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TRADE IN GOODS AND FACTORS WITH  
INTERNATIONAL DIFFERENCES IN TECHNOLOGY

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ABSTRACT

A general model of trade caused by international differences in production technology is developed using techniques of duality theory. For the case of product-augmenting differences in technology, it is shown that there is a positive correlation between net export and technological superiority, such that a country will "on average" export goods for which the country has superior technology. If some factors are permitted to be internationally traded, it is demonstrated via this correlation that the volume of trade must increase. Thus unlike trade caused by factor endowment differences, goods trade caused by product-augmenting differences in production technology is always in this sense complementary with factor trade. For factor-augmenting technology differences, in the absence of factor trade the goods trade pattern is as if it was caused by factor endowment differences. With factor trade, goods trade and factor trade can then be either complements or substitutes.

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## 1. Introduction

There has in recent years been a considerable amount of effort devoted to generalizing factor proportions models of international trade. Results have been produced in higher dimensional models where a "we-can't-say-anything" attitude previously prevailed (see Ethier (1983) for a recent survey). Dixit and Woodland (1982), for instance, use duality techniques to derive certain relations which generalize standard two-by-two results such as the Heckscher-Ohlin theorem.<sup>1</sup> While these results do not permit us to predict the direction of trade in individual goods, they do demonstrate, for example, that a country will "on average" export goods, which in some sense, use intensively the country's abundant factors.

Far less work has been done in the direction of generalizing models of alternative (non-factor proportions) determinants of trade such as differences in production technology, increasing returns to scale, imperfect competition, and domestic distortions. Differences in production technology, although viewed as important in explaining US-EEC-Japan trade, have received little attention past fixed-coefficient Ricardian models.<sup>2</sup> Other theories such as those based on economies of scale are in an even more rudimentary state of development.

The first purpose of this paper is to try to develop a more general approach to trade due to international differences in production technology. The goal is to derive some relations between the direction of trade and differences in technology.

The second task of the paper revolves around factor mobility. The traditional presumption derived from the Heckscher-Ohlin model is that factor

movements and commodity trade are substitutes; that is, factor movements lead to a reduction in the volume of commodity trade. Recently, there has been a great deal of work on the welfare effects of factor mobility and it turns out that, when there are trade distortions, these welfare effects are often closely related to the effects of factor movements on the volume of commodity trade. Thus, the substitutability between factor movements and commodity trade plays a key role in the welfare analyses of Bhagwati (1973), Brecher and Alejandro (1977) and Markusen and Melvin (1979).

More recently, Svensson (1982) and Markusen (1983) explicitly address the question of whether or not commodity trade and factor trade are substitutes or complements. Svensson analyzes a very general factor proportions model and shows that commodity and factor trade are not always substitutes, but may sometimes be complements, depending upon whether traded and nontraded factors are "cooperative" or "non-cooperative". Markusen takes a very different approach by analyzing a series of extremely simply non-factor-proportions model in order to show how complementarity can arise. While suggestive, Markusen's models give little hint as to the generality of the results.<sup>3</sup> The second purpose of the paper is thus to examine whether or not general results concerning factor mobility and the volume of trade can be derived in a model of technological differences.

It is perhaps not very surprising that few results can be obtained for any arbitrary differences in technology. On the other hand, fairly strong results can be obtained for product augmenting technical differences. Results include the following. First, we can indeed derive a correlation coefficient which relates the differences in technology to net trade flows. This coefficient is the inner product of the vector of product-augmenting technology parameters with

the vector of net exports, and it is shown to be positive. This result states that countries will on average export goods for which they have superior technology. Second, it is demonstrated that the value of this correlation coefficient increases when trade in some factors is allowed, indicating an increase in the volume of trade.

Thus unlike the case of trade based on differences in factor endowments, trade based on product augmenting differences in technology is always, in a specific sense, complementary to trade in factors. The intuition behind this result is straightforward. With trade caused by such differences in technology, each country tends to have relatively high prices for those factors used intensively in export (import) industries. In the post-factor movement equilibrium, each country will therefore be observed to be relatively well endowed with factors used intensively in its export industries. In the Heckscher-Ohlin model, this is of course the cause of trade in goods whereas in the present model it is the consequence of trade in factors.

We also consider factor-augmenting technical differences. In the absence of factor trade, it is straightforward to show that the pattern of goods trade is as if it was caused by factor endowment differences. With factor trade results are less clear cut. Goods trade and factor trade can be either complements or substitutes. Roughly, if traded and non-traded factors are cooperative, and if the demand for traded factors is inelastic, goods trade and factor trade tend to be substitutes.

The paper is organized as follows. Section 2 deals with trade in goods only. Section 3 considers product-augmenting technical differences. Section 4 introduces trade in factors. Section 5 covers factor-augmenting technical differences both with and without factor trade, and Section 6 contains a summary and some conclusions.

## 2. Trade in Goods

The model has  $n$  goods ( $i=1, \dots, n$ ) and  $m$  factors ( $j=1, \dots, m$ ). We let  $y = (y_i)$  and  $v = (v_j)$  denote the  $n$ - and  $m$ -vectors of goods outputs and factor inputs, respectively. We take  $\alpha$  to be a vector of technology parameters.  $Y(\alpha)$  will denote the production possibility set such that  $(y, v)$  is feasible if and only if  $(y, v) \in Y(\alpha)$ .  $p$  and  $r$  are  $n$ - and  $m$ -vectors of goods and factor prices respectively.  $c_i$  denotes the consumption of good  $i$  while  $x_i = y_i - c_i$  denotes the net export of good  $i$ .  $c$  and  $x$  are then  $n$ -vectors of consumption and net exports. There are two countries, the home and the foreign. Foreign variables will be denoted with a \* subscript.

Throughout the paper we will assume that there is perfect competition, non-increasing returns to scale, and no distortions in production. Then we can use the domestic product or revenue function, which is given by

$$(1) \quad G(p, v, \alpha) = \text{Max}\{py : (y, v) \in Y(\alpha)\},$$

where  $py$  denotes the inner product  $\sum_i p_i y_i$ . The domestic product function is assumed to be twice differentiable. Given the other assumptions about production, differentiability is assured if there are at least as many factors (later as many non-traded factors) as goods. The vector of price derivatives of  $G$  gives outputs as per the usual duality properties,

$$(2) \quad y = G_p(p, v, \alpha) = (\partial G / \partial p_i).$$

(Subindices will denote derivatives throughout the paper.)

The demand side of the economy is summarized by a twice differentiable expenditure function of prices and the welfare level  $u$ ,

$$(3) \quad E(p, u) = \text{Min}\{pc : U(c) > u\}.$$

Price derivatives of  $E$  give commodity demands,

$$(4) \quad c = E_p(p, u) = (\partial E / \partial p_i).$$

The equilibrium of the economy can then be represented by the budget constraint

$$(5) \quad E(p, u) = G(p, v, \alpha),$$

stating that expenditure equals domestic product. Equation (5) expresses the welfare level as an implicit function of  $(p, v, \alpha)$  and hence we can write

$$(6) \quad u = H(p, v, \alpha).$$

Equation (6) plus equations (2) and (4) allow us in turn to write the net export function as

$$(7) \quad X(p, v, \alpha) = G_p(p, v, \alpha) - E_p(p, H(p, v, \alpha)).$$

Trading equilibrium is then given by the condition that the two countries' net exports sum to zero,

$$(8) \quad X(p, v, \alpha) + X^*(p, v^*, \alpha^*) = 0.$$

Assume that the two countries have identical preferences, endowments, and technology ( $v = v^*$ ,  $\alpha = \alpha^*$ ). It then follows that (A)  $X(\ )$  and  $X^*(\ )$  are identical functions, (B)  $x = x^* = 0$  at the free-trade equilibrium, and (C) free-trade prices equal autarky prices. It will be very practical to start from this initial zero trade equilibrium.

To find how differences in technology influence the pattern of trade in goods, we now let the technology change by the vector  $d\alpha$  in the home country. Differentiating (7) and (8) we have respectively

$$(9) \quad X_p dp + X_\alpha d\alpha = dx, \text{ and}$$

$$(10) \quad X_p dp + X_\alpha d\alpha + X_p^* dp = 0; \quad X_\alpha d\alpha/2 = -X_p dp,$$

where the second equation in (10) follows from the fact that  $X_p = X_p^*$  initially. Substituting (10) into (9), the latter becomes

$$(11) \quad dx = X_\alpha d\alpha/2,$$

that is, home country net export will be one half of the initial excess supply at constant prices.

The result given in equation (11) is illustrated in Figure 1 for the two-goods case. Here  $X_1(\ ) = X_1^*(\ )$  are the two countries' initial identical net export supply curves for good 1, say. The initial equilibrium is at point A where there is no trade. Suppose that technical change in good 1 shifts the home country's export supply curve to the right as shown. The horizontal shift is equal to AB or  $X_{1\alpha} d\alpha$ . But there is now excess supply in the market for good 1 and its relative price must fall to reestablish equilibrium. With the countries initially identical, the local change in  $p_1/p_2$  must be such that each country absorbs exactly one half of the initial excess supply. Thus the new equilibrium occurs at price C and a value of  $x_1$  equal to CD or exactly half the initial excess supply. We then have the equilibrium  $x_1$  given by  $X_{1\alpha} d\alpha/2$  as in (11), following the price adjustment.

We now turn our attention to the underlying determinants of  $X_\alpha d\alpha$ . Differentiating (7) at constant prices gives us

$$(12) \quad X_\alpha d\alpha = G_{p\alpha} d\alpha - E_{pu} du = G_{p\alpha} d\alpha - (E_{pu}/E_u) E du.$$



Figure 1

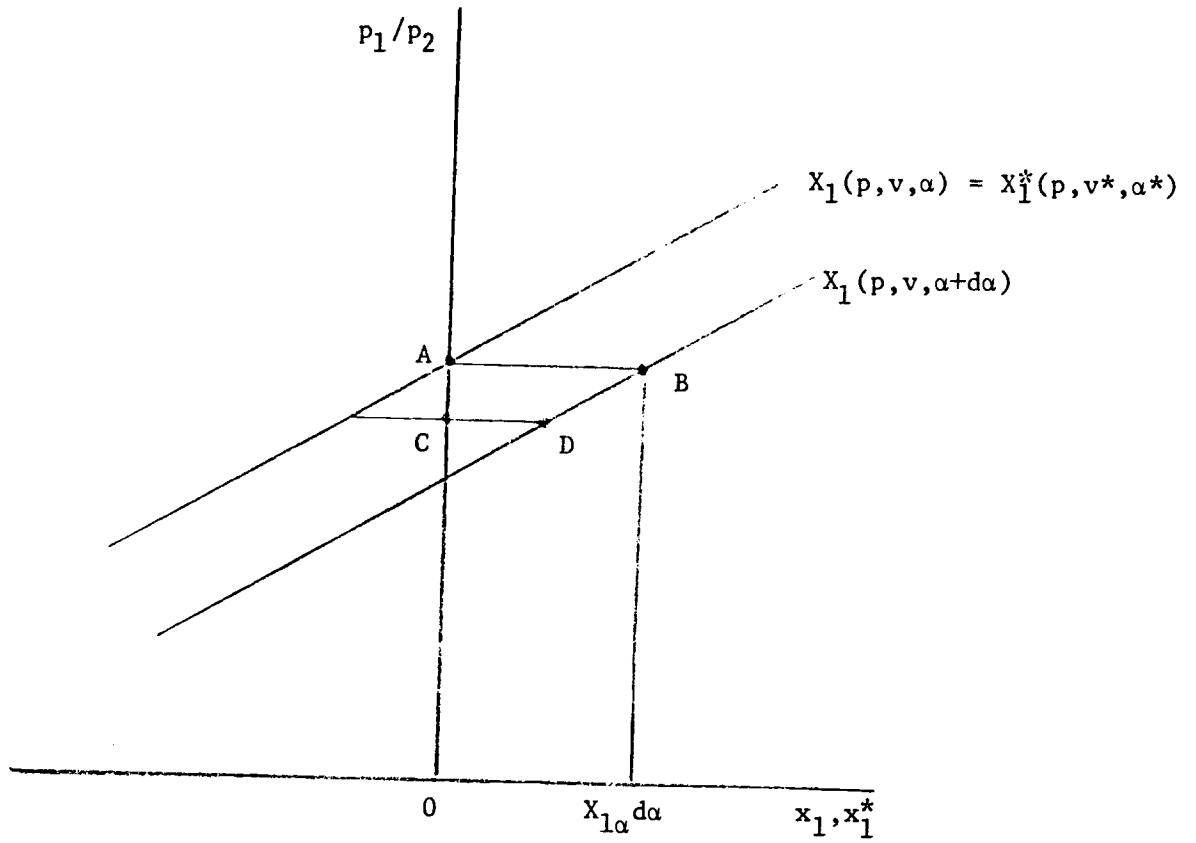
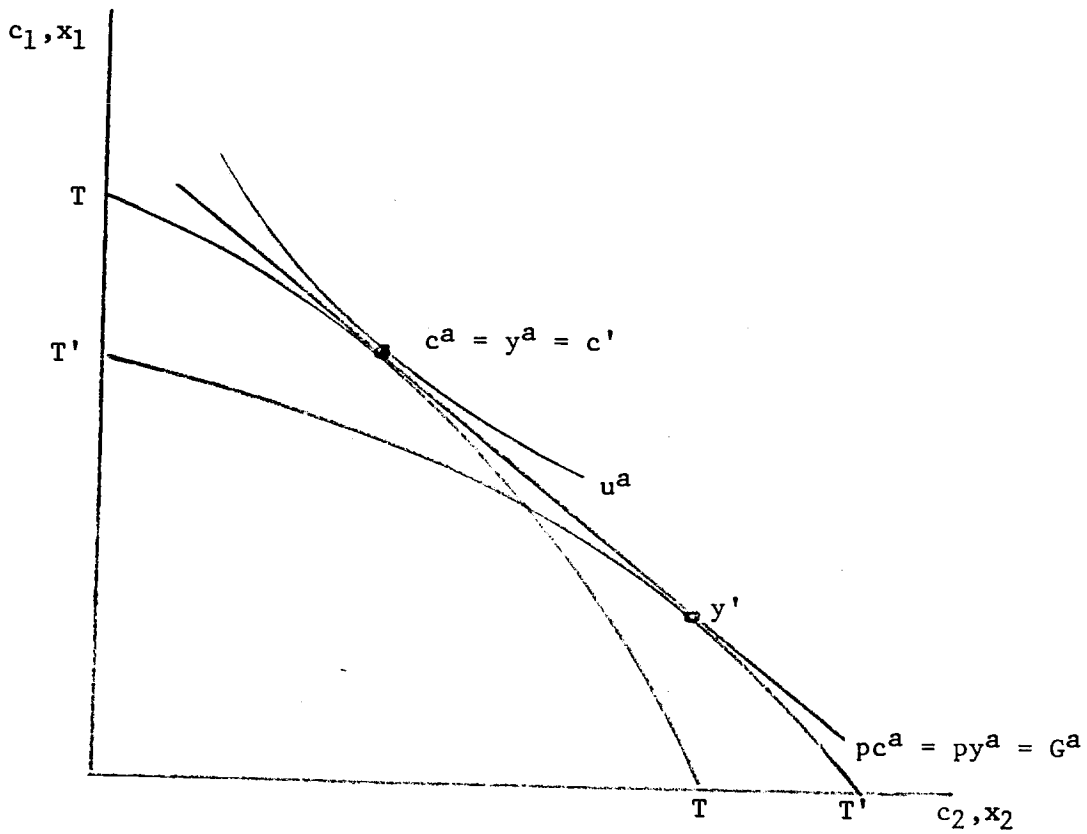


Figure 2



But  $(E_{pu}/E_u) = C_Y$  where  $C_Y$  denotes the vector of marginal propensities to consume. Differentiating (5) at constant prices we also have  $E_u du = G_\alpha d\alpha$  and so

(12) becomes

$$(13) \quad X_\alpha d\alpha = G_{p\alpha} d\alpha - C_Y G_\alpha d\alpha.$$

In order to abstract from consumption effects, let us consider only compensated changes in technology, defined as  $G_\alpha d\alpha = 0$ . We will in other words consider only changes which do not affect the value of production at constant prices. There is then no consumption response at constant prices so that by (13) (11) reduces to

$$(14) \quad dx = G_{p\alpha} d\alpha/2.$$

An interpretation of these compensated technology changes is given in Figure 2 for the two goods case. TT illustrates the initial production frontier for both countries with the initial equilibrium by  $c^a = y^a$ . Initial prices are given by the tangent to TT at  $c^a$  and the initial value of G is given by  $py^a$ . The assumption of compensating technical changes amounts to the assumption that the new production frontier must be tangent to  $py^a$ ; that is, at the initial prices the value of G must not change. Such a compensating change is illustrated in Figure 2 by the shift of TT to TT' and the corresponding shift of production to point  $y'$ . The value of G is thus unchanged at constant prices as is the consumption point ( $c' = c^a$ ) and the initial welfare level ( $u^a$ ). The net export vector at the initial prices is given by  $x' = y' - c'$ . The equilibrium net export vector will by (14) be half of this.

The assumption of compensating technical change eliminates demand effects, even without assuming homothetic preferences, and assures us a one-to-one

relationship between production changes and excess supply changes. This assumption is, of course, restrictive, but it may be justified on the grounds that we are inquiring into the role of production technology, not the role of demand, in determining trade flows. Perhaps we could also argue that this assumption is not significantly more restrictive than the assumption of homothetic preferences which is traditionally used in these types of problems to neutralize demand effects.<sup>4</sup>

The final point to note is that (14) unfortunately does not tell us a great deal about the relationship between technology and the direction of trade flows. This is true even if we are willing to assume no joint production, to identify technology parameters with production sectors (hence to let  $\alpha_i$  be a technology parameter in sector  $i$ ), and assume that all diagonal elements of  $G_{p\alpha}$  are positive ( $\partial y_i / \partial \alpha_i > 0$ ) and all off diagonal elements are negative ( $\partial y_i / \partial \alpha_j < 0, i \neq j$ ). This means that a technical improvement in sector  $i$  would increase output in sector  $i$  and decrease it in all other sectors. These assumptions do not, for example, imply that  $G_{p\alpha}$  is positive definite and more generally do not imply any systematic relationship between  $dx$  and  $d\alpha$ . More specific results will therefore require specific technology changes, a problem which will form the subject matter of Section 3.

### 3. Product-augmenting Technical Change

We shall introduce product-augmenting technical change in a somewhat general way, allowing for joint production. Let  $\bar{Y}$  be a given production possibility set with the corresponding domestic product function  $\bar{G}(p,v)$  and supply function  $\bar{y}(p,v) = \bar{G}_p(p,v)$ . Let  $\alpha = (\alpha_i)$  be an  $n$ -vector. We shall refer to a change  $d\alpha_i$  as a product-augmenting technical change in the production of good  $i$ . More precisely, let  $D(\alpha) = [\delta_{ij}\alpha_i : \delta_{ij} = 1, i = j; \delta_{ij} = 0, i \neq j]$  be the  $(n \times n)$  diagonal matrix whose diagonal consist of the vector  $\alpha$  ( $\delta_{ij}$  is the Kronecker delta.) Then define the production possibility set

$$(15) \quad Y(\alpha) = \{(y,v) : y = D(\alpha)\bar{y} = (\alpha_i\bar{y}_i), (\bar{y},v) \in \bar{Y}\}.$$

Then we define the domestic product function

$$(16) \quad G(p,v,\alpha) = \max\{p'y : (y,v) \in Y(\alpha)\},$$

where  $p'y$  denotes the inner product  $\sum_i p_i y_i$ , and the prime denotes transpose. Henceforth, all vectors will be taken as column vectors, to keep track of the matrix manipulations. We realize from the definition of  $Y(\alpha)$  that we have

$$(17) \quad G(p,v,\alpha) = \bar{G}(D(\alpha)p,v) \text{ and}$$

$$(18) \quad y(p,v,\alpha) = G_p(p,v,\alpha) = D(\alpha)\bar{G}_p(D(\alpha)p,v) = D(\alpha)\bar{y}(D(\alpha)p,v).$$

Hence, product-augmenting technical change enters very much as price changes, since  $D(\alpha)p = (\alpha_i p_i)$ .

Equation (14) above gave us an expression for the net export vector for goods. We differentiate (18) to get<sup>5</sup>

$$(19) \quad G_{p\alpha} = D(\bar{G}_p) + D(\alpha)\bar{G}_{pp}D(p) = D(y) + G_{pp}D(p),$$

where  $D(y)$  is the diagonal matrix of the vector  $y$ , etc., and where we without restriction have taken the initial value of all  $\alpha_i$  to equal unity, for which case  $\bar{G}_p = G_p = y$ ,  $D(\alpha) = [\delta_{ij}] = I$ , the identity matrix, and  $\bar{G}_{pp} = G_{pp}$ . (This can always be done by choosing physical units of goods appropriately.) Using (19), (14) now becomes

$$(20) \quad dx = G_{p\alpha} d\alpha/2 = D(y)d\alpha/2 + G_{pp} D(p)d\alpha/2.$$

Finally, we can pre-multiply both sides of (20) by the row  $n$ -vector

$$(2D(p)d\alpha)' = (2p_i \alpha_i)' \text{ to get}$$

$$(21) \quad 2d\alpha' D(p)dx = d\alpha' D(p)D(y)d\alpha + d\alpha' D(p) G_{pp} D(p)d\alpha > 0.$$

The first of the additive terms on the right-hand side of (21) equals  $\sum p_i y_i (d\alpha_i)^2$  which is strictly positive if  $y_i > 0$ , which we may safely assume. The second term is a quadratic form in  $G_{pp}$  which is a positive semi-definite matrix, since the domestic product function is convex in prices. This term is non-negative, and expression (21) is hence strictly positive.

The left-hand side of (21), which equals  $\sum_i d\alpha_i p_i dx_i$ , has a straightforward interpretation as a correlation coefficient relating differences in technology to the direction of trade. (21) notes that positive elements of  $d\alpha$  (technical superiority) are associated with positive elements of  $dx$  (net exports) and vice versa for negative  $d\alpha$ . (21) thus says that "on average" the country will export those goods ( $dx_i > 0$ ) for which it has superior technology ( $d\alpha_i > 0$ ). Hence we have a proposition about the direction of trade which is similar to propositions about the direction of trade with differences in factor endowments found in Dixit and Woodland (1982). Indeed, the present results are stronger or clearer than those of Dixit and Woodland in that differences in production technology can be unambiguously defined, while differences in factor intensities can unfortunately be defined only in a somewhat tautological fashion.

4. Trade in Goods and Factors

It is well known that factor prices are generally not equalized by trade when there are international differences in technology. From the standard properties of  $G$ , we have, with goods and no factor trade,

$$(22) \quad r = G_v(p, v, \alpha); \quad r^* = G_v^*(p, v^*, \alpha^*) \text{ and}$$

$$(23) \quad dr - dr^* = G_{vp} dp + G_{v\alpha} d\alpha - G_{vp}^* dp = G_{v\alpha} d\alpha,$$

since  $G_{vp} = G_{vp}^*$  initially. Differences in factor prices are determined by the matrix  $G_{v\alpha}$  whose elements depend on underlying factor intensities, elasticities of substitution, and the form of technical change.

Now, to introduce trade in factors as well as in goods, let  $v$  be decomposed into two sub-vectors  $k$  and  $\ell$  with corresponding prices  $r$  and  $w$ .  $k$  will be traded factors and will be henceforth referred to as capital, while  $\ell$  are assumed to be immobile factors and will be referred to as labour. Ownership of  $k$  is assumed to remain in the country of origin and hence all foreign factor income is assumed to be repatriated.  $k$  will denote the home country's endowment of capital while  $\tilde{k}$  will denote the capital actually used as input in the home country. Net exports of capital will then be given by  $z = k - \tilde{k}$ . Similar notation applies to the foreign country. Factor mobility requires us to define a modified revenue function, the national product function, as

$$(24) \quad \tilde{G}(p, r, k, \ell, \alpha) = \text{Max}\{G(p, \tilde{k}, \ell, \alpha) + r(k - \tilde{k}) : \tilde{k} \geq 0\}.$$

$\tilde{G}$  thus corresponds to the usual concept of national product while  $G$  (the same function employed above) corresponds to domestic product.

From (24) we can derive a capital input function  $\tilde{k}(p, r, \ell, \alpha)$  which gives the capital actually employed at home as a function of  $p$ ,  $r$ ,  $\ell$ , and  $\alpha$ . It will

fulfill

$$(25) \quad G_k(p, r, \bar{k}, \ell, \alpha) = r.$$

We assume that it is unique and differentiable.<sup>6</sup> This in turn gives us a capital export function,

$$(26) \quad Z(p, r, k, \ell, \alpha) = k - \bar{k}(p, r, \ell, \alpha).$$

The capital import and export functions are illustrated in Figure 3 for the case with one capital good.  $G_k$  gives the marginal product of capital with the capital input determined by the intersection of  $G_k$  and  $r$ . At constant prices, the endowment of capital ( $k$ ) determines net exports ( $k - \bar{k}$ ) but not the capital employed at home.

Note from (24) that we have the relationship

$$(27) \quad \tilde{G}(p, r, k, \ell, \alpha) = G(p, \bar{k}(p, r, \ell, \alpha), \ell, \alpha) + \tilde{r}(k - \bar{k}(p, r, \ell, \alpha)).$$

This in turn gives us

$$(28) \quad y = \tilde{G}_p = G_p + G_k \bar{k}_p - r \bar{k}_p = G_p \text{ and}$$

$$(29) \quad z = \tilde{G}_r = G_k \bar{k}_r + (k - \bar{k}) - r \bar{k}_r = k - \bar{k},$$

Where both equations follow from (25).

The budget constraint for the economy is now given by  $E = \tilde{G}$ , and the goods export function by  $\tilde{X} = G_p - E_p$ . Equilibrium conditions are given by

$$(30) \quad \tilde{X}(p, r, \alpha) + \tilde{X}^*(p, r, \alpha^*) = 0 \text{ and}$$

$$(31) \quad Z(p, r, \alpha) + Z^*(p, r, \alpha^*) = 0,$$

Figure 3

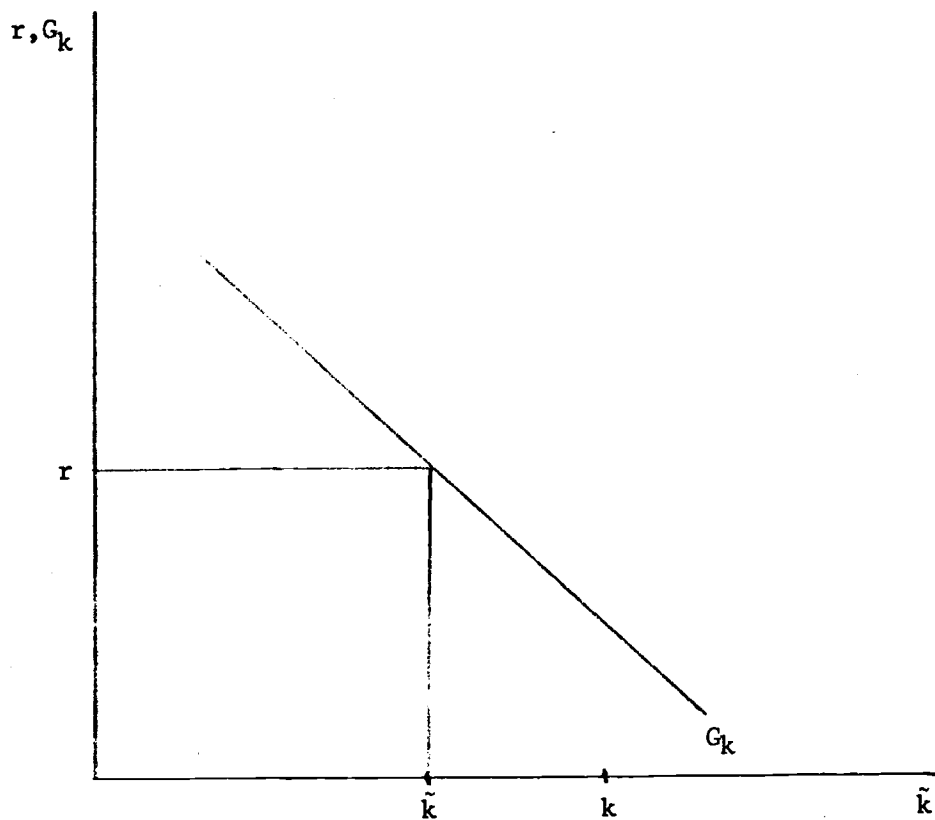
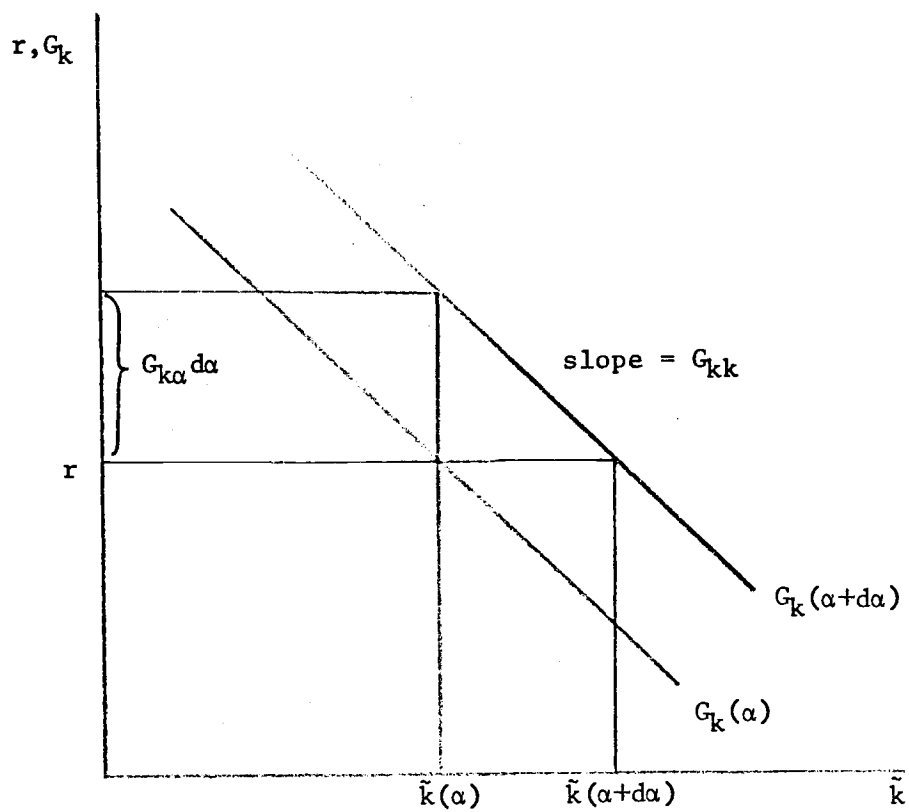


Figure 4





where for simplicity we have dropped  $k$  and  $l$  from the goods and capital export functions.

Differentiating  $\tilde{X}$  and  $Z$  we have

$$(32) \quad d\tilde{x} = \tilde{X}_p dp + \tilde{X}_r dr + \tilde{X}_\alpha d\alpha \text{ and}$$

$$(33) \quad dz = Z_p dp + Z_r dr + Z_\alpha d\alpha$$

Noting again that  $\tilde{x} = \tilde{x}^* = 0$  and  $z = z^* = 0$  initially, differentiation of (30) and (31) gives us

$$(34) \quad \tilde{X}_\alpha d\alpha/2 = -\tilde{X}_p dp - \tilde{X}_r dr \text{ and}$$

$$(35) \quad Z_\alpha d\alpha/2 = -Z_p dp - Z_r dr.$$

Substituting (34) and (35) into (32) and (33) we have equations similar to (11) above,

$$(36) \quad d\tilde{x} = \tilde{X}_\alpha d\alpha/2 \text{ and}$$

$$(37) \quad dz = Z_\alpha d\alpha/2 = -\tilde{k}_\alpha d\alpha/2.$$

We can now examine  $\tilde{X}_\alpha$ , recalling that  $\tilde{X} = \tilde{G}_p - E_p$ . We use the same procedure followed with equation (12) by considering only compensated changes such that there are no consumption effects ( $\tilde{G}_\alpha d\alpha = 0$ ). From (27), we see that

$$(38) \quad \tilde{G}_\alpha d\alpha = (G_k \tilde{k}_\alpha + G_\alpha - r\tilde{k}_\alpha) d\alpha = G_\alpha d\alpha,$$

so that the same set of compensating changes is being examined with and without factor movements. Similar to (14), this procedure allows us to rewrite (36) as

$$(39) \quad d\tilde{x} = \tilde{G}_{p\alpha} d\alpha/2.$$

Since  $\tilde{G}_p = G_p$  (equation (28)), we have from (27) that

$$(40) \quad d\tilde{x} = \tilde{G}_{p\alpha} d\alpha/2 = G_{p\alpha} d\alpha/2 + G_{pk} \tilde{k}_\alpha d\alpha/2.$$

Since the same set of compensating changes is being considered, we can subtract (14) from (40) to get the difference in goods trade with and without factor mobility,

$$(41) \quad d\tilde{x} - dx = G_{pk} \tilde{k}_\alpha d\alpha/2.$$

$G_{pk}$  can be thought of as a matrix of generalized Rybczynski effects (Dixit and Woodland (1982), Svensson (1982)). Element  $G_{ij}$  gives the change in the output of  $y_i$  at constant prices in response to a change in the amount of  $k_j$  used domestically.  $\tilde{k}_\alpha$  is a matrix giving the effects at constant  $(p,r)$  of technical change on the domestic use of capital. From (25) above, we get

$$(42) \quad \tilde{k}_\alpha = -G_{kk}^{-1} G_{k\alpha}.$$

assuming that  $G_{kk}$  is of full rank and invertible, an assumption generally valid given minimal substitution possibilities in production and consumption.

Substituting this last equation into (41), we have

$$(43) \quad d\tilde{x} - dx = -[G_{pk} G_{kk}^{-1} G_{k\alpha}] d\alpha/2$$

The relationship in (42) is illustrated in Figure 4, where the slope of  $G_k$  is  $G_{kk}$ .  $G_k$  is shifted up by  $\alpha$ , with the shift equal to  $G_{k\alpha} d\alpha$ . At a constant  $r$ , the horizontal shift  $\tilde{k}_\alpha d\alpha$  is then equal to  $G_{kk}^{-1} G_{k\alpha} d\alpha$ .

The bracketed matrix in (43), which is formed by the product of three matrices, is a matrix which maps the changes in technology into the changes in

goods trade.  $G_{kk}$  is by assumption negative definite and hence  $G_{kk}^{-1}$  is also negative definite. Unfortunately, there is no simple relationship between  $G_{pk}$  and  $G_{k\alpha}$  for arbitrary forms of technical change. Thus the bracketed matrix in (43) has no obvious properties which would allow us to advance a simple proposition about the relationship between technology and trade flows. We will therefore again examine the specific form of product-augmenting technical change in order to try to derive such a relationship.

Hence, assuming product-augmenting technical change, from (17), we know that

$$(44) \quad G_k = \bar{G}_k \text{ and } G_{k\alpha} = \bar{G}_{kp} D(p), \text{ hence}$$

$$(45) \quad \tilde{k}_\alpha = -G_{kk}^{-1} G_{kp} D(p),$$

where we exploit that  $\bar{G}_{kp} = G_{kp}$  since  $D(\alpha) = I$ . We can substitute (45) into (41) to give us

$$(46) \quad \tilde{dx} - dx = G_{pk} \tilde{k}_\alpha d\alpha / 2 = -G_{pk} G_{kk}^{-1} G_{kp} D(p) d\alpha / 2.$$

Similarly to (21), we can now pre-multiply (46) by  $2(D(p)d\alpha)'$  and get

$$(47) \quad 2d\alpha' D(p) (\tilde{dx} - dx) = -d\alpha' D(p) G_{pk} G_{kk}^{-1} G_{kp} D(p) d\alpha \\ = -(G_{kp} D(p) d\alpha)' G_{kk}^{-1} (G_{kp} D(p) d\alpha) > 0.$$

This expression is positive for all  $G_{kp} D(p) d\alpha \neq 0$ , since  $G_{kk}$  and hence  $G_{kk}^{-1}$  is negative definite given the assumption of full rank discussed above. We note from (44) that the condition  $G_{kp} D(p) d\alpha \neq 0$  is

$$(48) \quad G_{k\alpha} d\alpha \neq 0,$$

which is that the price  $r$  of capital, at constant goods prices and in the absence of trade in capital, should not be unchanged for the technical change considered. Hence, under (48) we rewrite (47) as

$$(49) \quad d\alpha \tilde{D}(p) dx > d\alpha D(p) dx > 0,$$

where the right-hand inequality is our result (21) above.

The inequalities in (49) give us a very strong result. They state that, with factor trade, the correlation between commodity exports and technology differences exceeds the same correlation without factor mobility. Since the same  $d\alpha$  is being considered in both cases, (49) states that  $d\alpha_i < 0$  is on average associated with a larger level of exports of  $x_i$  with factor mobility than without. Similarly  $d\alpha_i < 0$  is associated on average with a larger volume of imports of  $x_i$  with factor mobility. (49) thus implies that factor mobility leads to an increase in the volume of goods trade. In the present model, factor trade and commodity trade are therefore, in this sense, complements.

We can also briefly examine the pattern of factor trade by substituting the expression for  $\tilde{k}_\alpha$  in (45) into (37) to get

$$(50) \quad dz = \tilde{k}_\alpha d\alpha / 2 = G_{kk}^{-1} G_{kp} D(p) d\alpha / 2.$$

Now pre-multiply (50) by  $2(D(p)d\alpha)^{-1} G_{pk}$  to get

$$(51) \quad 2d\alpha \tilde{D}(p) G_{pk} dz = (G_{kp} D(p) d\alpha)^{-1} G_{kk}^{-1} (G_{kp} D(p) d\alpha) < 0,$$

which expression is negative for (48), by the previous argument.  $G_{pk}$  is the matrix of capital Rybczynski effects on output. In line with Dixit and Norman (1980) (see also Dixit and Woodland (1982) and Svensson (1982)) we might define

generalized factor intensities in a somewhat tautological way so that  $\partial^2 G / \partial p_i \partial k_j = \partial y_i / \partial k_j > 0$  is taken to mean that good  $i$  is intensive in the use of capital  $j$ . (51) can then be interpreted as stating that the home country will "on average" import factors ( $dz_j < 0$ ) used intensively in goods in which production the home country has technical superiority ( $da_i > 0$ ).

Consider for example a simple two-good two-factor example in which good 1 is capital intensive and is the good in which we have technical superiority ( $da_1 > 0$ ,  $da_2 < 0$ ,  $G_{k1} > 0$ ,  $G_{k2} < 0$ ). With only one factor (capital) mobile, we have

$$(52) \quad G_{kp} D(p) d\alpha = [(+)(-)] \begin{bmatrix} (+) \\ (-) \end{bmatrix} > 0$$

For (51) to hold, we must therefore have  $dz < 0$ . Factor mobility must lead to inflow of the factor used intensively in the production of the technically superior good (the export good). Following the factor movements, each country will be relatively well endowed with the factor used intensively in its export industry.

5. Factor-augmenting Technical Change

Finally we shall examine the case of factor-augmenting technical change.

Let  $\alpha = (\alpha_j)$  be an  $m$ -vector, where  $\alpha_j$  is the coefficient by which factor  $j$  is augmented. Then we can write

$$(53) \quad G(p, v, \alpha) = \bar{G}(p, D(\alpha)v),$$

where  $\bar{G}(p, v)$ , as in Section 3, is the domestic product function corresponding to a given productive possibility for  $\bar{Y}$ , and where we recall that  $D(\alpha)v = (\alpha_j v_j)$ .<sup>7</sup>

From (53) we get

$$(54) \quad G_p = \bar{G}_p \text{ and}$$

$$(55) \quad \bar{G}_{p\alpha} = \bar{G}_{pv} D(v).$$

Choosing the initial levels of the technology parameters equal to unity, so that  $D(\alpha) = I$ , we have  $\bar{G}_{pv} = G_{pv}$ , and the relation (14) will read

$$(56) \quad dx = G_{pv} D(v) d\alpha / 2.$$

This implies that, with no factor trade, the pattern of goods trade is exactly as if factor endowments have changed by  $dv = D(v)d\alpha = (v_j d\alpha_j)$  and the situation is as in Dixit and Woodland (1982). If we interpret the positive signs of the elements of Rybczynski matrix  $G_{pv}$  as indicating generalized factor intensities, we can interpret (56) as stating that the home country will export goods intensive in factors for which it has technological superiority.

We cannot derive any such straightforward result as that for product-augmenting technical differences. Let us however look at a simple case where with no factor trade, the equation

$$(57) \quad dx = G_{pk} D(k) d\alpha_k / 2 + G_{pl} D(l) d\alpha_l / 2$$

$(+) \quad (+) \quad (-) \quad (-)$

refers to a good (subset of goods) that is capital intensive ( $G_{pk} > 0$ ) and labor non-intensive ( $G_{pl} < 0$ ), say the two-good specific factors model, where (57) refers to good 1, capital is the factor specific to good 1 and labor is the factor specific to good 2. The vector of technology parameters is here decomposed into the subvectors  $\alpha_k$  and  $\alpha_l$ ,  $\alpha = (\alpha_k, \alpha_l)$ , corresponding to the decomposition of factor endowments  $v = (k, l)$  into traded factors, capital, and the non-traded factors, labor. Furthermore, let us assume that we have

$$(58) \quad d\alpha_k > 0 \text{ and } d\alpha_l < 0,$$

that is, the home country has a superiority for capital and disadvantage for labor. Clearly, it will export the capital intensive good.

How does the goods trade pattern differ with trade in capital? We have

$$(59) \quad d\tilde{x} - dx = G_{pk} \tilde{k}_\alpha d\alpha / 2.$$

$(+)$

If capital import increases ( $\tilde{k}_\alpha d\alpha > 0$ ), export of capital intensive goods will increase with factor trade, and factor trade and goods trade will be complements. If capital import decreases ( $\tilde{k}_\alpha d\alpha < 0$ ), factor trade and goods and goods trade will be substitutes.

To see what determines whether capital import increases or decreases, we note that from (53) we have

$$(60) \quad G_k = D(\alpha_k) \bar{G}_k \text{ and}$$

$$(61) \quad G_{k\alpha} d\alpha = D(\bar{G}_k) d\alpha_k + D(\alpha_k) [\bar{G}_{kk} D(k) d\alpha_k + \bar{G}_{kl} D(l) d\alpha_l].$$

Then we can, by using (42), write the change in capital import, at constant goods prices and rentals, as

$$\begin{aligned}
 (62) \quad \tilde{k}_\alpha d\alpha &= [-G_{kk}^{-1}D(r) - D(k)]d\alpha_k - G_{kk}^{-1}G_{k\ell}D(\ell)d\alpha_\ell \\
 &= [-G_{kk}^{-1}D(r) - D(k)]d\alpha_k + \tilde{k}_\ell D(\ell)d\alpha_\ell \\
 &\quad \quad \quad (+) \quad \quad \quad (-)
 \end{aligned}$$

Here we have used  $D(\alpha) = I$  as well as that the effect on capital input of changes in labor, at constant goods and capital prices, is given by

$$(63) \quad \tilde{k}_\ell = -G_{kk}^{-1}G_{k\ell}.$$

The bracketed expression on the righthand side is negative (positive) if the demand for capital is inelastic (elastic). Using the same definition as in Svensson (1982), capital and labor are cooperative (non-cooperative) if  $\tilde{k}_\ell$  is positive (negative).

We conclude that capital import will decrease, and goods and factor trade be substitutes, if capital demand is inelastic and capital and labor are cooperative, whereas goods and factor trade are complements if capital demand is elastic and capital and labor are non-cooperative.

The intuition behind the dependence on the elasticity of demand for capital can be simply explained as follows: Let there be only one capital good. From (60), we have

$$(64) \quad r = G_k = D(\alpha_k)\bar{G}_k(p, D(\alpha)v).$$

Equation (64) shows that an increase in  $\alpha_k$  has two conflicting influences on  $r$  and therefore on the direction of trade in capital. Formally, we have



$$(65) \quad \partial r / \partial \alpha_k = \bar{G}_k + \alpha_k G_{kk} k = r + k / \tilde{k}_r ,$$

since  $\alpha_k = 1$  initially. Hence,

$$(66) \quad \frac{\partial r}{\partial \alpha_k} > 0 \text{ if and only if } -r \tilde{k}_r / k < 1 .$$

If the demand for capital is inelastic ( $-r \tilde{k}_r / k < 1$ ) an increase in  $\alpha_k$  implies a fall in  $r$ . Capital then flows out of the country when factor trade is permitted leading to a smaller volume of exports of the capital intensive good than would have occurred with factor trade.

## 6. Summary and Conclusions

The first purpose of this paper was to develop a general model of trade caused by international differences in production technology and attempt to derive a simple relationship between the direction of trade and the differences in technology. The second purpose was to examine factor mobility within the context of the model since it is well known that factor prices are generally not equalized by trade in commodities alone when there are international differences in technology.

General expressions for the relationship between technology and goods trade with and without factor trade were derived. While these expressions are fairly simple in algebraic structure, they are not simple to interpret from an economic point of view. We therefore moved from the general case to the specific case of product-augmenting technical change in order to obtain clearer results.

For the product-augmenting case, a specific result was derived for the relationship between goods trade and technology. We showed that there is a positive correlation between net export and technical differences, in the sense that countries will on average export goods for which the country has technical superiority.

Very strong results were then obtained for the case in which some factors are allowed to move in response to international price differences. It was shown that factor mobility leads to an increase in the correlation between goods and factor trade, indicating an increase in the volume of goods trade relative to the no-factor-trade situation. Factor trade and commodity trade are thus complements as in Markusen (1983).

The intuition behind this result can be captured by considering a simple two-good, two-factor Heckscher-Ohlin model. With endowments initially equal,

the production changes generated by trade will bid up the price of the factor used intensively in each country's export industry (the industry in which the country has technical superiority). Factor mobility then leads to a direction of factor flows which acts to reinforce the pattern of comparative advantage and trade caused by differences in technology.

This example also helps provide the intuition behind our result relating the vector of factor trade with the vector of technology parameters. Our result indicates that countries will on average import those factors used intensively in industries in which the country has technical superiority. But these industries are, by earlier results, the export industries. Thus factor trade leaves each country relatively well endowed with the factors used intensively in the country's export industries. As noted in the introduction, this is the cause of trade in goods in the Heckscher-Ohlin model, whereas it is the consequence of trade in factors in the present model.

For the case with factor-augmenting technical change, results were not as straightforward. We could easily show that with no factor trade, the goods trade pattern is exactly as if there were factor endowment differences. With factor trade, goods and factor trade can be either substitutes and complements as in the case when trade is caused by endowment differences (Svensson (1982)). For both factor-augmenting technology differences and for factor endowment differences, goods trade and factor trade tend to be substitutes if traded and non-traded factors are cooperative. But in the factor-augmenting case, the substitute/complement relationship depends also on the elasticity of demand for traded factors due to the fact that factor-augmenting technical change has an ambiguous effect on a factors marginal product.

Footnotes

- \* We are grateful for comments from participants in an IIES seminar.  
Henrik Horn and Torsten Persson contributed specific comments.
1. See also Jones and Neary (1983) and Neary (1980).
  2. An important exception is Findlay and Grubert (1959).
  3. See also Purvis (1972), where it is shown that with technological differences in a Heckscher-Ohlin framework, capital mobility may lead to complete specialization in production and to increased volume of goods trade.
  4. See Dixit and Woodland (1982) and Svensson (1982) for a more complete discussion of the role of demand in this type of problem.
  5. This relation is derived by Dixit and Norman (1980), for the no joint production case.
  6. This amounts to assuming that the matrix  $G_{kk}$  is negative definite.
  7. This way of introducing factor-augmenting technical change is followed by Dixit and Norman (1980) and Woodland (1982).

References

- Bhagwati, Jagdish N., 1973. "The Theory of Immiserizing Growth: Further Applications," in Michael B. Connolly and Alexander K. Swoboda (eds.) International trade and money (Toronto: University of Toronto Press), 45-54.
- Brecher, Richard A. and Carlos Diaz Alejandro, 1977. "Tariffs, Foreign Capital, and Immiserizing Growth," Journal of International Economics 7, 317-322.
- Dixit, Avinash K. and Victor Norman, 1980. Theory of international trade (Cambridge: Cambridge University Press).
- Dixit, Avinash K. and Alan D. Woodland, 1982. "The Relationship Between Factor Endowments and Commodity Trade," Journal of International Economics 13, 201-214.
- Ethier, Wilfred J., 1983. "Higher Dimensional Trade Theory," in Ronald W. Jones and Peter B. Kenen (eds.) Handbook of international economics, Volume 1 (Amsterdam: North-Holland Publishing Co.), forthcoming.
- Findlay, Ronald and H. Grubert, 1959. "Factor Intensities, Technological Progress and the Terms of Trade," Oxford Economics Papers 11-1, 111-121.
- Jones, Ronald W. and J. Peter Neary, 1983. "The Positive Theory of International Trade," in Ronald W. Jones and Peter B. Kenen (eds.), Handbook of international economics, Volume 1 (Amsterdam: North-Holland Publishing Co.), forthcoming.
- Markusen, James R. and James R. Melvin, 1979. "Tariffs, Capital Mobility and Foreign Ownership," Journal of International Economics 9, 395-410.
- Markusen, James R., 1983. "Factor Movements and Commodity Trade as Complements," Journal of International Economics 13, forthcoming.

- Mundell, R., 1957. "International Trade and Factor Mobility," American Economic Review 47, 321-335.
- Neary, J. Peter, 1980. "International Factor Mobility, Minimum Wage Rates, and Factor-Price Equalization: A Synthesis," IIES Seminar Paper No. 158.
- Purvis, Douglas D., 1972. "Technology, Trade and Factor Mobility," Economic Journal 82, 991-999.
- Svensson, Lars E.O., 1982. "Factor Trade and Goods Trade," NBER Working Paper No. 999.
- Woodland, Alan D., 1982. International trade and resource allocation (Amsterdam: North Holland).