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Ellen McGrattan
Edward Prescott

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ABSTRACT

During the 1990s, market hours in the United States rose dramatically. The rise in hours occurred as gross domestic product (GDP) per hour was declining relative to its historical trend, an occurrence that makes this boom unique, at least for the postwar U.S. economy. We find that expensed plus sweat investment was large during this period and critical for understanding the movements in hours and productivity. Expensed investments are expenditures that increase future profits but, by national accounting rules, are treated as operating expenses rather than capital expenditures. Sweat investments are uncompensated hours in a business made with the expectation of realizing capital gains when the business goes public or is sold. Incorporating expensed and sweat equity into an otherwise standard business cycle model, we find that there was rapid technological progress during the 1990s, causing a boom in market hours and actual productivity.

Ellen McGrattan
Research Department
Federal Reserve Bank of Minneapolis
90 Hennepin Avenue
Minneapolis, MN 55401
and NBER
erm@ellen.mpls.frb.fed.us

Edward Prescott
Research Department
Federal Reserve Bank of Minneapolis
90 Hennepin Ave.
Minneapolis, MN 55401
and NBER
edward.prescott@asu.edu

During the 1990s, market hours in the United States increased dramatically. A possible explanation for the rise is that the 1990s were a period of rapid technological progress. But if that were so, then productivity—defined as GDP per hour worked—should have boomed as well. In fact, productivity fell relative to trend in the decade.[1] The question then is why, unlike in all U.S. postwar booms, did market hours boom in the 1990s while productivity did not?

One possibility is that some factor other than improvements in production efficiency was responsible for the dramatic increase in hours and the lackluster productivity growth. Another possibility is simpler, that there is a measurement problem. We find that at least the latter is true: there is a problem in measuring output growth, which in turn affects the measure of productivity growth. The source of this problem is unmeasured *intangible investment*, which increased dramatically during the 1990s. We test this hypothesis, and our test results support it. We find that properly accounting for these investments changes the picture of the 1990s dramatically: U.S. productivity did in fact boom along with market hours during the period.

We focus on two specific types of intangible investment that are not included in the national accounts. One is expenditures financed by owners of capital which, by national accounting rules, is *expensed* rather than capitalized. Examples of this type include research and development (R&D), advertising, and investments in building organization capital. The other type of intangible investment we consider is *sweat*, investment financed by worker-owners who allocate effort and time to their business and receive compensation at less than their market rate. This type of investment is made with the expectation of realizing future profits or capital gains when the business goes public or is sold.

There is compelling evidence that both of these types of unmeasured investment were abnormally high in the 1990s. Rapid technological advancements were being made in industries that are relatively intensive in producing intangible capital, such as those related to information technology (IT). According to Doms (2004), two notable pieces of evidence

[1] A common misperception is that growth in GDP per hour was high during the 1990s. This misperception is based on high productivity growth rates found in some industries.

are what was happening to Intel processor speeds, which increased 4.6 percent per month in the period 1997–2000, and fiber optic throughput, which rose from 2.5 gigabits to 400 gigabits per second between 1995 and 2000. In addition, data from the Current Population Survey of the U.S. Department of Labor show a shift of labor into IT-related and managerial occupations with greater opportunities for business owners to make capital gains on expensed and sweat investment. Other evidence is the National Science Foundation's report of R&D investment, which grew 30 percent faster than GDP between 1994 and 2000. R&D investment is expensed and thus lowers corporate profits. Compensation per hour was also low over this period, suggesting that sweat investment was abnormally high.

Total expensed and sweat investment is not directly measurable, but with the aid of growth theory, its magnitude can be inferred from U.S. national income and product account (NIPA) data. Specifically, we incorporate expensed and sweat investment into an otherwise standard growth model, use the extended model to infer the path for intangible investment, and show why including this type of investment is critical for understanding the dramatic rise in hours in the 1990s. We first show that a standard one-sector growth model without intangible investment generates predictions for market hours that are grossly at odds with U.S. data. We then extend the model by having two technologies available in the business sector: one for producing final goods and services and one for producing intangible capital. We allow the rates of technological progress to differ across these two technologies, thus allowing for a technology boom in the sector producing intangible capital. Our estimate of net intangible (expensed plus sweat) investment in the business sector is a little over 0.03 of GDP during the early 1990s and rises to over 0.08 of GDP at the peak of the boom.

Before we use the model to make predictions about what actually happened to U.S. economic activity in the 1990s, we test the model in two ways to check its reliability as a tool for this economic analysis. One test is a check on the internal consistency of the equilibrium of our model. We use first-order conditions of the model and observations from the NIPA to determine allocations of factor inputs and total factor productivities (TFPs) across sectors. We then ask, If households in the model economy were confronted with these time paths for sectoral TFPs, would the model's behavior be close to the behavior of the U.S. economy? The answer is yes.

This consistency test imposes a lot of discipline on the model. To demonstrate this, we consider an alternative hypothesis of what lay behind the hours boom in the 1990s. We posit that hours rose because of a reduction in labor distortions rather than an increase in intangible investment. We assume that TFPs in our two sectors change proportionally. Thus, the number of free parameters with this alternative hypothesis is the same as with the original. We find that variations in labor distortions cannot account for the behavior of the U.S. economy in this period, particularly the boom in hours.

For a second test of our extended model, we compare its prediction for factor incomes and capital gains to U.S. measures in, respectively, the NIPA and the U.S. Flow of Funds accounts. This test is demanding because neither the income data nor the capital gains data were used to infer our measures of sectoral TFPs. Because the incomes reported in the NIPA data do not include expensed and sweat equity, we need to compare them to our model's total incomes less intangible investments. The Flow of Funds accounts report *holding gains* on assets at market value, which is the change in the asset amount outstanding less net purchases during the period. We show that the holding gains of households, subtracting gains for real estate, should move with our model's intangible gains during the 1990s period. We find that the model's predictions of both incomes and gains are in line with U.S. observations.

Because the extended model clearly passes all of our tests of reliability, we can confidently use it to get a more accurate picture of the U.S. economy in the late 1990s. In particular, we use the model to compare current accounting measures for output, investment, and productivity with analogs that include expensed and sweat investment. Solow's (1987) remark that "you can see the computer age everywhere but in the productivity statistics" is pertinent for our findings. The model predicts lackluster productivity performance if current accounting measures are used and a boom for productivity if expensed and sweat investments are included. From this, we conclude that ignoring these two types of intangible investment gives a distorted picture of the U.S. economy in the 1990s.

The puzzling rise in hours during the 1990s that we attempt to understand here is not discussed in the business cycle literature. There are at least two reasons for this. First and foremost, business cycle research has been almost exclusively concerned with the statistical properties of fluctuations and not with individual cycles. That focus can blind researchers to certain puzzles in the data. Second, there is a common misperception that labor productivity was high during the hours boom, which would imply that this period is not puzzling for standard theories. This is certainly not the case for the aggregates: GDP per hour was below trend for the entire decade. The misperception is based in part on high growth rates in some industries and in part on the rapid recovery in U.S. business productivity that began in 1997.

Our findings show that standard productivity measures grossly understate the actual rise in labor productivity whether we consider the overall economy or the business sector. For example, after accounting for intangible investment, we find that the recovery in business productivity began earlier and was faster than standard statistics show. Over the period 1993–2000, the difference in labor productivity growth due to the inclusion of intangible investment is 1.2 percent per year. Thus, our analysis deals directly with the criticism of Brynjolfsson and Hitt (2000), who argue that intangible investment is "not well captured by traditional macroeconomic measurement approaches," by which they mean growth-accounting approaches such as those of Jorgenson and Stiroh (1999) and Oliner and Sichel (2000). Here, we explicitly model the intangible investment and use theory and data to infer its size.

The paper is organized as follows. In section 1, we review the standard theory and show that the prediction for hours is inconsistent with U.S. data in the 1990s. In section 2, we summarize some evidence motivating our extension of the standard theory. The extension, in section 3, includes expensed and sweat investment. In section 4, we conduct tests of this extended theory. In section 5, we reevaluate the performance of the U.S. economy in the 1990s through the lens of the extended theory. Conclusions are in section 6.

1. The Standard Model

We start with the standard growth model used in the study of business cycles. We derive from this model a prediction for hours of work and show that given actual data its prediction of hours worked in the 1990s is grossly at odds with the fact that U.S. hours rose dramatically during that period. We investigate the failure of the model and use the failure to motivate an extension with expensed and sweat equity.

1.1. The Economy

In the standard growth model, given initial capital stock k_0, the problem for the stand-in household is to choose consumption c, investment x, and hours h to maximize expected utility,

$$\max E \sum_{t=0}^{\infty} \beta^t U(c_t, h_t) N_t$$

subject to these constraints:

$$c_t + x_t = r_t k_t + w_t h_t - T_t \tag{1.1}$$

$$k_{t+1} = [(1-\delta)k_t + x_t]/(1+\eta), \tag{1.2}$$

where variables are written in per capita terms and $N_t = N_0(1+\eta)^t$ is the population in t. Capital is paid rent r_t, and labor is paid wage w_t. Households discount future utility at rate β, and capital depreciates at rate δ. The term T_t is the sum of all taxes less all transfers.

Firms in the economy use the following constant returns technology:

$$Y_t = A_t F(K_t, H_t) \tag{1.3}$$

to produce goods sold to the household. Capital letters in this case denote aggregates. The parameter A_t is the technology parameter that varies over time. The firms rent capital and labor. If profits are maximized, then the rental rates are equal to the marginal products.

If the goods market clears in this economy, then $N_t(c_t + x_t) = Y_t$. Here, c includes both private and public consumption and x includes both private and public investment.

1.2. The Intratemporal Condition

At an optimum, household decisions satisfy the intratemporal first-order condition equating the marginal rate of substitution between consumption and leisure and the after-tax real wage. Let τ_{ct} be the tax rate on consumption, and let τ_{ht} be the tax rate on wage income $w_t h_t$. If the utility and production functions are[2]

$$U(c,h) = \log c + \psi \log(1-h)$$
$$F(K,H) = K^{\theta} H^{1-\theta},$$

then the household's intratemporal condition is

$$\frac{\psi(1+\tau_{ct})c_t}{1-h_t} = (1-\tau_{ht})(1-\theta)\frac{y_t}{h_t}, \tag{1.4}$$

where y_t is per capita output. Only current period variables enter this relation.

1.3. A Bad Prediction

The household's intratemporal condition is the key equation for the model's prediction for hours of work. We use it and relevant data for the 1990s to make that prediction.

From the U.S. national accounts, we have data for total consumption of services and nondurables c, gross domestic product y, and the tax receipts needed to estimate tax rate τ_{ct} and τ_{ht}.[3] Solving (1.4) for h_t yields

$$h_t^{predicted} = \left[1 + \left(\frac{\psi}{1-\theta}\right)\left(\frac{1+\tau_{ct}}{1-\tau_{ht}}\right)\frac{c_t}{y_t}\right]^{-1}. \tag{1.5}$$

We use (1.5) to compare this predicted series to the actual hours of work per capita.[4]

In Figure 1, we plot this model's predicted per capita hours of work along with the U.S. actual, indexed so that 1990 equals 100. The difference between the series is striking.

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[2] These functional form assumptions are standard in the business cycle literature.

[3] The sources of national account data that we use are the Board of Governors (1945–2005) and the U.S. Department of Commerce (1929–2005). See Appendix A for more details. Tax rates are estimated as in Prescott (2004).

[4] The source of hours and population is the Current Population Survey and is described in detail by Prescott, Ueberfeldt, and Cociuba (2005). We set $\theta = 0.34$ and $\psi = 1.33$, but the results are not sensitive to these choices.

Actual hours per capita rose 8 percent between 1992 and 1999, more than 1 percent per year. The predicted series actually falls during this period, primarily because of a rise in the tax on wage income τ_{ht}. To account for an 8 percent rise in hours in (1.5), that tax rate would have had to fall 5 percentage points.

Why is the prediction for hours worked so bad? Since the prediction is derived from the household's intratemporal condition, another way of asking this question is, Why is there such a large deviation in the condition during the 1990s?[5] We turn next to evidence that suggests that the large deviation is due to the fact that we are using the wrong measure of output. Unmeasured intangible investment was abnormally large during this period. Standard measures of output growth are distorted when the importance of intangible investments grows, which can potentially explain why *measured* labor productivity was low at the same time that rapid technological advancements were being made. If true output (in (1.5)) is understated by GDP, then predicted hours will be too low.

2. Evidence of Increased Intangible Investment

We present two types of evidence that the unmeasured intangible investment was abnormally large during the 1990s. One type of evidence is related to the behavior of NIPA compensation and profits; the other, to the technology boom going on during the period. Because intangible investments are expensed in the NIPA, measurements of factor incomes are understated to a greater extent in periods when these investments are high. We show that that was true for the 1990s. We then present evidence that, during the technology boom, the level of investment was indeed high and led to large capital gains that are missed by the NIPA's income measure.

2.1. Low Compensation and Corporate Profits

If all incomes were included in the national accounts, we'd expect to see compensation

[5] There is also a deviation in the condition relating the marginal rate of substitution between consumption today and tomorrow and the marginal rate of transformation. But this intertemporal deviation is small in comparison to the intratemporal deviation.

and profits rise during a boom. The U.S. national accounts reveal that compensation and profits were low during the boom period, suggesting that unmeasured expensed and sweat investment was abnormally high.

In Figure 2, we plot average weekly hours of work for the noninstitutional population, aged 16 to 64 (the same number of hours as those in Figure 1). We also plot the wage rate corresponding to these hours, which is computed as follows. We take NIPA compensation and deflate it by the GDP deflator. We then detrend for population growth by dividing real compensation by this population. Finally, because there is technological growth, we divide the wage rate by the factor 1.02^t, where t indexes time. For all of the 1990s, NIPA real, detrended compensation per hour is below the 1990 level, despite the fact that there was a boom in hours.[6]

In Figure 3, we compare NIPA GDP and corporate profits, both deflated by the GDP deflator and detrended so that they do not grow with population or technology. We see that profits fall (relative to trend) in the late 1990s when GDP, R&D, and capital gains are high.

2.2. The Technology Boom

Although lower income is indirect evidence of increased expensed and sweat investment, there is also some direct evidence available related to the technology boom. The 1990s were a period of rapid technological advances. Companies were increasing R&D and the payoffs were evident in increased IPOs and mergers and acquisitions. There is also direct evidence that most of the rise in hours was concentrated in activities related to the technology boom.

One indicator of increased intangible investment is increased funding of R&D, which is expensed by corporations. The National Science Foundation (NSF) (1953–2003) reports that industry R&D increased 68 percent between 1994 and 2000, while GDP rose only 39

[6] In earlier work (McGrattan and Prescott 2005b), we abstract from sweat equity and treat NIPA compensation as true labor income. Doing so reduces the estimate of total intangible investment in both expensed and sweat equity.

percent. The NSF includes in this measure expenses for wages and salaries, fringe benefits for R&D personnel, materials and supplies, property and other taxes, maintenance and repairs depreciation, and an appropriate share of overhead. The NSF also reports that a significant fraction of the company-funded R&D is done by companies in information technology (IT) industries.

Another indicator of abnormally large intangible investment is the dramatic increase in IPOs and mergers and acquisitions. According to data from the Thomson Financial Securities Data Corporation (SDC) database (also available in Ritter and Welch 2002, Table 1), gross proceeds from IPOs were significantly higher in the 1990s than in the 1980s. Gross proceeds of IPOs averaged $8.2 billion over the period 1980–89 and $30.9 billion over the period 1990–99. Large increases in the value of existing equity, and therefore large capital gains, are typically associated with IPOs. Because these gains are not included in NIPA, NIPA incomes understate true income. Other related evidence available from the SDC database is the volume of announced mergers and acquisitions. The volume rose from $0.6 trillion in 1994 to $3.4 trillion by 2000. As in the case of IPOs, the accrued capital gains are not included in NIPA measures of income.

The evidence presented thus far suggests that during the 1990s boom, business owners made large unmeasured investments and made large gains on those investments. If that is true, then we should see the rise in hours concentrated in certain groups of occupations. Using Current Population Survey (CPS) data, we find that the large increase in hours is, in fact, concentrated among a small group of occupations.[7] According to the survey data, per capita hours for the entire noninstitutional population aged 16 to 64 rose 6.5 percent between 1992 and 2000. Half of this increase was due to hours worked of those with at least one year of college education in a select set of occupations. These occupations include most managers and proprietors, computer analysts, and certain financial occupations involved with IPOs and mergers and acquisitions.[8] Our aim here is to focus on occupations in

[7] Here, we are referring to data compiled from the March supplement of the CPS survey. See www.ipums.org for more details (Ruggles et al. 2004).

[8] Specifically, using data from IPUMS (www.ipums.org, Ruggles et al. 2004), we split workers into two groups: those with variable EDUCREC greater than or equal to 8 and variable OCC in the set {4,

which many workers make large unmeasured investments. Hours of the educated in these occupations accounted for only 10.3 percent of the hours in 1992, but rose 30 percent between 1992 and 2000. This change in hours contributed half of the overall change in hours.

In summary, we find compelling evidence that intangible investment was abnormally high during the 1990s. This motivates an extension of the standard model with both tangible and intangible investment and a reevaluation of the predictions of growth theory for the U.S. economy during this period.

3. Our Extended Model

We now modify the standard growth model by extending it to include intangible investment. Intangible investments are made by businesses, so we distinguish business and non-business activity. We start by describing the technology available to businesses, the optimal business size, and the aggregate production technology. The household problem remains the same except for an additional investment choice. We examine the model's prediction for hours and show that the extended model accounts for the puzzling U.S. observations during the 1990s.

3.1. Extensions

The aggregate production technology is characterized by the two aggregate production functions:

$$y_{bt} = A_t^1 (k_{Tt}^1)^{\theta_1} (k_{It})^{\phi_1} (h_t^1)^{1-\theta_1-\phi_1} \tag{3.1}$$

$$x_{It} = A_t^2 (k_{T,t}^2)^{\theta_2} (k_{It})^{\phi_2} (h_t^2)^{1-\theta_2-\phi_2}. \tag{3.2}$$

Firms produce y_b, which is business output, using their intangible capital k_I, tangible capital k_T^1, and labor h^1. Firms produce intangible capital—such as new brands, new products R&D, patents, etc.—using intangible capital k_I, tangible capital k_T^2, and labor

7, 9, 13, 14, 15, 18, 21, 22, 34, 37, 64, 65, 229, 23, 24, 25, 225} and the remainder.

h^2. Note that k_I is an input to both business sectors; it is not split between them as is the case for tangible capital and labor. The aggregation theory underlying this technology is developed in Appendix B.

Given (k_{T0}, k_{I0}), the stand-in household maximizes

$$\max E \sum_{t=0}^{\infty} \beta^t [\log c_t + \psi \log(1 - h_t)] N_t$$

subject to

$$\begin{aligned}
c_t + x_{Tt} + q_t x_{It} = {} & r_{Tt} k_{Tt} + r_{It} k_{It} + w_t h_t + \zeta_t \\
& - \tau_{ct} c_t - \tau_{ht}(w_t h_t - (1-\chi) q_t x_{It}) - \tau_{kt} k_{Tt} - \tau_{xt} x_{Tt} \\
& - \tau_{pt}\{r_{Tt} k_{Tt} + r_{It} k_{It} - \delta_T k_{Tt} - \chi q_t x_{It} - \tau_{kt} k_{Tt}\} \\
& - \tau_{dt}\{r_{Tt} k_{Tt} + r_{It} k_{It} - x_{Tt} - \chi q_t x_{It} - \tau_{kt} k_{Tt} \\
& \quad - \tau_{pt}(r_{Tt} k_{Tt} + r_{It} k_{It} - \delta_T k_{Tt} - \chi q_t x_{It} - \tau_{kt} k_{Tt}) - \tau_{xt} x_{Tt}\}
\end{aligned}$$

$$k_{Tt+1} = [(1 - \delta_T) k_{Tt} + x_{Tt}]/(1 + \eta) \tag{3.3}$$

$$k_{It+1} = [(1 - \delta_I) k_{It} + x_{It}]/(1 + \eta). \tag{3.4}$$

As before, all variables are in per capita units and there is growth in population at rate η. Consumption c includes both private and public consumption, and tangible investment x_T includes both private and public investment. The relative price of intangible investment and consumption is q. The rental rates for business tangible and intangible capital are denoted by r_T and r_I, respectively, and the wage rate for labor is denoted by w. Inputs are paid their marginal products. Other income is denoted by ζ and is exogenous in the household's decision problem. Other income includes government transfers and non-business capital income net of taxes and investment. Non-business labor income is included in wh.

We treat hours $\bar{h}_n$, investment $\bar{x}_n$, and output $\bar{y}_n$ in the non-business sector exogenously because this sector is quite small compared to the business sector and is not important for the issues being addressed. Doing this makes the exposition of the model much clearer. We simply set the levels of non-business hours, investment, and output equal to U.S. levels. Measured output y_m, which corresponds to GDP, is the sum of y_b and $\bar{y}_n$. Measured tangible investment x_m is the sum of business tangible investment x_T and non-business tangible investment $\bar{x}_n$. Measured hours h is the sum of business hours $h^1 + h^2$ and non-business hours $\bar{h}_n$.

The tax system in the model economy mimics the U.S. system. It includes taxes on consumption τ_c, measured wages τ_h, tangible capital (that is, property) τ_k, investment τ_x, profits τ_p, and distributions τ_d. Let χ denote the fraction of intangible investment financed by capital owners.[9] The amount $\chi q_t x_{It}$ is *expensed investment*, financed by the capital owners who have lower profits with increases in this type of investment. The amount $(1-\chi)q_t x_{It}$ is *sweat investment*, financed by workers who have lower compensation with increases in this type of investment.

Gross domestic product in the economy is the sum of total consumption (public plus private) and tangible investment (public plus private) for business and non-business; in per capita terms GDP is $y_{mt} = c_t + x_{Tt} + \bar{x}_{nt}$. Gross domestic income (GDI) is the sum of labor income less sweat investment $w_t h_t - (1-\chi)q_t x_{It}$, business capital income less expensed investment, $r_{Tt}k_{Tt} + r_{It}k_{It} - \chi q_t x_{It}$, and non-business capital income (which is found residually as the difference between GDP and the other components of GDI).

3.2. Predictions for Hours

We showed that a key failure with the standard theory was evident in the intratemporal condition (1.4), the predictions of which are inconsistent with U.S. data. The extended model with intangible investment has an analogous intratemporal condition, but labor productivity in this model must be measured differently. Therefore, we show the extended model has the potential to resolve the puzzling movements in hours and productivity during the 1990s.

[9] The choice is irrelevant without taxes. With taxes, the choice is all or none without risk which might optimally be shared between capital owners and worker-owners.

To see that, consider the intratemporal condition for the extended model, which is

$$\frac{\psi(1+\tau_{ct})c_t}{1-h_t} = (1-\tau_{ht})(1-\theta)\left(\frac{y_{bt}}{h_t^1}\right), \tag{3.5}$$

where $h_t = h_t^1 + h_t^2 + \bar{h}_{nt}$ and $\theta = \theta_1 + \phi_1$. Note that the real wage rate in the standard model is proportional to the standard definition of labor productivity, output of final goods and services divided by total hours. In the extended model, however, hours are split between two different types of activities. The real wage rate to labor producing final goods and services is proportional to the labor productivity given by y_{bt}/h_t^1. This ratio is larger than measured business output y_{bt} divided by *total* business labor input $h_t^1 + h_t^2$. As more time is allocated to the accumulation of intangible investment (h_t^2 increasing), the understatement of true wages becomes more severe.[10]

We use (3.5) to determine how large h_t^2 would have to be in order for (3.5) to hold. From (3.5), we derive the following formulas for h_t^1 and h_t^2:

$$h_t^1 = \left(\frac{1-\theta}{\psi}\right)\left(\frac{1-\tau_{ht}}{1+\tau_{ct}}\right)\left(\frac{y_{bt}}{c_t}\right)(1-h_t) \tag{3.6}$$

$$h_t^2 = h_t - h_t^1 - \bar{h}_{nt}. \tag{3.7}$$

Using observations on business value-added, consumption, total hours, non-business hours, and tax rates, we can directly infer the allocation of hours to production of final goods and services and to production of new intangible capital. In Figure 4, we plot the ratio h_t^2/h_t, which represents the fraction of hours devoted to producing intangible capital. These data are based on our model's expressions (3.6) and (3.7) with $\theta = 0.34$ and $\psi = 1.33$ and on U.S. data on tax rates, consumption, business value-added, and total hours. Clearly these hours would have had to increase dramatically, starting in 1992. The fraction h_t^2/h_t consistent with U.S. data rises from 2.7 percent to 7.7 percent by 2000.

If hours devoted to producing intangible capital rose as in Figure 4, then there is no deviation in the intratemporal condition (3.5). What caused this large increase in hours,

[10] In standard sticky wage models, the condition (1.4) is replaced by a dynamic equation relating the nominal wage to a markup over expected future marginal rates of substitution between consumption and leisure. As McGrattan (2005) shows, however, the impact of monetary shocks in these models is tiny. Given the magnitude of these shocks for the United States during the 1990s, these models cannot account for the dramatic rise in hours and output.

especially hours devoted to building intangible capital? The evidence presented earlier suggests that advances in technology that were particularly large in certain activities could have generated the increase. Next, we test this hypothesis in the context of our extended model.

4. Testing of Extended Model

Before we use the extended model to analyze the 1990s, we test it to see if it has the potential to resolve the puzzle of the 1990s. We put the model to two tests, and it passes them both.

One test is a check on the consistency of the model's equilibrium. We use a subset of equilibrium relations for the model to infer sequences for sectoral TFPs. Treating these sequences as exogenous, we compute the equilibrium path and compare it with U.S. data. The match between the two sets of time series is close. Thus, the model does extremely well on this test.

The other test is more demanding. We compare predictions of the model to observations that were not used when we inferred paths for sectoral TFPs. In particular, we compare the model's predictions for factor incomes and capital gains to analogs in the U.S. data. The model passes this test as well.

4.1. First Steps

Two steps are needed in order to conduct the tests. We have to revise the national accounts to be consistent with our extended model, and we have to choose the model's parameters.

4.1.1. Revising Data

We start with the tedious but essential step of revising the national accounts to make them consistent with our model. Four adjustments are necessary. We provide an overview of the process here and details in Appendix A.

Consumption Taxes. Unlike the NIPA, our model output does not include consumption taxes as part of consumption and as part of value-added. We thus subtract sales and excise taxes from the NIPA data on taxes on production and imports and from personal consumption expenditures. As a result, unlike the NIPA, our data use producer prices rather than a mixture of producer and consumer prices.

Financial Services. We treat some of the NIPA's financial services as intermediate rather than as final and subtract them from GDP and from consumption services. Specifically, we subtract personal business expenses for handling life insurance and pension plans from net interest and from personal consumption expenditures.

Fixed Asset Expenditures. We treat expenditures on all fixed assets as investment. Thus, spending on consumer durables is treated as an investment rather than as a consumption expenditure. We introduce a consumer durables services sector in much the same way as the NIPA introduces owner-occupied housing services. Households rent the consumer durables to themselves. A related adjustment is made for government capital.

Intangible Investment. Our output measure includes intangible investment. Thus, total product in the model is the sum of intangible investment and gross domestic product (which is the same concept as the NIPA's GDP after the adjustments are made for consumption taxes, intermediate financial services, and consumer durables). On the income side of our extended model accounts, we add capital gains $q_t x_{It}$. Fraction χ of capital gains is allocated to compensation, and fraction $1 - \chi$ to profits.

4.1.2. Setting Parameters

Next, we need to choose parameters for the extended model. Here, we report and motivate our choices.

For interest and growth rates, we use estimates based on U.S. trends. In particular, we set the interest rate at 4.1 percent and the annual growth in population at 1 percent ($\eta = .01$). We also assume that technology grows over time in the economy, so that per capita GDP and its components grow at 2 percent annually. These choices imply that

$\beta = .98$.

Household preferences depend on the parameter ψ. As we did earlier, in the standard model, here we choose $\psi = 1.33$ in order to match data for the fraction of time allocated to work (in U.S. business and non-business activities). This value implies that 27.5 percent of discretionary time is allocated to work.

Since changes in tax rates on capital were modest during the 1990s, we hold these constant. We set the profits tax rate $\tau_p = 0.35$ because most of the taxes on profits are corporate income taxes. We set the distribution tax $\tau_d = 0.15$, which is slightly less than our estimate in earlier work (McGrattan and Prescott 2005a) for corporate distributions; this is appropriate because noncorporate taxes are not taxed twice. We set the tax on investment $\tau_x = 0$ because depreciation allowances, investment tax credits, and investment taxes were negligible in the 1990s. Finally, we set the property tax rate $\tau_k = 0.016$, which is consistent with the NIPA non-sales taxes on production and imports.

We use the same series for the tax on consumption that we used in the standard model. But for the tax on labor τ_h, we need to make one adjustment. Since we want to assume that τ_h is the tax rate on labor income excluding capital gains, we can either subtract the capital gains tax receipts from receipts or add capital gains income to taxable income before constructing our estimate of the tax rate. We do the latter because the income data are available from the U.S. Department of Commerce.[11]

The share parameters and depreciation rates are chosen so that 1990 in the model simulations looked like 1990 in U.S. time series.[12] U.S. corporate and noncorporate business accounts for 75 percent of value-added. The remaining sector, the non-business sector, includes U.S. households, nonprofits, and government.[13]

We use the same input elasticities for producing both final goods and intangible capi-

[11] We add capital gains income to taxable income using data reported by the BEA in the table comparing NIPA personal income and the IRS's adjusted gross income.

[12] There is no way to determine δ_I. We chose 0 and experimented with other values to make sure our main results did not change.

[13] In Appendix A, we show specifically how we categorize business and non-business activity for U.S. national accounts.

tal. When we match the extended model and data for 1990, we find that $\theta_1 = \theta_2 = 0.254$, $\phi_1 = \phi_2 = 0.087$, and $\delta_m = 0.04$ for the business sector technologies.

The final parameter to be set is χ, the fraction of intangible investment that is financed by capital owners. As noted earlier, the only real ramification of this choice is for tax payments. But the evidence in Figures 2 and 3 indicates that some investment is being done by both shareholders and workers. We chose $\chi = 0.5$ and then experimented with other values. The main effect of varying χ is a change in the effective tax rates on labor and capital.

4.2. Two Tests

4.2.1. Internal Consistency

The first test of our extended model is to ask, Are there paths of sectoral TFPs $\{A_t^1, A_t^2\}$ that imply an equilibrium of the model that is consistent with U.S. observations? If the answer is no, then this model is not useful for predicting what was happening during the 1990s U.S. boom. If the answer is yes, then it may be. We find that the model is potentially useful for predicting the size of intangible investments and the patterns of actual output per hour.

Test Results. Above we showed that the intratemporal condition relating the marginal rate of substitution to the after-tax wage rate gives us an expression for sectoral hours in terms of U.S. observations. If we had observations on all investment and capital stocks, we could use (3.1) and (3.2) to find total factor productivity (TFP) for the two technologies, feed these TFPs into the model, and compute equilibrium responses of the household.

However, we do not have observations on all investments and capital. So, we must use additional equilibrium relations to determine the sequences of TFP. In Appendix C, we describe the steps used to derive sequences for A_t^1 and A_t^2. The main idea is to equate returns to capital in order to determine the sectoral allocation of capital stocks. The resulting sequences for TFPs are plotted in Figure 5 along with the standard measure of

TFP: GDP divided by $K_{mt}^{0.33} H_t^{0.67}$, where K_{mt} is total measured capital and H_t is total hours. All series are real and relative to trend.

In the figure, the three curves are quite different. The standard TFP measure falls slightly over the period, but stays close to trend. The implied TFPs for the model with intangible investment show large increases. In the sector producing final goods and services, the increase is about 6 percent. In the sector producing intangible capital, the increase is close to 17 percent.

We now examine U.S. data through the lens of the extended model, with intangible investment. We compute the model's equilibrium for the TFP series in Figure 5 along with the varying tax rates on market wages and consumption discussed above. In Figure 6, we display the results for per capita total hours worked. Unlike the comparable figure with the standard model's predictions (Figure 1), here, the predictions and the actual series track each other closely. The extended model predicts that hours used to produce final goods and services actually fall below trend somewhat in the 1990s. But because hours spent building intangible capital rise significantly, the model predicts a large overall increase in hours — which is what the data actually show.

The model and data paths for labor productivity in the business sector are also close. We plot these paths in Figure 7. The model's prediction for business value-added tracks the actual NIPA series closely, so its prediction for labor productivity does too. We find as well a close match for total labor productivity (GDP per hour).

There is a modest difference in the model and data paths of tangible investment. These series are plotted in Figure 8. The model prediction is sensitive to the choice of capital taxation, which is fixed over the period. However, even with the assumption of constant capital taxes, the model does well predicting the pattern, the rise and fall, of tangible investment.

Test Strength. If TFPs are determined from first-order conditions of the household's problem, is the consistency test much of a test? Here we consider an alternative version of the extended model that has the same number of free parameters as the model with different TFPs in producing final goods and intangible capital. The alternative has TFPs in (3.1) and (3.2) varying proportionally and τ_{ht} chosen to satisfy the intratemporal first-order condition. The hypothesis underlying this alternative version is that the boom in market hours is caused by a large reduction in labor distortions. Despite the fact that there are the same number of free parameters as in the original version of the extended model, we find that the alternative with reduced labor distortions is grossly at odds with U.S. data.

To see this, first note that labor productivities are equated in equilibrium. Thus, the intratemporal condition can be written as

$$\frac{\psi(1+\tau_{ct})c_t}{1-h_t} = (1-\tau_{ht})(1-\theta)\left(\frac{y_{bt}+\bar{q}x_{It}}{h_t}\right). \tag{4.1}$$

Here, the relative price q is fixed because TFPs are assumed to vary proportionally. Equation (4.1) has two unobserved variables: τ_{ht} and x_{It}. We can use (4.1) along with the intertemporal condition relating the marginal rate of substitution of consumption this period and next to the return on intangible capital in order to determine sequences of τ_{ht} and x_{It}.

The resulting sequences oscillate wildly. For example, the series for the labor distortion oscillates between 0 percent and 40 percent and displays little persistence. Such a pattern is inconsistent with the notion of reduced labor distortions and cannot generate an hours boom unless movements in capital tax rates and TFP are also oscillatory and offsetting.

We conclude that our consistency test is a strong test. This gives us a considerable degree of confidence in the hypothesis that increased growth in TFPs, especially in the sector producing intangible capital, resolves the puzzling U.S. observations during the 1990s. The next test builds our confidence further.

4.2.2. External Accuracy

Thus far, we have checked on the extended model's internal consistency using TFP sequences derived from the model's equilibrium relations. Now we consider a more demanding test, comparing model predictions to observations not used to infer the TFP paths.

In particular, we explore predictions for business wage compensation as measured in the NIPA and for business capital gains as measured in the Flow of Funds accounts. Neither series was used to derive our measures of TFPs. We find that the model predicts these series remarkably well.

To compare the model's prediction for NIPA wage compensation in the business sector, we need to construct wages as a national accountant would. Such an accountant, placed in the model economy, would estimate wage compensation in the business sector as $w_t(h_t^1 + h_t^2) - (1-\chi)q_t x_{It}$, in effect, subtracting part of intangible capital, the value of sweat equity. In Figure 9, we plot this predicted series along with the actual U.S. series. Both are real series, detrended by 2 percent annually, and set equal to 100 in 1990. The two are fairly close. Relative to the 1990 trend level, both the model prediction and the actual wages are up nearly 8 percent in 2000. We should note that our choice of $\chi = 0.5$ is relevant for this prediction. The value of χ determines the level of taxation on expensed versus sweat equity, which affects the equilibrium measured compensation. Higher values of χ increase the predicted value of compensation. Allowing for variation in χ would imply a better fit of these curves. However, we do not have independent evidence of the financing of expensed and sweat equity.

Next, we compare the model's predictions for estimates of the increase in capital gains from expensed and sweat equity to U.S. household holding gains reported in the Flow of Funds accounts. Those gains are the change in the value of assets outstanding (taken from Table L.100) less the net purchases during the period (taken from Table F.100). If Flow of Funds accountants recorded holding gains for our model households, they would compute differences in the total value of businesses (for which the household is the residual claimant). The value of all businesses in t, V_t, is composed of two parts:

$$V_t = (1-\tau_{dt})(1+\tau_{xt})K_{T,t+1} + [\chi(1-\tau_{dt})(1-\tau_{pt}) + (1-\chi)(1-\tau_{ht})]q_t K_{I,t+1}, \quad (4.2)$$

where capital letters denote aggregates. On the right side of (4.2), the first term is the value of tangible capital, and the second, the value of intangible capital. Notice that the price of intangible capital depends on χ, since income to capital and income to labor are taxed differently.

The change in the value V of businesses does not exactly reflect the additional income in the model economy. The additional income is $q_t X_{It}$ (in units of the final goods and services). However, during periods with large investments of intangible capital, the increase in holding gains, as defined in the Flow of Funds accounts, is a good approximation to the increase in intangible investment.

In Figure 10, we plot an estimate of U.S. real holding gains relative to GDP, using data from the Flow of Funds accounts and the NIPA. To illustrate that the late 1990s and early 2000s were special, we show the estimates annually back to 1953. The figure reveals a significant break in the series in 1995. Before that year, the series averages around 6 percent of GDP. In 1995 and thereafter, the average is 12 percent. A difference of 6 percent of GDP is large.

Because our theory does not provide an explanation for the huge swings in asset prices, we compare the model's predicted gains with the U.S. averages. We also have to make an adjustment for foreign gains because our model includes only domestic sectors. Since many domestic corporations have foreign subsidiaries, the value of U.S. corporations includes equity from foreign capital, and the holding gains include gains from this foreign capital. We estimate the gains by assuming that the ratio of after-tax foreign corporate profits (excluding gains) to after-tax domestic corporate profits (excluding gains) is equal to the ratio of foreign to domestic holding gains. With this assumption, our estimate of foreign gains relative to total gains is approximately 23 percent on average for the period 1990–2003.

In Figure 11, we show again the U.S. actual average real holding gains along with the extended model's prediction for them. Both curves rise significantly in the late 1990s. The rise is coincident with the dramatic rise in hours. We thus conclude that the model passes this demanding test.

We can now report with confidence the predictions of the extended model for the macroeconomy during the 1990s.

5. Predictions for the U.S. Macroeconomy

What does all of this mean for U.S. output, productivity, and investment? If some output is unmeasured relative to inputs, then GDP and productivity estimates are biased downward. If the mismeasurement is intangible investment, then the investment estimates are also biased downward. Our extended model's predictions for variables with and without intangible investment demonstrate how distorted standard data and models have been.

In Figure 12, we compare two predictions for output, both computed from the extended model. One is the model's prediction for gross domestic product, which includes consumption and tangible investment but not intangible investment. The other prediction is for what we will call total output, which is GDP plus intangible investment $q_t x_{It}$. Both series are detrended by 2 percent annually and set equal to 100 in 1990. Comparison of the predictions shows GDP growing slightly faster than 2 percent per year between 1991 and 1997. But total output takes off starting in 1994. At its peak, in 1999, total output was far above its 1990 trend, whereas GDP was only slightly higher than its trend.

In Figure 13, we show the same sort of comparison for business labor productivity: business value-added — with and without intangible investment — divided by total business hours. Again, both measures are detrended by 2 percent annually and set equal to 100 in 1990. Notice how different the predictions are. Measured labor productivity, which is what national accountants would record if put into the economy, shows a significant fall relative to trend up to 1997 and then a sharp increase through 2000. But total productivity, including intangible investment, fell only until 1993 and then, starting in 1994, grew very quickly. Over the period 1993–2000, the difference in growth rates for these two series is 1.2 percent per year.

In Figure 14, we compare the model's two measures of total investment: tangible investment and tangible plus intangible investment. Again, both are detrended by 2 percent annually and normalized to 100 in 1990. And again, the predictions—with and without intangible investment—are very different. Between 1991 and 1999, tangible investment alone rose almost 20 percent. Total investment, however, rose more than 30 percent.

Finally, in Figure 15, we display intangible investment as a share of total output, by which we mean GDP plus intangible investment. This figure displays the bottom line of our study, that the value of this investment is large and increased significantly in the 1990s. That is precisely what we see in Figure 15. We can, hence, say with some confidence that because the standard accounting measures and models do not take account of intangible investment, they do not accurately reflect what was going on in the U.S. economy during this period.

6. Conclusion

Why did U.S. market hours boom in the 1990s while productivity didn't? The answer is clear when attention expands beyond standard measures of output. Standard measures of output do not include a type of investment that rose significantly in the 1990s: intangible investment. In McGrattan and Prescott (2005a), we found that intangible capital was important for estimating the value of corporate equity. Here, we have considered its impact on hours, output, and productivity. To do this, we have modified a standard growth model by extending it to include intangible investment. Using data for the United States and our extended model, we conclude that intangible investment was a large contributor to the U.S. hours boom of the 1990s. Measurements of productivity based on these types of investment show that productivity boomed along with hours in the 1990s. More far-reaching is the implication of our work: ignoring this investment in the data and in the standard growth model has clearly produced a distorted view of the performance of the U.S. economy—and will continue to do so whenever intangible investment changes significantly.

Appendix A. Adjusting the National Accounts

In this appendix, we describe in more detail the adjustments that are made to the national accounts so that they are consistent with our theory.

In Table A1, we display the components of our measure of domestic business value-added. This measure is close to, but not exactly the same as, the sum of the value-added of corporate business, sole proprietorships and partnerships, and other private business as defined in the NIPA tables. In our table, we note the source of these NIPA series. Two adjustments, made in lines 20 and 25, together imply that our estimate of domestic business value-added is lower than NIPA's by an amount equal to 0.049 of GDP. The first adjustment (line 20) removes the personal business expense for handling life insurance and pension plans from net interest. We treat these financial services included in NIPA as intermediate rather than as final. The second adjustment (line 25) removes sales tax from taxes on production and imports. Our model's output does not include consumption taxes as part of consumption or as part of value-added, but the BEA does.

In Table A2, we display the components of our measure of domestic non-business value-added. This measure is close to, but not exactly the same as, the sum of value-added of households, nonprofits, general government, and government enterprises. Three adjustments are made. We add depreciation of consumer durables (line 5), subtract sales taxes (line 24), and add imputed capital services for consumer durables and government capital (line 25). Adjustments for consumer durables are necessary because we include consumer durables with investment while the BEA includes durables with consumption. Services for government capital are included because the BEA does not impute any value to them. We assume a rate of return equal to 4.1 percent, which is an estimate of the return on other types of capital.

In Table A3, we construct our measure of gross domestic product. The adjustments noted above are also included in product, so that income and product balance. We have also categorized tangible investment into business and non-business as in the case of incomes. That is, investments of corporations and noncorporate business are included with business investment, and investments of households, nonprofits, and government are included with non-business investment.

To be consistent, we also categorize hours from the Current Population Survey (CPS) as business or non-business. Using the March supplement (through www.ipums.org), we construct business hours as the sum of hours for the self-employed—both incorporated and unincorporated—and hours for private wage and salary workers less hours for employees in nonprofits. Because private wage and salary workers include employees at nonprofits, we use BEA data on compensation in nonprofits, and assuming an average wage rate equal to the economy-wide average, we can infer hours for nonprofits. Hours in the non-business sector are found by subtracting business hours from the total. We use the hours from the March supplement sample to compute the fractions of hours in business and non-business. For our final series, we multiply these fractions by total hours in the monthly CPS sample.

Table A. Revised National Accounts, Averages Relative to GDP, 1990–2003

A1. Domestic Business Value-Added

1	Domestic Business Value-Added	.700
2	Consumption of fixed capital	.082
3	Corporate business (NIPA 7.5)	.067
4	Sole proprietorships and partnerships (NIPA 7.5)	.013
5	Other private business (NIPA 7.5)	.003
6	Labor Income	.469
7	Compensation of employees	.421
8	Corporate business (NIPA 1.13)	.382
9	Sole proprietorships and partnerships (NIPA 1.13)	.036
10	Other private business (NIPA 1.13)	.002
11	70% Proprietors' income with IVA and CCadj (NIPA 1.13)	.049
12	Capital Income	.149
13	Corporate profits with IVA and CCadj (NIPA 1.13)	.073
14	30% Proprietors' income with IVA and CCadj (NIPA 1.13)	.021
15	Rental income of persons with CCadj (NIPA 1.13)	.006
16	Net interest and miscellaneous payments	.022
17	Corporate business (NIPA 1.13)	.014
18	Sole proprietorships and partnerships (NIPA 1.13)	.012
19	Other private business (NIPA 1.13)	.005
20	*Less*: Intermediate financial services[a] (NIPA 2.5.5)	.009
21	Taxes on production and imports[b]	.026
22	Corporate business (NIPA 1.13)	.056
23	Sole proprietorships and partnerships (NIPA 1.13)	.008
24	Other private business (NIPA 1.13)	.002
25	*Less*: Sales tax (NIPA 3.5)	.040

See footnotes at the end of the table.

Table A. Revised National Accounts (Cont.)

A2. Domestic Non-business Value-Added

1	Domestic Non-business Value-Added	.337
2	Consumption of fixed capital	.099
3	Households	.084
4	Excluding consumer durables (NIPA 7.5)	.012
5	Consumer durable depreciation (FOF F10)	.062
6	Nonprofits (NIPA 7.5)	.004
7	General government (NIPA 7.5)	.018
8	Government enterprises (NIPA 7.5)	.003
9	Labor Income	.154
10	Compensation of employees	.154
11	Households (NIPA 1.13)	.001
12	Nonprofits (NIPA 1.13)	.042
13	General government (NIPA 1.13)	.099
14	Government enterprises (NIPA 1.13)	.012
15	Capital Income	.083
16	Current surplus of government enterprises (NIPA 1.13)	.001
17	Rental income of persons with CCadj (NIPA 1.13)	.008
18	Net interest and miscellaneous payments	.033
19	Households (NIPA 1.13)	.031
20	Nonprofits (NIPA 1.13)	.002
21	Taxes on production and imports[b]	.004
22	Households (NIPA 1.13)	.011
23	Nonprofits (NIPA 1.13)	.001
24	*Less*: Sales tax (NIPA 3.5)	.007
25	Imputed capital services[c]	.038
26	Household, consumer durables	.013
27	Government capital	.025

See footnotes at the end of the table.

Table A. Revised National Accounts (Cont.)

A3. Domestic Value-Added and Product

1	Total Adjusted Domestic Income	1.043
2	Domestic Business Value-Added	.700
3	Domestic Non-business Value-Added	.337
4	Statistical Discrepancy	.006
5	Total Adjusted Domestic Product	1.043
6	Private consumption	.618
7	Personal consumption expenditures (NIPA 1.1.5)	.678
8	*Less*: Consumer durables (NIPA 1.1.5)	.083
9	*Less*: Intermediate financial services[a] (NIPA 2.5.5)	.009
10	*Less*: Sales tax, nondurables and services (NIPA 3.5)	.042
11	Consumer durable depreciation (FOF F10)	.062
12	Imputed capital services[c]	.013
13	Public consumption (NIPA 3.1)	.179
14	Government consumption expenditures (NIPA 3.1)	.154
15	Imputed capital services[c]	.025
16	Business tangible investment[d]	.112
17	Corporate gross private domestic investment (FOF F6)	.092
18	Noncorporate gross private domestic investment (FOF F6)	.020
19	Non-business tangible investment	.134
20	Household	.114
21	Excluding consumer durables (FOF F6)	.036
22	Consumer durables (NIPA 1.1.5)	.083
23	*Less:* Sales tax, durables (NIPA 3.5)	.005
24	Nonprofits (FOF F6)	.007
25	Government investment (NIPA 3.1)	.033
26	Net exports of goods and services (NIPA 1.1.5)	−.021

NOTE: IVA, inventory valuation adjustment; CCadj, capital consumption adjustment.

[a] Expense is for handling life insurance and pension plans.
[b] This category includes business transfers and excludes subsidies.
[c] Imputed capital services are equal to 4.1 percent times the current-cost net stock of government fixed assets and consumer durables goods.
[d] Ten percent of farm business is in corporate, with the remainder in noncorporate.

Appendix B. Aggregation in Extended Model

Here we develop the aggregation theory underlying the technology of our extended model. (See Section 3.)

A business is characterized by the stock of its (unmeasured) intangible capital, K_I. This capital can be used for two activities. One activity produces the composite output of the business Y_b, and the other produces intangible investment goods X_I.

Inputs of (measured) tangible capital K_T^i and hours H^i along with K_I produce an intermediate good Z^i via a standard constant returns to scale neoclassical production function f^i for $i \in \{1, 2\}$. In particular, the production functions are

$$Z^i = (K_T^i)^{\theta_i} K_I^{\phi_i} (H^i)^{1-\theta_i-\phi_i}, \quad i \in \{1, 2\}.$$

The quantity of Y_b produced is $g^1(Z^1)$, and the quantity of X_I produced is $g^2(Z^2)$. The functions g^i are increasing, initially strictly convex, then strictly concave, and they satisfy $g^i(0) = 0$. The slope of the maximal tangent ray from the origin is A^i. The point of tangency is $\hat{Z}^i$. The margin of adjustment is the number of units operated, which is variable. The capital stock K_I can be split over businesses through mergers, acquisitions, and spin-offs. All production units that are operated will have the same K_I. This K_I will depend upon the relative prices of the three inputs. Production units of type i will be operated at level $\hat{Z}^i$ and produce $g^i(\hat{Z}^i)$.

Appendix C. Deriving Total Factor Productivities

Here we describe how we derive sequences for the sectoral TFPs needed to compute equilibrium paths for our extended model.

We start with observables.[14] We have sequences for all total measured tangible investment x_{mt}, measured business tangible x_{bt}, and measured non-business tangible $\bar{x}_{nt}$. We have sequences for total business hours $h_t^1 + h_t^2$ and non-business hours $\bar{h}_{nt}$. Let y_{mt} be measured output (GDP), which is the sum of business output of final goods and services y_{bt} and non-business output $\bar{y}_{nt}$. We have sequences for all three of these output series. Finally, we have series for consumption, assumed to be the sum of private and public consumption, and for tax rates.

Now we are ready to use the model's equilibrium conditions to derive the two TFP series, A^1 and A^2. We use (3.6) and (3.7) to infer the allocation of hours within the business sector. Let $y_{It} = q_t x_{It}$. Equating wage rates implies that

$$y_{It} = \frac{(1-\theta_1-\phi_1)h_t^2}{(1-\theta_2-\phi_2)h_t^1} y_{bt}.$$

Given observables and $\{y_{It}\}$, the sequences for k_{It} and q_t are chosen to satisfy

$$y_{It}/q_t + (1-\delta_I)k_{It} = k_{It+1}$$

$$\begin{aligned} q_t(1/c_t)[(1-\chi)(1-\tau_{ht} + \chi(1-\tau_{pt})(1-\tau_{dt})]/(1+\tau_{ct}) \\ = \beta(1/c_{t+1})/(1+\tau_{ct+1}) \\ [q_{t+1}((1-\chi)(1-\tau_{ht+1}) + \chi(1-\tau_{pt+1})(1-\tau_{dt+1}))(1-\delta_T) \\ + (1-\tau_{pt+1})(1-\tau_{dt+1})(\phi_1 y_{bt+1} + \phi_2 y_{It+1})/k_{It+1}] \end{aligned}$$

given initial conditions for capital. Finally, we use the production technologies along with outputs, capital stocks, and hours to determine the time series for TFPs.

[14] For more details on how we construct the observable time series, see Appendix A.

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Figure 1

U.S. and Standard Model Per Capita Hours Worked

Indexed, 1990=100, Annually, 1990–2003

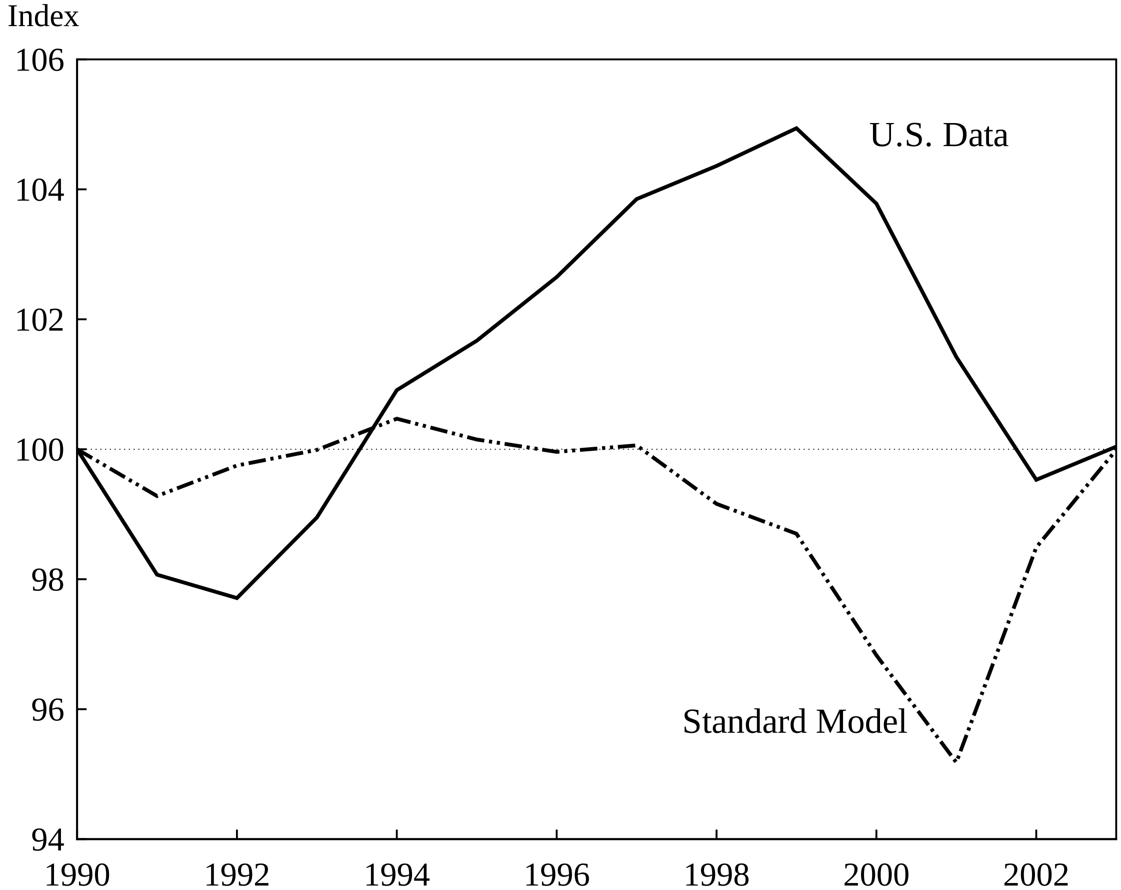

Figure 2

Average Weekly Hours Worked by Noninstitutional Population, Aged 16–64, and NIPA Real, Detrended Compensation Per Hour

Indexed, 1990=100, Annually, 1990–2003

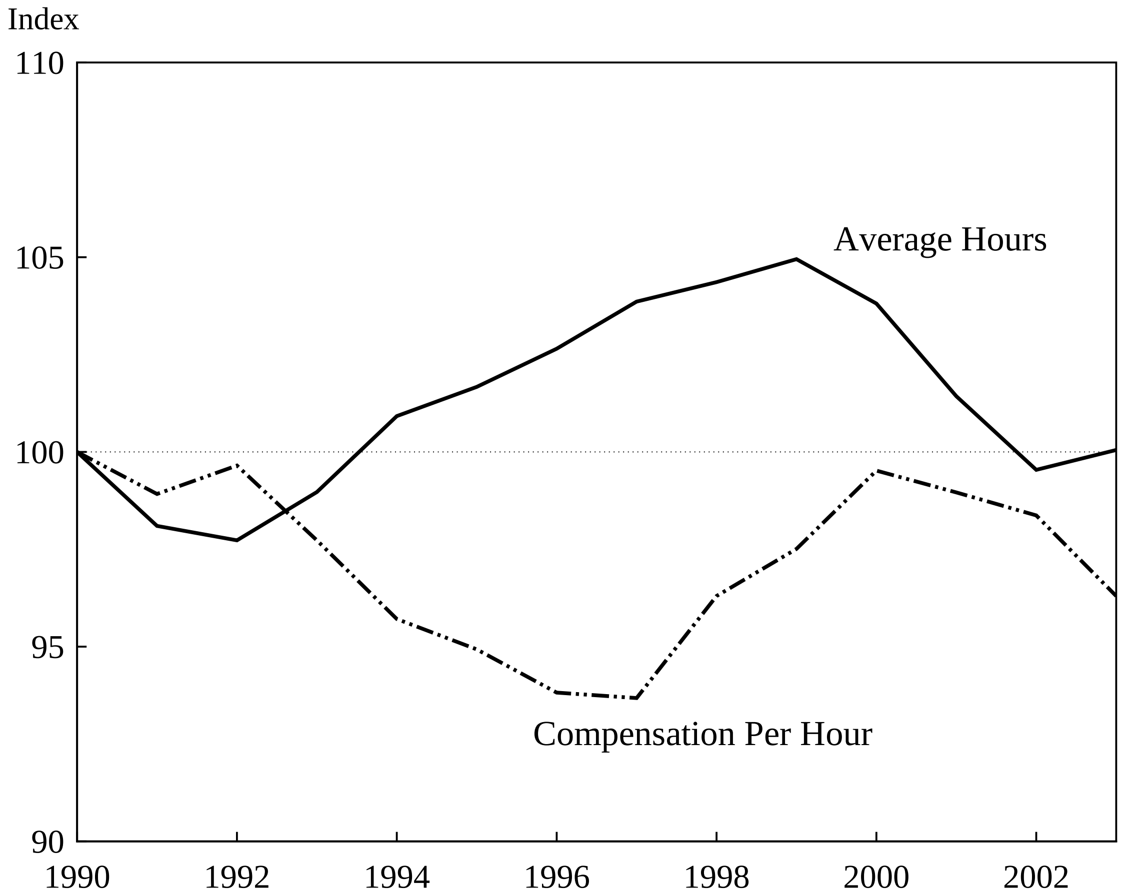

Figure 3

NIPA Real, Detrended GDP and Corporate Profits

Indexed, 1990=100, Annually, 1990–2003

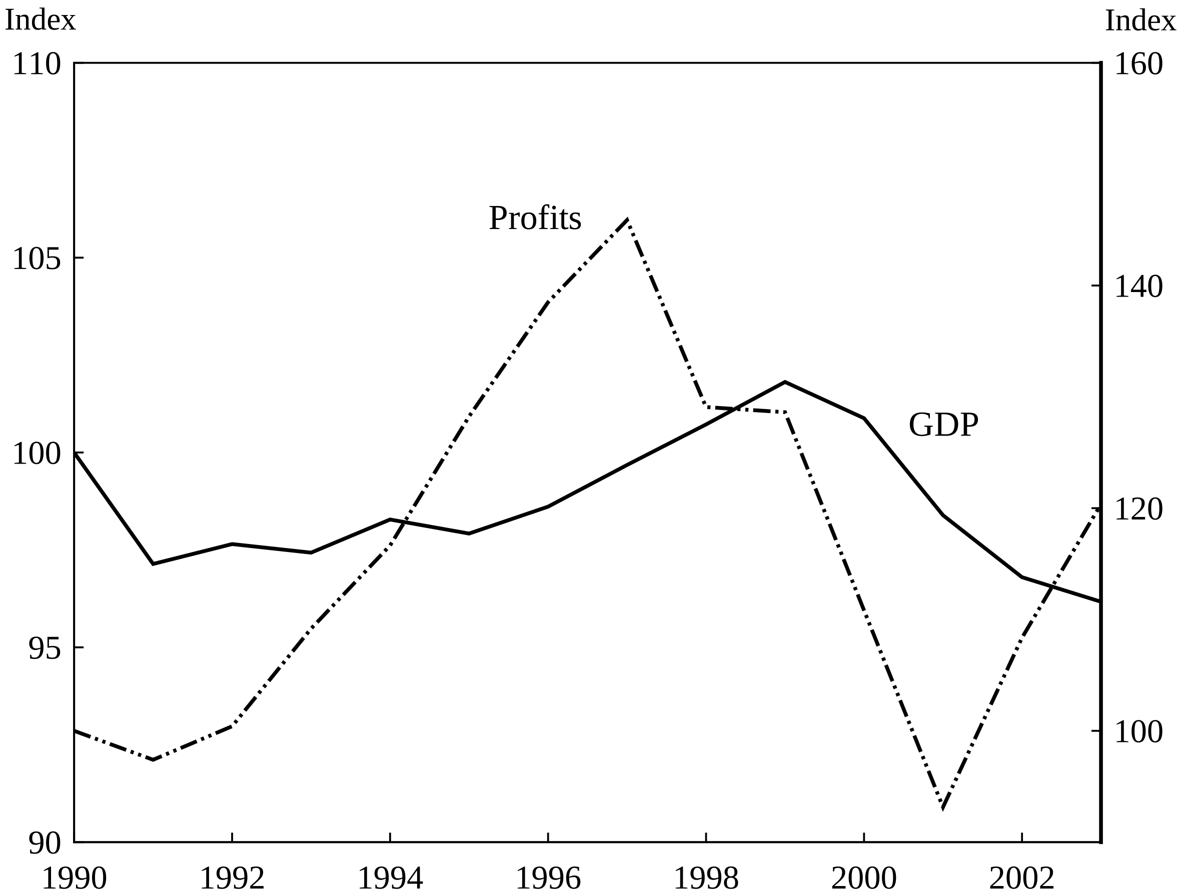

Figure 4

Extended Model Hours Producing Intangible Capital

% of All Hours Worked, Annually, 1990–2003

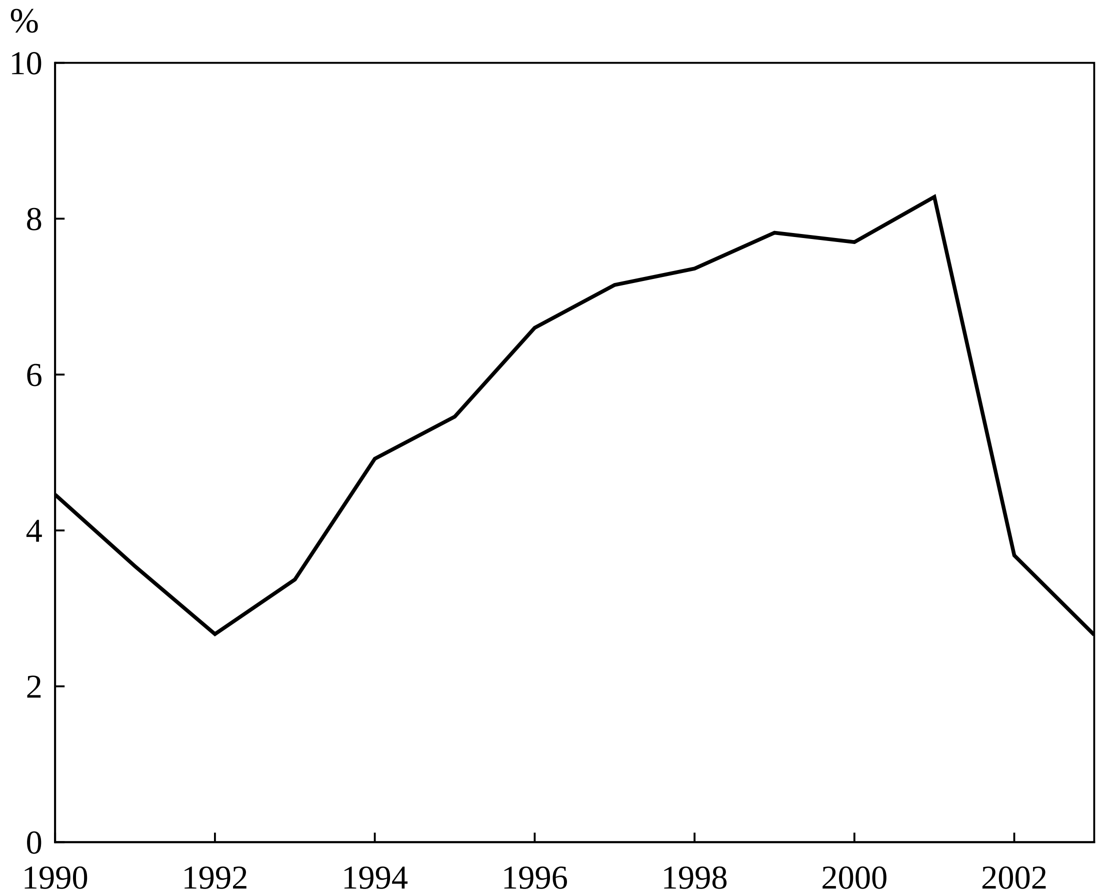

Figure 5

U.S. and Extended Model Total Factor Productivity

Real and Detrended, Indexed, 1990=100, Annually, 1990–2003

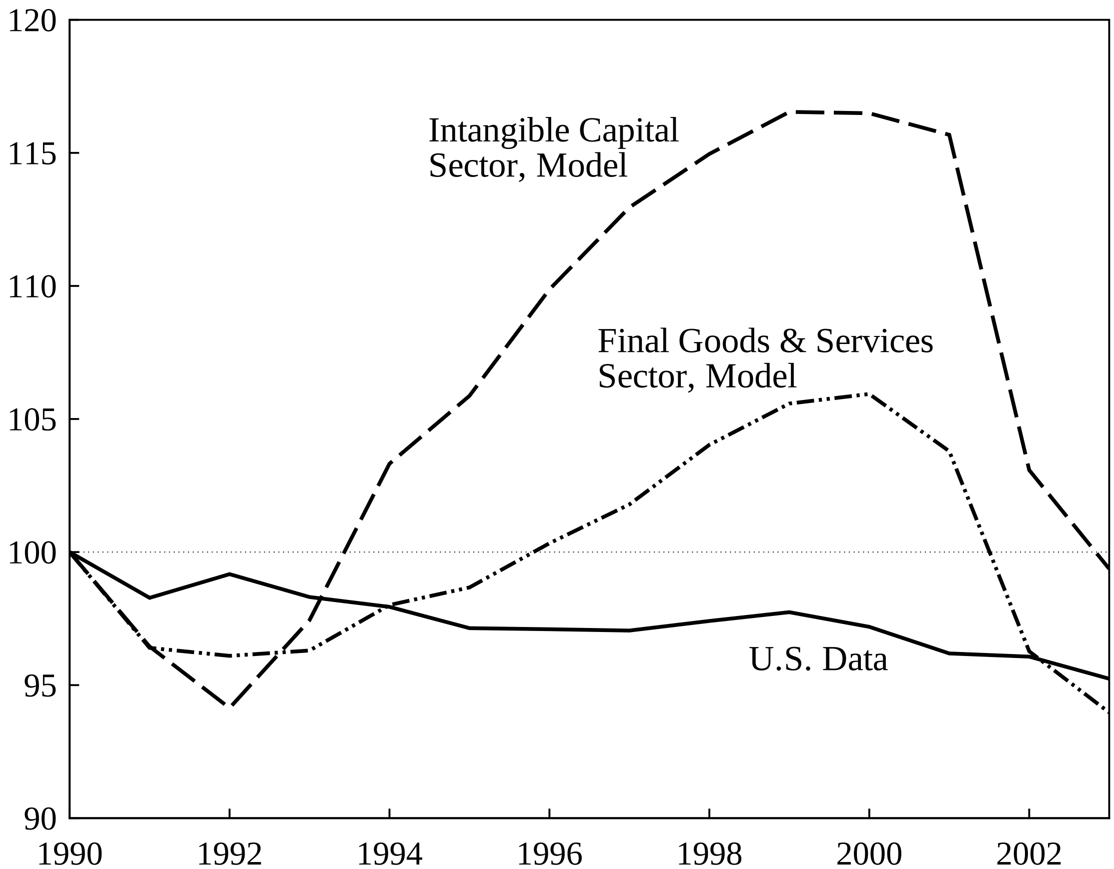

Figure 6

U.S. and Extended Model Per Capita Hours Worked

Indexed, 1990=100, Annually, 1990–2003

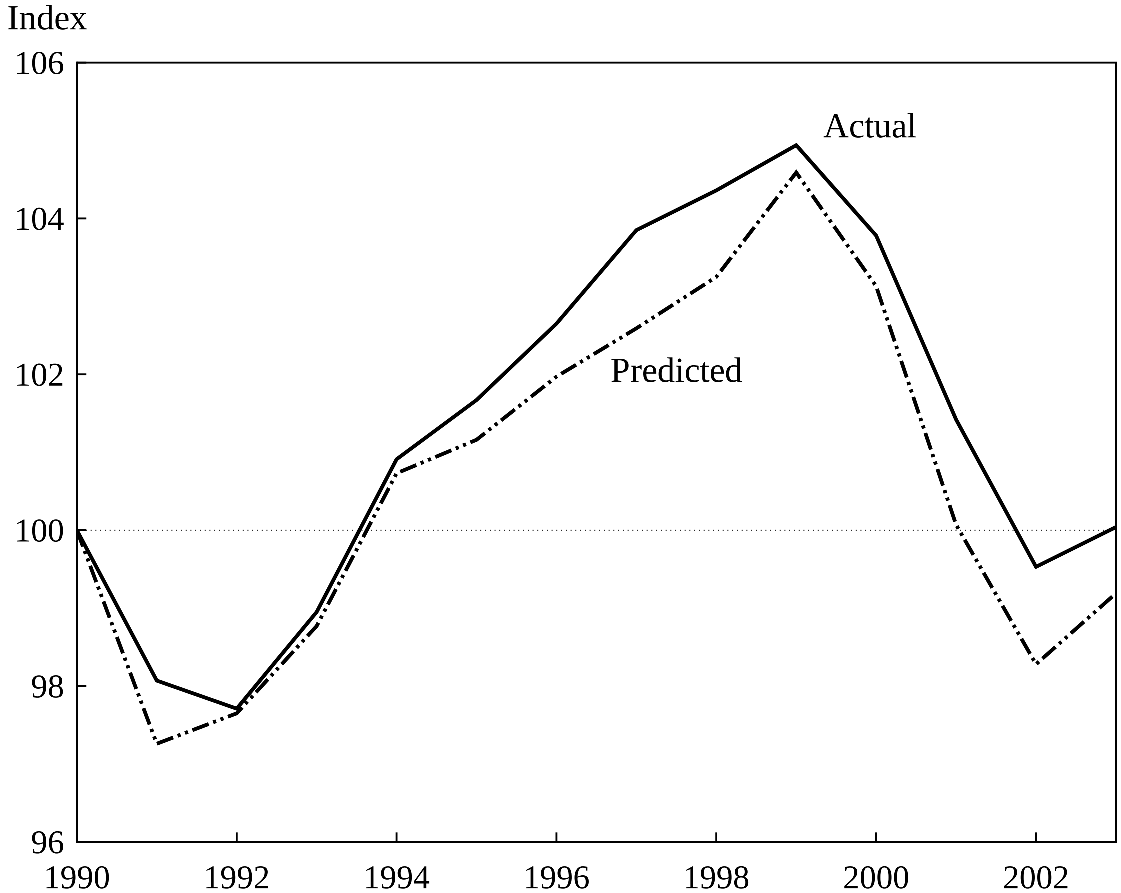

Figure 7

U.S. and Extended Model Business Labor Productivity

Real and Detrended, Indexed, 1990=100, Annually, 1990–2003

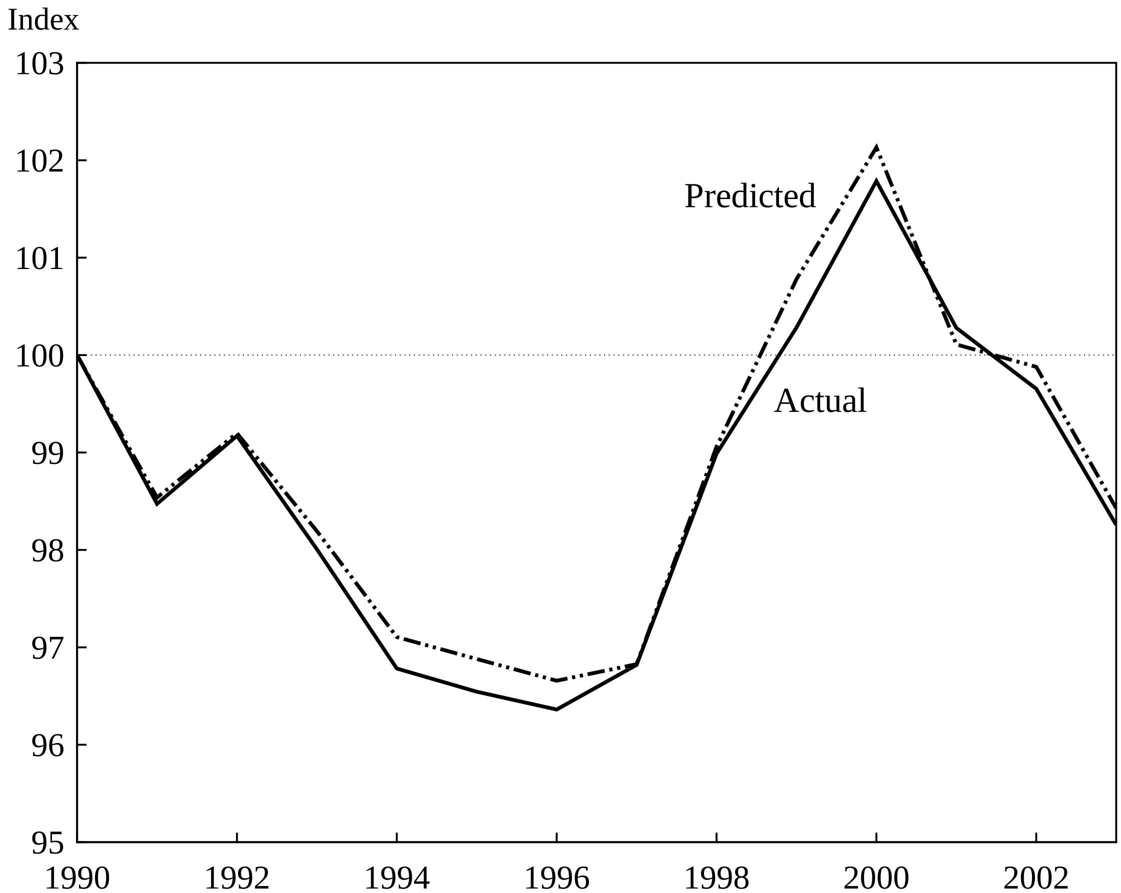

Figure 8

U.S. and Extended Model Tangible Investment

Real and Detrended, Indexed, 1990=100, Annually, 1990–2003

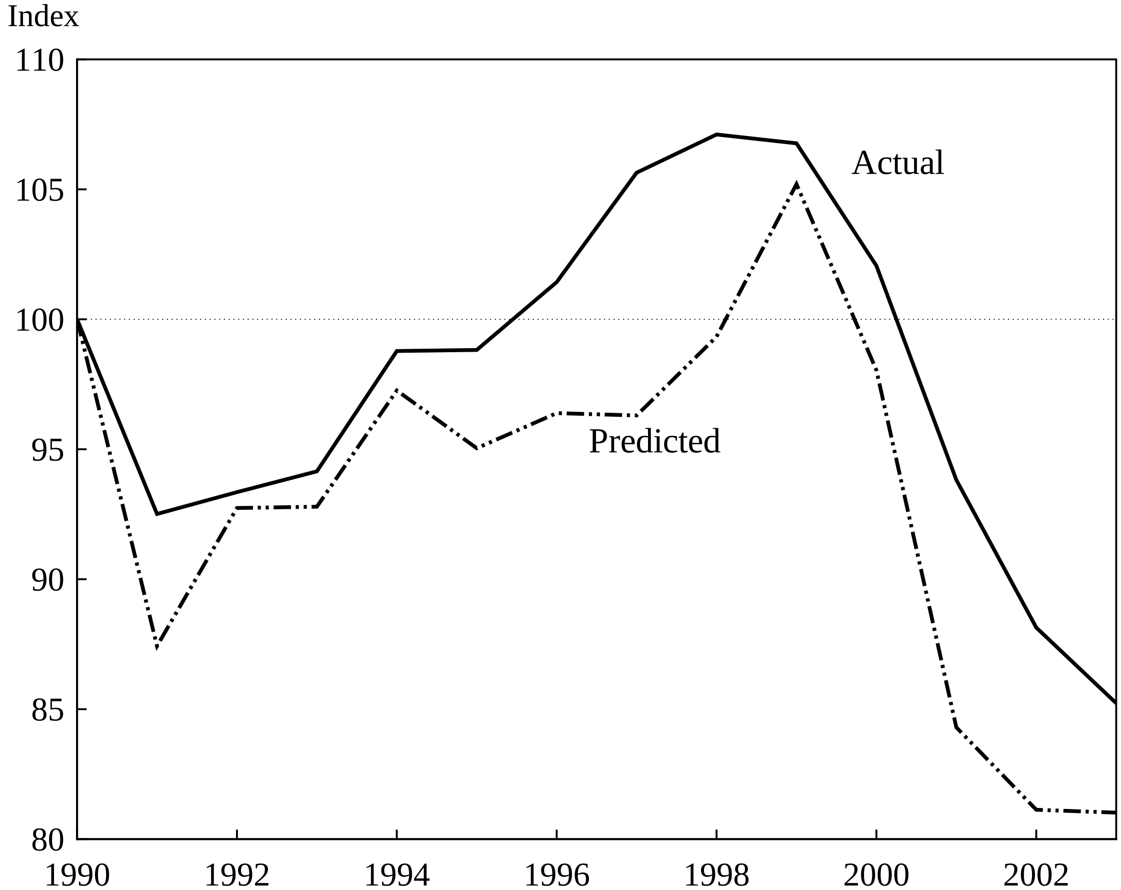

Figure 9

U.S. and Extended Model Business Compensation Less Sweat

Real and Detrended, Indexed, 1990=100, Annually, 1990–2003

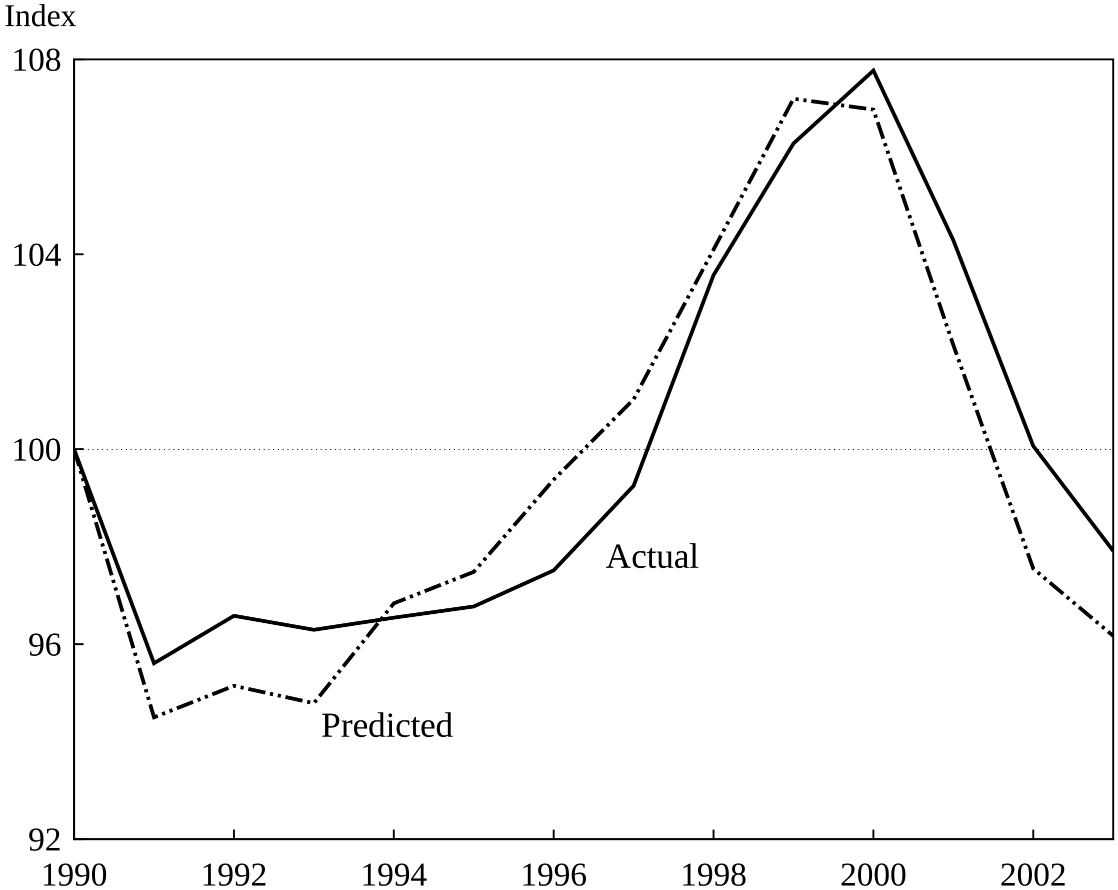

Figure 10

U.S. Household Real Holding Gains

Excludes Real Estate, % of Real GDP, Annually, 1953–2003

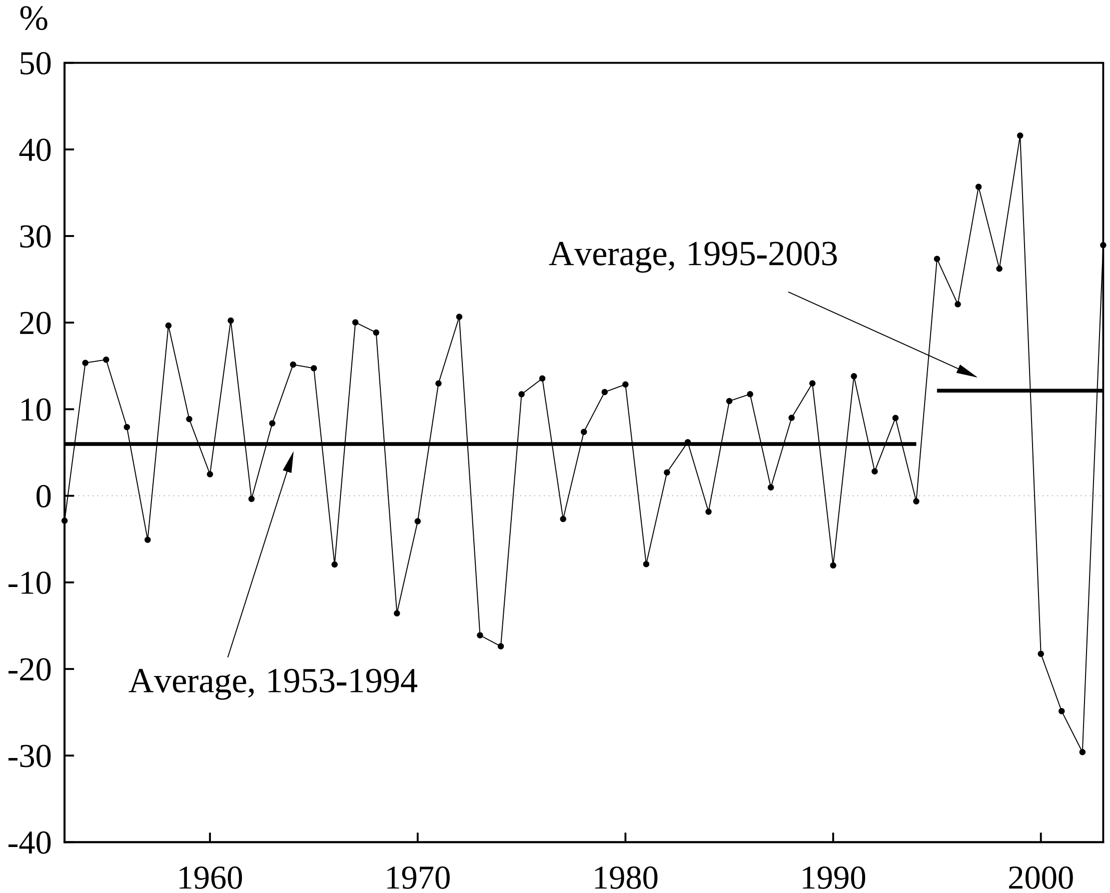

Figure 11

U.S. and Extended Model Real Holding Gains

U.S. Averaged and Excludes Real Estate, % of Real GDP, Annually, 1990–2003

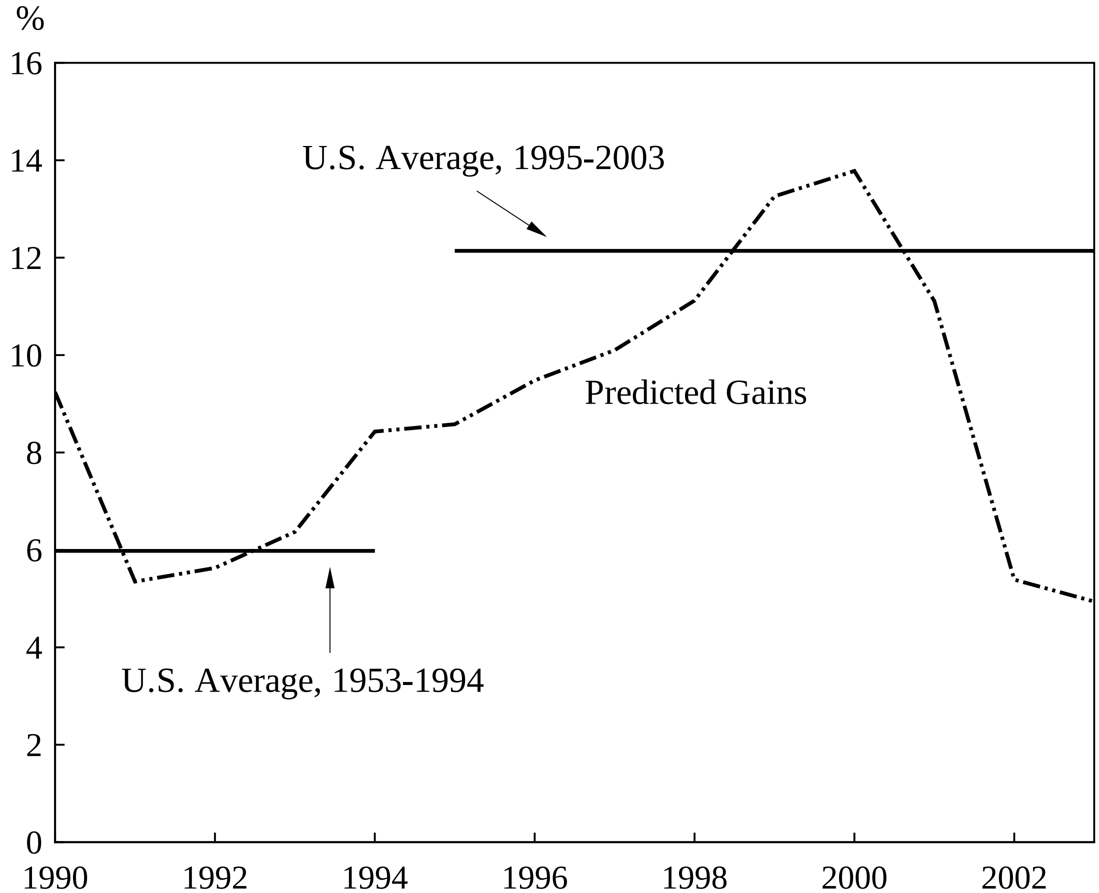

Figure 12

Extended Model Total and Measured Output

Real and Detrended, Indexed, 1990=100, Annually, 1990–2003

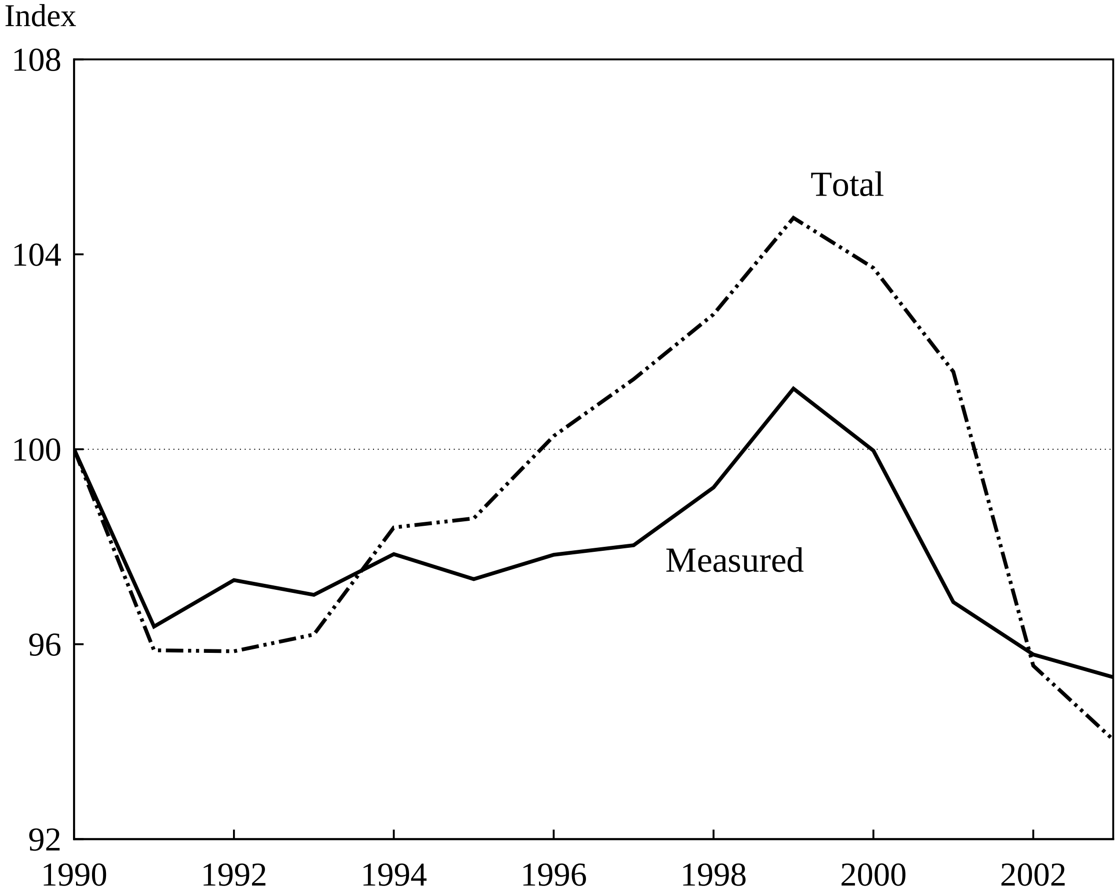

Figure 13

Extended Model Total and Measured
Business Labor Productivity

Real and Detrended, Indexed, 1990=100, Annually, 1990–2003

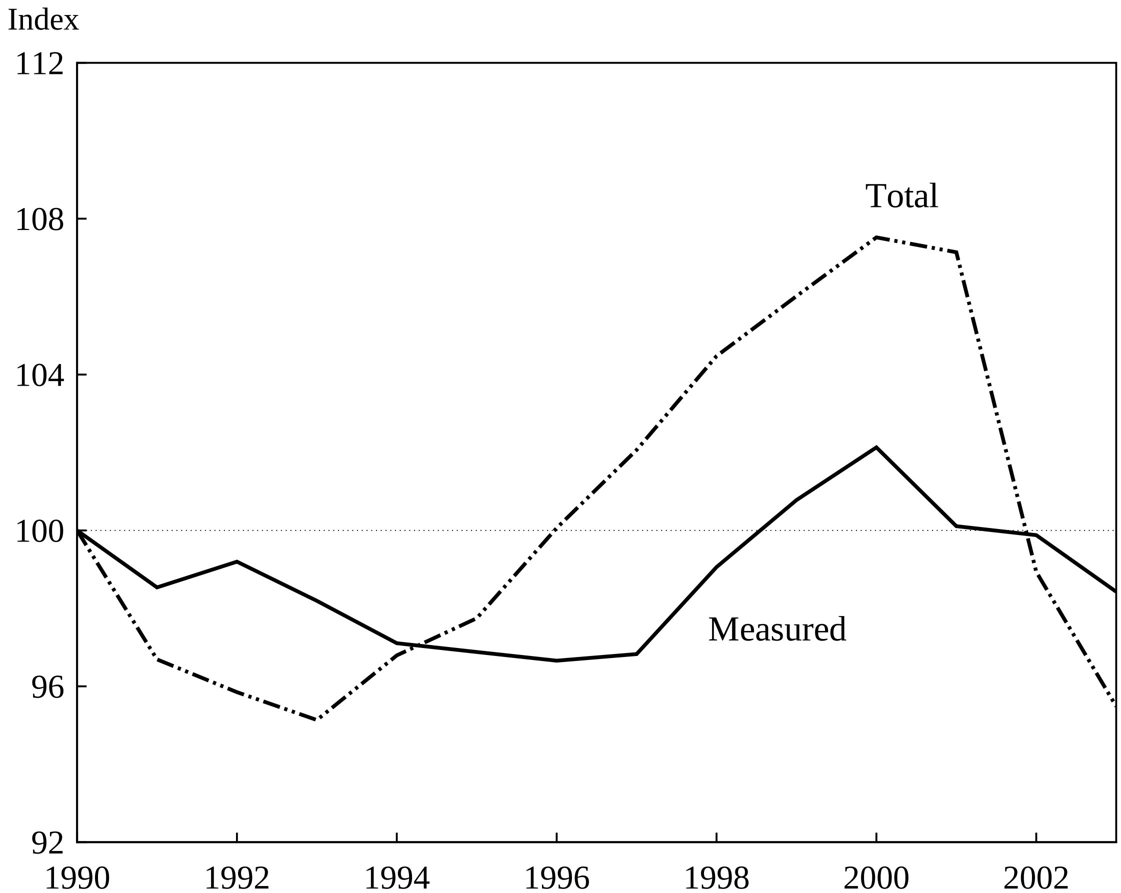

Figure 14

Extended Model Total and Measured Investment

Real and Detrended, Indexed, 1990=100, Annually, 1990–2003

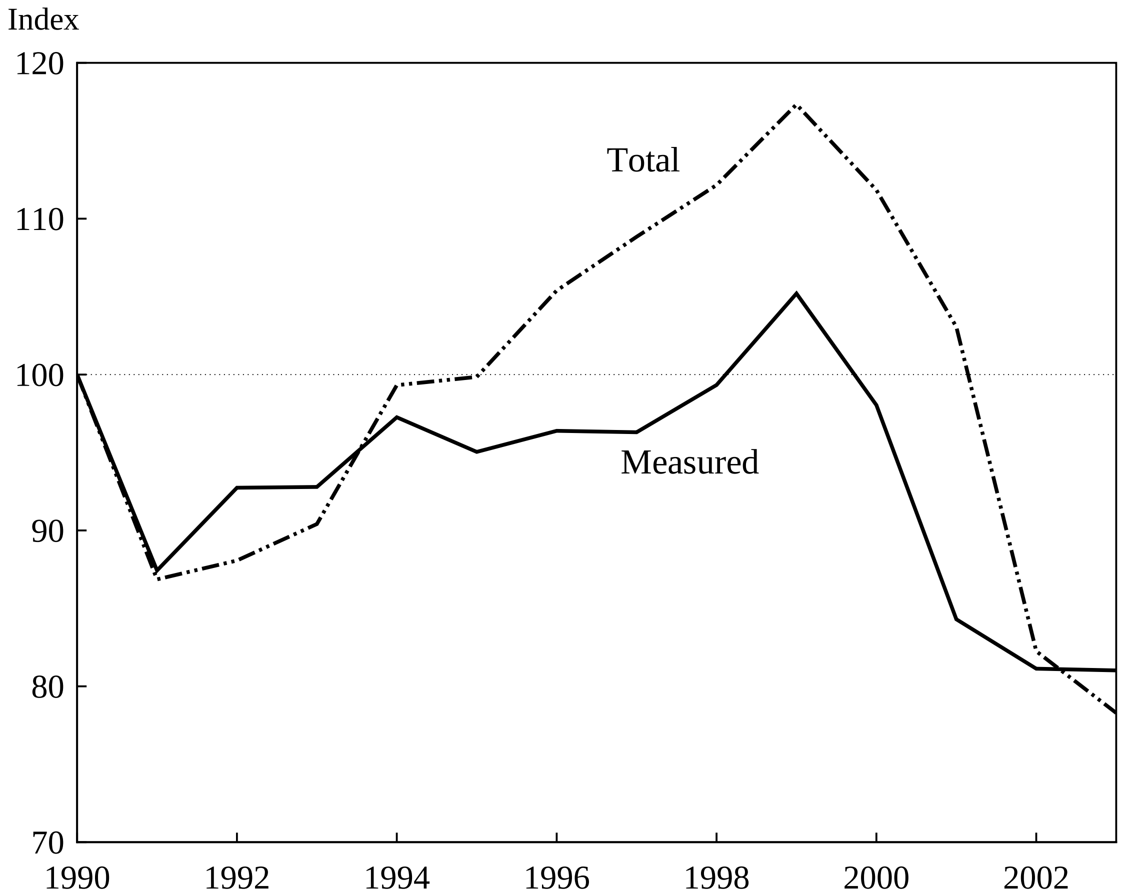

Figure 15

Extended Model Intangible Share of Total Output

Annually, 1990–2003