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INFLEXIBLE RELATIVE PRICES AND PRICE LEVEL INERTIA

Olivier J. Blanchard

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Inflexible Relative Prices and Price Level Inertia

Abstract

A decrease in aggregate demand at given prices and wages decreases output and employment. The decrease in employment exerts downward pressure on real wages. The decrease in production exerts downward pressure on mark ups. With perfectly synchronized price and wage decisions, nominal wages and prices decrease instantaneously until equilibrium is re-established at a lower price level and the initial relative prices. If, however, price and wage decisions are asynchronized, this process cannot take place instantaneously but rather takes place over time. If real wages and mark ups are rather insensitive to shifts in demand, the process of adjustment is slow, the effects of money on output are strong and lasting.

The paper formalizes this intuitive argument and characterizes the implications of asynchronization for the joint behavior of relative and nominal prices.

Olivier J. Blanchard
Department of Economics
Harvard University
Littauer Center
Cambridge, Massachusetts 02138

(617) 495-2119

Asynchronization of price decisions is a pervasive characteristic of decentralized economies. Macroeconomists have recently concentrated on a particular set of price decisions, namely contract wage decisions which are both set in nominal terms for long periods of time and staggered over time across contracts (Taylor [1980] for example). Asynchronization is, however, not limited to contract wages. Most prices, except for those determined in exchange or asset markets, are set for discrete periods of time, ranging from a few days to a few months. These price decisions are not all taken at the same time but spread through time rather uniformly.

As in the case of imperfect information, we have to decide whether asynchronization is a deviation from the standard frictionless model we may safely ignore or a deviation we want to build on to explain fluctuations. In a previous paper (Blanchard [1983]), I have shown that even if all price decisions are taken for short periods of time, but if the number of price decisions is large, asynchronization will lead to substantial price level inertia. This suggests that asynchronization may indeed help explain price level inertia and thus generate macroeconomic fluctuations. This paper goes further and addresses the following question: In the presence of asynchronization, how is the degree of price level inertia related to slopes of demand and supply curves in the various markets? The paper derives a relation between inflexibility of relative prices and inertia of nominal prices. More precisely, it argues that the less sensitive relative prices are to shifts in demand, the more slowly will nominal prices adjust to offset aggregate demand disturbances.

The rest of the paper is spent deriving, explaining and qualifying this result. It is organized in four sections. The first sets up the model but maintains the assumption of synchronization of price decisions.

The second introduces asynchronization and derives the relation between relative price inflexibility and price level inertia. The third examines the relation of the model to both "disequilibrium" models and to the "price wage mechanism" (Tobin [1970]) which underlies much of the empirical work on prices and wages. The fourth extends the initial model to allow for a larger menu of inputs and outputs: its purpose is to characterize the joint movement of relative and nominal prices in response to nominal disturbances. Until then, the time structure of price decisions is taken as given. Whether asynchronization may be stable if price setters are free to choose the timing of price decisions is discussed in the conclusion.

Section I. A Simple Model

The simplest model must have one relative price and thus two nominal prices. Consider the following:

$$\ell^s = b^{-1}(w-p) \qquad ; b > 0 \qquad (1)$$

$$y^d = c(m-p) \qquad ; c > 0 \qquad (2)$$

$$y = a\ell \qquad ; a \in [0,1] \qquad (3)$$

$$\Rightarrow \quad p = w \qquad \text{if } a = 1$$

$$y^s = (a/(1-a))(p-w) \qquad \text{if } a < 1 \qquad (4)$$

$$\ell^d = (1/(1-a))(p-w) \qquad \text{if } a < 1 \qquad (5)$$

All variables are in logarithms; ℓ and y denote labor and output respectively; m, w and p are nominal money, the nominal wage and the nominal price of output respectively. Labor supply is an increasing function of the real wage, output demand an increasing function of real money balances. Output is produced from labor with constant or decreasing returns to scale. The implied notional demand for labor and supply of output are given by (4) and (5). Constant terms are deleted from all equations for notational simplicity.

The competitive equilibrium is simply $w = p = m$ and $y = \ell = 0$. How is it reached? Asynchronization of price decisions presupposes the existence of price setters. Thus, before introducing asynchronization, we must introduce an explicit price setting mechanism:

The nominal wage is determined in the labor market given the nominal price. The nominal price is determined in the goods market given the nominal wage. To avoid issues of monopoly or monopsony power which are inessential to this argument, both wages and prices are assumed to be set competitively. Thus, although we shall think of workers as setting nominal wages and firms as setting nominal prices, who actually sets them in each market is irrelevant.

The supply and demand expressed in each market depend on the constraints perceived by agents in the other markets. It is assumed that if a market does not clear at the prevailing wage price pair, the outcome is demand determined; demand is therefore never constrained.

We can now characterize the wage and price relations, and the equilibrium. Suppliers of labor choose the nominal wage so that the supply of labor equals the derived demand for labor by firms:

$$b^{-1}(w-p) = a^{-1}y = a^{-1}c(m-p) \Rightarrow$$

$$w-p = a^{-1}cb(m-p) \quad \text{or} \tag{6}$$

$$w = \Theta p + (1-\Theta)m \quad ; \quad \Theta = 1 - a^{-1}cb \quad , \quad \Theta \leq 1. \tag{7}$$

The nominal wage is a linear combination of the price level and nominal money. The effect of the price level is a priori ambiguous: an increase in the price level decreases aggregate demand and the derived demand for labor, requiring a decrease in the real wage. The lower real wage and the higher price level imply that the nominal wage may increase or decrease. Θ is the first important parameter for what follows. Note that if labor supply is very elastic, i.e. if b is very small, Θ is close to unity.

Suppliers of output choose the nominal price of output so that the supply of output equals the demand for output. As by assumption, they are never constrained in their demand for labor, output supply is notional supply:

$$(a/(1-a))\ (p-w) \ = \ c(m-p) \quad \Rightarrow$$

$$p-w \ = \ ca^{-1}(1-a)(m-p) \tag{8}$$

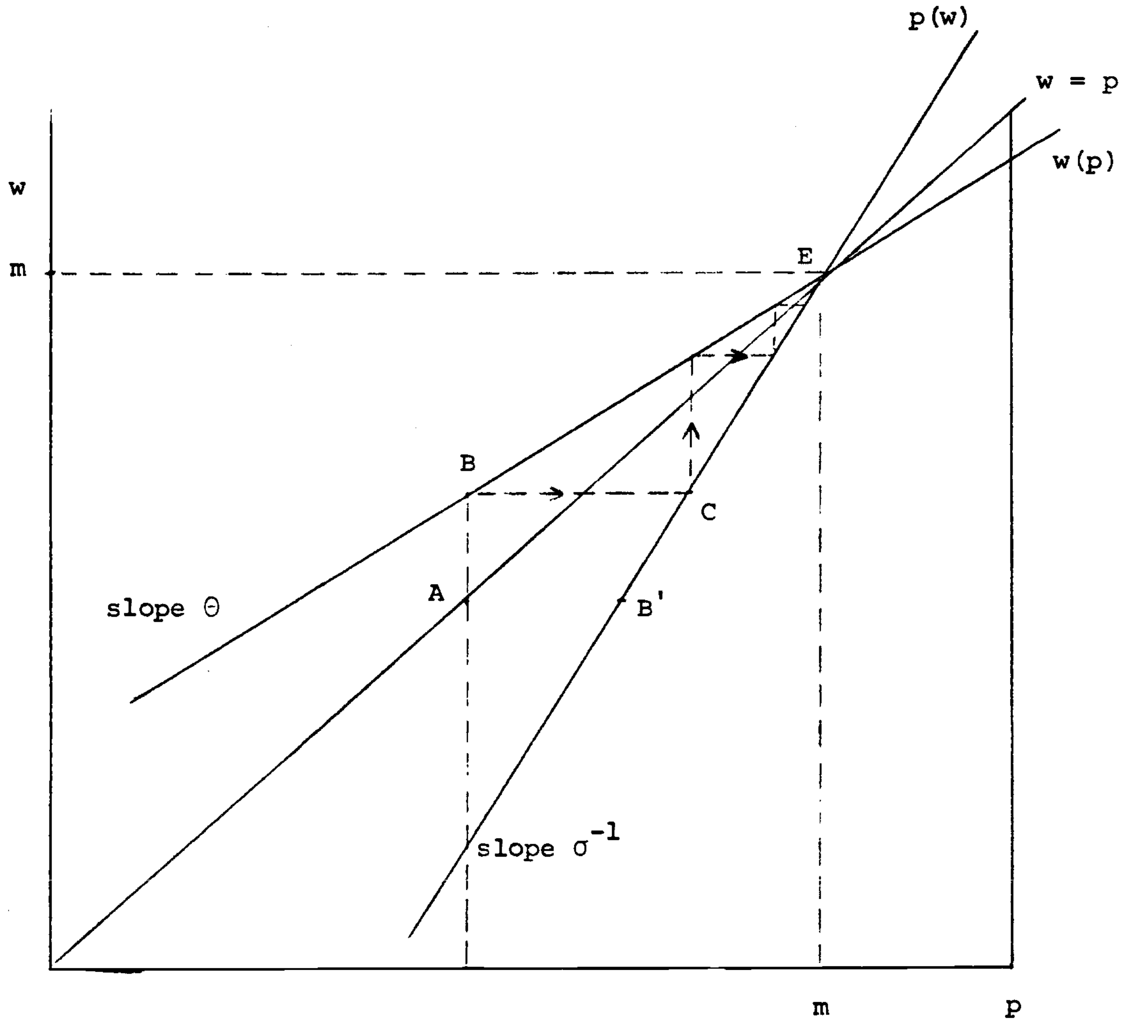
$$p \ = \ \sigma w + (1-\sigma)m \quad ; \quad \sigma \ \equiv \ a/(a+(1-a)c) \ , \ \sigma \in (0,1] \tag{9}$$

The price of output is a linear combination of the nominal wage and nominal money. σ is the second important parameter in what follows. Note that if returns to scale are nearly constant, so that notional supply is very elastic, a is close to unity and so is σ.

The wage and price loci are drawn in Figure 1. The wage relation has slope Θ, the price relation has slope σ^{-1}. Equilibrium requires that both relations hold, thus that $y = \ell = 0$ and $w = p = m$. The specific price setting assumptions may affect the way in which the equilibrium is attained but do not affect the equilibrium values of quantities and prices. Money is neutral; neutrality clearly does not depend on the values of Θ and σ.

It is useful for later reference to give a heuristic description of how this equilibrium may be reached through a tatonnement type process. Suppose that nominal money increases, so that in Figure 1, the initial equilibrium is at A and the new equilibrium at E: At initial values of w and p, aggregate demand increases, increasing output and employment. Workers, therefore, want a higher real wage. Given p, they want to go to point B. Firms want a higher mark up; given w, they want to go to point B'. These two claims, of a higher real wage and of a higher mark up, are clearly inconsistent. Attempts by both firms and workers to increase relative prices lead to increases in nominal prices and wages. This process ends when real money balances are back to their previous level: desired real wages and desired mark ups are then consistent

Figure 1



and equilibrium is reached. A possible path of adjustment in which wages and prices adjust in turn is ABC...E in Figure 1.

Section II. Asynchronization of Price and Wage Decisions

Most nominal prices are set for discrete periods of time. For each such period, price setters prefer the simplicity of a constant nominal price to a predetermined — but not necessarily constant — price path or to contingent price paths. Moreover, nominal price changes do not take place simultaneously but rather take place at a fairly uniform rate over time.

We shall for the moment take these facts as given and return later to what may determine the timing of price decisions by price setters. We formalize them in the following mechanical but convenient way:

Price decisions are taken every two periods, at even times. Thus prices are set at time t for periods t, t+1, and so on. Wage decisions are taken every two periods, at odd times, thus at t-1 for t-1 and t, and so on. Again, the unit period should be thought of as fairly short, of the order of a month, so that after two months all price and wage decisions have been freely revised.

Price decisions are now given by:

$$p_t = \frac{1}{2}(\sigma\, w_{t-1} + (1-\sigma)m_t) + \frac{1}{2}(\sigma\, E(w_{t+1}|t) + (1-\sigma)E(m_{t+1}|t)) \qquad (10)$$

$E(.|t)$ denotes the expectation of a variable conditional on information available at time t. p_t denotes the price chosen at t for t and t+1; similarly w_{t-1} denotes the wage chosen at t-1 for t-1 and t and is therefore the wage still prevailing at t. Equation (10) is the natural extension of (9); it states that the nominal price is a weighted average of its market clearing value at t, which depends on the current nominal wage and money, and of its expected market clearing value at t+1 which depends on expected nominal wage and money.[1] Weights are assumed equal for simplicity: discounting would not substantially affect the results. Equation (10) implies that for each interval during which the nominal price is fixed, the expected average mark up is an increasing function of expected average demand.

In a similar way, wage decisions are given, at t-1 for example, by:

$$w_{t-1} = \frac{1}{2}(\Theta\, p_{t-2} + (1-\Theta)m_{t-1}) + \frac{1}{2}(\Theta E(p_t|t-1) + (1-\Theta)E(m_t|t-1)) \qquad (11)$$

The nominal wage set at time t-1 for t-1 and t is a weighted average of its market clearing value at t-1 and its expected market clearing value for t. Equation (11) implies that for each interval during which the nominal wage is fixed, the expected average real wage is an increasing function of expected average aggregate demand.

The dynamic behavior of the economy is now characterized by equations (10) and (11), and a specification of the money process. The system is solved for a general money process in the appendix; in the text I focus on the effects of an unanticipated permanent increase in money. It is a counterfactual experiment but one which shows most clearly the dynamics of the system.

The Adjustment of the Price Level

Starting from steady state, nominal money increases at time t = 0, from zero to unity (by appropriate normalization). This increase is unanticipated and permanent. What are the effects on prices over time?

The answer is easy to derive under the assumption of static expectations and helps develop the intuition. Using (10) and (11) gives in this case:

$$\begin{aligned} p_0 &= (1-\sigma) \\ p_t &= \sigma\Theta\, p_{t-2} + (1-\sigma\Theta) \quad ; \quad t = 2,4,\ldots \end{aligned} \qquad (12)$$

At time $t = 0$, after the increase in money, wages have not yet increased: firms increase their nominal price only to the extent they want to increase their mark up to supply the higher level of output. Under constant returns, there is no increase in prices, as $\sigma = 1$. At time $t = 1$, wages increase both because prices are higher and because, unless $\Theta = 1$, a higher real wage is required to supply a higher level of labor. Under static expectations, the degree of price inertia is given by $\sigma\Theta$ and adjustment thus is slower, the more inflexible real wages and mark ups.

Under rational expectations, the price level path is given instead by (see appendix):

$$p_0 = 1 - \frac{1}{2}\sigma(1 - \frac{1}{4}\sigma\Theta(1+\lambda))^{-1} \quad , \quad 0 \leq p_0 \leq 1-\lambda \qquad (13)$$

$$p_t = \lambda p_{t-2} + (1-\lambda) \quad ; \quad t = 2,4,\ldots$$

$$\text{where } \lambda \equiv (1-(1-\sigma\Theta)^{1/2})/(1+(1-\sigma\Theta)^{1/2})$$

$$\lambda \leq \sigma\Theta \quad ; \quad \lambda = 0 \text{ if } \sigma\Theta = 0 \quad , \quad \lambda = 1 \text{ if } \sigma\Theta = 1$$

The initial jump in nominal prices is larger than under static expectations. The degree of persistence, or inertia, is now given by λ rather than $\sigma\Theta$. λ is an increasing function of $\sigma\Theta$, but smaller than $\sigma\Theta$. That the adjustment should be faster under rational expectations is obvious: at time $t = 0$ for example, price setters take into account not only the current increase in demand, but also the forthcoming increase in nominal wages at $t = 1$. Along the path, price and wage setters take account not only of demand and current wages or prices but also of expected increases in prices and wages. Although the adjustment is faster under rational expectations, there is still a direct relation between relative price inflexibility, measured by σ and Θ, and nominal price inertia, measured by λ.

Table 1

The Effects of An Increase in Money on Relative and Nominal Prices

σ = .99 Θ = .99

<table>
<tr><th>Time:</th><th>p</th><th>w</th><th>w-p</th><th>Average Real Wage*</th><th>Average Mark Up**</th></tr>
<tr><td>0</td><td>.133</td><td>.0</td><td>-.133</td><td>-.076</td><td rowspan="2">.009</td></tr>
<tr><td>1</td><td>.133</td><td>.247</td><td>.114</td><td rowspan="2">.007</td></tr>
<tr><td>2</td><td>.346</td><td>.247</td><td>-.100</td><td rowspan="2">.007</td></tr>
<tr><td>3</td><td>.346</td><td>.432</td><td>.085</td><td rowspan="2">.005</td></tr>
<tr><td>4</td><td>.508</td><td>.432</td><td>-.076</td><td rowspan="2">.006</td></tr>
<tr><td>5</td><td>.508</td><td>.572</td><td>.064</td><td rowspan="2">.003</td></tr>
<tr><td>6</td><td>.629</td><td>.572</td><td>-.057</td><td rowspan="2">.004</td></tr>
<tr><td>7</td><td>.629</td><td>.678</td><td>.049</td><td rowspan="2">.003</td></tr>
<tr><td>8</td><td>.721</td><td>.678</td><td>-.043</td><td rowspan="2">.003</td></tr>
<tr><td>9</td><td>.721</td><td>.757</td><td>.036</td><td rowspan="2">.002</td></tr>
<tr><td>10</td><td>.790</td><td>.757</td><td>-.033</td><td rowspan="2">.003</td></tr>
<tr><td>11</td><td>.790</td><td>.817</td><td>.027</td><td rowspan="2">.001</td></tr>
<tr><td>12</td><td>.842</td><td>.817</td><td>-.025</td><td rowspan="2">.002</td></tr>
<tr><td>13</td><td>.842</td><td>.862</td><td>.020</td><td rowspan="2">.000</td></tr>
<tr><td>14</td><td>.881</td><td>.862</td><td>-.019</td><td></td></tr>
</table>

* Average real wage : average value for the two period interval during which the nominal wage is fixed.

** Average markup: average value for the two period interval during which the nominal price is fixed.

An example of a path or adjustment, for $\sigma = \Theta = .99$ is given in Table 1.

Relative Prices During the Adjustment Process

We start with a puzzle. After the increase in money, price setters desire a higher mark up, wage setters a higher real wage. Under static expectations, both sides fail to take into account future movements in either prices or wages and are therefore systematically disappointed. This cannot happen under rational expectations; in particular, as there is no unanticipated movement in money after $t = 0$, the rational expectation path is a perfect foresight path. Thus, as equations (10) and (11) hold for actual values of wages and prices, workers must, for every interval during which nominal wages are fixed, obtain a higher real wage; firms must also, for every interval during which nominal prices are fixed, obtain a higher mark up. How can these be consistent?

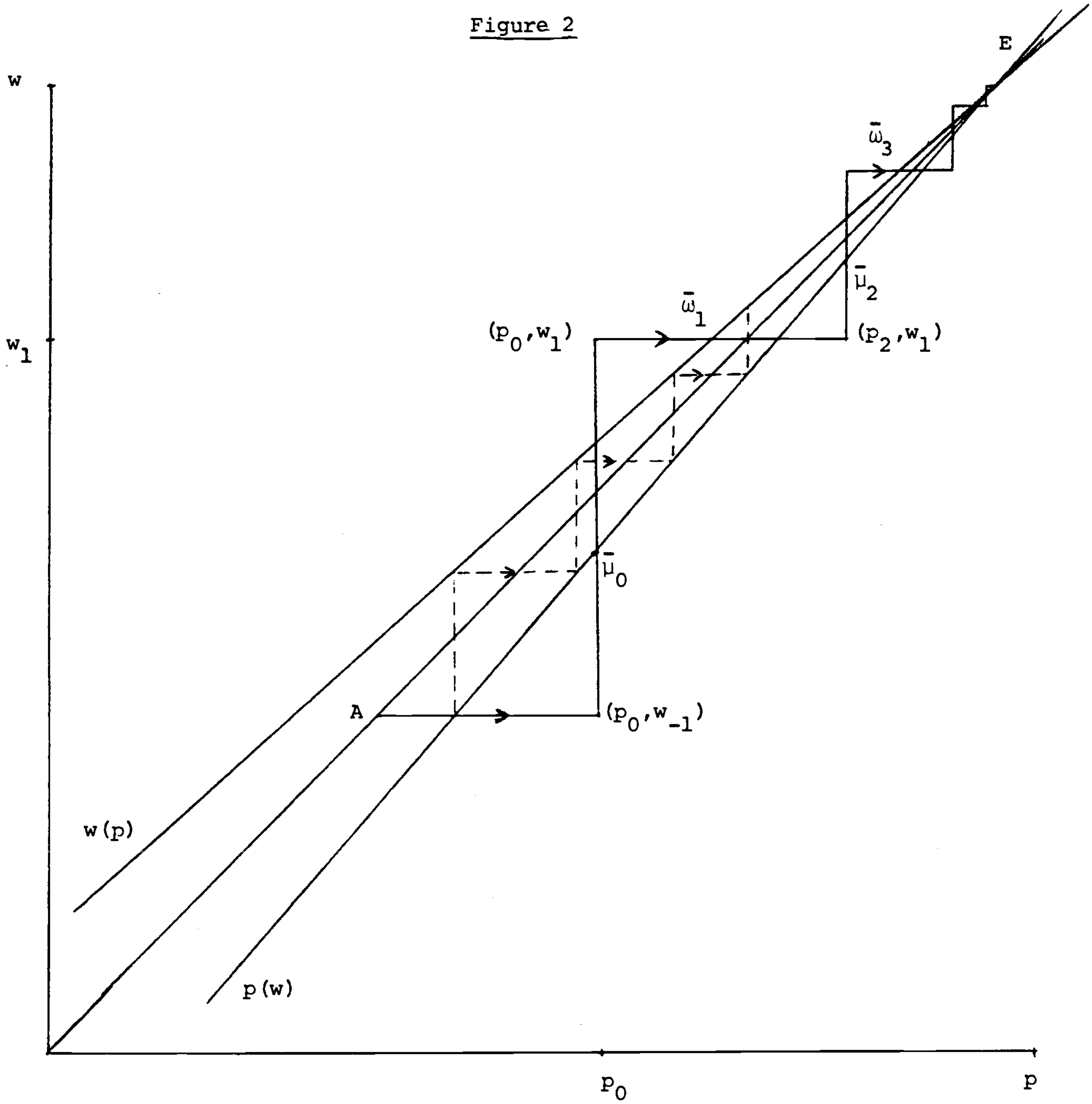
Table 1 shows that they can indeed be consistent (the algebra is derived in the appendix). They can be consistent because the two intervals described above do not coincide but overlap: There is then a path of increases in nominal prices and wages such that in turn the average mark up and real wage are higher. This paradoxical path is the result of the assumption of rational expectations; the relevant result is not this paradox but the fact that there exists a rational expectation path.

The other and directly related characteristic of the process of adjustment is that there is no persistent deviation of the real wage from its equilibrium value: The real wage simply oscillates around this value during the path of adjustment.

The two paths of adjustment, under static and rational expectations are represented in Figure 2, the static expectation path by the dotted line, the rational expectation path by the thick line. Note that the average wage (denoted by $\bar{\omega}$) is always on the w(p) locus, the average mark up ($\bar{\mu}$) always on the p(w) locus.

To summarize, movements in money create, at the initial pair of prices and wages, desired changes in relative prices. After an increase in money, both relative prices (the real wage and its inverse, the mark up) are too low; after a decrease in money, they are both too high. As workers and firms in turn re-establish their desired relative prices, nominal prices adjust until aggregate demand returns to its equilibrium value. If relative prices are relatively inflexible, the movement of nominal prices is slow.

Figure 2


w
E
$\bar{\omega}_3$
$\bar{\mu}_2$
$\bar{\omega}_1$
w_1
(p_0, w_1)
(p_2, w_1)
$\bar{\mu}_0$
A
(p_0, w_{-1})
w(p)
p(w)
p_0
p

Section III. Output Fluctuations and Disequilbrium

The model developed above generates output and employment fluctuations in response to movements in aggregate demand. Can it be seen as providing price dynamics for "disequilibrium" models, models in which agents face sustained quantity constraints (Barro Grossman [1972] for example)?

An obvious but unimportant difference is the assumption of demand determination used here as opposed to the minimum rule used in these models. Let's restrict ourselves to the case of a decrease in nominal money, for which output decreases under either demand determination or a minimum rule. A major difference still remains. In disequilibrium models, a decrease in money generates a decrease in output and a regime of "Keynesian" unemployment (Malinvaud [1977]). As long as this regime prevails, firms sell less output than they would like at the prevailing mark up, workers sell less labor than they would like at the prevailing real wage. There are therefore sustained quantity constraints. This is not the case, however, in the model developed here. Although a decrease in money also decreases output and employment, relative prices along the path of adjustment were shown in the previous section to be such that on average firms do not feel constrained in the amount of output they sell and workers do not feel constrained in the amount of labor they supply.[2,3] There is no equivalent in this model of a sustained period of "Keynesian" unemployment. Asynchronization of price decisions generates fluctuations but not "disequilibrium".

Further deviations from the standard model are needed to obtain disequilibrium. Suppose for example that the wage, instead of being determined in a spot market, is given by contracts which specify a constant real wage at any level of employment, even if spot labor supply is an increasing function of the real wage. Suppose alternatively that firms adopt a price rule which specifies price as a function of standard unit costs, even if marginal cost is an increasing function of the level of production. (Why such rules might be chosen is still the subject of much research and very much an unsettled issue. See for example Azariadis [1975] and Green-Kahn [1983] for derivations of such real wage employment loci, Nordhaus [1970] for a discussion of price rules). In the absence of asynchronization, these rules would clearly not imply nominal price inertia and would be consistent with neutrality of money. However, in the presence of asynchronization, they have two implications: the first is that, even if spot labor and output supply schedules were upward sloping, relative prices may be inflexible and lead, with asynchronization, to substantial price level inertia. The second is that although on average prices and wages will satisfy price and wage rules, firms and workers may not be supplying the amount they would like. In particular, after a decrease in money, firms will want to supply more than they can at the prevailing mark up and thus the economy may be in a regime similar to a Keynesian regime.

Thus, to generate the phenomena described by disequilibrium models, we need two sets of assumptions. The first and probably hardest to justify is that relative prices be more inflexible than would be implied by the equality of spot supply and demand. The second is the presence of asynchronization which gives price level inertia as the result of the rigidity of real wages and mark ups.[4]

Section IV. Movements in Relative Prices

In the above model, after an increase in money and at the initial level of wages and prices, firms want a higher mark up, workers a higher real wage. Along the path of adjustment the outcome is a draw and the real wage, except for oscillations, remains constant. Will it remain the case, when a larger menu of goods is introduced that, except for oscillations, there are no movements in relative prices, or should we expect systematic distortions in the structure of relative prices in response to nominal disturbances? The answer is interesting both in itself and because it provides for ways of testing this class of models.

The initial model can be extended by increasing the number of outputs and inputs, and by examining the effects of differences in lengths of time between price decisions across markets. I shall only consider the first extension, through three examples.

The first is a model with one output and many inputs. It is characterized as follows:

$$\ell_i^s = b_i^{-1}(w_i - p) \qquad i = 1,\dots,n$$

$$y^d = c(m-p)$$

$$y = \frac{1}{n}\sum_{i=1}^{n} \ell_i$$

$$\Rightarrow \quad p = \frac{1}{n}\sum_{i=1}^{n} w_i \equiv w \tag{14}$$

$$\ell_i^d = (p-w_i) + y \tag{15}$$

Definitions are the same as before. The supply of an input is an increasing function of its real price. Output is produced from n inputs according to a Cobb Douglas technology, under constant returns to scale, Equations (14) and (15) give the implications of competition and unconstrained profit maximization; equation (14) gives the price as a weighted average of input prices and is implied by the zero profit condition. The level of output supply is then indeterminate; given any level of output, input demand is given by (15). As before, input suppliers choose input prices to equalize input supply and derived input demand:

$$\ell_i^s = \ell_i^d \Rightarrow b_i^{-1}(w_i - p) = (p - w_i) + c(m-p)$$

$$\Rightarrow w_i - p = (1-\Theta_i)(m-p) \qquad \Theta_i \equiv (1+b_i(1-c))/(1+b_i)$$

Aggregating over inputs, this gives:

$$w - p = (1-\Theta)(m-p) \qquad \Theta \equiv \frac{1}{n}\sum_{i=1}^{n}\Theta_i \tag{16}$$

and

$$w_i - w = (\Theta-\Theta_i)(m-p) \tag{17}$$

Equations (14) and (16) are identical to those used in the previous section (under constant returns). Thus, when the same structure of asynchronization is introduced, similar results arise: The "real wage", w-p, oscillates around its equilibrium value. From (17), however, this is not true of "product wages" $(w_i - p)$. Inputs for which supply is more elastic $(\Theta_i > \Theta)$ have a lower relative price during the adjustment process following an increase in money. There is therefore a systematic distortion in the structure of relative prices.

The second example is a model with one input and many outputs. In addition to generating systematic distortions in the structure of prices, it also shows what other factors than supply elasticities affect the degree of inertia of the price level. It is characterized by:

$$\ell^s = nb^{-1}(w-p) \quad ; \quad p \equiv \frac{1}{n}\sum_{i=1}^{n} p_i \quad ; \quad b \geq 0$$

$$y_i^d = c_i(m-p) - d(p_i - p) \qquad i=1,\ldots,n \;;$$

$$y_i = a\ell_i \qquad i=1,\ldots,n \;;\; a < 1$$

$$\Rightarrow \quad y_i^s = (a/(1-a))(p_i - w) \qquad i=1,\ldots,n$$

$$\ell_i^d = (1/(1-a))(p_i - w) \qquad i=1,\ldots,n$$

There is a common labor market where labor supply is a function of the real wage. The demand for output i is a function of real money balances and of its price relative to the price level. Production of output i is carried out under decreasing returns. Notional output supply and labor demand are functions of product wages.

Wage setters choose the wage so as to equalize labor supply and derived labor demands. Thus (ignoring the difference between sum of logs and log of sums...):

$$\ell^s = \sum_{i=1}^{n} \ell_i^d \quad \Rightarrow \quad b^{-1}(w-p) = a^{-1}c(m-p) \qquad c \equiv \frac{1}{n}\sum_{i=1}^{n} c_i \tag{18}$$

In turn price setters choose their price so as to equalize notional supply to demand:

$$(a/(1-a))(p_i - w) = c_i(m-p) - d(p_i - p) \tag{19}$$

Aggregating over outputs gives:

$$(a/(1-a))(p-w) = c(m-p) \tag{20}$$

$$\Rightarrow \qquad (p_i - p) = ((1-a)/(a+d(1-a)))(c_i - c)(m-p) \tag{21}$$

Equations (18) and (20) are again identical to those used in the previous section. Thus, with the same structure of asynchronization, the real wage will remain on average constant following a change in money. From (21), however, this is not true of relative prices. Outputs with low demand elasticity with respect to money balances will have a lower relative price during the process of adjustment to an increase in money.

As there is no asynchronization between output price decisions and because of our assumption of perfect competition, the parameter d, which characterizes the degree of substitutability between outputs, plays no role in the determination of the path of the price level and the real wage. To see what asynchronization between price decisions imply, consider the case where there are two price setters, with prices p_1, p_2. From above we can express their price as a function of m, w and the other price:

$$p_i = \left(\frac{a}{1-a} + \frac{1}{2}(d+c_i) \right)^{-1} \left(\frac{a}{1-a} w + c_i m + \frac{1}{2}(d-c_i)p_j \right) \qquad i,j = 1,2$$

Given the prices p_1 and p_2, we can solve for the nominal wage w from (18). Replacing w in the above equation and rearranging gives:

$$p_i = \Theta_i p_j + (1-\Theta_i) \qquad i,j = 1,2$$

where $\Theta_i \equiv (a-cb+(1-a)(d-c_i))/(a+cb+(1-a)(d+c_i)) \qquad i = 1,2$

This system in (p_1, p_2) has the same structure as the original system in (w, p). Thus given asynchronization of price decisions for p_1, p_2, similar results follow and the degree of price level inertia depends on Θ_1 and Θ_2. The parameter d plays an important role. If a is less than unity, then the higher d, the closer Θ_1 and Θ_2 to unity, the slower the adjustment of nominal prices. (The model presented here is close to one developed by Akerlof [1969].

The previous two examples show that if demand or supply elasticities differ across inputs or outputs, relative prices or wages will be distorted during the process of adjustment. The last example shows that asynchronization itself may create distortions in the structure of relative prices. Returning to the initial model, assume that production is now carried out in n steps, each under constant returns to scale. The model then becomes (this model is analyzed in detail in Blanchard [1983]):

$$y_0^s = b^{-1}(p_0 - p_n)$$

$$y_n^d = c(m-p_n)$$

$$y_i = y_{i-1} \qquad i=1,\ldots,n$$

$$\Rightarrow \quad p_i = p_{i-1} \qquad i=1,\ldots,n$$

The output of step i is denoted y_i. The primary input is denoted y_0, final output y_n. The supply of primary input depends on its price relative to the price of final output, the price level p_n. Technology exhibits constant returns to scale. The implied zero profit condition is that all prices be equal (up to constants which have been left out). In the absence of asynchronization, primary input setters choose the input price such as to equalize supply and derived demand.

$$y_0^s = y_0^d \quad \Rightarrow \quad b^{-1}(p_0 - p_n) = c(m-p_n) \tag{22}$$

Each intermediate input setter in turn chooses its price such that

$$p_i = p_{i-1} \qquad i=1,\ldots,n$$

Assume now that the structure of price decisions is asynchronized in the following way: input producers with i even take price decisions at even times, input producers with i odd take price decisions at odd times. Thus, again, all prices are revised after two periods. Given that structure, the effect of a permanent, unanticipated increase in money at $t = 0$, is particularly easy to trace if $bc = 1$, as in this case (22) implies $p_0 = m$. This case corresponds to $\Theta = 0$, in the model of Section I; in that model, if $\Theta = 0$, the price level and nominal wage adjust fully to their new equilibrium value within two periods. Table 2 gives the cross-section time series characterization of the effects of money on prices in the case where $n = 20$. Two results emerge which did not in the simpler model:

The first, which is not the focus of this section, is that the degree of price level inertia is a function of the number of price decisions: it takes n periods, where n is the number of price decisions, for the price level p_n to reach its new equilibrium value. The second is that, although each price setter (for $i = 1,\dots,n$) realizes on average a zero mark up for each period during which its price is fixed, the structure of relative prices is distorted. The relative price of inputs early in the chain of production increases on average compared to the price level.

Table 2

Structure of Prices After an Increase in Money:

Time:	0	1	2	3	4	5	6
Prices:							
p_0	1.	1.	1.	1.	1.	1.	1.
p_1	.0	1.	1.	1.	1.	1.	1.
p_2	.5	.5	1.	1.	1.	1.	1.
p_3	.0	.75	.75	1.	1.	1.	1.
p_4	.375	.375	.875	.875	1.	1.	1.
p_5	.0	.625	.625	.937	.937	1.	1.
p_6	.312	.312	.782	.782	.968	.968	1.
.							
p_8	.273	.273	.711	.711	.929	.929	.992
.							
p_{10}	.246	.246	.656	.656	.890	.890	.977
.							
p_{12}	.225	.225	.612	.612	.853	.853	.959
:	:	:	:	:	:	:	:
p_{20}	.176	.176	.498	.498	.738	.738	.886

The prices are generated according to:

$$p_{it} = 0 \quad \text{if} \quad t = 0 \quad , \quad i \text{ odd}$$

$$p_{it} = \frac{1}{2}(p_{i-1,t-1} + p_{i-1,t+1}) \text{ otherwise}$$

Conclusion

The argument of the paper has been the following: The first effect of a movement in aggregate demand is to make initial relative prices mutually inconsistent. After a decrease in money, all relative prices are "too high"; they are all "too low" after an increase in money. In the presence of asynchronization, pressure on relative prices leads to a decrease of nominal prices in the first case, an increase of nominal prices in the second case. If pressure on relative prices is weak, the return to equilibrium is slow.

We have, however, relied until now on a mechanical asynchronization structure. Although this may be acceptable to characterize the movement of prices under a given regime, it prevents us from predicting the dynamic effects of a change in regime, such as for example a sudden decelleration of money growth. To do so requires a theory of asynchronization itself. Even if agents find it optimal to choose a fixed length between price decisions (an assumption we shall examine below), they certainly have the choice as to the timing of such decisions. In the examples above, wage setters probably have an incentive to synchronize decisions with price setters by shifting decisions from odd to even times: the asynchronization is not stable and might well disappear over time. Can we therefore construct asynchronization structures which would be stable in this sense? Consider the following two examples: The first relies on the existence of a large number of price decisions, the second on the existence of two types of disturbances.

In the first, suppose there are many types — more precisely, a continuum — of outputs, each of them produced from many types — again, a continuum — of inputs. Input price decisions are taken discretely and staggered over time so that the number of price decisions per unit time is constant; if input cost shares are equal, output producers who also take decisions discretely are therefore indifferent to the timing of their decision. Suppose that output price decisions are also staggered over time so that the number of output price decisions per unit time is constant. Thus input price setters, who are also the consumers, are indifferent to when they take their price decisions. This structure of asynchronization is stable: given the timing of other agents' decisions, no one has an incentive to change his own. It is, however, a knife edge stability. If a firm or a worker changes his timing, then all others have an incentive to do so until all decisions are synchronized.

The second example relies on the existence of two types of disturbances, aggregate and idiosyncratic. Let's first relax the assumption of a constant time interval between price decisions; the optimal rule is indeed more likely to be an (s,S) rule rather than a fixed length rule, implying that the time between decisions is random rather than constant (Sheshinski and Weiss [1981] have looked at pricing decisions in this context. If price setters follow (s,S) rules, and if disturbances were only idiosyncratic, changes in prices would be independent of each other and asynchronization would remain. If there are both idiosyncratic and aggregate disturbances, but if the size of idiosyncratic disturbances is large compared to that of aggregate disturbances, the above argument suggests that some asynchronization may remain. This line of reasoning is, however, difficult to formalize and is left for future research.[6,7]

The other task left for future research is a test of the model; the approach of the paper is only one of many which attempt to explain the effects of money on economic activity. A stringent test of its relevance is a test of its implications for the joint behavior of relative and nominal prices. This will extend work by Fischer [1981], Taylor [1981], Marquez and Vining [1982] among others.

Appendix. Derivation of Prices and Wages

I first solve for p_t for a general process for money, and then for the path of prices and wages corresponding to the path of money described in the text.

Using equation (11) for both t-1 and t+1, taking $E(w_{t+1}|t)$ using iterated expectations and replacing in equation (10) gives:

$$p_t = \frac{1}{4}\sigma\Theta(p_{t-2}+E(p_t|t-1) + p_t + E(p_{t+2}|t)) + \frac{1}{4}\psi_t \tag{A1}$$

where ψ_t is given by:

$$\psi_t \equiv \sigma(1-\Theta)m_{t-1} + \sigma(1-\Theta)E(m_t|t-1)$$

$$+ 2(1-\sigma)\ m_t + 2(1-\sigma)E(m_{t+1}|t)$$

$$+ \sigma(1-\Theta)E(m_{t+1}|t) + \sigma(1-\Theta)E(m_{t+2}|t)).$$

Equation (A1) must be solved in two steps. The first is to solve for $E(p_t|t-1)$, the second for p_t. Taking expectations on both sides of (A1) conditional on information available at t-1 and rearranging gives:

$$E\ (\sigma\Theta p_{t-2} + (2\sigma\Theta-4)p_t + \sigma\Theta p_{t+2} = -\psi_t\ \ |t-1). \tag{A2}$$

Let λ be the smallest root of:

$$\sigma\Theta\lambda^2 + (2\sigma\Theta-4)\lambda + \sigma\Theta = 0$$

It is given by:

$$\lambda = (1-(1-\sigma\Theta)^{1/2}) \ / \ (1+(1-\sigma\Theta)^{1/2})$$

This root is an increasing function of $\sigma\Theta$, equal to 0 for $\sigma\Theta = 0$ and to 1 for $\sigma\Theta = 1$. Solving equation (A2) by factorization, subject to the condition that p_t does not explode, gives:

$$E(p_t|t-1) = \lambda p_{t-2} + (\lambda/\sigma\Theta) \sum_{i=0}^{\infty} \lambda^i E(\psi_{t+2i}|t-1) \tag{A3}$$

The last step is to solve for p_t. Deriving $E(p_{t+2}|t)$ from (A3) and replacing $E(p_{t+2}|t)$ and $E(p_t|t-1)$ in (A1) gives p_t:

$$(4-\sigma\Theta(1+\lambda))p_t = \sigma\Theta(1+\lambda)p_{t-2} + \psi_t + \lambda \sum_{i=0}^{\infty} \lambda^i (E(\psi_{t+2i}|t-1) + E(\psi_{t+2i+2}|t))$$

I now consider a specific path for money. m increases at t = 0 from zero to unity; the increase is unanticipated and permanent.

To solve for the price level path, note that for $t > 0$, there is no remaining uncertainty and thus equation (A3) holds for actual values of p and ψ rather than for their expectations. Noting also that for $t > 0$:

$$(\lambda/\sigma\Theta) \sum_{i=0}^{\infty} \lambda^i \psi_{t+2i} = (1-\lambda) ,$$

we have:

$$p_{t+2} = \lambda p_t + (1-\lambda) \qquad \text{for } t \geq 0$$

This gives in particular, for t=0, p_2 as a function of p_0. Equation (A1) in turn, for t=0, gives another relation between p_0 and p_2. Noting that $E(p_0|-1) = 0$ and $E(p_2|0) = p_2$, we have:

$$(1 - \frac{1}{4}\sigma\Theta)p_0 = \frac{1}{4}\sigma\Theta p_2 + (1-\sigma) + \frac{1}{2}\sigma(1-\Theta).$$

Solving the two equations in p_0 and p_2 gives the following characterization of the path of the price level:

$$p_0 = 1 - \frac{1}{2}\sigma(1 - \frac{1}{4}\sigma\Theta(1+\lambda))^{-1} \quad ; \quad 0 \leq p_0 \leq 1-\lambda$$

$$p_t = \lambda p_{t-2} + (1-\lambda) \qquad \text{for } t = 2,4,\ldots$$

The Behavior of the Real Wage

Let x_t be the real wage at time t; from equation (11) at t-1 and t+1, and from equation (13):

$$x_0 \equiv w_0 - p_0 = -p_0 \qquad < 0$$

$$x_{t+1} \equiv w_{t+1} - p_t = [\tfrac{1}{2}\Theta(1+\lambda)-1](p_t-1).$$

$$\Theta,\lambda < 1 \Rightarrow \tfrac{1}{2}\Theta(1+\lambda) - 1 < 0 \Rightarrow x_{t+1} > 0$$

$$x_{t+2} \equiv w_{t+1} - p_{t+2} = [\tfrac{1}{2}\Theta(1+\lambda^{-1})-1]\ (p_{t+2}-1)$$

From the definition of λ:

$$(1+\lambda^{-1}) = 1 + (1+(1-\sigma\Theta)^{1/2})/(1-(1-\sigma\Theta)^{1/2})$$

$$= 2/(1-(1-\sigma\Theta)^{1/2})$$

$$\Rightarrow \tfrac{1}{2}\Theta(1+\lambda^{-1}) = \Theta/(1-(1-\sigma\Theta)^{1/2})$$

$$\sigma \leq 1 \Rightarrow \tfrac{1}{2}\Theta(1+\lambda^{-1}) \geq \sigma\Theta/(1-(1-\sigma\Theta)^{1/2})$$

$$\sigma\Theta = (1-(1-\sigma\Theta)^{1/2})(1+(1-\sigma\Theta)^{1/2} \Rightarrow$$

$$\tfrac{1}{2}\Theta(1+\lambda^{-1}) - 1 \geq (1+(1-\sigma\Theta)^{1/2}) - 1 = (1-\sigma\Theta)^{1/2} \geq 0$$

$$\Rightarrow x_{t+2} < 0$$

Footnotes

1. Equation (10) follows from the assumption that the expected geometric average price over the two periods must be the same under synchronization and asynchronization. A theoretically more appealing assumption is that the expected arithmetic average profit must be the same under synchronization and asynchronization. This leads, however, to a less tractable specification, which differs from the specification in the text in three ways:

   As firms are most of the time off their notional supply curve, the price has to be on average higher than under synchronization to maintain the same level of average profit.

   The weights on the two periods may not be equal, but will in general depend on the expected levels of demand in both periods.

   Finally, the covariance between wages and nominal money will appear in the price equation.

   These three points can be made clearer by deriving the price equation which would follow from imposing a zero arithmetic average profit condition, in the case of constant returns to scale ($\sigma = 1$). In this case (all letters denote levels, not logarithms), P_t would be given by:

$$P_t = (M_t/(M_t+EM_{t+1}))W_{t-1} + (EM_{t+1}/(M_t+EM_{t+1}))\left(\frac{E(M_{t+1}W_{t+1})}{EM_{t+1}}\right)$$

   as opposed to:

$$p_t = 1/2\ w_{t-1} + 1/2\ E(w_{t+1}) \text{ in the text.}$$

2. The precise statement is that the average supply by workers (firms) is consistent with the average real wage (mark up) where "average" is the geometric average over each two period interval during which nominal wages (nominal prices) are fixed.

3. This result is similar to that obtained in imperfect information models, where money may affect output and employment but where agents execute their desired trades at the perceived relative prices.

4. This suggests that explaining the behavior of relative prices, quite apart from nominal rigidities should be high on the research agenda. This appears indeed to have been the approach followed in the 1960's (as summarized by Tobin [1970]). Research was focused on documenting and explaining the inflexibility of mark ups, on documenting and explaining the slow and small reaction of wages to unemployment, the slope of the short-run Phillips curve. Nominal price and wage inertia was then obtained not from explicit asynchronization, but from assumptions about price and wage expectations.

5. Similar problems of stability of the asynchronization structure arise in another class of models. These models characterize the effects of money on activity when decisions about when to shift from interest bearing assets into money are asynchronized (Rotemberg [1982] and Grossman and Weiss [1982]).

6. Two other explanations have been suggested for asynchronization. The first is that asynchronization allows for a more efficient utilization of information in price decisions. The second is that asynchronization may be the outcome of a game between price setters; it is sometimes used to explain the timing structure of labor contracts. These have not, to my knowledge, been formalized.

7. An interesting attempt to explain asynchronization, which relies on two types of disturbances and a Nash equilibrium concept has been developed by Fethke and Policano [1982]. The structure of their model is, however, quite different from the one presented in the text.

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