

NBER WORKING PAPER SERIES

ON OVERBORROWING

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Working Paper 11913

<http://www.nber.org/papers/w11913>

NATIONAL BUREAU OF ECONOMIC RESEARCH

1050 Massachusetts Avenue

Cambridge, MA 02138

January 2006

I would like to thank Stephanie Schmitt-Grohé for comments and discussions and Arthur X. Liu and Javier Garcia-Cicco for research assistance. Newer versions of this paper are maintained at [www.econ.duke.edu/~uribe](http://www.econ.duke.edu/~uribe). The views expressed herein are those of the author(s) and do not necessarily reflect the views of the National Bureau of Economic Research.

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On Overborrowing  
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NBER Working Paper No. 11913  
January 2006, Revised March 2006  
JEL No. F4

**ABSTRACT**

This paper characterizes the equilibrium dynamics in an economy facing an aggregate debt ceiling. This borrowing limit is intended to capture an environment in which foreign investors base their lending decisions predominantly upon macro indicators. Individual agents do not internalize the borrowing constraint. Instead, a country interest-rate premium emerges to clear the financial market. The implied equilibrium dynamics are compared to those arising from a model in which the debt ceiling is imposed at the level of each individual agent. The central finding of the paper is that the economy with the aggregate borrowing limit does not generate higher levels of debt than the economy with the individual borrowing limit. That is, there is no overborrowing in equilibrium.

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# 1 Introduction

A central question in emerging-market macroeconomics is what factors lead countries to accumulate excessive levels of external debt. It is often argued by economic observers and policymakers that emerging markets tend to overborrow when the lending decisions of foreign financial institutions are guided by rough indicators of the emerging country's macroeconomic performance and not by careful assessment of individual borrowers' abilities to repay. This is because individual agents fail to internalize the effect their own borrowing decisions have on the country's aggregate credit conditions. Overborrowing, it is argued, makes emerging countries prone to balance-of-payments crises, or sudden stops, and calls for government policy aimed at putting sand in the wheels of external finance. The contribution of this paper is to investigate whether the type of lending practices described above indeed lead emerging countries to overborrow.

To this end, I characterize the equilibrium dynamics of a small open economy subject to an aggregate borrowing constraint. I have in mind a situation in which foreign lenders lack the ability or the incentives to monitor individual investment projects in the emerging country and instead base their lending decisions on observation of a few macroeconomic indicators, such as total external debt or output growth. Individual agents do not internalize the credit constraint. I assume that in this economy credit rationing is implemented through a market mechanism. Specifically, when the aggregate debt limit is reached, an interest-rate premium emerges in the domestic economy that ensures that individual borrowing decisions are collectively compatible with the aggregate credit constraint. I compare the equilibrium dynamics of this economy to those of an economy in which the borrowing limit is imposed at the level of each individual agent.

The specific question that my investigation aims to address is whether the economy with the aggregate debt limit tends to overborrow relative to the economy with debt limits imposed at the level of each individual agent. I

find that there is no overborrowing in equilibrium. The reason is that in the economy with the aggregate credit constraint, market incentives, conveyed by the interest rate, induce individual saving decisions that are identical to those caused by the imposition of agent-specific debt limits.

Two features of models with debt limits are key in generating no overborrowing. One feature is that when the debt constraint is internalized the opportunity cost of funds is independent of the household's debt position, even for levels of debt arbitrarily close to the ceiling. Only when the debt constraint is binding does the shadow cost of loans adjust upwards. The second theoretical feature that is important in generating the no-overborrowing result is that when the debt limit binds, it does so for all agents at the same time. I present two theoretical examples showing that once any of the aforementioned two features are absent aggregate debt limits may induce overborrowing in equilibrium.

The remainder of the paper is organized in five sections. Section 2 presents a simple model of a small open economy facing an aggregate borrowing ceiling. Section 3 presents an economy where the debt limit is imposed at the individual level. Section 4 establishes analytically the central result of no overborrowing. It shows that when the rents from financial rationing accrue to domestic residents, the equilibrium dynamics in the economy with the aggregate debt limit and in the economy with the individual debt limit are identical. Section 5 studies the case in which rents from financial rationing accrue to the foreign lenders. In this case, the economies with an individual and an aggregate debt limit can no longer be compared analytically. However, I establish numerically the absence of overborrowing. Section 6 puts the no-overborrowing result in perspective and explores modifications to the basic framework capable of inducing overborrowing in the presence of aggregate debt limits.

## 2 An Economy With An Aggregate Borrowing Ceiling

Consider an economy populated by a large number of identical households with preferences defined over consumption of a perishable good,  $c_t$ , and labor effort,  $h_t$ , and described by the utility function

$$E_0 \sum_{t=0}^{\infty} \theta_t U(c_t, h_t), \quad (1)$$

where  $U$  denotes the period utility function, which is assumed to be increasing in the first argument, decreasing in the second argument, strictly concave, and twice continuously differentiable. With the purpose of ensuring stationary equilibrium dynamics, I adopt a standard practice in modeling small open economies by assuming that the subjective rate of discount is a function of endogenous variables (see, for instance, Schmitt-Grohé and Uribe, 2003, and the references cited therein). Specifically, I let  $\theta_0 = 1$  and  $\theta_t/\theta_{t-1} = \beta(C_t, H_t)$ , where  $C_t$  and  $H_t$  denote, respectively, aggregate consumption and hours worked, and  $\beta$  is a function assumed to be decreasing in its first argument and increasing in its second argument. The household takes the evolution of  $C_t$  and  $H_t$  as given. The choice of aggregate variables as arguments of the discount factor simplifies the household's optimality conditions. It will become clear, however, that the central result of this paper is robust to assuming that the discount factor is a function of the individual levels of consumption and effort.

Output, denoted  $y_t$ , is produced with a technology that takes labor as the only input. Production is subject to an aggregate, stochastic, stationary productivity shock denoted by  $z_t$ . Formally,  $y_t = e^{z_t} F(h_t)$ . The production function  $F$  is assumed to be positive, strictly increasing, and strictly concave. Allowing for capital accumulation would not alter the main results of the paper.

The only financial asset available to households is a risk-free international bond. Letting  $a_t$  denote the household's debt due in period  $t$ , its sequential budget constraint is given by

$$\frac{a_{t+1}}{R_t} = a_t + c_t - e^{z_t} F(h_t), \quad (2)$$

where  $R_t$  denotes the gross interest rate on assets held between periods  $t$  and  $t + 1$ . Households are assumed to be subject to a no-Ponzi-game constraint of the form  $\lim_{j \rightarrow \infty} E_t \frac{a_{t+j+1}}{\prod_{s=0}^j R_{t+s}} \geq 0$ .

The household's problem consists in choosing contingent plans  $c_t$ ,  $h_t$ , and  $a_{t+1}$  so as to maximize (1) subject to (2) and the no-Ponzi-game constraint, given the processes  $R_t$  and  $z_t$  and the initial condition  $a_0$ . The first-order conditions associated with this problem are (2), the no-Ponzi-game constraint holding with equality, and

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = e^{z_t} F'(h_t), \quad (3)$$

$$U_c(c_t, h_t) = \beta(C_t, H_t) R_t E_t U_c(c_{t+1}, h_{t+1}).$$

Foreign lenders impose an aggregate borrowing limit on the domestic economy, which stipulates that the aggregate per capita level of external liabilities assumed by the country in any period  $t \geq 0$ , which I denote by  $A_{t+1}$ , be no greater than a ceiling  $\kappa > 0$ . That is,

$$A_{t+1} \leq \kappa.$$

Foreign lenders take the evolution of the country's external debt  $A_t$  as given. They interpret this variable as an indicator of the strength of the country's fundamentals and are willing to lend funds to domestic residents without restrictions as long as the country's external debt is below the threshold  $\kappa$ . Individual domestic households also take the evolution of  $A_t$  as exogenous.

In periods in which the aggregate borrowing ceiling is not binding, for-

eign investors lend to domestic residents at the world interest rate, which is assumed to be constant and equal to  $R^* > 1$ . When the aggregate borrowing limit is binding, the domestic interest rate may adjust upward to ensure market clearing in the domestic financial market. In this case the economy faces a country interest-rate premium, equal to  $R_t - R^*$ . It follows that  $R_t$  must satisfy  $R_t \geq R^*$  and  $(R_t - R^*)(A_{t+1} - \kappa) = 0$ .

## 2.1 The Rents From Financial Rationing

When the domestic interest rate,  $R_t$ , is above the world interest rate,  $R^*$ , a financial rent is generated. Values of  $R_t$  above  $R^*$  create pure rents because in this economy there is no default in equilibrium by assumption. The precise way in which these rents are allocated will in general have consequences for aggregate dynamics. Here, I consider two polar cases. In one case, all financial rents accrue to the foreign lenders. In the other case, financial rents accrue entirely to domestic residents.

When financial rents are appropriated by nonresidents, increases in the domestic interest rate entail a resource cost to the domestic economy as a whole. This cost is reflected in an aggregate resource constraint of the form  $A_{t+1}/R_t = A_t + C_t - e^{z_t} F(H_t)$ . Note that this expression features the domestic interest rate,  $R_t$ , instead of the world interest rate,  $R^*$ .

Alternatively, rents from credit rationing could accrue entirely to domestic residents. This case arises when, possibly because of competition among foreign lenders, domestic financial institutions borrow in the world financial market at the rate  $R^*$ . Thus, the country interest-rate premium represents a net rent to domestic financial intermediaries. I assume that these rents are distributed in a lump-sum fashion among domestic households, who own the domestic financial institutions in equal shares. In this case, the existence of an interest-rate premium does not introduce a resource cost to the domestic economy. The aggregate resource constraint is therefore given by  $A_{t+1}/R^* = A_t + C_t - e^{z_t} F(H_t)$ . Note that this expression features the world interest rate,

$R^*$ , instead of the domestic interest rate,  $R_t$ .

Because households are homogeneous, in equilibrium we have that individual and aggregate per capita variables are identical; thus  $C_t = c_t$ ,  $H_t = h_t$ , and  $A_t = a_t$ . We are ready to provide definitions of competitive equilibria when financial rents accrue to foreign lenders and when financial rents accrue to domestic residents:

**Definition 1 (Equilibrium When Rents Accrue Domestically)** *A stationary competitive equilibrium under an aggregate borrowing ceiling when rents from financial rationing accrue to domestic residents is a set of stationary stochastic processes  $\{c_t, h_t, a_{t+1}, R_t\}_{t=0}^{\infty}$  satisfying*

$$U_c(c_t, h_t) = \beta(c_t, h_t) R_t E_t U_c(c_{t+1}, h_{t+1}), \quad (4)$$

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = e^{z_t} F'(h_t), \quad (5)$$

$$R_t \geq R^*, \quad (6)$$

$$a_{t+1} \leq \kappa, \quad (7)$$

$$(R_t - R^*)(a_{t+1} - \kappa) = 0, \quad (8)$$

$$\frac{a_{t+1}}{R^*} = a_t + c_t - e^{z_t} F(h_t), \quad (9)$$

given the process  $\{z_t\}_{t=0}^{\infty}$  and the initial condition  $a_0$ .

**Definition 2 (Equilibrium When Rents Accrue to Foreigners)** *A stationary competitive equilibrium under an aggregate borrowing ceiling when rents from financial rationing accrue to foreign lenders is a set of stationary stochastic processes  $\{c_t, h_t, a_{t+1}, R_t\}_{t=0}^{\infty}$  satisfying conditions (4)-(8) and the resource constraint*

$$\frac{a_{t+1}}{R_t} = a_t + c_t - e^{z_t} F(h_t), \quad (10)$$

given the process  $\{z_t\}_{t=0}^{\infty}$  and the initial condition  $a_0$ .



I postpone the characterization of equilibrium in these economies until I described equilibrium in an economy with an internalized borrowing limit.

### 3 An Economy With An Individual Borrowing Ceiling

Suppose now that lenders impose a debt ceiling at the level of each individual household. That is,

$$a_{t+1} \leq \kappa. \tag{11}$$

Unlike in the economy described in the previous section, in this economy domestic agents internalize the borrowing constraint. Therefore, they will take this constraint into account in solving their intertemporal optimization problem. Accordingly, the household problem consists in maximizing (1) subject to (2) and (11). The optimality conditions of this problem consist of (2), (3), (11), and

$$U_c(c_t, h_t) \left[ \frac{1}{R_t} - \xi_t \right] = \beta(C_t, H_t) E_t U_c(c_{t+1}, h_{t+1}),$$

$$\xi_t \geq 0,$$

$$(a_{t+1} - \kappa) \xi_t = 0,$$

where  $\xi_t$  denotes the Lagrange multiplier associated with the debt constraint (11) divided by the marginal utility of consumption. When the debt ceiling is binding,  $\xi_t$  is strictly positive, and the household faces an effective (shadow) interest rate given by  $\tilde{R}_t \equiv R_t / (1 - R_t \xi_t)$ , which is greater than the market interest rate  $R_t$ . This effective interest rate reflects the fact that at the market interest rate the household would like to borrow beyond the limit  $\kappa$ .

Foreign lenders supply funds to domestic residents at the world interest

rate,  $R^*$ . Therefore,  $R_t$  equals  $R^*$  at all dates and states. The following definition of a competitive equilibrium then applies:

**Definition 3 (Equilibrium With An Individual Debt Ceiling)** *A stationary competitive equilibrium under an individual debt ceiling is a set of stationary stochastic processes  $\{c_t, h_t, a_{t+1}, \xi_t\}_{t=0}^\infty$  satisfying*

$$U_c(c_t, h_t) \left[ \frac{1}{R^*} - \xi_t \right] = \beta(c_t, h_t) E_t U_c(c_{t+1}, h_{t+1}), \quad (12)$$

$$-\frac{U_h(c_t, h_t)}{U_c(c_t, h_t)} = e^{z_t} F'(h_t), \quad (13)$$

$$\xi_t \geq 0, \quad (14)$$

$$a_{t+1} \leq \kappa, \quad (15)$$

$$(a_{t+1} - \kappa)\xi_t = 0. \quad (16)$$

$$\frac{a_{t+1}}{R^*} = a_t + c_t - e^{z_t} F(h_t), \quad (17)$$

given the process  $\{z_t\}_{t=0}^\infty$  and the initial condition  $a_0$ .

We are ready to compare equilibrium dynamics under aggregate and individual debt limits.

## 4 An Equivalence Result

In this section, I show that the equilibrium processes for debt, consumption, hours, and output in the economy with an individual debt ceiling and in the economy with an aggregate debt ceiling with rents from financial rationing accruing to domestic households are identical. To see this, consider the economy with an individual debt constraint. Definition 3 lists the equilibrium conditions corresponding to this economy. Equations (12) and (14) together with the fact that  $U_c(c_t, h_t) > 0$  imply that  $\xi_t \in [0, 1/R^*]$ . Define  $R_t = R^*/(1 - R^*\xi_t)$ . Clearly,  $\xi_t > 0$  iff  $R_t > R^*$ , and  $\xi_t = 0$  iff  $R_t = R^*$ . With these

results in mind, use the definition of  $R_t$  to eliminate  $\xi_t$  from the equilibrium conditions (12)-(17). It follows immediately that the resulting expressions are identical to the equilibrium conditions pertaining to the economy with an aggregate debt limit and rents accruing to domestic households, given by equations (4)-(9).

We conclude that in the simple economic environment studied here, the practice by foreign investors of basing their lending decisions on macroeconomic indicators—as opposed to individual solvency indicators—does not induce overborrowing. The individual incentives created by the market (i.e., by  $R_t$ ) in the economy with the aggregate debt limit are exactly the same as those emerging from an individual debt limit. Formally, as is clear in the derivation of the equivalence result, the market and social price of external funds are identical in the economy with the aggregate debt limit. The following proposition summarizes the no-overborrowing result:

**Proposition 1 (No Overborrowing)** *The equilibrium dynamics of  $c_t$ ,  $h_t$ ,  $y_t$ , and  $a_t$  are identical in the economy with an individual debt limit and in the economy with an aggregate debt limit with rents from financial rationing accruing to domestic households.*

This proposition is robust to a number of modifications of the basic model within which it was derived. For instance, it can be shown that the equivalence result continues to hold in the context of an economy with capital accumulation. The result can also be shown to be robust to alternative specifications of the discount factor. In particular, when the discount factor is assumed to depend on the individual levels of consumption and effort, as opposed to aggregate measures of these variables. Enriching the sources of uncertainty to include shocks to endowments, tastes, or the world interest rate would also leave the no-overborrowing result unaltered.

The no-overborrowing result stated in proposition 1 contrasts sharply with the findings of Fernández-Arias and Lombardo (1998). These authors conclude that when agents fail to internalize the debt limit, the economy

tends to overborrow. The structure of the model economy used by Fernández-Arias and Lombardo is similar to the one presented here, with three nonessential differences. Namely, their model is cast in perfect foresight and in continuous time, and output is assumed to take the form of an exogenous endowment. The central difference between the Fernández-Arias and Lombardo model and the one I study here has to do with the mechanism through which credit rationing is brought about in the economy with an aggregate debt limit. In the formulation I adopt in this paper, credit rationing is implemented through a market mechanism. The interest rate,  $R_t$ , adjusts to induce agents to borrow an amount that in the aggregate is in line with the credit limit imposed on the country as a whole. In the Fernández-Arias and Lombardo model, credit rationing is not implemented through the price system. Indeed, they assume that the domestic interest rate is always equal to the world interest rate ( $R_t = R^*, \forall t$ ). Instead they impose a credit constraint of the type  $a_t \leq a_\tau, t \geq \tau$ , at the level of each individual household, where  $\tau$  is the date at which the aggregate borrowing constraint becomes binding, which is known under perfect foresight. Agents do not internalize the fact that in equilibrium  $a_\tau$  must equal  $\kappa$ . Note that in the Fernández-Arias and Lombardo model agents internalize a substantial part of the credit limit, namely the fact that individual debts cannot grow beyond  $a_\tau$  after time  $\tau$ . The only aspect of the debt ceiling agents do not internalize is the ceiling  $\kappa$  itself. In the formulation adopted in the present paper, by contrast, agents do not internalize any component of the credit limit. They borrow and lend freely at the interest rate  $R_t$  (subject, of course, to the standard no-Ponzi-game constraint).

## 5 Resource Costs

When rents from financial rationing are appropriated by foreign lenders, it is no longer possible to compare analytically the dynamics of external debt

in the economies with the aggregate debt limit and in the economy with the individual debt limit. I therefore resort to numerical methods to characterize competitive equilibria.

To this end, I adopt the following functional forms for preferences and technologies:  $U(c, h) = [c - \omega^{-1}h^\omega]^{1-\sigma} / (1-\sigma)$ ,  $\beta(c, h) = [1 + c - \omega^{-1}h^\omega]^{-\psi}$ , and  $F(h) = k^\alpha h^{1-\alpha}$ , where  $\sigma$ ,  $\omega$ ,  $\psi$ ,  $k$ , and  $\alpha$  are fixed parameters. Table 1 displays the values I assign to these parameters. The time unit is meant to

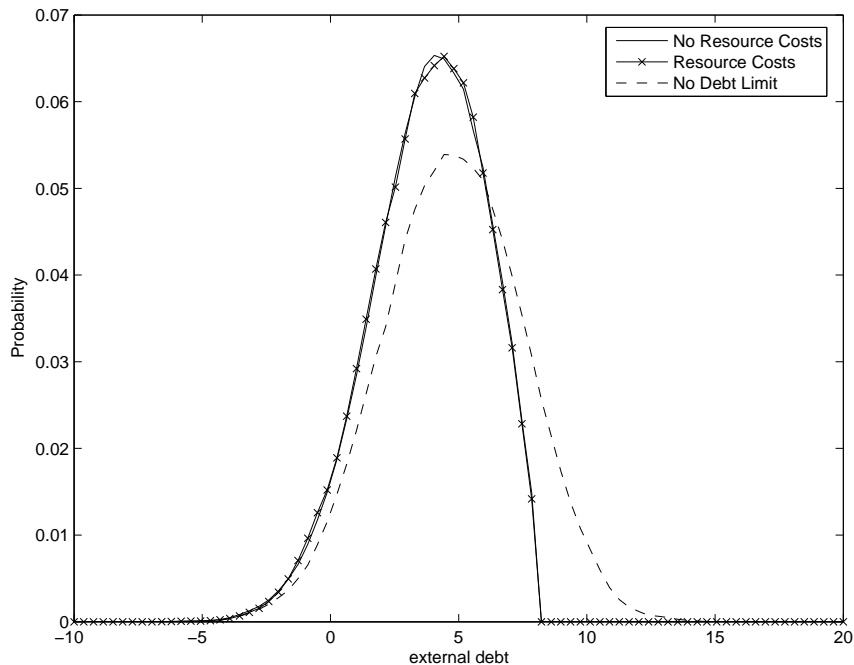
Table 1: Parameter Values

$\sigma$	$\omega$	$\psi$	$\alpha$	$R^*$	$\kappa$	$k^*$	$\pi_{HH} = \pi_{LL}$	$z^H = -z^L$
2	1.455	0.0222	0.32	1.04	7.83	78.3	0.71	0.0258

be one year. The values for  $\alpha$ ,  $\omega$ ,  $\sigma$ , and  $R^*$  are taken from Schmitt-Grohé and Uribe (2003). I set the parameter  $\psi$  so as to induce a debt-to-GDP ratio,  $a/y$ , of 50 percent in the deterministic steady state. The calibrated value of  $\kappa$  is such that in the economy without the debt limit, the probability that  $a_t$  is larger than  $\kappa$  is about 15 percent. The value assigned to the parameter  $k$  ensures that, if  $k$  is interpreted as a factor of production that is in fixed aggregate supply (such as land), then its market price in the deterministic steady state is unity. The productivity shock is assumed to follow a two-state symmetric Markov process with mean zero. Formally,  $z_t$  takes on values from the set  $\{z^1, z^2\}$  with transition probability matrix  $\pi$ , and  $z^1$ ,  $z^2$ , and  $\pi$  satisfy  $z^1 = -z^2$  and  $\pi_{11} = \pi_{22}$ . I set  $\pi_{11}$  equal to 0.71 and  $z^1$  equal to 0.0258. This process displays the same serial correlation (0.58) and twice as large a standard deviation (2.58 percent) as the one estimated for Canada by Mendoza (1991). My choice of a process for the productivity shock that is twice as volatile as the one observed in a developed small open economy like Canada reflects the view that to a first approximation what distinguishes business cycles in developed and developing countries is that the latter are about twice as volatile as the former (Kydland and Zarazaga, 1997).

I solve the model using the Chebyshev Parameterized Expectations method.

Figure 1: Equilibrium Distribution of External Debt



The state spaced is discretized using 1000 points for the stock of debt,  $a_t$ . The parameterization of expectations uses 50 coefficients. I compute the equilibrium for three model economies: An economy with no debt limit, an economy with a debt limit and financial rents accruing to domestic residents, and an economy with a debt limit and financial rents flowing abroad. The procedure approximates the equilibrium with reasonable accuracy. The DenHaan-Marcet test for 5-percent left and right tails yields, respectively, (0.047,0.046), (0.043,0.056), and (0.048,0.056). In conducting this test, I use 1000 simulations of 5000 periods (years) each, dropping the first 1000 periods. The Matlab code that implements the numerical results reported in this section are available on my website.

Figure 1 displays with a solid line the equilibrium probability distribution of external debt in the economy with an aggregate debt limit and financial rents from rationing accruing to domestic agents. According to proposi-

tion 1, this line coincides with the debt distribution in the economy with a household-specific debt limit. The figure shows with a dash-crossed line the distribution of debt in the economy with an aggregate debt limit and financial rents accruing to foreign lenders. As a reference, the figure also displays, with a dashed line, the debt distribution in an economy without a debt limit. The main result conveyed by the figure is that the distribution of debt in the economy with a debt limit is virtually unaffected by whether financial rents are assumed to flow abroad or stay within the country's limits.

The resource costs due to financial rents remitted abroad in the economy in which this rents belong to foreigners are fairly small. This implication is the result of two characteristics of the equilibrium. First, the economy seldom hits the debt limit. In effect, even though  $\kappa$  is calibrated to create a right tail of 15 percent probability in the debt distribution pertaining to the economy with no debt limit, in the economies with a debt limit this constraint binds less than once every one hundred years. This shift to the left in the distribution of debt is due to increased precautionary savings aimed to mitigate the likelihood of holding too much debt in periods in which the interest rate is above the world interest rate. Second, when the debt limit does bind, it produces a country interest-rate premium of less than 2 percent on average. Because the external debt is about 40 percent of GDP in the economies with a debt limit, it follows that the cost of remitting financial rents abroad is less than 0.008 percent of GDP per year on average.

The no overborrowing result continues to hold under a more stringent debt limit. I experimented lowering the value of  $\kappa$  by 25 percent, from 7.8 to 5.9. This smaller value of the debt limit is such that in the unconstrained economy the probability that  $a_t$  is larger than  $\kappa$  is about 30 percent. Under this parameterization, I continue to find no overborrowing. Specifically, the debt distribution in the economy with an aggregate borrowing limit and rents accruing to foreign lenders is virtually identical to the distribution of debt in the economy with an aggregate debt limit and rents accruing to

domestic households, which, as stated in proposition 1, is identical to the debt distribution in the economy with an individual borrowing limit.

## 6 Interpreting the No-Overborrowing Result

The analytical derivation of the no-overborrowing result makes it clear that the reason why households do not have a larger propensity to borrow in the economy with the aggregate debt limit is that in this economy the market and social prices of international liquidity are the same. Two features of the economy studied in this paper are crucial in generating this equality. First, when the borrowing limit is internalized the shadow price of funds, given by the pseudo interest rate  $R^*/(1 - R^*\xi_t)$ , is constant and equal to the world interest rate  $R^*$  except when the debt ceiling is binding. The shadow price of funds equals the world interest rate even as households operate arbitrarily close to the debt ceiling.

Second, in the economy with the individual debt constraint, when the debt ceiling binds, it does so for all agents simultaneously. This property is a consequence of the assumption of homogeneity across economic agents.

These two features are present in more general formulations of credit constraints than the simple one I consider in this paper.

Examples of environments in which the absence of either of the above-mentioned two features causes the market price of foreign funds to be below the social price thereby inducing overborrowing are not difficult to come by. In what follows, I spell out two such examples.

### 6.1 Debt-Elastic Country Premium

In this subsection, I present an example that illustrates that overborrowing is more likely to arise when in the economy in which credit frictions are in-



ternalized the (shadow) opportunity cost of funds increases as the individual household assumes larger debt positions.

Consider the small open economy model with a debt-elastic interest rate studied in Schmitt-Grohé and Uribe (2003). Specifically, let  $R_t = R(A_{t+1})$ , with  $R' > 0$ . Because individual households take the evolution of the aggregate debt position,  $A_t$ , as exogenous, atomistic households do not internalize the dependence of the interest rate on their individual debt positions. The reason why the cost of funds is debt elastic is unspecified in this simple setting, but it could be due to the presence of default risk as in models of sovereign debt. Let  $A^*$  denote the steady-state value of debt in this economy. Then  $A^*$  must satisfy the condition

$$1 = R(A^*)\beta,$$

where  $\beta$  is a constant subjective discount factor. Assume now that the debt-elastic interest-rate schedule is imposed at the level of each individual household, so that  $R_t = R(a_{t+1})$ . Let  $A^{**}$  denote the steady-state level of external debt in this economy. It can be shown that  $A^{**}$  is determined by the condition

$$1 - \frac{A^{**}R'(A^{**})}{R(A^{**})} = R(A^{**})\beta.$$

Clearly, the fact that  $R' > 0$  implies that, if a steady state with  $A^{**}$  exists, it must be the case that

$$A^* > A^{**}.$$

That is, the economy with the financial externality generates overborrowing. We note that in the economy with the aggregate debt limit the market price of foreign funds,  $R(A_{t+1})$ , is strictly lower than the social cost of foreign funds, given by  $R(A_{t+1})/(1 - A_{t+1}R'(A_{t+1})/R(A_{t+1}))$ . This discrepancy, which is key in generating overborrowing, is absent in the economy of the previous sections.

## 6.2 Heterogeneous Agents

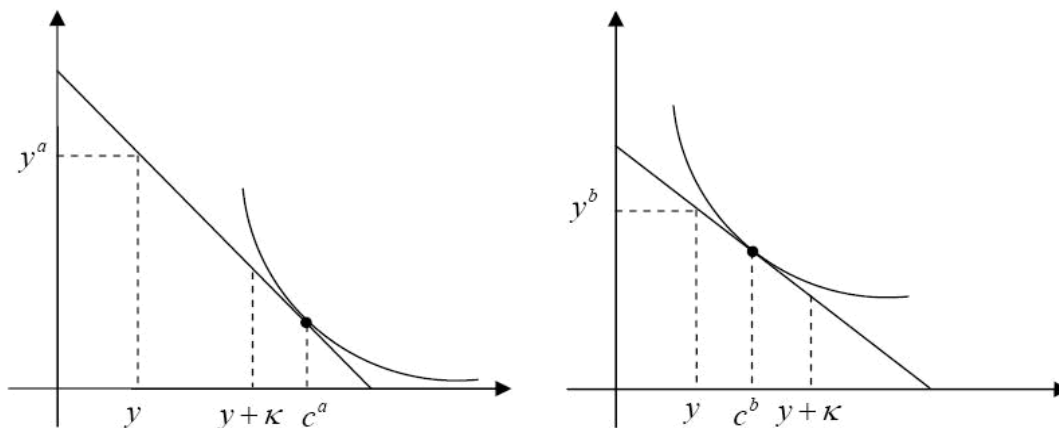
The following example describes a situation in which overborrowing occurs because debt limits do not bind for all agents at the same time. The example is in the context of a two-period, endowment economy without uncertainty. The economy faces a constant debt ceiling  $\kappa$  per capita. There is a continuum of agents of measure one, and agents are heterogeneous. The central result obtains under a variety of sources of heterogeneity, such as differences in endowments, preferences, or initial asset positions. Here, I assume that agents are identical in all respects except in their period-2 endowments. Specifically, all households receive the same endowment of  $y$  units of goods in period 1, but in period 2, half of the households receive an endowment of  $y^a > y$  and the other half receive a smaller endowment of  $y^b < y^a$ . Agents receiving the larger future endowment have a stronger incentive to borrow in period 1 to smooth consumption over time.

We have in mind a situation in which in the absence of a debt ceiling households with high expected endowment consume  $c^a > y + \kappa$  units in period 1 and the rest of the households consume  $c^b < y + \kappa$  units. Figure 2 depicts the equilibrium in the absence of a debt constraint. In this case, aggregate external debt per capita equals  $A^u = (c^a + c^b)/2 - y$ .

When the borrowing ceiling  $\kappa$  is imposed at the level of each individual household, half of the households—those with high period-2 endowment—are constrained and consume  $y + \kappa$  units, whereas the other half is unconstrained and consumes  $c^b$ . Aggregate external debt per capita equals  $A^i = (\kappa + c^b - y)/2 < A^u$ . Clearly, we also have that  $A^i < \kappa$ .

Now suppose that the debt ceiling is imposed at the aggregate level, If aggregate external debt in the unconstrained environment does not exceed the ceiling, i.e., if  $A^u \leq \kappa$ , then the equilibrium interest rate is the world interest rate  $R^*$ , and consumption of each agent equals the level attained in the absence of borrowing constraints. External debt is given by  $A^a = A^u > A^i$ . Alternatively, if the aggregate level of external debt in the unconstrained en-

Figure 2: Overborrowing in an Economy with Heterogeneous Agents



environment exceeds the ceiling (i.e., if  $A^u > \kappa$ ), then the economy is financially rationed, the domestic interest rate exceeds the world interest rate, and aggregate borrowing per capita is given by  $A^a = \kappa > A^i$ . Therefore, regardless of whether the aggregate debt limit is binding or not, external borrowing is higher when the debt ceiling is imposed at the aggregate level.

Therefore, the combination of heterogeneous consumers and debt limits imposed at the aggregate level induces overborrowing in equilibrium. Overborrowing occurs because of a financial externality. Specifically, the group of more frugal consumers provide a financial service to the group of more lavish consumers by placing comparatively less pressure on the aggregate borrowing constraint. This service, however, is not priced in the competitive equilibrium.<sup>1</sup>

<sup>1</sup>Interestingly, economic heterogeneity, although of a different nature, is also the root cause of overborrowing in the dual-liquidity model of emerging-market crisis developed by Caballero and Krishnamurthy (2001). In their model, there is heterogeneity in the provision of liquidity across assets. Some assets are recognized as liquid collateral by both domestic and foreign lenders, while other assets serve as collateral only to domestic lenders. Caballero and Krishnamurth show that in financially underdeveloped economies this type

### 6.2.1 Socially Optimal External Borrowing

The definition of overborrowing that I have applied thus far compares the levels of external debt in economies with individual and aggregate borrowing limits. When agents are homogeneous, this definition in most cases amounts to comparing the competitive equilibrium with an aggregate debt limit with the solution to a social planner's problem that endogenizes the presence of the aggregate credit limit. When agents are heterogeneous, however, this equivalence ceases to hold. For in this case the benevolent social planner will in general not wish to impose the same debt limit on each individual agent.

For the purpose of understanding overborrowing, it is important to limit the role of the social planner to the optimal allocation of a limited amount of foreign credit. In particular, it is important to abstract from issues of wealth redistribution. Therefore, we define the optimization problem of a social planner that cares equally about all households as maximizing the average level of welfare,  $U(c_1^a, c_2^a) + U(c_1^b, c_2^b)$ , subject to individual resource constraints,  $c_1^a + c_2^a/R^* = y + y^a/R^*$  and  $c_1^b + c_2^b/R^* = y + y^b/R^*$ , and the external credit constraint,  $c_1^a + c_1^b \leq 2(y + \kappa)$ . Formally, the nonredistributive nature of the social planner is reflected in the imposition of individual resource constraints, as opposed to a single consolidated resource constraint. The appendix presents the optimality conditions associated with this problem and works out a specific numerical example. The main conclusions of that analysis are: (a) It is not socially optimal to equalize intertemporal marginal rates of substitution in consumption across heterogeneous agents. By contrast, in the competitive equilibrium with an aggregate debt limit intertemporal marginal rates of substitution are identical across all households and equal to the domestic interest rate. (b) The aggregate level of external debt per capita in the competitive equilibrium with an aggregate debt limit is socially optimal. This is obvious from the fact that both environments feature the

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of heterogeneity produces an externality whereby the market price of international liquidity is below its social marginal cost.

same debt constraint,  $c_1^a + c_1^b \leq 2(y + \kappa)$ , which binds in the competitive equilibrium with an aggregate debt limit iff it binds in the social planner's problem. (c) There is overborrowing in the competitive economy with an aggregate debt limit in the sense that big borrowers borrow more and small borrowers borrow less than in the social planner's equilibrium. (d) There is underborrowing in the economy with debt limits imposed at the individual level, in the sense that big borrowers borrow less and small borrowers borrow more than in the social planner's equilibrium. That is,  $a_i^a < a_s^a$  and  $a_i^b > a_s^b$ .

## Appendix: Optimal Allocation of an Aggregate Debt Limit

Consider a social planner that cares equally about each household and does not pursue redistribution of wealth but limits its action to an efficient allocation of credit across households.

Specifically, in the example given in section 6.2, where the heterogeneity originates in half of the population having higher endowment in period two, the social planner's problem is given by:

$$\max \{U(c_1^a, c_2^a) + U(c_1^b, c_2^b)\}$$

subject to

$$c_1^a + \frac{c_2^a}{R^*} = y + \frac{y^a}{R^*},$$

$$c_1^b + \frac{c_2^b}{R^*} = y + \frac{y^b}{R^*},$$

$$c_1^a + c_1^b \leq 2(y + \kappa).$$

Note that the planner's problem features two intertemporal resource constraints, one for each type of agent, instead of a single consolidated resource constraint. This separation reflects the fact that we do not allow the social planner to pursue wealth redistribution. Letting  $\mu$  denote the Lagrange multiplier associated with the borrowing limit, the first-order conditions associated with the planner's optimization problem are:

$$U_1(c_1^a, c_2^a) - R^*U_2(c_1^a, c_2^a) = \mu$$

$$U_1(c_1^b, c_2^b) - R^*U_2(c_1^b, c_2^b) = \mu.$$

Consider the following numerical example. Let  $U(c_1, c_2) = -\frac{1}{2}(c_1 - \bar{c})^2 - \frac{\beta}{2}(c_2 - \bar{c})^2$  and  $R^* = 1/\beta$ . We adopt the following specific parameter values:

$y = 1$ ,  $y^a = 2$ ,  $y^b = 1.5$ ,  $R^* = 1/\beta = 1.1$ ,  $\bar{c} = 3$ , and  $\kappa = 0.3$ . The matlab program `heterogeneous.m` on my website implements the numerical exercise conducted here.

## Unconstrained Equilibrium

Let  $Y^a = y + y^a/R^* = 2.82$  and  $Y^b = y + y^b/R^* = 2.36$  denote the present discounted value of the endowment streams evaluated at the world interest rate. Then, the unconstrained equilibrium is given by

$$c_{u1}^a = c_{u2}^a = \frac{R^*}{1 + R^*} Y^a = 1.48,$$

$$c_{u1}^b = c_{u2}^b = \frac{R^*}{1 + R^*} Y^b = 1.24.$$

The individual levels of debt assumed in period 1 are:

$$a_u^a = c_{u1}^a - y = 0.48$$

$$a_u^b = c_{u1}^b - y = 0.24$$

and the aggregate level of debt per capita is

$$a_u = \frac{a_u^a + a_u^b}{2} = 0.36.$$

## Individual Debt Limits

Under an individual debt constraint, we have that individual and aggregate debt positions are given by:

$$a_i^a = \min\{a_u^a, \kappa\} = 0.3,$$

$$a_i^b = \min\{a_u^b, \kappa\} = 0.24,$$

$$a_i = \frac{a_i^a + a_i^b}{2} - y = 0.27.$$

### 6.3 The Social Planer's Problem

Under the assumed quadratic period utility function, the first-order condition of the social planer is given by

$$c_{s2}^a - c_{s1}^a = \mu,$$

$$c_{s2}^b - c_{s1}^b = \mu.$$

Using these expressions to eliminate  $c_{s2}^a$  and  $c_{s2}^b$  from the individual resource constraints, we obtain

$$\frac{1 + R^*}{R^*} c_{s1}^a + \frac{\mu}{R^*} = Y^a,$$

$$\frac{1 + R^*}{R^*} c_{s1}^b + \frac{\mu}{R^*} = Y^b.$$

Solving for consumption we obtain

$$c_1^a = \frac{R^*}{1 + R^*} \left( Y^a - \frac{\mu}{R^*} \right),$$

$$c_1^b = \frac{R^*}{1 + R^*} \left( Y^b - \frac{\mu}{R^*} \right).$$

Use these equations to eliminate first-period consumption from the aggregate credit constraint

$$\frac{R^*}{1 + R^*} \left[ Y^a + Y^b - \frac{2\mu}{R^*} \right] = 2(y + \kappa).$$

Solving for the Lagrange multiplier yields

$$\mu = \frac{R^*}{2} (Y^a + Y^b) - (1 + R^*)(y + \kappa).$$



## Competitive Equilibrium with an Aggregate Debt Limit

In a constraint and competitive equilibrium with an aggregate debt limit, we have:

$$c_{a1}^a - \bar{c} = \beta R_a (c_{a2}^a - \bar{c}).$$

Solving for future consumption yields

$$c_{a2}^a = (\beta R_a)^{-1} c_{a1}^a + [1 - (\beta R_a)^{-1}] \bar{c}.$$

We assume that rents from financial rationing are distributed lump-sum to households to ensure that in equilibrium the present discounted value of consumption evaluated at the world interest rate equals the present discounted value of endowments also evaluated at the world interest rate. Using the above equation to eliminate  $c_{a2}^a$  from such resource constraint we get

$$c_{a1}^a = \left[ 1 + \frac{(\beta R_a)^{-1}}{R^*} \right]^{-1} \left[ Y^a - \frac{[1 - (\beta R_a)^{-1}] \bar{c}}{R^*} \right] = 1.42.$$

Similarly,

$$c_{a1}^b = \left[ 1 + \frac{(\beta R_a)^{-1}}{R^*} \right]^{-1} \left[ Y^b - \frac{[1 - (\beta R_a)^{-1}] \bar{c}}{R^*} \right] = 1.18.$$

We report here the equilibrium values implied by our assumed parameterization. The equilibrium conditions cannot be expressed in closed form, so we resort to a numerical solution. The individual levels of debt assumed in period 1 are:

$$a_a^a = c_{a1}^a - y = 0.42,$$

$$a_a^b = c_{a1}^b - y = 0.18.$$

And the aggregate level of debt per capita is

$$a_a = \frac{a_a^a + a_a^b}{2} = 0.30.$$

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