpha y) + \delta U_\alpha(\alpha)]\}dy \\ &= -\int_{\mathcal{Y}} \mu_2(y)\left[yu_c(\alpha y) + \delta U_\alpha(\alpha)\right]dy, \end{aligned}$$

which implies:

$$V_{\alpha\alpha}(\alpha) = -\int_{\mathcal{Y}} \mu_2(y)\left[y^2u_{cc}(\alpha y) + \delta U_{\alpha\alpha}(\alpha)\right]dy.$$

The result is immediate. ■

Concavity in $U(.)$ and $V(.)$ in the above lemma will be used to show the following results. To simplify our analysis below, we assume the single outside blockholder is risk neutral, i.e., $v(c) = c$.

Proposition 10 *In an SMPPE, if both $U(\alpha)$ and $V(\alpha)$ are concave, then in equilibrium $\alpha'_y = \frac{\partial \alpha'}{\partial y}$ and $c_{my} = \frac{\partial c_m}{\partial y}$ have the same sign. Moreover, if in equilibrium $\mu_1 \frac{f_e(y|e)}{f(y|e)} + \frac{\mu_2(y)}{f(y|e)}$ is increasing in y, then both $c_m(y)$ and $\alpha'(y)$ are increasing in y.*

Proof. For the first claim, Proposition 8 implies:

$$V_\alpha(\alpha')u_c(c_m) = -U_\alpha(\alpha').$$

Differentiate it with respect to y, giving:

$$(V'_{\alpha\alpha}u_c + U'_{\alpha\alpha})\alpha'_y + V_\alpha u_{cc}c_{my} = 0,$$

from which, $V'_{\alpha\alpha} < 0$, $U'_{\alpha\alpha} < 0$, $V_\alpha < 0$, $u_c > 0$, and $u_{cc} < 0$ imply that c_{my} and α'_y have the same sign.

For the second claim, differentiating the Lagrangean in (17) with respect to $p(y)$ gives:

$$\frac{1}{u_c(c_m(y))} = \mu_1 \frac{f_e(y|e)}{f(y|e)} + \mu_2(y)\frac{1}{f(y|e)},$$

and the result is immediate. ■

The above proposition shows that when both $U(\alpha)$ and $V(\alpha)$ are concave, if the manager's shareholdings are increasing (decreasing) with output, so is his consumption level. With positive equity price, p, this rules out the possibility that the manager buys shares when output is low while selling shares when output is higher.

3.3 Discussion

In this subsection we discuss how the empirical implications of our model are related to existing empirical findings.

Our model predicts that managers are active traders in the stock market, and that their trades are not informative. That is, since the equilibrium stock prices are being set to induce the managers to hold a certain amount of shares, their trades are not conveying "inside information" to outside investors. These predictions, that managers trade and that such trade is not informative, are consistent with what we now about insider trading. Lakonishok and Lee (2001) study the insider trading activity of all firms traded on the NYSE, AMEX, and Nasdaq during the period 1975-1995. They find that "insiders are active and that there is at least some insider trading in more than 50% of the stocks in a given year" (p. 82). However, "In spite of the attention that insiders' activities receive, we do not observe any major stock price changes around the time of insider trading" (p. 82).

It should also be noted that we have not said whether the output shocks in the model are economy-wide shocks or idiosyncratic shocks. In our model, idiosyncratic risk is priced. The risk averse managers have concentrated holdings in the stocks of the firms where they work. Their utility is affected by nondiversified exposure to output shocks of their individual firms. Indeed, that is the point of the compensation arrangement with the manager. Outside investors respond to the output shocks in attempting to reset the managers' shareholdings to the optimal level. Recent empirical evidence is consistent with the existence of idiosyncratic risk being priced. Campbell, Lettau, Malkiel, and Xu (2001) show that there has been an increase in firm level stock price volatility, compared to market volatility, over the period 1962 to 1997.

Our model also predicts that the ownership structure of the firm matters. Generally, the model generates equity premia and excess equity return volatilities, depending on risk aversion, as well as the ownership structure. But, in particular, with single blockholder there is a higher equity premium and higher equity return volatilities. The numerical examples below display this in detail. The usual view of blockholders is that they reduce the cost of capital because of close monitoring of the manager. But, our model produces the opposite result. It is, however, consistent with the findings of Ashbaugh-Skaife, Collins, and LaFond (2004) that firms with more blockholders have a higher cost of equity.

4 Some Numerical Examples

The stock price dynamics of the model are difficult to analyze analytically, but can be illustrated with numerical examples. In this section, we provide some illustrative numerical examples to analyze the level of the stock price through time, as well as the equity premium, risk free rate, their volatilities, and their dynamics as functions of the initial ownership share of the manager, risk aversion of the manager and the outside investors, and the firm's ownership structure.

4.1 Assumptions and Methodology

We assume the following:

- Output, y, can only take one of two values, y_H or y_L;
- The effort choice $e \in [\underline{e}, \overline{e}]$, and the cost of effort is $g(e) = qe^2$ with q a positive constant;
- The probability $\Pr(y_H|e) = e$;
- $u(c_m) = \frac{\lambda_m c_m^{1-\gamma_m}}{1-\gamma_m}$ and $v(c_i) = \frac{\lambda_i c_i^{1-\gamma_i}}{1-\gamma_i}$, with λ_m, λ_i, γ_m, and γ_i all positive constants.

The investor's consumption, c_i, has two parts: labor income, c_i^0, and dividend and trading income, $(1-\alpha)y + (\alpha' - \alpha)p$. Similarly, the manager's consumption, c_m, has two parts: labor income, c_m^0, and dividend and trading income, $\alpha y - (\alpha' - \alpha)p$. We assume that each period the labor incomes are constant for both the manager and the investor.

The parameters for our base example are as follows:

$$\begin{aligned} y_H &= 0.5,\ y_L = 0.15,\ \delta = 0.98; \\ \underline{e} &= 0.2,\ \overline{e} = 0.8,\ q = 0.16; \\ \lambda_m &= 2 \times 10^{-4},\ c_m^0 = 0.1,\ \gamma_m = 5.0; \\ \lambda_i &= 2 \times 10^3,\ c_i^0 = 6.0,\ \gamma_i = 5.0. \end{aligned}$$

The parameter values are set based on the following concerns:

1. The choice of λ_i and λ_m is mainly for normalization to avoid too large or too small values for the purpose of numerical accuracy.
2. The value of q makes it happen that the fraction of equity held by the manager will change the effort choice substantially.
3. The risk aversion coefficients of the manager and the investor are a bit larger than what the literature suggests (about 3.0) to make the model effect on equity risk premium more salient.
4. The value of c_m^0 is low to capture the fact that entrepreneurs have a low labor income while with a large fraction of equity. However, a non-zero value of c_m^0 serves as a cushion to avoid an explosion in value of the manager's utility when α is close to zero.
5. We choose δ, y_H, y_L and c_i^0 to approximately match the level of the risk-free interest rate, dividend-consumption ratio, and consumption growth rate volatility (standard deviation) from the data.[6] With different levels of effort choices (in equilibrium, the effort choice is changing over time), we compare the model and the data in Table 1.

[6] For risk-free interest rate, we get monthly 1-Year Treasury Constant Maturity Rates (04/1954-06/2005) from

	Data	Model $e = 0.3$	Model $e = 0.5$	Model $e = 0.7$
Dividend-Consumption Ratio	0.033	0.041	0.051	0.062
Stand. Dev. of Cons. Growth Rate	0.036	0.036	0.039	0.036
Real Risk-Free Interest Rate	0.018	0.020		

Table 1: Risk-Free Interest Rate, Dividend-Consumption Ratio, and Consumption Growth Rate Volatility: Comparison of Real Data with the Model

Table 1 shows that with different levels of equilibrium effort choice of the manager, the chosen parameters generate reasonable consumption growth rate volatility and the risk-free rate, which are close to the real data.

We now describe the methods used to generate our numerical example.

Case 1: Continuum outside investors

Define a grid $\{\alpha_1, ..., \alpha_N\}$. Starting from some initial functions $U^0(\alpha)$, $V^0(\alpha)$ and $M^0(\alpha)$, we map $(U^t(\alpha), V^t(\alpha), M^t(\alpha))$ to $(U^{t+1}(\alpha), V^{t+1}(\alpha), M^{t+1}(\alpha))$ as follows:

- Step 1: For each α_k, $k = 1, ..., N$, given the output y, find the optimal α' to maximize $u(c_m)+\delta U^t(\alpha')$ with p determined by the competitive price formation condition in 4 using $M^t(\alpha')$;
- Step 2: Find the optimal effort choice, e, of the manager given the optimal α' and the resulting p for each y;
- Step 3: Define $U^{t+1}(\alpha) = -g(e) + E[u(c_m) + \delta U^t(\alpha')|e]$, $V^{t+1}(\alpha) = E[v(c_i) + \delta V^t(\alpha')|e]$, and $M^{t+1}(\alpha) = E[\frac{y+p}{p}\delta M(\alpha')|e]$.

Repeat Steps 1-3 until (U, V, M) converges, and then generate sample equilibrium paths based on the limit (U, V, M).

Case 2: Single outside block investor

Define a grid $\{\alpha_1, ..., \alpha_N\}$. Starting from some initial functions $U^0(\alpha)$ and $V^0(\alpha)$, we map $(U^t(\alpha), V^t(\alpha))$ to $(U^{t+1}(\alpha), V^{t+1}(\alpha))$ as follows:

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the website of FRED at St. Louis Fed. We deflate the nominal interest rates using CPI to get real interest rates.

The annual personal consumption data (1929-2004) come from the website of the Bureau of Economic Analysis. The annual dividend data (1926-2004) used are from CRSP NYSE_AMEX_NASDAQ index. We back out the dividends from the value-weighted index returns (including dividends and excluding dividends) and the market value of all stocks in NYSE, AMEX and NASDAQ.

For the standard deviation of annual consumption growth rate, we use the value from Mehra and Prescott (1985) over the period 1889-1978. The calculated value for the period 1929-2004 is higher, 0.0562.

- Step 1: For each α_k, $k = 1, ..., N$, given the output y, find the optimal (p, α') to maximize $v(c_i) + \delta V^t(\alpha')$ such that $u(c_m) + \delta U^t(\alpha') \geq u(\alpha y) + \delta U^t(\alpha)$;

- Step 2: Find the optimal effort choice, e, of the manager given the optimal (p, α') for each y;

- Step 3: Define $U^{t+1}(\alpha) = -g(e) + E[u(c_m) + \delta U^t(\alpha')|e]$ and $V^{t+1}(\alpha) = E[v(c_i) + \delta V^t(\alpha')|e]$.

Repeat Steps 1-3 until (U, V) converges, and then generate sample equilibrium paths based on the limit (U, V).[7]

4.2 Numerical Results

The first set of results are displayed in figures, using the parameter values and assumptions discussed above. Figures 1-3 report the results for the case of a continuum of investors in comparison to the benchmark case. Figures 4-6 report the results for the case of the single block investor, in comparison with the benchmark case. Recall that the benchmark case does not allow trade by the manager.

We will first explain the figures and then discuss the results. The horizontal axis in all the figures is the initial starting value of the manager's stock holdings, α. Each figure then shows the equilibrium value of a variable of interest as a function of α.

Start with Figure 1. The figures going down the left column show the benchmark base, indicated by a "0" subscript. In particular, the left panel of Figure 1 reports how the lifetime expected utility of the manager, U_0, the lifetime expected utility of the investor, V_0, the marginal value of shares for the investor, M_0, and the effort choice, e, vary with the fraction of equity owned by the manager, α, in the benchmark case. The figures going down the right column show the difference between the benchmark case and the case of many, small, dispersed investors.

In Figure 2, the top two graphs show the results of the trading behavior for the case with a continuum investors. The left graph reports the trading volume when output is high or low, respectively. The right graph reports the *expected* trading volume of the manager, which gives us the direction of the change of the state variable α in expectation. The left panel of the six graphs below this first row reports how the equity price, equity return, and bond return vary with the fraction of equity initially owned by the manager in the benchmark case of no trading by the manager. Again, the right panel reports the difference between the case of a continuum of investors and the benchmark case. With the subscript "0" denoting the benchmark case, the variable definitions are as follows:

[7] Since we already know the form of $U(.)$ in this case, we can replace U^t at each step with U_0. Indeed, they yield the same result.

p_H: equity price when output is high

p_L: equity price when output is low

r_H: the expected equity return conditional on that output is high

r_L: the expected equity return conditional on that output is low

r: the unconditional expected equity return

r_{bH}: the expected bond return conditional on that output is high

r_{bL}: the expected bond return conditional on that output is low

r_b: the unconditional expected bond return.

The left panel of Figure 3 reports how the equity premium, equity return volatility, and bond return volatility vary with the initial fraction of equity owned by the manager in the benchmark case. The right panel reports the difference between the case of a continuum of investors and the benchmark case. With the subscript "0" denoting the benchmark case, the variable definitions are as follows:

ep_H: the expected equity premium conditional on that output is high

ep_L: the expected equity premium conditional on that output is low

ep: the unconditional expected equity premium

vol_H: the equity return volatility (standard deviation) conditional on that output is high

vol_L: the equity return volatility (standard deviation) conditional on that output is low

vol: the unconditional equity return volatility (standard deviation)

vol_b: the bond return volatility (standard deviation).

Figures 4-6 display the results for the block investor case. They are the counterparts for Figures 1-3, and the variables are similarly defined.

What do the figures show? Figures 1 and 4 show that the effort choice of the manager peaks at a equity fraction of $\alpha \approx 0.09$ for both ownership structures. The single investor can manipulate the price to make the manager trade towards the optimal level of α. But, with a continuum of investors, the equity price is formed competitively, and the corporate governance effect of the price is weaker as α will stay around 0.86, which is too high for the manager to expend high effort. We can also see this from the value function of the investor, V, as a function of α, in Figures 1 and 4. $V(\alpha)$ peaks at $\alpha \approx 0.09$, and in the case with single investor, the investor has higher lifetime expected utility than in the case with a continuum of investors. At the same time, the manager's utility is higher in the case with a continuum of investors. These results confirm our conjecture that the case with the single investor allows the investor to negotiate with the manager more effectively.

In the case with a continuum of investors (Figure 1), trading can substantially weaken the incentive of the manager to expend effort for some values of α, and improve incentives for some other values of α. Trading has two opposing effects on the effort choice. On the one hand, trading smooths the manager's consumption, making his consumption less volatile, and this

has a negative effect on the effort choice. On the other hand, speculative trading (i.e., buy at high and sell at low) by the manager makes his future expected utility more volatile since his expected utility is increasing with α, and this has a positive effect on the effort choice. When α is large, the trading volume is large and the second effect dominates; when α is relatively small, the trading volume is small, and the first effect dominates. In the case with single investor (Figure 4), these two effects cancel and the effort choice is the same as in the benchmark case.

As shown in Figure 2, at most values of α (except when α is very close to zero) the manager is buying with high output and selling with low output to smooth his consumption. At $\alpha \approx 0.86$, the expected trading volume is zero. As we would expect, in equilibrium, α_t will be fluctuating around 0.86. We can also observe that, around $\alpha \approx 0.86$, the expected trading volume is very small in comparison with the trading volume, especially in the region with $\alpha < 0.86$. This implies that, when there is an output shock, after the trading, it takes a long time in expectation for α to go back to the original level. With the single block investor, things are different. As shown in Figure 5, when α is small, the manager is always buying. When α is relatively large the manager is buying with high output and selling with low output to smooth his consumption. At $\alpha \approx 0.09$, the expected trading volume is zero. In equilibrium, α_t will be fluctuating around 0.09.

Now we turn to examine asset prices, returns, and the equity premium. In the case of a continuum of investors Figures 2 and 3 show that around $\alpha \approx 0.86$, equity prices/returns are more volatile than in the benchmark case and the equity premium is higher. This is mainly because the investors' consumption becomes more volatile while the manager is smoothing his own consumption. Again, the case of the single block investor is different. Figures 5 and 6 show that, when $\alpha < 0.09$, the equity price is lower than in the benchmark, regardless the level of output. The manager always buys equity; when $\alpha > 0.09$, and there are large over-reactions to output. The price is very high given high output and the price is very low given low output. Further, the manager buys with high output and sells with low output. With this over-reaction to output, when output is high, the manager will not buy too much equity given the very high price and when output is low, the manager will have to sell a lot (relatively) to smooth his own consumptions. This creates a strong mechanism for the manager to keep his stock holdings around $\alpha \approx 0.09$. Figure 5 shows that even though the conditional trading volumes are smaller than in the case with a continuum of investors, the expected trading volume is much larger than in the case with continuum investors. Figure 6 demonstrates that, in the single investor case, there are higher volatilities in equity returns. Though the conditional equity premium is lower when there is a high output, the expected equity premium is larger than in the benchmark case due to the high conditional equity premium when output is low.

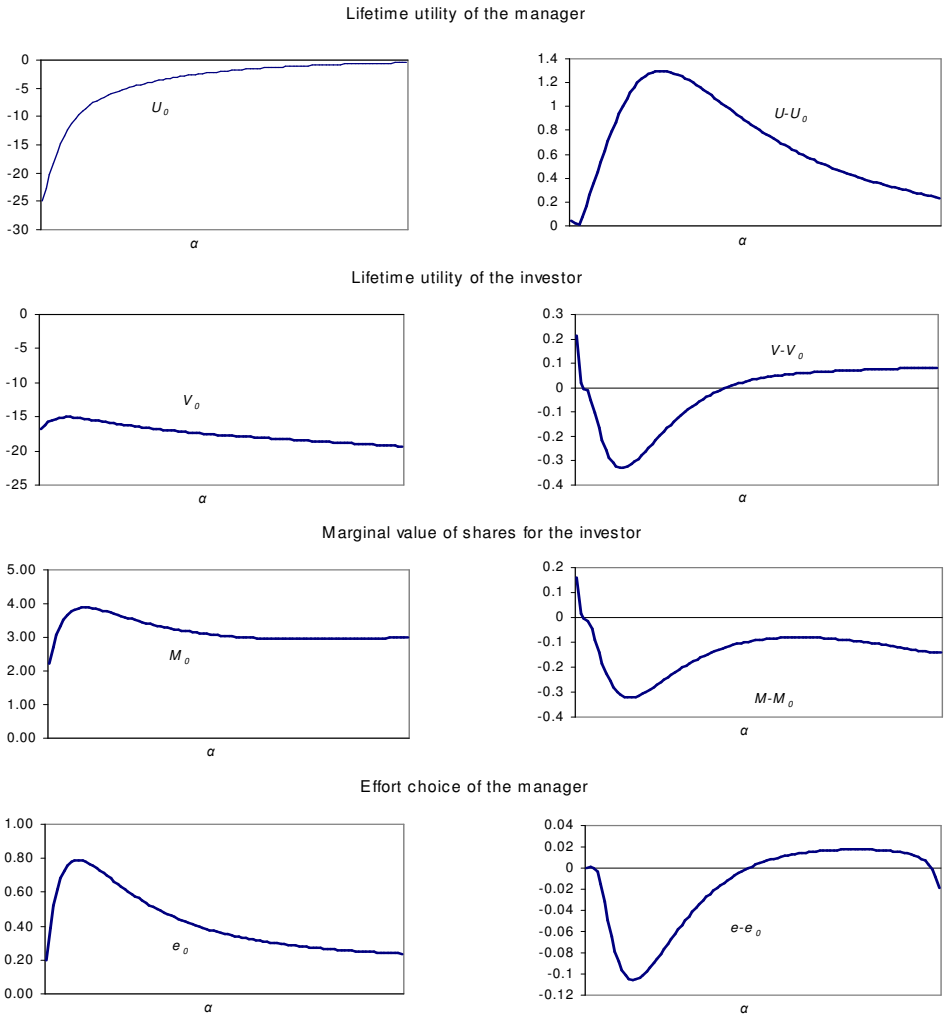

Figure 1: Continuum Investors: U, V, M, and e

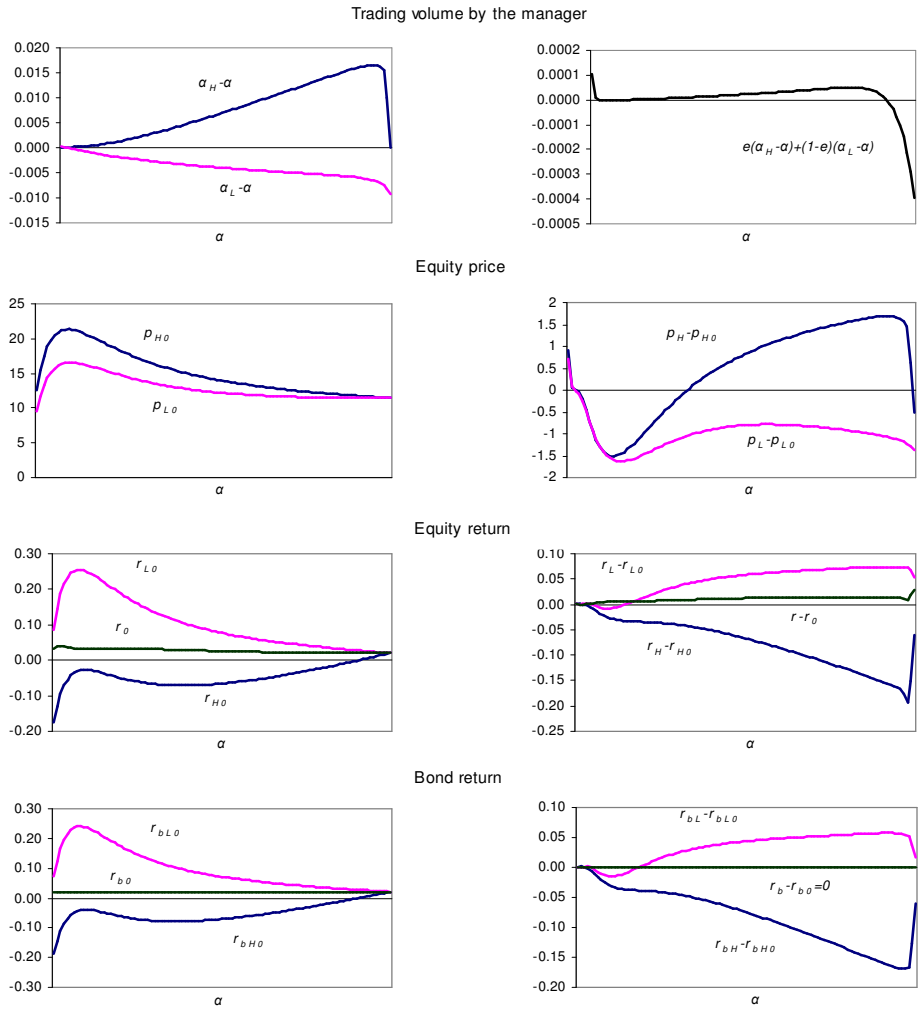

Figure 2: Continuum Investors: Trading Behavior and Asset Prices/Returns

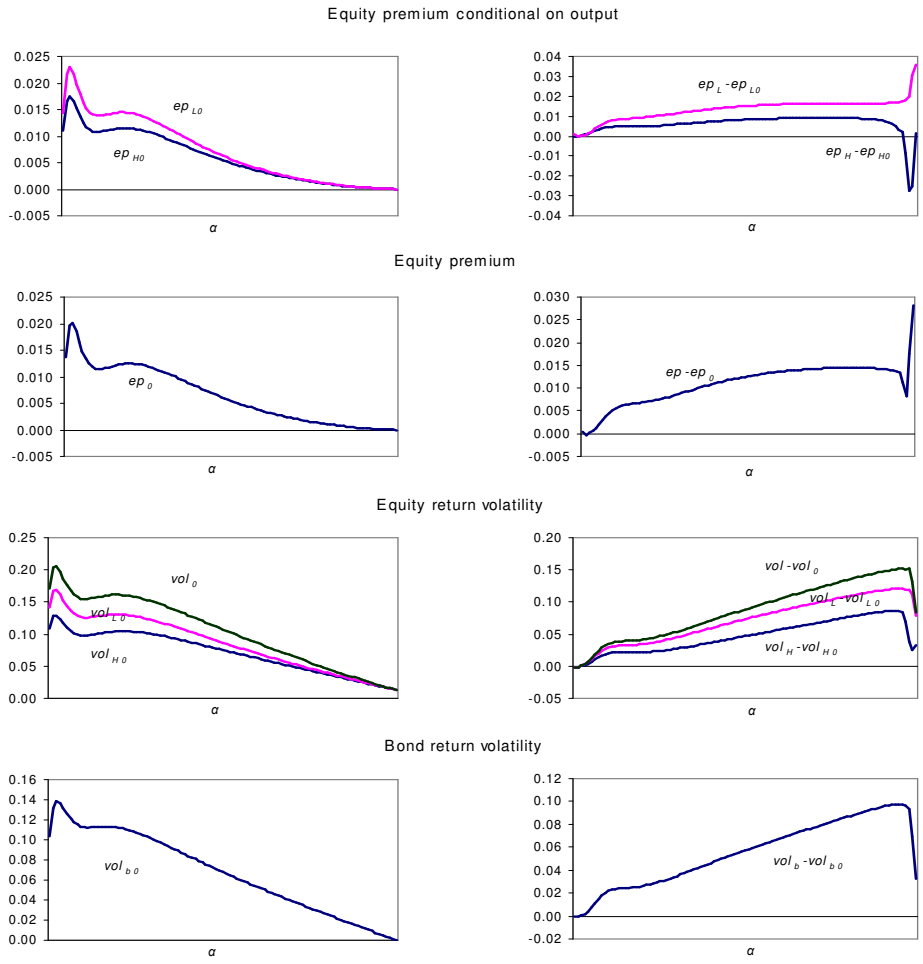

Figure 3: Continuum Investors: Equity Premium and Asset Return Volatilities

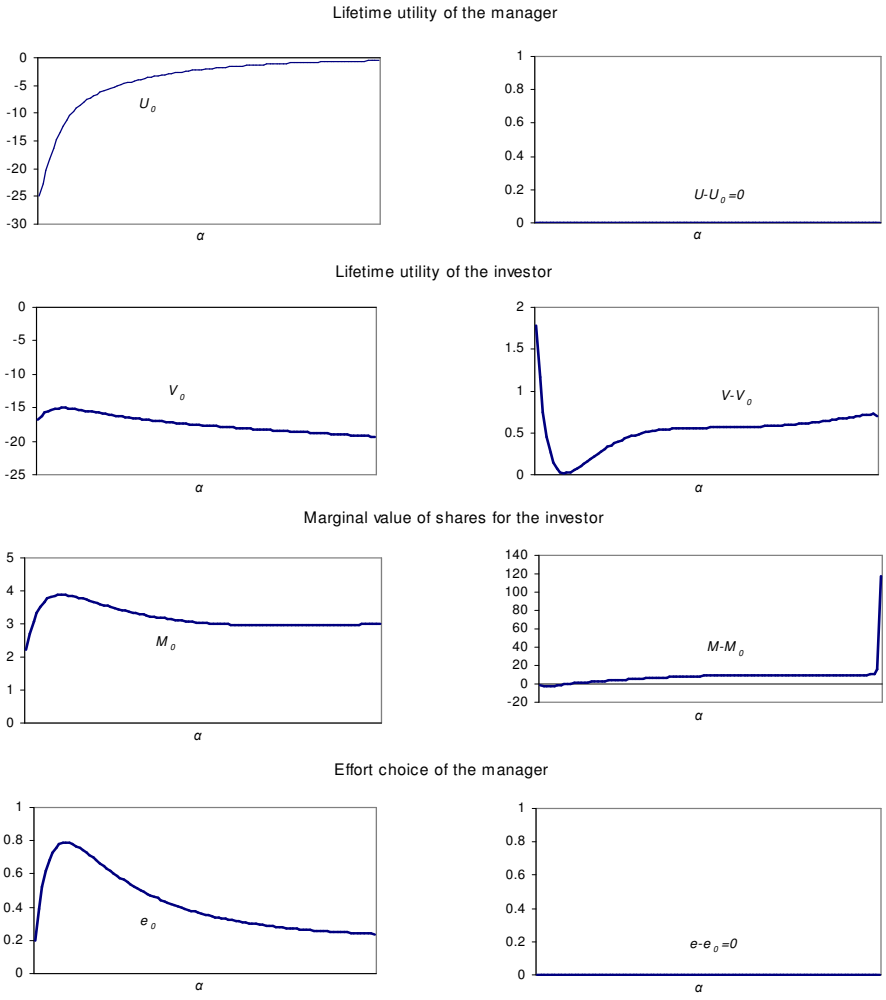

Figure 4: One Block-Investor: U, V, M, and e

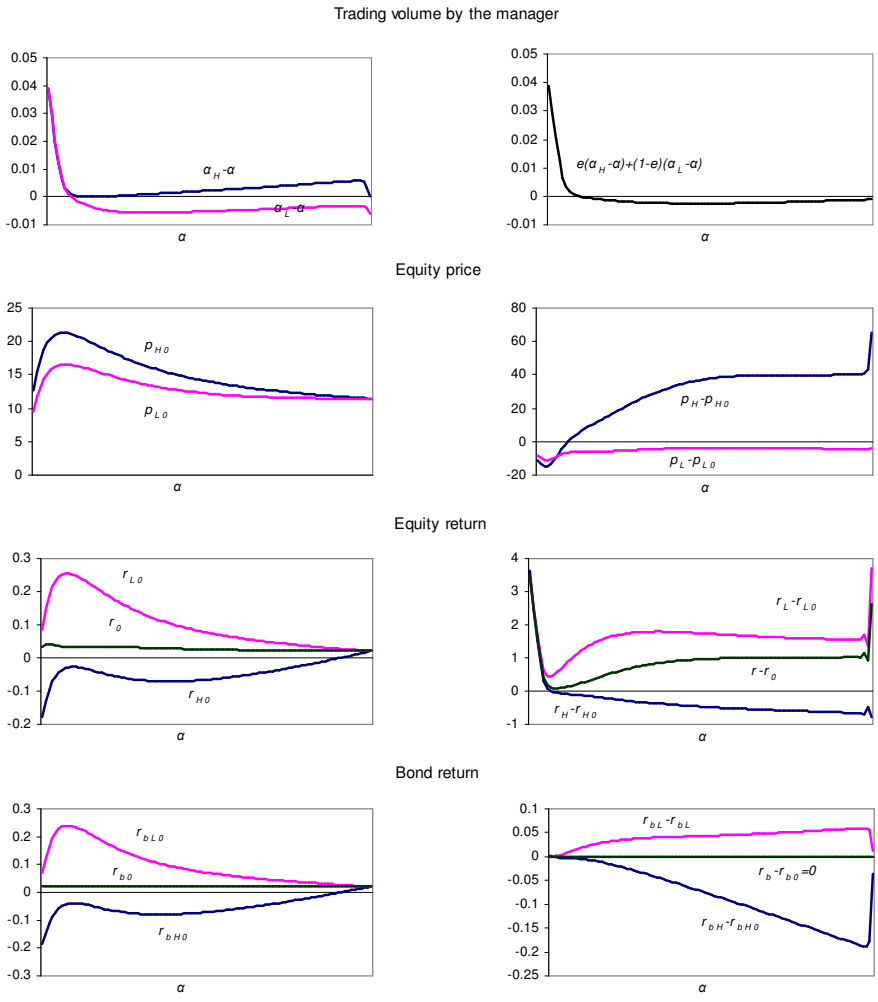

Figure 5: One Block-Investor: Trading Behavior and Asset Prices/Returns

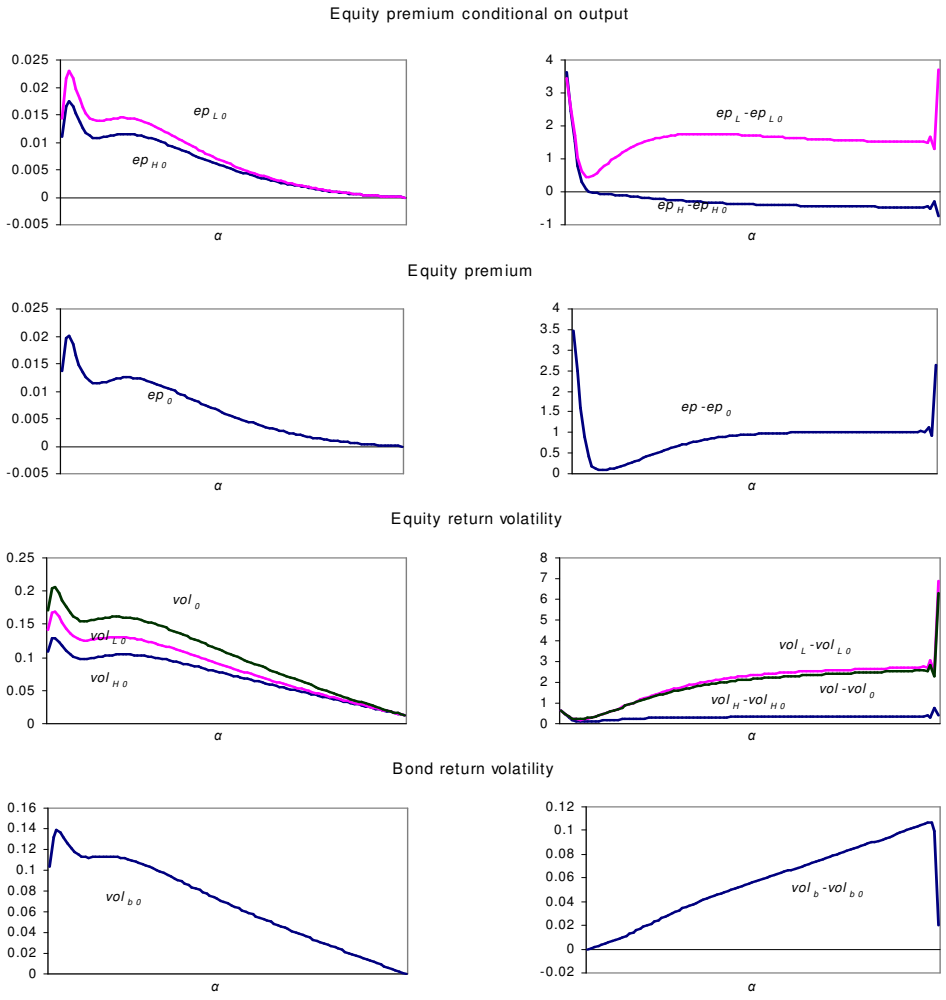

Figure 6: One Block-Investor: Equity Premium and Asset Return Volatilities

	Contin. Investors ($\alpha_0 = 0.8$)	Benchmark	Single Investors ($\alpha_0 = 0.088$)	Benchmark	Data (1954-2004)
Average α	0.7614	0.7614	0.08806	0.08806	–
Average e	0.2917	0.2713	0.7879	0.7882	–
Average r_s	0.04217	0.02174	0.1302	0.03199	0.088
Average r_b	0.02079	0.02041	0.02057	0.02041	0.018
Average $r_s - r_b$	0.01138	0.00133	0.1096	0.01158	0.070
$\sigma(r_s)$	0.2643	0.05390	0.5439	0.1554	0.18
$\sigma(r_b)$	0.1153	0.03068	0.1224	0.1134	0.021

Table 2: Simulation Results

4.3 Simulation

Once we have obtained the equilibrium value functions, we can simulate a sample equilibrium path for 1000 periods, and then repeat that 1000 times. For each simulated equilibrium path, we calculate the average α, average e, average equity return r_s, the volatility of r_s, average bond return r_b, and the volatility of r_b.[8] Then, we take the average of each statistic across the 1000 paths. Table 2 reports the summary statistics for two ownership structures, the continuum of investors case and the block investor case. For comparison, we also show the results for the benchmark case and the corresponding real data.

Note that in Table 2 there are two benchmark cases, unlike in the previous subsection. Here we first calculate the steady state level of α for the each ownership structure and then using that ownership level we calculate the statistics for the benchmark case. Consequently, there is a benchmark case for each ownership structure. This will allow us to compare the asset price dynamics holding initial ownership constant.

In the case of a continuum of investors, there is higher average stock return and higher volatility than in the benchmark case (with the same average α). The equity premium increases from 0.13% (benchmark case) to 2.13%. The average bond returns remain approximately the same, but the volatility is much higher than in the benchmark case. In the case of a single block holder, there is higher average stock return and higher volatility than in the benchmark case (with the same average α). The equity premium increases from 1.15% (benchmark case) to 11.0%. The average bond returns remain approximately the same, but the volatility is much higher than in the benchmark case. Compared with the continuum of investors case, the block holder case yields a higher equity premium and a higher volatility.

[8] The bond price is given by: $p_b(y) = \frac{1}{v_c(c_i(y))} E[\delta v_c(c_i'(y')|e]$.

To further explore the dynamic properties of the endogenous variables, we plot the autocorrelation coefficients (in Figure 7 and 9) and the periodograms (in Figure 8 and 10) for the fraction of equity owned by the manager, α, output, y, equity price, p, effort choice of the manager, e, equity return, r_s, and bond return, r_b.[9] We use ρ to denote the autocorrelation coefficients and S to denote the sample periodograms.

As we can see from Figures 7 and 9, α, e and p show a momentum pattern in the short run and a mean reverting pattern in the long run in both ownership cases. The autocorrelation coefficients are positive for short horizons but negative for long horizons. Figures 8 and 10 show that at very high frequencies (the short run) and at very low frequencies (the long run) contribute most to the variation of these variables.

The effects on asset returns and outputs are noisy. Figures 7 and 8 show that, in the case with a continuum of investors, output shows very weak patterns similar to those with α, e and p. Bond returns are pretty much white noise with no serial correlation and equity returns display a strong cyclical behavior with a period $\frac{2\pi}{\pi} = 2$ years. This is mainly due to the strong negative correlation between r_{st} and r_{st+1} (which is not shown in Figure 7). The behavior of the variables y, r_s and r_b are similar in the case with single investor, as shown in Figures 9 and 10, except that y is more like a white noise.

5 Conclusion

We study a framework in which corporate decisions interact with asset pricing because executive compensation depends on the levels of outputs and equity prices. Managers have incentives to trade in the stock market to smooth consumption, and, as a result, then outside shareholders' consumption is affected by both the effort choices and the trading behavior of the manager. In such a setting, we showed that stock prices are set not as passive reflections of future cash flows. Rather, trade in the secondary stock market sets prices in equilibrium to create incentives for those cash flows to be created through the effort choices of the managers. Risk sharing considerations prevent instantaneous adjustment of stock prices to obtain the optimal managerial shareholdings. Such adjustment takes time. Our numerical examples display the dynamic patterns of prices constantly being reset to monitor management.

[9] The periodogram is a transformation of the autocovariances of a stationary process. It demonstrates how much of the variation of the data can be attributed to periodic random components with a certain frequency. See, for example, Hamilton (1994).

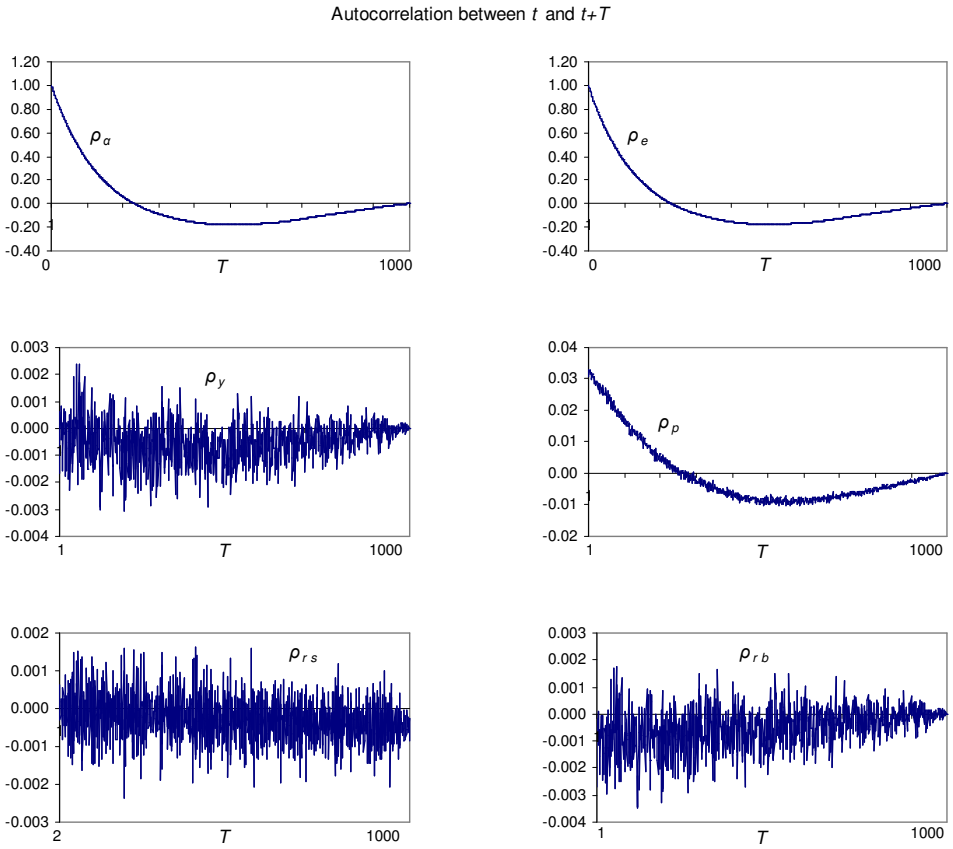

Figure 7: Continuum Investors: Autocorrelation Coefficients

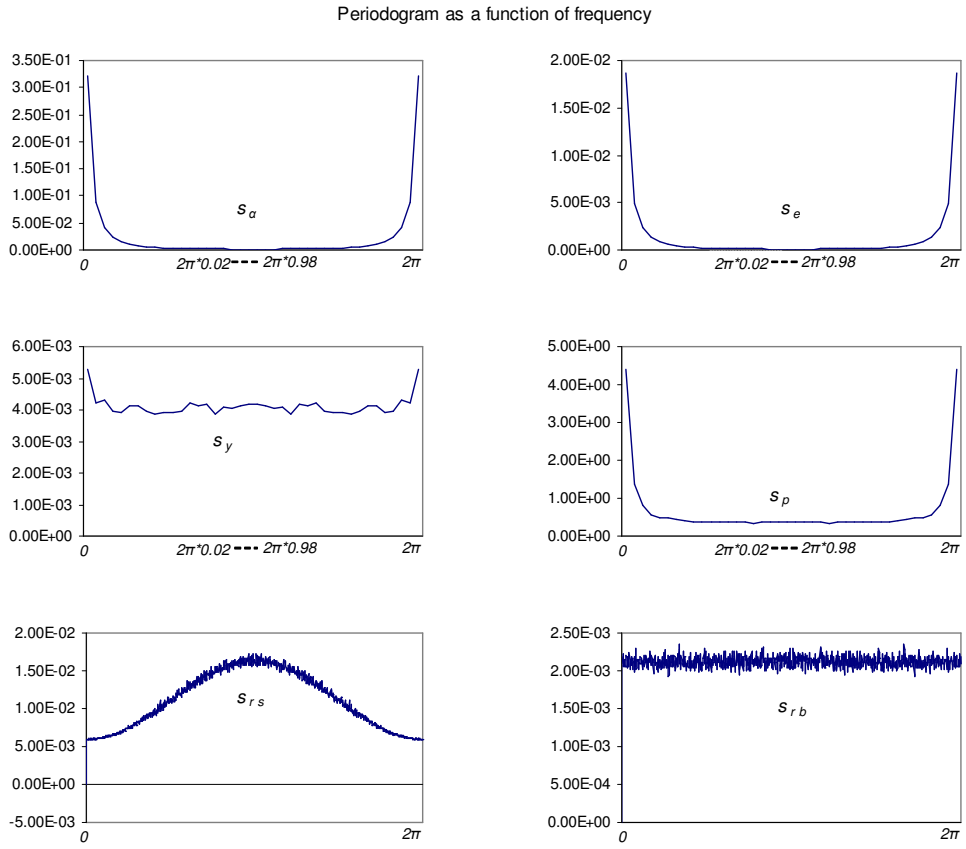

Figure 8: Continuum Investors: Sample Periodograms

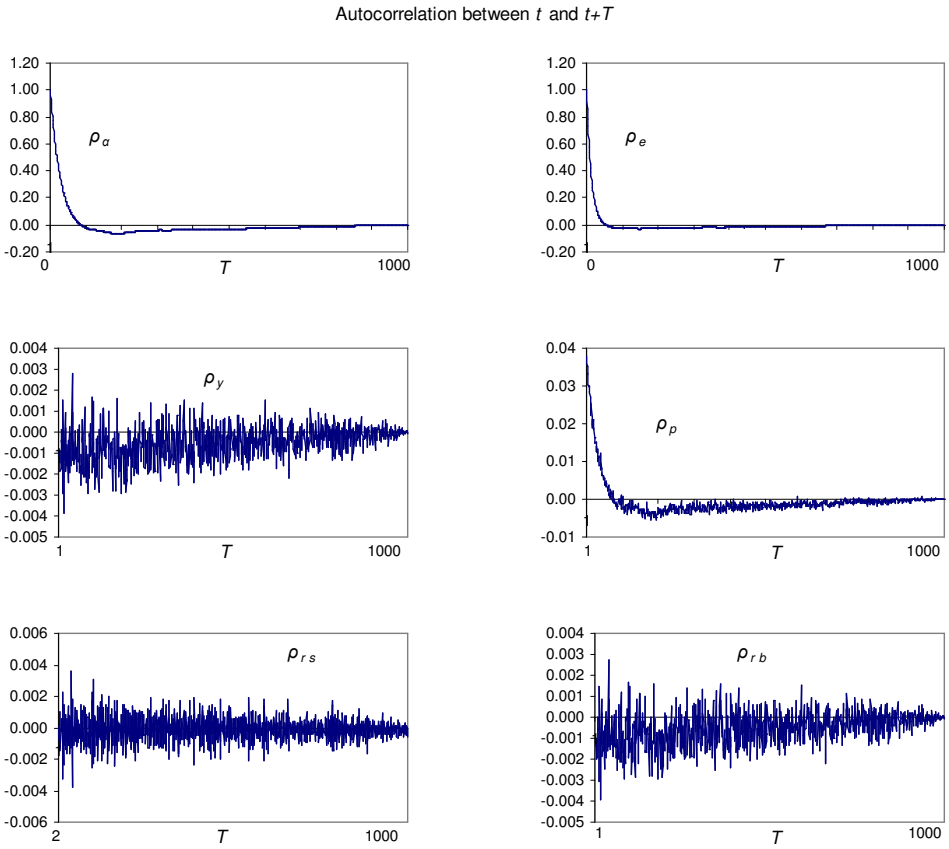

Figure 9: One Block-Investor: Autocorrelation Coefficients

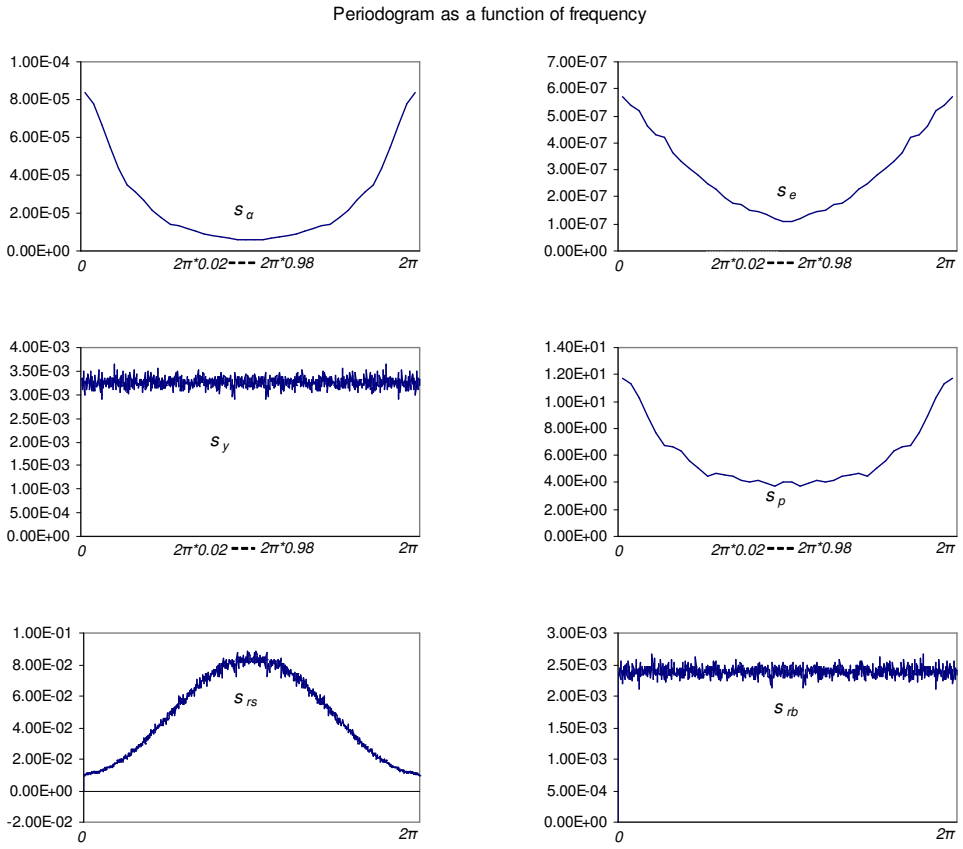

Figure 10: One Block-Investor: Sample Periodograms

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Appendix

Proof for Lemma 5

Proof. The representative investor's decision problem is:

$$\begin{aligned} & \max_{\{\alpha_t\}_{t=1}} E_0\left[\sum_{t=0}^{\infty}\delta^t v(c_{it})\right] \\ s.t.\ c_{it} &= \alpha_{it}(y_t+p_t)-\alpha_{it+1}p_t. \end{aligned}$$

Taking the derivative with respect to α_{it+1}, for $t \geq 0$, we have:

$$\begin{aligned} v_c(c_{it})p_t &= E_t[\delta(y_{t+1}+p_{t+1})v_c(c_{it+1})] \\ &= E_t[\delta y_{t+1}v_c(c_{it+1})+\delta E_{t+1}[\delta(y_{t+2}+p_{t+2})v_c(c_{it+2})]] \\ &= E_t[\sum_{s=t+1}^{\infty}\delta^{s-t}y_s v_c(c_{is})] \leq \sum_{s=t+1}^{\infty}\delta^{s-t}\overline{yv}_c = \frac{\delta}{1-\delta}\overline{yv}_c, \end{aligned}$$

which implies that $p_t \leq \frac{\delta}{1-\delta}\frac{\overline{yv}_c}{v_c(\overline{y})}$ for any $t \geq 0$. Consequently, we have:

$$\begin{aligned} M_{t+1} &= E_t[(y_{t+1}+p_{t+1})v_c(c_{it+1})] \\ &\leq \left(\overline{y}+\frac{\delta}{1-\delta}\frac{\overline{yv}_c}{v_c(\overline{y})}\right)\overline{v}_c < \infty. \end{aligned}$$

■

Proof for Lemma 6

Proof. The necessity part is trivial. We now prove sufficiency.
Assume $(c^*_{it}, \alpha^*_{it+1}) \in CE(\alpha^*_{it}, M^*_{t+1})$. Then, a necessary condition for equilibrium is:

$$(-p^*_t v_c(c^*_{it})+\delta M^*_{t+1})(\alpha_{it+1}-\alpha^*_{it+1}) \leq 0, \text{ for all } \alpha_{it+1} \in [0,1].$$

This implies that:

$$\begin{aligned} & E_t[(-p^*_t v_c(c^*_{it})(\alpha_{it+1}-\alpha^*_{it+1})|h^{t-1}] \\ \leq\ & -E_t[\delta M^*_{t+1}(\alpha_{it+1}-\alpha^*_{it+1})|h^{t-1}]. \end{aligned}$$

Add $E_t[(y_t+p^*_t)v_c(c^*_t)(\alpha_{it}-\alpha^*_{it})|h^{t-1}] = E_t[M^*_t(\alpha_{it}-\alpha^*_{it})|h^{t-1}]$ to each side of the above inequality, and using:

$$c_{it}-c^*_{it} = [\alpha_{it}(y_t+p^*_t)-\alpha_{it+1}p^*_t]-[\alpha^*_{it}(y_t+p^*_t)-\alpha^*_{it+1}p^*_t],$$

we have:

$$E[v_c(c^*_{it})(c_{it}-c^*_{it})|e_t] \leq E[M^*_t(\alpha_{it}-\alpha^*_{it})-\delta M^*_{t+1}(\alpha_{it+1}-\alpha^*_{it+1})|e_t].$$

By the concavity of $v(.)$, we have:

$$v(c_{it}) \leq v(c_{it}^*) + v_c(c_{it}^*)(c_{it} - c_{it}^*).$$

Therefore:

$$\begin{aligned} & \lim_{T\to\infty} E\left[\sum_{t=0}^{T} \delta^t [v(c_{it}) - v(c_{it}^*)]\right] \\ \leq & \lim_{T\to\infty} E\left[\sum_{t=0}^{T} \delta^t [M_t^*(\alpha_{it} - \alpha_{it}^*) - \delta M_{t+1}^*(\alpha_{it+1} - \alpha_{it+1}^*)]\right] \\ = & \lim_{T\to\infty} E\left[\delta^{T+1} M_{T+1}^*(\alpha_{iT+1} - \alpha_{iT+1}^*)\right] = 0. \end{aligned}$$

Since M_t^* is bounded from above by Lemma 5, $\lim_{T\to\infty} E[\sum_{t=0}^{T} \delta^t [v(c_{it}) - v(c_{it}^*)]] \leq 0$, and q^* is optimal. ■

Factorization of $\Gamma(\alpha)$ for the game with continuum investors

Let $W : [0,1] \to \mathcal{R}^3$ be an arbitrary value correspondence, which is compact and convex.

Definition 4 *A vector $\zeta_1 = (\alpha'(y), U'(y), V'(y), M'(y))$ is said to be consistent with respect to W at initial shareholdings $\alpha \in [0,1]$ if:*

1. *Generation:* $(U'(y), V'(y), M'(y)) \in W(\alpha)$.
2. *Short Sale Constraint:* $\alpha'(y) \in [0,1]$ *for any* $y \in \mathcal{Y}$.
3. *Investor Optimality:* $(\alpha_i'(y), c_i(y)) \in CE(1-\alpha, M'(y))$ *with price* $p(y)$, *where* $\alpha_i'(y) = 1 - \alpha'(y)$ *and* $c_i(y) = (1-\alpha)(y + p(y)) - (1 - \alpha'(y))p(y)$.

The investor optimality in a competitive market requires $-p(y)v_c(c_i(y)) + \delta M'(y) = 0$ (First order condition), which determines $p(y)$ once $M'(y)$ is set. Given α', for each $y \in \mathcal{Y}$ let:

$$\begin{aligned} \underline{U}(\alpha, y, \alpha') & \equiv \min_{(\widetilde{U'}, \widetilde{V'}, \widetilde{M'})} [u(\widetilde{c}_m) + \delta \widetilde{U'}] \\ s.t.\ \widetilde{\zeta}_1 & = (\alpha', \widetilde{U'}, \widetilde{V'}, \widetilde{M'}) \text{ is consistent with respect to } W \text{ at } \alpha, \\ \text{and } \widetilde{c}_m & = \alpha(y + \widetilde{p}) - \alpha' \widetilde{p}. \end{aligned}$$

$\underline{U}(\alpha, y, \alpha')$ gives us the worst possible payoff for the manager, and this is used to define the punishment value for the manager when he deviates from the equilibrium path.

Define $\overline{U}(\alpha, y) \equiv \max_{\widehat{\alpha'}(y)} \underline{U}(\alpha, y, \widehat{\alpha'}(y))$. $\overline{U}(\alpha, y)$ is the best alternative value to the manager when he deviates from the equilibrium.

Definition 5 *A consistent vector $\zeta = (e, \zeta_1)$ is said to be admissible with respect to W at initial share holding α if ζ_1 is consistent with W at α and*

$$\begin{aligned} E[-g(e) + u(c_m(y)) + \delta U'(y)|e] &\geq \max_{\widehat{e}} E[-g(\widehat{e}) + u(c_m(y)) + \delta U'(y)|\widehat{e}] \\ \text{and } u(c_m(y)) + \delta U'(y) &\geq \underline{U}(\alpha, y). \end{aligned}$$

Admissibility adds the manager's incentive constraint to the requirements for consistency; the first inequality implies that the manager will make the optimal effort choice given his trading behavior, and the second inequality requires the manager to trade optimally. When the manager trades an unexpected fraction of equity, the investor's beliefs are updated in the subsequent subgame so as to yield the worst possible payoff for the manager. This is without loss of generality. If an equilibrium is sustainable with another type of punishment upon deviation, then it must also be sustainable with the worst punishment upon deviation. This is the same concept as what Chari and Kehoe (1990) call a "sustainable equilibrium."

The vector $\zeta = (e, \alpha'(y), U'(y), V'(y), M'(y))$ gives the following values to the manager and the investor and for the marginal value of the shares for the investor:

$$\begin{aligned} \grave{U}(\alpha, \zeta) &\equiv E[u(c_m(y), e) + \delta U'(y)|e], \\ \grave{V}(\alpha, \zeta) &\equiv E[v(c_i(y)) + \delta V'(y)|e], \\ \grave{M}(\alpha, \zeta) &\equiv E[(y + p(y))v_c(c_i(y))|e]. \end{aligned} \tag{18}$$

Let $\Upsilon(\alpha, \zeta) \equiv (\grave{U}(\alpha, \zeta), \grave{V}(\alpha, \zeta), \grave{M}(\alpha, \zeta))$, and for a value correspondence W, define:

$$B(W)(\alpha) = \{\Upsilon(\alpha, \zeta) | \zeta \text{ is admissible with respect to } W \text{ at } \alpha\}.$$

Lemma 12 *If W has a compact graph, then $B(W)$ has a compact graph.*

Proof. First, we can show that $B(W)$ has a bounded graph. It is easy to see that $V(\alpha) \in [\frac{v(\underline{y})}{\delta}, \frac{v(\overline{y})}{\delta}]$ and $U(\alpha) \in [\frac{-g(\overline{e})+u(\underline{y})}{\delta}, \frac{-g(\underline{e})+u(\overline{y})}{\delta}]$. Also we have shown in Lemma 5 that $M \in (0, \overline{M})$ is bounded from below and above.

Second, we can show that $B(W)$ has a closed graph. Let $\{w_n, \alpha_n\}$ be a sequence in the graph of $B(W)$ which converges to a point (w, α). We need to show that (w, α) is also in the graph of $B(W)$. By the definition of $B(W)$, there exists a sequence of vectors $\zeta_n = \{e_n, \alpha'_n(y), U'_n(y), V'_n(y), M'_n(y)\}$ admissible with respect to W at α_n, and $w_n = \Upsilon(\alpha_n, \zeta_n)$ each of which satisfies the short-sale constraint, price consistency, and incentive compatibility, and $\Upsilon(s_n, U_n, V_n)(\alpha_n) = w_n$. Because the space of admissible stage strategies and value functions are bounded, we may assume this sequence converges to some limit point $\zeta = \{e, \alpha'(y), U'(y), V'(y), M'(y)\}$, where the convergence of functions $\alpha'_n(y)$, $U'_n(y)$, $V'_n(y)$, and $M'_n(y)$ is almost everywhere.

The convergence of the sequence $\{e_n\}$ is trivial to show. To show the almost-everywhere convergence of $\{\alpha'_n(y)\}$, $\{U'_n(y)\}$, $\{V'_n(y)\}$, and $\{M'_n(y)\}$, we proceed as follows. First, all

these functions are bounded on $\mathcal{Y}$, which is bounded itself. Therefore, these functions are in L_p-space. It is easy to find a Cauchy subsequence for each of them, say $\{g_n\}$, and we know that there exists a function g in L_p-space with a L_p-norm, s.t., $\lim_{n\to\infty} ||g_n - g||_p = 0$. Second, $\lim_{n\to\infty} ||g_n - g||_p \to 0$ in L_p-space implies that there exist a subsequence $\{h_n\}$ of $\{g_n\}$, such that $h_n \to g$ almost everywhere. This guarantee that g is a feasible function, i.e., it is constrained by the same bounds as that for the original function sequence, $\{\alpha'_n(y)\}$, $\{U'_n(y)\}$, $\{V'_n(y)\}$, or $\{M'_n(y)\}$.

By continuity of Υ, we know that $\Upsilon(\alpha, \zeta) = w$, and ζ is admissible with respect to W at α. Therefore, (w, α) is in the graph of $B(W)$. ■

Following Phelan and Stacchetti (2001), we can show that $\Gamma(\alpha) = B(\Gamma)(\alpha)$ for all α, and $\Gamma(\alpha)$ is the largest value correspondence W such that $W = B(W)(\alpha)$. For each α, define $W_\infty(\alpha) = B(B(...(B(W)))(\alpha) = B^\infty(W)(\alpha)$, and with the compactness property of $B(.)$, we can show $W_\infty(\alpha) = \Gamma(\alpha)$.