A THEORY OF GROWTH AND VOLATILITY AT
THE AGGREGATE AND FIRM LEVEL

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ABSTRACT

This paper presents an endogenous growth model that explains the evolution of the first and second moments of productivity growth at the aggregate and firm level during the post-war period. Growth is driven by the development of both (i) idiosyncratic R&D innovations and (ii) general innovations that can be freely adopted by many firms. Firm-level volatility is affected primarily by the Schumpeterian dynamics associated with the development of R&D innovations. On the other hand, the variance of aggregate productivity growth is determined mainly by the arrival rate of general innovations. Ceteris paribus, the share of resources spent on development of general innovations increases with the stability of the market share of the industry leader. As market shares become less persistent, the model predicts an endogenous shift in the allocation of resources from the development of general innovations to the development of R&D innovations. This results in an increase in R&D, an increase in firm-level volatility, and a decline in aggregate volatility. The effect on productivity growth is ambiguous.

On the empirical side, this paper documents an upward trend in the instability of market shares. It shows that firm volatility is positively associated with R&D spending, and that R&D is negatively associated with the correlation of growth between sectors which leads to a decline in aggregate volatility.

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1 Motivation

The literature on endogenous growth has made substantial progress in the past 15 years. In spite of these advances, however, there remains much to be learnt about the determinants of long-run productivity growth. In our opinion, the existing literature suffers from two main limitations.

First, state-of-the-art models (Aghion and Howitt [1998, Ch. 12], Dinopoulos and Thompson [1998], Jones [1995], Kortum [1997], Peretto [1998], Segerstrom [1998] and Young [1998]) predict a positive relationship between the growth rate of productivity and the share of research & development (R&D) in GDP. However, this prediction does not seem to be true for aggregate data from the post-war United States (US). Figure 1 illustrates the smoothed growth rate of productivity as well as the evolution of the share of private R&D in GDP as measured by the NSF. No clear relationship seems to exist between the two variables.[1,2] Jones and Willians [1998] also find no significant relationship between R&D intensity and TFP growth at the sector level in the US once sector-level fixed effects are introduced. At the firm level, however, Griliches [1980, 1986] and Griliches and Mairesse [1984] have examined the effect of these same measures of R&D intensity on productivity or TFP growth and have observed a strong positive association even after including firm-level fixed effects.

Second, in addition to having trouble explaining the first moments of growth, the existing theories have left the second moments out of their scope entirely, as though their determinants were orthogonal to the first moments' determinants.[3] This presumption, however, is debatable in light of the interesting dynamics of volatility during the post-war period. Two strands of the literature have characterized the evolution of volatility at the aggregate and firm level. McConnell and Perez-Quiros [1999] and Stock and Watson [2003] have shown that the volatility of aggregate variables such as output, hours worked and labor productivity growth has declined during the post-war period. At the firm level, however, these same variables have become more volatile (Comin and Mulani [2006], Comin and Philippon [2005] and this paper). Perhaps most importantly, these

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[1]Examination of TFP growth or output growth results in similar conclusions. Similarly, the upward trend in R&D also holds for the share of scientists and engineers in employment and for total R&D expenses in the US and in the OECD.

[2]Comin [2004] also claims that R&D expenditures, as defined by the NSF, only give a partial picture of growth since they can account for a small fraction of productivity growth in the US during the post-war period.

[3]There exists literature that has attempted to explore the effects of exogenous increases in aggregate volatility on growth (Ramey and Ramey [1995], Barlevy [2003]). A key difference between that literature and this paper is that here volatility (both aggregate and firm-level) is endogenous to analysis.

diverging trends are also true for the volatility of productivity growth. Figure 2 illustrates the time series of the volatility of productivity growth at the aggregate and firm level. The left axis plots the standard deviation of 10-year centered rolling windows of annual productivity growth. The right axis plots the evolution of the same variable averaged for firms in the COMPUSTAT data base.[4] The opposing trends are evident.

This paper builds an endogenous growth model that enhances our understanding of the determinants of productivity growth and that has implications for firm and aggregate volatility that are consistent with the evidence. Our model builds on the quality-ladder models of Aghion and Howitt [1992] and Grossman and Helpman [1991, Ch. 4]. In this context, standard R&D investments lead firms to develop new products that replace the current leading products. Such productivity improvements lead to substantial firm-level volatility since incumbents incur losses while entrants enjoy capital gains. These innovations are to a large extent sector specific, however, and have only a minor effect on aggregate volatility.

To explain the movements in aggregate growth and volatility, it is necessary to consider a second type of innovation - general innovations.[5] General innovations have two properties. First, they are applicable to several firms and sectors.[6] Second, a firm that develops a general innovation cannot appropriate the benefits other firms enjoy when they adopt it. This is the case because general innovations - such as managerial and organizational innovations, improved process controls, product development, testing practices and pre-production planning, new personnel and accounting practices, financial innovations, the use of electricity as the source of energy in a plant, etc. - are hard to patent and relatively easy to reverse-engineer.[7,8]

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[4]For each firm in COMPUSTAT, we compute its volatility in a given year as the centered standard deviation of 10 consecutive annual growth rates of the sales per worker. The firm volatility measure plotted in Figure 2 is the average volatility across firms.

[5]A large majority of the innovations that fit our notion of general innovations are explicitly excluded by the NSF from their definition of R&D. See Comin [2004] and section 3 for more on the NSF's definition of R&D.

[6]This feature links our general innovations to the General Purpose Technologies (GPT's) of Bresnahan and Trajtenberg [1995] and Helpman and Trajtenberg [1998]. Unlike GPT's, general innovations do not have to be revolutionary. An interesting difference from a modelling point of view is that the development of GPT's and their components is usually modelled as exogenous or supported by patents. Our general innovations are developed endogenously and are not patentable.

[7]See Table 1 for a longer, albeit incomplete, list of general innovations. For a description of the innovations, see Appendix 1.

[8]Hellwig and Irmen [2001] and Boldrin and Levine [2000] have also highlighted the importance of innovations that are not patentable.

These two properties have interesting implications. First, general innovations have a large effect on aggregate growth because they affect many firms symmetrically. Furthermore, since general innovations arrive randomly over time, investments in the development of general innovations may lead to substantial volatility in aggregate productivity growth. Second, the inability of innovators to appropriate the social value of general innovations means that their incentives to develop them depend on their firm's productivity gain from implementing the innovation. These productivity gains are larger for more valuable firms. As a result, general innovations are typically conducted by large, leading firms.

In equilibrium, there is a negative relationship between resources spent on R&D and resources spent on the development of general innovations. Since (1) R&D leads to turnover in market leaders and to a decline in the value of leading firms and (2) the private return to a general innovation increases in the value of the firm, a force that leads the economy to invest more in R&D may induce a decline in the rate of development of general innovations.

This trade off between R&D and general innovations accounts for the trends observed in growth and volatility at the aggregate level. First, productivity growth increases with the development of both R&D and general innovations. Since an increase in R&D intensity leads to a decline in the arrival rate of general innovations, however, the actual relationship between R&D and productivity growth is ambiguous. Second, aggregate volatility is primarily affected by the arrival rate of general innovations because this determines the co-movement of growth across sectors by causing simultaneous fluctuations. Hence, a decline in investments in general innovations leads to a decline in aggregate volatility. Finally, firm-level volatility is primarily driven by market turnover. An increase in R&D intensity leads to turnover in the market leader and increases firm-level volatility.

In addition to developing a new model of growth and volatility, this paper also provides empirical evidence of the forces and mechanisms emphasized by the model. First, it documents two new facts: (i) a very significant increase in the market turnover rate since World War II and (ii) a substantial decline in the correlation of productivity growth across sectors over the same period. Second, it shows that market turnover and firm volatility have increased by more in sectors where R&D intensity has increased by more. Finally, this paper establishes that R&D is negatively associated with aggregate volatility by showing that sectors that experienced greater increases in R&D also experienced greater declines in the correlation between their own growth and the rest of the economy's. Thus, the increase in R&D leads to lower aggregate volatility.

While there must be other forces that have contributed to the trends in firm-level and, specially,

aggregate volatility, the mechanisms emphasized in our model are quantitatively significant. A calibration of the model shows that it can account for (1) the lack of a relationship between R&D and productivity growth, (2) 75 percent of the increase in firm volatility and, (3) over 40 percent of the decline in aggregate volatility.

The rest of the paper is organized as follows. Section 2 presents the formal model and undertakes the comparative statics exercises. Section 3 discusses and evaluates predictions of the model in both qualitative and quantitative terms. Section 4 concludes.

2   Model

The following describes an endogenous technological change model that delivers endogenous growth and endogenous volatility at the aggregate and firm-level. To maximize the clarity of exposition, we present the basic trade off between R&D and general innovations in the context of a one sector model. We then extend this basic framework to a multisector economy to understand the determinants of the co-movement of growth across sectors, which is essential for the evolution of aggregate volatility.

2.1   Basic set up

Preferences

The representative consumer enjoys a utility flow that is linear on the units of final output consumed, c_t. The present discounted value of utility is represented as

$$U = \int_0^\infty c_t e^{-rt} dt, \tag{1}$$

where r denotes the instantaneous discount rate. Consumers inelastically supply a mass of L units of labor. They also pay some lump sum of taxes, T_t.

Production

We initially assume the economy is comprised of one sector that competitively produces y_s units of output with price p_s. Output is produced by combining $m + 1$ intermediate goods, where m is a constant. Each intermediate good is produced by one and only one producer. Intermediate goods can be of two types. The good with highest quality, q, is the leading intermediate good. Consumers perceive this as a differentiated intermediate good because of its superior technical properties. The rest of producers cannot compete with the leading intermediate good and must produce standard, undifferentiated intermediate goods.

Let x^l denote the number of units of the leading intermediate good employed to produce output. Similarly, let x_i^f denote the number of intermediate goods employed from the ith standard producer. Then the production function can be expressed as:

$$y_s = q \left(\beta \left(x^l \right)^\alpha + (1 - \beta) \left(\sum_{i=1}^{m} x_i^f \right)^\alpha \right)^{1/\alpha},\tag{2}$$

where q is the quality of the leading intermediate good, $\beta < 1$ is the market share of the leading intermediate good, and $\alpha \in (0,1)$ is the elasticity of substitution between the leading good and the composite of standard intermediate goods.[9]

The production of a unit of intermediate good requires a units of labor. a declines with the efficiency of the production process, h, such that

$$a = 1/h\tag{3}$$

Innovation

Intermediate good producers can undertake two types of innovations. First, they can attempt to develop an intermediate good of a quality higher than q. In particular, after spending a share n_i^q of aggregate output, they face a probability $\lambda_i^q = \bar{\lambda} \frac{n_i^q}{1 - s_{R\&D}}$ over an instantaneous time-interval dt of developing a new leading good with quality $\delta_q q$ $(\delta_q > 1)$.[10] In this formulation, $\bar{\lambda}$ measures the probability of succeeding in the development of a superior intermediate good per fraction of output spent on R&D. $s_{R\&D}$ denotes a R&D subsidy that is financed by the lump sum taxes paid by consumers. When a standard intermediate good producer succeeds in his R&D efforts, he becomes the new leading intermediate good producer, and the former market leader becomes a standard intermediate good producer.[11]

Second, intermediate goods producers can also invest in improving the production process of their intermediate good (i.e. reducing the cost of production, a). Specifically, he can invest a share n_i^h of aggregate output and face an instantaneous probability $\lambda_i^h = \bar{\lambda}^h \left(n_i^h \right)^{\rho_h}$, with $0 < \rho_h < 1$,

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[9]Note that this formulation incorporates an externality from the quality of the leading intermediate good to the productivity of the standard intermediate goods.

[10]Griliches [1984] finds evidence in favor of the linearity of the R&D production technology using firm-level data.

[11]This formulation of the R&D dynamics has several interesting features. First, the lower demand elasticity of the leading intermediate good is instrumental in generating cross-sectional variation in sales per worker. Second, by not having to carry around the distribution of qualities for intermediate goods, we make substantial progress towards an analytical solution of the model. Third, the absence of entry and exit simplifies the computation of firm-level moments.

of successfully increasing h to $\delta_h h$, with $\delta_h > 1$. We denote this type of production improvements general innovations (henceforth, GI). For future reference, it is useful to define $c(\lambda) \equiv \left(\lambda/\bar{\lambda}^h\right)^{\frac{1}{\rho_h}}$ as the share of aggregate output that a producer must invest to face a probability λ of developing a general innovation.

These two types of innovations differ in their appropriability. Firms that invent a new product or improve the quality of an existing product can patent the innovation and extract a substantial fraction of the surplus enjoyed by other firms from such an innovation. On the other hand, firms that develop GIs, such as improvements in management practices, cannot appropriate the benefits experienced by other firms that use the innovations. Appropriating this surplus is impossible because GIs are easy to reverse engineer and because they are difficult to patent, since most of them are not embodied in a good. These characteristics are reflected in the assumption that all producers immediately (and costlessly) adopt GIs.

A second difference between the two types of innovations that will be important in the multisector extension is their applicability. The impact of new or improved goods is often restricted to a small number of sectors, whereas GIs , such as improvements in management or in the organization of production mentioned above, can be applied to many different economic activities across a wide array of sectors.

Government

The government collects lump sum taxes from the consumers to finance the exogenous R&D subsidy at every instant.

2.2 Analysis

We start by exploring the pricing problem of intermediate good producers. The leading intermediate good producer faces an isoelastic demand function and sets a price, p_x^l, equal to the marginal cost times a markup given by the inverse of the elasticity of demand (i.e. $1/\alpha$). Bertrand competition between standard intermediate good producers brings the price of standard intermediate goods, p_x^f, down to their marginal cost of production. These arguments are reflected in the following expressions, where w denotes the wage rate.

$$p_x^l = \frac{aw}{\alpha} \tag{4}$$

$$p_x^f = aw \tag{5}$$

The price of sectoral output is then given by

$$p_s = \frac{wa}{\xi q}, \tag{6}$$

where $\xi \equiv \left[(\beta \alpha^\alpha)^{\frac{1}{1-\alpha}} + (1-\beta)^{\frac{1}{1-\alpha}} \right]^{\frac{1-\alpha}{\alpha}}$. Expression (6), together with the choice of numeraire, determines the wage rate.

Plugging the prices into the demand functions, we can solve for the share of each producer's sales in nominal output:[12]

$$\frac{p_x^l x_s^l}{p_s y_s} = \frac{1}{\alpha} \left(\frac{\alpha \beta}{\xi^\alpha} \right)^{\frac{1}{1-\alpha}} \equiv \varkappa^l \tag{7}$$

$$\frac{p_x^f x_{si}^f}{p_s y_s} = \frac{1}{m} \left(\frac{1-\beta}{\xi^\alpha} \right)^{\frac{1}{1-\alpha}} \equiv \varkappa^f \tag{8}$$

To explore the investment decisions, it proves useful to introduce some notation. Let v^l and v^f denote, respectively, the market value of the leading and standard intermediate good firms, both divided by nominal output.

Producers of standard intermediate goods can try to develop a new leading intermediate good by undertaking R&D investments. The share of output invested by standard intermediate good producers in developing R&D innovations is determined by the following arbitrage condition:

$$\overbrace{(1 - s_{R\&D})}^{\text{Marginal Cost}} = \overbrace{\bar{\lambda}(\delta_q v^l - v^f)}^{\text{Expected Mg. Benefit from R\&D Innovations}} \tag{Lq}$$

The left-hand side in (Lq) is the private cost of investing one percent of output in R&D, whereas the right-hand side is the expected marginal benefit from such an investment. With probability $\bar{\lambda}$ the follower experiences a capital gain given by the difference between the value of succeeding in developing a new leading good, $\delta_q v^l$, and the value of a follower in the absence of such an innovation, v^f.

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[12]Combining the demands for each intermediate good and the labor market clearing condition, allows us to solve for the number of units of each each type of intermediate goods sold:

$$x^l = \frac{1}{a \left[1 + ((1-\beta)\alpha/\beta)^{1/(1-\alpha)} \right]} L$$

$$x_i^f = \frac{((1-\beta)\alpha/\beta)^{1/(1-\alpha)}}{a \left[1 + ((1-\beta)\alpha/\beta)^{1/(1-\alpha)} \right]} \frac{L}{m}$$

As shown in Appendix 1, the optimal pricing and R&D investment decisions of standard interme-diate good producers imply that, in the symmetric equilibrium, the value of standard intermediate goods firms, v^f, is zero. Intuitively, since they charge a price equal to the marginal cost of pro-duction, they incur loses equal to the cost of undertaking innovations. The linearity in the R&D technology implies that the loses from the R&D investments are exactly compensated by the ex-pected capital gains from becoming market leaders, making the net value of a standard intermediate good producer zero.

The current market leader can also invest in R&D innovations. He faces the same marginal cost of innovation as followers, but the expected marginal benefit is $\bar{\lambda}(\delta_q - 1)v^l$ instead of $\bar{\lambda}\delta_q v^l$. Since $v^l > v^f = 0$, equation (Lq) implies that the expected marginal benefit of R&D innovations for the leader is lower than the marginal cost of conducting these innovations. As a result, the market leader does not conduct R&D innovations in equilibrium.[13]

The market leader does, however, have incentives to develop general innovations that reduce the marginal cost of producing intermediate goods for all producers. In an interior solution, the optimal investment in GIs by the leader results in the following equality:

$$\overbrace{c'(\lambda^h)}^{\text{Marginal Cost}} = \overbrace{(\delta_h - 1)v^l}^{\text{Mg. Benefit from General Innovations}} \tag{Lh}$$

The left-hand side of (Lh) is the cost of increasing the probability of developing a general innovation by one percent, whereas the right-hand side is the market leader's private benefit from the arrival of a GI. This is given by the market value of the leader times the gain in productivity from the arrival of the GI, $\delta_h - 1$. Note that, since GIs cannot be sold, their private return is proportional to the value of the firm that develops them.[14] Followers, in principle, can also come out with general improvements in productivity. In equilibrium, however, since the private value of these innovations is proportional to the value of the firm, and v^f is equal to zero, followers do not undertake general innovations.[15]

To close the model, we just need to determine the value of the market leader, v^l, which is

[13] This result is standard in Schumpeterian models. However, it is not critical for the mechanisms of the model that we use to explain the evolution of aggregate and firm volatility. The lack of R&D investments by leaders, though, simplifies the analysis. Comin and Ludvigson [2006] provide evidence of the larger propensity of followers to engage in R&D investments.

[14] This result parallels the logic of the span of control model in Lucas [1978].

[15] Table 1 and Appendix 2 provide evidence that GIs are developed mostly by market leaders.

determined by the following asset equation:

$$rv^l = (1-\alpha)\varkappa^l - c(\lambda_h) + \lambda^h(\delta_h - 1)v^l - \lambda^q v^l \tag{9}$$

Equation (9) says that the expected income generated by a license on the leading product during a unit interval, rv^l, is equal to the instantaneous profit flow net of the costs of investing in GI, $(1-\alpha)\varkappa^l - c(\lambda_h)$, plus the expected capital gain from succeeding in developing a GI, $\lambda^h(\delta_h - 1)v^l$, minus the expected capital loss from being replaced as market leader by a standard intermediate good producer, $\lambda^q v^l$.

Solving for v^l yields the following expression:

$$v^l = \frac{(1-\alpha)\varkappa^l - c(\lambda^h)}{r + \lambda^q - \lambda^h(\delta_h - 1)}, \tag{10}$$

where the numerator reflects the profit flow and the denominator reflects the time preference, r, the creative destruction effect, λ^q, and the expected gains from the development of GIs, $\lambda^h(\delta_h - 1)$.

The optimal investments in R&D (i.e. equation Lq) and general innovations (i.e. equation Lh) govern the dynamics of the economy. Note in particular that, since there is no state variable, the economy converges immediately to the new equilibrium (λ^h, λ^q) following any perturbation in a parameter.

We are interested in exploring the comparative statics of the investment intensities in R&D and GIs with respect to the R&D subsidy $(s_{R\&D})$ and the efficiency of R&D investments $(\bar{\lambda})$. To this end, we isolate v^l from condition (Lq) and obtain that $v^l = (1 - s_{R\&D})/(\bar{\lambda}\delta_q)$. Plugging this back in condition (Lh) results in the following expression for λ^h:

$$c'(\lambda^h) = \frac{(1 - s_{R\&D})(\delta_h - 1)}{\bar{\lambda}\delta_q}. \tag{11}$$

The convexity of $c(.)$ implies that the arrival rate of GIs, λ^h, decreases with $s_{R\&D}$ and with $\bar{\lambda}$. Intuitively, increases in $s_{R\&D}$ and $\bar{\lambda}$ reduce the marginal private cost of developing an embodied innovation. Restoring the equilibrium in the arbitrage condition requires a decline in the value of the market leader. This decline in v^l reduces the marginal private return from investing in developing GIs. As a result, the arrival rate of GIs (λ^h) declines.

To explore the response of λ^q to increases in $s_{R\&D}$ and $\bar{\lambda}$, we substitute the expression for v^l in the arbitrage equation (Lq) as follows:

$$(1 - s_{R\&D}) = \bar{\lambda}\delta_q \frac{(1-\alpha)\varkappa^l - c(\lambda^h)}{r + \lambda^q - \lambda^h(\delta_h - 1)} \tag{12}$$

Increases in $s_{R\&D}$ and $\bar{\lambda}$ require a decline in v^l to restore the arbitrage condition. The Envelope Theorem implies that $\partial v^l / \partial \lambda^h = 0$. Therefore, an increase in λ^q is the only way to bring down v^l and restore the arbitrage condition.

The arrival rate of R&D innovations, λ^q, depends both on the exogenous parameters $s_{R\&D}$ and $\bar{\lambda}$ and on the share of output private agents spend on R&D, n^q. To determine whether increases in $s_{R\&D}$ and $\bar{\lambda}$ lead to increases in the share of private R&D expenditures, we substitute $\frac{\bar{\lambda}}{1-s_{R\&D}} n_q$ for λ^q in (12) and rearrange as follows:

$$1 = \frac{\bar{\lambda}}{1 - s_{R\&D}} \; \frac{\delta_q[(1-\alpha)\varkappa^l - c(\lambda^h)]}{r + \frac{\bar{\lambda}}{1-s_{R\&D}} n_q - \lambda^h(\delta_h - 1)} \tag{13}$$

Increases in $s_{R\&D}$ or $\bar{\lambda}$ increase the productivity per share of sectoral output spent in R&D today, but also increase the productivity of the followers that will try to take over tomorrow's leader. These two forces are the same; the only difference between them is the timing. The new market leader benefits from the higher productivity of R&D expenses earlier than the producers that will take over in the future. Therefore, as long as the effective discount rate net of the turnover rate (i.e. $r - \lambda^h(\delta_h - 1)$) is positive, the first force dominates and the share of private R&D expenses in GDP, n^q, increases with $s_{R\&D}$ and $\bar{\lambda}$. We define this parametrization as Condition 1.

Condition 1: $r > \lambda^h(\delta_h - 1)$, *where λ^h is defined in (11).*

Proposition 1 summarizes our findings so far.

Proposition 1 *In response to increases in $s_{R\&D}$ or $\bar{\lambda}$, the arrival rate of general innovations, λ^h, declines while the arrival rate of R&D innovations, λ^q, increases. Further, if Condition 1 holds, the share of GDP spent on private R&D, n^q, also increases.*

It follows from Proposition 1 that $s_{R\&D}$ and $\bar{\lambda}$ cause the rate of R&D-driven and general innovations to move in opposite directions. This negative co-movement between R&D-driven and general innovations is one of the two key elements driving the post-war dynamics of growth and volatility at the aggregate and firm level. To introduce the second key element, we need to extend the analysis into a multisector setting.

2.3 Multisector economy

To move from the one-sector to the multisector economy, we need to determine how sectoral output is aggregated and the applicability of innovations across sectors.

The multisector economy is composed of N sectors. As above, sectoral output, y_s, is produced according to (2). Final output, y, results from competitively aggregating the N sectoral outputs in the following Cobb-Douglas way:[16]

$$y = \prod_{s=1}^{N} y_s^{1/N} \tag{14}$$

In terms of the innovations' applicability across sectors, we assume that R&D innovations are sector-specific while, for the time being, general innovations diffuse freely and immediately to all the sectors in the economy. This difference in the innovation's applicability is founded on both a priori knowledge and empirical evidence. Innovations in management, sales, personnel, distribution and similar fields can be applied to virtually all sectors of the economy because firms in all sectors need to manage, sell, motivate and coordinate workers and distribute their products and services. Hence, the generality of GIs. In contrast, R&D innovations eventually lead to the creation or improvement of a product that increases the productivity of a sector-specific task. Thus, R&D innovations are mostly sector-specific. In the empirical section, we explore the generality of GIs and R&D innovations by estimating the effect of R&D on the correlation of growth across sectors. The evidence presented there supports the greater generality of GIs over R&D innovations.

The generality of GIs implies that the development of a new GI leads both sectoral and aggregate output to increase by a factor of δ_h. The sector specificity of R&D innovations implies that the development of a R&D innovation in sector s leads to an increase in y_s by a factor δ_q, but leads to an increase in aggregate output by only a factor of $\delta_q^{1/N}$.

We denote the arrival rate of R&D innovations in sector s by λ_s^q. λ^h continues to denote the arrival rate of GIs in the economy. Following the same logic as in the one-sector version, we can derive arbitrage and optimal investment in GIs equations very similar to (Lq) and (Lh).[17] For the sake of brevity, we relegate the details of the derivations to Appendix 1. We can also use an asset equation similar to (9) to determine the market value of the producer of the leading intermediate good in any given sector as

$$v^l = \frac{(1-\alpha)\varkappa^l - c(\lambda^h/N)}{r + \lambda_s^q(1 - (\delta_q^{1/N} - 1)(N-1)) - \lambda^h(\delta_h - 1)} \tag{15}$$

[16]The Cobb-Douglas nature of the aggregate production function implies that the nominal output of each sector represents a share $1/N$ of GDP, regardless of the number of R&D and GIs innovations developed in the sector.

[17]The only difference that the multisector context introduces in the R&D arbitrage condition is that, because of the sectoral specificity of R&D innovations, the capital gain from the development of a R&D innovation now becomes $\delta_q^{1/N} v^l$, instead than $\delta_q v^l$.

This expression differs from the value of the leader in the one-sector economy in two important aspects. First, now GIs are also developed in other sectors. Therefore, the leader in sector s incurs only in the cost of developing a fraction $1/N$ of the GIs developed in the economy, but benefits from an arrival rate of λ^h. Second, now the leader in sector s benefits from the increase in aggregate demand associated with the development of R&D innovations in the other sectors. This reduces the effective discount rate by the term $(\lambda_s^q(\delta_q^{1/N} - 1)(N-1))$.

As before, the arbitrage and optimal investment in GIs equations together with the expression for v^l, govern the dynamics of the economy. Given the similarity to the one-sector model, it is not surprising that we can specify conditions such that $s_{R\&D}$ and $\bar{\lambda}$ cause λ_s^q and λ^h to move in opposite directions. For brevity's sake, these conditions are specified in Appendix 1, and are assumed to hold henceforth. As before, λ^h declines with $s_{R\&D}$ and $\bar{\lambda}$. The important difference with respect to the one sector economy is that, since some GIs are developed in other sectors, $\partial v^l/\partial \lambda^h > 0$ at the optimum. In this context, it could be the case that, in response to increases in $s_{R\&D}$ or $\bar{\lambda}$, the decline in λ^h is large enough to necessitate a decline (rather than an increase) in λ_s^q to equalize the marginal cost with the expected marginal benefit from R&D innovations. We regard this as a pathological case and impose conditions, stated in Appendix 1, that rule it out.

Next, we compute the first and second moments of the growth rates of output and productivity at the aggregate and firm level and explore their evolution in response to increases in $s_{R\&D}$ and $\bar{\lambda}$.

Aggregate moments

Growth is the result of both embodied and general innovations. For any given sector s, the growth rate of the sector's output (or productivity), γ_{y_s}, is equal to the number of embodied innovations in the sector times the log of their effect on sectoral output plus the number of general innovations developed in the entire economy times their effect on sectoral output. Formally,

$$\gamma_{y_s} = \gamma_{y/l_s} = \#q_s * \ln(\delta_q) + \#h * \ln(\delta_h),$$

where $\#q_s$ is the number of new embodied innovations developed in the sector during the period, and $\#h$ is the number of new general innovations developed in the economy.

The growth rate of the economy, γ_y, is the average of the sectoral growth rates:

$$\gamma_y = \gamma_{y/l} = \frac{\sum_{n=1}^{N} \gamma_{y_n}}{N} = \frac{\sum_{n=1}^{N} \#q_n}{N} * \ln(\delta_q) + \#h * \ln(\delta_h).$$

Given the Poisson arrival rates of new technologies, the average growth rate and average variance

of the growth rate of output at the sector and aggregate level are as follows:

$$E\gamma_{y_s} = \lambda_s^q \ln(\delta_q) + \lambda^h \ln(\delta_h) \tag{16}$$

$$E\gamma_y = \lambda_s^q \ln(\delta_q) + \lambda^h \ln(\delta_h) \tag{17}$$

$$V\gamma_{y_s} = \lambda_s^q (\ln(\delta_q))^2 + \lambda^h (\ln(\delta_h))^2 \tag{18}$$

$$V\gamma_y = \frac{\lambda_s^q}{N}(\ln(\delta_q))^2 + \lambda^h (\ln(\delta_h))^2 \tag{19}$$

Several conclusions can be drawn from these expressions. (i) Aggregation does not affect the expected growth rate of productivity since aggregate and sectoral expected growth rates (expressions 17 and 16, respectively) coincide. (ii) Increases in $s_{R\&D}$ or $\bar{\lambda}$ have ambiguous effects on expected growth. In particular, these parameter changes lead to increases in λ_s^q and declines in λ^h and hence to an ambiguous effect on the average growth rate of productivity both at the aggregate and sector levels. (iii) The variance of sectoral growth (expression 18) also responds ambiguously to increases in $s_{R\&D}$ and $\bar{\lambda}$. (iv) However, this ambiguity disappears when we explore their effect on the variance of aggregate growth (expression 19). R&D-driven innovations are sector specific and are averaged away at the aggregate level. Hence, their effect on aggregate volatility is smaller than on sectoral volatility. General innovations, on the other hand, are adopted across the economy. Thus, their impacts are the same at the aggregate and sectoral level. As a result, , for N sufficiently large, the decline in aggregate volatility driven by the decline in λ^h dominates the increase in volatility associated with the higher λ_s^q, and aggregate volatility declines in response to increases in $s_{R\&D}$ and $\bar{\lambda}$. (v) Hence, aggregation does affect the second moments of the growth rate of productivity, (expressions 18 and 19).

Firm-level moments

Expected firm-level sales growth $-$ denoted by $E\gamma_{salesi}-$ is affected by the rates of arrival of general innovations and R&D innovations in the economy through their effects on aggregate growth, $E\gamma_y$. In addition, producers of standard intermediate goods expect a higher growth rate of sales than market leaders because they invest in R&D, and with probability λ_s^q/m they will take over the market leader and his sales. Conversely, the current market leader does not invest in R&D and with probability λ_s^q will be taken over by a follower, experiencing a loss in sales. As can be observed below, the distribution of the expected growth rate of sales per worker $-$ denoted by $E\gamma_{salesi/L_i}-$ only differs from the distribution of the growth rate of sales in the size of the capital gain/loss from market turnover. Hence, at the firm level, the model predicts a positive relationship between R&D intensity and expected growth both of sales and sales per worker.

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$$E\gamma_{salesi} = \begin{cases} E\gamma_y - \lambda_s^q \ln(\beta m/((1-\beta))) \text{ for } i = l \\ E\gamma_y + \lambda_s^q/m \ln(\beta m/((1-\beta))) \text{ for } i = f \end{cases}$$

$$E\gamma_{salesi/L_i} = \begin{cases} E\gamma_y - \lambda_s^q \ln(1/\alpha) \text{ for } i = l \\ E\gamma_y + \lambda_s^q/m \ln(1/\alpha) \text{ for } i = f \end{cases}$$

The firm-level volatility of the growth rates of sales and sales per worker depends on the variance of the aggregate growth rate of the economy and the risk of turnover in the market leader. Expressions (20) and (21) present the average variances of the growth rate of sales and sales per worker.[18]

$$var(\gamma_{sales_i}) = var(\gamma_y) + \lambda_s^q \left(\frac{1+\beta(m-1)}{m}\right) \left(\ln(\frac{\beta m}{(1-\beta)})\right)^2 \tag{20}$$

$$var(\gamma_{sales_i/L}) = var(\gamma_y) + \lambda_s^q \left(\frac{1+\beta(m-1)}{m}\right) (\ln(1/\alpha))^2 \tag{21}$$

The variance of aggregate output in the US data is approximately two orders of magnitude smaller than the variance of firm-level volatility. Hence, the quantitatively important term is the latter, which is driven by the turnover rate, λ_s^q. An increase in $s_{R\&D}$ or $\bar{\lambda}$ leads to higher turnover, λ_s^q, both directly and through the increased investments in the development of R&D-driven innovations that it triggers. In this way, $s_{R\&D}$ and $\bar{\lambda}$ increase firm-level volatility.

3   Discussion and Evidence

We have just shown that the model predictions are consistent with the facts described in the introduction. It predicts the lack of a clear relationship between R&D intensity and productivity growth at the aggregate level, the positive association they have at the firm level, the upward trend in firm-level volatility and the downward trend in aggregate volatility. In this section, we do three things. First, we describe the increase in R&D subsidies during the post-war period. Second, we discuss further theoretical predictions of the model and bring them to the data in order to further check for the empirical relevance of the mechanisms described in the model. Third, we conduct calibration exercises to assess the power of the model to generate the dynamics of volatility and growth observed in the data.

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[18]Firm-level variances are weighted by the share of firm sales.

3.1 Driving forces

The R&D tax policy in the United States has been implemented through three main initiatives (Hall [1995]). The expensing rules for R&D, introduced in 1954, allowed US firms to expense most R&D expenditures against corporate income for tax purposes. The Economic Recovery Tax Act of 1981 allowed US firms to allocate all R&D expenses against income earned within the United States, even if a substantial part of their revenue was generated from foreign sales. In addition, the act introduced the Federal R&D tax credit which allowed firms to deduct from corporate income taxes, in proportion to the established credit rate, a portion of qualified R&D expenditures that exceeded a certain level.

State-level R&D tax credits followed soon after when, in 1982, Minnesota became the first state to introduce such a credit. Since then the number of states offering a R&D tax credit has steadily increased, and 31 states currently offer some form of a tax credit on general, company funded R&D. Not only has the number of states offering a tax credit increased, but the average value of these credits has also grown. Wilson [2005] calculated the effective value of all state-level credits for every year since their inception, taking into account the statutory credit rate, the base amount and whether the credit itself was taxable. He found that the effective average value of the state-level tax credits has grown approximately four-fold since their inception in 1982. Hall and Wosinka [1999] examined the benefit of these federal and state tax credits for US firms. They calibrated the effective R&D subsidy to range between .4 and .6 depending on whether the firm is subject to state taxation and whether it is eligible for the tax credits.

The increasing level of R&D subsidies leads, in our specification of the R&D technology, to an increasing turnover rate. In addition to the direct effect, R&D subsidies also induce higher private R&D expenses, according to Proposition 1. There is a literature devoted to test this prediction, which concludes that R&D tax credits have lead to a substantial increase in the share of private R&D in GDP both in the US (Hall [1993], Mamuneas and Nadiri [1996]) and in other OECD countries (Bloom, Griffith and Van Reenen [2002]).

It is important to emphasize that R&D subsidies apply only to R&D expenses. According to the NSF, "R&D consists of activities carried on by persons trained, either formally or by experience, in the physical sciences such as chemistry and physics, the biological sciences such as medicine, and engineering and computer science. R&D includes these activities if the purpose is to do one or more of the following things:

1. Pursue a planned search for new knowledge [...]. (Basic research)

2. Apply existing knowledge to problems involved in the creation of a new product or process [...]. (Applied research)

3. Apply existing knowledge to problems involved in the improvement of a present product or process. (Development)."[19]

The NSF also presents a list of activities closely related to our GIs, which are explicitly excluded from the definition of R&D. Among these we find social science expenditures, defined as those "devoted to further understanding [of] the behavior of groups of human beings or of individuals as members of groups [in the following areas]: personnel, economics, artificial intelligence and expert systems, consumer, market and opinion, engineering psychology, management and organization, actuarial and demographic..."

Therefore, investments in developing general innovations (by-and-large) do not benefit from R&D subsidies.[20]

Though harder to quantify, the growing trend of outsourcing services or the production of certain components has made it easier for followers to figure out how to improve the products and services provided by market leaders. This diffusion of knowledge beyond the boundaries of market leaders has increased the productivity of private R&D expenses, $\bar{\lambda}$.[21]

3.2 Productivity growth

Our model predicts an ambiguous effect of R&D on productivity growth at the aggregate and sector level because of the negative effect that R&D has on the number of GIs developed in the economy. Abdih and Joutz [2005] provide further details about the relationship between R&D and growth. They estimate cointegration relationships between R&D labor, patent applications (i.e. R&D output), and TFP. They find that, while there is a strong and significant positive relationship

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[19] Process innovation applies to the development of new industrial processes such as those that lead to the production of steel or chemical products. In our context, this is the same as standard R&D that leads to a new product or an improved version of an existing product.

[20] Rationalizing the R&D subsidies goes beyond the scope of this paper. Though R&D has some positive externalities, political considerations may also be involved. Congress, for example, has repeatedly failed to renew the Federal R&D tax credit for longer than one or two years. One rationale for this is that keeping the credit temporary can be used as a carrot for business, and it encourages corporations to make financial contributions to their representatives every year in order to preserve this feature of the tax law (*New York Times*, October 28, 1998).

[21] These changes in the flow of knowledge are unlikely to have a significant impact on the productivity of investments in developing GIs. This is the case because those that now more easily acquire knowledge are followers, and followers develop a small share of all the general innovations developed in the economy.

between R&D labor and patent applications, there is no statistical relationship between patents and TFP. That is, R&D investments produce patents but patent growth fails to have an effect on TFP growth. The lack of a relationship persists after allowing for different leads and lags. These results support the view presented by our model, as Abdih and Joutz [2005] recognize. In particular, they constitute indirect evidence in favor of the joint hypothesis that GIs are an important source of productivity growth and that R&D dampens the development of GIs.

We are not the first ones to highlight the importance of general innovations for productivity growth. After studying the importance of the innovations introduced during the last century in the US, Mokyr [2002] claims that "much of the productivity increase in the twentieth century was the result of the perfection of production techniques and process innovation. [...] These led to a continuous transformation in organizational methods, most obviously in mass production in manufacturing techniques but eventually in services and agriculture as well."

Unfortunately, direct measures on the intensity of investment in general innovations are not available. This makes it difficult to directly test the negative effect of R&D on the development of general innovations. One imperfect substitute to this exercise consists of creating a list of GIs and showing that most of them were introduced either before WWII or between the 50's and early 60's when firm turnover was low. Table 1 provides our (very incomplete) list of GIs, most of which were developed before 1970 by large firms that dominated their markets.[22] Below, we conduct a more systematic test of the negative effect of R&D on GIs investments based on the sectoral variation in second moments.

3.3 Firm volatility[23]

As described in section 2, our model rationalizes the increase in firm-volatility observed by Comin and Mulani [2006] and Comin and Philippon [2005] in the COMPUSTAT sample. It is worth making two remarks about the facts uncovered by Comin and Mulani [2006]. First, we show that the increase in firm volatility in COMPUSTAT is qualitatively and quantitatively robust to conditioning on a firm-level fixed effect, the age and size of the firm. Second, we show that the increase in firm volatility is not driven by a potential change in the composition of firms in the

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[22] A brief description of each technology and why they qualify as a general innovation is relegated to Appendix 1.

[23] Consistent with the data, the cross-sectional distribution of firm sizes, measured by employment or by relative sales, in the model is constant over time. However, we are more interested in the statistics of the growth rates rather than the levels.

COMPUSTAT sample by showing that the upward trend in firm volatility is robust to including a cohort fixed effect and cohort-specific age and size effects.

In our model, the increase in firm volatility is driven by an increase in the turnover of market leaders, λ_s^q. Comin and Philippon [2005] show that various measures of the turnover rate have increased very significantly in the data. Figure 3, for example, plots a measure of the inverse of the turnover rate for the sample of firms in the COMPUSTAT database. Specifically, for each two digit sector and year, firms are ranked by the level of sales per worker. After creating a vector of percentiles for every year in the post-war period, persistence in rankings is measured by computing the correlation between the vectors of rankings in two years, five and ten years apart (i.e. 1950 with 1955 and 1950 with 1960). Repeating the same exercise for all the years in the post-war period results in a time series for the turnover in market leadership.[24]

Both of these statistics indicate that there has been an increase in market turnover. In the early 50s, the correlation of rankings was 0.9 for the 5-year-apart measure and 0.8 for the 10-year-apart measure. These correlations have declined in a fairly monotonic manner reaching 0.71 and 0.66, respectively, at the end of the sample in 2002. These numbers can be used to compute, approximately, the turnover rates in our model, λ_s^q.[25] In the mid 50's, λ_s^q was approximately two percent while, in the mid 90s, it was 2.5 to 3 times higher. Comin and Philippon [2005] conduct similar exercises using other measures of market leadership, such as profit rates and market value. Specifically, they compute the probability that a firm currently ranked in the top 20th percentile of its sector by profits rate or market value is not in the top 20th percentile in five years. These exercises imply that the turnover rate has increased by a factor of five or six during the post-war period.

These estimates of λ_s^q can be used to calibrate the ability of the model to account for the upward trend in firm volatility. Recall that the variance of the growth rates of sales and sales per worker at the firm level depends on the variance of aggregate growth, $var(\gamma_y)$, and on the turnover rate as

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[24]This measure of turnover is unlikely to be affected by entry into the COMPUSTAT sample. This is the case because when there are more firms in sample, it is more likely that a firm is taken over by some other firm, but the decline in the percentile associated with this decline in the ranking will be smaller if there were fewer firms in sample.

[25]See Appendix 3 for the formal derivation.

follows:

$$var(\gamma_{sales_i}) = var(\gamma_y) + \lambda_s^q \left[\left(\frac{1 + \beta(m-1)}{m} \right) \left(\ln(\frac{\beta m}{(1-\beta)}) \right)^2 \right] \qquad (22)$$

$$var(\gamma_{sales_i/L}) = var(\gamma_y) + \lambda_s^q \left[\left(\frac{1 + \beta(m-1)}{m} \right) (\ln(1/\alpha))^2 \right] \qquad (23)$$

In the US, $var(\gamma_y)$ is approximately two orders of magnitude smaller than the variance of firm-level growth and hence irrelevant to the evolution of firm-level volatility. Our previous estimates imply that the turnover rate in COMPUSTAT 1950, λ_{s1950}^q, was approximately two percent. We can then calibrate the terms in square brackets in (22) and (23) to match the initial firm volatility in COMPUSTAT.[26] Based on the direct estimates in Comin and Philippon [2005] and on the evolution of the private R&D intensity in the US, the turnover rate at the end of the sample, λ_{s2000}^q, is at least between 2.5 and 3 times larger than the initial turnover rate.[27] Therefore, the model predicts an increase in firm-level variance by at least a factor of 2.5 or 3. Since in the data, firm variance has increased by a factor of approximately four in the post-war period, the model can explain at least 62 to 75 percent of the increase in the variance of firm-level growth.

Cross-sectional variation in relationship between R&D and firm volatility

Our model has testable predictions about the relationship between R&D and firm volatility. In the context of our model, variation in R&D intensity comes from variation in either $s_{R\&D}$ or $\bar{\lambda}$. First, in environments where R&D is not important (i.e. because $\bar{\lambda}$ is very low), Schumpeterian dynamics should not be very relevant in explaining firm volatility. In these contexts, firm volatility will be driven by other factors and we could observe patterns of the evolution of firm volatility other than the upward trend predicted by our model. Second, in sectors where R&D intensity has significantly increased (i.e. because $s_{R\&D}$ or $\bar{\lambda}$ have increased by more), we should observe larger increases in firm volatility and in the turnover rate. As we shall see next, the data supports these predictions. Third, in other countries where R&D has increased, we should observe a similar increase in firm volatility.

In the US, approximately 95 percent of private R&D expenditure is conducted by publicly traded firms. Non-publicly traded firms represent between 40 and 50 percent of value added, but conduct

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[26]This implies that the sales and sales per worker of market leaders are approximately 70 percent higher than sales or sales per worker of market followers.

[27]Private R&D intensity in the US has increased by a factor of three. In addition, the R&D subsidies have increased and the efficiency of R&D expenses has increased. The linearity of the production function for R&D innovations implies that the turnover rate must have increased by, at least, a factor of three.

a very small part of total R&D. The COMPUSTAT sample used to compute the firm-level volatility and turnover measures reported so far covers only the publicly traded firms. Consistent with our model, firm volatility has increased very significantly in publicly traded firms. Interestingly, Davis et al. [2006] have recently found that non-publicly traded firms have not become more volatile since the mid 1970s. They use employment information in firms that virtually cover all of the US economy and have shown that privately held firms have become less volatile since the mid 1970s. To explain the decline in the volatility of privately-held firms, it is necessary to consider mechanisms that drive down firm volatility and that are particularly relevant for privately held firms. One such force may be the improvement of financial markets that now allow privately-held firms to better insure their risks. Exploring this hypothesis is beyond the goal of this paper, but what is relevant for us is that the lack of an upward trend in the volatility of privately-held firms is consistent with the irrelevance of R&D-driven Schumpeterian dynamics for this group of firms.

One way to further explore the prediction that R&D is associated with firm volatility and turnover is by looking at the sectoral variation within the publicly traded firms. Comin and Philippon [2005] build a panel of annual R&D intensities, turnover rates and average firm volatility in 35 two-digit sectors that cover the US economy from 1950 until 1996. For each sector, they compute the ratio of R&D expenses to total sales, the median and average standard deviation of a 10-year rolling window of growth in sales and in sales per worker, and the persistence in the rankings of sales per worker as in Figure 3. Then they estimate the following regressions:

$$\lambda_{it}^q = \alpha_{0i} + \alpha_1 t + \beta * (R\&D/Sales)_{it} + \epsilon_{it}$$
$$\sigma_{it} = \alpha_{0i} + \alpha_1 t + \beta * (R\&D/Sales)_{it} + \epsilon_{it}$$

These specifications include both a sector-level fixed effect and a time trend to reduce the possibility of spurious correlations between R&D and volatility. In all the cases, they find a positive and statistically significant association between R&D and firm volatility and between R&D and turnover. These estimates are robust to substituting the time trend for time dummies. Further, the estimated coefficient is economically significant. The increase in R&D could account for 60 percent of the increase in firm volatility.

The mechanisms described in our model may also explain the volatility dynamics in other countries. Parker [2006] and Thesmar and Thoenig [2004] have found similar upward trends in the volatility of publicly-traded companies in the UK and France. Interestingly, the periods studied by these authors are periods where, as in the US, there was (i) an important increase in R&D subsidies,

(ii) an important increase in private R&D intensity and (iii) a decline in aggregate volatility.

3.4 Sectoral co-movement

To gain further insight into the evolution of aggregate volatility in our model economy, we can conduct a variance-covariance decomposition of the variance of aggregate growth. Recall that $\gamma_y \equiv \frac{\sum_{n=1}^{N} \gamma_{yn}}{N}$. Therefore,

$$V(\gamma_y) = \frac{V(\gamma_{y_s})}{N} + \frac{N(N-1)}{N^2} cov(\gamma_{y_n}, \gamma_{y_{n'}}) \tag{24}$$

where $cov(\gamma_{y_n}, \gamma_{y_{n'}})$ denotes the covariance between the growth rates of two generic sectors n and n'.

In expression (24), as the number of sectors, N, increases, the importance of the sectoral variance in aggregate variance declines, and aggregate volatility increasingly depends on the covariance of growth across sectors. Sectoral variance, $V(\gamma_{y_s})$, depends on the arrival rate of embodied innovations developed in the sector, λ_s^q, and the arrival rate of general innovations developed in the economy, λ^h. The sectoral covariance, on the other hand, is equal to $\lambda^h(\ln(\delta_h))^2$ and depends solely on the hazard rate for general innovations. Therefore, as the number of sectors increases, the variance of aggregate growth increasingly depends on the intensity of general innovations while the arrival rate of R&D-driven innovations becomes less relevant. Further, increases in $s_{R\&D}$ or $\bar{\lambda}$ lead, unambiguously, to declines in the average covariance of growth across sectors and, if the number of sectors in the economy is large, they also induce declines in aggregate volatility.

The covariance of sectoral growth can be trivially decomposed into the product of the standard deviations and correlation of sectoral growth:

$$cov(\gamma_{y_s}, \gamma_{y'_s}) = \sqrt{V(\gamma_{y_s})V(\gamma_{y_{s'}})} * corr(\gamma_{y_s}, \gamma_{y_{s'}})$$

When looking at actual data, the variance of growth in a sector typically depends on factors such as the sector size and age. To filter out these effects, it is useful to explore the model implications for the correlation of growth across sectors. The correlation of growth between sectors s and s' depends on λ^h and λ_s^q as follows:

$$corr(\gamma_{y_s}, \gamma_{y_{s'}}) = \frac{\lambda^h(\ln(\delta_h))^2}{\lambda_s^q(\ln(\delta_q))^2 + \lambda^h(\ln(\delta_h))^2} \tag{25}$$

Note that the sectoral correlation is increasing in λ^h and decreasing in λ_s^q. It follows from our previous analysis that increases in $s_{R\&D}$ and $\bar{\lambda}$ lead to declines in the correlation of sectoral growth.

Has the correlation of sectoral growth declined?

To explore empirically the evolution of the correlation of growth across sectors, we proceed as follows. First, $corr([\gamma_{s,\tau}]_{t-4}^{t+5}, [\gamma_{j,\tau}]_{t-4}^{t+5})$ is defined as the correlation between the annual growth rate in sectors s and j during the 10-year period centered at t. Then, for every sector s, the average correlation with the rest of the sectors is computed as follows:

$$corr_{s,t}^{\text{sec}} = \sum_{j \neq s} \frac{\omega_j^{\text{sec}}}{\sum_{h \neq s} \omega_h^{\text{sec}}} corr([\gamma_{s,\tau}]_{t-4}^{t+5}, [\gamma_{j,\tau}]_{t-4}^{t+5}) \ , \tag{26}$$

where ω_j^{sec} denotes the average share of sector $j's$ sale in the total sales of the economy. Finally, aggregate correlation is defined as a weighted average of the sectoral correlations:

$$corr_t^a = \sum_s \omega_s^{\text{sec}} corr_{s,t}^{\text{sec}}$$

Figures 4 and 5 show a clear downward trend in the average correlation, $corr_t^a$, of productivity and TFP growth across sectors during the post-war period.[28] Comin and Philippon [2005] show that the decline in the correlation of sectoral growth is driven by a decline in the covariance of growth across sectors, as opposed to a decline in the variance of sectoral growth. This evidence provides further support to our model, which predicts an unambiguous decline in the covariance of growth across sectors and an ambiguous evolution of the variance of growth at the sector level in response to increases in $s_{R\&D}$ and $\bar{\lambda}$.

Imperfect diffusion of GIs

The basic version of the model predicts no cross-sectional variation in the correlation of growth between sectors because GIs are adopted immediately in all sectors. Now we enrich the model by relaxing the assumption that GIs are applicable to all the sectors in the economy. Specifically, we introduce two new assumptions: (i) the intermediate goods producers of a given sector can freely adopt all the general innovations developed in the sector and (ii) the random variable that determines whether a general innovation is suitable to be adopted in a sector other than the one in which it was developed follows a Bernoulli distribution that is independent across sectors and innovations.

Let ψ denote the probability that a general innovation is adopted in a sector other than the one in which it was developed. The previous assumptions imply that the arrival rate of general innovations in sector n is equal to $\lambda_n^h + \psi(N-1)\overline{\lambda}_{(-n)}^h$, where λ_n^h denotes the rate of development of

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[28]See Comin and Philippon [2005] for more on this downward trend.

GIs in sector n, and $\overline{\lambda}^h_{(-n)}$ denotes the average rate of development of GIs in the sectors other than n.[29] The covariance of growth in two sectors, n and n', depends on how frequently they adopt the same GIs. Clearly, the probability of such a coincidence is higher for the technologies developed in either of the sector than for technologies developed in other sectors. Specifically, the probability that a technology developed in n (or n') is suitable for adoption in n' (or n) is ψ. The probability that a technology developed in a sector other than n and n' is suitable for adoption in n and n' is $\psi^2 < \psi$. Thus, the covariance between the growth in sectors n and n' is

$$cov(\gamma_{y_n}, \gamma_{y_{n'}}) = \left[\psi(\lambda^h_n + \lambda^h_{n'}) + \psi^2(N-2)\overline{\lambda}^h_{-(n,n')} \right] (\ln(\delta_h))^2,$$

where $\overline{\lambda}^h_{-(n,n')}$ denotes the average rate of development of general innovations in the sectors other than n and n'. Averaging over all the sectors n', the average covariance of the growth of sector n with the growth rate in the other sectors is

$$cov_n = \left[\psi(\lambda^h_n + \overline{\lambda}^h_{(-n)}) + \psi^2(N-2)\overline{\lambda}^h_{(-n)} \right] (\ln(\delta_h))^2 \qquad (27)$$

To explore the cross-section variation in this covariance, suppose, for example, that the efficiency of investments in the development of embodied innovations, $\overline{\lambda}$, varies across sectors. We know that, in sectors with higher values of $\overline{\lambda}$, leading firms have fewer incentives to develop GIs. Given the imperfect diffusion of GIs, those sectors with a higher $\overline{\lambda}$ adopt fewer GIs and co-vary less with the rest of the sectors. Hence, there is a negative cross-sectoral relationship between $\overline{\lambda}$ and the average covariance of a sector. We also know from our previous analysis that there is a positive relationship between $\overline{\lambda}$ and R&D intensity. Therefore, the model implies a negative cross-sectional relationship between R&D intensity in a given sector and the average covariance of growth in that sector.

Using the same logic as in section 2.4, it follows that the variance of growth in sector n is

$$var_n = \lambda^q_n (\ln(\delta_q))^2 + (\lambda^h_n + \psi\overline{\lambda}^h_{-n}) (\ln(\delta_h))^2 \qquad (28)$$

The average correlation of growth between sector n and the other sectors, then, is

$$corr_n = \frac{cov_n}{\sqrt{var_n \overline{var_{-n}}}} \qquad (29)$$

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[29]If GIs do not diffuse perfectly across sectors, sectors that develop fewer GIs also implement fewer GIs. As a result, the model predicts that in sectors with more R&D investments, the contribution to growth from GIs will be lower. Since R&D investments have a direct positive effect on growth, the resulting relationship between R&D and growth will be ambiguous. This is consistent with the insignificant relationship found between R&D intensity and TFP growth found by Jones and Williams [1998] in a panel of sectors.

where $\overline{var_{-n}}$ is the average variance across sectors other than n. Given the negative effect of $\bar{\lambda}$ on cov_n and the positive effect of $\bar{\lambda}$ on var_n, the model implies a negative cross-sectional relationship between R&D intensity in sector n and the sectoral correlation of growth (expression 29).

This prediction is very important because it allows us to test (albeit indirectly) the negative effect that R&D has on the investments in the development of GIs. To test this prediction, we estimate the following specification:

$$corr_{s,t}^{\text{sec}} = \alpha_s + \beta t + \gamma RD_{s,t} + \epsilon_{st} \tag{30}$$

where $corr_{s,t}^{\text{sec}}$ is defined in expression (26), and $RD_{s,t}$ denotes the R&D intensity in sector s at time t. The first and seventh columns in Table 2 report the estimate of γ when $corr_{s,t}^{\text{sec}}$ is measured by the correlations of productivity and TFP growth, respectively. In both cases, R&D is associated with a significant decline in correlation. Specifically, the estimates of γ are -3.3 for productivity and -2.5 for TFP growth, with $p-values$ of two percent. This implies that the increase in R&D is associated with a decline of between 7.5 and 10 percentage points of the 10 and 25 percentage point decline observed in the sectoral correlation of TFP or productivity growth. These estimates are robust to replacing the time trend with year dummies.

Columns 2 and 8 of Table 2 replace R&D intensity with a sector's firm-level volatility as the explanatory variable. Consistent with the model, higher firm-level volatility in a sector is also associated with lower correlation of sectoral growth with other sectors.[30] This shows that the trends in firm and aggregate volatility are not simply a coincidence: A common component can account for an important part of both trends. This is not the case in current models of firm heterogeneity[31] because the interactions between firms embedded in these models are not adequate: most of them are partial equilibrium models and treat firms as independent entities. Even though more recent versions of these models have incorporated general equilibrium interactions, they seem insufficient to generate the co-movement patterns that drive the diverging trends in volatility. In this sense, our model emphasizes a particular mechanism that introduces strong interactions between firms and that has aggregate implications for first and second moments of growth.

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[30]These results are robust to restricting the sample to the private sectors, using other variables to measure firm volatility, using the median instead of the average to measure the firm volatility in the sector, using a measure of turnover in the sector as the independent variable instead of a measure of firm volatility and including a time trend or no trend at all instead of the year fixed effects.

[31]For example, Bertola and Caballero [1990] and Gabaix [2005].

In principle, the estimated effect of R&D on sectoral correlation could be driven by omitted variable bias. For example, it could be argued that R&D intensity may be related to the sensitivity of sectors to aggregate shocks. However, to the extent that this sensitivity has not changed significantly over time, this effect should be captured by the sector fixed effect. One kind of aggregate shock that has been related to the decline in aggregate volatility is oil price shocks. To test if the omission of the sensitivity of the sector to oil prices is biasing our estimates of γ towards significance, we run regression (30), controlling for the share of energy in the sector. Columns 3, 4, 9 and 10 show that including the share of energy in the control set has no effect on the estimates of the effect of R&D or firm volatility on sectoral correlation. Further, in columns 5, 6, 11 and 12, we show that these results hold when we restrict our sample to the sectors other than the energy sector.

Another explanation for the decline in aggregate volatility is proposed by Thesmar and Thoenig [2004]. Building on Arrow [1971], they claim that financial innovation can lead to greater risk taking by firms, but also to fewer aggregate credit crunches. Their analysis implies that sectors that benefit more from financial innovation are going to experience larger declines in their correlation with the rest of the economy because of the lower exposure to credit crunches and binding collateral constraints (Bernanke, Gertler and Gilchrist [1996]). Lower exposure to financial stress will lead to lower aggregate volatility. Comin and Philippon [2005] empirically explore this hypothesis by including in regression (30) two additional controls that proxy for the degree of financial dependence in the sector: the amount of debt and equity issued in the sector, each divided by the total sales in the sector. In contrast to R&D, both measures of financial market dependence are positively associated with the correlation of sectoral growth (albeit this relationship is statistically insignificant). Therefore, improvements in financial markets do not seem to be a major force decreasing aggregate volatility. More importantly for our purposes, the negative effect of R&D on the correlation of sectoral growth is not driven by the omission of measures of external financial dependence.

In summary, the existing theories proposed to explain the decline in aggregate volatility do not seem to be driving the negative relationship between R&D and the correlation of sectoral growth. This reinforces the view that, as suggested by our model, this relationship is causal.[32]

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[32]Philippon [2003] argues that an increase in competition in the goods market leads firms to adjust their prices faster, which reduces the impact of aggregate demand shocks. While intuitively appealing, Philippon [2003]'s is a within-sector explanation with no implication for the evolution of sectoral co-movement.

3.5 Calibration

Section 3.3 showed econometric evidence in favor of the model's mechanisms. In what follows, we use a calibration to assess the model's quantitative ability to generate the observed evolutions of aggregate growth and volatility. One way of doing this would be to calibrate all the parameters of the technology to develop general innovations and use them in the model along with the evolution of R&D-style innovations to pin down the evolution of λ^h. However, the lack of independent information to calibrate ρ_h and $\bar{\lambda}^h$ makes this route unfeasible.

Alternatively, we assume that the post-war decline in the correlation of productivity growth across sectors is driven by the decline in the development of general innovations. We then use this information to pin down the evolution of λ^h and explore the model's implications for the evolution of productivity growth and aggregate volatility in 1950 and 2000.

Specifically, we use the following 6-step procedure:[33]

(i) Calibrate the initial turnover rate, λ^q_{s1950}, to match the initial correlation of rankings in Figure 3. As shown in Appendix 3, this yields an estimate for λ^q_{s1950} of two percent.

(ii) Using the value of λ^q_{s1950} and the initial correlation and variance of sectoral growth (0.5 and 0.0005, respectively), pin down the values for $\lambda^h_{1950} * (\ln(\delta_h))^2$, $\lambda^q_{s1950} * (\ln(\delta_q))^2$ and $\ln(\delta_q)$.

(iii) Using the average initial growth rate of productivity (0.025), calibrate $\ln(\delta_h)$ and λ^h_{1950}.

(iv) Calibrate the final turnover rate, λ^q_{s2000}, to 2.5 times the initial turnover rate (i.e. five percent).

(v) Using the final correlation of sectoral growth (0.25) and the calibrated value of $\ln(\delta_h)$, compute the final arrival rate for general innovations (λ^h_{2000}).

(vi) With this information and the number of sectors (35), compute the final expected growth rate of productivity, $E\gamma_{y2000}$, and the initial and final variance of aggregate productivity growth ($V\gamma_{y1950}, V\gamma_{y2000}$).

Table 3 shows the actual data, as well as the model's predictions for the final expected growth rate of labor productivity and the initial and final standard deviations of aggregate productivity growth.

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[33] A more detailed explanation of this calibration is presented in Appendix 2.

Moment	Data	Model
$E\gamma_{y2000}$	0.02	0.017
$V\gamma_{y1950}$	$4*10^{-4}$	$2.56*10^{-4}$
$V\gamma_{y2000}$	$1.44*10^{-4}$	$1.44*10^{-4}$
Increment in $V\gamma_y$	$-2.56*10^{-4}$	$-1.12*10^{-4}$

Table 3

This simple calibration illustrates two things. First, the model can easily explain the lack of a relation between R&D and productivity growth at the aggregate level. Despite the substantial increase in R&D expenses, the model predicts a small decline in expected productivity growth for the year 2000.[34] Second, the mechanisms emphasized by the model can account for a significant fraction of the decline in aggregate volatility. The model underpredicts the initial level of aggregate volatility; however, this is not surprising given that the only type of aggregate disturbances are technology shocks, a scenario that is clearly unrealistic. The predicted decline in the variance of aggregate productivity growth, however, represents over 40 percent of the observed decline in aggregate volatility. This estimate must be taken with caution because of the identification assumption that the decline in the co-movement of sectoral growth is entirely driven by the decline in the development of general innovations. However, this assumption may not be far from the truth, given the important negative effects of R&D on sectoral correlation that we estimated above. Moreover, this rough estimate of the contribution of our endogenous technological change mechanisms to the decline in aggregate volatility are consistent with Stock and Watson [2003]'s conclusion: after considering the effects of a more active monetary policy and lower commodity price shocks, 50 percent of the decline in aggregate volatility must be due to less volatile technology shocks.

4 Conclusion

A thorough understanding of the forces that drive growth in the US is an essential prerequisite for undertaking informed policy recommendations. This paper has presented a new growth theory for the US that is superior to current models because it overcomes two hurdles that we believe any valid theory must overcome. First, it explains the relationship between R&D and productivity growth

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[34]This calibration implies that about 90 percent of aggregate productivity growth was driven by general innovations in 1950. this fraction declined to 67 percent by 2000.

at the firm-level, as well as the lack of a relationship between the two at the sector and aggregate level. Second, it explains the evolution of the second moments of productivity growth at the firm and aggregate level. In particular, it explains the diverging trends in aggregate volatility and in the volatility of publicly traded firms, and the fact that the decline in aggregate volatility is due to a decline in the correlation of sectoral growth.

In addition to being consistent with these facts, this paper has also provided evidence on the importance of the mechanisms emphasized by the model. In particular, it has showed that firm volatility and market turnover are positively associated with R&D. Perhaps most importantly, it has showed that sectors that have experienced higher increases in R&D have also experienced greater declines in the correlation of their growth with the rest of the economy. This indicates that there is a strong connection between aggregate and firm volatility. Furthermore, it supports the view that this connection operates mainly through the effect of R&D on the decline in the co-movement of growth across sectors. By no means does this imply that all of the decline in aggregate volatility (or increase in firm volatility) is driven by this common component associated with R&D; however, it does show that this component is an important piece of the puzzle.

Finally, our model suggests that sectoral co-movement is driven by the development of general innovations, and the decline of their importance in growth is at the root of the observed dynamics for the first and (to some extent) second moments of aggregate productivity growth. Since general innovations are, by-and-large, not included in the NSF measure of R&D and since there is no measure of the investments made to develop them or the number of general innovations developed, we are unable to directly explore the determinants of general innovations. Instead, in this paper, we have evaluated our theory of general innovations by exploring the validity of its implications for the second moments of growth. In this way, studying the second moments of the growth process can make up for the lack of the current data about general innovations.

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Appendix 1: Mathematical Details

First order conditions for the multisector case

In the symmetric equilibrium, the conditions that determine the investment in R&D and GIs by each type of firm are the following:

$$1 - s_{R\&D} = \bar{\lambda}(\delta_q^{1/N} v^l - v^f) \tag{LqM}$$

$$c'(\lambda^h/N) = (\delta_h - 1)v^l \tag{LhM}$$

Proof that $v^f = 0$

The arbitrage condition for R&D implies that

$$0 = \frac{\lambda_s^q}{m}\left(\frac{-(1 - s_{R\&D})}{\bar{\lambda}} + \left(\delta_q^{1/N} v^l - v^f\right)\right). \tag{31}$$

Note that $\frac{\lambda_s^q(1 - s_{R\&D})}{m\bar{\lambda}}$ is equal to the cost of R&D investments for the follower.

In the symmetric equilibrium, the value of a standard intermediate good producer can be expressed as:

$$rv^f = -\frac{\lambda_s^q(1 - s_{R\&D})}{m\bar{\lambda}} + \frac{\lambda_s^q}{m}(\delta_q^{1/N} v^l - v^f) + \lambda_s^q(\frac{m-1}{m})(\delta_q^{1/N} - 1)v^f + \lambda_s^q(N-1)(\delta_q^{1/N} - 1)v^f + \lambda^h(\delta_h - 1)v^f \tag{32}$$

Plugging expression (31) into (32) yields:

$$rv^f = \lambda_s^q(\frac{m-1}{m})(\delta_q^{1/N} - 1)v^f + \lambda_s^q(N-1)(\delta_q^{1/N} - 1)v^f + \lambda^h(\delta_h - 1)v^f$$

This equation implies that $v^f = 0.\square$

Multisector

As in the one sector model, λ^h declines with $\bar{\lambda}$ and $s_{R\&D}$. To show this, we just have to combine equations (LqM) and (LhM) to obtain the following expression for λ^h :

$$c'(\lambda^h/N) = \frac{(\delta_h - 1)(1 - s_{R\&D})}{\bar{\lambda}\delta_q^{1/N}} \tag{33}$$

Plugging in the functional form specified for $c(.)$, results in the expression for the rate of arrival of general innovations:

$$\lambda^h = N\bar{\lambda}^h\left(\frac{(1 - s_{R\&D})\rho_h\bar{\lambda}^h(\delta_h - 1)}{\bar{\lambda}\delta_q^{1/N}}\right)^{\frac{\rho_h}{1 - \rho_h}} \tag{34}$$

Plugging back the expression for λ^h (34) into equation (LqM) and using expression (15) allows us to solve for the arrival rate of embodied innovations at the sector level, λ_s^q:

$$\lambda_s^q = \frac{\left[\frac{\bar{\lambda}\delta_q^{1/N}}{1 - s_{R\&D}}(1 - \alpha)\varkappa^l - r + \lambda^h(\delta_h - 1)\left(1 - \rho_h/N\right)\right]}{1 - (\delta_q^{1/N} - 1)(N - 1)} \tag{35}$$

Substituting in (34) for λ^h we obtain

$$\lambda_s^q = \frac{\left[\bar{\lambda}\delta_q^{1/N}/(1-s_{R\&D})\right](1-\alpha)\varkappa^l + (N-\rho_h)\left[\rho_h\bar{\lambda}^h(\delta_h-1)\left[(1-s_{R\&D})/(\bar{\lambda})\right]^{\rho_h}\right]^{1/(1-\rho_h)} - r}{1-(\delta_q^{1/N}-1)(N-1)} \quad (36)$$

Differentiating with respect to $s_{R\&D}$ or $\bar{\lambda}$ it follows that λ_s^q increases with these parameters iff Condition 2 holds.

Condition 2: $\dfrac{(1-\alpha)\varkappa^l - \left(\frac{\rho_h\theta^h(\delta_h-1)(1-s_{R\&D})}{\theta^q\,\delta_q^{1/N}}\right)^{1/(1-\rho_h)}\left(\frac{N-\rho_h}{1-\rho_h}\right)}{1-(\delta_q^{1/N}-1)(N-1)} > 0$

Private R&D expenditures as a share of aggregate output, n_q, is equal to $\frac{\lambda_s^q(1-s_{R\&D})}{\bar{\lambda}}$. Multiplying expression (36) by $(1-s_{R\&D})/\bar{\lambda}$ and differentiating with respect to $s_{R\&D}$ or $\bar{\lambda}$ it follows that n_q increases with these parameters iff

$$\frac{r - \frac{(N-\rho_h)(\delta_h-1)\bar{\lambda}^h}{(1-\rho_h)}\left(\frac{(1-s_{R\&D})\rho_h\bar{\lambda}^h(\delta_h-1)}{\bar{\lambda}\delta_q^{1/N}}\right)^{\frac{\rho_h}{1-\rho_h}}}{1-(\delta_q^{1/N}-1)(N-1)} > 0$$

Using the expression for λ^h in equilibrium (34) this can be expressed as Condition 3

Condition 3: $\dfrac{r-\lambda^h\frac{(N-\rho_h)(\delta_h-1)}{N(1-\rho_h)}}{1-(\delta_q^{1/N}-1)(N-1)} > 0.$

Proposition 2 summarizes the conclusions from this analysis.

Proposition 2 *In response to increases in $s_{R\&D}$ and $\bar{\lambda}$, the arrival rate of general innovations, λ^h declines; if Condition 2 holds, the arrival rate of R&D innovations, λ_s^q, increases; if Condition 3 holds, the share of GDP spent in private R&D, n_q, increases.*

Appendix 2: Discussion of General Technologies

We present here several examples of inventions that meet the two criteria that characterize our notion of general technologies. First, while these innovations originated in a particular context, the general nature of the idea underlying them meant they were applied to many economic activities across industries and sectors. Second, the disembodied nature of these innovations meant that they could not be patented. As a result, firms could not appropriate the benefit from these innovations when competitors, whether within or across industries, adopted them. As the model predicts, in many cases, these GIs were undertaken by the market leaders.

I. Production Design

A. Mass production of cars and Ford's assembly line

Mass production first originated in the automobile industry in the United States in 1901. American car manufacturer Ransome Eli Olds (1864-1950) invented the basic concept of the assembly line and mass produced the first automobile, the Curved Dash Oldsmobile. Henry Ford (1863-1947) invented an improved version of the assembly line by installing the first conveyor belt-based assembly line in his car factory in Ford's Highland Park, Michigan plant, around 1913-14. The assembly line reduced production costs for cars by reducing assembly time.

The philosophy of mass production was simple. Fixed overhead costs were spread out over larger and larger volumes of production, thus lower and lower prices became possible. This strategy that characterized mass production was to become the defining characteristic of American industry throughout the twentieth century. The Ford Motor Company was of course, at the time, one of the top two car manufacturers in the country.

B. Scientific Management

Scientific management is the study of relationships between workers and machines. Frederick Taylor, regarded as the Father of Scientific Management, published Principles of Scientific Management in 1911, in which he proposed work methods designed to increase worker productivity. Taylor realized that organization productivity could be increased by enhancing the efficiency of production processes. This involved breaking down each task to its smallest unit and to figure out the one best way to do each job. Emphasis was laid on ensuring the worker indulged in only those motions essential to the task. Taylor looked at interaction of human characteristics, social environment, task, and physical environment, capacity, speed, durability, and cost. The overall goal was to remove human variability.

The results were profound. Productivity under Taylorism went up dramatically. In a famous

experiment on the output of a worker loading pig iron to a rail car, Taylor increased the worker's output from 12 to 47 tons per day. New departments arose such as industrial engineering, personnel, and quality control. There was also growth in middle management as there evolved a separation of planning from operations. Rational rules replaced trial and error; management became formalized and efficiency increased. This model, based on merit and unquestioned authority, was a dramatic improvement over earlier models of organization.

C. Management Consulting

McKinsey and Co. was one of the first management consulting firms established in 1923 in Chicago. While the consulting industry had originated before then, the introduction of McKinsey's innovative approach to analyzing and solving problems constituted an important general technology. The McKinsey way of consulting can be decomposed in the following three steps. First the consultant gathers as much factual information about the client's organization as possible. Second, after a thorough analysis of the facts, an initial hypothesis is determined, to be tested with the client. Finally, a set of recommendations are presented to the client. These recommendations are limited to what can be realistically done given the resources of the client, the consulting firm and the amount of time required. Further, the recommendations are proposed along with milestones to be achieved as intermediate steps towards the ultimate target.

D. Multi-Divisional Structure

Faced by stiff competition from Ford Motors, General Motors, at the time the leading car manufacturer in the country, helped pioneer the Multi-divisional organizational structure in the 1920's. The organization was divided into several divisions, each responsible for the production of the car and its marketing to the assigned market segment. Each was to have its own managerial team with complete autonomy over its operational decisions. The central office's role would be restricted to evaluate each divisions performance and coordinate overall strategy. The system helped General Motors transition from a chaotic organization into a streamlined and efficient competitor in the automobile industry. As a result of the organizational change, GM's market share grew to 45 percent in 1940 from 11 percent in 1921. The multi-divisional structure has since become a standard organizational feature of the corporate world, enabling many companies to efficiently produce a wide array of products.

E. Just-in-Time Manufacturing

Toyota, the leading automobile manufacturer in Japan and one of the largest car manufacturers in the world, introduced the 'Just-in-Time' system of manufacturing in the 1950's. Elimination of

the inventories meant that Toyota had to tighten coordination between successive stages of production. The lack of inventories to buffer disruptions between adjacent stages of production meant improvements in the reliability of every step of the process. The new system meant fewer interruptions in the production process, faster identification of flaws in the cars and better communication with suppliers. The success of its manufacturing system has helped it and other corporations achieve world success in their respective industries.

II. Human Resources Management

A. The Hawthorne Studies

Beginning in 1924 and continuing until 1933, the Western Electric Company sponsored a series of experiments for studying worker productivity and morale at its Hawthorne Works near Chicago. As a market leader – the company was the manufacturing arm of AT&T, the leading long distance provider for most of the 20th century – the company initiated these studies to determine the effect of working conditions on productivity.

The studies collectively highlighted the importance of positive worker attitude and provided information about factors other than physical working conditions that contribute to productivity. In particular, researchers found that a group norm regarding the rate of productivity significantly affects individual performance, and that informal authority from influential group members often overrode formal authority from the supervisor. A major outcome of the interviews was to teach supervisors how to handle employee complaints. Smaller work groups and greater freedom were found to be the greatest drivers of the observed increase in productivity. These findings on the relationship between improvements in productivity and better employee morale were applied to a wide ranging group of employment settings.

B. Industrial Psychology

Industrial psychology involved the testing of morale and efficiency at businesses, industrial and military organizations. Edwin A. Fleishman (1953) undertook what was a typical project of its time at the International Harvester Company, one of the leading industrial corporations in the United States. Fleishman studied the relationship of training programs on the leadership of supervisors and their sensitivity to and consideration of the needs and feelings of subordinates. While supervisors showed an initial response to the training program by being more considerate towards their subordinates, in due course, they reverted back to their original behavior. The reversal of the behavior was attributed to the culture or climate of the department the subjects came from. In what came to be known as a critical point in organizational change, the study highlighted the difference

between focusing on the individual and focusing on contextual variables (such as group norms and organizational culture).

C. Survey Feedback

The organizational survey feedback method first showed up in the late 1940's. Questionnaires were being used to systematically assess employee morale and attitudes in organizations. Floyd Mann's study in 1957, guided by Rensis Likert, went a long way in developing what we now know as the Survey Feedback method. The method involved data collection by questionnaire to determine employee's perceptions of the management of the organization. The second aspect of the method was reporting the results back to the employees who answered the questionnaire. Once the results of the survey had been conveyed, managers, using the help of the subordinates, would chart out a plan to undertake positive changes in areas of concern as reflected in the survey results. The study emphasized that the effectiveness of the method relied on what the manager did with the information from the survey. Positive changes occurred when the manager discussed the results with his subordinates

D. Sensitivity Training

Sensitivity training refers to small group discussions where the primary, almost exclusive source of learning is the behavior of the group members themselves. Participants receive feedback from one another regarding their behavior in the group. Sensitivity training, also known as T-groups, became the earliest tool of what came to be known as organizational development. Kurt Lewin discovered the concept when undertaking a training workshop in Connecticut in 1946. He was asked to conduct a workshop that would help improve community leadership in general and interracial relationships in particular. Lewin brought in trainers and researchers and along with the participants engaged in lectures, role play and general group discussions. In the evenings, the trainers and researchers would evaluate the events of the day. The workshop acquired its significance however when participants happened to observe and participate in the evaluations as well. Participants began to object to the interpretation of their behavior on several occasions. The observation by the participants resulted in the three-way discussion among the researchers, trainers and participants. The participants in turn became more sensitive to their own behavior in terms of how they were being perceived by others and the impact their behavior was having on others. Carl Rogers labeled this mode of learning as "perhaps the most significant social invention of the century".

III. Credit/banking

Improvements in the credit and banking sector have, both directly and indirectly, resulted in

improvements in businesses across all sectors of the economy.

A. Credit card

The credit card industry began in the United States in the 1930s when oil companies and hotel chains began issuing credit cards to customers for purchases made at their own gas stations and hotels. The bank credit card was introduced in the 1950s. While store or book credit allowed irregular repayment and installment loans required regular repayment, the credit cards of the early 1950s combined both types of credit. In 1951, Franklin National Bank released the first revolving charge card. At the time, it was one of the largest banks in the United States. Using the revolving card a customer could borrow money, repay it and borrow again as long as the borrower remained under their credit limit. The organizations that are now called Visa and MasterCard sprang up to create interchange, a nation-wide system designed to settle credit card transactions between banks, merchants and customers.

Today, with help from Visa and MasterCard, financial institutions are marketing credit cards to people all over the world. Credit cards have allowed consumers to carry debt, something that previously required a bank loan – a much more intensive process than a credit-card approval. Credit cards have been the primary instrument that fueled international consumerism and high consumer debt, each of which has spurred multiple trickle-down industries.

B. Credit Reporting

In Manhattan during the 1830s, Lewis Tappan developed extensive credit records while handling credit in his brother's wholesale silk business. He then extended this aspect of the business to other suppliers who needed information. He contracted with agents and correspondents throughout the country to "gossip" about the solvency, prospects, and character of local businesses. He established R. G. Dun & Co. (which later on merged with its biggest rival Bradstreet to form Dun & Bradstreet, the largest credit reporting entity in the world), an information hub that could rapidly service new inquiries and add new information and in the process helped found the business of credit reporting in the United States. The credit reporting system and improvements in the same have helped firms minimize risk. With access to the credit history of their customers, firms could target only consumers meeting their criteria of their acceptable levels of risk. It has helped institutions reduce bad debts and streamline their bottom lines.

C. ERMA and MRCI

During the 1950s, Bank of America, one of the largest banks in the nation at the time, initiated the Electronic Recording Method of Accounting computer processing system or ERMA, a

project to computerize the banking industry. ERMA computerized the manual processing of checks and account management and automatically updated and posted checking accounts. MICR, the magnetic ink character recognition, was also part of ERMA. MICR allowed computers to read special numbers at the bottom of checks that allowed computerized tracking and accounting of check transactions. These inventions led to a more efficient banking system.

D. Electronic money

The widespread use of electronic currency began with the automated clearinghouse (ACH), set up by the US Federal Reserve in 1972 to provide the US Treasury and commercial banks with an electronic alternative to check processing. Payments made today in nearly all of the deposit currencies in the world's banking systems are handled electronically through a series of inter-bank computer networks.

Although banks have been able to move currency electronically for decades, only recently has the average consumer had the capability to use electronic transfers in any meaningful way. The increasing power and decreasing cost of computers — coupled with advancements in communication technology that make global interaction available at vastly reduced costs — have together made the digital transfer of funds a reality for millions of individuals around the world.

IV. Computer / Software / Internet

While innovations in this category clearly exhibit the characteristics of general technologies, they are included in the NSF's definition of Basic Research. In this sense, they are exceptions to the rule: general technologies are not R&D.

A. Arpanet

Arpanet was created during the Cold War to meet the need for large powerful computers in the country that were networked with each other to overcome geographic differences. Four computers were the first connected in the original ARPAnet. As the network expanded, different models of computers were connected, creating compatibility problems. The solution rested in a better set of protocols called Transmission Control Protocol/Internet Protocol (TCP/IP) designed in 1982. To send a message on the network, a computer broke down its data into IP (Internet Protocol) packets, like individually addressed digital envelopes. TCP (Transmission Control Protocol) ensured the packets were delivered from client to server and reassembled in the right order. Several other innovations occurred under ARPAnet - email (or electronic mail), the ability to send simple messages to another person across the network (1971); telnet, a remote connection service for controlling a computer (1972); and file transfer protocol (FTP), which allowed information to be sent from one

computer to another in bulk (1973). Each of these inventions has made it significantly easier for businesses to communicate and share information both across and within each other.

B. Fortran

John Backus and a group started to design the FORmula TRANslator System, or FORTRAN at IBM in 1954. IBM has of course been the leading computer technology corporation of the 20th century. Prior to the introduction of FORTRAN, computers were slow and unreliable and all programming was done in machine or assembly code. The authors of FORTRAN claimed that the resulting code would be as efficient as handcrafted machine code. Work on FORTRAN was completed in 1957 and for many years after, FORTRAN dominated programming, and was the common tongue for computer programmers.

C. Computers

Conrad Zuse invented the first freely programmable computer, the Z1 Computer, in 1936. However, the computers that are an integral part of all commercial activity today are the result numerous related innovations since then. From the creation of the transistor in 1947, the first commercial computer in 1951 to the introduction of the integrated circuit in 1958 and the microprocessor in 1971, several innovations have come together to integrate the use of computers in our lives. This general technology has had an unparalleled impact on all commercial activity – from the organization of businesses, to record keeping, to communication and the speedy automation of otherwise time consuming tedious tasks. Every business regardless of industry has adopted the use of computers in order to improve production and increase efficiency.

D. Internet Search Engines

The first Internet search engine, called 'Archie', was created in 1990 by Alan Emtage, a student at McGill University. Since then numerous search engines have enabled people to search for and gather information in a more inexpensive and convenient manner than ever before. Information is used to produce virtually any good and service. Search engines increase the efficiency in the process of gathering information. Thus, search engines increase productivity in a wide range of sectors. Whether innovations in search engines are appropriable is more debatable. Clearly, they are not embodied and non-patentable. However, the effectiveness of the search engine and the advertising revenues depend in part on the number of users. Since users may respond to innovations in the search engine a part of the revenues created by these innovations will be appropriable. Having said that, we still believe that, the lack of patents makes the concept of search engines a general innovation.

V. Trade

The introduction of malls and department stores constitute a general technology because improvements in the distribution of goods and services benefited a variety of industries in the economy.

A. The Mall

The first shopping mall was the Country Club Plaza, founded by the J.C. Nichols Company and opened near Kansas City, MO, in 1922. J.C. Nichols Company was a prominent commercial and residential real estate developer. The first enclosed mall called Southdale opened in Edina, Minnesota in 1956. In the 1980s, giant mega malls were developed. Mega malls revolutionized the retail industry. The geographical concentration of hundreds of stores offering goods and services catering to every walk of life meant consumers could now indulge in a one-stop shopping experience. Since their inception, mega malls have helped all retail outlets, independent of their industry, cater to a much larger population of consumers.

B. Department Stores

In 1877, John Wanamaker opened "The Grand Depot", a six story round department store in Philadelphia. He is credited with creating the first White Sale, modern price tags, and the first in-store restaurant. He also pioneered the use of money-back guarantees and newspaper ads to advertise his retail goods. Along with the retail giants of the day including, Marshall Field in Chicago, Alexander T. Steward in New York, Wanamaker was one of the first to discover the vast power of buying wholesale and how it could cut costs to reduce retail prices.

C. Internet Shopping

Shopping on the internet has opened a new portal for doing business for virtually every type of business in every industry. Every day, millions of dollars are transacted in exchange for every imaginable product or service through the internet. The wide applicability of this invention is evident. Similar to internet search engines, shopping on the internet is also not perfectly non-appropriable. Specific websites that create a brand image in creating a market for purchase and sale of goods and services (e.g. ebay, shopping.com) are able to extract a revenue stream from the transactions. However, the concept of a website used to create a virtual marketplace for transactions is a general innovation because it is not patentable and any individual or business is free to create such a website.

VI. Marketing

A. Coupons

Coupons were first introduced in 1895 by Asa Candler, owner of the Coca-Cola Company, one of

the largest manufacturer, distributor and marketer of nonalcoholic beverages at the time in the US. Candler placed coupons in newspaper for a free Coke from any fountain - to help promote the new soft drink. Today coupons are an integral part of promotion campaigns for every business. Cut-out coupons are included in newspapers as an advertising tool. They are also embedded in products so as to encourage repeat purchases. Over the years, coupons have been adopted as marketing tool across industries to help businesses build a brand image and target their customers in a more efficient manner.

B. Mail Order Catalog

Aaron Montgomery Ward invented the idea of a mail order catalog. As a traveling salesman, he realized that his rural customers could be better served by mail-order, a revolutionary idea at the time. The first catalog consisted of a single sheet of paper with a price list, 8 by 12 inches, showing the merchandise for sale with ordering instructions. Today, mail-order catalogs are an integral part of major retail businesses. They have helped businesses across sectors to tap into the market of consumers who are unwilling or unable to access the retail outlets. Serving as an effective marketing medium, mail order catalogs have opened up new segments of consumers previously unavailable to these businesses.

VII. Chemical Engineering

Arthur D. Little, Inc., one of the first consulting firms founded in 1886 that became a pioneer and industry leader in the chemical consulting industry, introduced the concept of the 'unit operations' in 1915. It referred to activities such as mixing, heating, filtering, verizing among others that featured in any chemical process. Chemical engineering research was directed towards the improvement of such processes. The concept of unit operations was instrumental to the success of Pre-production Planning. Pre-production made possible the transition from the confines of the laboratory to large scale production and was critical to the development of chemical engineering. In its stages of infancy, chemical engineering research was applied to the paper and pulp industry and contributed to the at the time new sulfite process of converting wood pulp into paper. In more recent times, advances in the field have had a substantial impact across several sectors, perhaps most noticeably on the petrochemical industry.

Appendix 3: Discussion of Calibration

In this appendix, we discuss in greater detail the calibration conducted in section 3.5 to explore the model predictions for aggregate volatility and growth. In particular, we explain each of the 6 steps.

(i) and (iv) calibrate the turnover rates (λ^q_{s1950} and λ^q_{2000}) to match the initial correlation of rankings in figure 3.

We proceed in two steps. First, we use the model to compute the productivity percentiles of the leader and the followers in a sector. Second, we use the model to compute the expected correlation of the percentiles over time as a function of λ^q_s.

At any given moment in time, the market leader has higher productivity than the m followers. These in turn have the same level of sales per worker. The percentile of the leader $p_l = 1/(2(m+1))$, while the percentile of the followers $p_f = (m+2)/(2(m+1))$. Let's denote by $\overrightarrow{p_t}$ the $(m+1)$ x 1 vector that contains the percentile of each firm at year t. The mean and variance of $\overrightarrow{p_t}$ are constant and given by $\mu_p = 0.5$ and $Var_p = m/(2(m+1))^2$, respectively.

The correlation of percentiles between years t and $t+1$ is given by the following expression:

$$
\begin{aligned}
Corr(\overrightarrow{p_t}, \overrightarrow{p_{t+1}}) &= \frac{Cov(\overrightarrow{p_t}, \overrightarrow{p_{t+1}})}{Var_P} \qquad (37) \\
&= \frac{E\left[\sum_{i=1}^{m+1}(p_{it} - \mu_p)(p_{it+1} - \mu_p)/(m+1)\right]}{Var_p},
\end{aligned}
$$

where E denotes the expectation of $\overrightarrow{p_{t+1}}$ conditional on $\overrightarrow{p_t}$.

With probability $1 - \lambda^q_s$, no firm will take over the market leader and $\overrightarrow{p_{t+1}}$ will be the same as $\overrightarrow{p_t}$. In this event, $\sum_{i=1}^{m+1}(p_{it}-\mu_p)(p_{it+1}-\mu_p)/(m+1) = \sum_{i=1}^{m+1}(p_{it}-\mu_p)^2/(m+1) = Var_p$. With probability λ^q_s, one firm will take over the market leader and they will swap their percentiles at year $t+1$. For the market leader, $(p_{it} - \mu_p) = -m/(2(m+1))$, while for the followers, $(p_{it} - \mu_p) = 1/(2(m+1))$. Hence,

$$
\begin{aligned}
Cov(\overrightarrow{p_t}, \overrightarrow{p_{t+1}}) &= (1 - \lambda^q_s)Var_p + \lambda^q_s\left[\frac{m-1}{m+1}\frac{1}{(2(m+1))^2} - \frac{2}{m+1}\frac{m}{(2(m+1))^2}\right] \\
&= (1 - \lambda^q_s)Var_p - \frac{2\lambda^q_s Var_p}{m(m+1)} \\
&\simeq (1 - \lambda^q_s)Var_p,
\end{aligned}
$$

where the last approximation holds when m is sufficiently large. Substituting into (37), it follows that

$$
Corr(\overrightarrow{p_t}, \overrightarrow{p_{t+1}}) \simeq (1 - \lambda^q_s)
$$

It also follows that for small λ_s^q,

$$Corr(\overrightarrow{p_t}, \overrightarrow{p_{t+5}}) \simeq (1 - 5\lambda_s^q).$$

Since in 1950 $Corr(\overrightarrow{p_t}, \overrightarrow{p_{t+5}}) \simeq 0.9$, we calibrate λ_{s1950}^q to 0.02. Similarly, since in 2000 $Corr(\overrightarrow{p_t}, \overrightarrow{p_{t+5}}) \in (0.7, 0.75)$, we calibrate λ_{s2000}^q to (0.05, 0.06).

(ii) Using the value of λ_{s1950}^q and the initial correlation and variance of sectoral growth, pin down the values for $\lambda_{1950}^h * (\ln(\delta_h))^2$, $\lambda_{s1950}^q * (\ln(\delta_q^{1/N}))^2$ and $\ln(\delta_q^{1/N})$.

In the multisector version of the model, we have seen that the variance of sectoral growth and the correlation of sectoral growth are given by the following expressions:

$$V\gamma_{y_s} = \lambda_s^q(\ln(\delta_q^{1/N}))^2 + \lambda^h(\ln(\delta_h))^2 \tag{38}$$

$$corr(\gamma_{y_s}, \gamma_{y_{s'}}) = \frac{(\delta_h)^2\lambda^h}{(\delta_q^{1/N})^2\lambda_s^q + (\delta_h)^2\lambda^h} \tag{39}$$

It follows that:

$$\lambda_s^q(\ln(\delta_q^{1/N}))^2 = V\gamma_{y_s}/(1 + \Phi),$$

where

$$\Phi \equiv \frac{corr(\gamma_{y_s}, \gamma_{y_{s'}})}{1 - corr(\gamma_{y_s}, \gamma_{y_{s'}})}.$$

It also follows from (38) and (39) that $\lambda^h(\ln(\delta_h))^2 = \Phi V\gamma_{y_s}/(1 + \Phi)$ and (trivially) $\ln(\delta_q^{1/N}) = \sqrt{\lambda_s^q(\ln(\delta_q^{1/N}))^2/\lambda_s^q}$.

We calibrate $corr(\gamma_{y_s}, \gamma_{y_{s'}})_{1950}$ to 0.5 (figure 6) and $V_{\gamma_{ys}}(1950)$ to 0.0005 both computed using the Jorgenson and Stiroh 35-KLEM dataset. That pins down $\lambda_{s1950}^q(\ln(\delta_q^{1/N}))^2$, $\lambda_{1950}^h(\ln(\delta_h))^2$ and $\ln(\delta_q^{1/N})$, which is assumed to be constant.

(iii) Using the average initial growth rate of productivity, calibrate $\ln(\delta_h)$ and λ_{1950}^h.

The expected growth rate of the economy is given by the following expression:

$$E\gamma_y = \lambda_s^q \ln(\delta_q^{1/N}) + \lambda^h \ln(\delta_h) \tag{40}$$

It follows that:

$$\ln(\triangle h) = \frac{\lambda^h(\ln(\delta_h))^2}{E\gamma_y - \lambda_s^q \ln(\delta_q^{1/N})}. \tag{41}$$

Further, once $\ln(\delta_h)$ is known, $\lambda^h = \lambda^h (\ln(\triangle h))^2 / (\ln(\delta_h))^2$. We use BLS data reported in figure 1 to calibrate $E\gamma_{y1950}$ to 0.025 and then use expression (41) to pin down $\ln(\delta_h)$ and λ^h_{1950}.

(v) Using the final correlation of sectoral growth and the calibrated value of $\ln(\delta_h)$, compute the final rate of arrival of general innovations (λ^h_{2000}).

From expression (39), it follows that

$$\lambda^h = \Phi\lambda^q_s (\ln(\delta_q^{1/N}))^2 / (\ln(\delta_h))^2.$$

Substituting in (i) Φ_{2000} , which we set to 0.25 based on figure 6, (ii) λ^q_{s2000}, which we have set to 0.05 based on the discussion above and (iii) the calibrated values of $\ln(\delta_q^{1/N})$ and $\ln(\delta_h)$, we can pin down λ^h_{2000}.

(vi) With this information and the number of sectors (35), compute the final expected growth rate of productivity $(E\gamma_{y2000})$, the initial and final variance of aggregate productivity growth $(V\gamma_{y1950}, V\gamma_{y2000})$.

This follows by evaluating the following two expressions at $\lambda^q_{s1950}, \lambda^h_{1950}, \lambda^q_{s2000}, \lambda^h_{2000}$.

$$\begin{aligned}
E\gamma_y &= \lambda^q_s \ln(\delta_q^{1/N}) + \lambda^h \ln(\delta_h) \\
V\gamma_y &= \frac{\lambda^q_s}{N}(\ln(\delta_q^{1/N}))^2 + \lambda^h(\ln(\delta_h))^2
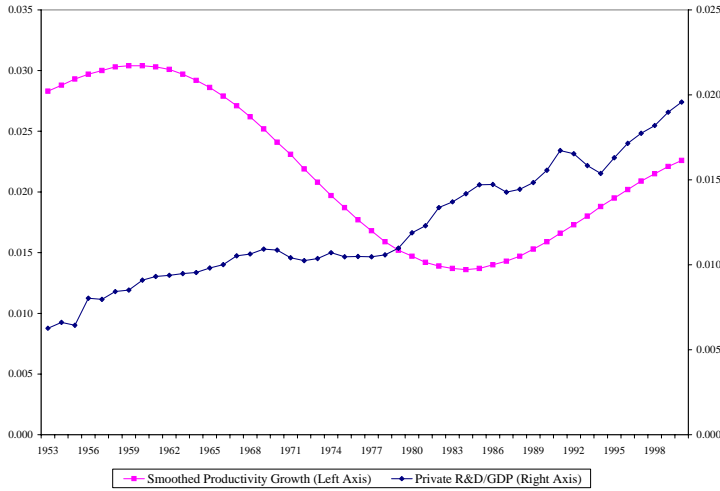\end{aligned}$$

Figure 1: Evolution of (Smoothed) Productivity Growth and Private R&D share in GDP. Note: Productivity growth series obtained from the BLS. The productivity growth series has been smoothed with a Band-Pass filter that keeps fluctuations associated with cycles of period greater than 30 years. Private R&D expenses comes from the NSF.
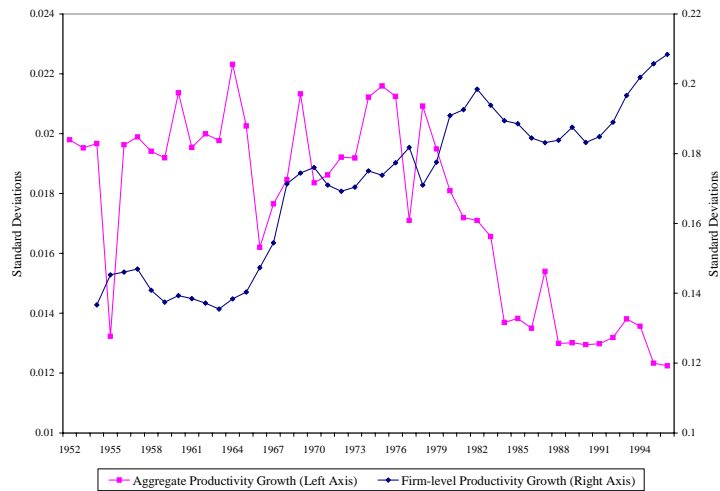

Figure 2: Evolution of the Aggregate and Firm-level Volatility of productivity. Note: Aggregate productivity growth comes from the BLS. Firm-level sales per worker obatined from COMPUSTAT. Firm and aggregate volatility series are computed as indicated in the text.
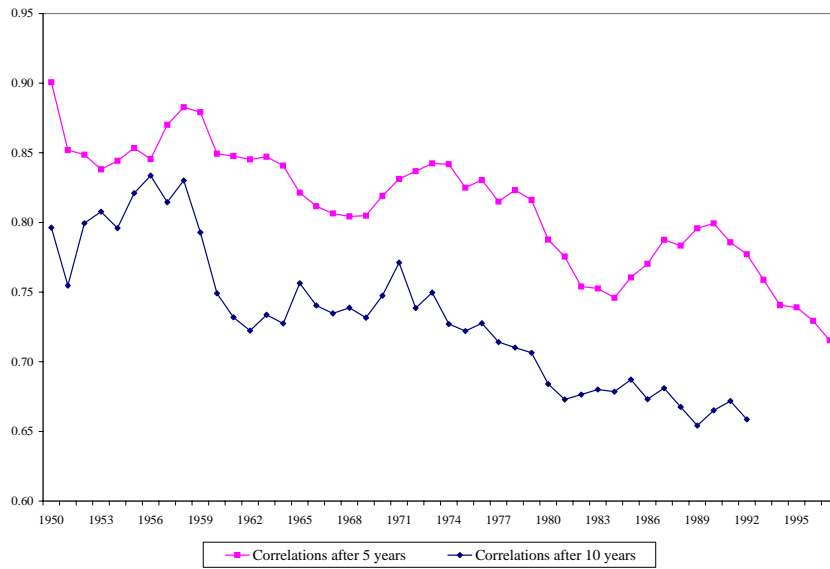
Figure 3: Correlations of firm percentiles by sales per worker. Note: Source COMPUSTAT.

Figure 4: Evolution of Sectoral Correlation of Productivity Growth. Note: Data source Jorgenson and Stiroh KLEM data sets.

Figure 5: Correlation of Sectoral TFP Growth. Note: Data source is KLEM Jorgenson and Stiroh data set.

Table 1
Examples of General Innovations

Innovation	Date	Importance	Market Leader
Management and Production Design			
Mass Production	1900	Fixed costs spread out over larger volumes meant lower costs.	Yes
Ford Assembly Line	1913	Shorter assembly time resulted in lower production costs.	Yes
Scientific Management	1911	Used a scientific approach to production processes to improve productivity.	
McKinsey Management Consulting	1923	Introduced a streamlined approach to consulting services.	
Multi-Divisional Structure	1920's	Introduced the idea of autonomous divisions responsible for pursuing goals, independent of each other.	Yes
Just-in-Time Manufacturing	1950's	Improved synergies between adjacent production processes to minimize inventories.	Yes
Human Resource Management			
Hawthorne Studies	1924-1933	Highlighted the importance of the relationship between the employee morale and productivity.	Yes
Industrial Psychology	1940's-50's	Emphasized contextual variables for purposes of training and positive organization change.	Yes
Survey Feedback	1940's	Highlighted the importance of sharing feedback with employees.	
Sensitivity Training	1946	Focused on the importance of open discussion in small groups.	
Credit/Banking			
Credit card	1950	Helped businesses and consumers undertake credit transactions in a more extensive and systematic manner.	Yes
Electronic Recording Method of Accounting	1950's	Helped computerize the banking industry.	Yes
Magnetic Ink Character Recognition	1950's	Allowed computerized tracking and accounting of check transactions.	Yes
Electronic Money	1972	Introduced an electronic alternative to check processing.	Yes
Computer / Software / Internet			
Hypertext	1945	Basis of the eventual World Wide Web.	
Arpanet	1969	Enabled the exchange of information over large geographic distances.	
Fortran	1957	High-level programming language that made for improved scientific, engineering and mathematical applications.	Yes
Computers	1936	Enabled the automation of an assortment of functions.	
Internet Search Engines	1990	Greatly reduced cost and increased convenience of gathering of information.	
Internet Shopping	1990's	Provided firms with a new avenue to conduct their business.	
Trade			
Mall	1922	Started the modern-day one-stop shop for all consumers.	Yes
Department Store	1877	Improved the efficiency of retail and distribution.	
Marketing			
Coupons	1895	Effective promotion/marketing tool.	Yes
Mail order catalog	1872	Enabled businesses to target consumers that did not access to retail outlets.	
Chemical Engineering			
Chemical Engineering	1920's	Improved the design and control of similar operations at plants in several different industries.	Yes

Table 2: R&D, Firm-level Volatility and Sectoral Co-movement

	Dependent variable											
	correlation in sectoral productivity growth						correlation in sectoral TFP growth					
R&D	-3.28		-3.11		-3.39		-2.49		-3.11		-3.1	
	(1.42)		(1.44)		(1.45)		(1.09)		(1.06)		(1.07)	
Firm level volatility		-0.297		-0.287		-0.27		-0.237		-0.239		-0.25
		(0.122)		(0.121)		(0.121)		(0.083)		(0.084)		(0.085)
Energy share			-0.49	-0.35	-0.077	0.076			-0.057	0.065	-0.18	-0.038
			(0.37)	(0.36)	(0.36)	(0.36)			(0.19)	(0.18)	(0.24)	(0.22)
N	1011	1011	1011	1011	982	982	1011	1011	1011	1011	982	982
Sectors	All	All	All	All	Non-energy		All	All	All	All	Non-energy	

Notes:

Newey-West standard errors are reflected in parentheses.

Firm volatility is measured by the sectoral average of the firm-level variance of the growth rate of sales.

All regressions include sector and year fixed effects.