THE EMPIRICAL RISK-RETURN RELATION:
A FACTOR ANALYSIS APPROACH

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NBER Working Paper No. 11477
July 2005
JEL No. G12, G10

ABSTRACT

A key criticism of the existing empirical literature on the risk-return relation relates to the relatively small amount of conditioning information used to model the conditional mean and conditional volatility ox excess stock market returns. To the extent that financial market participants have information not reflected in the chosen conditioning variables, measures of conditional mean and conditional volatility - and ultimately the risk-return relation itself - will be misspecified and possibly highly misleading. We consider one remedy to these problems using the methodology of dynamic factor analysis for large datasets, whereby a large amount of economic information can b summarized by a few estimated factors. We find that three new factors, a "volatility", "risk premium", and "real" factor, contain important information about one-quarter ahead excess returns and volatility that is not contained on commonly used predictor variables. Moreover, the factor-augmented specifications we examine predict and unusual 16-20 percent of the one-quarter ahead variation in excess stock market returns, and exhibit remarkably stable and strongly statistically significant out-of-sample forecasting power. Finally, in contrast to several pre-existing studies that rely on a small number of conditioning variables, we find a positive conditional correlation between risk and return that is strongly statistically significant, whereas the unconditional correlation is weakly negative and statistically insignificant.

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1   Introduction

Financial economists have long been interested in the empirical relation between the condi-
tional mean and conditional volatility of excess stock market returns, often referred to as the
risk-return relation. The risk-return relation is an important ingredient in optimal portfolio
choice, and is central to the development of theoretical models aimed at explaining observed
patterns of stock market predictability and volatility. Among those theoretical models that
have become standard-bearers in finance, a positive risk-return relation is the benchmark
prediction, so that times of predictably higher risk coincide with times of predictably higher
excess returns, and vice versa. Unfortunately, the body of empirical evidence on the risk-
return relation is mixed and inconclusive. Some evidence supports the theoretical prediction
of a positive risk-return tradeoff, but other evidence suggests a strong negative relation. Yet
a third strand of the literature finds that the relation is unstable and varies substantially
through time. (We summarize the existing evidence below.)

Several criticisms of the existing empirical literature relate to the relatively small amount
of conditioning information used to model the conditional mean and conditional volatility of
excess stock market returns. First, the conditional expectations underlying the conditional
mean and conditional volatility are typically measured as projections onto predetermined
conditioning variables; but, as Harvey (2001) points out, the decision of *which* predeter-
mined conditioning variables to use in the econometric analysis can influence the estimated
risk-return relation. In practice, researchers are forced to choose among a few conditioning
variables because conventional statistical analyses are quickly overwhelmed by degrees-of-
freedom problems as the number rises. Such practical constraints introduce an element
of arbitrariness into the econometric modeling of expectations and may lead to omitted-
information estimation bias, since a small number of conditioning variables is unlikely to
span the information sets of financial market participants. If investors have information
not reflected in the chosen conditioning variables used to model market expectations, mea-
sures of conditional mean and conditional volatility will be misspecified and possibly highly
misleading.[1]

A second and related criticism of the existing empirical literature is that the estimated
relation between the conditional mean and conditional volatility of excess returns often de-
pends on the parametric model of volatility, e.g., GARCH, EGARCH, stochastic volatility, or
kernel density estimation (Harvey (2001)). Such procedures may impose potentially restric-
tive parametric assumptions and they often suffer from a curse-of-dimensionality problem
that constrains their ability to entertain large datasets of conditioning information.

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[1] This point was made forcibly by Hansen and Richard (1987) in the context of estimating and testing
dynamic asset pricing models.

Finally, the reliance on a small number of conditioning variables exposes existing analyses to problems of temporal instability in the underlying forecasting relations being modeled. For example, it is commonplace to model market expectations of future stock returns using the fitted values from a forecasting regression of returns on to a measure of the market-wide dividend-price ratio. A difficulty with this approach is that the predictive power of the dividend-price ratio for excess stock market returns is unstable and exhibits statistical evidence of a structural break in the mid-1990s (Lettau, Ludvigson, and Wachter (2004)).

In this paper we consider one remedy to these problems using the methodology of dynamic factor analysis for large datasets. Recent research on dynamic factor models finds that the information in a large number of economic time series can be effectively summarized by a relatively small number of estimated factors, affording the opportunity to exploit a much richer information base than what has been possible in prior empirical study of the risk-return relation. In this methodology, "a large number" can mean hundreds or, perhaps, even more than one thousand economic time series. By summarizing the information from a large number of series in a few estimated factors, we eliminate the arbitrary reliance on a small number of exogenous predictors to estimate the conditional mean and conditional volatility of stock returns, and make feasible the use of a vast set of economic variables that are more likely to span the unobservable information sets of financial market participants. In the words of Stock and Watson (2004), dynamic factor analysis permits us to turn dimensionality from a curse into a blessing.

Dynamic factor analysis allows us to escape the limitations of existing empirical analyses discussed above, on several fronts. First, if a large amount of information can be effectively summarized by a relatively few common factors, then a natural remedy to the omitted information problem is to augment fitted conditional moments with estimated factors. We do so here by including estimated factors in the construction of fitted mean and volatility. Second, by combining dynamic factor analysis with a nonparametric approach to modeling volatility—an approach referred to hereafter as *realized volatility*—we avoid relying on potentially restrictive parametric structures while at the same time insuring that our measure of conditional volatility effectively summarizes a large amount of information that may be important for predicting the variance of the stock market. Third, there is some evidence (discussed below) that dynamic factor analysis provides robustness against the temporal instability that often plagues low-dimensional forecasting regressions. Indeed, our application appears supportive of this evidence, since the factor-augmented predictive relations we employ are remarkably stable over time, despite the observed temporal instability of many commonly used predictor variables over the sample period we study.

An important question of our study is the degree to which estimated common factors add information about the conditional mean and conditional volatility of stock returns that is not

already contained in commonly used predictor variables. If, on the one hand, we find that the factors provide new information, then we have evidence that previous estimates of conditional moments are misspecified and the estimated risk-return relation potentially contaminated. On the other hand, if we find that the information provided by the factors is largely contained in commonly used predictor variables, then we have evidence that previous estimates are likely to be well specified. Either way, our study contributes to the empirical literature on the risk-return relation by evaluating both the potential role of omitted information in the estimated risk-return relation, as well as the robustness of previous results to conditioning on richer information sets.

We estimate common factors from two quarterly post-war datasets of economic activity using the method of principal components. The first dataset is comprised of 209 primarily macroeconomic indicators; the second dataset is comprised of 172 financial indictors. As a result of investigating these data, we find a number of results particularly interesting.

First, in modeling the conditional mean of excess stock market returns, we introduce two new financial factors that are particularly important for forecasting quarterly excess returns on the aggregate stock market. In doing so, we contribute to the continuing debate over the predictability of stock market returns.[2] The first financial factor is the square of the first common factor of the dataset comprised of financial indicators. This factor explains almost 80 percent of the contemporaneous variation in squared stock market returns, so we label it a "volatility factor." The second financial factor is the third common factor from the dataset comprised of financial indicators and is highly correlated with a linear combination of three state variables widely used in the empirical asset pricing literature to explain cross-sectional variation in risk premia. These state variables are the Fama-French factors SMB_t, HML_t, and the market return (Fama and French (1993)). Thus, our second factor connects the time series with the cross-section of expected excess stock market returns. For this reason, we call this second factor a "risk premium factor." When the volatility and risk premium factors are included with the consumption-wealth variable cay_t, found elsewhere to predict quarterly stock returns (Lettau and Ludvigson (2001a)), the statistical model predicts an unusually high 16 percent of the variation in one-quarter ahead excess returns. Moreover, the two factors on their own exhibit remarkably stable, strongly statistically significant out-of-sample forecasting power for quarterly excess returns that is found to be strongest in data *after* 1995, a period in which the predictive power of many traditional forecasting variables was exceptionally poor.

Second, in modeling the conditional volatility of excess stock market returns, we find

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[2]See for example, Campbell and Yogo (2002), Campbell and Thompson (2004), Goyal and Welch (2004), Lewellen (2004).

one macroeconomic factor that, when combined with other predictor variables, is especially useful for forecasting stock market volatility. This factor is the first common factor from the macroeconomic dataset, known to be a "real factor," since it is highly correlated with measures of real output and employment but not highly correlated with prices (Stock and Watson (2002b)).

Third, we find that distinguishing between the *conditional* correlation (conditional on lagged mean and lagged volatility) and *unconditional* correlation between the conditional mean stock return and its conditional volatility is crucial for understanding the empirical risk-return relation. This finding is consistent with that of Brandt and Kang (2004) who argue that the distinction may explain the disagreement in the literature about the contemporaneous correlation between risk and return. In contrast to some previous studies, however, (e.g., Brandt and Kang (2004), Lettau and Ludvigson (2003)) we find a positive conditional correlation that is strongly statistically significant, whereas the unconditional correlation is weakly negative and statistically insignificant. We show here that the findings in Lettau and Ludvigson (2003) can be attributed to the omission of the volatility and risk premium factors, which contain important information about one-quarter ahead returns.

Finally, our results imply that the conditional Sharpe ratio has an unmistakable countercyclical pattern, increasing sharply in recessions and declining at the onset of expansions. These findings are consistent with those in Brandt and Kang (2004) and Lettau and Ludvigson (2003).

The rest of this paper is organized as follows. In the next section we briefly review related literature. Section 3 lays out the econometric framework, discusses the use of principal components analysis to estimate common factors, and explains how factors are chosen for modeling the conditional mean and conditional volatility of stock returns. Section 4 explains the empirical implementation and describes the data. We move on in Section 5 to present our empirical findings, including the results of one quarter ahead predictive relations, and our results for the estimated risk-return relation. Two additional analyses are performed as robustness checks: out-of-sample investigations, and small-sample inference. Section 6 concludes.

2 Related Literature

Our empirical investigation is related to several disparate strands of economic literature. On the methodology side, our use of dynamic factor analysis is an application of statistical procedures developed elsewhere for the case where both the number of economic time series used to construct common factors, N, and the number of time periods, T, are large and

converge to infinity.[3] The presumption of the dynamic factor model is that the covariation among economic time series is captured by a few unobserved common factors. Stock and Watson (2002b) show that consistent estimates of the space spanned by the common factors may be constructed by principal components analysis. Bai and Ng (2005) show that the least squares estimates from factor-augmented forecasting regressions are $\sqrt{T}$ consistent and asymptotically normal, and that pre-estimation of the factors does not affect the consistency of the second-stage parameter estimates or their standard errors. Stock and Watson (2002b) and Stock and Watson (2004) find that predictions of real economic activity and inflation are greatly improved relative to low-dimensional forecasting regressions when the forecasts are based on the estimated factors of large datasets. An added benefit of this approach, mentioned above, is that the use of common factors can provide robustness against the structural instability that plagues low-dimensional forecasting regressions. Stock and Watson (2002a) provide both theoretical arguments and empirical evidence that the principal components factor estimates are consistent even in the face of temporal instability in the individual time series used to construct the factors. The reason is that such instabilities may "average out" in the construction of common factors if the instability is sufficiently dissimilar from one series to the next.

Our use of realized volatility to model return volatility is motivated by recent findings in the volatility modeling literature. Andersen, Bollerslev, and Diebold (2002) and Andersen, Bollerslev, Diebold, and Labys (2003) argue that nonparametric volatility measures such as realized volatility benefit from being free of tightly parametric functional form assumptions and provide a consistent estimate of ex-post return variability. Realized volatility, in turn, permits the use of traditional time-series methods for modeling and forecasting, making possible the employment of estimated common factors from large datasets to measure conditional, or expected, volatility. Earlier studies of realized stock market volatility include French, Schwert, and Stambaugh (1987) and Schwert (1989).

Finally, our work is connected to a large literature examining the empirical relation between the conditional mean and conditional volatility of excess stock market returns. Bollerslev, Engle, and Wooldridge (1988), Harvey (1989), Campbell and Hentschel (1992) and Ghysels, Santa-Clara, and Valkanov (2005) find a positive risk-return relation, while Campbell (1987), Breen, Glosten, and Jagannathan (1989), Pagan and Hong (1991), Glosten, Jagannathan, and Runkle (1993), Whitelaw (1994), Lettau and Ludvigson (2003) and Brandt and Kang (2004) find a negative relation. French, Schwert, and Stambaugh (1987) find a negative relation between returns and the unpredictable component of volatility, a result they

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[3]Dynamic factor analysis with large N is preceded by a literature studying classical factor analysis for the case where N is relatively small and fixed. See for example, Sargent and Sims (1977); Sargent (1989), and Stock and Watson (1989, 1991).

interpret as indirect evidence that *ex ante* volatility is positively related to *ex ante* excess returns. Campbell (1987), Harvey (1989) and Kandel and Stambaugh (1990) argue that the relation between the conditional mean and conditional volatility varies over time. Yet, most of these studies use a small number of predetermined conditioning variables to form estimates of the conditional mean and the conditional volatility, potentially subjecting the findings to the omitted information problems emphasized by Hansen and Richard (1987) and Harvey (2001). One study that does not rely on predetermined conditioning variables is Brandt and Kang (2004) who model the conditional mean and conditional volatility as latent state variables identified only from the history of return data. An advantage of this approach is that it eliminates the reliance on a few arbitrary conditioning variables in forming estimates of conditional moments. A corresponding disadvantage is that potentially useful information is discarded. Perhaps more important, even the latent state variable methodology is not immune to the general criticism of omitted information, since the latent variables must in practice be modeled as following low-order, linear time series representations of known probability distribution. For example, Brandt and Kang assume the conditional mean and conditional volatility evolve according to first-order Gaussian vector autoregressive processes. If the true representation is of higher order, nonlinear, or non-Gaussian, we again face an omitted information problem.

3   Econometric Framework

In this section we describe our econometric framework, which involves estimating common factors from large datasets of macroeconomic and financial information. Such estimation is carried out using principal component analysis, a procedure that has been described and implemented elsewhere for forecasting measures of macroeconomic activity and inflation (e.g., Stock and Watson (2002b), Stock and Watson (2002a), Stock and Watson (2004)). We refer the reader to those papers for a detailed description of this procedure; here we only outline how the implementation relates to our application.

The goal of our procedure is to estimate the conditional mean and conditional volatility of excess stock market returns and, ultimately, the relation between these two variables. For $t = 1, \ldots T$, let m_{t+1} denote continuously compounded excess returns in period $t + 1$ and let VOL_{t+1} be an estimate of their volatility. The objective is to estimate $E_t m_{t+1}$, the conditional mean of m_{t+1}, and conditional volatility $E_t VOL_{t+1}$, using information up to time t. We confine ourselves to estimation of $E_t m_{t+1}$ and $E_t VOL_{t+1}$ using linear parametric models.

First consider estimation of the conditional mean $E_t m_{t+1}$. A standard approach is to

select a set of K predetermined conditioning variables at time t, given by the $K \times 1$ vector Z_t, and then estimate

$$m_{t+1} = \beta' Z_t + \epsilon_{t+1} \tag{1}$$

by least squares. The estimated conditional mean is then the fitted value from this regression, $\widehat{m}_{t+1|t} = \hat{\beta}' Z_t$. The issue at hand is whether we can go beyond (1) to make use of the substantially more information that is available to market participants. That is, suppose we observe a $T \times N$ panel of data with elements $x_{it}, i = 1, \ldots N, t = 1, \ldots, T$, where the cross-sectional dimension, N, is large, and possibly larger than the number of time periods, T. How to use this information is not immediately obvious because, unless we have a way of ordering the importance of the N series in forming conditional expectations (as in an autoregression), there are potentially 2^N possible combinations to consider. Furthermore, letting x_t denote the $N \times 1$ vector of panel observations at time t, estimates from the regression

$$m_{t+1} = \gamma' x_t + \beta' Z_t + \epsilon_{t+h}$$

quickly run into degrees-of-freedom problems as the dimension of x_t increases, and estimation is not even feasible when $N + K > T$.

The approach we consider is to posit that x_{it} has a factor structure taking the form

$$x_{it} = \lambda_i' f_t + e_{it}, \tag{2}$$

where f_t is a $r \times 1$ vector of latent common factors, λ_i is a corresponding $r \times 1$ vector of latent factor loadings, and e_{it} is a vector of idiosyncratic errors.[4] The crucial point here is that $r << N$, so that substantial dimension reduction can be achieved by considering the regression

$$m_{t+1} = \alpha' F_t + \beta' Z_t + \epsilon_{t+1}, \tag{3}$$

where $F_t \subset f_t$. Equation (1) is nested within the factor-augmented regression, making (3) a convenient framework to assess the importance of x_{it} via F_t, even in the presence of Z_t. But the distinction between F_t and f_t is important, because factors that are pervasive for the panel of data x_{it} need not be important for predicting m_{t+1}.

As common factors are not observed, we replace f_t by $\widehat{f}_t$, estimates that, when $N, T \to \infty$, span the same space as f_t. (Since f_t and λ_i cannot be separately identified, the factors are

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[4]We consider an *approximate* dynamic factor structure, in which the idiosyncratic errors e_{it} are permitted to have a limited amount of cross-sectional correlation. The specification limits the contribution of the idiosyncratic covariances to the total variance of x as N gets large:

$$N^{-1} \sum_{i=1}^{N} \sum_{j=1}^{N} |E(e_{it} e_{jt})| \leq M.$$

only identifiable up to an $r \times r$ matrix.) In practice, f_t are estimated by principal components analysis (PCA).[5] Let the Λ be the $N \times r$ matrix defined as $\Lambda \equiv (\lambda'_1, ..., \lambda'_N)'$. Intuitively, the estimated time t factors $\widehat{f}_t$ are linear combinations of each element of the $N \times 1$ vector $x_t = (x_{1t}, ..., x_{Nt})'$, where the linear combination is chosen optimally to minimize the sum of squared residuals $x_t - \Lambda f_t$.

To determine the composition of $\widehat{F}_t$, we form different subsets of $\widehat{f}_t$, and/or functions of $\widehat{f}_t$ (such as $\widehat{f}^2_{1t}$). For each candidate set of factors, $\widehat{F}_t$, we regress m_{t+1} on $\widehat{F}_t$ and Z_t and evaluate the corresponding BIC and $\bar{R}^2$. Following Stock and Watson (2002b), minimizing the BIC yields the preferred set of factors $\widehat{F}_t$. The final model for returns is based on Z_t plus this optimal $\widehat{F}_t$. That is,

$$m_{t+1} = \alpha' \widehat{F}_t + \beta' Z_t + \epsilon_{t+1}. \tag{4}$$

To conserve notation, we use $\widehat{F}_t$ to denote the factors used in the final model, but it should be understood that the components of $\widehat{F}_t$ are selected using formal statistical procedures. In what follows, we denote the fitted conditional mean

$$\mu_t \equiv \widehat{m}_{t+1|t} = \widetilde{\alpha}' \widehat{F}_t + \widetilde{\beta}' Z_t.$$

Under the assumption that $N, T \to \infty$ with $\sqrt{T}/N \to 0$, Bai and Ng (2005) showed that (i) $(\hat{\alpha}, \hat{\beta})$ obtained from least squares estimation of (4) are $\sqrt{T}$ consistent and asymptotically normal, and the asymptotic variance is such that inference can proceed as though f_t is observed, (ii) the estimated conditional mean, $\mu_t = \widehat{F}'_t \hat{\alpha} + Z'_t \hat{\beta}$ is $\min[\sqrt{N}, \sqrt{T}]$ consistent and asymptotically normal, and (iii) the h period forecast error $m_{t+h} - m_{t+h|t}$ from (4) is dominated in large samples by the variance of the error term, just as if f_t is observed. The importance of a large N must be stressed, however, as without it, the factor space cannot be consistently estimated however large T becomes.

Given a measure, VOL_t, of the volatility of excess returns at time t, estimation of conditional volatility is carried out in the same way as estimation of the conditional mean, and the same asymptotic results for conducting inference apply. That is, we estimate a final model for volatility based on based on Z_t plus an optimally chosen (by the BIC criterion) set of factors $\widehat{F}_t$,

$$VOL_{t+1} = a' \widehat{F}_t + b' Z_t + u_{t+1}, \tag{5}$$

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[5] To be precise, the $T \times r$ matrix $\widehat{f}$ is $\sqrt{T}$ times the r eigenvectors corresponding to the r largest eigenvalues of the $T \times T$ matrix $xx'/(TN)$ in decreasing order. Let Λ be the $N \times r$ matrix of factor loadings $(\lambda'_1, ..., \lambda'_N)'$. Λ and f are not separately identifiable, so the normalization $f'f/T = I_r$ is imposed, where I_r is the r-dimensional identity matrix. With this normalization, we can additionally obtain $\widehat{\Lambda} = x'\widehat{f}/T$, and $\widehat{\chi}_{it} = \widehat{\lambda}'_i \widehat{f}_t$ denotes the estimated common component in series i at time t. The number of common factors, r is determined by the panel information criteria developed in Bai and Ng (2002).

where it should be noted that the variables in $\widehat{F}_t$ and Z_t may differ from those in (4). In what follows, we denote the fitted conditional volatility

$$\sigma_t \equiv \widehat{VOL}_{t+1|t} = \widehat{a}' \widehat{F}_t + \widehat{b}' Z_t.$$

Our analysis is based on quarterly data. To obtain a measure of quarterly volatility for excess returns, we follow French, Schwert, and Stambaugh (1987) and Schwert (1989) and use the time-series variation of daily returns:

$$VOL_t = \sqrt{\sum_{k \in t} (R_{sk} - \overline{R}_s)^2}, \tag{6}$$

where VOL_t is the sample volatility of the market return in quarter t, R_{sk} is the daily return minus the implied daily yield on the three-month Treasury bill rate, $\overline{R}_s$ is the mean of R_{sk} over the whole sample, and k represents a day. Following Andersen, Bollerslev, and Diebold (2002) and Andersen, Bollerslev, Diebold, and Labys (2003) we call this measure *realized volatility*. Andersen, Bollerslev, Diebold, and Labys (2003) demonstrate, using the theory of quadratic variation, that realized volatility is an unbiased estimator of actual volatility, and often performs better than parametric GARCH or stochastic volatility models at capturing volatility. Most important for our application, realized volatility permits us to use the estimated common factors from large datasets to model conditional volatility, by constructing these estimates as fitted values from statistical models of the form (5).

The final aspect of our econometric framework is a reduced-form linear equation for the conditional mean as a function of the contemporaneous conditional volatility and lags of the two:

$$\mu_t = \delta + \beta_1 \sigma_t + \beta_2 \sigma_{t-1} + \alpha \mu_{t-1} + \varepsilon_t. \tag{7}$$

This is a generalization of the more common *volatility-in-mean* model that relates the conditional mean to the conditional volatility of returns. Here, we follow Whitelaw (1994) and Brandt and Kang (2004) and include lags of μ_t and σ_t in modeling the risk-return relation. Both Whitelaw and Brandt and Kang find important lead-lag interactions between conditional mean and conditional volatility. Since Whitelaw uses a small number of exogenous predictors to model these moments, an important question is whether his results are specific to the exogenous predictors he used. The results of Brandt and Kang, which do not rely on exogenous predictors, suggest that this may not be the case, since some of their findings are similar. Our application provides further evidence on this question, by exploiting a vast database of information in forming conditional moments. The coefficient β_1 measures the volatility-in-mean effect; the coefficient β_2 measures the *lag-volatility-in-mean* effect.[6]

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[6]We have also studied an analogous mean-in-volatility equation taking the form

$$\sigma_t = \delta + \alpha_1 \mu_t + \alpha_2 \mu_{t-1} + \beta \sigma_{t-1} + \xi_{t+1}.$$

Notice also that the procedure described above explicitly recognizes the possibility that the conditional mean may not be proportional to conditional volatility. If they were proportional, as in the capital asset pricing model of Sharpe (1964) and Lintner (1965), then any and all variation in the conditional mean excess return would be driven by variation in the conditional variance of the excess return. In this case the risk-return relation could be estimated by regressing *ex post* excess returns on a measure of *ex ante* volatility. But in more general models that produce countercyclical variation in the conditional Sharpe ratio μ_t/σ_t, the conditional mean is not perfectly correlated with conditional volatility.[7] This motivates our search for possibly distinct state variables to forecast mean and volatility, as well as our use of *ex ante* rather than *ex post* excess returns on the left-hand-side of (7). Below, we estimate equations of the form (7) using either ordinary least squares (OLS) or two-stage least squares (2SLS), where in the latter we instrument for σ_t with variables known at time $t - 1$.

4   Empirical Implementation and Data

A detailed description of the data and our sources is given in the Appendix. We study quarterly data spanning the period 1960:1 to 2002:4. The continuously compounded excess return m_{t+1} is the log return on the Center for Research in Security Prices (CRSP) value-weighted price index, in excess of three month Treasury bill rate. Our measure of volatility, VOL_t, (6), uses the daily CRSP return minus the implied daily yield on the three-month Treasury bill rate.

We estimate two sets of factors from two quarterly post-war datasets, one comprising of 209 series of macroeconomic indicators, and one comprising of 172 series financial indicators, both spanning the period 1960:1 to 2002:4. Following Stock and Watson (2002b) and Stock and Watson (2004), the macro series were selected to represent broad categories of macroeconomic time series: real output and income, employment and hours, real retail, manufacturing and trade sales, consumer spending, housing starts, inventories and inventory sales ratios, orders and unfilled orders, compensation and labor costs, capacity utilization measures, price indexes, and foreign exchange measures. The financial database consist of a broad number of indicators measuring the aggregate time-series behavior of the stock market as well as the behavior of a broad cross-section of asset returns. The data include valuation ratios such as

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The empirical results lead to the same conclusions about the risk-return relation as the volatility-in-mean equation (7). We therefore omit those results to conserve space.

[7]The conditional Sharpe ratio varies over time if changing risk or risk aversion provide good descriptions of dynamic asset market behavior (e.g., Constantinides (1990), Constantinides and Duffie (1996), Campbell and Cochrane (1999), Chang and Sundaresan (1999)).

the dividend-price ratio and earnings-price ratio, growth rates of aggregate dividends and prices, default and term spreads, yields on corporate bonds of different ratings grades, yields on Treasuries and yield spreads, a broad cross-section of industry equity returns, following Fama and French (1992), returns on 100 portfolios of equities sorted into 10 size and 10 book-market categories, and a group of variables we call "risk-factors," since they have been used in cross-sectional or time-series studies to uncover variation in the market risk-premium. These risk-factors include the three Fama and French (1993) risk factors, namely the excess return on the market MKT_t, the "small-minus-big" (SMB_t) and "high-minus-low" (HML_t) portfolio returns,[8] the momentum factor UMD_t,[9] the consumption-wealth variable cay_t of Lettau and Ludvigson (2001a),[10] the bond risk premia factor of Cochrane and Piazzesi (2005),[11] and the small stock value spread $R15 - R11$[12]. We also include the small-stock value spread as a risk-factor in the financial dataset, the difference between returns in the smallest size/highest book-market quintile and returns in the smallest size/lowest book-market quintile. Campbell and Voulteenaho (2005) use the small stock value spread to predict monthly stock market returns. The complete list of series is given in the Appendix, where for the macro variables a coding system indicates how the data were transformed so as to insure stationarity. All of the raw data in x_t are standardized prior to estimation.

Since we decompose our time-series information into two panel datasets, we postulate two factor structures of the form (2) above. For the macro dataset, we follow the notation introduced above. That is, denote the estimated factors formed from the macro dataset as $\widehat{f}_{it}$, $i = 1, ..., r_f$, where r_f is the number of common factors for the macro dataset and $\widehat{f}_t$ is a $r_f \times 1$ vector of these latent common factors. The subset $\widehat{F}_t \subset \widehat{f}_t$ comprises those estimated factors from the macro dataset that are used in modeling fitted mean and fitted

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[8] SMB is the difference between the returns on small and big stock portfolios with the same weight-average book-to-market equity. HML is the difference between returns on high and low book-equity/market-equity portfolios with the same weighted-average size. Further details on these variables can be found in Fama and French (1993).

[9] This factor is available from Kenneth French's Dartmouth web page. It is created from portfolios, formed monthly, that are the intersections of 2 portfolios formed on size (market equity) and 3 portfolios formed on prior (2-12 month) return. UMD (Up Minus Down) is the average return on the two high prior return portfolios minus the average return on the two low prior return portfolios

[10] The variable cay_t is measured as a cointegrating residual between log consumption, log asset wealth, and log labor income, all in real per capita terms. The presence of labor income accounts for the role of human capital in aggregate wealth; see Lettau and Ludvigson (2001a) for details.

[11] The bond risk factor is a linear combination of forward rates of different maturities, here measured as the quarterly average of monthly data.

[12] This variable is created from 25 size and book-market sorted portfolio returns taken from Kenneth French's Dartmouth web site, by subtracting the portfolio return in the smallest size and lowest book to market category ($R11$), from the return in the smallest size and highest book-market category ($R15$).

volatility. To distinguish the factors estimated from the financial dataset from these macro factors, we introduce a new notation for financial factors that is directly analogous to the notation for macro factors. Denote the estimated factors formed from the financial dataset $\widehat{g}_{it}$, $i = 1, ..., r_g$, where r_g is the number of common factors for the financial dataset and $\widehat{g}_t$ is a $r_g \times 1$ vector of these latent common factors. The subset $\widehat{G}_t \subset \widehat{g}_t$ comprises those estimated factors from the financial dataset that are used in modeling fitted mean and fitted volatility. We then form estimates of the conditional mean and conditional volatility by computing the fitted values from regressions of mean and volatility on both sets of factors:

$$m_{t+1} = \alpha_1' \widehat{F}_t + \alpha_2' \widehat{G}_t + \beta' Z_t + \epsilon_{t+1}, \tag{8}$$

and

$$VOL_{t+1} = a_1' \widehat{F}_t + a_2' \widehat{G}_t + b' Z_t + u_{t+1}, \tag{9}$$

where, as described above, minimizing the BIC over models with different combinations of the variables in $\widehat{F}_t$, $\widehat{G}_t$, and Z_t yields the preferred specification. Notice that, in using the BIC criterion to choose the best model, we include many of the predictors used elsewhere to forecast returns or volatility *both* in the set of financial data used to estimate the factors $\widehat{G}_t$, and in the set of possible independent predictors, Z_t. This permits us to assess the extent to which the factors contain information independent of that contained in commonly used predictive variables.

In estimating the time-t common factors, we face a decision over how much of the time-series dimension of the panel to use. We use the full sample of time-series information to estimate the common factors at each date t, instead of using data only up to date t (recursive estimates). This approach can be thought of as providing smoothed estimates of the latent factors, and ultimately smoothed estimates of μ_t and σ_t, as in Brandt and Kang (2004).[13] The advantage of this approach over recursive information is that estimates of f_t are available for the entire sample $t = 1, ...T$.[14] More important, smoothed estimates of the latent factors, f_t, are the most efficient means of summarizing the covariation in the data x because the estimates use do not discard information in the sample. Exploiting this efficiency is appropriate for our application, since we are not interested in real-time forecasting *per se*,

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[13] In addition, the same smoothed estimate approach is taken in Bernanke, Boivin, and Eliasz (2005), who use common factor analysis to summarize the information in the Federal Reserve's time-t policy reaction function.

[14] Recursive estimates would significantly restrict the sample over which we could obtain observations on μ_t and σ_t. Recursive estimation requires estimation of (8) and (9) over some initial number, R, of observations of our full data set, with fitted values formed over the remaining $T - R$ observations, using one-step-ahead recursive regressions. Thus, observations on μ_t and σ_t would be available only over the last $T - R$ periods of our sample rather than over the full sample.

but rather in an accurate estimate of the population risk-return relation. We do, however, assess the robustness of our forecasting results to an out-of-sample investigation in which the predictor factors are reestimated recursively each period using data only up to time t. A description of this procedure is given below.

In estimating (8) and (9), a question also arises as to what variables should be included in Z_t. The empirical asset pricing literature has uncovered a number of variables that have been shown, in one sample or another, to contain predictive power for excess stock returns. Shiller (1981), Fama and French (1988), Campbell and Shiller (1989), Campbell (1991), and Hodrick (1992) find that the ratios of price to dividends or earnings have predictive power for U.S. excess returns, and Harvey (1991) finds that similar financial ratios predict stock returns in many different countries. Thus we often include the dividend-price ratio in Z_t (results using the earnings-price ratio are similar). Campbell (1991) and Hodrick (1992) find that the relative T-bill rate (the 30-day T-bill rate minus its 12-month moving average) predicts returns, thus we often include a quarterly version of it (the three-month Treasury bill rate minus its 4 quarter moving average) in Z_t. We denote this variable $RREL_t$. Fama and French (1988) study the forecasting power of the term spread (the 10-year Treasury bond yield minus the one-year Treasury bond yield) and the default spread (the difference between the BAA and AAA corporate bond rates). Thus, we also consider specifications in which these variables, denoted TRM_t, and DEF_t respectively, are part of Z_t. Lettau and Ludvigson (2001a) find that the consumption-wealth variable cay_t is a strong predictor of quarterly excess returns, and Lettau and Ludvigson (2001b) find that it is a predictor of portfolio returns, therefore we include this variable in some specifications of Z_t. Finally, in addition to several of the variables already discussed, Whitelaw (1994) finds that the one year Treasury yield, $YIELD_t$, has predictive power for volatility at both monthly and quarterly horizons.

5   Empirical Results

Table 1 presents summary statistics for our estimated factors $\widehat{f}_t$ and $\widehat{g}_t$. The number of factors, r_f, and r_g, are determined by the information criteria developed in Bai and Ng (2002). The criteria indicate that the factor structures of both datasets are well described by 8 common factors. The first factor explains the largest fraction of the total variation in the panel of data x, where total variation is measured as the sum of the variances of the individual x_{it}. The second factor explains the largest fraction of variation in x, controlling for the first factor, and so on, where recall that the estimated factors are mutually orthogonal by construction. Table 1 reports the fraction of variation in the data explained by factors 1

to i.[15] Table 1 shows that a small number of factors account for much of the variance in the two panel datasets we explore. The first five common factors of the macro dataset account for almost 60 percent of the variation in the macroeconomic series, and the first five factors of the financial dataset account for almost 80 percent of the variability in the financial series.

To get an idea of the persistence of the estimated factors, Table 1 also displays the first-order autoregressive (AR(1)) coefficient for each factor. None of the factors have a persistence greater than 0.85, but there is considerable heterogeneity across estimated factors, with coefficients ranging from slightly negative (first factor of the financial dataset), to positive in excess of 0.8 (the second factor of both datasets).

As mentioned, we formally choose among a range of possible specifications for the conditional mean and conditional volatility using these variables and the estimated common factors (and possibly nonlinear functions of those factors such as $\widehat{f}_{1t}^2$) using the BIC criterion. Given the large number of possible specifications, we report only the subset of those specifications analyzed that are most interesting.[16] We present those results next.

5.1   One-quarter Ahead Predictive Regressions

Tables 2 and 3 present results from estimating various specifications for the models (8) and (9). For each specification, the regression coefficient, heteroskedasticity and serial-correlation robust t statistics, adjusted R^2 statistic, and BIC criterion are reported.

We begin with the results in Table 2, predictive regressions for excess returns. As benchmarks, columns a through d of Table 2 report the results of specifications for forecasting one-quarter ahead excess returns, without including any estimated factors. Column a shows that cay_t is a strong predictor of quarterly excess returns, explaining eight percent of the variation in one-quarter ahead returns with a t statistics in excess of four. These results are essentially the same as those reported in Lettau and Ludvigson (2001a). Unlike studies using older data, however, the dividend-price ratio displays little predictive ability for future returns in this sample (column b). It is well known that data from the 1990s have substantially weakened the forecasting power of the dividend-price ratio for returns. Columns c and d include lagged realized volatility VOL_t as an additional predictor, along with cay_t and $RREL_t$. All three variables have marginal predictive power and together explain 12 percent of variation in next quarter's return. This is consistent with the findings of Guo (2005), who reports that predictive regressions that include cay_t along with a measure of aggregate stock

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[15]This is given as the the sum of the first i largest eigenvalues of the matrix xx' divided by the sum of all eigenvalues.

[16]Specifications that include lagged values of the factors beyond the first were also examined, but additional lags were found to contain very little information for either returns or volatility that was not already contained in the one-period lag specifications.

market volatility as predictor variables exhibit strong out-of-sample forecasting power for quarterly excess returns. However, there is little evidence that either the term spread TRM_t or the default spread, DEF_t have important predictive power for returns, as studies using previous samples of data have found.

The remaining columns of Table 2 include estimated common factors as predictive variables, in addition to several of the exogenous predictors discussed above. Column e shows that several factors have marginal predictive power for returns when included without non-factor predictor variables, for example, factors 2 and 5 from the macro dataset and factors 3 and 6 from the financial dataset. But much of the information about future returns that is contained in those factors is subsumed by cay_t and VOL_t (column f). The exception is $\widehat{G}_{3t}$, which has statistically significant predictive power beyond that contained in cay_t and VOL_t.

A number of specifications using various polynomial bases of the estimated factors were also considered. Two factors in particular stand out as containing important information about future returns that is not already contained in commonly used predictor variables. These are, the square of the first estimated factor from the financial database, $\widehat{G}_{1t}^2$, and the third estimated factor $\widehat{G}_{3t}$ from the financial database. Column g shows that these two factors alone explain an unusual 9 percent of next quarter's excess return, and they retain their marginal predictive power no matter what other commonly used predictor variables are included in the regression. The information in these two factors is largely independent of that in the consumption-wealth variable cay_t. Thus, when combined with cay_t, the regression model explains 16 percent of one-quarter ahead excess stock market returns, achieving the lowest BIC criterion of all the models studied. In addition to these two factors, the product of the third and fourth estimated factors from the financial database, and the product of the third and sixth estimated factors from the macro database, contain information about future returns that is not already contained in any of $\widehat{G}_{1t}^2$, $\widehat{G}_{3t}$, $RREL_t$ or cay_t (column l). This statistical model explains a striking 19 percent of one-quarter ahead excess returns, but the BIC criterion gives a higher penalty for the additional variables. As a consequence, the model ranks lower than the more parsimonious three-factor specification that includes only cay_t, $\widehat{G}_{1t}^2$, and $\widehat{G}_{3t}$. By contrast, a four-factor specification that includes cay_t, $\widehat{G}_{1t}^2$, $\widehat{G}_{3t}$ and the product $\widehat{F}_{3t} \cdot \widehat{F}_{6t}$ has a BIC statistic that is almost as small as the three-factor specification (-2.10 versus -2.11) but has an $\overline{R}^2$ that is slightly higher (0.17 versus 0.16), thus we consider both statistical models of future returns when forming estimates of μ_t below.

A similar analysis is conducted for stock market volatility, with results reported in Table 3. Column a of Table 3 shows that cay_t has predictive power for quarterly volatility, as found in Lettau and Ludvigson (2003), explaining about 8 percent of one–quarter ahead volatility. The dividend price ratio also has predictive power for future volatility, explaining 10 percent

of one-quarter ahead volatility. The predictive coefficients in the volatility equation are both negative for cay_t and $d_t - p_t$. Since these variables are positively related with future returns, the finding that they are negatively related with future volatility may at first suggest that the conditional mean is negatively correlated with conditional volatility. However, as we shall see, such a conclusion ignores the information contained in the estimated factors for future returns and future volatility. We show below that this information is important for properly identifying the risk-return relation.

Table 3 shows that a number of estimated common factors contain information about future volatility. The factors $\widehat{F}_{1t}$, $\widehat{F}_{6t}$ and $\widehat{F}_{7t}$ together explain about 11 percent of one-quarter ahead volatility (column c), and the $\widehat{G}_{1t}$, $\widehat{G}_{6t}$ and $\widehat{G}_{7t}$ explain about 18 percent of one-quarter ahead volatility (column d). All six together explain 25 percent of one-quarter ahead volatility. Of course, stock market volatility is known to be persistent, and its lags explain a large fraction of the future volatility. This can be seen in the results reported in columns f through n where one- and sometimes two-period lagged volatility is shown to be strongly statistically significant and their inclusion increases the adjusted R-squared statistic considerably. Columns h and i include the estimated common factors of columns c and d, respectively, along with a number of other variables used elsewhere to predict volatility: cay_t, $d_t - p_t$, DEF_t, $YIELD_t$, and lagged volatility. The estimated factors $\widehat{G}_{1t}$, $\widehat{G}_{6t}$ and $\widehat{G}_{7t}$ appear to contain much of the same information contained in these other predictor variables, as they no longer have marginal predictive power when included along with the other predictors. cay_t, DEF_t and $YIELD_t$ are also driven out of the regression that includes all the variables mentioned above, including the factors. By contrast, the factors constructed from the macro dataset $\widehat{F}_{1t}$, $\widehat{F}_{6t}$ and $\widehat{F}_{7t}$ retain marginal predictive power, while cay_t and DEF_t are never important once these factors and the dividend-price ratio are included. The last six columns of Table 3 show that a number of nonlinear functions of estimated factors also contain information about future volatility. But, in some cases, the information is largely common to that in the dividend-yield or one-year Treasury bill (T-bill) rate, and the best specifications according to the BIC criterion contain just $\widehat{F}_{1t}$ and $\widehat{F}_{6t}$, along with $d_t - p_t$, $YIELD_t$ and VOL_t. This model explains almost 40 percent of one-quarter ahead volatility. But a model containing just $\widehat{F}_{1t}$, $d_t - p_t$, $YIELD_t$ and VOL_t, performs equally well, so we often use this more parsimonious statistical model when forming estimates of σ_t. (The conclusions are unchanged if we include $\widehat{F}_{6t}$.)

In summary, the results reported in Tables 2 and 3 indicate that good forecasts of returns and volatility can be made with only a few factors, and that the best forecasts often contain combinations of factors and commonly used conditioning variables. It is reassuring that many of estimated factors are found to contain information that is largely common to that in many predictor variables that have long served as conditioning variables in asset pricing

applications, suggesting that the standard variables do indeed summarize a large body of information about economic and financial activity. At the same time, however, the evidence in Tables 2 and 3 suggests that the information in commonly used predictors is still incomplete, because a few factors contain important information about future returns and future volatility that is not contained in these other variables.

Can we given an economic interpretation to the common factors? Because the factors are only identifiable up to a $r \times r$ matrix, a detailed interpretation of the individual factors is inappropriate. Nonetheless, it is useful to briefly characterize the factors that emerge from our our formally chosen specification procedure that are most important for forecasting returns and volatility. For forecasting returns, the results above indicate that two financial factors standard out, $\widehat{G}_{1t}^2$ and $\widehat{G}_{3t}$. $\widehat{G}_{1t}^2$ explains almost 80 percent of the contemporaneous variation in squared stock market returns, so we label this factor a "volatility factor." We adopt this naming convention even though the factor is more highly correlated with squared returns than realized volatility VOL_t. $\widehat{G}_{1t}^2$ is still relatively highly correlated with VOL_t, explaining 40 percent of its quarterly variation. The second important factor, $\widehat{G}_{3t}$, is highly correlated with a linear combination of three state variables used in the empirical asset pricing literature to explain cross-sectional variation in risk premia. When $\widehat{G}_{3t}$ is regressed on the three Fama-French cross-sectional risk factors, MKT, SMB_t, and HML_t, we find the following results (heteroskedasticity and serial correlation robust t-statistics in parentheses):

$$\widehat{G}_{3t} = \underset{(3.87)}{0.212} + \underset{(4.26)}{0.052} \cdot MKT_t - \underset{(-17.39)}{0.322} \cdot SMB_t - \underset{(-23.27)}{0.404} \cdot HML_t + u_t, \qquad \overline{R}^2 = 86\%.$$

In addition, if the three-month Treasury bill rate, RF_t, is included as an explanatory variable, the four variables together explain 96% of the variation in the factor $\widehat{G}_{3t}$ in our sample:

$$\widehat{G}_{3t} = \underset{(-8.19)}{-0.460} + \underset{(7.07)}{0.062} \cdot MKT_t - \underset{(-29.65)}{0.320} \cdot SMB_t - \underset{(-33.65)}{0.403} \cdot HML_t + \underset{(12.50)}{1.41} \cdot RF_t + u_t, \qquad \overline{R}^2 = 96\%.$$

The factor loads heavily on cross-sectional asset pricing factors, and therefore connects the time-series and the cross-section of expected stock returns. It is of interest that this factor is so well explained by time-variation in well-established cross-sectional risk factors, since factors that spuriously explain the cross-section are unlikely to also spuriously explain the time-series. Thus, we call this second factor a "risk premium factor."

For forecasting volatility, the results above indicate that at least one macroeconomic factor is a useful predictor when it is combined with other predictor variables, such as the dividend-yield, the one-year Treasury yield, and lagged volatility. This factor, F_{1t}, is the first common factor from the dataset comprised of macroeconomic indicators. (Stock and Watson (2002b)) formed factors from similar datasets of monthly data, and found that the first common factor is a "real factor," that is highly correlated with measure of real output

and employment but not highly correlated with prices. This is also the case in our application using quarterly data, where the first macro factor explains 73 percent of the contemporaneous variation in an index of manufacturing output. Thus, we follow this naming convention and call $\widehat{F}_{1t}$ a "real factor."

5.2 The Empirical Risk-Return Relation

We now turn our attention to the estimated risk-return relation, modeling the conditional mean and conditional volatility using the state variables chosen with the BIC criterion, as described above. Based on the BIC criterion, the conditional mean is measured by fitted values from a regressions of excess returns on the state variables in $MOD1$ and conditional volatility by fitted values from a regression of realized volatility on the state variables in $SPEC1$. We also consider two alternative models of the conditional mean and conditional volatility, denoted $MOD2$ and $SPEC2$, respectively. $MOD2$ uses the three state variables used in $MOD1$, (cay_t, $\widehat{G}_{1t}^2$ and $\widehat{G}_{3t}$), but adds the product of two estimated macro factors $\widehat{F}_{3t} \cdot \widehat{F}_{6t}$ (column o of Table 2). $SPEC2$ uses the same state variables used in $SPEC1$, ($d_t - p_t$, $YIELD_t$, $\widehat{F}_{1t}$, and VOL_t), but omits the real factor $\widehat{F}_{1t}$. Econometric models of the form (7) are used to evaluate the empirical risk-return relation. In addition to (7), we follow Ghysels, Santa-Clara, and Valkanov (2005) and consider regressions of the form

$$\mu_t = \delta + \beta_1 \sigma_t + \beta_2 \sigma_{t-1} + \alpha \mu_{t-1} + \gamma m_{t-1} + \varepsilon_{t+1}, \tag{10}$$

where the lagged excess return is included as an additional explanatory variable.

The conditional correlation between the conditional mean and conditional volatility is positive in all cases. Table 4 shows that, conditional on lagged volatility and lagged mean, the coefficient on contemporaneous volatility is positive and strongly statistically significant regardless of which specification is used to model mean or volatility. Moreover, the t-statistics for β_1 are all in excess of 4. This positive contemporaneous trade-off between risk and return is consistent with the results of French, Schwert, and Stambaugh (1987), Bollerslev, Engle, and Wooldridge (1988), Harvey (1989), Campbell and Hentschel (1992) and Ghysels, Santa-Clara, and Valkanov (2005) but not those of Campbell (1987), Breen, Glosten, and Jagannathan (1989), Pagan and Hong (1991), Glosten, Jagannathan, and Runkle (1993), Whitelaw (1994), Lettau and Ludvigson (2003) and Brandt and Kang (2004). The coefficient on m_{t-1} in (10) is also strongly statistically significant and negative, but the inclusion of m_{t-1} does not alter the strong positive contemporaneous relation between conditional mean and conditional volatility. Thus, we find a strong positive volatility-in-mean effect for all specifications. By contrast, the lag-volatility-in-mean effect is strongly negative. Both lagged volatility and lagged mean are important explanatory variables for the conditional mean,

and all the same conclusions arise if σ_t is used as the left-hand-side variable instead of μ_t. The conditional correlation points to significant lead-lag interactions in the relation between conditional mean and conditional volatility, again consistent with Brandt and Kang (2004) and also Whitelaw (1994), and suggests that μ_t and σ_t are highly persistent.

Although we both emphasize the importance of lead-lag interactions in the risk-return relation, our results on the sign of the contemporaneous relation between μ_t and σ_t contrast with those of Brandt and Kang (2004) and also Lettau and Ludvigson (2003), who report a negative correlation between conditional mean and conditional volatility. Moreover, although Brandt and Kang find a negative *conditional* correlation between μ_t and σ_t (conditional on lagged mean and volatility), they find a positive *unconditional* correlation between μ_t and σ_t. These results are the opposite of those reported in Table 4, where the conditional correlation is positive and the unconditional correlation (omitting lags of μ_t and σ_t) is negative. Thus, somewhat ironically, like Brandt and Kang (2004), we find that distinguishing between the *conditional* (on lagged mean and lagged volatility) and *unconditional* correlation between the conditional mean stock return and its conditional volatility is crucial for understanding the empirical risk-return relation.

There are a number of possible reasons why our results differ from those of Brandt and Kang. First, as mentioned, the econometric methodologies differ. Brandt and Kang use a latent VAR approach to model the conditional mean and conditional volatility, assuming that these variables follow first-order, linear, Gaussian processes. This approach relies on the history of returns to infer μ_t and σ_t and does not condition upon the vast set of exogenous conditioning variables we employ in this study. Second, Brandt and Kang model the log moments, whereas we follow the bulk of the literature and model the relation between the mean and volatility in levels.[17] Third, our sample size and data frequency differ: Brandt and Kang studied monthly data from January 1946 through December 1998, while we study quarterly data from the first quarter of 1960 to the second quarter of 2003. Several variables that are important for predicting returns and volatility (e.g., cay_t) are only available at quarterly frequency and the predictable dynamics may vary from monthly to quarterly horizons.

Our econometric approach is more closely related to that of Lettau and Ludvigson (2003), who consider a wide range of commonly used predictor variables for returns and volatility in modeling the risk-return relation. Yet, unlike Lettau and Ludvigson, we find a strong positive conditional correlation between μ_t and σ_t, whereas they report a negative relation. Since Lettau and Ludvigson survey a broad range of models studied in the literature (distin-

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[17]Brandt and Kang use their assumption that the log moments are bivariate normally distributed to infer the relation between the level moments, which, under this assumption must be bivariate log-normally distributed. With this distributional assumption, they approximate the correlation between the level moments and conclude that the level moments also display a negative conditional correlation.

guished by the particular conditioning variables used to measure conditional moments), the differences in our results suggest that the conditioning information introduced by our estimated factors may be especially important for properly measuring the risk-return relation in methodologies that rely on exogenous predictors.

To illustrate this point, Table 5, presents the results from estimating the risk-return relation using the best fitting specifications (according to the BIC criterion) *omitting* estimated factors. The table shows that the findings in Lettau and Ludvigson (2003) can be largely attributed to the omission of the volatility and risk premium factors $\widehat{G}_{1t}^2$ and $\widehat{G}_{3t}$, which contain important information about one-quarter ahead returns. First, consider rows 3 and 4, which report the relation between μ_t and σ_t. These rows reproduce the qualitative findings of Lettau and Ludvigson (2003), who focused on the unconditional contemporaneous correlation: the coefficient in a regression of μ_t on σ_t is negative and statistically different from zero. This result holds any time the estimated volatility and combination factors are omitted when modeling μ_t and σ_t. By contrast, the results in Table 4, based on models that make use of information in $\widehat{G}_{1t}^2$ and $\widehat{G}_{3t}$, show that the contemporaneous relation is slightly negative but not statistically different from zero. Instead, in Table 4, it is the *conditional* correlation between the mean and volatility that is strongly statistically significant, but this correlation is not negative but positive. Interestingly, row 1 of Table 5 shows that this positive conditional correlation carries over to the case where the estimated factors are omitted in modeling μ_t and σ_t, but differs from the case where estimated factors are employed in that this result is not robust to the inclusion of lagged returns m_t as a right-hand-side variable. These results suggest that the factor-augmented specifications of μ_t and σ_t are important for properly identifying the empirical risk-return relation.

The results discussed so far are all based on OLS estimation. We also estimated the specifications (7) and (10) using two-stage least squares, instrumenting for σ_t using lagged variables as instruments: $\mu_{t-1}, \mu_{t-2}, \sigma_{t-1}, \sigma_{t-2}, m_{t-1}, VOL_{t-1}$. Because the results from 2SLS estimation are very similar to those using OLS estimation, we present only one set of findings from 2SLS estimation in Table 4, reported in row 13. As with the OLS estimation, the conditional correlation between μ_t and σ_t is found to be strongly statistically significant and positive, indicating a positive volatility-in-mean relationship. By contrast, the lag-volatility-in-mean relation is also strongly negative, again the same finding using OLS estimation.

Figure 1 depicts graphically over time variation in the conditional mean, based on the factor-augmented specification $MOD1$, along with 95% confidence intervals formed from 10,000 bootstrapped observations on our exogenous predictors and factors. The conditional mean rises in all seven of the NBER recession periods in our sample. This countercyclical pattern in the conditional mean is consistent with findings in Fama and French (1989), Lettau and Ludvigson (2003) and Brandt and Kang (2004). The conditional mean itself varies

between -0.04 and 0.08 over most of the sample, and there are a number of negative observations. As one would expect, negative observations on the conditional mean are common in linear empirical models (e.g., Harvey (2001), Lettau and Ludvigson (2001b), Lettau and Ludvigson (2003)). One might be more comfortable with fewer negative observations, but it should be noted that an occasional negative risk premium on stock market wealth is not necessarily inconsistent with equilibrium asset pricing models (Boudoukh, Richardson, and Whitelaw (1997); Whitelaw (2000)).

The dynamics of conditional volatility, based on $SPEC1$, are displayed in Figure 2. In most recessions, conditional volatility tends to be high and increasing, consistent with the results of Schwert (1989) and Schwert (1990). But there are a few recessions for which this is not the case using our measure of σ_t, notably the recessions of early 1960 and 2001. The cyclical movements in conditional volatility are, however, swamped by large, low-frequency fluctuations. This feature likely contributes to the weak unconditional risk-return relation in our sample, since the conditional mean appears far less persistent. Figure 3 plots the conditional volatility and conditional mean on the same graph, displaying the weak negative contemporaneous correlation in our sample. Interestingly, the negative correlation appears largely attributable to the period between 1995 and 2000, when conditional mean trended up and conditional volatility trended down. The countercyclical variation in volatility is not as distinct as that in the conditional mean; as a result, the conditional Sharpe ratio (the ratio of conditional mean to conditional volatility) is distinctly countercyclical (Figure 4). In particular, the Sharpe ratio rises sharply in every recession, with noticeable spikes in the 1970-71 and 1990-91 recessions. Such countercyclical variation in the Sharpe ratio arises naturally in models with countercyclical risk or risk aversion.

In summary, we find a strongly positive linear relation between conditional mean and conditional volatility, once empirically important lags of these variables are controlled for in the regression analysis. These findings support the theoretical prediction of a positive risk-return tradeoff, but also indicate a negative relation between mean and lagged volatility. Our next two subsections present additional results that pertain to the robustness of our underlying forecasting relations: out-of-sample analysis and small-sample inference.

5.3   Out-of-Sample Analysis

In the analysis above, we formally selected models for estimating the conditional mean and conditional volatility of stock returns using the BIC criterion from predictive regressions over the full sample. In this section we report results on the out-of-sample forecasting power of our formally selected models. This procedure involves fully recursive factor estimation and parameter estimation using data only through time t for forecasting at time $t + 1$.

For forecasting excess returns, we focus on the predictive power of the volatility and risk-premium factors in isolation, since the out-of-sample forecasting power of cay_t has been studied elsewhere (see Lettau and Ludvigson (2001a); Guo (2005)). The results from this two-factor model are compared to a constant expected returns benchmark. For forecasting volatility, we focus on the predictive power of one of our formally chosen specifications that includes the real factor along with other predictor variables. The real factor alone is not a significant predictor of volatility, but is when combined with the variables in column m of Table 3. Since volatility is known to be persistent, this model is compared with a first-order autoregressive benchmark.

Table 6 reports results from one-quarter-ahead out-of-sample forecast comparisons of log excess returns, m_{t+1} and volatility, VOL_{t+1}. For the purpose of this out-of-sample analysis, the factors G_{1t}^2 and G_{3t} are formed recursively from the financial dataset omitting cay_t. Since the cointegrating coefficients in cay_t are estimated over the full sample, omitting cay_t insures that the time-t factors used to forecast returns at time $t+1$ are formed using data only up to time t. $SPEC1$ denotes a forecasting model for volatility that uses the following variables as predictors: the CRSP log dividend-price ratio $d_t - p_t$, the one year Treasury bill yield $YIELD_t$, the real factor $\widehat{F}_{1t}$, and lagged volatility VOL_t. This corresponds to the model in column m of Table 3. For each forecast, MSE_u denotes the mean-squared forecasting error of the unrestricted model including predictor factors; MSE_r denotes the mean-squared forecasting error of the restricted benchmark model that excludes additional forecasting variables. In the column labeled "MSE_u/MSE_r", a number less than one indicates that the models that uses the additional forecasting variables have lower forecast error than the benchmark to which it is compared.

Results for three forecast samples are reported: 1975:1-2003:2; 1985:1-2003:2; 1995:1-2003:2. The results for the first forecast sample are reported in Rows 1 and 4. Here the parameters and factors were estimated recursively, with the initial estimation period using only data available from 1960:1 through 1975:1. Next, the forecasting regressions were run over the period $t =$1960:1,...,1975:1, and the values of the regressors at $t =$1975:1 were used to forecast $m_{1975:2}$ (row 1) or $VOL_{1975:2}$. All parameters and factors are then reestimated from 1960:1 through 1975:2, and forecasts were recomputed for $m_{1975:3}$ and $VOL_{1975:3}$, and so on, until the final out-of-sample forecast is made for $m_{2003:2}$ and $VOL_{2003:2}$. The same procedure is used to compute results reported in rows 2, 3, 5, and 6 where the initial estimation period is either $t =$1960:1,...,1985:1 (rows 2 and 5) or $t =$1960:1,...,1995:1 (rows 3 and 6). The column labeled "Test Statistic" in Table 6 reports the ENC-NEW test statistic of Clark and McCracken (2001) for the null hypothesis that the benchmark model encompasses the unrestricted model with additional predictors. The alternative is that the unrestricted model contains information that could be used to improve the benchmark model's forecast. "95%

Asympt. CV" gives the 95th percentile of the asymptotic distribution of the ENC-NEW test statistic.

Consider the forecasts of excess returns in rows 1 through 3 of Table 6. The two-factor model that uses only G_{1t}^2 and G_{3t} as predictors improves substantially over the constant expected returns benchmark. These models have a forecast error variance that is 92 and 91 percent of the constant expected returns benchmark for the forecast periods 1975:1-2003:2 and 1985:1-2003:2, respectively. For the period 1995:1-2003:2 the model has a forecast error variance that is only 79 percent of the constant expected returns benchmark. This is rather surprising, since it implies that the two-factor model exhibits the greatest relative improvement over the benchmark during a period in which the forecasting power of many conventional predictor variables breaks down. No matter what subperiod the model is evaluated over, the ENC-NEW test statistic always indicates that the improvement in forecast power is strongly statistically significant, at the one percent or better level. These results show that relative forecast improvement afforded by the estimated factors is stable over time and both statistically and economically significant. Figure 5 gives a graphical impression of the predictive power of these two factors by plotting the forecasted value of excess returns along with the actual value over the period 1975:1-2003:2. Naturally the fitted value is less volatile than actual value, but the figure shows that the estimated factors do a remarkable job of forecasting the increase in excess returns in the late 1990s and the decline in early 2000 through 2002.

Rows 4 through 6 report the result of out-of-sample volatility forecasts. As for excess returns, there is substantial improvement in forecasting power relative to the autoregressive benchmark that is strongly statistically significant. Interestingly, as for returns, the model outperforms the benchmark by the largest margin in the period 1995:1-2003:2, displaying a forecast error variance that is just 70 percent of the benchmark. These results are displayed graphically in Figure 6. The model does an excellent job of capturing the low-frequency shift upward in volatility over the period 1995-2003.

5.4 Small Sample Inference

According to the asymptotic theory for PCA estimation discussed in Section 2, heteroskedasticity and autocorrelation consistent standard errors can be used to obtain robust t statistics that are asymptotically $N(0, 1)$. To guard against inadequacy of the asymptotic approximation in finite samples, we also consider bootstrap inference for two of our formally chosen specifications, those in column p of Table 2 (returns) and column m of Table 3 (volatility). The model in column p of Table 2 uses cay_t, $\widehat{G}_{1t}^2$ and $\widehat{G}_{3t}$ as state variables for estimating the conditional mean; we refer to this model as $MOD1$ for conditional expected returns.

The volatility model in column m of Table 3 uses $d_t - p_t$, $YIELD_t$, $\widehat{F}_{1t}$, and VOL_t as state variables for estimating conditional volatility; we refer to this specification as $SPEC1$ for conditional volatility. Small sample inference is especially important when the right-hand-side variables are highly persistent (e.g., Ferson, Sarkissian, and Simin (2003)) but, as Table 1 demonstrates, none of the factors from our preferred specifications are highly persistent. Nevertheless, we proceed with a bootstrap analysis as a robustness check, by generating bootstrap samples of the exogenous predictors Z_t, as well as of the estimated factors $\widehat{F}_t$ and $\widehat{G}_t$.

We consider two bootstrap procedures. First, we take the factors as given and generate bootstrap samples of Z_t, $\widehat{F}_t$ and $\widehat{G}_t$ from univariate first-order autoregressive models. This allows us to isolate the potential role of persistence in contaminating inference. Bootstrap samples of m_{t+1} are obtained in two ways, first by imposing the null hypothesis of no predictability, (i.e. α_1, α_2 and β in (8) are zero vectors and residuals from regression on a constant are resampled), and second, without imposing the null by resampling the residuals of (8). A regression using the bootstrap data gives new estimates of α_1, α_2, and β, and new $\bar{R}^2$ statistics. A directly analogous investigation is conducted for the volatility equation (9). This is repeated $B = 10,000$ times. The results are reported in Table 7.

In the second procedure, we take into account the pre-estimation of the factors by re-sampling the $T \times N$ panel of data, x_{it}. This creates bootstrapped samples of the factors themselves. For each i, least squares estimation of $\widehat{e}_{it} = \rho_i \widehat{e}_{it-1} + v_{it}$ yields the estimates $\widehat{\rho}_i$ and $\widehat{v}_{it}$, $t = 2, \ldots T$, where recall that $\widehat{e}_{it} = x_{it} - \widehat{\lambda}'_i \widehat{f}_t$. Then $\widehat{v}_{it}$ is re-sampled (while preserving the cross-section correlation structure) to yield bootstrap samples of $\widehat{e}_{it}$. In turn, bootstrap values of x_{it} are constructed by adding the bootstrap estimates of the idiosyncratic errors, $\widehat{e}_{it}$, to $\widehat{\lambda}'_i \widehat{F}_t$. Estimation by the method of principal components on the bootstrapped data then yields a new set of estimated factors. Together with bootstrap samples of Z_t (based on AR(1) models as above), this delivers a set of bootstrap regressors. Samples of m_{t+1} are again obtained in two ways, either by imposing or not imposing the null of no predictability, and bootstrap $\bar{R}^2$ and t statistics are obtained by performing a regression on the bootstrap data. Bootstrap confidence intervals for the parameter estimates and $\bar{R}^2$ statistics can be calculated from $B = 10,000$ replications. A directly analogous investigation is conducted for the volatility equation (9). These results are reported in Table 8.

Both Tables 7 and 8 indicate that the results based on bootstrap inference are consistent with those based on asymptotic inference. Using either procedure, the coefficients on the exogenous predictors and estimated factors are statistically different from zero at the 95% level and are well outside the 95% confidence interval under the no-predictability null. The three factor model for returns generates an adjusted R-squared statistic of 16% in historical data; by contrast, using bootstrapped data, the 95% bootstrapped confidence interval for

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this statistic under the no-predictability null ranges from just -2% to 4%. Similarly, the four factor model for volatility generates an adjusted R-squared statistic of 37% in historical data; by contrast, using bootstrapped data, the 95% bootstrapped confidence interval for this statistic under the no-predictability null ranges from just -2.1% to 4.4%. In short, the magnitude of predictability found in historical data is too large to be accounted for by sampling error in samples of the size we have.

Finally, the top panel of Tables 7 and 8 reports a 95% bootstrapped confidence interval of a $\chi^2(2)$ statistic for the null hypothesis that the estimated volatility and combination factors are jointly equal to zero. The corresponding $\chi^2(2)$ estimate from historical data of 16.87 is well outside the 95% confidence interval of $(0.52, 7.65)$ under the no-predictability null. The statistical relation of these factors to future returns is strong, even accounting for the small sample distribution of standard test statistics.

6   Conclusion

A large and growing body of empirical work is devoted to estimating the relation between risk and return in the U.S. stock market. Although theory typically predicts a positive relation, empirical findings are mixed and often suggest a negative relation. An important limitation of existing empirical work, however, pertains to the relatively small amount of conditioning information used to estimate the conditional mean and conditional volatility of excess stock market returns. In turn, the use of such sparse information sets in the construction of fitted moments can translate into omitted information bias in the estimated risk-return relation.

In this paper, we consider one approach to this omitted information problem by employing a methodology for incorporating a large amount of conditioning information in our estimates of the conditional mean and conditional volatility of excess stock market returns. Recent research on dynamic factor models finds that the information in a large number of economic time series can be effectively summarized by a relatively small number of estimated factors, affording the opportunity to exploit a rich base of information more likely to span the information sets of financial market participants than in previous analyses. In doing so, our study contributes to the empirical literature by evaluating both the potential role of omitted information in the estimated risk-return relation, as well as the robustness of previous results to conditioning on richer information sets.

Some of our results support the findings of pre-existing studies. For example, we find that the conditional mean return and conditional Sharpe ratio are strongly countercyclical, and that lead-lag dynamics are important elements of the risk-return relation. But other key aspects of our results differ from previous work, suggesting that the factor-augmented

approach is important for properly identifying the empirical risk-return relation. In particular, we introduce several new factors that contain significant information about either future returns or future volatility. Two factors stand out as particularly important for quarterly excess returns: a volatility factor that is highly correlated with squared returns, and a risk-premium factor that is highly correlated with well-established risk factors for explaining the cross-section of expected returns. Using the information contained in these estimated factors, we find that the contemporaneous relation between the conditional mean and conditional volatility is strongly positive, once we control for lags of these variables. Our findings therefore support the standard theoretical prediction of a positive volatility-in-mean effect. We also find a strongly negative lag-volatility-in-mean effect, about which there is much less theoretical precedent. Finally, we find that the improvement in out-of-sample forecasting power afforded by our estimated factors is strongly statistically significant and remarkably stable over time, even though the relationship between many commonly used predictors and future returns is unstable over our sample period, especially during the last half of the 1990s. This evidence reinforces the notion that dynamic factor analysis can provide robustness against the temporal instability that plagues low-dimensional forecasting regressions.

There are several possible directions for future work. The investigation here could be extended to study the potential role of nonlinearities in the risk-return relation (e.g., Harvey (2001)), or to assess the degree of time variation in the risk-return relation (e.g., Campbell (1987), Harvey (1989)). In addition, the approach taken here—in which common factors from large datasets are combined with measures of realized volatility—could be extended to model conditional covariances, or conditional betas, as in the work of Andersen, Bollerslev, Diebold, and Wu (2005b) and Andersen, Bollerslev, Diebold, and Wu (2005a). All of these issues lend themselves well to study using large datasets of conditioning information, summarized by a few estimated factors.

Data Appendix

This appendix describes our data. The data are quarterly and span 1960:1 to 2002:4.

Macro Data

Below we list the data used to construct the macro factors. The format is, series number, series mneonic; transformation code and brief series description. All macro data are from DRI-Global Insight, Basic Economics Database. The data transformation (DT) codes are, 1=no transformation; 2 = first difference; 4 = log; 5 = log first difference. GY = national income.

Category		DT	Description
FX (FX)			
1	BPAUS	2	U.S. ASSETS ABROAD (NET)
2	BPB	2	BAL OF P'MENT:BALANCE ON MERCHANDISE TRADE,MIL.$ SA
3	GDFXFC	5	CHAIN-TYPE QUANTITY INDEX - EXPORTS OF GOODS AND SERVICES
4	GNET	2	NET EXPORTS OF GOODS AND SERVICES
5	GRFIW	5	RECEIPT FACTOR INCOME FROM REST OF WORLD(BIL.$,SAAR)(T1.9)
6	GXIM	1	% CHG FRM PRECEDING PERIOD: IMPORTS (CURRENT$)
7	GXMDQF	5	EXPORTS-DURABLE GOODS,SRC:BEA,BIL OF 1996 DOLLARS
8	GXMNQF	5	EXPORTS-NONDURABLE GOODS,SRC:BEA,BIL OF 1996 DOLLARS
9	GXMQF	5	EXPORTS-GOODS(FIXED),SRC:BEA,BIL OF 1996 DOLLARS
10	GDFMFC	5	CHAIN-TYPE QUANTITY INDEX - IMPORTS OF GOODS AND SERVICES
Consumption (Cons)			
11	GDFCDC	5	CHAIN-TYPE QUANTITY INDEX - PCE, DURABLE GOODS
12	GXDAQF	5	AUTO OUTPUT-EXPORTS,SRC:BEA,BIL OF 1996 DOLLARS
13	GXPC	1	% CHG FROM PRECEDING PERIOD:PERSONAL CONSUMPTION EXPENDS.(CURR.$)
14	GDFCFC	5	CHAIN-TYPE QUANTITY INDEX - PERSONAL CONSUMPTION EXPENDITURES
Prices (Pri)			
15	EXRJAN	5	FOREIGN EXCHANGE RATE: JAPAN (YEN PER U.S.$)
16	EXRUK	5	FOREIGN EXCHANGE RATE: UNITED KINGDOM (CENTS PER POUND)
17	EXRUS	5	UNITED STATES;EFFECTIVE EXCHANGE RATE(MERM)(INDEX NO.)
18	GD	5	IMPLICIT PR DEFLATOR: GROSS NATIONAL PRODUCT
19	GDC	5	IMPLICIT PR DEFLATOR: PERSONAL CONSUMPTION EXPENDITURES
20	GDCD	5	IMPLICIT PR DEFLATOR: DURABLE GOODS,PCE
21	GDCN	5	IMPLICIT PR DEFLATOR: NONDURABLE GOODS,PCE
22	GDCS	5	IMPLICIT PR DEFLATOR: SERVICES, PCE
23	GDEX	5	IMPLICIT PR DEFLATOR: EXPORTS OF GDS & SER
24	GDEXIM	5	TERMS OF TRADE
25	GDFCC	5	CHAIN-TYPE PRICE INDEX - PERSONAL CONSUMPTION EXPENDITURES
26	GDFCNC	5	CHAIN-TYPE PRICE INDEX - PCE, NONDURABLE GOODS
27	GDFCSC	5	CHAIN-TYPE PRICE INDEX - PCE, SERVICES
28	GDFDCF	5	CHAIN-TYPE PRICE INDEX - NATL DEFENSE EXPENDITURES & GROSS INVESTM
29	GDFDFC	5	CHAIN-TYPE PRICE INDEX - PCE, DURABLE GOODS
30	GDFDPC	5	CHAIN-TYPE PRICE INDEX- PRODUCERS' DURABLE EQUIPMENT
31	GDFEXC	5	CHAIN-TYPE PRICE INDEX - EXPORTS OF GOODS AND SERVICES
32	GDFGEC	5	CHAIN-TYPE PRICE INDEX - GOVT CONSUMPTION EXPENDITURES & GROSS INV
33	GDFGFC	5	CHAIN-TYPE PRICE INDEX - FED CONSUMPTION EXPEND & GROSS INVESTMENT
34	GDFGOC	5	CHAIN-TYPE PRICE INDEX - NONDEF CONS EXPENDITURES & GROSS INVESTME
35	GDFGSC	5	CHAIN-TYPE PRICE INDEX - S&L CONSUMPTION EXPEND & GROSS INVESTMENT
36	GDFICF	5	CHAIN-TYPE PRICE INDEX - PRIVATE FIXED INVESTMENT

37	GDFIMC	5	CHAIN-TYPE PRICE INDEX - IMPORTS OF GOODS AND SERVICES
38	GDFIRC	5	CHAIN-TYPE PRICE INDEX - RESIDENTIAL
39	GDFISC	5	CHAIN-TYPE PRICE INDEX - NONRESIDENTIAL STRUCTURES
40	GDFNRC	5	CHAIN-TYPE PRICE INDEX - NONRESIDENTIAL
41	GDGF	5	IMPLICIT PR DEFLATOR: FED GOV'T PURCH OF GDS & SER
42	GDIS	5	IMPLICIT PR DEFLATOR: PRIVATE NONRESINDENTIAL STRUCTURES
43	LBGDPU	5	IMPLICIT PRICE DEFLATOR: NONFARM BUSINESS (1982=100,SA)
44	PMCP	5	NAPM COMMODITY PRICES INDEX (PERCENT)
45	PSCCOM	5	SPOT MARKET PRICE INDEX:BLS & CRB: ALL COMMODITIES(1967=100)
46	PSCFOO	5	SPOT MARKET PRICE INDEX:BLS & CRB: FOODSTUFFS (67=100,NSA)
47	PSCMAT	5	SPOT MARKET PRICE INDEX:BLS & CRB: RAW INDUSTRIALS(67=100,NSA)
48	PUCX	5	CPI-U: COMMODITIES LESS FOOD (82-84=100,SA)
49	PUCXX	5	CPI-U:COMMODITIES LESS FOOD AND ENERGY (82-84=100,SA)
50	PUHS	5	CPI-U: SHELTER (82-84=100,SA)
51	PUNEW	5	CPI-U: ALL ITEMS (82-84=100,SA)
52	PUXF	5	CPI-U: ALL ITEMS LESS FOOD (82-84=100,SA)
53	PUXHS	5	CPI-U: ALL ITEMS LESS SHELTER (82-84=100,SA)
54	PUXM	5	CPI-U: ALL ITEMS LESS MIDICAL CARE (82-84=100,SA)
55	PUXX	5	CPI-U: ALL ITEMS LESS FOOD AND ENERGY (82-84=100,SA)
56	PWFCSA	5	PRODUCER PRICE INDEX:FINISHED CONSUMER GOODS (82=100,SA)
57	PWFPSA	5	PRODUCER PRICE INDEX:CAPITAL EQUIPMENT (82=100,SA)
58	PWFSA	5	PRODUCER PRICE INDEX: FINISHED GOODS (82=100,SA)
59	PWIMSA	5	PRODUCER PRICE INDEX:INTERMED MAT.SUPPLIES & COMPONENTS(82=100,SA)
60	PWMND	5	PRODUCER PRICE INDEX: TOTAL NONDURABLE GOODS (82=100,NSA)
61	PWSA2X	5	PRODUCER PRICE INDEX:INTERMED MAT.LESS FOODS & FEEDS(82=100,SA)
62	PWSA3X	5	PRODUCER PRICE INDEX:FINISHED CONSUMER GODS EXCL FOODS(82=100,SA)
63	PWSA4	5	PRODUCER PRICE INDEX:CONSUM NOND GODS LESS FOOD(82=100,SA)
64	PWSA5	5	PRODUCER PRICE INDEX:CONSUMER DURABLE GOODS (82=100,SA)

Fixed Investment (Inv)

65	GFINO	5	FIXED INVEST:PRODUCER DURABLE EQUIP. OTHER(BIL$SAAR)(T5.4)
66	GIFQF	5	FIXED INVEST, TOTAL(FIXED), SRC:BEA, BIL OF 1996 DOLLARS
67	GINQF	5	FIXED INVEST, NONRESIDENTIAL(FIXED), SRC:BEA, BIL OF 1996 DOLLARS
68	GIRQF	5	FIXED INVEST, RESIDENTIAL(FIXED), SRC:BEA, BIL OF 1996 DOLLARS
69	GISQF	5	PUR OF NONRES STRUCT-TOTAL, SRC:BEA, BIL OF 1996 DOLLARS
70	GPIQF	5	GROSS PRIV DOMESTIC INVEST(FIXED),SRC:BEA,BIL OF 1996 DOLLLARS
71	GXIFN	1	% CHG FRM PRECEDING PERIOD:NONRESIDENTIAL FIXED INVEST.(CURR.$)
72	GXIFR	1	% CHG FRM PRECEDING PERIOD:RESIDENTIAL FIXED INVESTMENT(CURR$)
73	GXIPD	1	% CHG FRM PRECEDING PERIOD: NONRESID.PRODUCERS'DUR.EQUIP(CURR$)
74	GXIS	1	% CHG FRM PRECEDING PERIOD: NONRESIDENTIAL STRUCTURES (CURRENT$)
75	GXPI	1	% CHG FRM PRECEDING PERIOD:GROSS PRIV.DOM.INVESTMNT(CURR.$)
76	GDFFIC	5	CHAIN-TYPE QUANTITY INDEX - PRIVATE FIXED INVESTMENT
77	GDFIFC	5	CHAIN-TYPE QUANTITY INDEX - GROSS PRIVATE DOMESTIC INVESTMENT

Output & Income (Out)

78	GDFDEC	5	CHAIN-TYPE QUANTITY INDEX - NATL DEF EXPENDITURES & GROSS INVESTME
79	GDFEOC	5	CHAIN-TYPE QUANTITY INDEX - NONDEF CONS EXPEND & GROSS INVESTMENTS
80	GDFFGC	5	CHAIN-TYPE QUANTITY INDEX - FED CONSUMPTION EXPEND & GROSS INVESTM
81	GDFGGC	5	CHAIN-TYPE QUANTITY INDEX - GOVT CONSUMPTION EXPENDITURES & GROSS
82	GDFGLC	5	CHAIN-TYPE QUANTITY INDEX - S&L CONSUMPTION EXPEND & GROSS INVESTM
83	GDFINC	5	CHAIN-TYPE QUANTITY INDEX - NONRESIDENTIAL
84	GDFNFC	5	CHAIN-TYPE QUANTITY INDEX - PCE, NONDURABLE GOODS
85	GDFPDC	5	CHAIN-TYPE QUANTITY INDEX - PRODUCERS' DURABLE EQUIPMENT
86	GDFRFC	5	CHAIN-TYPE QUANTITY INDEX - RESIDENTIAL

87	GDFSFC	5	CHAIN-TYPE QUANTITY INDEX - PCE, SERVICES
88	GDFSTC	5	CHAIN-TYPE QUANTITY INDEX - NONRESIDENTIAL STRUCTURES
89	GDPQF	5	GDP (FIXED),SRC:BEA,BIL OF 1996 DOLLARS
90	GPBQF	5	GDP-BUS,SRC:BEA,BIL OF 1996 DOLLARS
91	GPY	5	PERSONAL INCOME, TOTAL
92	GWY	5	NAT'L INCOME: WAGES AND SALARIES
93	GXNP	1	% CHANGE FROM PRECEDING PERIOD, GNP CURRENT $
94	GXSAV	5	PERSN'L INCOME: PERS SAVING RATE, GPSAV AS % OF GYD
95	GXYD	1	% CHG FRM PRECEDING PERIOD: DISP. PERSONAL INCOME (CURRENT $)
96	GYDPCQ	5	DISPOSABLE PERSONAL INCOME PER CAPITA IN CHAINED (1996) DOLLARS
97	GYFIR	5	GY BY IND DIV: FINANCE, INSUR AND REAL ESTATE
98	GYGGE	5	GY BY IND DIV: GOV'T AND GOV'T ENTERPRISES
99	GYM	5	GY BY IND DIV: MANUFACTURING INDUSTRY
100	GYMD	5	GY BY IND DIV: DURABLE GOODS MANUFACTURING INDUSTRY
101	GYMN	5	GY BY IND DIV: NONDURABLE GOODS MANUFACTURING INDUSTRY
102	GYS	5	GY BY IND DIV: SERVICE INDUSTRIES
103	GYT	5	GY BY IND DIV: TRANSPORTATION INDUSTRY
104	GYUT	5	GY BY IND DIV: ELECTRIC, GAS AND SANITARY SEW INDUSTRY
105	LIPM	5	OUTPUT INDEX - MANUFACTURING (PC) (INDEX,92=100,SA)
106	LIPMD	5	OUTPUT INDEX - DURABLE MANUFACTURING (PC) (INDEX,92=100,SA)
107	LIPMN	5	OUTPUT INDEX - NONDURABLE MANUFACTURING (PC) (INDEX,92=100,SA)
108	LOUTM	5	OUTPUT PER HOUR OF ALL PERSONS, INDEX - MANUFACTURING (PC) (INDEX,92=100,SA)
109	LOUTMD	5	OUTPUT PER HOUR OF ALL PERSONS, INDEX - DURABLE MANUFACTURING (PC) (INDEX,92=1
110	LOUTMN	5	OUTPUT PER HOUR OF ALL PERSONS, INDEX, NONDURABLES;INDEX: 1992=100,SA;SRC: BLS

Sales, Ortders, Purchases (SOP)

111	GNSAQF	5	AUTO OUTPUT-FINAL SALES,SRC:BEA,BIL OF 1996 DOLLARS
112	GNSQF	5	FINAL SALES OF DOMESTIC PRODUCT (BILL 1996 $, SAAR)
113	GNSQF	5	FINAL SALES OF DOMESTIC PRODUCT (BILL 1996 $, SAAR)
114	GODSQF	5	FINAL SALES OF DURABLES,SRC:BEA,BIL OF 1996 DOLLARS
115	GONSQF	5	FINAL SALES OF NONDURABLES,SRC:BEA,BIL OF 1996 DOLLARS
116	GXNPD	1	GROSS DOM PURCH:CURRENT DOLLARS(%,CHANGE)(T8.1)
117	GXNS	1	FINAL SALES OF DOM PROD:CURRENT DOLLARS(%,CHANGE)(T8.1)
118	GXNSD	1	FINAL SALE TO DOM PURCH:CURRENT DOLLARS(%,CHANGE)(T8.1)
119	MOCMQ	5	NEW ORDERS (NET) - CONSUMER GOODS & MATERIALS, 1996 DOLLARS (BCI)
120	MSONDQ	5	NEW ORDERS, NONDEFENSE CAPITAL GOODS, IN 1996 DOLLARS (BCI)
121	MTQ	5	SALES - MANUFACTURING & TRADE, CHAINED 1996 DOLLARS (BCI)
122	PMNO	5	NAPM NEW ORDERS INDEX (PERCENT)
123	RISMAT	1	Real Inventory-Sales Ratios - Manufacturing and trade industries, Ratio, SAAR
124	RZTRU	5	RETAIL SALES: NEW MOTOR TRUCKS, TOTAL (# OF UNITS,NSA)
125	SMB	5	SHIPMENTS - ALL MANUFACTURING INDUSTRIES NAICS (M3)
126	SMU	5	SHIPMENTS - ALL MANUFACTURING INDUSTRIES NAICS (M3)
127	LBOUT	5	OUTPUT PER HOUR ALL PERSONS: BUSINESS SEC(1982=100,SA)
128	LBOUTU	5	OUTPUT PER HOUR ALL PERSONS: NONFARM BUSINESS(82=100,SA)

Employment and Hours (EMP)

129	LBMN	2	HOURS OF ALL PERSONS: BUSINESS SECTOR (1982=100,SA)
130	LBMNU	2	HOURS OF ALL PERSONS: NONFARM BUSINESS SEC (1982=100,SA)
131	LHEL	2	INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS (1967=100;SA)
132	LHEM	5	CIVILIAN LABOR FORCE: EMPLOYED, TOTAL (THOUS.,SA)
133	LHEMF	5	CIVILIAN LABOR FORCE: EMPLOYED, WOMEN, 16 YEARS + (THOUS.,SA)
134	LHEMM	5	CIVILIAN LABOR FORCE: EMPLOYED, MEN, 16 YEARS + (THOUS.,SA)
135	LHEMPA	5	RATIO, CIV.EMPLOYMNT/TOTAL NONINST.POPUL.,INC.ARMED FORCES(SA)
136	LHME25	2	EMPLOYED PERSONS: MALES, 25 TO 54 YEARS (THOUS.,SA)

137	LHMP20	2	LABOR FORCE PARTICIPATION RATE: MEN, 20 YRS.+ (%,SA)
138	LHMU25	2	UNEMPLOYMENT RATE: MEN, 25 TO 54 YEARS (%,SA)
139	LHMUR	2	UNEMPLOYMENT RATE: MEN, 20 YEARS & OVER (%,SA)
140	LHTPTA	2	LABOR FORCE PARTICIPATION RATE: BOTH SEXES,16-19 YRS.(%,SA)
141	LHTUR	2	UNEMPLOYMENT RATE: BOTH SEXES, 16-19 YEARS (%,SA)
142	LHU14	2	UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS (THOUS.,SA)
143	LHU15	2	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS + (THOUS.,SA)
144	LHU26	2	UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS (THOUS.,SA)
145	LHU27	2	UNEMPLOY.BY DURATION: PERSONS UNEMPL.27 WKS + (THOUS,SA)
146	LHU5	2	UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5 WKS (THOUS.,SA)
147	LHU680	2	UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS (SA)
148	LHUFR	2	UNEMPLOYMENT RATE: WOMEN, 16 YEARS AND OVER (%,SA)
149	LHUMR	2	UNEMPLOYMENT RATE: MEN, 16 YEARS AND OVER (%,SA)
150	LHUR	2	UNEMPLOYMENT RATE: ALL WORKERS, 16 YEARS & OVER (%,SA)
151	LHURM	2	UNEMPLOYMENT RATE: MARRIED MEN, SPOUSE PRESENT (%,SA)
152	LHURMF	2	UNEMPLOYMENT RATE: MARRIED WOMEN, SPOUSE PRESENT (%,SA)
153	LLCPB	5	UNIT LABOR COST: NONFINANCIAL CORP (1982=100,SA)
154	LMNM	2	HOURS OF ALL PERSONS, INDEX - MANUFACTURING (PC) (INDEX,92=100,SA)
155	LMNMD	2	HOURS OF ALL PERSONS, INDEX - DURABLE MANUFACTURING (PC) (INDEX,92=100,SA)
156	LMNMN	2	HOURS OF ALL PERSONS, INDEX - NONDURABLE MANUFACTURING (PC) (INDEX,92=100,SA)
157	LUINC	2	AVG WKLY INITIAL CLAIMS,STATE UNEMPLOY.INS.,EXC P.RICO(THOUS;SA)
158	LURSP	2	INSURED UNEMPLOYMENT AS % COVERED EMPLOY.,EXC P.RICO(%,SA)
159	LZHUR	2	UNEMPLOYMENT RATE: TOTAL, 16 YRS AND OVER (%,NSA)

Compensation and labor cost per hour (CHI)

160	LBCP	5	COMPENSATION PER HOUR: BUSINESS SECTOR (1982=100,SA)
161	LBCPU	5	COMPENSATION PER HOUR: NONFARM BUSINESS SEC(1982=100,SA)
162	LBLCP	5	UNIT LABOR COST: BUSINESS SECTOR (1982=100,SA)
163	LBLCPU	5	UNIT LABOR COST: NONFARM BUSINESS SEC (1982=100,SA)
164	LCPM	5	COMPENSATION PER HOUR INDEX - MANUFACTURING (PC) (INDEX,92=100,SA)
165	LCPM7	5	COMPENSATION PER HOUR (REAL), INDEX-MANUFACTURING (PC) (INDEX,92=100,SA)
166	LCPMD	5	COMPENSATION PER HOUR INDEX - DURABLE MANUFACTURING (PC) (INDEX,92=100,SA)
167	LCPMD7	5	COMPENSATION PER HOUR (REAL), INDEX - DURABLE MANUFACTURING (PC) (INDEX,92=100
168	LCPMN	5	COMPENSATION PER HOUR INDEX - NONDURABLE MANUFACTURING (PC) (INDEX,92=100,SA)
169	LCPMN7	5	COMPENSATION PER HOUR (REAL), INDEX - NONDURABLE MANUFACTURING (PC) (INDEX,92=
170	LLCPM	5	UNIT LABOR COSTS INDEX-MANUFACTURING (PC) (INDEX,92=100,SA)
171	LLCPMD	5	UNIT LABOR COSTS INDEX - NONDURABLE MANUFACTURING (PC) (INDEX,92=100,SA)
172	LLCPMN	5	UNIT LABOR COSTS INDEX, NONDURABLES;INDEX: 1992=100, SA;SRC: BLS

Capacity Utilization (Util)

173	UTL11	2	CAPACITY UTILIZATION - MANUFACTURING (SIC)
174	UTL15	2	CAPACITY UTILIZATION - NONMETALLIC MINERAL PRODUCT NAICS=327
175	UTL17	2	CAPACITY UTILIZATION - FABRICATED METAL PRODUCT NAICS=332
176	UTL21	2	CAPACITY UTILIZATION - MOTOR VEHICLES AND PARTS NAICS=3361-3
177	UTL22	2	CAPACITY UTILIZATION - AEROSPACE AND MISCELLANEOUS TRANSPORTATION EQ.
178	UTL29	2	CAPACITY UTILIZATION - PAPER NAICS=322
179	UTL31	2	CAPACITY UTILIZATION - PETROLEUM AND COAL PRODUCTS NAICS=324
180	UTL32	2	CAPACITY UTILIZATION - CHEMICAL NAICS=325
181	UTL33	2	CAPACITY UTILIZATION - PLASTICS AND RUBBER PRODUCTS NAICS=326
182	UTL44	2	CAPACITY UTILIZATION - PRIMARY & SEMIFINISHED PROCESSING (CAPACITY)
183	UTL45	2	CAPACITY UTILIZATION - FINISHED PROCESSING (CAPACITY)

Housing (Hous)

| 184 | GSVNT | 5 | PRIVATE CONSTRUCTION:NONRESIDENTIAL TOTAL(BIL$,SAAR) |
| 185 | HS6FR | 4 | HOUSING STARTS: TOTAL NEW PRIV HOUSING UNITS (THOUS.,NSA) |

186	HSMW	5	HOUSING STARTS:MIDWEST(THOUS.U.)S.A.
187	HSNE	5	HOUSING STARTS:NORTHEAST (THOUS.U.)S.A.
188	HSSOU	5	HOUSING STARTS:SOUTH (THOUS.U.)S.A.
189	HSWST	5	HOUSING STARTS:WEST (THOUS.U.)S.A.

Indexes (Ind)

190	DCOINC	5	COMPOSITE INDEX OF 4 COINCIDENT INDICATORS(87=100,SA)
191	DLAGG	5	COMPOSITE INDEX OF 7 LAGGING INDICATORS(87=100,SA)
192	DLDF1P	1	DIFFUSION INDEX:12 LEAD INDICATOR COMPONENT(% RISING +1-MO SPAN)
193	DLDF6P	1	DIFFUSION INDEX:12 LEAD INDICATOR COMPONENT(% RISING +6-MO SPAN)
194	DLEAD	2	COMPOSITE INDEX OF 11 LEADING INDICATORS(87=100,SA)
195	DRATE	2	RATIO, COINCIDENT INDEX TO LAGGING INDEX(87=100,SA)
196	HHSNTN	5	U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83)
197	PMDEL	5	NAPM VENDOR DELIVERIES INDEX (PERCENT)
198	PMEMP	5	NAPM EMPLOYMENT INDEX (PERCENT)
199	PMI	5	PURCHASING MANAGERS' INDEX (SA)
200	PMP	5	NAPM PRODUCTION INDEX (PERCENT)

Other (Other)

201	CCIPY	2	RATIO, CONSUMER INSTAL CREDIT TO PERSONAL INCOME (%,SA)(BCD-95)
202	FDLALT	5	DELINQ.RATE 1-4 U.RESID.MTGE:ALL LOAN;TOTAL PAST DUE(%,SA)
203	FMD	5	MORTGAGE DEBT OUTST'G:ALL PROPERTIES (MIL$,EOQ,NSA)
204	GGOFS	2	GOVT CURRENT SURPLUS/DEFICIT-OTHER,SRC:BEA,BILLIONS OF 1992 DOLLARS
205	GJJPAT	5	CORP PROFIT AFTER TAXES WITH IVA & CCA (BCD 79)
206	GJPATX	5	RATIO,PROFITS(AFT TAXES)WITH IVA&CCA/CORP DOMES.INCOME(SA)(BCD81)
207	LBPB	5	UNIT PROFITS: NONFINANCIAL CORP (1982=100,SA)
208	PMNV	5	NAPM INVENTORIES INDEX (PERCENT)
209	PZRP67	5	PURCH POWER CONSUMER $,URBAN WAGE EARNERS,CLER WKRS(67=$1,NSA)

Financial Data

We list the financial data used to construct factors, the endogenous variables, and other conditioning variables used to predict returns or volatility. The format is, series number, mnumonic, source, and brief description. CRSP = Center for Research in Security Prices, University of Chicago; FED = Federal Reserve Board; Shiller = Robert Shiller's Yale web page http://www.econ.yale.edu/~shiller/data/ie_data.xls; French = Kenneth French's Dartmouth web page http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/; LL = Sydney Ludvigson's NYU web page http://www.econ.nyu.edu/user/ludvigsons/; CP = John Cochrane and Monika Piazzesi, University of Chicago GSB. At the bottom we list portfolios of equity returns sorted into size (market capitalization) and book-market categories. Portfolios with missing data during the sample 1960:1-2002:4 are omitted from the analysis.

Category	**Source**	**Description**
Prices, Yield, Dividends (PYD)		
1	D_log(DIV)	CRSP	Log difference of the sum of the dividends in the last 4 quarters (divs are not reivested)
2	D_log(P)	CRSP	Log difference of the CRSP portfolio price when dividends are not reinvested
3	D_DIVreinvest	CRSP	Log difference of the sum of the dividends in the last 4 quarters (divs are reinvested)
4	D_Preinveste	CRSP	Log difference of the CRSP portfolio price when dividends are reinvested
5	d-p	CRSP	DIVreinveste - Preinveste = log(DIV) - log(P)
6	P/E	Shiller	Price/earnings ratio
Interest rates and Spread (IRS)		
7	RREL	FED	Difference b/w Risk free and its last 4 quarters average
8	Yield10y	FED	Quarterly yield of bonds with maturity 10 years
9	TRM10y-3m	FED	Difference b/w 10yTbonds rate and the risk free rate
10	Yield1y	FED	Yield from a t-bond with maturity one year (secondary mkt, nominal)

11	TRM10y-1y	FED	Term spread b/w 10years and 1 year t-bonds.
12	AAA	FED	AAA corporate bonds yield (Moody's seasoned)
13	BAA	FED	BAA corporate bonds yield (Moody's seasoned)
14	DEF	FED	AAA-BAA yield: risk default spread
15	RF	French	One-month Treasury bill rate from French dataset

Risk Factors (RiF)

16	R15-R11	French	Small stock value spread constructed from French database
17	factor	CP	Piazzesi-Cochrane risk factor, quarterly average (Cochrane and Piazzesi, 2005)
18	CAY	LL	Lettau-Ludvigson risk factor (Lettau and Ludvigson 2001a)
19	Mkt-RF	French	Fama-French market risk factor (Fama and French, 1993)
20	SMB	French	Fama-French risk factor (Fama and French, 1993)
21	HML	French	Fama-French risk factor (Fama and French, 1993)
22	UMD	French	Momentum risk factor, French data set

Industries (Industry Returns 4 digit SIC codes, French data set)

23	Agric	French
24	Food	French
25	Soda	French
26	Beer	French
27	Smoke	French
28	Toys	French
29	Fun	French
30	Books	French
31	Hshld	French
32	Clths	French
33	MedEq	French
34	Drugs	French
35	Chems	French
36	Rubbr	French
37	Txtls	French
38	BldMt	French
39	Cnstr	French
40	Steel	French
41	Mach	French
42	ElcEq	French
43	Autos	French
44	Aero	French
45	Ships	French
46	Mines	French
47	Coal	French
48	Oil	French
49	Util	French
50	Telcm	French
51	PerSv	French
52	BusSv	French
53	Comps	French
54	Chips	French
55	LabEq	French
56	Paper	French
57	Boxes	French
58	Trans	French
59	Whlsl	French
60	Rtail	French

61	Meals	French
62	Banks	French
63	Insur	French
64	RlEst	French
65	Fin	French
66	Other	French
Size/BM		(Returns on stock portfolios sorted by size and book-to-market ratio, French dataset)
67	1_2	French
68	1_4	French
69	1_5	French
70	1_6	French
71	1_7	French
72	1_8	French
73	1_9	French
74	1_High	French
75	2_Low	French
76	2_2	French
77	2_3	French
78	2_4	French
79	2_5	French
80	2_6	French
81	2_7	French
82	2_8	French
83	2_9	French
84	2_High	French
85	3_Low	French
86	3_2	French
87	3_3	French
88	3_4	French
89	3_5	French
90	3_6	French
91	3_7	French
92	3_8	French
93	3_9	French
94	3_High	French
95	4_Low	French
96	4_2	French
97	4_3	French
98	4_4	French
99	4_5	French
100	4_6	French
101	4_7	French
102	4_8	French
103	4_9	French
104	4_High	French
105	5_Low	French
106	5_2	French
107	5_3	French
108	5_4	French
109	5_5	French
110	5_6	French
111	5_7	French

112	5_8	French
113	5_9	French
114	5_High	French
115	6_Low	French
116	6_2	French
117	6_3	French
118	6_4	French
119	6_5	French
120	6_6	French
121	6_7	French
122	6_8	French
123	6_9	French
124	6_High	French
125	7_Low	French
126	7_2	French
127	7_3	French
128	7_4	French
129	7_5	French
130	7_6	French
131	7_7	French
132	7_8	French
133	7_9	French
134	8_Low	French
135	8_2	French
136	8_3	French
137	8_4	French
138	8_5	French
139	8_6	French
140	8_7	French
141	8_8	French
142	8_9	French
143	8_High	French
144	9_Low	French
145	9_2	French
146	9_3	French
147	9_4	French
148	9_5	French
149	9_6	French
150	9_7	French
151	9_8	French
152	9_High	French
153	10_Low	French
154	10_2	French
155	10_3	French
156	10_4	French
157	10_5	French
158	10_6	French
159	10_7	French

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Table 1: Summary Statistics for $\widehat{f}_{it}$ and $\widehat{g}_{it}$

i	$\mathrm{AR1}(\widehat{f}_{it})$	R_i^2	$\mathrm{AR1}(\widehat{g}_{it})$	R_i^2
1	0.678	0.245	-0.03	0.657
2	0.806	0.431	0.844	0.700
3	0.565	0.499	0.166	0.739
4	0.369	0.538	0.284	0.774
5	0.039	0.569	0.762	0.792
6	0.590	0.598	0.492	0.807
7	0.195	0.622	0.429	0.820
8	0.221	0.643	0.262	0.831

For $i = 1, \ldots 8$, $\widehat{f}_{it}$ is estimated by the method of principal components using a panel of data with 209 indicators of economic activity from t=1960:1-2002:4 (172 time series observations). The data are transformed (taking logs and differenced where appropriate) and standardized prior to estimation. $\widehat{g}_{it}$ is estimated using a panel of 159 series consisting primarily of financial data. $\mathrm{AR1}(F_{it})$, $\mathrm{AR1}(\widehat{g}_{it})$ are the first order autocorrelation coefficients for factors i. The relative importance of the common component, R_i^2, is calculated as the fraction of total variance in the data explained by factors 1 to i.

Table 2: Regressions of Quarterly Excess Returns on Lagged Conditioning Variables and Factors

Row	Regressor	(a)	(b)	(c)	(d)	(e)	(f)
Model: $m_{t+1} = \alpha_0 + \alpha_1'\widehat{F}_t + \alpha_2'\widehat{G}_t + \beta'Z_t + \epsilon_{t+1}$,
1	cay_t	**1.98**		**2.09**	**2.15**		**1.55**
	(t-stat)	(4.01)		(4.52)	(4.22)		(2.67)
2	$d_t - p_t$		0.03		0.00		
	(t-stat)		(1.62)		(0.89)		
3	$RREL_t$			**-5.86**	**-7.31**		-4.13
	(t-stat)			(-2.29)	(-2.47)		(-1.69)
4	VOL_t			**0.43**	**0.44**		**0.46**
	(t-stat)			(2.18)	(2.21)		(2.58)
5	DEF_t				0.64		
	(t-stat)				(0.11)		
6	TRM_t				-2.12		
	(t-stat)				(-0.61)		
7	$\widehat{F}_{2t}$					**-0.02**	-0.01
	(t-stat)					(-2.92)	(-1.35)
8	$\widehat{F}_{5t}$					**-0.01**	-0.01
	(t-stat)					(-2.43)	(-1.09)
9	$\widehat{G}_{3t}$					**0.02**	**0.02**
	(t-stat)					(3.49)	(2.96)
10	$\widehat{G}_{6t}$					**-0.02**	-0.01
	(t-stat)					(-2.80)	(-1.26)
11	$\overline{R}^2$	0.08	0.01	0.12	0.11	0.10	0.15
12	BIC	-2.07	-2.00	-2.06	-2.01	-2.02	-2.01

Notes: See next page.

Table 2, Continued:

Regressions of Quarterly Excess Returns on Lagged Conditioning Variables and Factors

Model: $m_{t+1} = \alpha_0 + \alpha_1' \widehat{F}_t + \alpha_2' \widehat{G}_t + \beta' Z_t + \epsilon_{t+1},$										
Row	Regressor	(g)	(h)	(i)	(j)	(k)	(l)	(m)	(n)	(o)	(p)
13	cay_t			**1.70**		**1.84**	**1.68**	**1.79**	**1.64**	**1.90**	**1.82**
	(t-stat)			(3.20)		(4.04)	(3.89)	(4.00)	(3.48)	(4.24)	(3.94)
14	$RREL_t$			**-5.59**		**-5.40**	**-5.05**	**-5.42**			
	(t-stat)			(-2.27)		(-2.26)	(-2.39)	(-2.28)			
15	VOL_t			0.19		0.14					
	(t-stat)			(0.79)		(0.60)					
16	$\widehat{G}_{1t}^2$	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**	**0.01**
	(t-stat)	(4.11)	(4.06)	(2.02)	(3.73)	(2.15)	(3.40)	(3.68)	(3.57)	(3.91)	(3.88)
17	$\widehat{G}_{3t}$	**0.02**	**0.02**	**0.02**	**0.02**	**0.02**	**0.02**	**0.02**	**0.02**	**0.02**	**0.02**
	(t-stat)	(3.73)	(3.78)	(3.11)	(3.40)	(3.04)	(2.98)	(3.11)	(2.90)	(3.17)	(3.13)
18	$\widehat{G}_{6t}$		**-0.01**	-0.01							
	(t-stat)		(-2.44)	(-0.82)							
19	$\widehat{G}_{3t} \cdot \widehat{G}_{4t}$				**0.01**		**0.01**		**0.01**		
	(t-stat)				(3.77)		(3.09)		(2.80)		
20	$\widehat{F}_{3t} \cdot \widehat{F}_{6t}$						**-0.01**			**-0.01**	
	(t-stat)						(-3.00)			(-2.90)	
21	$\overline{R}^2$	0.09	0.10	0.16	0.12	0.16	0.19	0.17	0.17	0.17	0.16
22	BIC	-2.06	-2.05	-2.04	-2.07	-2.07	-2.08	-2.10	-2.10	-2.10	-2.11

Notes: The table reports estimates from OLS regressions of excess stock returns on lagged variables named in column 2. The dependent variable m_{t+1} is the log return on the CRSP value-weighted stock market index over the three-month Treasury-bill rate. The regressors $\widehat{F}_{it}$ and $\widehat{G}_{it}$ are estimated by the method of principal components using a panel of data with 209 and 159 individual series, respectively, over the period 1960:1-2002:4. $\widehat{F}_{it}$ are constructed from a panel of data on economic activity, $\widehat{G}_{it}$ from a panel of data on financial returns. The exogenous conditioning variables in Z_t are cay_t, the consumption-wealth variable of Lettau and Ludvigson (2001), $d_t - p_t$ the CRSP log dividend-price ratio, $RREL_t$, the yield on the 3-month Treasury bill rate minus its 4 quarter moving average, DEF_t, the BAA corporate bond rate minus the AAA corporate bond rate, TRM_t, the difference between the 10-year Treasury bond yield and the 3-month Treasury bill yield. Newey and West (1987) corrected t-statistics are reported in parentheses. Coefficients that are statistically significant at the 5% level are highlighted in bold. A constant is always included in the regression even though its estimate is not reported in the Table. The sample spans the period from the first quarter of 1960 to the fourth quarter of 2002.

Table 3: Regressions of Quarterly Volatility on Lagged Conditioning Variables and Factors

	Model: $VOL_{t+1} = a_0 + a_1'\widehat{F}_t + a_2'\widehat{G}_t + b'Z_t + u_{t+1},$								
Row	Regressor	(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)
1	cay_t	**-0.702**							-0.167	-0.302
	(t-stat)	(-2.776)							(-1.303)	(-1.694)
2	$d_t - p_t$		**-0.026**						**-0.031**	**-0.033**
	(t-stat)		(-2.652)						(-4.829)	(-2.333)
3	DEF_t								2.237	3.822
	(t-stat)								(1.0156)	(1.095)
4	$YIELD_t$								**1.046**	1.033
	(t-stat)								(3.566)	(1.863)
5	VOL_t						**0.380**	**0.306**	**0.284**	**0.254**
	(t-stat)						(5.525)	(5.143)	(4.956)	(3.442)
6	VOL_{t-1}						**0.198**	**0.211**	0.075	**0.168**
	(t-stat)						(2.895)	(3.621)	(1.505)	(2.587)
7	F_{1t}			**0.007**		**0.005**	0.002		**0.004**	
	(t-stat)			(3.650)		(3.233)	(1.603)		(2.860)	
8	$\widehat{F}_{6t}$			**-0.005**		**-0.005**	**-0.004**		**-0.004**	
	(t-stat)			(-2.393)		(-2.617)	(-2.436)		(2.728)	
9	$\widehat{F}_{7t}$			**-0.006**		**-0.005**	**-0.004**		**-0.004**	
	(t-stat)			(-2.558)		(2.273)	(-2.295)		(-2.0243)	
10	$\widehat{G}_{1t}$				**-0.004**	**-0.004**		-0.002		-0.004
	(t-stat)				(-2.177)	(-2.193)		(1.139)		(1.772)
11	$\widehat{G}_{6t}$				**0.009**	**0.007**		**0.004**		-0.001
	(t-stat)				(3.116)	(2.454)		(2.372)		(-0.418)
12	$\widehat{G}_{7t}$				**0.009**	**0.009**		**0.005**		-0.002
	(t-stat)				(3.673)	(4.198)		(2.517)		(-0.437)
13	$\overline{R}^2$	0.08	0.10	0.11	0.18	0.25	0.32	0.32	0.40	0.36
14	BIC	-4.17	-4.19	-4.16	-4.24	-4.25	-4.37	-4.38	-4.41	-4.35

Notes: See next page.

Table 3, continued:

Regressions of Quarterly Volatility on Lagged Conditioning Variables and Factors

	Model: $VOL_{t+1} = a_0 + a_1'\widehat{F}_t + a_2'\widehat{G}_t + b'Z_t + u_{t+1},$						
Row	Regressor	(i)	(j)	(k)	(l)	(m)	(n)
15	cay_t			-0.147			
	$(t\text{-stat})$			(-1.083)			
16	$d_t - p_t$			**-0.026**	**-0.035**	**-0.036**	**-0.033**
	$(t\text{-stat})$			(-4.169)	(-6.945)	(-6.717)	(-5.030)
17	$YIELD_t$			**1.165**	**1.291**	**1.152**	**1.299**
	$(t\text{-stat})$			(3.852)	(4.554)	(3.592)	(3.854)
18	VOL_t		**0.365**	**0.245**	**0.345**	**0.347**	**0.399**
	$(t\text{-stat})$		(5.141)	(3.845)	(5.025)	(4.706)	(4.332)
19	VOL_{t-1}		**0.139**	0.070			
	$(t\text{-stat})$		(2.329)	(1.577)			
20	$\widehat{F}_{1t}$			**0.003**	**0.005**	**0.005**	
	$(t\text{-stat})$			(2.135)	(3.802)	(4.015)	
21	$\widehat{F}_{6t}$			**-0.004**	**-0.005**		
	$(t\text{-stat})$			(-2.349)	(-2.854)		
22	$\widehat{F}_{7t}$			-0.003			
	$(t\text{-stat})$			(-1.715)			
23	$\widehat{G}_{1t}^2$	**0.002**	-0.001	0.000			
	$(t\text{-stat})$	(2.525)	(-1.407)	(0.176)			
24	$\widehat{G}_{2t} \cdot \widehat{G}_{7t}$	**-0.006**	**-0.004**	-0.001			
	$(t\text{-stat})$	(-6.192)	(-5.053)	(-1.285)			
25	$\widehat{G}_{3t}^2$	**0.005**	**0.003**	**0.002**			
	$(t\text{-stat})$	(4.892)	(2.662)	(2.545)			
26	$\widehat{G}_{5t} \cdot \widehat{G}_{7t}$	**0.010**	**0.007**	**0.004**			
	$(t\text{-stat})$	(3.023)	(2.628)	(1.982)			
27	$\overline{R}^2$	0.25	0.35	0.41	0.39	0.37	0.35
28	BIC	-4.30	-4.40	-4.35	-4.49	-4.49	-4.48

Notes: See next page.

Notes: The table reports estimates from OLS regressions of excess stock returns on lagged variables named in column 2. The dependent variable VOL_{t+1} is realized volatility for the CRSP value-weighted stock market index. The regressors $\widehat{F}_{it}$ and $\widehat{G}_{it}$ are estimated by the method of principal components using a panel of data with 209 and 159 individual series, respectively, over the period 1960:1-2002:4. $\widehat{F}_{it}$ are constructed from a panel of data on economic activity, $\widehat{G}_{it}$ from a panel of data on financial returns. The exogenous conditioning variables in Z_t are cay_t, the consumption-wealth variable of Lettau and Ludvigson (2001), $d_t - p_t$ the CRSP log dividend-price ratio, DEF_t, the BAA corporate bond rate minus the AAA corporate bond rate, $YIELD_t$, the one year Treasury bill yield. Newey and West (1987) corrected t-statistics are reported in parentheses. Coefficients that are statistically significant at the 5% level are highlighted in bold. A constant is always included in the regression even though its estimate is not reported in the Table. The sample spans the period from the first quarter of 1960 to the fourth quarter of 2002.

Table 4: Relation Between Conditional Mean and Conditional Volatility

		Model: $\mu_t = \delta + \beta_1\sigma_t + \beta_2\sigma_{t-1} + \alpha\mu_{t-1} + \gamma m_{t-1} + \varepsilon_{t+1}$								
		Regressor							$\overline{R}^2$	BIC
Row	Regressand $\mu_t =$	$\sigma_{1,t}$ (t-stat)	$\sigma_{1,t-1}$ (t-stat)	$\mu_{1,t-1}$ (t-stat)	$\sigma_{2,t}$ (t-stat)	$\sigma_{2,t-1}$ (t-stat)	$\mu_{2,t-1}$ (t-stat)	m_{t-1} (t-stat)		
1	$\mu_{1,t}$	**1.40** (4.80)	**-1.46** (-7.67)	**0.62** (10.03)					0.41	-4.23
2	$\mu_{1,t}$	**1.07** (4.14)	**-1.18** (-7.25)	**0.71** (11.32)				**-0.11** (-3.40)	0.46	-4.30
3	$\mu_{1,t}$	-0.05 (-0.14)							-0.01	-3.76
4	$\mu_{1,t}$				**1.31** (4.57)	**-1.38** (-7.09)	**0.63** (10.60)		0.42	-4.25
5	$\mu_{1,t}$				**1.00** (4.05)	**−1.14** (-7.35)	**0.71** (11.71)	**-0.11** (-3.39)	0.47	-4.32
6	$\mu_{1,t}$				-0.08 (-0.22)				0.00	-3.76
7	$\mu_{2,t}$	**1.40** (4.80)	**-1.45** (-7.52)	**0.62** (9.53)					0.36	-4.07
8	$\mu_{2,t}$	**1.08** (4.17)	**-1.19** (-7.11)	**0.70** (10.46)				**-0.11** (-3.22)	0.41	-4.13
9	$\mu_{2,t}$	-0.04 (-0.11)							-0.01	-3.75
10	$\mu_{2,t}$				**1.31** (4.66)	**-1.39** (-7.23)	**0.63** (10.11)		0.42	-4.24
11	$\mu_{2,t}$				**1.02** (4.15)	**-1.15** (-7.51)	**0.71** (10.84)	**-0.10** (-3.16)	0.46	-4.30
12	$\mu_{2,t}$				-0.07 (-0.18)				-0.00	-3.75
13-2SLS	$\mu_{1,t}$	**2.22** (4.86)	**-2.15** (-5.53)	**0.68** (11.58)					0.36	–

Notes: See next page.

Notes: This table reports regressions of estimated conditional mean excess returns $\mu_t \equiv \widehat{E}_t (m_{t+1})$, $m_{t+1} \equiv r_{t+1} - r_{f,t+1}$, on the CRSP value-weighted stock market index over the 3-month Treasury-bill rate on estimated conditional volatility $\sigma_t \equiv \widehat{E}_t (VOL_{t+1})$ and one-period lags of these variables. The conditional mean and volatility are estimated as fitted values from regressions of excess returns and realized volatility on informaton variables known at time t. $\mu_{1,t}$ denotes the fitted value from a regression of excess returns on the information variables cay_t, G_{1t}^2, and G_{3t}. $\mu_{2,t}$ denotes the fitted value from a regression of excess returns on the information variables used to form $\mu_{1,t}$ plus $F_{3t} \cdot F_{6t}$. $\sigma_{1,t}$ denotes the fitted value from a regression of realized quarterly volatility, VOL_{t+1}, on the information variables $d_t - p_t$, the CRSP log dividend-price ratio, $YIELD_t$, the one year Treasury bill yield, F_{1t} and VOL_t. $\sigma_{2,t}$ denotes the fitted value from a regression of realized quarterly volatility, VOL_{t+1}, on the information variables $d_t - p_t$, the CRSP log dividend-price ratio, $YIELD_t$, the one year Treasury bill yield, and VOL_t. Newey and West (1987) corrected t-statistics are reported in parentheses. Coefficients that are statistically significant at the 5% level are highlighted in bold. A constant is always included in the regression even though its estimate is not reported in the table. All estimation is by OLS except for results reported in row 13, where two-stage least squares (2SLS) is used with instruments $\mu_{1,t-1}, \mu_{1,t-2}, \sigma_{1,t-1}, \sigma_{1,t-2}, m_{t-1}, VOL_{t-1}$. The sample spans the period from the first quarter of 1960 to the fourth quarter of 2002.

Table 5:

Relation Between Conditional Mean and Conditional Volatility, Omitting Factors

		Model: $\mu_t = \delta + \beta_1\sigma_t + \beta_2\sigma_{t-1} + \alpha\mu_{t-1} + \gamma m_{t-1} + \varepsilon_{t+1}$					
		Regressor				$\overline{R}^2$	BIC
Row	Regressand $\mu_t =$	$\sigma_{2,t}$ (t-stat)	$\sigma_{2,t-1}$ (t-stat)	$\mu_{3,t-1}$ (t-stat)	m_{t-1} (t-stat)		
1	$\mu_{3,t}$	**0.33** (2.67)	**-0.36** (-2.81)	**0.85** (20.28)		0.72	-5.66
2	$\mu_{3,t}$	0.02 (0.20)	-0.07 (-0.76)	**0.93** (26.39)	**-0.10** (-6.42)	0.81	-6.00
3	$\mu_{3,t}$	**-0.44** (-2.36)				0.10	-4.55
4	$\mu_{3,t}$	**-0.49** (-2.57)			**-0.04** (-2.00)	0.12	-4.54

Notes: This table reports regressions of estimated conditional mean excess returns $\mu_t \equiv \widehat{E}_t(m_{t+1})$ on the CRSP value-weighted stock market index over the 3-month Treasury-bill rate on estimated conditional volatility $\sigma_t \equiv \widehat{E}_t(VOL_{t+1})$ and one-period lags of these variables. The conditional mean and volatility are estimated as fitted values from regressions of excess returns and realized volatility on informaton variables known at time t. $\mu_{3,t}$ denotes the fitted value from a regression of excess returns on cay_t, the best fitting conditional mean specification omitting factors, according to the BIC criterion. $\sigma_{2,t}$ is the best fitting conditional volatility specification omitting factors, according to the BIC criterion, and denotes the fitted value from a regression of realized quarterly volatility, VOL_{t+1}, on the information variables $d_t - p_t$, the CRSP log dividend-price ratio, $YIELD_t$, the one year Treasury bill yield, and VOL_t. Newey and West (1987) corrected t-statistics are reported in parentheses. Coefficients that are statistically significant at the 5% level are highlighted in bold. A constant is always included in the regression even though its estimate is not reported in the table. The sample spans the period from the first quarter of 1960 to the fourth quarter of 2002.

Table 6: Out-of-Sample Predictive Power

Row	Forecast Sample	Comparison	MSE_u/MSE_r	Test Statistic	95% Asympt. CV
		Excess Returns			
1	1975:1-2003:2	$\left(\widehat{G}_1^2, \widehat{G}_3\right)'$ v.s. $const$	0.918	11.56**	2.79
2	1985:1-2003:2	$\left(\widehat{G}_1^2, \widehat{G}_3\right)'$ v.s. $const$	0.911	7.69**	1.90
3	1995:1-2003:2	$\left(\widehat{G}_1^2, \widehat{G}_3\right)'$ v.s. $const$	0.786	5.97**	1.03
		Volatility			
4	1975:1-2003:2	$SPEC1$ v.s. AR	0.902	16.58**	3.46
5	1985:1-2003:2	$SPEC1$ v.s. AR	0.821	9.75**	4.35
6	1995:1-2003:2	$SPEC1$ v.s. AR	0.706	10.04**	1.28

**Significant at the one percent or better level.

Notes: The table reports results from one-quarter-ahead out-of-sample forecast comparisons of log excess returns, m_{t+1} and volatility, VOL_{t+1}. $(G_1^2, G_3)'$ denotes a predictive model for excess returns that uses the squared value of G_{1t} and G_{3t} as predictive variables. These factors are formed from the financial data set with cay_t removed. $SPEC1$ denotes a forecasting model for volatility that includes as predictive variables $d_t - p_t$, the CRSP log dividend-price ratio, $YIELD_t$, the one year Treasury bill yield, F_{1t}, and VOL_{t-1}. Rows 1 through 3 report forecast comparisons of an unrestricted model, which includes $(G_1^2, G_3)'$ as predictors for excess returns, with the constant expected returns benchmark ($const$). Rows 4 through 6 report forecast comparisons of the unrestricted model $Spec1$ for predicting volatility, with a first-order autoregressive benchmark (AR). MSE_u is the mean-squared forecasting error of the unrestricted model; MSE_r is the mean-squared forecasting error of the restricted model that excludes additional forecasting variables. In the column labeled "MSE_u/MSE_r", a number less than one indicates that the models that use the additional forecasting variables have lower forecast error than the benchmark to which it is compared. In Rows 1 and 4, the parameters and factors were estimated recursively, using only data available from 1960:1 through 1975:1. The forecasting regressions were run for $t =$1960:1,...,1975:1, then the values of the regressors at $t =$1975:1 were used to forecast $m_{1975:2}$ (row 1) or $VOL_{1975:2}$. All parameters and factors are then reestimated from 1960:1 through 1975:2, and forecasts were recomputed for $m_{1975:3}$ and $VOL_{1975:3}$, and so on, until the final out-of-sample forecast is made for $m_{2003:2}$ and $VOL_{2003:2}$. The same procedure is used to compute results reported in rows 2, 3, 5, and 6 where the initial estimation period is either $t =$1960:1,...,1985:1 (rows 2 and 5) or $t =$1960:1,...,1995:1 (rows 3 and 6). The column labeled "Test Statistic" reports the ENC-NEW test statistic of Clark and McCracken (2001) for the null hypothesis that the benchmark model encompasses the unrestricted model with additional predictors. The alternative is that the unrestricted model contains information that could be used to improve the benchmark model's forecast. "95% Asympt. CV" gives the 95th percentile of the asymptotic distribution of the test statistic.

Table 7: Small Sample Inference: Bootstrapping the Regressors

Excess Returns

x_{it}	$\hat{\beta}$	Unrestricted model 95% CI	Unrestricted model 90% CI	Under the null 95% CI	Under the null 90% CI
c	-0.001	(-0.016 0.013)	(-0.013 0.011)	(-0.006 0.025)	(-0.003 0.023)
$\widehat{G}^2_{1t}$	0.011	(0.003 0.018)	(0.004 0.017)	(-0.008 0.008)	(-0.007 0.006)
$\widehat{G}_{3t}$	0.019	(0.007 0.032)	(0.009 0.030)	(-0.013 0.014)	(-0.011 0.012)
cay_t	1.822	(0.793 2.836)	(0.986 2.659)	(-1.135 1.109)	(-0.936 0.927)
R^2	0.170	(0.068 0.287)	(0.081 0.266)	(0.001 0.055)	(0.002 0.046)
$\bar{R}^2$	0.16	(0.051 0.274)	(0.065 0.253)	(-0.017 0.038)	(-0.016 0.029)

Volatility

x_{it}	$\hat{\beta}$	Unrestricted model 95% CI	Unrestricted model 90% CI	Under the null 95% CI	Under the null 90% CI
c	-0.106	(-0.177 -0.047)	(-0.165 -0.059)	(-0.017 0.141)	(-0.002 0.126)
$d_t - p_t$	-0.036	(-0.056 -0.021)	(-0.053 -0.024)	(-0.022 0.020)	(-0.017 0.016)
$YIELD_t$	1.152	(0.446 2.022)	(0.584 1.853)	(-0.907 0.974)	(-0.758 0.791)
$\widehat{F}_{1t}$	0.005	(0.001 0.010)	(0.002 0.009)	(-0.005 0.005)	(-0.004 0.004)
VOL_t	0.347	(0.188 0.451)	(0.210 0.428)	(-0.158 0.138)	(-0.138 0.110)
R^2	0.386	(0.189 0.694)	(0.223 0.652)	(0.003 0.065)	(0.004 0.056)
$\bar{R}^2$	0.371	(0.170 0.686)	(0.204 0.644)	(-0.021 0.043)	(-0.020 0.033)

Let x_{it} denote the state variables for summarizing the conditional value of y_t (either returns or volatility). For each state variable $x_{it}, i = 1, \ldots K - 1$, we estimate $x_{it} = \rho_i x_{it-1} + v_{it}$. Let $v_{.,t}$ be the $1 \times K$ vector of residuals. Let $\tilde{x}_{1,.} = x_{1,.}$. For $t = 2, \ldots T$, $\tilde{x}_{it}$ is generated as $\tilde{x}_{it} = \rho_i \tilde{x}_{it-1} + \tilde{v}_{it}$, where $\tilde{v}_{.,t}$ is sampled (with replacement) from $v_{.,t}, t = 2, \ldots T$. Unrestricted samples of y_t are generated recursively as $\tilde{y}_t = \tilde{X}(t, 1 : K - 1)\hat{\beta}(1 : K - 1) + \hat{\beta}(K)\tilde{y}_{t-1}\tilde{e}(t)$, where $\hat{\beta}$ are the least squares estimates reported in column 2, and $\tilde{e}$ are resampled from $\hat{e}$, the least squares residuals. Samples under the null are generated as $\tilde{y} = \bar{y} + \tilde{e}^0$, where $\tilde{e}^0$ is resampled form $\hat{e} = y - \bar{y}$.

Table 8: Small Sample Inference: Bootstrapping the Panel Data and Regressors

Excess Returns

x_{it}	$\hat{\beta}$	Unrestricted Model		Under the null	
		95% CI	90% CI	95% CI	90% CI
c	-0.001	(-0.015 0.013)	(-0.013 0.011)	(-0.006 0.025)	(-0.003 0.023)
$\widehat{G}^2_{1t}$	0.011	(0.004 0.017)	(0.005 0.016)	(-0.008 0.007)	(-0.007 0.006)
$\widehat{G}_{3t}$	0.019	(0.007 0.032)	(0.009 0.029)	(-0.013 0.013)	(-0.011 0.011)
cay_t	1.822	(0.802 2.835)	(0.988 2.654)	(-1.124 1.109)	(-0.929 0.917)
R^2	0.170	(0.077 0.297)	(0.092 0.278)	(0.001 0.056)	(0.002 0.046)
$\bar{R}^2$	0.16	(0.060 0.284)	(0.076 0.265)	(-0.017 0.039)	(-0.016 0.029)
$\chi^2(2)$	16.87	(6.373 44.108)	(8.010 39.600)	(0.052 7.651)	(0.104 6.117)

Volatility

x_{it}	$\hat{\beta}$	unrestricted model		under the null	
		95% CI	90% CI	95% CI	90% CI
c	-0.106	(-0.177 -0.048)	(-0.163 -0.060)	(-0.017 0.141)	(-0.002 0.125)
$d_t - p_t$	-0.036	(-0.056 -0.021)	(-0.053 -0.024)	(-0.022 0.020)	(-0.017 0.016)
$YIELD_t$	1.152	(0.459 2.009)	(0.596 1.855)	(-0.918 0.962)	(-0.744 0.768)
$\widehat{F}_{1t}$	0.005	(0.002 0.010)	(0.002 0.009)	(-0.005 0.005)	(-0.004 0.004)
VOL_t	0.347	(0.191 0.453)	(0.211 0.428)	(-0.158 0.138)	(-0.139 0.110)
R^2	0.386	(0.208 0.709)	(0.242 0.674)	(0.003 0.066)	(0.004 0.056)
$\bar{R}^2$	0.371	(0.189 0.702)	(0.224 0.666)	(-0.021 0.044)	(-0.020 0.034)

Let x_{it} denote the state variables for summarizing the conditional value of y_t (either returns or volatility). Let $z_{it}, i = 1, \ldots N, t = 1, \ldots T$ be standardized data from which the factors are extracted. By definition, $z_{it} = \lambda'_i F_t + u_{it}$. Let $\hat{\lambda}_i$ and $\hat{F}_t$ be the principal components estimators of λ_i and F_t, and let $\hat{u}_{it}$ be the estimated idiosyncratic errors. For each $i = 1, \ldots N$, we estimate an AR(1) model $\hat{u}_{it} = \rho_i \hat{u}_{it-1} + w_{it}$. Let $\tilde{u}_{1,.} = u_{1,.}$. For $t = 2, \ldots T$, let $\tilde{u}_{it} = \hat{\rho}_i \tilde{u}_{it-1} + \tilde{w}_{it}$, where $\tilde{w}_{i,t}$ is sampled (with replacement) from $\hat{w}_{.,t}, t = 2, \ldots T$. Then $\tilde{z}_{it} = \hat{\lambda}'_i \hat{F}_t + \tilde{u}_{it}$. Estimation by principal components on the data $\tilde{z}$ yields $\tilde{F}_t$. The remaining regressors (other than the factors and the lagged dependent variable) are obtained by first estimating an AR(1), and then resampling the residuals of the autoregressions. Unrestricted samples of y_t are generated as $\tilde{y} = \tilde{X}\hat{\beta} + \tilde{e}$, where $\hat{\beta}$ are the least squares estimates reported in column 2, and $\tilde{e}$ are resampled from $\hat{e}$, the least squares residuals, and $\tilde{X}$ is a set of bootstrapped regressors with $\hat{F}_t$ replaced by $\tilde{F}_t$. Samples under the null are generated as $\tilde{y} = \bar{y} + \tilde{e}^0$, where $\tilde{e}^0$ is resampled form $\hat{e} = y - \bar{y}$. The row labeled "$\chi^2(2)$" in the top panel reports the Wald statistic for the null hypothesis that the coefficients on G^2_{1t-1} and G_{3t-1} are jointly zero. The Wald statistic has an asymptotic $\chi^2(2)$ distribution.
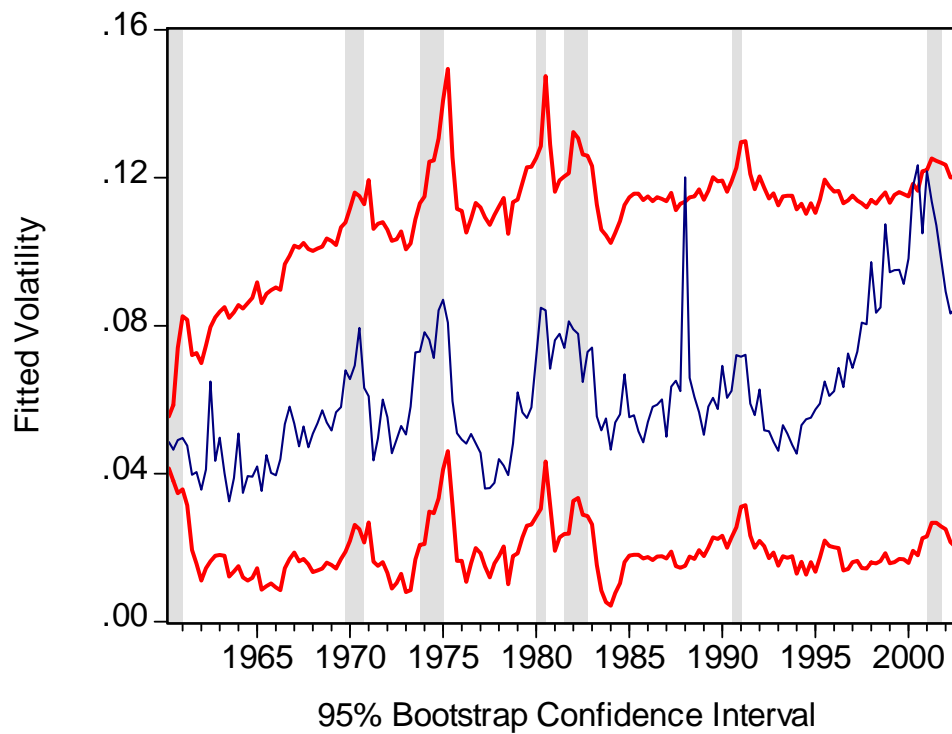
Figure 1
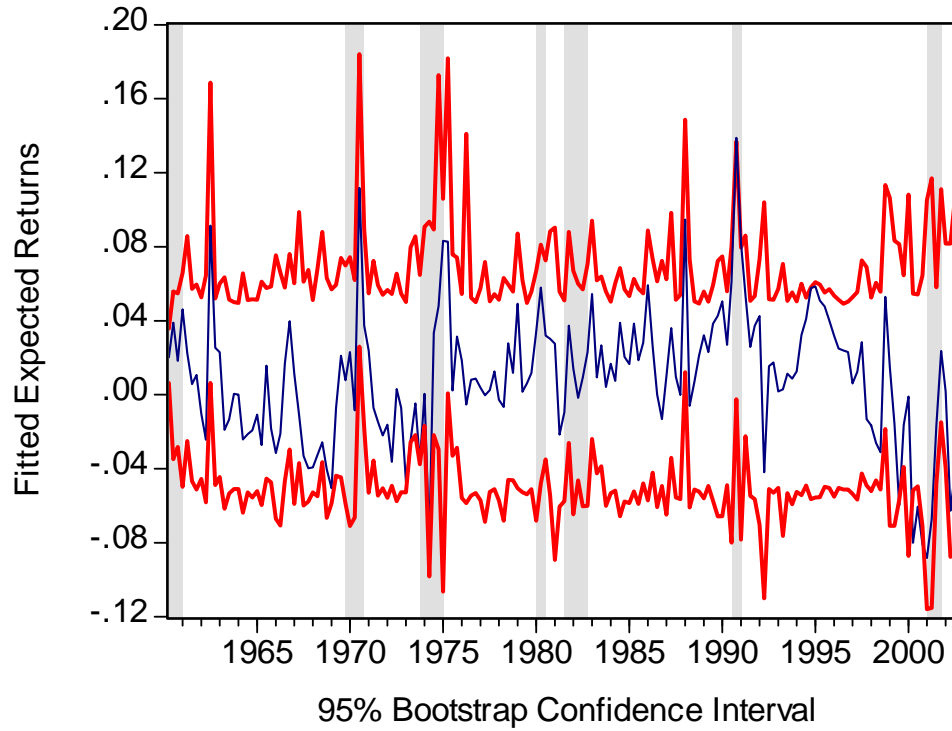
95% Bootstrap Confidence Interval
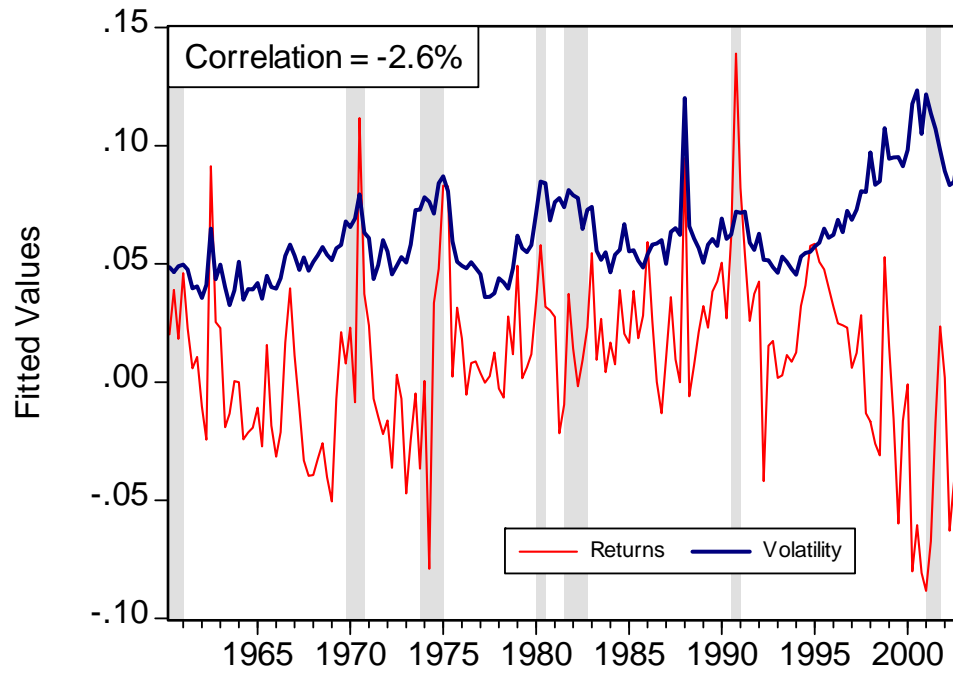
Estimate Based on CRSP-VW Index Data

Note: Shading denotes quarters designated recessions by the NBER.
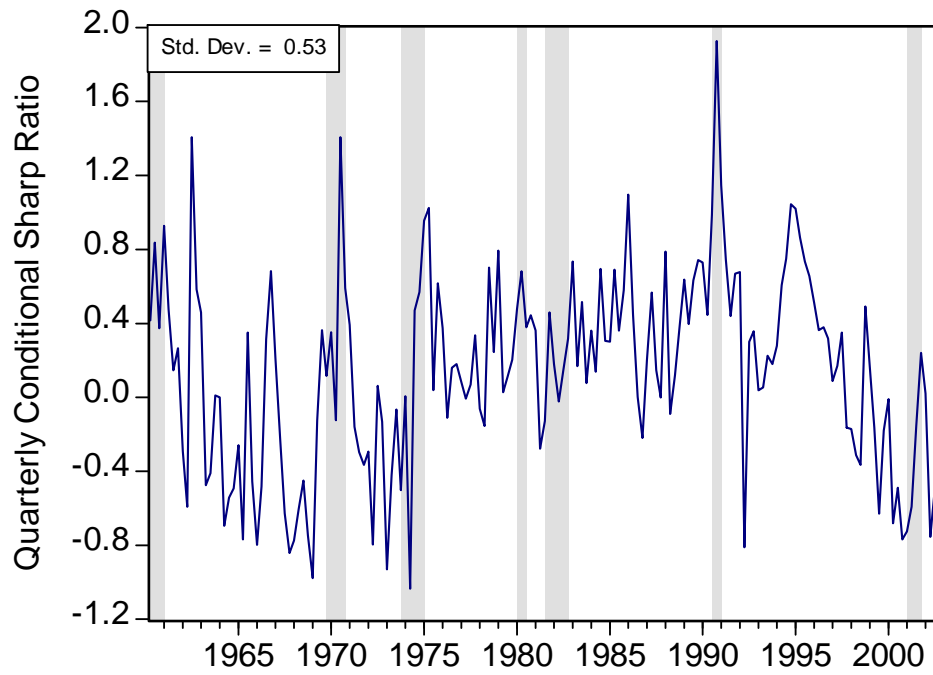Source: Authors' Calculations.

Figure 2

Note: Shading denotes quarters designated recessions by the NBER.
Source: Authors' Calculations.

Figure 3

Note: Shading denotes quarters designated recessions by the NBER.
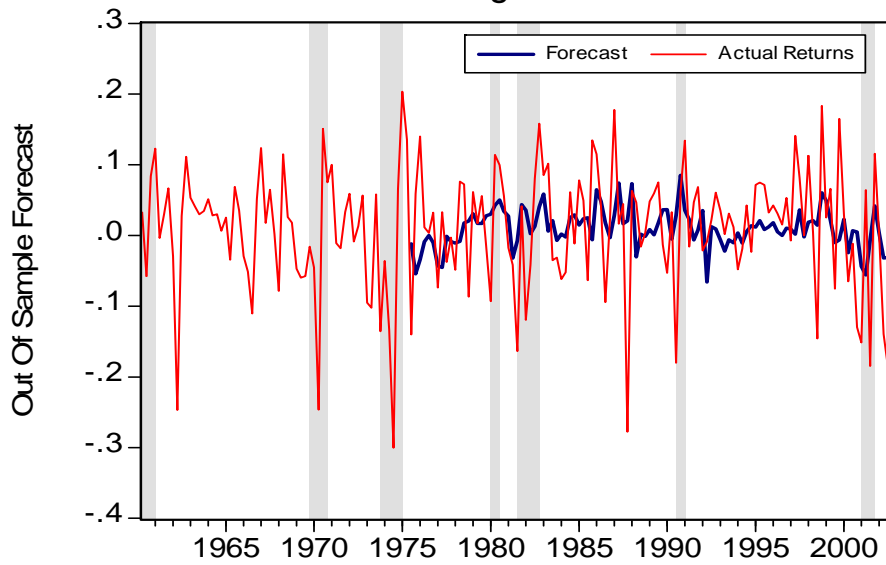Source: Authors' Calculations.

Figure 4

Estimate Based on CRSP-VW Index Data

Note: Shading denotes quarters designated recessions by the NBER.
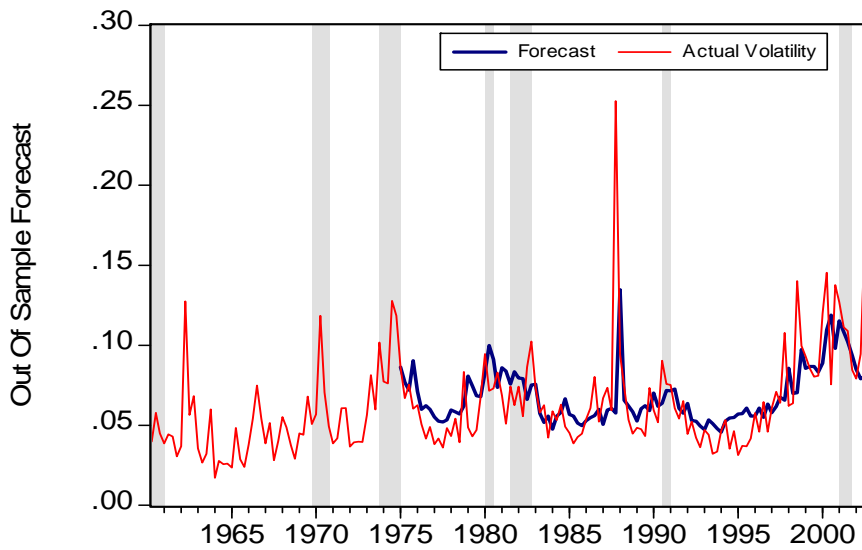Source: Authors' Calculations.

Figure 5

Note: Shading denotes quarters designated recessions by the
NBER. Forecasts are based on volatility and combination
factors only, constructed from financial dataset
excluding *cay*.

Source: Authors' Calculations.

Figure 6

Note: Shading denotes quarters designated recessions by the
NBER. Forecasts are based on the dividend-price ratio,
one-year Treasury yield, one-quarter lagged volatility
and the real factor from the macro dataset.

Source: Authors' Calculations.