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MIMICKING PORTFOLIOS WITH CONDITIONING INFORMATION

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ABSTRACT

Mimicking portfolios have long been useful in asset pricing research. In most empirical applications, the portfolio weights are assumed to be fixed over time, while in theory they may be functions of the economic state. This paper derives and characterizes mimicking portfolios in the presence of predetermined state variables, or conditioning information. The results generalize and integrate multifactor minimum variance efficiency (Fama, 1996) with conditional and unconditional mean variance efficiency (Hansen and Richard (1987), Ferson and Siegel, 2001). Empirical examples illustrate the potential importance of time-varying mimicking portfolio weights and highlight challenges in their application.

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I. Introduction

Mimicking portfolios for economic factors are important in both asset pricing theory and empirical work. Breeden (1979) showed how portfolios can replace the state variables in Merton's (1973) intertemporal asset pricing model. These portfolios "hedge" the state variable risks, and their returns describe the risk premiums. Mimicking portfolios are needed to identify the factor risk premiums when the risk factors in the model are not traded assets. Huberman, Kandel and Stambaugh (1987) provide characterizations of mimicking portfolios that can replace the risk factors in empirical multiple-beta asset pricing models.

Many studies rely on mimicking portfolios. Breeden, Gibbons and Litzenberger (1989) use them for aggregate consumption growth; Chen, Roll and Ross (1986) for a number of macroeconomic factors. Ferson and Harvey (1991) use mimicking portfolios to assess the amount of asset-return predictability captured by asset pricing models. Eckbo, Masulis and Norli (2000) use them to measure the risk exposures of post-seasoned-equity-offering stock returns. Lamont (2001) forms “tracking” portfolios to study the relation of stock returns to a number of economic variables. Ferson and Harvey (1993) employ mimicking portfolios to make inferences about the premiums attached to risk factors in international equity markets. Fama and French (1993, 1996) propose a three-factor model, which Fama (1998) interprets as a specification of mimicking portfolios for the state variables in a Merton (1973) style multibeta model.

Studies that employ mimicking portfolios typically use methodologies in which the weights that define the portfolios are either fixed over time, or vary over time in ad hoc ways (e.g., a rolling regression is used). However, such an approach is not optimal when predetermined variables are available that are related to expected future returns and second moments. This paper studies mimicking portfolios with weights that vary optimally over time in the presence of such conditioning information.

Mimicking portfolios with conditioning information may be characterized with an extension of "unconditional" mean variance efficiency, as described by Hansen and Richard (1987) and Ferson and Siegel (2001). The portfolios they describe use conditioning information to achieve efficiency with respect to the unconditional mean and variance. They are a subset of the "conditionally efficient "portfolios that optimize with respect to the conditional moments. We show how these ideas extend to the "multifactor minimum variance" portfolios described by Fama (1996).

Using a sample of asset returns, economic factors and conditioning information similar to Lamont (2001), we explore the empirical advantage to constructing mimicking portfolios with the optimal, time-varying weights. An example with a single risky asset and a riskless asset shows that we can more than double the correlation with an inflation factor. In Lamont's sample of industry portfolios and bond returns the potential improvement in the correlations is more than 20%, abstracting from estimation error. Our analysis also illustrates how estimation error and errors in specifying the form of the data generating process present challenges for future research and applications.

The remainder of this paper is organized as follows. Section II provides an overview of the central ideas. Section III presents and interprets the main analytical results. Section IV contains the empirical analysis. Section V offers some additional results and discusses the relation of mimicking portfolios with conditioning information to utility maximization. Section VI concludes.

II. Overview of the Concepts and Results

The primitives of the problem are a vector of N asset returns, R_t, a vector of L lagged instruments, Z_{t-1}, (the conditioning information) and a K-vector of economic state variables or factors, F_t. The conditioning information may include the lagged values of the returns or F_t. We assume there is a joint probability distribution for $\{R_t, F_t, Z_{t-1}\}$, for which the relevant first and second moments exist. To keep the notation simple, we will drop the time subscripts when they are not needed for clarity.

Motivating Time-varying Weights

In asset pricing models, mimicking portfolios arise when there are stochastic changes in the consumption-investment opportunity set. In the classical model of Merton (1973) an investor's optimal demand for assets may be expressed as a mean-variance component plus a component that hedges or mimicks, the unexpected shocks to the state variables that describe future investment opportunities. The hedge portfolio weights are proportional to conditional multiple regression coefficients of the factor on the vector of asset returns, and therefore depend on the conditional covariances of the assets' returns and the conditional covariances of returns with the factors.

In a dynamic economy, the conditional means, covariances and the return covariances with the factors are functions of current information about the economic state. These moments will typically vary over time as information about the state of the economy changes. Indeed, a huge empirical literature documents time-variation in the conditional first and second moments of asset returns and their covariances with economic variables. Despite these obvious arguments that mimicking portfolios should be dynamic, empirical applications often assumed fixed weights.

Conditional and Unconditional Efficient Portfolios

Hansen and Richard describe portfolios with weights that may depend on the information, yet minimize unconditional variance for a given unconditional mean. Such portfolios are *unconditionally efficient* (UE) with respect to the information. Hansen and Richard show that UE portfolios are a subset of the *conditionally efficient* (CE) portfolios, which minimize conditional variance for a given conditional mean. Ferson and Siegel (2001) derive solutions for UE portfolios and show that they are optimal for agents with quadratic utility, in a single-period model.

As characterized by Fama (1996), the optimal portfolios in Merton's (1973) dynamic multiperiod model minimize variance, for a given expected return **and** given covariances with the state variables or factors. Because of the restriction on factor covariances, the portfolios are not minimum variance efficient. Fama calls the optimal portfolios *multifactor minimum variance*. We derive conditional and unconditional versions of these portfolios.

When all of the moments are conditioned on the information, Z, we have *conditional multifactor minimum variance* portfolios (CMMV). These are the portfolios described by Merton (1973) and Fama (1996). When the portfolios use the information Z, yet minimize unconditional variance for a given unconditional mean and covariances with the factors, the result is *unconditionally multifactor minimum variance* with respect to the information Z (UMMV). These portfolios generalize the UE portfolios of Hansen and Richard (1987).

Mimicking Portfolio Concepts

A **portfolio is UMMV with respect to Z if and only if** its weights, x(Z) solve the problem:

$$\text{Min}_{x(Z)} \text{ Var}[x(Z)'R] \text{ subject to } E[x(Z)'R]=c,\ E[Fx(Z)'R]=d \text{ and } x(Z)'\underline{1}=1, \qquad (1)$$

where $\underline{1}$ is an N-vector of ones and c and d are constants. A **portfolio is CMMV with respect to Z if and only if** its weights solve the problem:

$$\text{Min}_{x(Z)} \text{ Var}[x(Z)'R\,|\,Z] \text{ s.t. } E[x(Z)'R\,|\,Z]=c(Z),\ E[Fx(Z)'R\,|\,Z]=d(Z) \text{ and } x(Z)'\underline{1}=1, \qquad (2)$$

where c(Z) and d(Z) are known functions of Z. Note that when the mean of the portfolio return is constrained, the constraint on the uncentered second comoment with the factors is equivalent to a constraint on the portfolio's covariance with the factors.

III. Analytical Results

We present solutions for UMMV and CMMV portfolios and interpret their properties. UMMV portfolios are a subset of CMMV portfolios. A subset of UMMV and CMMV portfolios maximize the correlation with a factor, in the set of all possible portfolio rules with weights that may depend on Z.

Proposition 1:

If a portfolio is UMMV it must be CMMV (almost surely in Z), but the converse is not true. (Proofs are in the Appendix.)

Proposition I extends a well-known result from Hansen and Richard (1987) to a multi-factor setting. Hansen and Richard show that UE portfolios are a subset of CE portfolios. This result has a striking implication for tests of the conditional CAPM, which Cochrane (2001) dubs the "Hansen-Richard Critique." Assume that the conditional CAPM implies that the market portfolio is mean variance efficient, conditional on an information set, Ω, that cannot be observed. The econometrician can only test efficiency conditional on an observable subset of information, $Z \subset \Omega$. If you reject efficiency given Z, it does not imply that you reject efficiency given Ω, because portfolios that may use Ω and are efficient given Z, are a subset of the portfolios that are efficient given Ω. Thus, in principle the conditional CAPM could be true despite all of the tests in the literature that have rejected the efficiency of the market portfolio. The Hansen-Richard critique applies even if it was possible to measure the "true" market portfolio.

Using similar logic, Proposition I implies a multi-factor version of the Hansen-Richard critique. Assume, following Fama (1996) that Merton's (1973) model implies the market portfolio is CMMV, and further assume that the relevant conditioning information, Ω, cannot be observed. With a subset of the information, Z, we can reject that the market is CMMV with respect to Z, but this does not imply that we reject Merton's model. The multi-factor Hansen-Richard critique applies even if it is possible to measure both the true market portfolio *and* the relevant risk factors.[1]

Explicit Solutions

We provide a constructive solution for UMMV portfolios, which from Equation (1) solve the minimization:

$$\text{Min}_{x(Z)}\ E\{[x(Z)'R]^2\} \text{ subject to } E[x(Z)'R]=c,\ E[Fx(Z)'R]=d \text{ and } x(Z)'\underline{1}=1, \qquad (3)$$

Letting $\Lambda \equiv [E(RR'|Z)]^{-1}$ and $\mu(Z)=E(R|Z)$, write the Lagrangian for this problem as:

$$\text{Min}_{x(Z)}\ E\{\ x(Z)'\Lambda^{-1}x(Z) + 2\lambda_1[x(Z)'\mu(Z)-c] + 2[x(Z)'E(RF'|Z)-d']\ \lambda_2 + 2\gamma(Z)[x(Z)'\underline{1}-1]\}.$$

where the scalars $\lambda_1>0$ and $\gamma(Z)>0$ (almost surely in Z) and the K-vector $\lambda_2>0$, are the multipliers. Consider a perturbation w(Z) = x(Z) + *a*y(Z), where x(Z) is the optimal solution and y(Z) is any other portfolio weight function. If the weight function x(Z) is optimal, the derivative of the Lagrangian for w(Z) with respect to *a* must be zero when

[1] An interesting solution to this problem occurs when the model is closed by assuming that the lagged values of the observed risk factors describe the relevant information, Ω. Then the model is testable.

evaluated at *a*=0. The first order condition may be written as:

$$E\{\, y(Z)'[\Lambda^{-1}x(Z) + \lambda_1 \mu\ (Z) + E(RF' \mid Z)\ \lambda_2\ +\ \gamma(Z)\underline{1}] \,\} = 0. \tag{4}$$

Since Equation (4) must hold for all y(Z), it implies that the term in square brackets must be zero, almost surely in Z. Solving for the optimal weight, we have:

$$x(Z)\ =\ -\lambda_1 \Lambda\, \mu\ (Z) - \Lambda\, E(RF' \mid Z)\ \lambda_2 -\ \gamma(Z)\, \Lambda\ \underline{1}. \tag{5}$$

Evaluating the multiplier $\gamma(Z)$ by imposing the constraint that $x(Z)'\underline{1} = 1$, the solution may be expressed as:

$$x(Z)\ =\ \frac{\Lambda \underline{1}}{\underline{1}'\Lambda \underline{1}}\ +\ \left[\Lambda - \frac{\Lambda \underline{1}\underline{1}'\Lambda}{\underline{1}'\Lambda \underline{1}}\right]\left\{-\lambda_1 \mu(Z) - E(RF'|Z)\lambda_2\right\} \tag{6}$$

The CMMV solution is derived in similar fashion, except that the multipliers associated with c(Z) and d(Z) are functions of the information, $\lambda_1(Z)$ and $\lambda_2(Z)$:

$$x(Z)\ =\ \frac{\Lambda \underline{1}}{\underline{1}'\Lambda \underline{1}}\ +\ \left[\Lambda - \frac{\Lambda \underline{1}\underline{1}'\Lambda}{\underline{1}'\Lambda \underline{1}}\right]\left\{-\lambda_1(Z)\mu(Z) - E(RF'|Z)\lambda_2(Z)\right\} \tag{7}$$

Maximum Correlation Portfolios

Consider a portfolio that has maximum squared correlation with a single factor, F. The portfolio variance must be minimal among all portfolios that have the same mean and the same covariance with the factor, because correlation is covariance divided by the product of the standard deviations. It follows that there are values for the multipliers in Equations (6) and (7) for which the solutions maximize the unconditional and conditional correlations, respectively, with the factor. Thus, maximum correlation portfolios are special cases of CMMV and UMMV portfolios.

Proposition 2:

The portfolio that maximizes its squared conditional correlation with a particular factor F, subject to portfolio weights that sum to 1.0, has weights given by equation (7), with:

$$\lambda_1(Z) = \frac{-\gamma_1(Z)[E(F|Z) - \gamma_{\mu F}(Z)] + \gamma_\mu(Z)\gamma_F(Z)}{\gamma_\mu(Z)[E(F|Z) - \gamma_{\mu F}(Z)] + \gamma_F(Z)[\gamma_{\mu\mu}(Z)-1],}$$

$$\lambda_2(Z) = \frac{-\gamma_1(Z)[\gamma_{\mu\mu}(Z)-1] - \gamma_\mu(Z)^2}{\gamma_\mu(Z)[E(F|Z) - \gamma_{\mu F}(Z)] + \gamma_F(Z)[\gamma_{\mu\mu}(Z)-1],}$$

where:

$$\gamma_1(Z) = 1/(\underline{1}'\Lambda\underline{1}), \quad \gamma_\mu(Z) = \underline{1}'\Lambda\mu(Z)/(\underline{1}'\Lambda\underline{1}), \quad \gamma_F(Z) = \underline{1}'\Lambda E(RF'|Z)/(\underline{1}'\Lambda\underline{1}),$$

$$\Omega(Z) = [\Lambda - \Lambda\underline{1}\,\underline{1}'\Lambda/(\underline{1}'\Lambda\underline{1})], \quad \gamma_{\mu\mu}(Z) = \mu(Z)'\Omega(Z)\mu(Z),$$

$$\text{and} \quad \gamma_{\mu F}(Z) = \mu(Z)'\Omega(Z)E(RF'|Z).$$

Portfolios that maximize the squared unconditional correlation with a factor, over all portfolio weights that may depend on Z, may similarly be obtained as a special case of Equation (6).

Corollary:

The portfolio that maximizes its squared unconditional correlation with a particular factor F, subject to portfolio weights that sum to 1.0, has weights given by equation (6), with:

$$\lambda_1 = \frac{-\gamma_1[E(F)-\gamma_{\mu F}]+\gamma_\mu\gamma_F}{\gamma_\mu[E(F)-\gamma_{\mu F}]+\gamma_F[\gamma_{\mu\mu}-1],}$$

$$\lambda_2 = \frac{-\gamma_1[\gamma_{\mu\mu}-1]-\gamma_\mu{}^2}{\gamma_\mu[E(F)-\gamma_{\mu F}]-\gamma_F[\gamma_{\mu\mu}-1],}$$

where:

$$\gamma_1 = E(\gamma_1(Z)),\quad \gamma_\mu = E(\gamma_\mu(Z)),\quad \gamma_F = E(\gamma_F(Z))$$
$$\gamma_{\mu\mu} = E(\gamma_{\mu\mu}(Z)),\ \text{and}\ \ \gamma_{\mu F} = E(\gamma_{\mu F}(Z)).$$

Interpreting the Solutions

To interpret the various terms in the UMMV and CMMV solutions it is convenient to express equation (7) as the sum of three terms:

$$x(Z) = x_{GMS}(Z) - \lambda_1(Z)x_{MV}(Z) - w_H(Z)\lambda_2(Z), \tag{8}$$

$$x_{GMS}(Z) = \Lambda\underline{1}/(\underline{1}'\Lambda\underline{1})$$
$$x_{MV}(Z) = \Omega(Z)\mu(Z)$$
$$w_H(Z) = \Omega(Z)E(RF'|Z).$$

Equation (6) may be decomposed using the same three terms, but replacing $\lambda_1(Z)$ and $\lambda_2(Z)$ with the constants, λ_1 and λ_2. Equation (8) reveals that the solutions satisfy a K + 2 fund separation theorem. That is, any solution can be expressed as a combination of K + 2 time-varying weight vectors, where the particular combinations depend on the values of the multipliers associated with the constraints. (This is stated formally in Proposition 3.) Assuming that all values for the constraints are feasible, any solution can be obtained by selecting particular values for the multipliers. The values correspond to choosing particular targets for the mean return and the factor covariances. The two N-vectors $x_{GMS}(Z)$ and $x_{MV}(Z)$, with the K columns of $w_H(Z)$ form a basis for the set of all CMMV (and thus UMMV) solutions. In particular, by setting some of the multipliers to zero we obtain the solutions that relax the corresponding constraints.

If we set both $\lambda_1(Z)$ and $\lambda_2(Z)$ to zero the solution, $x_{GMS}(Z)$, minimizes the conditional second moment of returns subject only to the condition that the weights sum to 1.0. This is the global minimum conditional second moment portfolio; hence the notation, x_{GMS}. The GMS portfolio's conditional mean return is $E[x_{GMS}(Z)'R|Z] = \underline{1}'\Lambda\mu(Z)/(\underline{1}'\Lambda\underline{1}) = \gamma_\mu(Z)$. Its unconditional mean return is $E(\gamma_\mu(Z)) = \gamma_\mu$.

Other coefficients in Proposition 2 may be interpreted in terms of the K + 2 spanning portfolios. The global minimum second moment portfolio must have the same conditional second co-moment with every risky asset[2], equal to $1/(\underline{1}'\Lambda\underline{1}) = \gamma_1(Z)$. The conditional second co-moment of this portfolio with the factors are $\underline{1}'\Lambda E(RF'|Z)/(\underline{1}'\Lambda\underline{1}) = \gamma_F(Z)$. The unconditional second co-moments are $E[\gamma_F(Z)] = \gamma_F$.

If we set $\lambda_2(Z)$ to zero in Equation (8) we drop the constraint on covariances with the factors. The resulting solution, $x_{GMS}(Z) - \lambda_1(Z)x_{MV}(Z)$, minimizes the conditional second moment of return for a given conditional mean. This solution is therefore CE, as defined previously. In the special case where $\lambda_1(Z) = \lambda_1$ is a constant, the solution is the UE solution studied by Ferson and Siegel (2001).

Note that $\Omega(Z)$ is an NxN matrix such that , if multiplied by any N-vector, will produce a set of portfolio weights that sum to zero. In particular $x_{MV}(Z) = \Omega(Z)\mu(Z)$ and $w_H(Z) = \Omega(Z)E(RF'|Z)$ are vectors of portfolio weights that sum to zero. Applied to the vector of returns R, weights that sum to zero produce *excess returns*. The implied

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[2] If a portfolio has unequal conditional second moments on any two assets it cannot be the global minimum, since by reducing the weight in the asset with the higher comoment and increasing the weight on the asset with the lower comoment, the portfolio's second moment could be reduced.

portfolio is "long" those assets with positive weights and "short" those assets with negative weights. The portfolio return is therefore the return to the long positions in excess of those to the short positions. It follows that $E[x_{MV}(Z)'R|Z] = \mu(Z)'\Omega(Z)\mu(Z) = \gamma_{\mu\mu}(Z)$, and $E\{\gamma_{\mu\mu}(Z)\} = \gamma_{\mu\mu}$, are the conditional and unconditional expected excess returns of the $x_{MV}(Z)$ portfolio.

Finally, consider the NxK matrix of weights, $w_H(Z)$. These weights deliver the excess returns of the K hedging portfolios for the state variables or factors. Their conditional mean excess returns are $\mu(Z)'\Omega(Z)E(RF'|Z) = \gamma_{\mu F}(Z)$ and the unconditional means are $E[\mu(Z)'\Omega(Z)E(RF'|Z)] = \gamma_{\mu F}$.

A Numerical Example

We provide further intuition about the solutions using a simple, special case. The example is similar to one in Brown and Warner (1980). Assume that we are given two assets: A riskless asset with return r_f, a risky asset with return R, a single factor F, and a single instrument, Z. The vector $(R_{t+1}, F_{t+1}, Z_t)'$ has a trivariate normal distribution with mean $(\mu_R, \mu_F, \mu_Z)'$ and covariance matrix

$$\begin{pmatrix} \sigma_R^2 & \sigma_{RF} & \sigma_{RZ} \\ \sigma_{RF} & \sigma_F^2 & \sigma_{FZ} \\ \sigma_{RZ} & \sigma_{FZ} & \sigma_Z^2 \end{pmatrix}.$$

Our objective is to find a portfolio weight function $w(Z)$ such that the portfolio return $R_w = r_f + w(Z)(R - r_f)$ minimizes $E(R_w^2)$ subject to the constraints $E(R_w) = c$ and $E(R_W F) = d$. The weight function represents the amount invested in the risky asset while investing $[1 - w(Z)]$ in the risk-free asset. The optimal weight function is given by:

$$w(Z)=\frac{\lambda_1 E\left(R-r_f \mid Z\right)+\lambda_2 E\left[\left(R-r_f\right)F \middle| Z\right]}{E\left[\left(R-r_f\right)^2 \middle| Z\right]}, \tag{9}$$

where

$$E\left(R-r_f \mid Z\right)=\mu_R - r_f + \frac{\sigma_{RZ}}{\sigma_Z^2}\left(Z-\mu_Z\right)$$

$$E\left[\left(R-r_f\right)F \middle| Z\right]=\sigma_{RF}-\frac{\sigma_{RZ}\sigma_{FZ}}{\sigma_Z^2}+\left[\mu_R - r_f + \frac{\sigma_{RZ}}{\sigma_Z^2}\left(Z-\mu_Z\right)\right]\left[\mu_F + \frac{\sigma_{FZ}}{\sigma_Z^2}\left(Z-\mu_Z\right)\right]$$

$$E\left[\left(R-r_f\right)^2 \middle| Z\right]=\sigma_R^2-\frac{\sigma_{RZ}^2}{\sigma_Z^2}+\left[\mu_R - r_f + \frac{\sigma_{RZ}}{\sigma_Z^2}\left(Z-\mu_Z\right)\right]^2$$

and the constants λ_1 and λ_2 are chosen to achieve the constraints. (The Appendix provides a derivation.)

The weight w in Equation (9) is a ratio of two quadratic polynomials in Z, where the denominator has no real roots (assuming that the risky asset is not perfectly correlated with Z). Therefore the weight function is continuous and has a single horizontal asymptote as $Z \to \pm\infty$. This may be compared and contrasted to the optimal weights for UE portfolios (Ferson and Siegel, 2001) which do not constrain the covariance with a factor. In this special case, obtained by setting $\lambda_2 = 0$ in Equation (9), the optimal weight function is the ratio of a linear to a quadratic function, implying a single horizontal asymptote with a value of zero. Thus, Ferson and Siegel showed that the UE solution exhibits a "conservative" response to extreme realizations of the information Z. As Z gets large the weight in the risky asset approaches zero.[3]

Equation (9) illustrates that the hedging component of the UMMV solution

[3] The conservative behavior of the UE solution contrasts with CE solutions, such as under exponential utility and conditional normality, where the portfolio weight is linear in Z. In such cases the optimal weight is unbounded for large values of Z. Ferson and Siegel (2003) employ UE weights in a version of the Hansen and Jagannathan (1991) bounds, and find that the conservative response to large realizations translates into superior finite-sample properties of the estimated weights.

modifies the conservative response of the UE component, but the solution remains bounded for large values of Z. Thus, the hedging component of the solution approaches a constant for large values of Z in this example. We may therefore anticipate that estimates of the UMMV solution could be more "robust" to extreme data observations and have better sampling properties, compared with the CMMV solution.

We illustrate Equation (9) using monthly data for the Standard and Poors stock index return as the risky asset and inflation as the factor. The lagged instrument is a percentage term spread; the data are described below. The parameters are $r_f = 0.003690$, $\mu_R = 0.009376$, $\mu_F = 0.003375$, $\mu_Z = 0.503563$, $\sigma_R = 0.040540$, $\sigma_F = 0.003949$, $\sigma_Z = 0.503743$, $\sigma_{RF} = 0.000003380$, $\sigma_{RZ} = -0.001781$, and $\sigma_{FZ} = 0.0004730$.

Consider a portfolio with time-varying weights that has minimum (unconditional) variance subject to the constraints that it have the same unconditional expectation as the risky asset $\left(\mu_R = E\left(R_w\right) = 0.009376 = c\right)$ and the same unconditional covariance with the factor as the risky asset $\left(\sigma_{RF} = Cov\left(R_w, F\right) = 0.000003380 = d - c\mu_F\right)$. The values of the multipliers are $\lambda_1 = -0.079850$ and $\lambda_2 = 87.122801$, found with an iterative approximation method using numerical integration to evaluate the expectations. We find that the variance has been reduced from $\sigma_{\mathrm{R}}^2 = 0.001643$ to $\sigma_{\mathrm{Rw}}^2 = 0.001480$. The weight function that achieves this is shown in Figure 1. The x-axis extends above and below μ_Z by $3.89\sigma_Z$, representing 99.99% of Z values. Over this range the optimal weight is a concave function of the conditional mean of the risky asset.[4]

The behavior of the optimal weight function is shown in Figure 2 for extended Z values. There is a single asymptotic limit for both large and small Z, but for large Z values the asymptote is approached from below.

If we seek to maximize the correlation between the portfolio and the factor in

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[4] Concavity arises because $\sigma_{\mathrm{RZ}}\sigma_{\mathrm{FZ}} < 0$ in this example. If the product is positive the function is convex in Z. Brown et al (2004) describe situations where optimizing portfolio managers may prefer concave investment strategies.

this example, we more than double it, increasing the correlation from 0.0211 (between the risky asset and the factor) to 0.0481 by using the optimal portfolio weight with $\lambda_1 = -0.33740$ and $\lambda_2 = 100$.[5]

IV. Empirical Results

The main feature of our solutions to the mimicking portfolio problem is portfolio weights that vary over time with the conditioning information. The following empirical examples therefore focus on the effects of time-varying weights.

The Data

The data are taken from Lamont (2001), who studies the period from January of 1947 through December of 1994.[6] Mimicking portfolios are most important in empirical asset pricing when the factors are not traded assets. Our examples therefore focus on the subset of Lamont's factors that are not traded assets. These are US inflation, labor income growth and industrial production growth.[7] The lagged conditioning variables include a one-month Treasury bill rate, a term spread for one-year bonds, a "default" spread, or the difference between low grade and high grade corporate bond yields, a spread between commercial paper and Treasury rates, the lagged growth rates of industrial production and inflation (for the year ending one month prior to the monthly return), and the lagged annual excess return on the stock market index.[8] The base assets used to form the mimicking portfolios also follow

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[5] Any multiple of the vector (λ_1, λ_2) will achieve this same maximal correlation.

[6] We are grateful to Owen Lamont for making his data available to us.

[7] Lamont also studies US consumption expenditure growth, but these data are limited to a shorter monthly sample, so we do not use consumption.

[8] Lamont also includes a long-term yield spread and a market dividend yield. Neither of these produce t-ratios larger than two in the regressions reported in Lamont's Table 1 for the three factors we study. Furthermore, with the full set of instruments his covariance matrices are sometimes singular (see his footnote 6), and we find the same result. We therefore exclude these two instruments.

Lamont (2001). These are monthly returns for eight industry-grouped common stock portfolios, shown in Table 1, a broad market index, three maturities of Treasury bonds and a "junk" bond.[9]

Table 1 presents monthly regressions for the base asset returns and the factor growth rates on the lagged instruments. The purpose of this table is to summarize the data and to introduce the parameterization of our simulation exercises. The fitted regressions are used to represent the conditional means of the returns, $\mu(Z)$. The sample covariances of the residuals from these regressions define the conditional covariances when they are assumed to be fixed over time (heteroskedastic models are described below). The other parameters of the simulations are constructed from these building blocks. For example, for each value of Z, the matrix $\Lambda(Z) = E(RR'|Z)^{-1} = [Cov(R|Z) + \mu(Z)\ \mu(Z)']^{-1}$ is constructed with Cov(R|Z) and $\mu(Z)$. For a given factor the vector $E(RF|Z) = Cov(R,F|Z) + \mu(Z)E(F|Z)$ is constructed in a similar way.

The regressions in Table 1 show that the fitted conditional means represent about 10% of the variance of the monthly returns on average, with the R-squares ranging between 5% for the energy industry to 50% for the one-year bond return. For the monthly factors the R-squares are smaller, ranging from 0.2% to 13 %, and based on these results we assume constant factor means in some of the exercises to follow. The coefficients seem generally consistent with previous studies, such as Fama and Schwert (1977), Keim and Stambaugh (1986), Fama (1990) and Ferson and Harvey (1991).

Panels C and D of Table 1 use overlapping monthly observations for quarterly and annual future growth rates, respectively. The adjusted R-squares increase with the horizon of the growth rates, but because of the overlapping data the R-squares for the longer

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[9] We use CRSP data for the market index (the Standard and Poors 500 total return), the Treasury bond returns (a one-year, a seven year and a 30 year return) and the industry portfolios. The industry portfolios are courtesy of Ken French, and are combinations of the 48 industry groups used in Fama and French (2000).

horizons are likely to be biased. The standard errors in Table 1 are Newey-West (1987) heteroskedasticity and autocorrelation-consistent, using 24 moving average terms as in Lamont (2001) The t-ratios based on these standard errors do not indicate that the explanatory power is greater for the longer-horizon growth rates.

Relation to Lamont's (2001) Tracking Portfolios

Lamont (2001) forms "tracking portfolios" as the linear combination of returns, $b'R_t$, where the weights, b, are estimated by a linear regression of the future growth rates of an economic factor from t to t+k, on returns and lagged conditioning variables:[10]

$$F_{t+k} = a + b'R_t + c'Z_{t-1} + \varepsilon_{t+k}. \tag{10}$$

Lamont interest is in maximizing the squared correlation of changes in expectations of the future factor growth rate, $E_t(F_{t+k})-E_{t-1}(F_{t+k})$, with the unexpected excess returns, $R_t - E_{t-1}(R_t)$, where $E_{t-1}(.)$ denotes the conditional expectation at time t-1. He notes (in his footnote 1) that a useful extension would allow conditional coefficients in the tracking portfolios. Our examples explore this extension.

Lamont's (2001) tracking portfolios maximize the (squared) correlation, assuming that fixed weights are optimal. A special case, dropping the Z_{t-1} term from the regression (10), maximizes the unconditional correlation under the same assumptions. We compare our methods empirically to these two fixed-weight approaches.

To see how the results in this paper refine the approach of Lamont, write $F_{t+k} = E_t(F_{t+k}) + v$, with $E_t(v)=0$, and assume as in Lamont that $E_{t-1}(.)=E(.|Z_{t-1})$, so that the lagged instruments proxy for the available information at time t-1. Let $R_{pt} = b'R_t$ be the tracking

[10] Lamont uses excess return in his regressions, but we use raw returns for comparability. The weights, b, are not constrained to sum to 1.0 in Lamont's regression, unlike our portfolio solutions. When the weights are fixed over time this does not affect the correlations of $b'R_t$ with the factors.

portfolio return. Then:

$$\begin{aligned} & \text{Cov}\{ E_t(F_{t+k}) - E_{t-1}(F_{t+k}), R_{pt} - E_{t-1}(R_{pt}) \} && (11) \\ = \ & \text{Cov}\{ F_{t+k} - E(F_{t+k} | Z_{t-1}), R_{pt} - E(R_{pt} | Z_{t-1}) \} \\ = \ & \text{Cov}\{ F_{t+k}, R_{pt} \} - \text{Cov}\{ E(F_{t+k} | Z_{t-1}), E(R_{pt} | Z_{t-1}) \} \\ = \ & E[\text{Cov}\{F_{t+k}, R_{pt} | Z_{t-1}\}]. \end{aligned}$$

Thus, Lamont's approach focusses on the expected value of the conditional covariance. We refine this approach in two ways. First, our solutions allow time-varying weights to replace the fixed coefficient, b. The CMMV solution in particular, focusses on the conditional covariance, $\text{Cov}\{F_{t+k}, R_{pt} | Z_{t-1}\}$, which may vary over time with Z_{t-1}. Maximizing the squared conditional correlation for each value of Z implies maximizing the expected squared conditional correlation.[11] Thus, with time-varying weights we should obtain a higher expected conditional correlation. Of course, the solution with the maximum expected conditional correlation need not have the maximum unconditional correlation.

Our second refinement relates to the unconditional correlation. For intuition, consider the unconditional covariance:

$$\begin{aligned} & \text{Cov}\{ F_{t+k}, R_{pt} \} \\ = \ & E[\text{Cov}\{F_{t+k}, R_{pt} | Z_{t-1}\}] + \text{Cov}\{ E(F_{t+k} | Z_{t-1}), E(R_{pt} | Z_{t-1}) \}. && (12) \end{aligned}$$

The unconditional covariance between portfolio returns and a factor depends on two terms. Lamont's approach controlls the first term. Our UMMV solution allows the time-varying

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[11] Let the maximum squared conditional correlation be f(x(z)). Then for any other solution, f(y(z)), we have $f(x(z)) \geq f(y(z))$ almost surely in z, implying that the integral against a positive measure and thus the expectation, is also higher.

weights to optimally control the covariation of the conditional means in the second term as well.[12]

In summary, our solutions should attain larger correlations with a factor than a fixed-weight approach. By allowing for time-varying weights the average conditional (squared) correlations should be higher. By controlling the conditional means with the time-varying weights, the unconditional correlation should also be higher.

Potential Improvements Using Time-varying Weights

Our first exercise examines the potential improvements that time-varying weights could provide in the absence of estimation error. We abstract from estimation error in a simulation, by forming the mimicking portfolios with full knowledge of the form and parameters of the data generating process. We keep the initial simulations simple by holding $E(F|Z)$ fixed at the sample means and assuming homoskedasticity in the conditional covariances. Heteroskedasticity and time-varying factor means present additional sources of time-variation, which increase the potential advantage of our solutions relative to a fixed-weight approach.

We simulate a sample with one million observations from the data generating process, described more completely in the Appendix. The parameters that involve expectations, such as $\gamma_1 = E\{1/(\underline{1}'\Lambda\underline{1})\}$ and the regression (10), are estimated from the one million observations. The parameter estimates should essentially be at their probability limits with so many observations, there by abstracting from estimation error (10,000 observations produces similar results).[13]

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[12] The maximum correlation solution controls the time-varying expected portfolio return to minimize the unconditional return variance, given its covariance with the factor. Thus, the UMMV solution refines the fixed-weight solution even if we assume $E(F|Z)$ is a constant.

[13] While a one million observation sample effectively abstracts from estimation error, there will still be come simulation error due to numerical imprecision. For example, we compute expectations by summing over one million observations and dividing the result by one million. These errors typically amount to less than

We examine the unconditional correlations of mimicking portfolio returns with the factors, and also the expected conditional correlations. Given the parameters of the data generating process, the squared unconditional correlations may be computed directly. For Lamont's approach the solution is $[b'\mathrm{Cov}(R,F)]^2/[b'\mathrm{Cov}(R)b\mathrm{Var}(F)]$ where b and the variances and covariances are the estimates from the one-million-observation artificial sample. Letting $E(R_p) = \gamma_\mu - \lambda_1\gamma_{\mu\mu} - \lambda_2\gamma_{\mu F}$ and $\gamma_{FF} = E\{E(FR'|Z)\Omega(Z)E(RF|Z)\}$ the squared unconditional correlation for the UMMV solution given in the corollary to Proposition 2 is:

$$\rho_u^2 = \frac{\{\gamma_F - \lambda_1\gamma_{\mu F} - \lambda_2\gamma_{FF} - E(F)E(R_p)\}^2}{\mathrm{Var}(F)\{\gamma_1 + \lambda_1^2\gamma_{\mu\mu} + \lambda_2^2\gamma_{FF} + 2\lambda_1\lambda_2\gamma_{\mu F} - E(R_p)^2\}} \tag{13}$$

For a given portfolio weight, x(Z), the expected squared conditional correlation is:

$$\rho_c^2 = E\left\{\frac{[x(Z)'\mathrm{Cov}(R,F|Z)]^2}{\mathrm{var}(F|Z)x(Z)'\Sigma(Z)x(Z)}\right\}. \tag{14}$$

To compute the expected conditional correlation we take the average of the expression inside the expectation in (14), over the one million simulated Z observations, where the x(Z) are formed using the true parameter values, and the conditional covariances are the true parameters of the data generating process.

There are two timing conventions for the monthly factors shown in Table 2. When k=1 Lamont's regression, Equation (10), produces a portfolio, $b'R_t$, to track the future factor F_{t+1}. However, in some settings, such as hedging, we may be interested in a portfolio that is maximally correlated with the contemporaneous factor, F_t. Our solutions,

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3% of the figures reported in tables 2 and 3, as is illustrated below.

as well as Lamont's regression, can accommodate either timing convention and we present both versions. For the longer horizon factors, we follow Lamont and present the mimicking return, R_{pt}, for the factor growth rate from month t to t+k.

Table 2 shows the potential correlations, abstracting from sampling error. The unconditional correlations of the fixed-weight solution are obtained by dropping the lagged instruments from regression (10). The value is 13.5%, averaged across the factors and horizons. The average expected conditional correlation of Lamont's solution is 17.5%. These two figures would be identical if there was no information in the lagged Z about the future returns and factors.

The potential improvements from time-varying weights can be substantial. Averaged across the examples in the table, the absolute unconditional correlations are *16.5%* using the UMMV solution with time-varying weights. This represents an improvement of about 22% over the fixed-weight solution's 13.5%. The expected conditional correlation for Lamont's solution and the CMMV solutions are identical. This reflects the assumptions under which the two solutions are generated. It can be shown that if conditional mean returns are linear in Z and conditional covariances are fixed over time, then the CMMV solution that maximizes the squared conditional correlation has fixed weights and is equivalent to Lamont's solution.

Heteroskedasticity

To illustrate how time-varying weights in a CMMV solution can offer improved conditional correlations, we introduce a simple form of conditional heteroskedasticity. The heteroskedasticity is driven by time-varying factor betas, which are assumed to be linear functions of Z. For asset i the conditional beta is $B_i(Z) = b_{oi} + b'_{1i}Z$. We calibrate the betas to regressions with interaction terms. When the returns and factors are measured contemporaneously the regression is: $R_{it+1} = a_{oi} + a'_{1i}Z_t + b_{oi}F_{t+1} +$

b'_{1i} $(Z_t F_{t+1}) + v_{it+1}$. The conditional covariances are then modelled as functions of Z: $Cov(R_{it+1} R_{jt+1} | Z_t) = B_i(Z_t) B_j(Z_t) \sigma_F^2 + Cov(v_i, v_j)$, and $Cov(R_{it+1}, F_{t+1} | Z_t) = B_i(Z) \sigma_F^2$, where σ_F^2 and $Cov(v_i v_j)$ are fixed parameters.

Using the simple model of heteroskedasticity there are signs of overfitting the moments in Lamont's data. When we use all seven lagged instruments we find that the fitted conditional covariances appear unreasonably large. The simulated potential correlations of the time-varying weight solutions approach 99% in a few cases. To provide a more realistic comparison we use a subset of the lagged instruments for the experiments in Table 3. The one-year term spread, used in the numerical example above, is the first choice. Additional experiments also include the risk-free rate and default spread.

Table 3 shows that under conditional heteroskedasticity, time-varying weights can produce larger expected conditional correlations than the fixed-weight approach. Averaged across the horizons and instruments, the conditional correlations of the fixed-weight solutions are 32.9% for industrial production, 33.5% for labor income and 24.3% for inflation. The time-varying weight solutions average 49% for industrial production, 53.2% for labor income and 36.0% for inflation. The average ratio of the conditional correlation for a time-varying solution to the fixed-weight solution is 1.57, so the potential improvement is about 57% on average.

Sample Results

The next two tables present the actual sample performance of the mimicking portfolios. In Table 4 the parameters are estimated assuming that the homoskedastic process with linear conditional mean returns is correct, and estimating the parameters using the Generalized Method of Moments (GMM, Hansen, 1982). The "average conditional correlations" are the sample correlations of the residuals from regressing the fitted mimicking portfolio returns and the factors on the lagged conditioning variables.

Under the homoskedastic model assumptions the average conditional correlations produced by the CMMV solution and Lamont's regression approach are identical. These range from 14.7% to 31.1% depending on the factor and the horizon, with no clear patterns across factors or horizons.

The unconditional correlations are the sample correlations between a fitted mimicking portfolio and its factor. The regression approach delivers a larger unconditional correlation than the UMMV solution in every example. In the case of labor income and at shorter horizons the differences are often small. However, at the annual horizon the regression performs markedly better.

In Table 5 the sample performances are compared again, this time assuming that the data generating process features linear conditional means and heteroskedasticity, driven by time-varying betas as described above. The results with all seven instruments are reported in the first of the two rows of figures for each example. In the first row of figures only the lagged term spread is used as an instrument. The fixed-weight approaches obviously produce the same results as in the previous table when all seven conditioning variables are used.

Under the heteroskedastic model the UMMV solution performs better than under the homoskedastic model. The unconditional correlations are larger than in Table 4, often substantially so. The UMMV solution also slightly outperforms the regression method. For example, averaged across the horizons and instrument choices, the UMMV solution delivers correlations of 24.9% for industrial production, 30.1% for labor income and 26.7% for inflation. The corresponding figures for the fixed-weight regression are 20.7%, 29.0% and 26.1% respectively. Together with the results in Table 4 this shows that with a more accurate data generating process it is possible to obtain better results with the UMMV solution.

The conditional correlations in Table 5 also present some interesting patterns. First, the CMMV solution performs much worse than the simpler approach of Lamont. Many of the CMMV conditional correlations are insignificantly different from zero, and the averages are about 1/3 of the values produced by Lamont's approach. Second, the choice of instruments has a pronounced effect on the CMMV solutions, and overfitting is apparent when all seven are used. Assuming heteroskedasticity, the CMMV correlation is larger using the term spread alone than using all seven instruments, in all but three examples.

In summary, the UMMV solution performs about as well, or better, when all seven conditioning variables are used as when the single instrument is used. The CMMV, in contrast, typically performs less well with seven instruments. This is interesting given our previous observation that the heteroskedastic model appears overfit with all seven instruments. When the conditional second moment matrix is overfit its inverse is less stable numerically, which hurts the sampling properties of the estimators. The UMMV solution should be more robust than the CMMV solution to extreme values in the sample, as are likely to be produced by overfitting.

Estimation Error

Estimation error in some form must explain why the optimal time-varying weight solutions do not fully deliver their potential performance in the sample. Estimation error enters the problem in several ways. First, the mimicking portfolio weights depend on parameters that can only be estimated with error. This introduces randomness in repeated samples that affects the sampling distribution of the mimicking portfolio returns. For example, even if the asset returns are normally distributed, the products of returns with the estimated weights are nonnormal.[14]

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[14] This problem was encountered in different contexts by Brown and Warner (1980), who examine the use of "control portfolios" in event studies, and by Dybvig and Ross (1985), who study the optimal portfolios of informed investment managers.

The second effect of estimation error relates to our ability to evaluate the various approaches. In finite samples our estimates of the expected conditional and unconditional correlations are subject to estimation error.

The third effect relates to the specification of the form of the data generating process. The "true" generating process for the data is difficult to discover, and might be more complex than our examples imagine.

We present additional simulation exercises to address these issues. We can control the second effect in a simulation setting, as described below. This leaves the effects of parameter estimation error and specification error in the data generating process. The time-varying-weight approaches involve more estimated parameters, and they require that the functional form of the conditional means and covariances be specified. If parameter estimation error can explain the gap between the potential and realized performance of the mimicking portfolios it suggests that research can be profitably directed at obtaining better parameter estimates. If not, we are left with the problem of better specifying the form of the generating process. Perhaps, both issues deserve our attention.[15]

Parameter Estimation and Data Generating Process Errors

The next experiments focus on parameter estimation error and errors in specifying the form of the data generating process (DGP), either in isolation or taken together. The first type of error is present in all of the approaches, although perhaps the performance is affected to varying dergrees. It is interesting to see which approaches are more affected by estimation error. The fixed-weight approaches do not need to specify the

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[15] Of course the two issues interact in general. If we could know the data generating process down to the form of the probability distribution, we could estimate the parameters by maximum likelihood and attain the Cramer-Rao efficiency bound. Our interest here focuses on the central parameters of the problem—the conditional means and covariance matrix—and their functional relation to the lagged conditioning variables.

form of the DGP and thereby avoid the second type of error. If the time-varying weight approaches specify the wrong DGP they are no longer the optimal solutions, so it is interesting to see how sensitive their results are to DGP error. Finally, the two types of errors may interact, so we are interested in the combined effects.

The first experiment isolated estimation error in the parameters of the mimicking portfolio weights, abstracting from errors in specifying the form of the data generating process. Estimation error is captured by simulating artificial samples with 576 observations, matching the sample of Table 1. The heteroskedastic DGP is used to generate the data and the correct form of the process is assumed when estimating the parameters. We abstract from estimation error in our evaluation of the results by extending each of the 300 simulated samples to one million observations. The mimicking portfolio solutions continue to rely on the noisy parameter estimates based on the first 576 observations. However, the factor correlations with the mimicking portfolio returns are estimated using the one million observations. The results of the first experiment are shown in the first two columns of tables 6 and 7.

The second experiment isolates the effects of errors in specifying the DGP, abstracting from parameter estimation error. Here we generate data from the heteroskedastic process, but the "artificial analyst" in the simulations assumes the homoskedastic process. We abstract from parameter estimation error by using an artificial sample with one million observations.

The third experiment combines the effects of parameter estimation and DGP error. This experiment is essentially the same as the experiment with parameter estimation error, except that the artificial analyst incorrectly assumes that the homoskedastic DGP is correct.

Tables 6 and 7 summarize the results by reporting the ratio of each expected absolute correlation (as a percent) to the potential absolute correlation, computed as in

Table 3. We concentrate on the industrial production factor.

Focussing on the pure effects of parameter estimation error, the first two columns of table 6 show the impact on the unconditional correlations for the fixed-weight and UMMV approaches. The time-varying weight UMMV solution delivers smaller fractions (15% - 68%) of its potential unconditional correlation than the fixed-weight solution, which delivers 54% - 92%. This makes sense given that the UMMV solution requires the estimation of more parameters. However, the potential correlations of the UMMV solution are larger, as shown in Table 3. Multiplying the two figures shows that the UMMV solution is expected to deliver slightly larger correlations on average (13.7%, versus 13.1% for the regression approach).

Table 7 shows the impact of estimation error on the expected conditional correlations, comparing Lamont's fixed-weight approach with the time-varying-weight CMMV approach. The fractions are in favor of the fixed-weight approach, which range from 72% to almost 100% of the potential correlations. The time-varying-weight approach delivers 33% - 89% of its potential. This makes sense given the larger number of parameters in the CMMV solution. The fact that the percentages are closer together than in Table 6 also makes sense, as the numbers of parameters to be estimated in the two approaches is more similar in this experiment.

Average across the examples in Table 7, and considering the potential conditional correlations shown in Table 3, the expected conditional correlations are 29% for Lamont's method, versus 36% for the CMMV approach. Thus, while parameter estimation error hurts the time-varying-weight approaches more in percentage terms, the higher potential correlations of the optimal solutions can offset the effect.

The middle column of Table 6 focusses on DGP error in isolation. Only the UMMV solution is shown, because the fixed-weight regression is not affected by DGP error and thus delivers 100%.[16] Similarly, the results of this experiment are not reported in Table 7, as both solutions deliver 100%. (Recall that the CMMV solution is identical to Lamont's solution when it assumes linear expected asset returns and homoskedasticity.) The UMMV solution delivers between 82% and 98% of its potential correlation under pure DGP error.

The right-hand column of Table 6 combines DGP error and parameter estimation error. Only the UMMV solution is shown, because the fixed-weight solution produces the same results as in the first column of the table. (In Table 7, the results for this experiment are the same as for the case with parameter error only, for the reason described previously.) Compared to the second column with parameter error only, the overall percentages are similar. In some cases, the UMMV solution actually performs better than in the second column. This illustrates an interaction between DGP error and parameter estimation error. With the much smaller number of parameters in the homoskedastic process, the reduction in parameter estimation error can offset the effects of DGP error, and result in better performance even when the DGP is wrong.

Step-Ahead Comparisons

We conduct step-ahead comparisons of the approaches. We use the first T_e observations to estimate the parameters, then we apply the estimated weights to the returns, R_{T_e+1}, to form a mimicking portfolio return for $T_e + 1$. We roll the entire procedure forward one period, and repeat until the end of the sample. With T observations in the sample, we produce $T - T_e$ step-ahead returns. The rolling window is $T_e = 120$ months ($T_e = 60$ results in similar conclusions). These mimicking portfolios are feasible in the sense that they could have been estimated using the available data at the

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[16] We ran the analysis, and the variation of the results around 100% illustrates the effects of the numerical errors described previously. These are never more than 2.2 % of the figures in Table 3.

portfolio formation date. However, a rolling procedure may convey an "unfair" advantage to a fixed-weight approach. In each window the "fixed" weights can be different, and the weights will therefore vary slowly over time. We use the simple homoskedastic model assumptions consistent with Table 2. We also examine an example where we assume heteroskedasticity, driven by linear conditional betas. The results are similar using all seven instruments, slightly better when only a single instrument is used.

The step-ahead performance of all the fixed and time-varying-weight approaches are poor. Using the approximation $1/\sqrt{T-T_e}$ to the standard error of a sample correlation, only 15 out of 42 cases examined, are significantly different (two standard deviations) from zero. The "fixed" weight solution outperforms the time-varying weight solution in 19 out of 24 comparisons.

V. Additional Results

This section presents additional results that include a generalization of the representation of UE and CE portfolios in Hansen and Richard (1987). We also describe the utility functions for which UMMV portfolios are optimal, and present an alternative representation of the CMMV solution.

Characterizations

Hansen and Richard (1987) show that it is possible to represent all UMV and CMV portfolios in terms of two returns that are conditionally minimum variance. The first return, R^*_0, is a portfolio with weights that sum to 1.0. The second, R^*_e, is an excess return with weights that sum to 0.0. They show that all CMV portfolio returns may be expressed as $R^*_0 + w(Z)R^*_e$, where w(Z) is a scalar function of Z, while all UMV portfolios may be expressed as $R^*_0 + w\, R^*_e$, where w is a constant. Our solutions generalize this result for UMMV and CMMV portfolios.

Proposition 3:

Any CMMV portfolio may be expressed as $R^*_0 + w(Z)\, R^*_e + \Sigma_{j=1,...,K} v_j(Z)\, R^*_{ej}$, where R^*_e and R^*_{ej} are CMMV excess returns, with weights that sum to 0.0, and the $v_j(Z)$ are a scalar functions of Z. Any UMMV portfolio may be expressed as $R^*_0 + w\, R^*_e + \Sigma_{j=1,...,K} v_j\, R^*_{ej}$, where w and the v_j, j=1,...,K are constants.

Relation to Utility Maximization

Fama (1996) describes how under the assumption of normality, agents in Merton's (1973) model choose multifactor minimum variance portfolios. This refers to CMMV portfolios given the agents' information sets. The optimization problem at time t-1 is to choose the consumption expenditures C_{t-1} and the portfolio weight $x(Z_{t-1})$ to:

$$\text{Max } u(C_{t-1}) + E\{J(W_t,F_t) \mid Z_{t-1}\}, \text{ subject to } W_t=(W_{t-1}-C_{t-1})x(Z_{t-1})'R_t,\ x(Z_{t-1})'\underline{1}=1, \qquad (15)$$

where J(W,F) is the indirect utility function for wealth, which depends at time t on the state variables or factors, F_t. The first order condition for this problem with respect to the portfolio choice is:

$$(W_{t-1}-C_{t-1})E\{\, J_w(W_t,F_t)\, R_t \mid Z_{t-1}\} + \gamma_1 (Z_{t-1})\underline{1} = 0, \qquad (16)$$

where $J_w(.,.)$ denotes the partial derivative with respect to the first argument and $\gamma_1 (Z_{t-1})$ is a Lagrange multiplier. Expand the expectation of the product into the product of the expectations plus the covariance, and use Stein's (1973) Lemma to write $Cov(J_w,R \mid Z) = E(J_{ww} \mid Z)\, Cov(W,R \mid Z) + E(J_{wF} \mid Z)'\, Cov(F,R \mid Z)$. Now, $Cov(W_t,R_t \mid Z_{t-1}) =$

$(W_{t-1}-C_{t-1})\Sigma(Z)x(Z)$, where $\Sigma(Z)$is the conditional covariance matrix of R_t. The solution for x(Z) can be expressed as:

$$x(Z) = a_1(Z)\Sigma(Z)^{-1}\underline{1} + a_2(Z)\Sigma(Z)^{-1}\mu(Z) + \Sigma(Z)^{-1}E(RF'|Z)b(Z) \qquad (17)$$

for scalar functions of the information $a_1(.)$, $a_2(.)$ and K-vector b(.). The solution therefore minimizes the conditional variance of $x(Z)'R_t$ subject to constraints on $E[x(Z)'R|Z]$, $E[Fx(Z)'R|Z]$ and $x(Z)'\underline{1}=1$. Minimizing conditional variance, for each Z, subject to constraints that fix the conditional mean, implies minimizing the conditional mean of the squared return. Therefore the solution in Equation (17) also solves the problem in Equation (2), and is CMMV. Thus, Merton's (1973) agents choose CMMV portfolios, and the solution for the optimal portfolio weight is the same as given by Equation (7). The Appendix provides a representation for UMMV portfolios, analogous to Equation (17), written in terms of covariances instead of second moment matrices.

Since UMMV portfolios are a special case of CMMV portfolios, the question remains as to which subset of agents would find UMMV portfolios optimal. Ferson and Siegel (2001) show that UMV portfolios are optimal for agents with quadratic utility functions in a single period model. The next proposition generalizes this finding for UMMV portfolios in an intertemporal model.

Proposition 4:

In an intertemporal model, agents with the indirect utility function given by $J(W,F) = W - a W^2 - b'FW$, where a and b are constants, and who observe the conditioning information Z, choose optimal portfolios that are UMMV.

Proposition 4 establishes that a class of generalized quadratic utility investors will find UMMV portfolios to be optimal choices. Unlike our derivation of

Equation (17), the proposition does not assume normal distributions. The utility function may be interpreted as augmenting the quadratic utility with a preference for covariance with the factor, via the product term. Alternatively, the utility function may be written as $[1-b'F]W - a W^2$, a quadratic utility with "state dependent" utility that depends on F.

V. Conclusions

Mimicking portfolios for economic risk factors have long been useful in asset pricing research. In many applications the weights are assumed to be fixed over time, while in theory they should be functions of the economic state. We study mimicking portfolios where the weights depend on predetermined state variables, or conditioning information. This leads to extensions of results on conditional and unconditional mean variance efficiency, as developed by Hansen and Richard (1987), generalizations of the closed-form solutions for unconditionally mean variance efficient portfolios in Ferson and Siegel (2001), and conditional and unconditional versions of multifactor minimum variance portfolios, as described by Fama (1996). The relation between conditional and unconditional multifactor minimum variance portfolios is analogous to the relation between conditional and unconditional minimum variance efficient portfolios.

Under the assumption of normality, agents in Merton's (1973) model choose multifactor minimum variance portfolios given the agents' information set. We show that a class of generalized quadratic utility investors will choose unconditional multifactor minimum variance portfolios. The utility function may be interpreted as having state dependent risk aversion that depends on the factor. These results do not require normality.

Special cases of our solutions are maximum correlation portfolios that refine the "economic tracking portfolios" studied by Lamont (2001). In this case the correlation with an economic factor is maximized over the set of portfolio weight functions that may vary over time, depending on lagged conditioning variables.

We provide empirical examples to illustrate the implications of our results. Using a sample of asset returns, economic factors and conditioning information similar to Lamont (2001), we explore the potential advantage to constructing mimicking portfolios with the optimal, time-varying weights. The potential advantages can be large. A numerical example with a single risky asset and a riskless asset shows that we can more than double the correlation with an inflation factor. In Lamont's sample the potential improvement is more than 20%, abstracting from estimation error.

Estimation error in mimicking portfolio weights presents challenges for future research and for practical applications. We present simulation experiments that isolate the effects of parameter estimation error and error in specifying the form of the data generating process (DGP). The first type of error is present in all the approaches, but to a greater extent in the time-varying optimal solutions because they involve more parameters. The fixed-weight approaches (based on OLS regression) do not need to specify the DGP. If the time-varying weight approaches use the wrong DGP they are no longer optimal.

We find that the time-varying weight solutions are affected more by parameter estimation error than the fixed-weight approaches, and their performance suffers when the wrong DGP is assumed. The tradeoff is thus similar to the tradeoff between efficiency and robustness that is frequently encountered in econometric practice. We found examples where the higher correlations delivered by an optimal solution more than offset the negative effects of parameter estimation error. However, the actual sample performance of the fixed-weight approaches is typically superior. This leads us to conclude that in a setting where the correct DGP is known or specified as part of the model (e.g., when the forcing processes are a part of the model to be examined) the optimal solutions may be preferred in practice. Where robustness to the form of the DGP is also at issue, the simpler regression methods may be expected to perform better.

Appendix

Proof of Proposition 1:

We show that UMMV implies CMMV almost surely by contradiction. Assume that the portfolio R_p, with weights $w_p(Z)$, is not almost surely CMMV. Let Ω be the subset of possible values of Z for which R_p is not CMMV, a set with positive measure. Define the portfolio $R_w = w(Z)'R$, where the weights w(Z) are defined as follows. If $z \varepsilon \Omega$, then w(z) is CMMV with $w(z)'E(R|z) = E(R_p|z)$ and $w(z)'E\{RF|z\} = E(R_pF|z)$. Otherwise, $w(z) = w_p(z)$ for all values of z not in Ω. It follows that $Var(R_w|z) \leq Var(R_p|z)$ for all z, with strict inequality for $z \varepsilon \Omega$, while $E(R_p|Z)=E(R_w|Z)$ and $E(FR_p|Z)=E(FR_w|Z)$. By iterated expectations, this implies $E(R_p)=E(R_w)$ and $E(FR_p)=E(FR_w)$. Since the conditional means of the two portfolios are identical, $Var(R_w|Z) < Var(R_p|Z)$ implies $E(R_w^2|Z) < E(R_p^2|Z)$ with positive probability, which implies $E(R_w^2) < E(R_p^2)$, which implies $Var(R_w) < Var(R_p)$. Thus, R_p cannot be UMMV, and we have established that UMMV implies CMMV almost surely.

To show that CMMV does not imply UMMV, it suffices to find a portfolio that is CMMV but not UMMV. The example has three assets with returns $\{R_1,R_2,R_3\}$. The information set is chosen to have two points, so that Z=1 with probability 0.5 and Z=2 with probability 0.5. The asset returns are chosen to be conditionally independent of each other, given Z. Each asset has two possible returns, each value observed with conditional probability 0.5. When Z=1 the possible values are: $R_1=\{5\%, 15\%\}$, $R_2=\{4\%, 14\%\}$, $R_3=\{6\%, 16\%\}$. When Z=2 the possible values are: $R_1=\{5\%, 15\%\}$, $R_2=\{6\%, 16\%\}$, $R_3=\{4\%, 14\%\}$. For the factor, we choose the first asset, so that $F=R_1$. Consider the CMMV portfolio with the choice $c(Z)=E(R_2|Z)$, $d(Z)=E(FR_2|Z)$ and $x(Z)'\mathbf{1}=1$. This imposes three constraints for each Z. With three assets, the solution is generally unique. Thus R_2 is CMMV with this choice of c(Z) and d(Z). While R_2 is CMMV, it is not UMMV because it is dominated by the equally-weighted portfolio formed from assets 2 and 3 (which is UMMV). The variance of R_2 is 0.0026, while the variance of $(R_1+R_2)/2$ is 0.00125, a reduction of more than half. The constraints are satisfied in the sense that $E(R_2)=E\{c(Z)\}=E\{(R_1+R_2)/2\}=10\%$, and $E(FR_2)=E\{d(Z)\}=E\{F(R_1+R_2)/2\}=0.01$. QED

Proof of Proposition 2:

The proof follows by maximizing the squared conditional correlation, or equivalently $x(Z)'Cov(RF|Z)/[x(Z)'Cov(R|Z)x(Z)]^{1/2}$, over the choice of weight functions x(Z). The search for the optimal weights may be restricted to weight functions that satisfy Equation (7). Since the squared correlation is invariant to a constant linear transformation, the search may be limited, for each z, to the space of random variables of the form $d(z) + c(z)x(z)'R = d(z) + c(z)R'\Lambda\underline{1}/(\underline{1}'\Lambda\underline{1}) - \lambda_1(z)c(z)R'\Omega(z)\mu(z) - \lambda_2(z)c(z)R'\Omega(z)E(RF\,|\,z) = d(z) + c(z)U_1 - \lambda_1(z)c(z)U_2 - \lambda_2(z)c(z)U_3$, where $U_1 = R'\Lambda\underline{1}\,/\,(\underline{1}'\Lambda\underline{1})$, $U_2 = R'\Omega(z)\mu(z)$ and $U_3 = R'\Omega(z)E(RF\,|\,z)$. For given values of z, this expression may be recognized as the fitted values of a regression equation to predict the factor, F, with the regressors U_1, U_2 and U_3. The coefficients are $b(z)= [d(z),c(z),-\lambda_1(z)c(z),-\lambda_2(z)c(z)]'$. Letting $X=(1,U_1,U_2,U_3)'$ the coefficients satisfy $b(z) = [E\{XX'\,|\,Z\}]^{-1}E\{XF\,|\,Z\}$. The coefficients given in the proposition are the solution to this system of equations. QED.

The proof of the Corollary is a special case of Proposition 2, and proceeds by simply replacing d(z), c(z), $\lambda_1(z)$ and $\lambda_2(z)$ by the constants d, c, λ_1 and λ_2. QED.

Proof of Proposition 3:

The proposition follows directly from inspection of Equations (6) and (7). The first term of these equations gives the weight for $R_0^* = x_1(Z)'R$, with $x_1(Z) = \Lambda\underline{1}/(\underline{1}'\Lambda\underline{1})$. This is a portfolio with weights that sum to 1.0. In particular, R_0^* is the global minimum conditional second moment portfolio. The global minimum conditional second moment portfolio has some conditional mean return and some conditional covariance function with each factor. From equation (7) it is a CMMV portfolio, with target conditional mean and covariance with the factors equal to the values that correspond to the "unconstrained" global minimum second moment solution ($\lambda_1(Z)=\lambda_2(Z)=0$). The remaining terms of equation (6) and (7) describe portfolio weights that sum to zero, as can be seen by premultiplying the terms in square brackets by $\underline{1}'$. These two terms define the excess returns R_e^* and R_e^{*fj}, j=1,...,K. The scalar weights {$\lambda_1(Z)$ and $\lambda_{2j}(Z)$, j=1,...,K} on these excess returns are functions of Z in equation (7), when the solution is CMMV. In the UMMV solution of equation (6), these scalars are constants.

Proof of Proposition 4:

The proof of Proposition 4 does not require normality. First, we establish that if a portfolio weight function x(Z) maximizes the conditional expectation $E\{J(W_t,F_t)|Z_{t-1}\}$, subject to the constraints, then it must also maximize the unconditional expectation $E\{J(W_t,F_t)\}$, subject to the constraints. Suppose, by contradiction that another solution y(Z) maximizes the unconditional expectation and implies the indirect utility J(y), while x(Z) maximizes the conditional expectation and implies J(x). Then $E\{J(x) - J(y)|Z\} \geq 0$, implying $E\{J(x) - J(y)\} \geq 0$, contradicting that y(Z) maximizes the unconditional expectation, unless y(Z) = x(Z), almost surely. Using the specific form of the indirect utility we have $E\{J(W,F)\} = E\{W - aW^2 - b'FW\} = E(W) - a[Var(W) + E(W)^2] - b'[Cov(W,F) - E(W)E(F)]$. Maximizing this function is equivalent to minimizing the unconditional variance for a given unconditional mean return and covariances with the factors. The solution is therefore UMMV. QED.

Derivation of Equation (9):

The Lagrangian is

$$L(w) = \frac{1}{2}E\left[\left(R_w - r_f\right)^2\right] - \lambda_1\left[E\left(R_w\right) - c\right] - \lambda_2\left[E\left(R_w F\right) - d\right]$$
$$= \frac{1}{2}E\left[w^2(Z)\left(R - r_f\right)^2\right] - \lambda_1\left[r_f + E\left[w(Z)\left(R - r_f\right)\right] - c\right] - \lambda_2\left[r_f\mu_F + E\left[w(Z)\left(R - r_f\right)F\right] - d\right].$$

We proceed by perturbation, assuming that $w(Z)$ is the optimal solution and allowing $y(Z)$ to be any function for which expectations exist. Then we have $\frac{\partial}{\partial a}L\left(w + ay\right)\Big|_{a=0} = 0$, and hence

$$E\left[w(Z)y(Z)\left(R - r_f\right)^2\right] - \lambda_1 E\left[y(Z)\left(R - r_f\right)\right] - \lambda_2 E\left[y(Z)\left(R - r_f\right)F\right]$$
$$= E\left\{y(Z)E\left[w(Z)\left(R - r_f\right)^2 - \lambda_1\left(R - r_f\right) - \lambda_2\left(R - r_f\right)F \middle| Z\right]\right\} = 0$$

for all such functions $y(Z)$, which in turn implies that

$$E\left[w(Z)\left(R - r_f\right)^2 - \lambda_1\left(R - r_f\right) - \lambda_2\left(R - r_f\right)F \middle| Z\right] \overset{a.s.}{=} 0$$

from which we obtain the functional form of the optimal solution:

$$w(Z) = \frac{\lambda_1 E\left(R - r_f \mid Z\right) + \lambda_2 E\left[\left(R - r_f\right)F \middle| Z\right]}{E\left[\left(R - r_f\right)^2 \middle| Z\right]}.$$

Now observe that the conditional distribution of (R, F) given Z (using the joint normal distribution assumption) is

$$\left[\begin{pmatrix} R \\ F \end{pmatrix} \middle| Z\right] \sim N\left[\begin{pmatrix} \mu_R + \frac{\sigma_{RZ}}{\sigma_Z^2}\left(Z - \mu_Z\right) \\ \mu_F + \frac{\sigma_{FZ}}{\sigma_Z^2}\left(Z - \mu_Z\right) \end{pmatrix}, \begin{pmatrix} \sigma_R^2 - \frac{\sigma_{RZ}^2}{\sigma_Z^2} & \sigma_{RF} - \frac{\sigma_{RZ}\sigma_{FZ}}{\sigma_Z^2} \\ \sigma_{RF} - \frac{\sigma_{RZ}\sigma_{FZ}}{\sigma_Z^2} & \sigma_F^2 - \frac{\sigma_{FZ}^2}{\sigma_Z^2} \end{pmatrix}\right].$$

From this we immediately have

$$E\left(R-r_f \mid Z\right)=E\left(R \mid Z\right)-r_f=\mu_R-r_f+\frac{\sigma_{RZ}}{\sigma_Z^2}\left(Z-\mu_Z\right),$$

$$\begin{aligned} E\left[\left(R-r_f\right)F\middle|Z\right] &= Cov\left(R,F \mid Z\right)+E\left(R-r_f \mid Z\right)E\left(F \mid Z\right) \\ &= \sigma_{RF}-\frac{\sigma_{RZ}\sigma_{FZ}}{\sigma_Z^2}+\left[\mu_R-r_f+\frac{\sigma_{RZ}}{\sigma_Z^2}\left(Z-\mu_Z\right)\right]\left[\mu_F+\frac{\sigma_{FZ}}{\sigma_Z^2}\left(Z-\mu_Z\right)\right], \end{aligned}$$

and

$$\begin{aligned} E\left[\left(R-r_f\right)^2\middle|Z\right] &= Var\left(R \mid Z\right)+\left[E\left(R-r_f \mid Z\right)\right]^2 \\ &= \sigma_R^2-\frac{\sigma_{RZ}^2}{\sigma_Z^2}+\left[\mu_R-r_f+\frac{\sigma_{RZ}}{\sigma_Z^2}\left(Z-\mu_Z\right)\right]^2. \end{aligned}$$

Alternative Characterization:

Equation (17) provides an alternative representation for the CMMV solution of Equation (7), expressed in terms of covariances instead of second moment matrices. Here we provide a corresponding alternative formula for the UMMV optimal weight function. Let the conditional covariance matrix of the returns be $\Sigma = \Sigma(Z) = \mathrm{Cov}(R|Z)$.

The weight function (6) may be written as[17]

$$\begin{aligned} x(Z) = {} & \Sigma^{-1}\left(\underline{1} \quad \mu(Z)\right)\begin{pmatrix} \underline{1}'\Sigma^{-1}\underline{1} & \underline{1}'\Sigma^{-1}\mu(Z) \\ \mu'\Sigma^{-1}\underline{1} & 1+\mu'(Z)\Sigma^{-1}\mu(Z) \end{pmatrix}^{-1}\begin{pmatrix} 1 \\ -\lambda_1 \end{pmatrix} \\ & -\left[I-\Sigma^{-1}\left(\underline{1} \quad \mu(Z)\right)\begin{pmatrix} \underline{1}'\Sigma^{-1}\underline{1} & \underline{1}'\Sigma^{-1}\mu(Z) \\ \mu'(Z)\Sigma^{-1}\underline{1} & 1+\mu'\Sigma^{-1}\mu(Z) \end{pmatrix}^{-1}\begin{pmatrix} \underline{1}' \\ \mu'(Z) \end{pmatrix}\right]\Sigma^{-1}[\mathrm{Cov}(R,F'|Z)-\mu(Z)E(F'|Z)]\lambda_2. \end{aligned} \tag{A.1}$$

Simulation Details

[17] A derivation of this characterization is available by request to the authors.

The simulations are calibrated to the sample moments of the actual data, using the sample means and covariances and the sample regression functions on the lagged variables as parameters of the data generating process. One prominent feature of the data is the high degree of persistence in the lagged instruments. We capture this through a first order vector autoregression: $(Z_t - \bar{Z}) = A(Z_{t-1} - \bar{Z}) + U_{Zt}$, where the matrix A becomes a parameter of the simulation. We generate the artificial data using a parametric bootstrap approach. For a given factor, the $T \times (n + L + 1)$ matrix of sample residuals, $u = \{(u_{Rt}, u_{Zt}, u_{Ft})\}_t$ serves as the population of shocks, where the u_{Ft} are the unexpected factor realizations and the u_{Rt} are the regression residuals for the asset returns on the lagged instruments. We randomly resample rows from u with replacement. The lagged instruments are then built up recursively using the matrix A. The returns are formed as their conditional means, given the lagged Z, plus a shock.

When the data generating process features conditional heteroskedasticity driven by linear beats, we modify the simulations as follows. The regression $u_{Rt} = b_o u_{Ft} + B(Z_{t-1} - \bar{Z}) \otimes u_{Ft} + v_t$ determines the parameters (b_o,B) for the linear conditional betas. We resample from the residuals $\{(v_t, u_{Zt}, u_{Ft})\}_t$ and generate the draw for u_{Rt} from the regression function.

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Table 1

Regressions to Determine the Parmeters of the conditional means in the Simulations. The sample period is January 1947 through December, 1994 (576 months). Continuously compounded returns and factors are regressed monthly on the seven lagged instruments indicated in the columns. rflag is the lagged Treasury bill rate, term1y is the one-year term spread, defbond is the default-related bond yield spread, defcp is the spread of commercial paper over short term Treasury yields. Laginf, lagipx and lagrm are the lagged values over the preceding year for inflation, industrial production growth and the stock market return, respectively. The coefficient estimates are shown on the first line and Newey-West (1987) standard errors with 24 moving average terms are shown on the second line. Rsq is the adjusted coefficient of determination. Quarterly and annual factors are measured as overlapping monthly observations. All variables are in natural decimal units. The intercepts are not shown.

	rflag	term1y	defbond	defcp	lagipx	laginf	lagrm	Rsq
Panel A: Monthly returns								
market return	-2.82	-0.729	2.51	-0.022	-0.040	0.031	0.072	0.117
	0.877	0.462	0.547	0.454	0.023	0.043	0.012	
basic industries	-2.87	-0.965	2.72	-0.195	-0.048	0.058	0.083	0.108
	1.16	0.448	0.645	0.527	0.026	0.055	0.014	
capital goods	-3.08	-0.831	2.57	-0.296	-0.061	0.022	0.068	0.082
	1.16	0.531	0.826	0.506	0.036	0.069	0.015	
construction	-4.82	-0.958	2.91	-0.294	-0.123	0.170	0.105	0.086
	2.57	0.909	1.25	0.851	0.055	0.125	0.025	
consumer goods	-2.07	-1.01	3.12	-0.507	-0.067	-0.031	0.065	0.134
	0.951	0.406	0.605	0.595	0.027	0.047	0.014	
energy	-3.71	-0.588	1.36	0.101	-0.049	0.22	0.071	0.050
	1.30	0.692	0.685	0.508	0.033	0.07	0.022	
finance	-2.94	-1.17	3.22	-0.189	-0.043	0.032	0.079	0.110
	1.09	0.571	0.743	0.546	0.035	0.062	0.016	
transportation	-3.49	-1.61	3.81	-0.580	-0.028	0.064	0.080	0.104
	1.40	0.664	0.889	0.767	0.039	0.080	0.018	
utilities	-1.50	-1.60	2.80	0.211	-0.025	-0.061	0.035	0.112
	0.87	0.480	0.648	0.317	0.023	0.036	0.010	
longbond	-0.049	-1.88	2.47	0.737	0.051	-0.125	-0.009	0.166
	0.911	0.270	0.397	0.281	0.016	0.044	0.009	
intermediate bond	0.455	-1.54	1.80	0.456	0.041	-0.088	-0.013	0.247
	0.600	0.203	0.278	0.190	0.0107	0.0301	0.007	
one year bond	0.876	-0.467	0.607	0.148	0.012	-0.027	-0.004	0.508
	0.182	0.087	0.105	0.057	0.004	0.010	0.002	
junk bond	-0.826	-1.44	2.79	-0.133	0.034	-0.010	0.001	0.193
	0.672	0.263	0.391	0.203	0.013	0.027	0.007	

table 1, page 2

	rflag	term1y	defbond	defcp	lagipx	laginf	lagrm	Rsq
Panel B: Monthly Factor Growth Rates								
industrial production	-1.15	0.0453	0.511	0.183	-0.024	-0.044	-0.003	0.076
	0.266	0.125	0.174	0.129	0.011	0.022	0.005	
labor income	-0.622	0.274	0.022	0.151	-0.014	-0.026	0.002	0.002
	0.285	0.110	0.132	0.082	0.007	0.013	0.003	
inflation	0.410	0.136	-0.160	0.0343	-0.000	0.019	0.001	0.131
	0.175	0.0549	0.096	0.0671	0.005	0.015	0.001	
Panel C: Quarterly Factor Growth Rates								
industrial production	-3.10	-3.8E-05	1.43	0.408	-0.067	-0.120	-0.012	0.127
	0.726	0.322	0.423	0.345	0.028	0.062	0.014	
labor income	-1.56	0.392	0.181	-0.129	-0.036	-0.051	-0.006	0.026
	0.638	0.268	0.328	0.216	0.018	0.038	0.008	
inflation	1.28	0.342	-0.436	0.0487	-0.001	0.053	0.002	0.185
	0.512	0.178	0.287	0.194	0.015	0.044	0.004	
Panel D: Annual Factor Growth Rates								
industrial production	-9.29	1.35	3.30	-2.28	-0.272	-0.362	0.069	0.401
	2.60	0.810	1.46	0.859	0.096	0.222	0.027	
labor income	-5.39	0.371	2.01	-0.469	-0.038	-0.289	0.054	0.432
	1.58	0.391	0.846	0.550	0.049	0.095	0.017	
inflation	5.37	0.547	-1.75	-0.246	0.051	0.481	0.009	0.522
	1.74	0.392	0.919	0.466	0.034	0.136	0.013	

Table 2

Potential correlations of Mimicking Portfolios. The absolute correlations, in percent (%), between mimicking portfolios and economic factors are shown. The computations abstract from sampling error in the estimation of the mimicking portfolios weights, via a simulation with one million time-series observations. The data generating process features linear conditional expected returns, fixed factor means and homoskedasticity. The simulations are calibrated to the sample in Table 1. The Fix-weight solutions use a simple regression of the factor on returns where unconditional correlations are shown, and follow Lamont (2001), where expected conditional correlations are shown. The TV-weight solutions allow for optimal, time-varying weights that depend on the conditioning variables.

	Industrial Production		Labor Income		Inflation	
	Fix-weight	TV-weight	Fix-weight	TV-weight	Fix-weight	TV-weight
Panel A: Contemporaneous Monthly Factors						
Unconditional Correlation	9.0	15.3	26.0	30.4	17.9	18.3
Expected Conditional Correlation	16.3	16.3	31.0	31.0	21.1	21.1
Panel B: Monthly future Factors						
Unconditional Correlation	11.4	13.3	13.8	19.8	10.9	13.0
Expected Conditional Correlation	13.9	13.9	20.8	20.8	14.1	14.1
Panel C: Quarterly future Factors						
Unconditional Correlation	12.7	14.7	18.7	21.7	10.5	12.7
Expected Conditional Correlation	15.1	15.1	22.2	22.2	14.0	14.0
Panel D: Annual future Factors						
Unconditional Correlation	6.2	10.2	8.1	10.7	16.9	17.0
Expected Conditional Correlation	11.3	11.3	11.8	11.8	18.3	18.3
Averages:						
Unconditional	9.8	13.4	16.7	20.7	14.1	15.3
Conditional	14.2	14.2	21.5	21.5	16.9	16.9

Table 3

Potential correlations of Mimicking Portfolios. The expected absolute conditional correlations, in percent (%), between mimicking portfolios and economic factors are shown. The computations abstract from sampling error in the estimation of the mimicking portfolios weights, via a simulation with one million time-series observations. The data generating process features linear conditional expected returns, and heteroskedasticity driven by linear conditional betas. The simulations are calibrated to the sample in Table 1. The Fix-weight solutions follow Lamont (2001). The TV-weight solutions allow for optimal, time-varying weights. L refers to the number of lagged instruments. When L = 1 the term spread is the only instrument. When L = 2 the lagged, risk-free rate is included. When L = 3 we also include the lagged default spread.

	Industrial Production		Labor Income		Inflation	
	Fix-weight	TV-weight	Fix-weight	TV-weight	Fix-weight	TV-weight
Panel A: Contemporaneous Monthly Factors						
L = 1	17.8	25.0	28.8	36.4	28.8	36.3
L = 2	35.6	53.4	29.6	64.7	31.6	44.2
L = 3	24.4	48.7	28.5	70.0	26.1	45.3
Panel B: Monthly future Factors						
L = 1	17.9	22.8	20.2	25.6	20.7	23.5
L = 2	34.9	58.9	35.4	52.6	16.3	37.0
L = 3	28.0	60.8	30.1	56.6	15.0	42.2
Panel C: Quarterly future Factors						
L = 1	21.1	25.3	25.9	29.1	27.0	28.5
L = 2	50.4	61.7	39.7	62.5	15.0	27.9
L = 3	39.5	60.7	35.1	66.9	14.2	31.9
Panel D: Annual future Factors						
L = 1	35.3	37.0	36.3	39.6	38.3	39.1
L = 2	47.8	68.3	49.1	67.6	28.5	37.5
L = 3	38.0	67.1	43.8	67.1	30.0	38.0
Averages	32.9	49.1	33.5	53.2	24.3	36.0

Table 4

Sample performance of mimicking portfolios. The sample is the same as in Table 1. The estimates assume that the data generating process features linear conditional expected asset returns and homoskedasticity. The Fixed solution estimates the weights with an OLS regression of the factor on the vector of contemporaneous returns where unconditional correlations are shown, and uses the method of Lamont (2001) where average conditional correlations are shown. The TVW solutions allow weights that depend on the lagged conditioning variables in the previous month. The unconditional correlations are the sample correlations between the fitted mimicking portfolio returns and the factors. The Average conditional correlations are the correlations between the regression residuals of the returns and the factors, regressed on the lagged instruments.

	Industrial Production		Labor Income		Inflation	
	Fixed	TVW	Fixed	TVW	Fixed	TVW
Panel A: Monthly Contemporaneous Factors						
Unconditional Correlation	15.6	11.8	31.1	31.1	23.2	11.9
Average Conditional Correlation	17.1	17.1	31.3	31.3	22.8	22.8
Panel B: Monthly future Factors						
Unconditional Correlation	15.4	10.5	20.6	19.4	21.0	8.6
Average Conditional Correlation	14.5	14.5	20.9	20.9	15.1	15.1
Panel C: Quarterly future Factors						
Unconditional Correlation	19.2	14.3	26.4	24.1	23.5	7.4
Average Conditional Correlation	16.0	16.0	22.8	22.8	15.5	15.5
Panel D: Annual future Factors						
Unconditional Correlation	32.6	4.4	37.8	6.5	36.8	10.1
Average Conditional Correlation	14.7	14.7	15.6	15.6	25.6	25.6
Averages:						
Unconditional	20.7	10.3	29.0	20.3	26.1	9.5
Conditional	15.6	15.6	22.7	22.7	19.8	19.8

Table 5

Sample performance of mimicking portfolios. The sample is the same as in Table 1. The estimates assume that the data generating process features linear conditional expected asset returns, and heteroskedasticity driven by linear conditional betas. The Fixed solution estimates the weights with an OLS regression of the factor on the vector of contemporaneous returns where unconditional correlations are shown, and uses the method of Lamont (2001) where average conditional correlations are shown. The TVW solutions allow weights that depend on the lagged conditioning variables in the previous month. The unconditional correlations are the sample correlations between the fitted mimicking portfolio returns and the factors. The Average conditional correlations are the sample correlations between the regression residuals of the returns and the factors, regressed on the lagged instruments. The first row of numbers is based on the lagged term spread only. For the second row all seven lagged instruments are used.

	Industrial Production		Labor Income		Inflation	
	Fixed	TVW	Fixed	TVW	Fixed	TVW
Panel A: Monthly Contemporaneous Factors						
Unconditional	15.6	17.7	31.1	33.3	23.2	25.2
Correlation	15.6	27.0	31.1	30.7	23.2	29.3
Average Conditional	15.6	0.8	31.1	5.5	22.4	0.7
Correlation	17.1	0.1	31.3	4.9	22.8	19.6
Panel B: Monthly future Factors						
Unconditional	15.4	19.7	20.6	22.5	21.0	20.9
Correlation	15.4	23.0	20.6	32.6	21.0	24.8
Average Conditional	15.4	0.9	20.5	22.6	19.0	1.1
Correlation	14.5	1.5	20.9	3.2	15.1	0.3
Panel C: Quarterly future Factors						
Unconditional	19.2	22.7	26.4	28.2	23.5	27.2
Correlation	19.2	28.8	26.4	31.7	23.5	23.8
Average Conditional	18.9	21.4	26.3	6.2	21.3	6.3
Correlation	16.0	4.2	22.8	0.0	15.5	2.5
Panel D: Annual future Factors						
Unconditional	32.6	32.6	37.8	38.7	36.8	39.9
Correlation	32.6	27.8	37.8	23.2	36.8	22.6
Average Conditional	32.4	32.1	36.8	3.4	34.7	17.7
Correlation	14.7	3.1	15.6	5.8	25.6	4.5
Averages:						
Unconditional	20.7	24.9	29.0	30.1	26.1	26.7
Conditional	18.1	8.0	25.7	6.5	22.1	6.6

Table 6

Effects of parameter estimation error and error in specifying the form of the data generating process on the unconditional correlations of mimicking portfolios. The table shows the ratio of actual to potential absolute unconditional correlations with an industrial production growth factor, in percent. The potential correlations follow Table 3, assuming that the data generating process (DGP) features linear conditional mean asset returns and heteroskedsticity driven by linear conditional betas. In the colums labelled "Parameter error only," the simulations incorporate estimation error but the correct form of the DGP is used. In the columns labelled "DGP Error only," the simulations abstract from estimation error by using a sample with one million observations, but the portfolios incorrectly assume that the DGP features linear conditional asset returns and homoskedasticity, as in Table 2. In the columns labelled "Both DGP and Parameter Error," there is estimation error and the homoskedastic process is incorrectly assumed. The simulations are calibrated to the sample in Table 1. When L=1 only the lagged term spread is used as a conditioning variable. When L=2 the lagged risk-free rate is also included and when L=3 the lagged default spread is included as well. The FIX solution estimates the weights with an OLS regression of the factor on the vector of contemporaneous returns. The TVW solutions are the UMMV solutions that allow weights to depend on the lagged conditioning variables in the previous month.

	Parameter Error Only		DGP Error Only	DGP and Parameter Error
	FIX	TVW	TVW	TVW
Panel A: Contemporaneous Monthly Factor				
L=1	70.7	15.4	93.0	15.8
L=2	57.7	20.3	88.9	21.1
L=3	57.6	43.2	84.2	44.4
Panel B: Monthly future Factor				
L=1	71.4	26.2	94.7	59.6
L=2	56.4	23.1	85.3	63.3
L=3	54.5	30.2	83.9	30.9
Panel C: Quarterly future Factor				
L=1	79.5	42.6	93.2	41.4
L=2	67.3	34.4	84.5	37.0
L=3	60.2	45.4	82.4	46.1
Panel D: Annual future Factor				
L=1	92.2	68.2	98.1	67.7
L=2	70.7	17.4	83.8	16.5
L=3	58.8	19.0	85.3	12.7

Table 7

Effects of parameter estimation error on the expected conditional correlations of mimicking portfolios. The table shows the ratio of actual to potential absolute unconditional correlations with an industrial production growth factor, in percent. The potential correlations follow Table 3, assuming that the data generating process features linear conditional mean asset returns and heteroskedsticity driven by linear conditional betas. The simulations incorporate estimation error, and the correct form of the DGP is used. When L=1 only the lagged term spread is used as a conditioning variable. When L=2 the lagged risk-free rate is also included and when L=3 the lagged default spread is included as well. The FIX solution estimates the weights using the regression approach of Lamont. The TVW solutions are the CMMV solutions that allow the weights to depend on the lagged conditioning variables in the previous month.

	Parameter Error Only	
	FIX	TVW
Panel A: Contemporaneous Monthly Factor		
L=1	71.9	65.2
L=2	79.1	72.1
L=3	88.9	66.3
Panel B: Monthly future Factor		
L=1	73.2	33.5
L=2	89.3	75.0
L=3	97.9	74.8
Panel C: Quarterly future Factor		
L=1	77.3	72.1
L=2	75.4	78.4
L=3	95.1	74.7
Panel D: Annual future Factor		
L=1	90.7	88.8
L=2	85.1	78.1
L=3	82.3	77.5

Figure 1

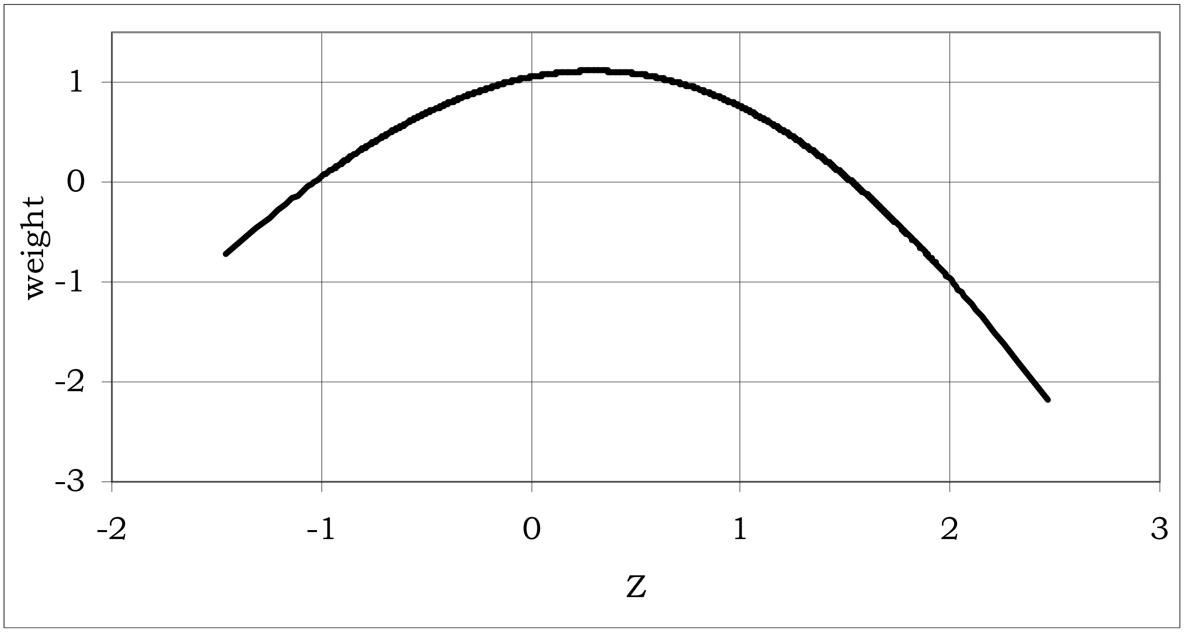

Figure 2