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DYNAMIC SCORING:
A BACK-OF-THE-ENVELOPE GUIDE

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ABSTRACT

This paper uses the neoclassical growth model to examine the extent to which a tax cut pays for itself through higher economic growth. The model yields simple expressions for the steady-state feedback effect of a tax cut. The feedback is surprisingly large: for standard parameter values, half of a capital tax cut is self-financing. The paper considers various generalizations of the basic model, including elastic labor supply departures from infinite horizons, and non-neoclassical production settings. It also examines how the steady-state results are modified when one considers the transition path to the steady state.

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Introduction

To what extent does a tax cut pay for itself? This question arises regularly for economists working at government agencies in charge of estimating tax revenues. Traditional revenue estimation, called static scoring, assumes no feedback from taxes to national income. The other extreme, illustrated by the renowned Laffer curve, suggests that tax cuts can generate so much economic growth that they completely (or even more than completely) pay for themselves. Most economists are skeptical of both polar cases. They believe that taxes influence national income but doubt that the growth effects are large enough to make tax cuts self-financing. In other words, tax cuts pay for themselves in part, and the open question is the magnitude of the effect.

In 2002 the staff of the Joint Tax Committee, prompted by several members of Congress, started work on the difficult task of dynamic scoring of tax policy. That is, they started developing a set of economic models that might be used to estimate the feedback effects of tax proposals. Dynamic scoring also received prominent discussion in a 2003 report by the Congressional Budget Office and the 2004 *Economic Report of the President*. The task of dynamic scoring is formidable, because there is little agreement about how best to model long-run economic growth and the effect of taxes on the economy.

The purpose of this paper is to investigate what the neoclassical growth model can contribute to this endeavor. The neoclassical growth model, first introduced by Ramsey (1928), is the most widely taught model of capital accumulation and long-run growth and is the workhorse of modern growth theory. For example, see the popular graduate-level textbooks by Romer (2001) and Barro and Sala-i-Martin (1999). This model is also widely used for thinking about issues in public finance (Chamley 1986; Judd 1985). Here we use the neoclassical growth model to consider the revenue effects of changes in tax rates on capital and labor income. One virtue of the model is that it sheds light on the key parameters that govern these revenue effects. The model also yields simple formulas for how much the dynamic estimates of these revenue effects differ from the static estimates.¹

These formulas permit some illuminating back-of-the-envelope calculations. For conventional parameter values, the model implies substantial feedback effects in the steady state. For example, suppose that the initial tax rates on capital and labor are 25 percent, the production function is Cobb-Douglas, the capital share is one-third, and labor supply is inelastic. Then, in the steady

¹In addition to the works already cited, our analysis is related to several strands of the literature on fiscal policy. One prominent example is Auerbach and Kotlikoff (1987), who analyze tax changes using computer-based simulations of overlapping-generations models. The subset of the literature closest to this paper has typically focused on Laffer effects: the possibility that tax cuts can be fully self-financing (e.g., Pecorino 1995, Ireland 1994, and Novales and Ruiz 2002). McGrattan (1994) uses a framework similar to ours but is primarily concerned with the impact of changing taxes on explaining economic fluctuations; the feedback effect of tax rates on tax revenue is implicit in her analysis. Despite the large related literature, we know of no research that derives analytical expressions for the feedback effects that are the focus of this paper.

state, the dynamic effect of a cut in capital income taxes on government revenue is only 50 percent of the static effect. That is, one-half of a capital tax cut pays for itself.

There are various ways in which the benchmark Ramsey model can be generalized. One is to include elastic labor supply. We show that this generalization has only minor effects on the analysis of capital income taxes, but it has significant effects on the analysis of labor income taxes. We assume a form of preferences that yields no trend in hours worked, as the uncompensated elasticity of labor supply is zero. The compensated (constant-consumption) elasticity of labor supply, however, need not be zero. If this elasticity is one-half and the other parameters are as described above, then the steady-state feedback from a labor income tax cut rises from zero to 17 percent. The model shows that, regardless of the labor supply elasticity, if capital and labor tax rates start off at the same level, cuts in capital taxes have greater feedback effects in the steady state than cuts in labor taxes.

Many economists are skeptical of the Ramsey model because of its assumption of an infinite-horizon consumer. We therefore introduce finite horizons in two ways. We first add some rule-of-thumb households that consume their entire labor income in each period, but we find that this has no effect on the steady-state results. The infinite-horizon consumers dominate in the long run, as is suggested by the earlier work of Judd (1985), Smetters (1999), and Mankiw (2000). Alternatively, if all consumers have finite horizons, as in Blanchard (1985), the results change. Yet the changes are quantitatively modest for plausible parameter values. For example, if households have an expected horizon of 50 years, then the fraction of a capital tax cut paid for by growth falls from 50 percent to 45 percent.

We also consider two widely discussed departures from the neoclassical production setting. We first consider the impact of imperfect competition, for Judd (2002) has shown that market power can substantially change the analysis of optimal tax policy. We find that market power can raise the ability of tax cuts to be self-financing, but only if there are substantial economic profits not dissipated by the fixed costs associated with entry. We also examine the possibility that there are positive externalities to capital accumulation, as suggested by Romer (1987) and DeLong and Summers (1991). In this case, the dynamic effects of tax changes are much larger than they are in the standard model.

The neoclassical model yields particularly simple expressions for steady-state feedback effects, but it is also important to consider the transition path to the steady state. We therefore consider a log-linearized version of the model for the special case of unitary intertemporal elasticity of substitution. For our canonical parameter values, we find that the immediate revenue feedback effects are quite similar for capital and labor taxes: slightly more than 10 percent of a tax cut immediately pays for itself through higher labor supply and national income. For both types of taxes, the feedback grows over time toward their steady-state values, reaching halfway after about ten years.

In all experiments that we consider, the government budget constraint is satisfied, as it must be in any well-specified model. Throughout the paper, we

assume that some form of lump-sum transfers adjusts in response to the tax changes. We have in mind such spending programs as welfare, social security, and farm subsidies. The dynamic scoring question that we are proposing, then, is how much such transfer spending needs to fall to offset a cut in tax rates.

Implicit in our use of a model of long-run growth is that we ignore any short-term effects of tax cuts that arise from traditional Keynesian channels. Many government and private-sector analysts have instead emphasized the power of tax cuts to stimulate a weak economy. Although we abstract from these effects in this study, we do not mean to suggest that such effects are insignificant. Integrating a model of long-run growth with a model of short-run business cycles remains a challenge for future research on dynamic scoring.

The paper is organized as follows. Section 1 presents the basic model and previews results. Section 2 derives and solves a more general version of the model which includes elastic labor supply. Section 3 discusses how the results change if we relax the assumption of infinite horizons, and Section 4 investigates departures from the neoclassical production setting. Section 5 considers the transition path. Section 6 concludes.

1 The Basic Ramsey Model

Before delving into the details of a more general model, which we do in the next section, it will be useful for many readers to preview our results for a familiar special case—the steady state of the Ramsey growth model.² We modify this model by including taxation at a rate τ_k on capital income and τ_n on all labor income. The population is normalized to one, and labor is supplied inelastically. Using conventional notation, we can write the steady state of the economy as follows:

$$r = f'(k). \tag{1}$$

$$w = f(k) - kf'(k). \tag{2}$$

$$(1 - \tau_k)r = \rho + \gamma g. \tag{3}$$

$$R = \tau_k rk + \tau_n w. \tag{4}$$

This system of four equations fully specifies the steady-state values of the four endogenous variables: k is capital per efficiency unit of labor, w is the wage rate, r is the before-tax rate of return to capital, and R is total tax revenue per efficiency unit. In addition, $f(k)$ is total output per efficiency unit, γ is the curvature coefficient in our instantaneous utility function (the reciprocal of the intertemporal elasticity of substitution), g is the rate of labor-augmenting technological change, and ρ is the subjective discount rate. Throughout this article, we assume that the production function is Cobb-Douglas:

$$y = f(k) = k^\alpha$$

²For standard introductions to the Ramsey model, we refer the reader to Romer (2001), chapter 2, or to Barro and Sala-i-Martin (1999), chapter 2.

where y is output per efficiency unit and the parameter α is capital's share of income. (Appendix 1 considers two generalizations of this production function).

Our goal is to estimate the impact of a tax change on steady-state tax revenue R . A conventional scoring assuming no dynamic effects from the tax cut yields the following results:

$$\left. \frac{dR}{d\tau_k} \right|_{static} = rk = \alpha y.$$

$$\left. \frac{dR}{d\tau_n} \right|_{static} = w = (1 - \alpha)y.$$

These equations show the impact of a tax change on tax revenue, assuming that national income and other macroeconomic variables are held constant. Notice that each of these derivatives equals the tax base of the respective tax.

Dynamic scoring estimates the impact of a tax change, taking into account the tax change's consequence for growth. By fully differentiating equations (1) through (4), we obtain the following results:

$$\left. \frac{dR}{d\tau_k} \right|_{dynamic} = \left[1 - \frac{\alpha\tau_k + (1 - \alpha)\tau_n}{(1 - \tau_k)(1 - \alpha)} \right] \left. \frac{dR}{d\tau_k} \right|_{static}. \quad (5)$$

$$\left. \frac{dR}{d\tau_n} \right|_{dynamic} = \left. \frac{dR}{d\tau_n} \right|_{static}. \quad (6)$$

These equations show the impact of a tax change on tax revenue, including the feedback from taxes to national income.

The central goal of this paper is to compare these dynamic and static revenue estimates. In this conventional Ramsey model, with its assumption of inelastic labor supply, the revenue impact of a change in the labor income tax rate is the same under dynamic and static scoring. This explains equation (6). The more interesting analysis pertains to result (5), the impact of a change in the capital tax rate τ_k on tax revenue. Consider the empirically plausible parameter values of $\tau_k = \tau_n = \frac{1}{4}$ and $\alpha = \frac{1}{3}$. Then, (5) yields

$$\left. \frac{dR}{d\tau_k} \right|_{dynamic} = \frac{1}{2} \left. \frac{dR}{d\tau_k} \right|_{static}.$$

A capital income tax cut has a long-run impact on revenue that is only half of its static impact. In other words, growth pays for 50 percent of a capital income tax cut in the steady state.³

³The feedback depends critically on the tax rate. If the capital tax rate were 0.40 instead of 0.25, and all other parameter values are the same, the feedback from a capital tax cut would be 75 rather than 50 percent.

The literature on taxation in the United States suggests that our choice of $\tau_k = \tau_n = 0.25$ is within the range of plausible estimates, although perhaps a bit conservative. Mendoza, Razin, and Tesar (1994) estimate a 40.7 percent capital tax rate (applied to corporate and non-corporate capital) for the United States in 1988, the last year of their series. This is above

This simple example illustrates two lessons. First, dynamic and static revenue estimation can lead to very different results. Second, the steady state of the Ramsey model yields simple expressions that can provide useful benchmarks for the task of dynamic scoring. In the sections that follow, we develop more general models to examine the robustness of these conclusions.

2 Elastic Labor Supply

In this and the next three sections, we extend the basic Ramsey model along a number of dimensions. In this section we allow for the possibility of elastic labor supply and present a more detailed derivation of our results.

To allow for elastic labor supply, we use a form of preferences over consumption and labor proposed by King, Plosser, and Rebelo (1988). King-Plosser-Rebelo preferences have the property that the uncompensated elasticity of labor supply is zero. This feature has the appealing implication that long-run growth caused by technological progress does not lead to a trend in hours worked. The compensated (constant-consumption) elasticity of labor supply need not be zero, however. This parameter, which we will call σ , will have a significant role in some of our results.

2.1 Firms

We begin with production. Assume there are many identical firms in competitive input and output markets, producing output with constant returns to scale technology according to the production function

$$Y = F(K, N),$$

where Y is the total amount of output, K is the total amount of capital, and N is the total labor input, including the adjustment for labor-augmenting technological change. That is, if n is the labor input supplied by the representative household and g is the rate of labor-augmenting technological change, then $N = ne^{gt}$. With these conventions, we can write the production function as

$$y = f(k, n).$$

where $y = Y/e^{gt}$ is output per efficiency unit and $k = K/e^{gt}$ is capital per efficiency unit.

To keep our analysis simple, we assume that the production function is Cobb-Douglas:

$$y = k^\alpha n^{1-\alpha}. \tag{7}$$

the estimate given by Gravelle (2004), who reports a rate of 33 percent for all capital in that year. Gravelle extends her estimates through 2003, by which point the capital tax rate had fallen to 23 percent. Mendoza et al. estimate a labor tax rate of 28.5 percent in 1988, together with a consumption tax of about 5 percent; these tax rates would combine to be equivalent to a tax on labor of about 31 percent.

This functional form is both realistic and widely-used in the analysis of economic growth.

Given competitive markets, firms earn zero profits and capital earns a before-tax rate of return r equal to its marginal product:

$$r = f_k(k, n) = \alpha k^{\alpha-1} n^{1-\alpha}. \quad (8)$$

Each efficiency unit of labor is paid a wage w equal to its marginal product,

$$w = f_n(k, n) = (1 - \alpha) k^\alpha n^{-\alpha}. \quad (9)$$

Below, in Section 4, we consider generalizations to non-competitive production settings.

2.2 Households

We use a conventional, infinitely-lived representative household. The household's instantaneous utility function takes the isoelastic form with curvature parameter γ . To incorporate elastic labor supply, we add labor n to the household's utility function. This labor variable should be interpreted broadly to include both time and effort.

The household's utility function is

$$U = \int e^{-\rho t} \frac{(ce^{gt})^{1-\gamma} e^{(1-\gamma)v(n)} - 1}{1 - \gamma} dt,$$

where $v(n)$ is a differentiable function of labor supply and all other variables are defined as before. This functional form was introduced by King, Plosser, and Rebelo (1988) and has been more recently explored by Kimball and Shapiro (2003).

We can write the household's dynamic budget constraint in per efficiency unit terms:

$$\begin{aligned} \dot{k} &= (1 - \tau_n)wn + (1 - \tau_k)rk - c - gk + T, \\ \lim_{t \rightarrow \infty} k e^{(-r+g)t} &= 0. \end{aligned}$$

where \dot{k} is the time derivative of the capital stock per efficiency unit and T represents lump-sum transfers from the government. The second equation is the standard transversality condition.

Household maximization yields the following first-order conditions:

$$\begin{aligned} v'(n) &= \frac{-(1 - \tau_n)w}{c}, \\ r &= \frac{1}{1 - \tau_k} \left[\rho + \gamma \left(\frac{\dot{c}}{c} + g \right) + (1 - \gamma)v'(n) \cdot \dot{n} \right], \end{aligned} \quad (10)$$

Equation (10) is the static condition determining the allocation of time between work and leisure. From this equation, one can derive an expression for the constant-consumption elasticity of labor supply, which we will denote σ :

$$\sigma = \frac{v'(n)}{v''(n) \cdot n}.$$

In the steady state, consumption per efficiency unit c and the wage per efficiency unit w are both constant. As a result, labor supply n is constant as well. The intertemporal first-order condition therefore reduces to

$$r = \frac{\rho + \gamma g}{1 - \tau_k}. \quad (11)$$

This is the same as in Section 1.

In the steady state, $\dot{k} = 0$, and we can write the steady-state level of consumption as:

$$c = f(k, n) - gk. \quad (12)$$

Equations (7) through (12) fully determine the steady-state values of six variables: y , k , n , r , w , and c .

2.3 Government

Total tax revenue per efficiency unit, denoted R , is the sum of taxes paid on capital income and labor income:

$$R = \tau_k r k + \tau_n w n. \quad (13)$$

The first term on the right of (13) is the capital tax rate times capital income, and the second term is the labor tax rate times labor income. The government collects this revenue and distributes it in the form of lump-sum transfers to households. For most of our results, the timing of these rebates is irrelevant, as the consumer is infinitely-lived. (Later, when we consider models with finite horizons, we assume that rebates occur immediately upon receipt of the tax revenue.)

2.4 Dynamic and Static Steady-State Scoring

A conventional scoring assuming no dynamic effects from the tax cut yields the following results for this model :

$$\left. \frac{dR}{d\tau_k} \right|_{static} = rk = \alpha f(k, n).$$

$$\left. \frac{dR}{d\tau_n} \right|_{static} = wn = (1 - \alpha)f(k, n).$$

By contrast, to find the true impact of the tax change on steady-state revenue, one would use all of the steady-state conditions. This yields the following:

$$\frac{dR}{d\tau_k} \Big|_{dynamic} = \left[1 - \frac{\alpha\tau_k + (1-\alpha)\tau_n}{(1-\alpha)(1-\tau_k)} - \frac{\alpha\tau_k + (1-\alpha)\tau_n}{(\rho + \gamma g) - \alpha(1-\tau_k)g} \frac{\sigma}{1+\sigma} g \right] \frac{dR}{d\tau_k} \Big|_{static} . \quad (14)$$

$$\frac{dR}{d\tau_n} \Big|_{dynamic} = \left[1 - \frac{\alpha\tau_k + (1-\alpha)\tau_n}{(1-\alpha)(1-\tau_n)} \frac{\sigma}{1+\sigma} \right] \frac{dR}{d\tau_n} \Big|_{static} . \quad (15)$$

Note that the labor supply elasticity σ enters results (14) and (15). In the case of a capital tax cut, the labor supply elasticity plays only a small role. If $g = 0$, then equation (14) is identical to equation (5) from the basic Ramsey model. In the case of a labor tax cut, however, the elasticity of labor supply plays a key role. The larger the elasticity of labor supply, the smaller the dynamic revenue impact of a labor tax cut. From these equations, one can show that, if the two tax rates are the same, a capital tax cut will always have a larger feedback effect than a labor tax cut.

To illustrate the effect of elastic labor supply, consider the following plausible parameter values: $\tau_k = \frac{1}{4}$, $\tau_n = \frac{1}{4}$, $\alpha = \frac{1}{3}$, $\gamma = 1$, $g = .02$, $\rho = .05$, and $\sigma = \frac{1}{2}$. These parameters yield:

$$\frac{dR}{d\tau_k} \Big|_{dynamic} = 0.47 \frac{dR}{d\tau_k} \Big|_{static} .$$

$$\frac{dR}{d\tau_n} \Big|_{dynamic} = 0.83 \frac{dR}{d\tau_n} \Big|_{static} .$$

Under these assumptions, a capital tax cut has a long-run impact on revenue of only 47 percent of its static impact. That is, growth pays for 53 percent of the static revenue loss. A labor tax cut has a long-run impact on revenue of only 83 percent of its static impact, and growth pays for 17 percent of the tax cut.

2.5 The Compensated Elasticity of Labor Supply

Equation (15) shows that the feedback effect for a labor tax cut depends crucially on the compensated elasticity of labor supply. This is a parameter over which there is substantial uncertainty.

Kimball and Shapiro (2003) present a recent, extensive discussion of this parameter, including references to a broad literature. As they note, it is important to recognize that there are different notions of the compensated elasticity. Kimball and Shapiro conclude that the Frisch (constant marginal utility) elasticity is about one, and that the constant-consumption elasticity is about 1.0 to 1.5. If σ is increased from 0.5 to 1.5 in our calculation, the revenue feedback effect of a labor tax cut rises from 17 percent to 30 percent.

Kimball and Shapiro point out there are economists with preferred values on both sides of their estimates. Labor economists analyzing micro data (e.g., Angrist, 1991 and Blundell, Duncan, and Meghir, 1998) tend to argue for smaller elasticities. A survey of labor economists conducted by Fuchs, Krueger, and Poterba (1998) found that the median labor economist believes the compensated elasticity of labor supply is 0.18 for men and 0.43 for women. By contrast, macroeconomists working in the real business cycle literature often choose parameterizations that imply larger values. Prescott (2004) examines cross-country data on hours worked and marginal tax rates and finds that these two variables are strongly correlated. He concludes that this international variation suggests a compensated elasticity of labor supply around 3. Precise dynamic scoring of labor tax changes will require better estimation of this key parameter.⁴

A related literature is associated with Feldstein (1999). He estimates the elasticity of taxable income with respect to one minus the tax rate to be between 1.0 and 1.5. In our model, this estimate would correspond most closely to σ , because this parameter governs how labor supply, and thus labor income, responds to taxes. Feldstein, however, believes his estimate includes more than the effects on labor supply. He points out that traditional estimates of responses to tax changes have "ignored the effect of higher income tax rates on tax avoidance through changes in the form of compensation (e.g., employer-paid health insurance) and through changes in the patterns of consumption (e.g., owner-occupied housing)." Future research should consider integrating this effect into dynamic models of economic growth and revenue estimation.

3 Finite Horizons

The results we have obtained so far rely on the neoclassical growth model with its assumption of a representative household who optimizes over an infinite planning horizon. This model is widely used and a natural benchmark. As Barro (1974) famously noted, the infinite-horizon household can be viewed as the result of generations' being linked via altruistic bequests.

Nonetheless, some economists are skeptical of the model's empirical realism. This raises the question: Would alternative models of household behavior lead to substantially different conclusions about dynamic scoring? It turns out that our results regarding steady-state feedback effects are surprisingly robust.

⁴One possible way of reconciling these differing estimates is to generalize the neoclassical model to include a role for work norms, as suggested by Blomquist (1993) and Grodner and Kniesner (2003). Suppose an individual's disutility from supplying labor depends on how much other people are working. That is, working long hours when others are doing so is not as onerous as working long hours while others are enjoying substantial leisure. In this case, as Glaeser, Sacerdote, and Scheinkman (2002) point out, a "social multiplier" causes aggregate elasticities to exceed individual elasticities. The larger aggregate elasticity would be the relevant one for the purposes of dynamic scoring.

3.1 Rule-of-thumb consumers

A large part of the consumption literature has suggested that current income exerts a greater influence on consumer spending than is predicted by the model of the infinitely-lived consumer. Campbell and Mankiw (1989) suggested that about half of income goes to households who follow the rule of thumb of consuming their current income. A prominent role for current income has also been documented by Shea (1995), Parker (1999), and Souleles (1999).

To see the implications of such behavior for dynamic scoring, suppose the model is the same as the one presented in Section 1, except that a fraction of households always consume their current income. How would our previous results change? The answer is, not at all.

Here is the logic. Equations (1) through (4) pin down the steady state in the neoclassical growth model. These equations would continue to hold, even if some consumers spend their current income. The only equation that comes from household behavior is equation (3). This equation would still obtain: It would be derived from the intertemporal first-order condition for the subset of maximizing consumers. As Judd (1985), Smetters (1999), and Mankiw (2000) have previously noted, as long as some households behave according to the neoclassical growth model, the steady state is not at all affected by a subset of households who do not.

3.2 The Blanchard Model

Blanchard (1985) suggested another way to relax the Ramsey model's assumption of infinite horizons. According to Blanchard's model, all households face a constant probability p of dying off every period and being replaced by a new household. Households respond to this risk by annuitizing all of their wealth. There are no bequests.⁵

To keep things simple, we consider the same special case that Blanchard emphasizes. In particular, we assume inelastic labor supply ($\sigma = 0$), log utility ($\gamma = 1$), and no technological progress ($g = 0$). In this case, the following equation determines the steady-state interest rate:

$$r = \frac{1}{1 - \tau_k} \left[\rho + p(\rho + p) \frac{k}{y} \right]. \quad (16)$$

We skip the derivation of this equation, as it follows immediately from Blanchard's equation (12). Note that for the special case of $p = 0$, the consumer faces an infinite horizon, and we obtain equation (5) from section 1.

The remainder of the Blanchard model is similar to the Ramsey model. The steady state of the economy is determined by equation (16) together with

⁵ Annuity markets play a crucial role in the Blanchard model. We have worked out a version of the Blanchard model in which, instead of annuitizing, households leave accidental bequests, which we assume are distributed as lump-sum payments to the newly born households. This alternative model yields the same dynamic feedback effects as the Ramsey model. Most likely, reality lies somewhere between the model with no annuitization and the model with full annuitization.

equations (1), (2), and (4). Because labor supply is inelastic, labor taxes do not yield interesting dynamic effects. Capital taxes, however, yield the following:

$$\frac{dR}{d\tau_k} \Big|_{dynamic} = \left\{ 1 - \left(\frac{\alpha}{1-\alpha} \right) \cdot \frac{[\alpha\tau_k + (1-\alpha)\tau_n] 2p(\rho+p)}{\rho^2 + 4p(\rho+p)\alpha(1-\tau_k) - \rho\sqrt{\rho^2 + 4p(\rho+p)\alpha(1-\tau_k)}} \right\} \cdot \frac{dR}{d\tau_k} \Big|_{static} \quad (17)$$

In the limit as p approaches 0, this simplifies to equation (5).

This equation shows how finite horizons as modeled by Blanchard affect our results regarding dynamic feedback effects. Figure 1 illustrates how the feedback effect varies with the value of p . Recall that for $p = 0$, we found that 50 percent of a capital tax cut pays for itself in steady state. If $p = .02$, so the average time horizon is fifty years, the dynamic feedback effect falls from 50 to 45 percent. If $p = .05$, so the average time horizon is twenty years, the feedback effect falls to 39 percent.

The bottom line is that the Blanchard generalization of the Ramsey model does alter our results. For plausible parameter values, however, the changes are only modest in size.

4 Departures from Neoclassical Production

So far, we have assumed a neoclassical production setting. In this section, we explore the implications of two departures from this assumption: imperfect competition and positive externalities to capital investment.

4.1 Imperfect Competition

Many markets in the economy are imperfectly competitive. Because of patents, copyrights, and fixed costs, prices can remain above marginal costs for long periods. Over the past several decades, models of monopolistic competition have become increasingly central in the theories of international trade, economic growth, and the business cycle. Judd (2002) has recently proposed that these models might also be important for the analysis of tax policy. Here we see whether adding imperfect competition to our generalized Ramsey model in section 2 alters our results about dynamic scoring.

To incorporate imperfect competition, it is useful to imagine an economy that produces in two stages. In the first stage, a competitive sector with constant returns to scale produces an intermediate good using capital and labor inputs. Competition ensures that price equals marginal cost. In the second stage of production, firms use the intermediate good to produce a final good that can be used for investment or consumption. Firms in this second stage produce one

unit of the final good from one unit of intermediate good and sell the final good at a markup over marginal cost. They may also face fixed costs of entry. We let μ equal the ratio of price to marginal cost in the final good industry.

With this market structure, the price of the final good, P , is

$$P = \mu P_M = \mu MC$$

where P_M is the price of the intermediate good, and MC is the marginal cost of producing the intermediate good. Hereafter, we let the final good be the numeraire, so $P = 1$.

Because the intermediate good is produced with both capital and labor, its marginal cost can be computed from the marginal product of either factor. That is,

$$MC = \frac{w}{f_n} = \frac{r}{f_k}$$

The two equations above yield equilibrium factor prices:

$$r = \frac{f_k}{\mu}. \quad (18)$$

$$w = \frac{f_n}{\mu}. \quad (19)$$

These two equations replace (8) and (9) from section 2.

The existence of a markup raises the possibility of economic profit. The final goods producers buy a quantity $f(k, n)$ of the intermediate good and then earn operating profits of $\left(\frac{\mu-1}{\mu}\right) f(k, n)$. We let θ be the fraction of operating profits that accrue to the owners of the firms as pure economic profits. That is, economic profits are

$$\pi = \theta \left(\frac{\mu-1}{\mu}\right) f(k, n).$$

This formulation allows for the possibility that all of the operating profits accrue to firm owners ($\theta = 1$), that all of the operating profits are dissipated through the fixed costs associated with entry ($\theta = 0$), and a range of intermediate cases.

Because households own firms, economic profits enter the household budget constraint. As a result, the steady-state equation (12) becomes:

$$c = \frac{f(k, n)}{\mu} + \pi - gk. \quad (20)$$

Economic profits are assumed to be taxed at a rate τ_π , so equation (13) becomes:

$$R = \tau_k r k + \tau_n w n + \tau_\pi \pi. \quad (21)$$

In the perfect competition case, the steady state of the economy was described by equations (7) to (13); the comparable system now includes equations (7), (18), (19), (10), (11), (20), and (21).

Analysis as before yields the following results:

$$\left. \frac{dR}{d\tau_k} \right|_{dynamic} = \left[1 - \frac{\alpha\tau_k + (1-\alpha)\tau_n + \theta(\mu-1)\tau_\pi}{(1-\alpha)(1-\tau_k)} - \frac{\alpha\tau_k + (1-\alpha)\tau_n + \theta(\mu-1)\tau_\pi}{(1+\theta(\mu-1))(\rho+\gamma g) - \alpha g(1-\tau_k)} \frac{\sigma}{1+\sigma} g \right] \left. \frac{dR}{d\tau_k} \right|_{static} \quad (22)$$

$$\left. \frac{dR}{d\tau_n} \right|_{dynamic} = \left[1 - \frac{\alpha\tau_k + (1-\alpha)\tau_n + \theta(\mu-1)\tau_\pi}{(1-\alpha)(1-\tau_n)} \frac{\sigma}{1+\sigma} \right] \left. \frac{dR}{d\tau_n} \right|_{static} \quad (23)$$

These equations are the counterparts to equations (14) and (15).

To see how imperfect competition affects the analysis of dynamic scoring of capital taxes, we can compare equations (22) and (14). Of course, if $\mu = 1$, this more general model collapses to the earlier one. Note, however, that the two models become identical also if there are no economic profits ($\theta = 0$). Thus, for imperfect competition to have important implications for dynamic scoring, it is crucial that not all profits be dissipated by the fixed costs associated with entry.

To get some sense of the magnitude of the effects that imperfect competition might generate, consider our standard parameter values $\alpha = \frac{1}{3}$, $\tau_k = \tau_n = \frac{1}{4}$, $\rho = .05$, $g = .02$, $\sigma = \frac{1}{2}$, $\gamma = 1$, and let $\tau_\pi = \frac{1}{4}$. For a 25 percent markup ($\mu = \frac{5}{4}$) and no profit dissipation ($\theta = 1$), the feedback effect from a capital tax cut is now 65 percent, compared to 53 percent under perfect competition.⁶

Similar conclusions hold for labor tax changes. By comparing equation (23) with equation (15), we can see that two cases become identical if there are no markups ($\mu = 1$), if there are no economic profits ($\theta = 0$), or if economic profits are not taxed ($\tau_\pi = 0$). On the other hand, if $\theta = 1$, $\mu = \frac{5}{4}$, and $\tau_\pi = \frac{1}{4}$, the feedback effect of a labor tax cut is 21 percent, compared to 16.7 percent in the benchmark case.

Thus, imperfect competition raises the ability of tax cuts to be self-financing only if it generates pure economic profits. If the fixed costs associated with entry dissipate all profits, then imperfect competition as modeled in this section acts like an adverse shift in the production function, lowering capital accumulation, consumption, and welfare, but having no effect on dynamic scoring of tax changes.

⁶Notice that this calculation holds the tax on economic profit constant at a rate of 25 percent. If economic profits were not taxed ($\tau_\pi = 0$), then the dynamic feedback would be almost identical to our base case with perfect competition: it would be 52 rather than 53 percent. Alternatively, one might assume that economic profits were taxed at the same rate as capital income. If so, then cutting the capital tax would also entail cutting the economic profits tax, which would reduce the dynamic feedback substantially—from 53 percent to 37 percent. It is unclear which assumption is best, as the economic profits generated by market power could accrue either to the owners of capital or to the suppliers of labor via union contracts.

4.2 Externalities to Capital

In the model examined in Section 2, capital earns its marginal product. Some economists, however, have suggested that the social marginal product of capital exceeds its private marginal product. DeLong and Summers (1991) estimate that "the social [rate of] return to equipment investment in well-functioning market economies is on the order of 30 percent per year," which is more than twice the private rate of return. Romer (1987, pp 165-166) suggests "The correct weight on the growth of capital in a growth accounting exercise may be closer to 1 than to 0.25. The true elasticity of output with respect to changes in capital may be greater than the share of capital in total income because of positive externalities associated with investment." In this section, we modify the model in section 2 to include such externalities to capital.

Suppose that each firm's production y_i is a function not only of its own capital k_i , but also of the general pool of knowledge, κ :

$$y_i = \kappa k_i^\alpha n_i^{1-\alpha}.$$

Each firm takes κ as given. However, κ is assumed to be an increasing function of the average firm's level of capital, k :

$$\kappa = k^\beta. \tag{24}$$

The parameters α and β measure the direct (private) and indirect (social) benefits of capital. That is, α determines the distribution of income between capital and labor, but $\alpha + \beta$ determines the rate at which diminishing returns set in for economy-wide capital accumulation.

In addition to (24), the steady-state conditions for this economy are as follows:

$$y = \kappa k^\alpha n^{1-\alpha}. \tag{25}$$

$$r = \alpha \kappa k^{\alpha-1} n^{1-\alpha}. \tag{26}$$

$$w = (1 - \alpha) \kappa k^\alpha n^{-\alpha}. \tag{27}$$

$$v'(n) = \frac{-(1 - \tau_n) w}{c}. \tag{28}$$

$$r = \frac{\rho + \gamma g}{1 - \tau_k}. \tag{29}$$

$$c = \kappa k^\alpha n^{1-\alpha} - gk. \tag{30}$$

Equations (24) through (30) fully determine the steady-state values of seven variables: κ , y , k , n , r , w , and c . The equation for tax revenue remains the same as equation (13):

$$R = \tau_k r k + \tau_n w n. \tag{31}$$

In this setting, the dynamic feedback effects for the model of Section 2 are as follows:

$$\frac{dR}{d\tau_k} \Big|_{dynamic} = \left[1 - \frac{\alpha\tau_k + (1-\alpha)\tau_n}{(1-\alpha-\beta)(1-\tau_k)} \frac{\alpha+\beta}{\alpha} - \frac{\alpha\tau_k + (1-\alpha)\tau_n}{(\rho+\gamma g) - \alpha(1-\tau_k)g} \frac{(1-\alpha)}{(1-\alpha-\beta)} \frac{\sigma}{(1+\sigma)} g \right] \cdot \frac{dR}{d\tau_k} \Big|_{static} \quad (32)$$

$$\frac{dR}{d\tau_n} \Big|_{dynamic} = \left[1 - \frac{\alpha\tau_k + (1-\alpha)\tau_n}{(1-\alpha-\beta)(1-\tau_n)} \frac{\sigma}{(1+\sigma)} \right] \frac{dR}{d\tau_n} \Big|_{static} \quad (33)$$

These results are analogous to equations (14) and (15).

The quantitative effects of externalities are potentially large. As before, consider the canonical values $\tau_k = \frac{1}{4}$, $\tau_n = \frac{1}{4}$, $\alpha = \frac{1}{3}$, $\gamma = 1$, $g = .02$, $\rho = .05$, and $\sigma = \frac{1}{2}$. Recall that in our Ramsey model of Section 2, 53 percent of a capital tax cut and 17 percent of a labor tax cut are self-financing. Suppose $\beta = \frac{1}{12}$, so that the externality from capital raises the return to capital by one-quarter (much smaller than DeLong and Summers (1991) estimate). Equations (32) and (33) yield

$$\frac{dR}{d\tau_k} \Big|_{dynamic} = .26 \frac{dR}{d\tau_k} \Big|_{static} \quad .$$

$$\frac{dR}{d\tau_n} \Big|_{dynamic} = .81 \frac{dR}{d\tau_n} \Big|_{static} \quad .$$

In this case, growth pays for 74 percent of a capital tax cut and 19 percent of a labor tax cut. These calculations indicate that modest externalities to capital slightly raise the dynamic feedbacks associated with labor income taxes and significantly raise the feedbacks associated with capital income taxes.

At this point, we should acknowledge that the existence and magnitude of these externalities are both speculative and controversial. Our analysis suggests that measuring their magnitude is crucial for the task of dynamic scoring.

5 Transitional Dynamics

The results presented so far in this paper consider only the economy's steady state. This section examines how our steady-state results from Section 2 are affected by considering the transition paths of labor supply and capital. After a tax cut, the capital stock, which is initially fixed, will gradually increase to its new steady-state level. Labor supply will immediately jump and then approach its new steady state.

We derive our results from a log-linearized version of a system of differential equations that describe the model dynamics. Readers who wish to see the derivation of the results of this section are referred to the Appendix. To keep

things simple, we assume log utility ($\gamma = 1$) and no technological change ($g = 0$).

To compute the path of tax revenues, we need the transition paths of n and k and the values of n and k at three points in time: prior to the tax cut, immediately after the tax cut, and in the long-run steady state after the tax cut. Denote the levels of n and k at these three points as n_0 , n_ε , and n^* , and k_0 , k_ε , and k^* . Similarly, let R_0 , R_ε , and R^* denote tax revenues per period prior to, immediately after, and in the long run after a tax cut. Using this notation, the transition paths can be written as:

$$\ln n_t - \ln n^* = (\ln n_\varepsilon - \ln n^*) e^{\lambda t}, \quad (34)$$

$$\ln k_t - \ln k^* = (\ln k_\varepsilon - \ln k^*) e^{\lambda t}. \quad (35)$$

where λ is equal to the negative eigenvalue of the characteristic matrix of the system of differential equations. Both n and k , and thus R , transition from their jump values to their steady-state values at this rate.

Tax revenues at any time t can be written as:

$$R_t = [\alpha\tau_k + (1 - \alpha)\tau_n] k_t^\alpha n_t^{1-\alpha}. \quad (36)$$

This allows us to compute tax revenue at any point in time. With equations (34)-(36) and the model's steady-state conditions from Section 2, we can calculate how much of the static impact of a tax cut is paid for over any given period. Table 1 shows these calculations for selected points along the transition path. We continue to assume our canonical values for the other parameters: $\tau_k = \frac{1}{4}$, $\tau_n = \frac{1}{4}$, $\alpha = \frac{1}{3}$, $\sigma = \frac{1}{2}$.

Recall that, for a capital tax cut, the feedback effect in the steady state is 50 percent. By contrast, the immediate feedback is 10.6 percent, as labor supply jumps up in response to the tax cut. The feedback is 21.3 percent by the fifth year, 29.1 percent by the tenth year, and 41.9 percent by the twenty-fifth year.

For a labor tax cut, the feedback effect in the steady state is 16.7 percent. The immediate feedback is 12.3 percent. The feedback is 13.5 percent by the fifth year, 14.3 percent by the tenth year, and 15.7 percent by the twenty-fifth year.

The immediate jump in labor supply plays a vital role in the timing of the feedback effects. The elasticity of labor supply determines the size of this initial jump. If $\sigma = 3$, the instantaneous feedback of a labor tax cut is 32.6 percent, compared to a steady-state feedback of 38 percent. For the case of a capital tax cut, the instantaneous feedback is 30.4 percent, while the steady-state feedback remains 50 percent.

One way to summarize the transition to the steady state is by calculating the present value of the reduced tax revenue, using the after-tax return to capital along the transition path to discount the revenue streams. By comparing the present value of the dynamic and static estimates, we obtain the "present-value" feedback effect, which is, in essence, a weighted average of the feedback effects along the entire path. For the canonical parameter values we have been using,

the present-value feedback of a capital tax cut is 33 percent, compared to a steady-state feedback of 50 percent. The present-value feedback of a labor tax cut is 14.8 percent, compared to a steady-state feedback of 16.7 percent.

Overall, the analysis of transitional dynamics demonstrates that the task of dynamic scoring is particularly important over longer time horizons. The time horizon is critical when analyzing capital taxes. In practical discussions of budget policy, scoring windows are only five or ten years. The generalized Ramsey model shows that many significant effects occur outside of this window.

6 Conclusion

This paper has examined the issue of dynamic scoring using the textbook neo-classical growth model and some generalizations of it. Our goal has been to provide theoretical guidance for economists interested in estimating the revenue effects of tax changes. The simple formulas we have derived permit back-of-the-envelope calculations that illustrate the degree to which tax cuts are self-financing.

In all of the models considered here, the dynamic response of the economy to tax changes is too large to be ignored. In almost all cases, tax cuts are partly self-financing. This is especially true for cuts in capital income taxes.

Not surprisingly, the results of this exercise depend on a number of key parameters. Because the values of some of these parameters are open to debate, reasonable people can disagree about the magnitude of the feedback effects. Two crucial parameters are the compensated elasticity of labor supply and the externality to capital accumulation. Unfortunately, the empirical literature does not give clear guidance about their magnitudes. The degree of imperfect competition may also be important, but only to the extent that market power leads to pure economic profits. Finally, the time horizon of consumers, although important for many questions in economics, appears not to be crucial for the task of dynamic scoring.

Although we have explored several variations of the basic Ramsey model to evaluate the robustness our conclusions, there are surely issues still to be addressed. As we noted earlier, some economists have emphasized the short-run Keynesian effects of tax policy, and these effects may be important for dynamic scoring. In addition, much of the literature on economic growth has stressed the role of human capital, which is absent from the models considered here. How tax policy affects human capital accumulation and how human capital affects economic growth are hard questions, but they may be crucial for revenue estimation, especially over longer time periods. In light of all the open questions, the results presented in this paper should be viewed only as first steps.

Policy economists will need to focus the next steps on evaluating which generalizations of the basic model are most salient and then estimating the key parameters. The task is pressing. In 2003, the U.S. House of Representatives adopted a rule that requires the staff of the Joint Committee on Taxation to analyze the macroeconomic impact of any major tax bill before the House can

consider the bill. One conclusion is impossible to escape: difficult as it may be, the subject of dynamic scoring should remain a high priority for those economists advising lawmakers on issues of tax policy.

Appendix 1: Production Function Generalizations

The results throughout this paper assume the production function is Cobb-Douglas. This Appendix extends the results in Section 1 in two ways.

Case 1: CES Production

Suppose the production function has a constant elasticity of substitution between capital and labor equal to ξ . Then, result (5) generalizes to become

$$\left. \frac{dR}{d\tau_k} \right|_{dynamic} = \left[1 - \frac{\alpha\tau_k + (1-\alpha)\tau_n}{(1-\tau_k)(1-\alpha)} \xi \right] \left. \frac{dR}{d\tau_k} \right|_{static}.$$

Greater substitutability between capital and labor (higher ξ) increases the extent to which capital tax cuts are self-financing. For example, if the elasticity of substitution is 1.5 rather than 1.0, the dynamic feedback effect rises from 50 percent to 75 percent.

As Ventura (1997) and Mankiw (1995) point out, international trade in goods can affect the degree of substitutability between capital and labor. In traditional Heckscher-Ohlin trade theory, a nation can move resources between industries with varying degrees of capital intensity. When a country's stock of capital increases, it can export capital-intensive goods and import labor-intensive goods, avoiding changes in the returns to either capital or labor. In other words, international trade raises the effective elasticity of substitution in an economy. One corollary of this line of analysis is that international trade increases the extent to which capital tax cuts pay for themselves.

Case 2: Depreciation of Capital

Consider now the introduction of exponential depreciation into the model. That is, we assume that gross output is produced with a Cobb-Douglas production function and net output is produced according to:

$$f(k) = k^\alpha - \delta k,$$

where δ is the rate of depreciation. All other equations of the model in Section 1 remain the same.

The result analogous to equation (5) is:

$$\left. \frac{dR}{d\tau_k} \right|_{dynamic} = \left[1 - \frac{\alpha'\tau_k + (1-\alpha)\tau_n}{(1-\tau_k)(1-\alpha)} \right] \left. \frac{dR}{d\tau_k} \right|_{static}.$$

where we use α' to indicate the capital share adjusted for depreciation. Specifically,

$$\alpha' = \alpha - \frac{\delta}{\left(\frac{\rho + \gamma g}{(1 - \tau_k)} + \delta\right)} = \alpha - \frac{\delta k^*}{\alpha f(k^*)},$$

These results indicate that the adjusted capital share can be thought of as capital's share of gross income, α , less the share of capital depreciation in gross capital income.

For our canonical parameter values and no depreciation, the steady-state feedback of a capital tax cut is 50 percent. If depreciation is $\delta = .03$, the adjusted capital share α' is 0.09 (compared to $\alpha = \frac{1}{3}$), and the feedback effect is reduced to 38 percent. That is, depreciation reduces the extent to which capital tax cuts pay for themselves.

It would be straightforward to extend the results in the body of the paper to include exponential depreciation, but we exclude depreciation to keep the model simple.

Appendix 2: Analysis of Transitional Dynamics

This appendix presents the technical details regarding the transitional dynamics of the model, as outlined in Section 5. We derive differential equations that describe the time paths of the capital stock, consumption, and the labor supply from the model in Section 2. We need the values of these variables at three points in time: the steady state before the tax cut, immediately after the tax cut, and in the new steady state. Combined with the rate of transition to the new steady state, these values will allow us to calculate tax revenue at any point in time.

We have nine values of these three variables to calculate, which we denote $c_0, n_0, k_0, c_\varepsilon, n_\varepsilon, k_\varepsilon, c^*, n^*, k^*$. The initial and new steady-state values can be calculated with the standard steady-state conditions. For the jump values, note first that the capital stock is momentarily fixed, so $k_0 = k_\varepsilon$. We can derive the remaining two jump values, c_ε and n_ε , with the differential equations that describe the transition path of the model from Section 2.

To derive these differential equations, we begin with the Hamiltonian describing the household's maximization problem:

$$H = e_t^{-\rho t} \frac{(ce^{gt})^{1-\gamma} e^{(1-\gamma)v(n)} - 1}{1-\gamma} + \varphi(t) [(1 - \tau_n)wn + (1 - \tau_k)rk - c - gk + T].$$

We assume $\gamma = 1$ and $g = 0$ for simplicity. Performing the household's maxi-

mization, we can derive the following results.

$$\frac{\dot{c}}{c} = (1 - \tau_k)\alpha k^{\alpha-1}n^{1-\alpha} - \rho, \quad (\text{A1})$$

$$\frac{\dot{n}}{n} = \frac{\rho + \tau_k\alpha k^{\alpha-1}n^{1-\alpha} - \alpha\frac{c}{k}}{\frac{1}{\sigma} + \alpha}. \quad (\text{A2})$$

where σ is the constant-consumption elasticity of labor supply.

Capital accumulation is determined by

$$\frac{\dot{k}}{k} = k^{\alpha-1}n^{1-\alpha} - \frac{c}{k}. \quad (\text{A3})$$

We now linearize the system (A1)-(A3) around the steady state. Rewrite the system in terms of natural logs:

$$\frac{d \ln c}{dt} = \alpha(1 - \tau_k)e^{(\alpha-1)(\ln k - \ln n)} - \rho, \quad (\text{A4})$$

$$\frac{d \ln n}{dt} = \frac{1}{\alpha + \frac{1}{\sigma}} \left[\alpha\tau_k e^{(\alpha-1)(\ln k - \ln n)} - \alpha e^{(\ln c - \ln k)} + \rho \right], \quad (\text{A5})$$

$$\frac{d \ln k}{dt} = e^{(\alpha-1)(\ln k - \ln n)} - e^{(\ln c - \ln k)}. \quad (\text{A6})$$

These equations can be used to solve for steady-state expressions in terms of parameters.

$$e^{(\alpha-1)(\ln k^* - \ln n^*)} = \frac{\rho}{\alpha(1 - \tau_k)},$$

$$e^{(\ln c^* - \ln k^*)} = \frac{\rho}{\alpha(1 - \tau_k)}.$$

Next, we take a first-order approximation of the log-linear system (A4)-(A6) around the log deviations of c, n, k from their steady-state values:

$$\begin{bmatrix} \frac{d \ln c}{dt} \\ \frac{d \ln n}{dt} \\ \frac{d \ln k}{dt} \end{bmatrix} = \begin{bmatrix} 0 & -(\alpha-1)\rho & (\alpha-1)\rho \\ \frac{-\rho}{(\alpha + \frac{1}{\sigma})(1 - \tau_k)} & \frac{-(\alpha-1)\tau_k\rho}{(1 - \tau_k)(\alpha + \frac{1}{\sigma})} & \frac{(1 + (\alpha-1)\tau_k)\rho}{(\alpha + \frac{1}{\sigma})(1 - \tau_k)} \\ -\frac{\rho}{\alpha(1 - \tau_k)} & \frac{-(\alpha-1)\rho}{\alpha(1 - \tau_k)} & \frac{\rho}{1 - \tau_k} \end{bmatrix} \begin{bmatrix} \ln\left(\frac{c}{c^*}\right) \\ \ln\left(\frac{n}{n^*}\right) \\ \ln\left(\frac{k}{k^*}\right) \end{bmatrix}.$$

Call the matrix above A . We can use the eigenvalues and eigenvectors associated with A to derive the transition paths of our variables to their steady-state levels. There are three eigenvalues, or roots to this equation, that can be calculated with standard mathematical software or derived analytically. It turns out that one of these is positive, one is negative, and one is zero. We will call them ϕ , λ , and ω , respectively. Call the matrix of eigenvectors V . Then, we can describe the paths of the log values of c, n, k with

$$\begin{aligned}
\ln c &= \ln c^* + v_{11}e^{\phi t}b_1 + v_{12}e^{\lambda t}b_2 + v_{13}e^{\omega t}b_3, \\
\ln n &= \ln n^* + v_{21}e^{\phi t}b_1 + v_{22}e^{\lambda t}b_2 + v_{23}e^{\omega t}b_3, \\
\ln k &= \ln k^* + v_{31}e^{\phi t}b_1 + v_{32}e^{\lambda t}b_2 + v_{33}e^{\omega t}b_3.
\end{aligned}$$

where v_{ij} is the i,j th component of the matrix of eigenvectors, and b_1, b_2, b_3 are coefficients determined by boundary conditions.

For the boundary conditions, consider the limit as $t \rightarrow \infty$. Because $\phi > 0$ and $\lim_{t \rightarrow \infty} c_t = c^*$, we know that $b_1 = 0$. Similarly, because $\omega = 0$, $b_3 = 0$ as well. That leaves us with:

$$\begin{aligned}
\ln c &= \ln c^* + v_{12}e^{\lambda t}b_2, \\
\ln n &= \ln n^* + v_{22}e^{\lambda t}b_2, \\
\ln k &= \ln k^* + v_{32}e^{\lambda t}b_2.
\end{aligned}$$

Because $k_0 = k_\varepsilon$,

$$b_2 = \frac{(\ln k_0 - \ln k^*)}{v_{32}} = \frac{(\ln k_\varepsilon - \ln k^*)}{v_{32}}.$$

We can use this to rewrite our system:

$$\ln c - \ln c^* = \frac{(\ln k_\varepsilon - \ln k^*)}{v_{32}} v_{12} e^{\lambda t}, \quad (\text{A7})$$

$$\ln n - \ln n^* = \frac{(\ln k_\varepsilon - \ln k^*)}{v_{32}} v_{22} e^{\lambda t}, \quad (\text{A8})$$

$$\ln k - \ln k^* = (\ln k_\varepsilon - \ln k^*) e^{\lambda t}. \quad (\text{A9})$$

Thus, at $t = \varepsilon$,

$$\begin{aligned}
(\ln n_\varepsilon - \ln n^*) e^{-\lambda \varepsilon} &= \frac{(\ln k_\varepsilon - \ln k^*)}{v_{32}} v_{22}, \\
\ln n - \ln n^* &= (\ln n_\varepsilon - \ln n^*) e^{\lambda t}. \quad (\text{A10})
\end{aligned}$$

Results (A9) and (A10) imply that the transition from the levels of k and n immediately after the tax cut to their new steady-state levels is at rate λ .

Equations (A9) and (A10) allow us to calculate the level of the capital stock and labor supply at any point in time. To use (A9) and (A10), we need the initial and new steady-state levels of our key variables. These can be derived with the steady-state conditions.

$$\begin{aligned}
y &= k^\alpha n^{1-\alpha}, \\
r &= \alpha k^{\alpha-1} n^{1-\alpha}, \\
w &= (1-\alpha)k^\alpha n^{-\alpha}, \\
v'(n) &= \frac{-(1-\tau_n)w}{c}, \\
r &= \frac{\rho}{1-\tau_k}, \\
c &= y.
\end{aligned}$$

For purposes of numerical implementation, we posit that $v(n)$ takes an isoelastic functional form.

With our values for n and k before the tax cut, immediately after the tax cut, and in the new steady state, we can use equations (A9) and (A10) and the eigenvectors generated by matrix A to calculate the values of the capital stock and labor supply at any time. This allows us to calculate tax revenue at any point in time, since

$$R = [\alpha\tau_k + (1-\alpha)\tau_n] k^\alpha n^{1-\alpha}.$$

The calculations given in the main text result from a numerical implementation of these results.

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Figure 1: Dynamic feedback in the Blanchard model

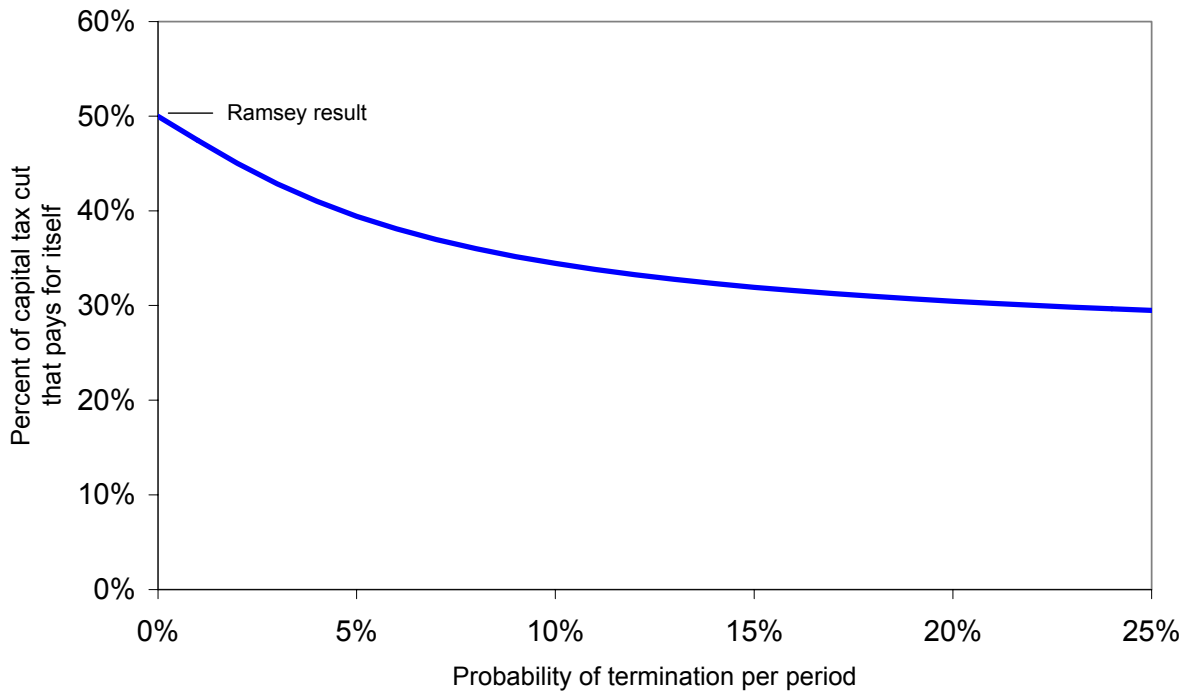


Table 1: Dynamic Feedback Effects along the Transition Path

Percent of static revenue impact offset by higher growth

Time	Capital taxes	Labor taxes
Immediate impact	10.6	12.3
1 year	13.0	12.6
3 years	17.4	13.0
5 years	21.3	13.5
10 years	29.1	14.3
25 years	41.9	15.8
50 years	48.4	16.5
Steady-state impact	50.0	16.7