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A RELATIONSHIP BETWEEN REGRESSION AND VOLATILITY TESTS OF MARKET EFFICIENCY

Jeffrey A. Frankel

James H. Stock

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Cambridge MA 02138

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Abstract

Volatility tests are an alternative to regression tests for evaluating the joint null hypothesis of market efficiency and risk neutrality. A comparison of the power of the two kinds of tests depends on what the alternative hypothesis is taken to be. By considering tests based on conditional volatility bounds, we show that if the alternative is that one could "beat the market" using a linear combination of known variables, then the regression tests are at least as powerful as the conditional volatility tests. If the application is to spot and forward markets, then the most powerful conditional volatility test turns out to be equivalent to the analogous regression test in terms of asymptotic power. In other applications, the volatility test will be less powerful than regression tests against our chosen alternative. However, these results are not inconsistent with the observation that volatility tests may be more powerful against other alternative hypotheses, such as that risk-averse investors are rationally maximizing the present discounted utility of future consumption, with a time-varying discount rate.

Jeffrey A. Frankel
Department of Economics
University of California
Berkeley, CA 94720

(415) 642-8084

James Stock
Department of Economics
University of California
Berkeley, CA 94720

(415) 642-1955

I. Introduction

There are two ways to go about testing the joint hypothesis of efficiency and risk neutrality in a particular financial market. First, regression tests compute conditional first moments; they look for bias conditional on a given information set. For example, in a forward or futures market (e.g. commodities or foreign exchange), the deviation of the next period's realized spot rate from the current one-period forward rate should be uncorrelated with variables known currently. An analogous condition holds in a longer-term asset market (e.g. stocks or bonds): the deviation of the present discounted value, assuming it is observable, of realized future returns (dividends or coupon payments), from the current asset price should again be uncorrelated with variables known currently. Second, the volatility tests introduced by Shiller (1979) and LeRoy and Porter (1981) compute second moments; they compare variances.[1] In a forward market this would mean comparing the variance of the spot rate and the variance of the forward rate. The joint null hypothesis of market efficiency and risk neutrality implies that the forward rate is less volatile than the spot rate. In a longer term asset market it would mean comparing the variance of the return with the variance of the asset price. The hypothesis implies that the asset price is less volatile than the return, in a specific sense.

A natural question to ask is which kind of tests, the regression tests or the volatility tests, is more powerful, is better able to reject the hypothesis in the event that it is false. As is often the case with questions of power, the answer depends on what the alternative hypothesis is. In this paper we take the alternative to be a particular failure of rational expectations or market efficiency. The alternative hypothesis is that one could "beat the market" on average, using a linear combination of data in a particular information set. We show that in all cases the regression tests are at least as powerful against this alternative as the volatility tests.

In the case of spot and forward rates a comparison of simple unconditional variances tells us very little. Empirically, the unconditional sample variance of the spot rate differs negligibly from the unconditional sample variance of the forward rate. We argue in Section II that such considerations suggest comparing the variances conditional on some particular information set, which is analogous to what we do in regression tests. The most powerful test will compute a variance conditional on an optimal linear combination of known variables. It is perhaps intuitive and unsurprising that the optimal linear combination turns out to be the same as the estimates one would get from a regression on the same set of known variables. It is perhaps more surprising that this most powerful volatility test also turns out to be equivalent to the analogous regression test in terms of asymptotic power. That is, as the number of observations becomes large, the volatility test is no more and no less likely to reject the variance inequality than the coefficients in the regression test are to differ significantly from zero. We prove this central result of the paper in Section III.

In the case of longer-term assets the power equivalence result does not hold. In Section IV we apply the principle that generated the class of volatility tests for spot and forward rates to the analogous variance relationship between the one-period interest rate and the one-period holding yield on a long-term coupon bond. In this case, the variance test that is most powerful against the alternative that the deviation can be predicted by a linear function of the information set, will not be as powerful as the corresponding regression test.

The difference in results between the spot/forward case and the longer-term asset case has a simple explanation. Upper bounds on the volatility of long-term asset prices and returns take into account restrictions imposed by the present discounted value formula. When the alternative hypothesis (that one could use the information set to "beat the market") looks only one period ahead, it is not surprising that a regression test designed to detect this alternative does better than a volatility test that is burdened by the extra baggage of the present value relation. However, the present value relation for the forward rate involves only one future date. Thus the extra baggage of the long-term discounted summation is absent in the case of spot and forward rates. As a result, it makes sense that regression tests do better, relative to the corresponding volatility tests, when applied to longer-term asset markets than when applied to spot and forward markets.[2]

Do these results constitute an indictment of volatility tests as being inferior to regression tests? Absolutely not. We stated at the outset that we take the alternative to the null hypothesis to be a particular failure of rational expectations, that one could beat the market given a particular information set. All our power results refer to this alternative. But there are other possible alternative hypotheses. One could question instead the risk-neutrality component of the joint null hypothesis. In particular, Grossman and Shiller (1981) and LeRoy and LaCivita (1981) consider the alternative hypothesis that risk-averse investors are rationally maximizing the present discounted value of future consumption, subject to a time-varying discount rate. What is extra baggage when seeking to diagnose the pathology of market inefficiency, is just the kit the doctor ordered when seeking to diagnose risk-aversion.[3]

Thus our results are consistent with Shiller's (1981) view of the relationship between volatility tests and regression tests: "My initial motivation for considering volatility measures in the efficient markets models was to clarify the basic smoothing properties of the models....[T]he procedures ought not to be regarded as just 'another test' of market efficiency" (p. 291). The volatility tests are optimally designed to test the degree to which asset markets are able to smooth out the path of consumption. When the null hypothesis has no intrinsic smoothing property (i.e. unbiasedness in the forward rate), volatility tests will detect nothing more than will regression tests (our Section III result). When the null hypothesis does embody the smoothing property (i.e. asset prices as the present discounted value of future returns), but the alternative hypothesis bears no relationship to smoothing, then volatility tests are able to detect less than regression tests (our Section IV result).

II. Volatility Bounds for Spot and Forward Rates

The rational expectations/efficient markets hypothesis is commonly taken to imply that

$$S_{t+1} = F_t + \varepsilon_{t+1}\ , \quad E_t\varepsilon_{t+1} = 0 \tag{1}$$

where $E_t(\cdot) \equiv E(\cdot|I_t)$ is the expectation conditional on the information set I_t. This implies a simple variance inequality:

$$\text{var } S_{t+1} = \text{var } F_t + \text{var } \varepsilon_{t+1} + \text{cov } F_t \varepsilon_{t+1}$$

(2)

$$\geq \text{var } F_t \text{ , since cov } F_t \varepsilon_{t+1} = 0 \text{ under the hypothesis.}$$

One might be tempted to test this bound. However, a casual glance at the sample variances for selected exchange rates (Table 1) indicates that the sample variances corresponding to (2) are almost equal; although no formal test is performed, it seems very unlikely that the inequality (2) would be rejected.[4] This finding will not be surprising to anyone who has ever seen a plot of the spot and forward rate over time. The two fluctuate enormously, but in tandem. There may be a finite component of the one-period change in the spot rate that is correctly foreseen by the forward rate; but if so it is dwarfed by the magnitude of the total change in the spot rate, and the very similar magnitude of the change in the forward rate.

This observation suggests pursuing the course discussed in the introduction, that is, developing a more powerful volatility test of market efficiency. A reasonable class of tests to consider, which generalizes that based on (2), looks at deviations around a mean conditional on an available information set. This is analogous to regression tests, in which we compute means conditional on particular information sets; the larger the information set, the more powerful the test.[5] Specifically, if Z_t is in I_t , then

$$\text{var}(S_{t+1}-Z_t) = \text{var}(F_t-Z_t+\varepsilon_{t+1})$$

$$= \text{var}(F_t-Z_t) + \text{var}(\varepsilon_{t+1}) + 2\text{cov}(F_t-Z_t,\varepsilon_{t+1}) \ .$$

Under the null hypothesis (1), $\text{cov}(Z_t,\varepsilon_{t+1}) = 0$. This results in the bound:

Table 1: variances around the sample mean
June 1973-April 1982

Currency	Spot Rate	Forward Rate
Canadian dollar	.00566866	.00561098
French franc	.00046331	.00047980
German mark	.00402365	.00431700
Japanese yen	.00000036	.00000038
Pound sterling	.0637202	.0632473

(2')
$$\mathrm{var}(F_t-Z_t) \le \mathrm{var}(S_{t+1}-Z_t) \ .$$

For the tests considered in the paper, we take this notion of examining deviations about a nonconstant variable Z_t one step further. From the familiar decomposition that mean square error is variance plus the square of the bias, a reasonable generalization of (2') is to consider a mean square error bound; that is, to consider a bound based on moments that in general could be noncentral, rather than the simple central moments examined so far. We now consider noncentral moments. Since

$$S_{t+1}-Z_t = F_t-Z_t+\varepsilon_{t+1} \quad \text{and} \quad E_tF_t\varepsilon_{t+1} = E_tZ_t\varepsilon_{t+1} = 0 \ ,$$

we have $E_t(S_{t+1}-Z_t)^2 = E_t(F_t-Z_t+\varepsilon_{t+1})^2 = E_t(F_t-Z_t)^2 + E_t\varepsilon_{t+1}^2$.
Thus, under the null hypothesis,

(3)
$$E(F_t-Z_t)^2 \le E(S_{t+1}-Z_t)^2 \ .$$

This inequality provides a basis for developing more exacting volatility tests of (1), since it explicitly employs the assumption that Z_t is in I_t . Furthermore, the inequality (2) is a special case of (3) in which $Z_t = E(S_{t+1}) = E(F_t)$ is constant.

It is interesting to note that (3) can also be arrived at by an altogether different line of reasoning than the motivation of increasing the power of the test. An important cause for concern related to any statistical implementation of the bound (2) falls under the general rubric of nonstationarity. Nonstationarity comes in many flavors; two of the most popular among econometricians are the existence of a time-dependent mean and the nonstationarity associated with a process having unit roots, so that the variance of the process is infinite. These two variants of nonstationarity seem particularly applicable to the foreign exchange data at hand. In the first case, the strong trends exhibited by exchange rates over the 1970s could be modeled as deterministic, although they may logically stem from nondeterministic factors such as inflation. In the second case Meese and Rogoff (1983) demonstrate that spot exchange rates cannot be modeled better than by a random walk. Even if the spot rate process in reality has finite variances, this suggests difficulty in estimating variances of the process in any finite sample. Both of these concerns suggest deriving bounds with conditional means and computing sample moments around means that vary over the sample period; in other words, the bound (3) can be seen as a simple way to defend against the perils of nonstationarity.[6]

As an example of a volatility bound implied by (3) which also seems to be a reasonable correction for this possible nonstationarity, let Z_t be the lagged spot rate. Thus, assuming lagged spot rates are in the information set, (1) implies that

$$E(F_t - S_{t-1})^2 \leq E(S_{t+1}-S_{t-1})^2 . \tag{4}$$

The sample variances associated with this bound are presented in Table 2. For this data, the bound is satisfied in all cases considered, so no formal test of significance is necessary to see that market efficiency as embodied in (4) cannot be rejected.

Can we devise a still more difficult volatility test of market efficiency than (4)? Indeed we can. If we define the test statistic

Table 2: Variances around the lagged spot rate

Currency	Spot Rate Mean $(S_{t+1} - S_{t-1})^2$	Forward Rate Mean $(F_t - S_{t-1})^2$
Canadian dollar	.00028153	.00015118
French franc	.00008562	.00004632
German mark	.00046910	.00039569
Japanese yen	4.40×10^{-8}	2.20×10^{-8}
Pound sterling	.00811008	.00376687

$$R(Z) \equiv 1 - E(S_{t+1}-Z_t)^2/E(F_t-Z_t)^2 ,$$

then (3) can be rewritten as

(3') $$R(Z) \leq 0 .$$

A value of the test statistic significantly above zero would constitute a rejection of the null hypothesis: forward rates would be too volatile relative to spot rates. Given the nature of the null hypothesis, a reasonable choice for Z_t (which plays the role of the conditional mean of S_{t+1}) is that $Z_t = F_t + \beta X_t$, where X_t is a mean-zero, nonconstant, univariate series assumed to belong to I_t. Since the bound (3') holds for all scalar β, we should select the value of β for which a test based on (3') is as likely as possible to reject the null hypothesis. Letting

$$\hat{R}(Z) \equiv 1 - \Sigma(S_{t+1}-Z_t)^2/\Sigma(F_t-Z_t)^2 ,$$

this suggests testing (3') using the statistic based on the solution to

(5) $$\max_{\beta} \hat{R}(F+\beta X) .$$

Letting β^* be the value of β which solves (4), a somewhat surprising result obtains:

(6) $$\hat{R}(Z) = \hat{\rho}^2_{X\varepsilon} ,$$

where $\hat{\rho}_{X\varepsilon} = \Sigma X_t \varepsilon_{t+1}/(\Sigma X_t^2 \Sigma \varepsilon_{t+1}^2)^{1/2}$ is the sample correlation coefficient and where $\varepsilon_{t+1} = S_{t+1} - F_t$.

The proof of (6) is easy: since $\hat{R}(F+\beta X) = 1 - \Sigma(\varepsilon_{t+1}-\beta X_t)^2/\beta^2 \Sigma X_t^2$, to solve (5) it is merely necessary to solve:

$$\min_{\beta} \Sigma(\beta^{-1}\varepsilon_{t+1}-X_t)^2$$

which has the solution $\beta^{*-1} = \Sigma\varepsilon_{t+1}X_t/\Sigma\varepsilon_{t+1}^2$. Substituting this statistic into the definition of $\hat{R}(F+\beta X)$ yields the result. It thus appears that the most discerning volatility test based on a statistic of the form $\hat{R}(F+\beta X)$ is equivalent to the correlation coefficient, which arises from considering regression tests! Of course, this argument is not based on formal power considerations. However, as is shown in the next section, among this class of volatility tests the "most discerning" test is in fact asymptotically most powerful against the (local) alternative that X_t and ε_{t+1} are correlated. Intuitively, the question whether the correlation coefficient is significantly greater than zero is the same as the question whether the regression coefficients are significantly greater than zero.[7]

III. Formal Statement of the Result

In this section we examine the power of the volatility tests of the previous section against the alternative that ε_{t+1} and X_t are correlated. The proof uses asymptotic statistical arguments. Specifically, it compares asymptotic approximations to the power functions of test statistics based on $\hat{R}(F+\tilde{\beta}X)$, where $\tilde{\beta}$ is permitted to be any function of data as long as $\tilde{\beta}^{-1}$, when standardized, has a limiting distribution with all its mass on the real line. Since the power of a test based on the statistic (6) will go to one when the covariance between X_t and ε_{t+1} is bounded away from zero, we adopt the conventional asymptotic approach of considering a local alternative under which this covariance tends towards zero as the sample size tends towards infinity.

The proof itself has two parts. First, the class of random variables $\tilde{\beta}^{-1}$ that need to be considered is narrowed down to those which tend to zero in probability under the local alternative. Second, it is possible to appeal to the results of the previous section to show that, of the variables with this property, the solution to the maximization problem (4) does indeed yield the asymptotically most powerful test.

For the statement of the result, it is convenient to reparameterize the problem. Let the local alternative be $\sigma_{X\varepsilon}^{(T)} = T^{1/2}\delta$, where T is the number of observations and δ is some nonzero, finite fixed number. Let $\phi = \beta^{-1}$. Let Φ be the set of all random variables $\tilde{\phi}$ which are functions of the data (possibly degenerate--that is, possibly a constant) and are such that $T^{1/2}(\tilde{\phi}-\phi)$ has a limiting distribution on the real line. Also, let ϕ^* be that element of Φ such that the one-sided test of the restriction (3) has the greatest local asymptotic power of all tests of level α based on $\hat{R}(F+\tilde{\phi}^{-1}X)$. Let $Y = (\varepsilon'\varepsilon/T \;\; \varepsilon'X/T \;\; X'X/T)'$ and let $\mu = (\sigma_\varepsilon^2 \;\; \sigma_{X\varepsilon} \;\; \sigma_X^2)'$, where ε' denotes the transpose of the column vector formed from the observations of ε_1 , ε_2 , ..., ε_{T-1} . Also, assume that $T^{1/2}(Y-\mu)$ has a limiting normal distribution with positive definite covariance matrix Σ . We now have:

Proposition.

The level α test based on $\hat{R}(F+\phi^{*-1}X)$ is asymptotically equivalent to the level α test based on the t-statistic of the slope coefficient in the OLS regression of ε_{t+1} on X_t . Furthermore, $\phi^* = X'\varepsilon/\varepsilon'\varepsilon$.

Proof

First we use the "delta method" to find the limiting distribution of the standardized random variable based on $\hat{R}(F+\tilde{\phi}^{-1}X)$. Let $\phi = \text{plim}\tilde{\phi}$ and let $a = \partial\hat{R}/\partial Y\big|_{Y=\mu,\ \tilde{\phi}=\phi}$. Also, let $r(\phi) = \text{plim}(\hat{R}(F+\tilde{\phi}^{-1}X))$, which will exist by the assumption that Y when standardized will have a limiting distribution and because $\hat{R}(\cdot)$ is continuous in Z . Then

$$T^{1/2}(\hat{R}-r) \xrightarrow{d} N(0,\tau(\phi)^2) \tag{7}$$

where $\tau(\phi)^2 = a'\Sigma a$ and $\phi = \text{plim}\tilde{\phi}$. Since a is continuous in ϕ , $\tau^2(\phi)$ is continuous in ϕ .

Since the null hypothesis is that $R < 0$, we wish to find the statistic of the form (7) that has the greatest chance of $\hat{R}$ exceeding zero under the local alternative. One approach to this problem is to compute $\tau(\phi)^2$ directly for many statistics $\tilde{\phi}$, and to compare the limiting behavior under the local alternative. However, this would be difficult, since the candidates $\tilde{\phi}$ must be specified in advance.

This problem can be sidestepped by noting that a necessary condition for a test of the form (7) to have nonnegligible power is that $r \geq 0$; otherwise $P(\hat{R} > 0) \to 0$ as $T \to \infty$ by definition of convergence in probability. Thus we can restrict our attention to those $\tilde{\phi}$ which result in $T^{1/2}\hat{R}$ having a limit which is bounded in probability away from $-\infty$.

It is easy to see that in fact $T^{1/2}\hat{R}$ must be bounded in probability (be $O_p(1)$) . By definition,

$$T^{1/2}\hat{R} = T^{1/2}(1 - (\tilde{\phi}^2\varepsilon'\varepsilon-2\tilde{\phi}\varepsilon'X+X'X)/X'X) \tag{8}$$

$$= 2\tilde{\phi}\frac{\varepsilon'X/T^{1/2}}{X'X/T} - (T^{1/2}\tilde{\phi})\tilde{\phi}\frac{\varepsilon'\varepsilon/T}{X'X/T} .$$

By assumption, $\tilde{\phi} \overset{p}{\to} \phi$, $X'X/T \overset{p}{\to} \sigma_X^2$, and $\varepsilon'\varepsilon/T \overset{p}{\to} \sigma_\varepsilon^2$. Also, under the local alternative, $\varepsilon'X/T^{1/2}$ has a limiting law on the line. Thus, by Slutsky's Theorem, $T^{1/2}\hat{R}$ is bounded above in probability for all $\tilde{\phi}$, so $r \le 0$. Thus we can restrict attention to $\tilde{\phi}$ such that $r = 0$, i.e. such that $T^{1/2}\hat{R} = 0_p(1)$. But, by (8), this will occur only if $T^{1/2}\tilde{\phi} = 0_p(1)$ which in turn implies that $\phi = 0$.

The result follows from this requirement, since it implies that, for all $\tilde{\phi}$ yielding nonnegligible power against the local alternative, $T^{1/2}\hat{R} \overset{d}{\to} N(0,\tau(0)^2)$ under the null hypothesis. Furthermore, since $\tau(\phi)$ is continuous, the variance of the limiting distribution of $T^{1/2}\hat{R}$ under the local alternative will be $\tau(0)^2$ for all contenders $\tilde{\phi}$. Thus the problem reduces to finding the function $\tilde{\phi}$ such that $\hat{R}$ is maximal for all $\tilde{\phi}$ satisfying $T^{1/2}\tilde{\phi} = 0_p(1)$. Since $\tilde{\phi} = \varepsilon'X/\varepsilon'\varepsilon$ was shown to solve this problem among <u>all</u> functions of the data, and since under the local alternative $T^{1/2}\varepsilon'X/\varepsilon'\varepsilon = 0_p(1)$, we have $\phi^* = \varepsilon'X/\varepsilon'\varepsilon$.

The asymptotic equivalence to the regression test follows from noting that, under the null hypothesis, the t-statistic for the slope coefficient of the OLS regression satisfies $T^{-1}t^2 = (\varepsilon'X)^2/(X'X)(u'u)$, where $u = \varepsilon - \hat{\gamma}X$, with $\hat{\gamma} = \varepsilon'X/X'X$. However, under the local alternative, $(\varepsilon'\varepsilon - u'u)/T$ converges to zero in probability. Thus $T^{-1}t^2$ is asymptotically equivalent to $\hat{\rho}_{X\varepsilon}^2 = \hat{R}(F+\phi^{*-1}X)$.

IV. Conditional Variance Bound Tests for Long Term Bonds

Will the "model specific" conditional variance bound tests of Section II be equivalent to regression tests when the tests are applied to the present discounted valuation of longer term assets rather than to forward/spot relationships? The results of this section suggest that they will not.[8]

This is demonstrated by considering an upper bound on the volatility of one-period holding yields, defined as capital gains plus interest payments, on an infinitely-lived coupon bond. This bound, originally derived by Shiller (1979), rests on a linearized expression for the long-term interest rate as a weighted average of expected future short-term interest rates:

$$R_t^{(n)} = (1-\gamma^n)^{-1}(1-\gamma)\Sigma_{k=0}^{n-1} \gamma^k E_t r_{t+k} \tag{9}$$

where $R_t^{(n)}$ is the n-period interest rate, r_t is the one-period interest rate, and γ is a constant with $0 < \gamma < 1$.[9] Shiller presents a formula linearly approximating the one-period holding yield on this bond $(H_t^{(n)})$:

$$H_t^{(n)} = (R_t^{(n)} - \gamma_n R_{t+1}^{(n-1)})/(1-\gamma_n) \tag{10}$$

where $\gamma_n = (1-\gamma^{n-1})/(1-\gamma^n)$. For an infinitely-lived bond, using (9) and (10) and letting $R_t \equiv R_t^{(\infty)}$ and $H_t \equiv H_t^{(\infty)}$, one obtains

$$H_t = r_t - [E_{t+1}(\Sigma_{k=1}^{\infty} \gamma^k r_{t+k}) - E_t(\Sigma_{k=1}^{\infty} \gamma^k r_{t+k})] \quad . \tag{11}$$

This relates the holding period yield to the current short rate and revisions in expectations of future short rates. Intuitively, if expectations of future rates are revised upwards, there is a capital loss on bonds this period that reduces the current ex post holding yield below the current interest rate.

The (approximate) market efficiency requirement (11) readily yields a bound on the volatility of H_t . Letting $\delta_t \equiv E_{t+1} - E_t$,

$$\begin{aligned} var(H_t) &= var(r_t) + var(\Sigma_1^\infty \gamma^k \delta_t r_{t+k}) \\ &= var(r_t) + \Sigma\gamma^{2k} var(\delta_t r_{t+k}) \\ &\le var(r_t) + \Sigma\gamma^{2k} var(r_{t+k}) \\ &= (1-\gamma^2)^{-1} var(r_t) \end{aligned}$$

where the assumption of stationarity was used in the last line. However, (11) can also be used to derive a conditional bound. Assuming Z_t to be in the information set,

(12) $$\begin{aligned} E(H_t - Z_t)^2 &= E((r_t - Z_t) - \Sigma_1^\infty \gamma^k \delta_t r_{t+k})^2 \\ &= E(r_t - Z_t)^2 + \Sigma\gamma^{2k} var(\delta_t r_{t+k}) \\ &\le E(r_t - Z_t)^2 + \gamma^2 (1-\gamma^2)^{-1} var(r_t) \end{aligned}$$

which provides a bound on the volatility of H_t around time-varying variable Z_t.

Clearly the bound (12) is not as tight as its analog pertaining to spot and forward rates (3) because of the last term. Thus a simple model-specific volatility test based on (12) with, say, $Z_t = r_t + \beta X_t$, where X_t is presumed to be in the information set, cannot be expected to perform as well as a regression test of the restriction $cov(H_t - r_t, X_t) = 0$ when the alternative hypothesis is that this covariance is nonzero.

V. Conclusion

In this note we examined second moment bounds of two types: those based on the fact that the variance of a conditional expectation (the forward rate) is no more than the unconditional variance of the random variable (the spot rate), and those derived from a net present discounted valuation relation which place an upper bound on holding yields for a long-term asset. We find that volatility tests of the first type can be expected to do no better than conventional regression tests of market efficiency. At best, when the volatility test is modified to be conditional on available information, it does as well as regression tests with the same set of information. However, this does not appear to be the case for tests of the second type of bound, since these bounds incorporate structure from the present discounted valuation relation which is untested by simple regression tests. In this case the volatility tests are less powerful than regression tests against the alternative that once can beat the market using a linear combination of variables in the information set. However, the volatility tests may be more powerful against other alternatives, such as the hypothesis of Grossman and Shiller (1981) that risk-averse investors are rationally maximizing the present discounted utility of future consumption, with a time-varying discount rate.

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FOOTNOTES

1. Other papers on volatility tests include Flavin (1982), Grossman and Shiller (1981), LeRoy and LaCivita (1981), Michener (1982), Shiller (1981a, b) and Singleton (1980).
2. Geweke (1980) also examines the behavior of volatility tests against an alternative of this type. He demonstrates that there are regions of the parameter space in which regression tests will reject but volatility tests will not. Our results differ from his in two ways. First, we consider an expanded class of volatility bounds (3). Second, we demonstrate that there is a conditional volatility test with the same asymptotic power as the corresponding regression test against this particular alternative. In fact, his conclusion that regression tests dominate unconditional volatility tests is implied by the Proposition in Section III.
3. This alternative is also discussed by Michener (1982). Stock (1982) shows that volatility tests can indeed be more powerful than regression tests when the alternative hypothesis is that risk-averse investors are rationally maximizing the present discounted utility of future consumption. As a result, the discount rate in the asset valuation relation varies over time with the marginal utility of consumption. Yet another alternative hypothesis raised by Shiller (1981), and further examined by Summers (1982), is that asset prices are subject to slowly changing fads.
4. Our forward rates are 30-day forward. Both spot and forward rates are bid rates, 10 a.m., last day of the month, in dollars per national currency, obtained originally from D.R.I. Flood (1981, p. 220) comments on the "striking fact" that spot and forward exchange rates "have about

the same degree of volatility." However, his computations use a measure of the variance somewhat different from ours.

5. In the terminology of Fama (1970), the larger the information set, the "stronger form" is the test. For one of many such regression studies of the forward exchange market, and for references to others, see Frankel (1980).

6. Meese and Singleton (1980) point out the perils of performing naive comparisons of unconditional sample variances of exchange rates when the theoretical variances may be infinite.

7. The results of this paper hold for the case that X_t is one-dimensional. If instead X_t is k-dimensional and β is a k-vector, then a result analogous to that of this section holds: letting β^* be the vector which maximizes $\hat{R}(F+X\beta)$, it can be shown that $\hat{R}(F+X\beta^*) = R^2$, where R^2 is the ratio of the explained to the total sum of squares from the ordinary least squares regression of ε_{t+1} on X_t. Thus our results generalize in a straightforward way to the multi-dimensional case. However, for simplicity we limit the discussion in the paper to the one-dimensional case.

8. The crucial difference is not that we are now thinking of the bond market whereas in the previous sections we were thinking of spot and forward markets in foreign exchange or commodities. The results of the previous sections <u>would</u> be applicable to bond markets. To see this, simply think of the spot price of one-period Treasury bills as a function of the one-period interest rate

$$S_t = \frac{1}{1+r_t} \approx 1 - r_t ,$$

and the one-period forward rate as a function also of the two-period interest rate

$$F_t = \frac{1+r_t}{1+R_t^{(2)}} \approx 1 + r_t - R_t^{(2)}$$

$$S_{t+1} = F_t + \varepsilon_{t+1} \quad .$$

If we have direct data on spot and futures prices of Treasury bills, or if we compute them from one-period and two-period interest rates, we can apply our previous results.

9. We have set to zero the maturity premium explicitly included by Shiller.