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WHY ARE REAL INTEREST RATES SO HIGH?

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Abstract

This paper applies the Capital Asset Pricing Model to help explain the anomalous behavior of real interest rates during the last several years. Specifically, we are able to show that the increased volatility of bond prices since the change in Federal Reserve operating procedure in October 1979 has substantially increased the required real risk premium on long term bonds. We also consider and reject the possibility that increased risk alone accounts for the recent increase in the short-term real rate. Finally, we use the model to simulate the financial effects of a Federal debt maturity management operation.

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WHY ARE REAL INTEREST RATES SO HIGH?

The recent behavior of the real interest rate and the term structure of nominal interest rates has been a major puzzle to most macroeconomists. Throughout the 1970's, analysts explained the historically high nominal interest rates as being the result of a high expected rate of inflation, assuming that real interest rates remained at their relatively low historical levels.[1] In the early 1980's, however, the expected real short-term interest rate apparently rose, as the nominal short-term rate remained high while the inflation rate declined.[2] The behavior of the real long-term interest rate has been less clear, since there is no accepted measure of long-term expected inflation. Many analysts argued that the long-term nominal interest rate was high in the early 1980's because the long-term expected inflation rate was high.[3] However, we argue in this paper that a significant portion of the rise in long-term nominal interest rates (see Figure 1) was due to an increase in the real risk premium on long-term bonds, apparently resulting from the increased volatility of interest rates since the change in Federal Reserve operating procedure in October, 1979.

The theoretical model we employ is a modified version of the capital asset pricing model (CAPM), which has become the standard financial model of capital market equilibrium over the past decade, but which to our knowledge has not been employed to explain macroeconomic phenomena of the sort investigated here. Standard macroeconomic models have been employed to explain the rise in the short-term real interest rate since October, 1979. These models, for example the IS-LM model, typically do not account for the possibility that changes in the return variability on bonds can affect the short-term real rate, and do not explain shifts in the term structure of interest rates. Our empirical results, using the CAPM, indicate that our model by itself cannot account for the recent high real rates of return on short-term nominal bonds, though it does explain higher long term rates quite well. Thus, our model explains the seeming anomaly of the flat nominal term structure observed in the early 1980's despite a reduction in the expected future inflation rate.

The use in this paper of the CAPM to forecast the term structure of real risk premia on bonds stands in sharp contrast to more usual methods of explaining the term structure. For example, the expectations hypothesis implies that high long-term yields could be explained only by high expected future short-term yields. The inconsistency between the expectations hypothesis and modern financial theory has been noted (e.g., see Cox, Ingersoll, and Ross, 1981). What appears not to be widely appreciated, however, is that whatever the structural explanation of the term structure, expected rates of return on bonds should obey an equilibrium asset pricing relationship such as the CAPM.

Our model also permits us to explore another issue in macroeconomics, namely how the maturity composition of the outstanding government debt affects relative yields on short and long-term debt and, more importantly, the required return on equity. Since Tobin's (1963) paper on debt management, this has been a central concern of monetary theorists and policy makers. Using our model, we are able to quantify the effects on yield spreads of shifts in the relative supplies of short-term vs. long-term government debt. Friedman (1982) and Roley (1982) have examined this issue using a different methodology: empirically estimated structural demand and supply equations for different assets. Our methodology, using an equilibrium pricing relationship, is quite different from theirs.[4]

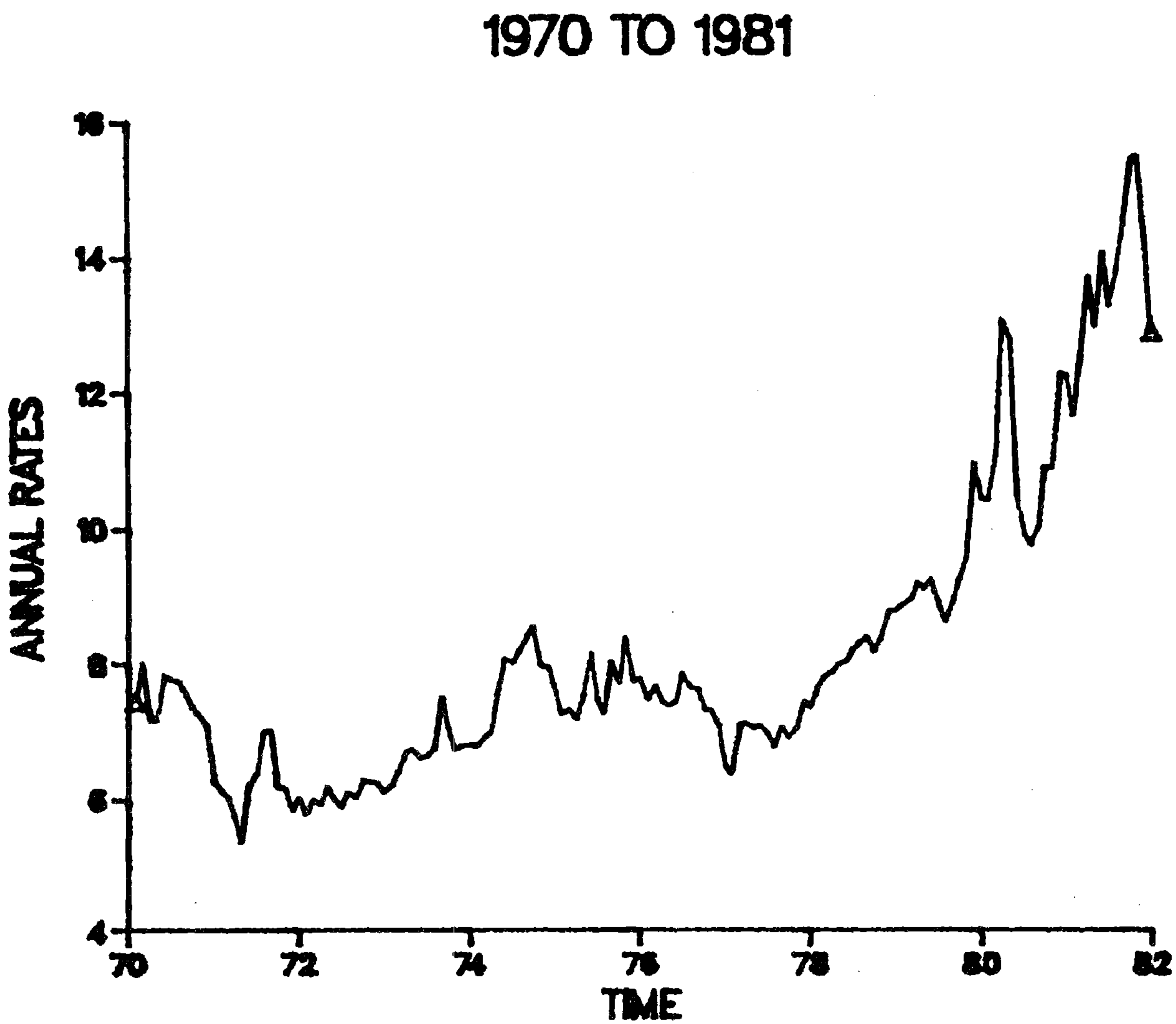
The structure of the paper is as follows: in the first section, we briefly explain the theoretical model and its two main implications for equilibrium risk premia. In Section 2, we explain how we estimated the inputs necessary to derive numerical values for the equilibrium risk premia from our model. Section 3 presents the estimated risk premia and shows how their behavior has evolved over time. We also consider the sensitivity of our estimates to several of our key assumptions about aggregate risk aversion and the stochastic process generating real returns. Finally, in Section 4 we explore the model's implications for the conduct of debt-management policy. In particular, we show how changes in the relative supplies of short and long-term government debt can affect the equilibrium risk premium on equity, thus influencing the rate of capital formation in the economy. An appendix discusses the construction of the data.

1. Theoretical Model

A. Determining the Risk Premia on Assets

Our basic model of portfolio selection is that of Markowitz (1952) as developed by Sharpe (1964), Lintner (1964), and Mossin (1966), and extended to a full intertemporal context by Merton (1969, 1971). Merton has shown that when asset prices follow a Geometric Brownian Motion in continuous time and portfolios can be continuously revised, then as in the original Markowitz model, only variances and covariances of the joint distribution of returns are needed to compute equilibrium risk premia. Investors are assumed to have homogeneous expectations about the values of these parameters. Furthermore,

FIGURE 1 --YIELD TO MATURITY ON DURATION 5 BONDS



Source: CRSP Government Bond File.

we assume that all n assets are continuously and costlessly traded and that there are no taxes.[5] In this context it is straightforward to show that the real risk premium on asset i relative to asset j, $R_i - R_j$, must in equilibrium be given by[6]

$$R_i - R_j = \delta(\sigma_{iM} - \sigma_{jM}) \tag{1}$$

where σ_{iM} = Covariance between the real rate of return on asset i and the real rate of return on the market portfolio

δ = Harmonic mean of investors' degree of relative risk aversion, weighted by investors' share of aggregate wealth.

Equation (1) holds for real rates of return whether or not there is a risk-free asset, and whether or not there is inflation; in this paper, we will be assuming there is no real risk-free asset. By choosing R_j appropriately, it is possible to derive the various versions of the CAPM-type asset pricing relationships, such as the standard CAPM when there is a risk-free asset, and the Black (1972) "zero-beta" version when there is not. Because the focus in this paper is on explaining risk premia on bonds, we will use the rate of return on Treasury bills as our benchmark asset.

In the standard version of the CAPM, the measure of aggregate relative risk aversion, δ, is eliminated in order to write the equilibrium pricing relationship solely in terms of (presumably) observable variables, such as beta and the risk premium on the market portfolio. In this paper, we are interested in explaining risk premia, so we leave the equation in the form (1). Our strategy is to estimate the necessary covariances and to compute risk premia for various levels of risk aversion. To our knowledge, no one has seriously suggested that the average level of relative risk aversion is below one or above six, so we restrict our calculations to that range.[7]

In standard discussions of the CAPM, the "market portfolio" is usually taken to mean a portfolio of equities. In this paper, we take as the market portfolio the total quantity of stocks and bonds which must be held. In theory, the market portfolio should represent the total quantity of claims which must be held by the household sector, so that our measure is preferable to using equities alone.

It should be emphasized that equation (1) does not give yields to maturity; rather, equation (1) gives the expected instantaneous holding period yield for any asset. This distinction is especially crucial for bonds, since a long term bond with an expected holding period yield appropriate for long-term bonds, will someday become a short-term bond with a different expected holding period yield. The expected yield to maturity will be an average of future expected holding period yields. For the purposes of this paper, the important point is that, ceteris paribus, an increase in the expected holding period yield on a long-term asset will also raise its yield to maturity.

B. The Shadow Risk-Free Rate

Given the equilibrium pricing relationship (1), it is possible to ask the following question: if a risk-free asset were introduced in zero aggregate supply, what rate of return would it have to have if equilibrium risk premia were to remain undisturbed? We call this rate the shadow risk-free rate.[8] From (1), it is easy to see that the shadow risk-free rate would satisfy

$$R_0 - R_f = \delta\sigma_{0M} \qquad (2)$$

where R_0 is the expected rate of return on zero-duration assets (Treasury bills), and σ_{0M} is the covariance between Treasury bills and the Market. As long as this covariance is unchanged, the risk premium on Treasury bills over the shadow risk-free rate will not change.

One reason for focusing on the shadow risk-free rate is the possibility that it remains relatively constant over time, and thus serves to tie down the absolute level of interest rates over time and not just the risk premia. In a consumption-based asset pricing model (see, for example, Cox, Ingersoll, and Ross, 1978), the shadow risk-free rate would depend on the rate of time preference and the expected rate of change in the marginal utility of consumption. Policy changes which changed neither of these would leave the shadow risk-free rate constant. One should think of the shadow risk-free rate as the rate that would prevail if the risk-free asset was in zero net supply. The more risk-averse investors would lend to the more risk-tolerant investors, and those with average risk aversion would hold none of the risk-free asset.

II. The Data

In this section we describe in some detail how we estimated covariances and market weights. It should be emphasized that the goal is not to test the CAPM, but instead to derive its implications for the change in risk premia due to the change in asset return variability over the last several years.

A. Assets Included

Obviously it is necessary to limit the number of assets studied. We assumed that there were ten classes of assets: stocks; Treasury-bills; and nominally risk-free bonds, of durations one through eight years. In measuring how long-term a particular debt issue is, we are careful throughout the paper to use duration instead of the more common measure of maturity date.[9] Duration is a measure of the time until the average payment to the bondholder occurs, and may be used to consistently compare bonds with very different payment streams, for which maturity date may provide a misleading comparison. The distinction between maturity and duration is important since the duration of bonds of a given maturity shortened considerably in the late 1970's.

Stocks are an obvious choice for inclusion, and we include a variety of bonds because it is bond yields we are interested in explaining. Some of the exclusions, however, are noteworthy: land, residential housing, and consumer durables together account for about 40 percent of household net worth as measured in the Flow-of-Funds Sector Balance Sheets. Unfortunately, there are no reliable rate of return series for these assets, so we were forced to exclude them from our market portfolio. Human capital is excluded for the same reason.

There are also a variety of assets which are not literally common stock or governments bonds, but which we assume are good substitutes for those assets. Time and Demand deposits, for example, are taken to have the same rate of return as Treasury bills[10]; corporate bonds and municipal bonds are assumed to be "like" government bonds; and non-corporate equity is assumed to have the same characteristics as equity. We decided that these assumptions (and others to be outlined below) were preferable to ignoring these assets. We could not separately include these other assets because acceptable rate of return data could not be obtained.

B. Real Rates of Return

We estimated covariances using monthly real rates of return for stocks and the nine categories of government bonds described above. We used monthly data because, as Merton (1980) has shown, the accuracy of variance and covariance estimates is improved by using frequent observation periods, and because one month is the shortest interval for which inflation data is available and hence for which real rates of return could be computed. Estimates of the mean, on the other hand, are not improved by increasing the observation frequency within a given time span. The price index used was the Bureau of Labor Statistics' Consumer Price Index, excluding the cost of shelter.

Real returns on Treasury Bills are from Ibbotson and Sinquefield (1982), while bond data are from the U.S. Government bond-file of the Center for Research in Security Prices (CRSP). Stock returns are from the NYSE monthly CRSP file. We divided the bonds into nine categories based on duration.

In all cases the rate of return was measured as the natural logarithm of the monthly real wealth relatives. On the assumption that these returns are lognormally distributed, the log of the wealth relative over a discrete time interval is normally distributed with mean μ_i and variance σ_i^2, where

$$\mu_i = R_i - \frac{\sigma_i^2}{2}$$

All reported means were converted to annual rates by multiplying them by 12 and the standard deviations by multiplying them by $\sqrt{12}$. This makes them comparable to the means and standard deviations of the continuously compounded rates of return one would obtain using a one year holding period.

C. Market Weights

As mentioned, the market portfolio consists of a number of different assets. In order to compute the covariance of the return on the market with that on other assets, it is necessary to know the composition of the market portfolio (the market weights), which we denote w_M. Theoretically, w_M should measure the percentage of household net worth invested in assets which comprise the market portfolio. We used the Flow-of-Funds Sector Balance Sheets to obtain this breakdown for broad categories of assets for 1976 and 1980. The Treasury Department's Monthly Statement of the Public Debt was used to determine the relative quantities of government bonds of different maturities outstanding in those two years. We assume in the calculations that government bonds are net worth to the household sector. The details of the procedure for determining the weights is relegated to an appendix.

Weights were computed by calculating the percentage of household net worth (taken from the Flow of Funds Balance Sheet) held in each kind of asset, under two alternative assumptions about financial intermediaries: first, that the liabilities of intermediaries are treated as assets by the household sector; and second, that financial intermediaries are a veil, so that households perceive themselves as directly holding the assets of intermediaries. Intermediaries are completely netted out in the second procedure. The chief difference in the two cases is that households hold more short-term assets in the first case. This is to be expected, since intermediaries typically borrow short and lend long. Weights computed under the two assumptions are presented in the Appendix.

There is one important class of assets held by households for which duration data is not available: pension fund and life insurance reserves.

Together, these constitute almost one-fifth of the financial net worth of households in both 1976 and 1980. Consequently our results are potentially sensitive to the allocation of these assets across durations. We elected to compute asset weights under each of two assumptions about these assets: first, that they were spread evenly across durations one through eight (we also used this assumption to allocate mortgages across durations); and second, that these reserves are predominantly long term. In the second case, we used the sum-of-the-years digits method to allocate these assets "triangularly" across durations. The second case is probably more reasonable, since for households pension reserves represent a long-term, nominally fixed claim. Nevertheless, we computed results under both sets of assumptions.

D. Computing Rate of Return Covariances

The most important input to the model is the covariance matrix of asset returns. In order to estimate a covariance matrix, it is necessary to create a time-series of residuals--deviations of actual rates of return from some mean or expected rate of return. We estimated covariances by computing the mean in two different ways:

1. Unconditional Covariance. Here we simply computed the covariance matrix of asset returns by taking deviations of actual rates of return from the historical mean rate of return over the same period. This is objectionable only if the conditional expected mean real rate of return changed (or was perceived to have changed) over the period in question.

2. Conditional Covariance. In this procedure, we allowed for a time-varying expected rate of return for each asset and computed deviations from this mean in estimating the covariance matrix. To do this, we performed

the following procedure: an expected real rate of return on one-month Treasury bills was computed in one of several ways (see below), and this process was used to generate a one-month ahead expected real rate of return on Treasury bills. Given this forecast of R_0, and the covariance matrix estimated over the previous twenty-four months, it was possible to estimate the expected rate of return on every asset using (1). With both the expected rate of return on every asset and the actual return, we were able to compute forecast errors. The latest forecast error (conditional on the forecast of the Treasury bill rate) was then entered into the data matrix and used the next period in computing the covariance matrix.

Thus, the entries in the conditional covariance matrix were computed as

$$\sigma_{ij}(T) = \frac{1}{N} \sum_{t=1}^{N} (\tilde{R}_{i,\,T-t} - \hat{R}_{i,T-t})(\tilde{R}_{j,T-t} - \hat{R}_{j,T-t}) \qquad (3)$$

where $\tilde{R}$ represents the realized real rate of return and $\hat{R}$ is the estimated expected real rate of return, computed using equation (1) and the previous month's covariance estimate.

An important advantage of this procedure is that the computed expected rates of return for each period are consistent with the model. The only part of this procedure which is *ad hoc* is the specification of the process generating expected rates of return on Treasury bills. The measure of covariance upon which the CAPM is based is the covariance of holding period rate of return deviations from expected rates of return. This is precisely what our procedure measures.

An obvious alternative way to compute the conditional covariance would be to calculate an expected rate of return based on a rolling ARIMA regression, or using some other kind of forecasting procedure. However, under the null hypothesis that the CAPM correctly describes asset market equilibrium, the ARIMA regression would likely fail to generate true expected rates of return, since it ignores the relationship between the covariance structure and expected rates of return.

We computed expected real rates of return on Treasury bills using four different methods:

a) $R_0 = i - \pi_{-1}$. The expected real yield over the coming month is the current nominal yield on a 30-day Treasury bill, i, less last month's inflation rate, π_{-1}.

b) $R_0 = a + bR_{0,\,-1}$. The expected real yield on Treasury bills follows an AR1 process. We experimented with computing a and b in three different ways: a and b were reestimated every period using only the last 24 months of data, and using all available data; and they were constrained to be the numbers reported in Ibbotson and Sinquefield (1982), i.e., a = 0, b = .63.

3. Results

A. Unconditional Covariances

We estimated the covariance matrix of asset returns over two periods: January, 1973 through September 1979, and January, 1980 through December, 1981. Summary statistics for these periods are presented in Table 1a. Table 1a has a number of striking features. First, except for stocks and bills, the ex post real rate of return on all assets is negative over both periods. We take this to demonstrate the point made above, that the mean cannot reliably be estimated over a relatively short span of time, and that the use of the ex post mean to estimate the ex ante mean would be futile over this time period.

Table 1a
Unconditional Means, Standard Deviations, and Correlations

January 1973-September 1979

	Common Stocks	1month Bills	Bonds (by duration in years) 1	2	3	4	5	6	7	8
Mean	-4.20	-1.61	-1.38	-1.55	-1.73	-2.81	-2.56	-3.59	-2.00	-2.75
Standard Deviation	17.39	1.04	2.01	2.94	3.62	4.66	4.82	4.49	4.89	5.24
Number of Observations	81	81	81	81	81	81	81	68	72	79
Correlation Coefficients:										
Stocks		.31	.39	.42	.34	.36	.19	.22	.32	.33
Bills			.56	.41	.34	.39	.16	.10	.21	.18
Bonds 1				.74	.74	.70	.71	.71	.73	.61
2					.90	.83	.74	.64	.73	.70
3						.83	.84	.69	.79	.75
4							.73	.63	.76	.66
5								.85	.84	.80
6									.89	.78
7										.83

January 1980-December 1981

	Common Stocks	1month Bills	Bonds (by duration in years) 1	2	3	4	5	6	7	8
Mean	2.63	2.83	3.40	-0.55	-1.68	-2.28	-3.86	-9.53	-3.90	-10.17
Standard Deviation	17.24	1.46	5.35	9.72	12.56	14.45	16.98	18.39	19.42	20.07
Number of Observations	24	24	24	24	24	24	24	24	24	24
Correlation Coefficients:										
Stocks		-.06	.26	.27	.22	.26	.26	.23	.35	.40
Bills			.58	.52	.53	.46	.48	.48	.44	.45
Bonds 1				.96	.92	.89	.87	.79	.83	.89
2					.95	.94	.90	.84	.88	.91
3						.98	.95	.87	.90	.92
4							.96	.87	.92	.93
5								.89	.93	.90
6									.92	.88
7										.94

Table 1b
Conditional Means, Standard Deviations, and Correlations

January 1977-September 1979

	Common Stocks	1month Bills	Bonds (by duration in years) 1	2	3	4	5	6	7	8
Standard Deviation	17.46	1.24	2.26	3.43	4.04	5.25	5.02	4.64	5.38	5.43
Correlation Coefficients:										
Stocks		.37	.33	.37	.31	.31	.07	.21	.23	.25
Bills			.69	.64	.56	.58	.37	.31	.40	.36
Bonds 1				.78	.79	.76	.76	.74	.79	.69
2					.90	.88	.76	.69	.74	.74
3						.93	.87	.76	.82	.85
4							.84	.74	.78	.80
5								.85	.86	.89
6									.90	.80
7										.86

January 1980-December 1981

	Common Stocks	1month Bills	Bonds (by duration in years) 1	2	3	4	5	6	7	8
Standard Deviation	17.55	1.00	5.15	9.60	12.5	14.4	16.9	18.5	19.4	20.4
Correlation Coefficients:										
Stocks		.13	.29	.29	.23	.28	.27	.24	.36	.41
Bills			.56	.49	.51	.47	.44	.36	.48	.43
Bonds 1				.95	.91	.88	.85	.74	.82	.86
2					.95	.94	.90	.83	.88	.90
3						.98	.95	.86	.90	.91
4							.96	.86	.92	.93
5								.89	.93	.90
6									.92	.88
7										.93

Instead, we use the covariance data to infer the pattern of expected risk premia across assets.

Comparing the two sample periods, it is apparent that the standard deviation on stocks and bills changed little between 1973-1979 and 1980-1981. The change in rate of return standard deviations on long-term bonds is dramatic, however, with the standard deviations on the longest-term bonds increasing four-fold, and exceeding the standard deviation on the rate of return on equity.

The pattern of correlation coefficients is especially important for our purposes, since we wish to examine the effect of Government debt management policies on relative asset yields. The sign and strength of correlations determines the relative substitutability of assets. For example, if the covariance between the rate of return on equity and duration 8 bonds is large, then an increase in the supply of duration 8 bonds would be expected to increase the yields on both duration 8 bonds and equity, while if the covariance is low, the effect on equity would be less pronounced.

The pattern of correlation coefficients in Table 1a does not seem to suggest that long-term bonds are more like bills than like equity, which was argued by Tobin (1963). Between the earlier and later periods, bills and stocks became essentially uncorrelated, while the correlation of stocks with the longer duration bonds rose, and that for shorter duration bonds fell. It is worth noting that covariances between the longer duration bonds and stocks did in general increase from the earlier to the later sample period, since both standard deviations and correlations increased. Thus the importance of debt management policy should be greater in the later than in the earlier period.

The covariance matrix was then used to compute risk premia, using equation (1). The level of risk aversion, δ, was set at 1, 3.5, and 6, to represent extreme plausible values of the level of aggregate risk aversion. Because the risk premium is proportional to the level of risk aversion, it is easy to compute the result of a change in this parameter. In Table 2 we present the results using 1980 market weights, computed under the assumption that pension fund and life insurance reserves are predominantly long-term assets.

The results are striking: the increase in the variability of rates of return on long-term assets from the seventies to the eighties raised the risk premium on long-term nominal bonds by a minimum of 110 basis points, and possibly by over 700 basis points, depending on aggregate risk aversion. Thus, the puzzle of the flat term structure in 1980 and 1981 may not be a puzzle at all. It is entirely possible that the Fed's restrictive action lowered expected future long-term inflation rates, and that the short-term interest rate rose for standard IS-LM reasons. The seeming anomaly that long-term interest rates also rose, may be explained by a rise in the real risk premium, which more than offset the decline in the long-term inflation premium.

Table 2 also shows that changes in covariances alone cannot explain the rise in the real short-term interest rate in the early 1980's. The risk premium for duration zero bonds relative to the shadow risk-free rate (the bottom row in Table 2) actually fell from 1973-79 to 1980-81, and was less than 25 basis points in both subperiods. This suggests that changes in the riskiness of the real return on Treasury bills cannot account for the rise in the short-term real rate. To explain the rise in the short-term real rate, it is necessary to explain a rise in the shadow risk-free rate, which this model

Table 2
Annual Risk Premia on Ten Assets (Percent)

Asset		Covariance Estimated 1973-1979			Covariance Estimated 1980-1981			Increase in Risk Premium, Evaluated at $\delta = 3.5$
		$\delta=1$	$\delta=3.5$	$\delta=6$	$\delta=1$	$\delta=3.5$	$\delta=6$	
Stocks		1.93	6.76	11.59	1.99	6.99	11.91	.230
Bond of Duration								
	0	-	-	-	-	-	-	-
	1	.056	.196	.337	.238	.834	1.43	.638
	2	.110	.385	.661	.458	1.62	2.71	1.23
	3	.110	.385	.661	.513	1.82	3.08	1.44
	4	.167	.583	1.00	.673	2.36	4.04	1.77
	5	.080	.280	.480	.775	2.71	4.65	2.43
	6	.088	.308	.529	.775	2.71	4.65	2.40
	7	.157	.548	.940	1.09	3.82	6.55	3.27
	8	.172	.601	1.03	1.24	4.34	7.44	3.74
$R_0 - R_f$		.040	.141	.242	.008	.027	.047	-.114

Note: Risk Premia calculated relative to rate of return on duration zero assets. Market weights are those for 1980, with pension fund reserves weighted toward long durations.
$R_0 - R_f$ computed using equation (2).

FIGURE 2A--RISK PREMIUM ON DURATION 4 BONDS

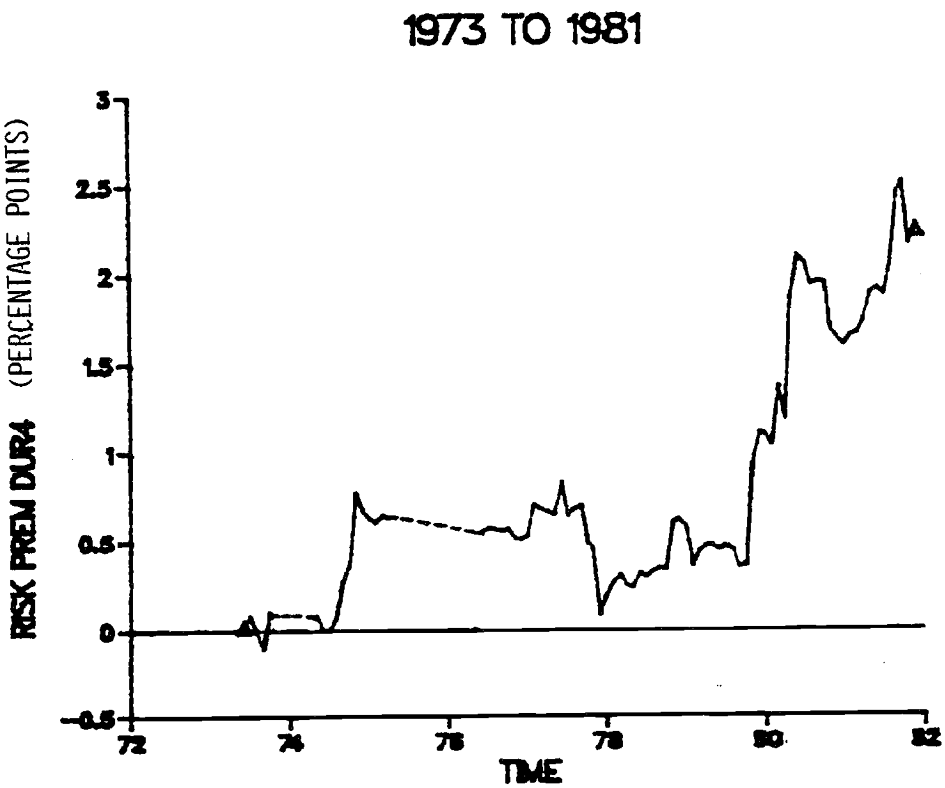
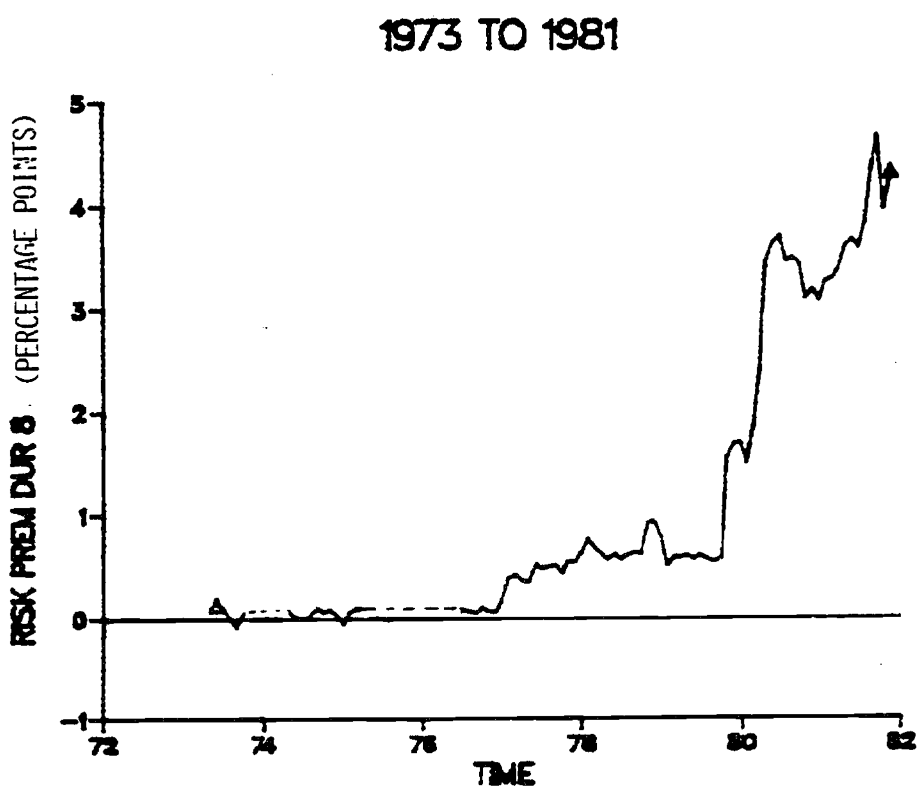


FIGURE 2B--RISK PREMIUM ON DURATION 8 BONDS



is not equipped to do.

B. Conditional Covariances

A possible objection to the use of unconditional covariance estimates to infer risk premia is that the levels of expected rates of return are changing over time. In this case, estimating unconditional covariances can give misleading results. To account for this possibility, we estimated conditional covariances as described above, using four different methods for calculating the expected real rate of return on Treasury bills.

All of the different methods for computing conditional covariances gave virtually identical results, so we report only those in which the expected real return on bills was taken to be the nominal bill rate less last month's inflation rate.

Table 1b reproduces Table 1a, except that all computations are for deviations of actual returns from expected returns. The covariances are computed using (3). It is clear that the conditional covariance estimates are virtually identical to the unconditional covariance estimates; the patterns of correlation coefficients and standard deviations are similar.

Because the conditional and unconditional covariances are similar, the forecasted real risk premia are similar in the two cases. Figures 2a and 2b perform the same kind of simulation as in Table 2. They present a time series of the risk premia (relative to Treasury bills) on duration 4 and duration 8 bonds. For both bonds, the risk premium shows a dramatic increase in 1980, demonstrating that at least some of the increase in nominal long-term yields was due to an increase in the required real rate of return. This risk premium is computed monthly, using the covariance matrix estimated from the previous 24 months of data. Aggregate risk aversion is set at 3.5.

This method of generating real risk premia can be criticized on the grounds that the measured covariance lags behind the true expected covariance; the expected covariance matrix in November, 1979, was probably not equal to the measured covariance matrix for the previous 24 months. Nevertheless, it is notable how quickly the risk premia in Figure 2 rise following October, 1979.

The gaps in the graphs (dotted lines) are the result of missing data. Very long-term government bonds did not exist for some time periods in the early and mid 1970's, so we simply omitted those periods, instead of arbitrarily assigning the market weights from missing bonds to other bonds.[11] When there were gaps, the covariance matrix was computed using the last 24 months for which a full data set was available. The expected real rate on Treasury bills, on the other hand, was always computed using the most recent data.

C. Sensitivity Analysis

The market weights used in the computations are at best rough approximations; consequently, it is important to know the sensitivity to changes in the market weights of the estimated risk premia. Table 3 contains risk premia estimated under two alternative assumptions about weights. All risk premia are computed under the assumption that the coefficient of relative risk aversion is 3.5.

Column 2 of Table 3 contains risk premia estimated using weights constructed under the assumption that financial intermediaries are a veil.

Table 3
Risk Premia Computed Under
Different Assumptions About Market Weights

Asset	Col. 6 From Table 2	Financial Intermediaries A Veil (1980 weights)	Pension Liabilities Spread Evenly, 1976 Weights
Stocks	6.95	7.22	6.66
Bonds of Duration 0	-	-	-
1	.834	1.04	.787
2	1.62	1.99	1.48
3	1.82	2.34	1.66
4	2.36	2.97	2.15
5	2.71	3.43	2.48
6	2.71	3.41	2.46
7	3.82	4.61	3.53
8	4.34	5.19	4.03
$R_0 - R_f$	.027	.053	.023

Note: All Risk Premia computed using 1980-81 covariance matrix, and assuming that δ=3.5.
$R_0 - R_f$ computed using Equation (2).

This results in a large redistribution of the weights from short to long-term assets, while keeping the weight on equity at the same level. While the risk premium on long-term bonds rises appreciably, the overall pattern and levels of risk premia remain much the same. This is also true for the risk premia presented in the third column, which uses the set of weights which most differed from the weights in Column 1, and which assumed intermediaries not to be a veil.

It is clear that the risk premium on equity is not particularly sensitive to the pattern of weights across bonds. An increase in the market weights assigned to long-term bonds, however, can appreciably raise the required rate of return on long-term bonds. For our purpose, which is examining the effect of covariance changes on risk premia, the pattern of risk premia appears sufficiently unaffected by small changes in the weights that it is unnecessary to worry about the exact assumptions used in constructing the weights or about variations in the weights over time.

4. Debt Management and Relative Risk Premia

An important strand in macroeconomic thought for the last 20 years has been the importance of government financing and monetary policy as determinants of relative rates of return on assets, and hence of real activity. This idea, due to Tobin (1964), stresses the myriad possible outcomes from various combinations of debt management policy and monetary management. The model in this paper actually provides a way to quantify the possible effect of alternative government debt policies on relative rates of return. There may be offsetting effects, however, so that the estimates obtained in this way should be thought of as maximum possible effects.

Before examining Table 4, which contains the results of a simple experiment in which the Treasury shifts the composition of debt towards long-term debt, we should examine the extent to which the government can actually affect the market weights. At the most (when financial intermediaries are considered a veil), government debt (excluding agency securities) constitutes about nine percent of household financial net worth. About a quarter of this is short-term government debt. Thus, if the Federal government were to replace all short-term debt with long-term debt, a shift of two percent would occur from short- to long-term bond weights. As we have already seen, a change of this magnitude will not change risk premia significantly. The government has other tools at its disposal, however, since it regulates financial institutions. Thus, by exerting regulatory pressure, it might be possible to reduce the percentage of short-term debt still further.

Table 4 shows the results of a maturity change in which the government increases the percentage of duration zero debt by 5 percentage points, and reduces the percentage in durations 3 through 8.[12] Even for this rather large change in the weights, the risk premium on equity is reduced by only fifteen basis points. The risk premium on the very longest-term debt is reduced somewhat more, by almost fifty basis points.

Some care is required in interpreting the results in Table 4. As Tobin (1963) emphasized, it is the risk premium on equity which matters for capital formation. The correct risk premium, however, is that on unlevered equity, while the risk premium we have actually measured is that on levered equity. To measure the effect of a government maturity change on the risk premium of unlevered equity, one must unlever the number in Table 4. This can be done by computing a weighted average of the risk premia in Table 4, where the weights

Table 4
Effect of "Operation Twist"
on Risk Premia

	Before	After
Stocks	6.95	6.80
Bonds of Duration 0	-	-
1	.834	.720
2	1.62	1.35
3	1.82	1.47
4	2.36	1.96
5	2.71	2.27
6	2.71	2.24
7	3.82	3.30
8	4.34	3.81
$R_0 - R_f$	.027	.038

Note: Uses 1980-81 covariance matrix, and 1980 weights assuming that financial intermediary liabilities are household assets. $\delta = 3.5$
Risk premia relative to duration zero bonds.
$R_0 - R_f$ computed using equation (2).

are the percentage of investment financed by equity and by each kind of debt. This weighted average approximates the unlevered cost of equity capital. Because all of the risk premia in Table 4 fell, it is clear that the required risk premium on unlevered equity would also have fallen.

Friedman (1981) and Roley (1982) have also studied the effects of a federal debt management operation on relative security yields. They both conclude that much smaller Federal debt management operations than those considered here would affect relative yields by roughly the same order of magnitude as in our simulation. An important difference between their models and ours, however, is that we assume that investors immediately adjust their portfolio composition in response to the change in relative yields, while Friedman and Roley both use a model in which adjustment takes time. To some extent, therefore, their results measure transient impact effects (which are greater than the long-run relative price changes) and thus are not directly comparable to our results.

This analysis is valid even though it ignores the difference between interest-bearing and non-interest bearing debt, which is of course at the heart of any theory of monetary policy. Our framework is partial equilibrium, taking the level of the Treasury bill rate as given. Monetary policy should be thought of as shifting the general level of interest rates up or down. The relative pattern of risk premia should not be affected, however, except to the extent that a change in monetary policy changes the pattern of covariances, as it appears to have in 1979. The model in this paper can really say nothing about the effects of monetary policy per se, though it can be used to analyze the effects of debt management.

Having discussed what the government can in principle do, it is worth noting that the government may not be able to have much effect. One argument--due to Barro (1974)--is simply that if households capitalize future tax liabilties, then a change in the risk composition of assets that households must hold (due to a change in the maturity structure of outstanding government debt) would simply be offset by a change in the risk composition of future tax liabilities. There are a number of reasons for finding this argument unconvincing, which are detailed in Tobin (1980).

5. Summary and Conclusions

This paper has applied financial theory to help explain the anomalous interest rate behavior of the last several years. A number of commentators have been surprised that nominal long-term interest rates remained high even when expected future inflation rates were thought to have declined. Our explanation is that even if expected future inflation rates did decline, the high long-term nominal yields can be explained as the result of increased real risk premia. We are able to show that for long-term bonds, the increased variance of the rate of return on long-term bonds and the increased correlation with other assets, should have induced a rise in the real risk premium on long-term bonds between 1973-1979 and 1980-1981.

While our model is able to account for an upward-sloping term structure of real expected rates of return, it is unable to account for the rise in the real rate of return on short-term bonds. We are able to show that the risk premium on bills relative to the shadow risk-free rate should not have increased in the 1980's, since the covariance between bills and the market changed little.

Our model also provides a novel way to analyze the effects of a debt-management operation, such as "Operation Twist." Instead of estimating a supply and demand equations for different assets, we simply use the CAPM, which is an equilibrium pricing relationship, to explore the effects of changes in the relative supplies of different assets.

We used only the one-factor CAPM in this paper; an obvious next step would be the use of a consumption based asset-pricing model, as in Breeden (1979), to compute risk premia. Unfortunately, for a variety of reasons the consumption-based approach is more difficult to implement, so we did not attempt it in this paper.

Appendix: Computing the Market Weights

First we calculated the quantities of government bonds outstanding for each duration. This was done using the Monthly Statement of the Public Debt and then converting maturity into durations. Results of these calculations are in Table A-1.

The following sums were computed to calculate w_i for each asset category:

$$w_0 = C + D + MMF + \eta_0 G + 1/2\ cCB + L + .025MU$$

$$w_1 = \eta_1 G + 1/2\ cCB + 1/8\ M + .025MU + a_1 P$$

$$w_n = \eta_n \left[G + \frac{1 - c}{1 - \eta_0 - \eta_1}\ CB\right] + \frac{1}{8}M + \eta_n\left[\frac{.95MU}{1 - \eta_0 - \eta_1}\right] + a_n P;\ n = 2,\ldots,8$$

where

E = Investment Co. Shares + Other Corporate Equity + Equity in non-Corporate Business

GB = Government Bonds

MU = State and Local Obligations

CB = Corporate Bonds + Foreign Bonds

D = Demand Deposits + Time & Savings Deposits

C = Currency

MMF = Money Market Funds

L = Open Market Paper + Loans + Consumer and Installment Credit + Trade Credit

M = Mortgages (These are assumed to be spread evenly across durations 1-8. Defined to include Federal Agency securities.)

η_i = Percentage of government bonds which are of duration i, i=0, 1,...,8
(Duration 0 is 0-90 days, 1 is 90-540 days, 2 is 1 1/2 - 2 1/2 years, etc.)

c = Percentage of corporate liabilities which are duration one year or less (It is assumed that 1/2 c is duration 0 and 1/2 c is duration 1.) $c_{1976} = .506$; $c_{1980} = .564$

a_i = Percentage of pension fund and life insurance reserves allocated to each duration

P = Pension Fund and Life Insurance Reserves

The market weights were then computed under two alternative assumptions about financial intermediaries: that the liabilities of financial intermediaries are treated as assets by the household sector, and that financial intermediaries are a "veil," so that households behaved as if they directly held the assets of intermediaries. The first is more coherent under the finance paradigm; nevertheless we computed both sets of weights as a sensitivity check. Notice that the biggest difference between the two computations is in the percentage of duration zero bonds held by households. Under the intermediaries-a-veil assumption, there are no financial institutions to perform the function of transforming long-term assets into short-term liabilities. Households hold no demand or time deposits, with the result that they instead directly hold assets such as mortgages. The aggregate debt-equity ratio is insensitive to the assumption that intermediaries are a veil, because financial institutions hold debt and are predominantly debt-financed. Tables A-2 and A-3 contain the results of both computations.

Table A-1
Government Bond Weights

Duration	1976 Corresponding Maturity (Years)	1976 Weight	1980 Corresponding Maturity (Years)	1980 Weight
0	0- .25	.275	0- .25	.219
1	.25- 1.58	.357	.25- 1.60	.342
2	1.58- 2.65	.123	1.60- 2.75	.128
3	2.65- 4.00	.083	2.75- 4.20	.092
4	4.00- 5.45	.037	4.20- 5.75	.032
5	5.45- 7.13	.064	5.75- 7.80	.049
6	7.13- 8.80	.006	7.80-10.90	.031
7	8.80-10.60	.017	10.90-13.70	.021
8	10.60+	.036	13.70+	.087

Note: Flower bonds were omitted from the sample. The maturity date was taken to be first call date if the bond sold at a premium, and maturity otherwise.

Source: Weights by maturity from Monthly Statement of the Public Debt, May 1976, May 1980. Conversion to duration by using data on CRSP Government Bond Files.

Table A-2
Market Weights for the
Household Sector

Assets	Pension Fund Reserves Spread Evenly over Durations 1-8		Pension Fund Reserves Weighted Toward High Durations	
	1976	1980	1976	1980
Stocks	.620	.640	.620	.640
Bonds = 0	.264	.262	.264	.262
1	.031	.026	.015	.007
2	.022	.017	.008	.003
3	.016	.013	.008	.005
4	.010	.007	.007	.005
5	.014	.009	.016	.012
6	.006	.007	.014	.016
7	.008	.006	.021	.020
8	.010	.013	.028	.032

Table A-3
Market Weights for the Household Sector,
Treating Financial Intermediaries as a Veil

Assets	1976	1980
Stocks	.632	.646
Bonds = 0	.108	.105
1	.062	.061
2	.054	.045
3	.039	.035
4	.023	.018
5	.032	.023
6	.011	.018
7	.015	.015
8	.022	.034

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Footnotes

1. Ibbotson and Sinquefield (1982) show that the average annual real rate of return on Treasury bills was close to zero, and that on long-term government bonds was about one percent, over the period 1926 to 1976.

2. See Martin Feldstein, "Why Short-Term Interest Rates Are High," Wall Street Journal, June 8, 1982.

3. See for example Robert E. Lucas, Jr., "Deficit Finance and Inflation," The New York Times, August 26, 1981.

4. Roley (1979) and Walsh (1983) also study the theoretical effects of debt management operations in a mean variance model. Walsh also shows how changes in the stock of assets outstanding can affect the covariance of security price changes.

5. The assumption that there are no taxes does not appear to be important for our qualitative results but can affect the magnitude of the risk premium, depending on the assets being compared. There are two problems, one relating to means, the other to covariances. To convert the risk premia to the difference in nominal before-tax expected yields, it is necessary only to divide the risk premium by one minus the tax rate if both assets are taxed at the same rate. In this case, the reported risk premia understate the before-tax nominal risk premia. If the two assets are taxed at different rates (as with stocks and bonds) it is necessary to know rate of return levels and not just risk premia in order to infer the difference in before-tax nominal yields.

   The covariance correction is more involved. For 30-day bills, the conditional real rate of return variance is unaffected by taxes since the nominal after-tax rate of return is certain, and the real rate of return is obtained by subtracting the stochastic inflation rate. For stocks and long-term bonds, there is uncertainty about the nominal yield, and hence about the part of the return which will be taxed. It is then necessary to adjust the covariance for the reduction in the real return variance due to taxes. (The deviation of actual from expected nominal returns will typically be taxed at the capital gains rate, at least to a first approximation.) The net effect of making all the adjustments is ambiguous.

6. It is shown in Bodie, Kane, and McDonald (1983) that the equilibrium expected rate of return on any asset may be written

$$R_i = R_{min} + \delta(\sigma_{iM} - \sigma^2_{min})$$

where δ is a measure of aggregate risk aversion, σ_{iM} is the covariance of the real rate of return on asset i with that of the market, and r_{min} and σ^2_{min} are the expected rate of return and the variance of the rate of return on the minimum variance portfolio. This relationship implies (1).

7. Friend and Blume (1975) estimate δ to be 2, while Friend and Hasbrouck obtain an estimate of 6. Grossman and Shiller (1981), using a different method, obtain an estimate of 4.

8. What we call the "shadow risk-free rate" is also the rate of return on the zero-beta portfolio in the Black (1972) version of the CAPM. Using arguments along the lines of Cox, Ingersoll, and Ross (1978), it is straightforward to show that in a model with one state variable and no risk-free asset, the zero beta portfolio has a rate of return equal to the rate of time preference, less the expected percentage change in the marginal utility of consumption.

It is also shown in Cox, Ingersoll, and Ross (1978), and in Lintner (1969) that the shadow risk-free rate equals the Lagrange multiplier associated with the wealth constraint in the portfolio maximization problem.

A possible reason for the non-existence of indexed bonds is discussed in Bodie, Kane and McDonald (1983), where it is shown that the welfare gain from introducing such an asset would be small.

9. Duration, as defined by Macaulay (1938) is a weighted average of the years to maturity of each of the cash flows from a security. The weights are the present value of each year's cash flow as a proportion of the total present value of the security. Duration equals final maturity only in the case of pure discount bonds. For coupon bonds and mortgages, duration is always less than maturity. The difference between maturity and duration for ordinary coupon bonds and mortgages is greater the longer the final maturity and the higher the level of interest rates. In our sample of bonds this difference rose steadily over the 1953-1981 period due to the rising trend in interest rates. The most pronounced differences were in the 8 year duration category. In 1953 the average maturity of the bonds in our 8 year duration portfolio was just under 9 years whereas in 1981 the average maturity of the 8 year duration portfolio was 23 years. This variation over the last 30 years calls into question the appropriateness of a bond return series with a constant maturity of 20 years, such as the one tabulated by Ibbotson and Sinquefield (1982).

10. Currency and bank deposits are taken to have a non-measurable "service yield" over and above their explicit, nominally-fixed rate of return.

11. Other long-term nominally fixed securities such as corporate bonds and mortgages did exist over these periods, so simply setting the long-term market weights to zero would have been inappropriate.

Ibbotson and Sinquefield use flower bonds in constructing their rate of return series for long-term government bonds. We exclude them, however, on the grounds that their true maturity is likely to be much shorter than the stated maturity.

12. The weights on duration 3 and 4 bonds are set to zero, and those for durations 5 through 8 are reduced by one percentage point.