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Craig Burnside
Domenico Fanizza

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ABSTRACT

In this paper we discuss fiscal and monetary policy issues facing heavily-indebted poor countries (HIPCs) who receive debt reduction via the enhanced HIPC initiative. This debt relief program is distinguished from previous ones by its conditionality: freed resources must be used for poverty reduction. We argue that (i) this conditionality severely limits the extent to which the initiative provides significant debt relief; (ii) depending on the response of monetary policy to an increase in social spending there could be a short-run increase in inflation in HIPC countries and (iii) the keys to long-run fiscal sustainability in the HIPCs are significant fiscal reforms by their governments, and the effectiveness of their poverty reduction programs in raising growth.

Craig Burnside
Department of Economics
Duke University
Box 90097
Durham, NC 27708-0097
and NBER
burnside@econ.duke.edu

Domenico Fanizza
Middle East and Central Asia Department
International Monetary Fund
700 19th Street, NW
Washington, DC 20431
dfanizza@imf.org

The enhanced HIPC initiative has acquired increasing prominence in the policy debate in developing countries.[1] A significant number of countries are already enjoying debt relief under the HIPC initiative. While an abundant literature developed during the 1980s on the macroeconomic impact of debt overhang and debt write-offs, little attention has been devoted to the macroeconomic impact of debt relief proposed under the enhanced HIPC initiative.[2] Debt relief under the HIPC initiative differs from previous major debt relief initiatives, such as the Baker and Brady plans, in that it concerns official rather than commercial debt and in that it imposes well-defined conditionality. In particular, it requires that budgetary resources no longer needed for debt service be used for poverty reduction purposes. This also distinguishes the enhanced HIPC initiative from previous official debt relief programs.

We focus on fiscal and monetary policy issues connected with debt reduction under the enhanced HIPC initiative. The issues we highlight stem from two distinguishing aspects of the initiative's design. First, the HIPC initiative relieves debt through forgiveness of a substantial fraction of a country's debt service payments. The initiative requires that the resources freed from debt service be used to increase government spending on poverty reduction programs.[3] Second, the initiative has a finite life—the increase in government spending and the forgiven debt service take place over a floating period whose length depends on the country's success in implementing a comprehensive anti-poverty strategy.[4] Given these features of the HIPC initiative we make four main points.[5]

First, we argue that, due to its conditionality, the HIPC initiative actually provides

[1] HIPC is an acronym for heavily-indebted poor country.

[2] See, among many others, Sachs (1984, 1989), Frenkel, Dooley and Wickam (1989), Claessens, Diwan and Fernandez-Arias (1992) and Fernández-Arias (1992).

[3] For a concise description of the HIPC initiative see Van Trotsenburg and MacArthur (1999) and the World Bank's HIPC website: www.worldbank.org/hipc.

[4] After a country qualifies for assistance, the international community commits debt relief to reach a target for the ratio of the net-present value of debt to exports (150 percent) or to government revenue (250 percent). This assistance is delivered over a variable period during which there is conditionality requiring increased poverty reduction spending. After a country has implemented a comprehensive poverty reduction strategy, creditors supply, without further conditionality, "top-up" debt relief needed to reach specific debt targets.

[5] In what follows we will frequently refer to the enhanced HIPC initiative as "the initiative," its characteristic conditionality as "HIPC conditionality," governments and organizations providing debt relief as "donors" and countries receiving debt relief as "recipient countries," or simply "HIPCs."

only limited debt relief; that is, there is only a modest relaxation of the government's lifetime budget constraint.[6] The reason for this is fairly straightforward: under the main provisions of the initiative, the government swaps one type of spending commitment for another of equal value. The modest amount of debt relief the initiative does provide is discussed below.

Second, we argue that monetary policy makers face a trade-off between stabilizing inflation and achieving permanent debt reduction. This trade-off is related to the standard one faced by any central bank when confronted by a temporary increase in domestic demand.

Third, we discuss three factors that can improve the finances of HIPC governments. First, if the poverty reduction programs initiated under debt relief substantially increase economic activity, governments will reap additional tax revenue. We show that under generous assumptions about the growth-enhancing effects of the initiative this increase in revenue is modest. Second, there is a provision of the HIPC initiative under which donors provide a "top-up" amount of relief that comes with no conditionality. Typically, this amount is modest compared to a country's initial indebtedness, so we do not discuss it further in this paper. Third, we therefore argue that HIPC governments must act on their own to implement significant additional fiscal reforms if they are to attain long-run fiscal sustainability.

Last, we attempt to rationalize the design of the initiative in an optimal policy framework. We argue that the initiative is consistent with donors whose preferences put a lot of weight on poverty reduction but little weight on debt reduction or fiscal sustainability. Thus, we find it hard to rationalize the emphasis put on the latter in public policy discussions.

Our analysis does not focus on assessing the effectiveness of poverty reduction spending in reducing poverty. Rather, we focus our analysis on the budgetary impact of the HIPC initiative. However, the impact of this spending on output is important in our analysis because of its secondary impact on tax revenue. To highlight the role of different aspects of the initiative, we decompose its effect on the government budget into two components: the direct effect, which ignores effects on tax revenue, and the

[6] For an early analysis of the financial impact of the HIPC initiative, see World Bank (2001).

indirect effect, which takes them into account. In measuring the indirect effect, we take as given Burnside and Dollar's (2000) results suggesting that aid is effective in raising growth in recipient countries with good macroeconomic policies. Thus our results give a plausible upper bound on the positive *budgetary* impact of the HIPC initiative.

In Section 1 we focus on our first point related to the relaxation of the government budget constraint. We illustrate the impact of debt relief with HIPC-conditionality on a government's finances using a standard model of the government budget constraint. This model only allows us to discuss the direct effects of debt relief with HIPC-conditionality on the budget, which we argue are nil. We argue that other things equal, after debt relief, the government must still raise the same amount of revenue, from all sources, as it did prior to receiving debt relief.

In Section 2 we develop a simple monetary model based on the standard Cagan money demand function, in order to fully characterize the equilibrium dynamics of prices, inflation, debt and seigniorage during and after the implementation of a debt relief initiative. In Section 3 we simulate our model under different monetary policies. By doing this we are able to highlight our second point. We find that, with what we describe as passive monetary policy, debt-relief with HIPC conditionality could have a short-term impact on money creation and inflation, which would eventually be reversed. Under what we describe as a more active monetary policy, the government can act to stabilize inflation, but it can only do so by raising its long-run indebtedness relative to what it would be under passive policy. This is because keeping inflation stable requires the government to issue more new debt to sterilize the monetary impact of the short-run increase in government spending under HIPC-conditionality.

Section 3 also explores the role of (i) any growth that results from debt relief and (ii) additional fiscal reforms implemented by a HIPC's government. We show that both tend to improve the government's lifetime budget constraint, and lessen the short-run monetary impact of the increase in government spending. Under generous assumptions about growth, we show that the indirect effects of the initiative on fiscal sustainability are modest. For this reason, we suggest that the essential ingredients for lasting fiscal sustainability in HIPC countries are significant fiscal reforms that are not met by reduced aid commitments from donors.

Our results beg the following question: if the HIPC initiative provides only limited debt relief, what might the designers of the initiative have had in mind in crafting it? Without fully articulating a model of optimal debt relief, in Section 4 we argue that, given the initiative's design, there is an implicit donor-side objective function that puts a great deal of weight on the costs of poverty, while putting little or no weight on the costs associated with high indebtedness and fiscal insolvency. We show how the initiative might simply be an indirect way to ensure that increased aid flows are used to increase government spending on programs favored by donors, as opposed to programs that might be preferred by local policy makers. While our sketched model can rationalize the design of the initiative, it cannot simultaneously rationalize the emphasis that, in public policy discussions, is put on fiscal sustainability. It is for this reason that we presume that the limited debt relief feature of the initiative may have been overlooked.

In Section 5 we discuss possible extensions to our model, and provide some concluding remarks.

1. Budget Constraints and HIPC Conditionality

We begin by discussing the fiscal implications of debt relief with "HIPC-conditionality" within a standard model of the government's intertemporal budget constraint. For convenience we work in continuous time.[7] In our simple model, there is only one good, whose price is P_t. The government issues only one type of debt, D_t, whose value is indexed in terms of that good. Thus we eliminate implicit default, through unanticipated inflation, by assumption.

We assume, for simplicity, that the net real interest rate on government debt is some constant r. The government finances its interest payments, rD_t, and its purchases of goods and services, G_t, in four ways: by raising tax revenue (net of transfers), Ω_t, through the issuance of base money, M_t, by receiving aid, A_t, or through

[7] In a separate appendix, available upon request, we show that this approach is equivalent to working with the simple static accounting framework familiar to students of monetary theory and policy.

the issuance of new debt. The government raises funds by issuing base money via seigniorage revenue, $\dot{M}_t/P_t$, where P_t is the price level and $\dot{M}_t$ is the time derivative of the money stock.[8] Hence, the government's flow budget constraint is given by

$$\dot{D}_t = rD_t + G_t - \Omega_t - A_t - \dot{M}_t/P_t, \tag{1.1}$$

where all variables are measured in units of local currency.

Writing the budget constraint in terms of D_0 and iterating, we obtain

$$D_0 = \int_0^\infty (\Omega_t - G_t + A_t + \dot{M}_t/P_t)e^{-rt}dt \tag{1.2}$$

where we have imposed the no-Ponzi scheme condition that

$$\lim_{t\to\infty} e^{-rt}D_t = 0.$$

We interpret the HIPC initiative using equations (1.1) and (1.2). Our starting point is to think of a working definition of a heavily indebted government. One interpretation of a heavily indebted government at time 0 is as follows: given the initial stock of debt, D_0, and the likely paths of government purchases and foreign aid receipts, $\{A_t, G_t\}_{t\in[0,\infty)}$, the combined present values of taxes and seigniorage revenue required to close the government budget constraint is very large, i.e.:

$$D_0 + \int_0^\infty (G_t - A_t)e^{-rt}dt = \int_0^\infty (\Omega_t + \dot{M}_t/P_t)e^{-rt}dt \gg 0.$$

In other words, we could think of HIPC governments as ones which, in order to be solvent, would need to rely on either (i) significant future seigniorage revenue, obtained at the cost of high inflation, or (ii) punitively high future tax revenues. An alternative, of course, is that the government might default on its debt. We discuss this possibility later.

Our next step is to see what the impact of the HIPC initiative would be on the government's lifetime budget constraint. To do this, we must characterize the initiative in terms of its effect on the various items in the government's lifetime budget constraint (1.2).

[8] We generically indicate time derivatives, $\partial Z_t/\partial t$, as $\dot{Z}_t$.

1.1. Direct Effects

As we described in the introduction, one feature of the HIPC initiative is that donors forgive some fraction of the scheduled debt service payments pertaining to a government's existing stock of external debt. At the decision point, which we refer to as date 0, the present value of the existing scheduled debt service is calculated, and is expressed as a percentage of the country's exports. The debt relief given under the HIPC initiative would forgive enough of this debt service to reduce this *NPV of debt-to-exports ratio* to no more than 150 percent.[9] To take an example, suppose the NPV of debt-to-exports ratio at date 0 was 260 percent.[10]. Then, over the life of the initiative, the country would reduce its service payments, *on debt existing at time 0*, by about 42 percent in present value terms.

There are several ways we could build this kind of debt relief into equations (1.2) and (1.1). One way would be to assume that before debt relief, D_0, on the left-hand side of (1.2), takes on some value. We could then assume that after debt relief, there is simply a change in the left-hand side of (1.2), to a new value $D'_0 = (1-\theta)D_0$, where θ represents the fraction of the country's debt that is effectively cancelled by the forgiven debt service.

An alternative interpretation is that at date 0 the country receives an announcement from donors stating that the present value of the future path of $\{A_t\}_{t\in[0,\infty)}$ will be higher by the amount θD_0 $(0 < \theta < 1)$, than it would have been in the absence of the initiative.

Notice that these two interpretations are isomorphic to one another in terms of their implications for the remaining items in the government's lifetime budget constraint. In particular, under the first interpretation the initial stock of debt is $(1-\theta)D_0$. Holding the present value of aid flows constant, this implies that $\int_0^\infty(\Omega_t - G_t + \dot{M}_t/P_t)e^{-rt}dt$ will fall by θD_0 under the debt-relief initiative. On the other hand, under the second interpretation the initial stock of debt is D_0, and the present value

[9] In some cases debt relief is calculated with reference to the present value of debt service relative to government revenue.

[10] This was the average figure across the 22 HIPCs that had reached the decision date by 1999. See Development Committee (2001).

of future aid flows rises by θD_0. Again, this implies that $\int_0^\infty(\Omega_t - G_t + \dot{M}_t/P_t)e^{-rt}dt$ will fall by θD_0 under the initiative. We find the latter interpretation to be more convenient notationally, and use it throughout the rest of this paper.

The next important feature of the HIPC initiative is its conditionality, requiring that the savings from reduced debt service be used to increase social spending. We model this conditionality as being equivalent to the government making an announcement at date 0 that the present value of the future path of $\{G_t\}_{t\in[0,\infty)}$ will be higher by the amount θD_0.

We refer to the effects of the HIPC initiative on the paths of A_t and G_t as its *direct* effects. We have characterized these effects as a change in the anticipated paths of aid and government spending to $\{A'_t, G'_t\}_{t\in[0,\infty)}$ from $\{A_t, G_t\}_{t\in[0,\infty)}$, where

$$\int_0^\infty A'_t e^{-rt} = \int_0^\infty A_t e^{-rt} + \theta D_0 \tag{1.3}$$

$$\int_0^\infty G'_t e^{-rt} = \int_0^\infty G_t e^{-rt} + \theta D_0. \tag{1.4}$$

Since (1.2) must be satisfied for all possible future paths, we have

$$D_0 = \int_0^\infty (\Omega'_t - G'_t + A'_t + \dot{M}'_t/P'_t)e^{-rt}dt, \tag{1.5}$$

where $\{\Omega'_t, M'_t, P'_t\}_{t\in[0,\infty)}$ are the post-debt relief paths of taxes, the money supply and the price level. Given (1.3) and (1.4) we can rewrite (1.5) as

$$D_0 = \int_0^\infty (\Omega'_t + \dot{M}'_t/P'_t)e^{-rt}dt + \int_0^\infty (A_t - G_t)\,e^{-rt}dt.$$

It is clear from (1.2), however, that this implies

$$\int_0^\infty (\Omega'_t + \dot{M}'_t/P'_t)e^{-rt}dt = \int_0^\infty (\Omega_t + \dot{M}_t/P_t)e^{-rt}dt, \tag{1.6}$$

where $\{\Omega_t, M_t, P_t\}_{t\in[0,\infty)}$ are the paths of taxes, the money supply and the price level that would have prevailed in the absence of debt relief.

Notice, from (1.6), that debt relief with HIPC conditionality does not relax the government budget constraint in the following sense. To satisfy its budget constraint, the government must raise just as much seigniorage and tax revenue after receiving debt relief as it needed to in the absence of debt relief.

In Sections 2 and 3 we consider a case where $\int_0^\infty \Omega_t' e^{-rt} dt = \int_0^\infty \Omega_t e^{-rt} dt$, i.e. there is no change in tax revenues induced by the initiative. In this setting, (1.6) implies that the initiative can have no impact on the present value of seigniorage revenue. On the other hand, depending on how monetary policy responds to increased government spending, and depending on the timing of that increased spending, the inflation rate can rise, fall, or remain unchanged in the short-run. We will see, later, in some model-based experiments, that if there is a temporary rise in inflation, it will later fall below its initial value. If, for some reason, inflation were to decline in the short run, it would rise above its initial value in the long-run.[11] On the other hand, it is possible that inflation could remain unchanged.[12] To achieve a desired path for inflation, the government must choose an appropriate monetary policy. In Section 2 we will see that the monetary policy consistent with a stable inflation path is an active one—in the sense that the central bank must neutralize the monetary injection resulting from increased government spending.

1.2. Indirect Effects

Of course, eventually we must modify our analysis to take account of the indirect effects of debt relief on the budget. We do this in detail in Section 2, but here we simply note where that analysis will lead us. Suppose that the increased social spending required under the HIPC initiative has a positive impact on growth in recipient countries. In this case, there will be some relaxation of the government's budget constraint. It will still be true that the combined present values of taxes and seigniorage revenue will be unchanged. I.e., (1.6) will still hold. However, now suppose the government leaves tax *rates* unchanged when it receives debt relief. In this case, the present value of tax revenue will rise, allowing the present value of seigniorage revenue to be lower. Furthermore, if there is an increase in money demand resulting from an economic expansion, this seigniorage revenue could be raised at a

[11] This basic point lies at the core of models of currency crises in which governments that are unable to close their lifetime budget constraints without resort to seignorage revenue are required to eventually abandon any exchange rate arrangement that limits their access to it. See, for example, Krugman (1979), Flood and Garber (1984), Obstfeld (1986), Drazen and Helpman (1987), and Burnside, Eichenbaum and Rebelo (2001).

[12] Anticipating our later results, this would be the case if $M_t' = M_t$ for all t.

lower steady state rate of inflation. Thus, if we measured the government's fiscal health by how much inflation it would need to generate to close its budget, both of these effects would be beneficial. In order to quantify these indirect effects we need a fully specified model, such as the one we present in Section 3.

1.3. Allowing for Default

The model we have outlined so far assumes that governments always close the lifetime budget constraint, through raising sufficient taxes and seigniorage revenue. In this section we modify our analysis of the government budget constraint to explicitly allow for default, but we maintain our assumption that debt is denominated in real terms. What we mean by explicit default is that the government can announce paths, $\{\Omega_t, G_t, M_t\}_{t\in[0,\infty)}$, that along with the path, $\{A_t\}_{t\in[0,\infty)}$, lead to a violation of (1.2). A version of (1.2) still holds, but in this version the left hand side variable, D_0^M, is a measure of the market value (at the discount rate r) of the government's future primary surpluses inclusive of aid and seigniorage revenue

$$D_0^M = \int_0^\infty (\Omega_t + A_t + \dot{M}_t/P_t - G_t)e^{-rt}dt. \tag{1.7}$$

Essentially, (1.7) recognizes the fact that governments often do not raise sufficient funds to honor (1.2).[13]

This naturally leads to an alternative interpretation of a heavily indebted government: given the government's announced paths for $\{\Omega_t, G_t, M_t\}_{t\in[0,\infty)}$ and the donors' announced path for $\{A_t\}_{t\in[0,\infty)}$, the market value of the government's debt is substantially lower than its face value:

$$D_0^M \ll D_0.$$

Under this interpretation we can think of HIPC governments as ones which are unable (or unwilling) to raise sufficient revenues to satisfy (1.2).

[13] By market value we do not mean to suggest that there is an active market in which the debt of HIPC countries is traded. Rather, by market value, we refer to the value of a claim to the entire future stream of the government's actual primary surpluses, inclusive of seigniorage revenue, using the discount rate r to compute present values.

How would we measure the impact of the HIPC initiative in this setting? In the previous subsection, we gave two interpretations of the HIPC initiative. In one interpretation, the initiative directly reduces the initial stock of debt to $D_0' = (1-\theta)D_0$. It also increase the present value of G_t by θD_0. In this case, D_0^M falls by θD_0. According to our other interpretation, D_0 doesn't change, but the present values of A_t and G_t both increase by θD_0. Therefore, D_0^M doesn't change either. According to both of these interpretations the degree of the government's "solvency" does not change: $D_0' - (D_0^M)' = D_0 - D_0^M$.

We wish to emphasize that our conclusions regarding the impact of the HIPC initiative on the government's budget are not specific to our modeling the initiative as an increase in aid flows, with those flows being used to make debt service payments. Our results are unchanged if we introduce separate notation for the debt service payments associated with the initial debt stock, D_0, and we examine a reduction in the magnitude of these payments.[14]

2. Extending the Model

In this section we extend our model so that we can describe (i) the indirect effects of the HIPC initiative on the government's lifetime budget constraint and (ii) the dynamic inflationary implications of different policy responses to debt relief. Our extensions consist of a money demand function, and a simple model of how debt service savings redirected to poverty reduction spending map into increased economic activity.

2.1. The Money Demand Function

We model the demand for local currency using the familiar Cagan (1956) money demand function

$$\ln(M_t/P_t) = a + \ln Y_t - \eta(r + \pi_t) \tag{2.1}$$

where a is some constant, Y_t represents the level of output, π_t is the inflation rate (i.e. $\pi_t = \dot{P}_t/P_t$) and $\eta > 0$. We do not believe that the implications of the Cagan

[14] In a separate appendix, available upon request, we show this in a setting where the original stock of debt has an arbitrary maturity structure.

specification differ substantially, in any *qualitative* manner, from those derived from alternative monetary models. However, the Cagan specification has computational advantages.

We can use the fact that $\pi_t = \dot{P}_t/P_t$ to derive a generic solution for the price level under the assumption that it is a continuous function of time.[15] We can rewrite (2.1) as

$$p_t = \eta r - a + \ln(M_t/Y_t) + \eta \dot{p}_t, \tag{2.2}$$

where $p_t = \ln P_t$. This implies that

$$p_t = \eta r - a + \frac{1}{\eta}\int_t^\infty e^{-(s-t)/\eta} \ln(M_s/Y_s)ds, \tag{2.3}$$

where we have assumed that $\lim_{t\to\infty} e^{-t/\eta} \ln P_t = 0$.[16]

2.2. Allowing for Output Effects

As we mentioned in Section 1, debt service savings directed to increased spending on poverty reduction may have a significant impact on the government budget through their effect on output. Let the additional government spending on poverty reduction under the initiative be given by $\hat{G}_t$. A simple way to allow for output effects is to assume that some fraction $0 < \alpha \leq 1$ of this spending has an investment component, where investment is defined broadly.

We denote the stock of capital specifically built by this investment as K_t. Initially we have $K_0 = 0$. We let K_t evolve according to

$$\dot{K}_t = \alpha \hat{G}_t - \delta K_t \text{ for } t \geq 0. \tag{2.4}$$

To keep our model as simple as possible, we assume that the level of output, absent debt relief, is some constant, Y, and that the level of output with debt relief is $Y_t = Y + \rho K_t$ for some $\rho \geq 0$. We assume that tax revenue is proportional to Y_t, i.e. $\Omega_t = \omega Y_t$ for all t.

One of advantage of our model of the output effects of debt relief is its tractability. A disadvantage is that we ignore some channels through which the initiative might

[15] This is a standard assumption in any one good model where the price level and the exchange rate must be continuous to avoid predictable arbitrage opportunities.

[16] This solution is easily verified by differentiating the right hand side of (2.3) with respect to t.

affect output. In neoclassical models, such as the one sketched by Barro (1997) an increase in government expenditure on goods and services has a direct impact on the supply of output, such as the one we have described above, but it also has indirect wealth and substitution effects that can theoretically enhance or offset the direct effect in determining the equilibrium level of output. In Keynesian models increases in government expenditure on goods and services, whether on consumption or investment goods, can have a direct expansionary effect on output through the conventional multiplier mechanism. These sorts of models are beyond the scope of this paper, but they do point out an important shortcoming of our analysis. In the end, when we calibrate our model of output effects we do so bearing in mind these shortcomings. Our quantitative version of the model, discussed in Section 3, is consistent, we think, with an upper bound on the possible impact of debt relief on real activity.

2.3. An Initial Steady State

We assume that at time 0 the economy is initially in a steady state where $Y_t = Y$, $G_t = G$, $\Omega_t = \omega Y$, $A_t = A$, $\pi_t = \pi$, $M_t/P_t = m = e^{a-\eta(r+\pi)}Y$. These assumptions imply that

$$\dot{D}_t = rD_t + G - \omega Y - A - \pi m.$$

We also assume that the stock of debt in the initial steady state is constant, so that $\dot{D}_t = 0$, and

$$\pi m + \omega Y = rD_0 + G - A. \tag{2.5}$$

This steady state version of the government budget constraint illustrates, once again, our interpretation of a heavily indebted government. Holding G and A fixed, the higher the level of the government's debt, D_0, the higher its inflation rate must be (over the range $\pi < 1/\eta$, πm is increasing in π), or the more punitive its taxes must be, to avoid default.

We should point out that our analysis does not depend in any crucial way on the initial steady state assumption. Rather, this assumption lends us some analytical convenience without affecting our basic message.[17]

2.4. Characterizing the HIPC Initiative

We interpret the HIPC initiative as an attempt to improve the fiscal position of the recipient government relative to the initial steady state position. In terms of the analysis in Section 1 we will work with our first interpretation of the HIPC initiative and its conditionality, though we reiterate that our conclusions are not sensitive to that choice, which is merely notational. We first imagine that, at time 0, an economy is in a steady state as described by the previous subsection. Then the country receives a previously unanticipated injection of aid that lasts until period T—this captures the finite lifetime of the initiative, though in reality the completion date is often uncertain. In particular we assume that

$$A_t = \begin{cases} A(1+\psi) & \text{for } 0 \leq t < T \\ A & \text{for } t \geq T, \end{cases} \tag{2.6}$$

where $\psi > 0$ determines the increase in aid.[18] We interpret the conditionality of the HIPC initiative as requiring that government expenditure increase by as much as the aid flow until date T.[19] I.e. we assume that

$$G_t = \begin{cases} G + A\psi & \text{for } 0 \leq t < T \\ G & \text{for } t \geq T. \end{cases} \tag{2.7}$$

Given our notation, above, this means $\hat{G}_t = A\psi$ for $0 \leq t < T$ and $\hat{G}_t = 0$ for $t \geq T$.

Since the increase in A_t, in our model, plays the role of decreased debt service payments in the budget constraint, we assume that debtors set ψ in order achieve a

[17] Our assumption that the economy is initially in a steady state matters only in interpreting the quantitative simulations, in Section 3. The post-debt relief paths of inflation, debt, and other variables are invariant to whether one assumes, or not, that the pre-debt relief economy is in steady state. Of course, comparisons between the pre- and post-debt relief paths do depend on the steady state assumption.

[18] Notice that after period T aid flows revert to their former levels. This ignores the possibility, pointed to by Cohen (2001), that donors will decrease their non-debt relief aid once the HIPC initiative is well under way. Given our results, this situation would imply a net worsening of the government's fiscal position.

[19] Alternatively we could assume that transfers from the government to the private sector increase over the implementation period of the initiative. The equivalence of government purchases and transfers would not carry over to a general equilibrium model.

particular debt reduction target. In particular, notice that the present value of the debt relief implied by (2.6) is

$$\int_0^\infty (A_t - A)e^{-rt}dt = \int_0^T (A\psi)e^{-rt}dt = A\psi\frac{1-e^{-rT}}{r} \tag{2.8}$$

If θ represents the fraction of a government's debt service that is effectively forgiven, then

$$\theta = A\psi\frac{1-e^{-rT}}{rD_0}. \tag{2.9}$$

2.5. The Path of Output

Our assumptions so far are sufficient to determine the paths of output, Y_t, and tax revenues, Ω_t. We have assumed that $\dot{K}_t = \alpha\hat{G}_t - \delta K_t$ for $t \geq 0$, $K_0 = 0$, $\hat{G}_t = \psi A$ for $0 \leq t \leq T$, $\hat{G}_t = 0$ for $t > T$, and $Y_t = Y + \rho K_t$. With these assumptions we can easily solve for the path of output with debt relief. In the appendix we show that

$$K_t = \begin{cases} \int_0^t \alpha\hat{G}_t e^{\delta(s-t)}ds = \left(1-e^{-\delta t}\right)(\alpha/\delta)\psi A & \text{for } 0 \leq t \leq T, \\ K_T e^{-\delta(t-T)} = e^{-\delta t}\left(e^{\delta T}-1\right)(\alpha/\delta)\psi A & \text{for } t > T. \end{cases} \tag{2.10}$$

Of course, this implies that

$$Y_t = \begin{cases} Y + \left(1-e^{-\delta t}\right)(\rho\alpha/\delta)\psi A & \text{for } 0 \leq t \leq T, \\ Y + e^{-\delta t}\left(e^{\delta T}-1\right)(\rho\alpha/\delta)\psi A & \text{for } t > T. \end{cases} \tag{2.11}$$

We are now able to measure the increase in the present value of tax revenue due to the effect of the debt relief initiative on output. Using (2.11), it is straightforward to show that the increase in tax revenue is

$$\int_0^\infty \omega(Y_t - Y)e^{-rt}dt = \omega\rho\alpha\frac{1-e^{-rT}}{(\delta+r)\,r}\psi A = \frac{\omega\rho\alpha}{\delta+r}\theta D_0. \tag{2.12}$$

Notice that (2.12) relates the increase in the present value of tax revenues to the magnitude of the debt relief package.

2.6. Monetary Policy

To close the model, and describe the paths of prices, inflation and debt under the initiative, we must describe monetary policy after date 0. Given that we have determined the paths of A_t, G_t and Ω_t, there are infinitely many paths of the money supply that are consistent with the lifetime budget constraint, (1.7).

To illustrate the effects of different monetary policies, we assume that

$$M_t = \begin{cases} M_0(Y_t/Y_0)e^{\gamma t} & \text{for } 0 \leq t < T \\ M_T(Y_t/Y_T)e^{\bar{\gamma}(t-T)} & \text{for } t \geq T. \end{cases} \tag{2.13}$$

We have restricted ourselves to the class of monetary policies in which money grows at a constant rate relative to real GDP, over each of the two subintervals. Within this class of policies there are still infinitely many that satisfy the government's lifetime budget constraint, but we focus on two interesting cases.

In what follows it is useful to have an expression for the equilibrium inflation rate given that monetary policy is described by (2.13). In the appendix we show that (2.3) implies

$$\pi_t = \dot{p}_t = \begin{cases} \gamma + (\bar{\gamma} - \gamma)e^{(t-T)/\eta}. & \text{for } 0 \leq t < T \\ \bar{\gamma} & \text{for } t \geq T, \end{cases} \tag{2.14}$$

given (2.13).

Active Monetary Policy In our first example, we call monetary policy *active*, because the monetary authority tries to keep inflation stable across the two intervals: $0 \leq t \leq T$ and $t > T$. I.e. the monetary authority chooses γ and $\bar{\gamma}$ so that $\pi_t = \bar{\pi}$, for all $t \geq 0$, for some constant $\bar{\pi}$. From (2.14) it is clear that this requires $\pi_t = \bar{\pi} = \gamma = \bar{\gamma}$ for $t \geq 0$. In doing this, the monetary authority must ensure that the government's lifetime budget constraint, (1.2), is satisfied. Notice that (1.2) can be rewritten as

$$\int_0^\infty \omega(Y_t - Y)e^{-rt}dt = \int_0^\infty (\pi m - \dot{M}_t/P_t)e^{-rt}dt. \tag{2.15}$$

We know π and m, and have already determined the left-hand side of (2.15). Given our other results, the path of $\dot{M}_t/P_t$ is completely determined by γ. Because (2.15) is a nonlinear equation in γ, we solve it numerically as described in the appendix.

Passive Monetary Policy We label our second example *passive* monetary policy. In what follows we describe the mechanics of the example and explain why the labeling is natural. We have assumed that government spending increases from G to $G_t = G + \psi A$ in the period $0 \leq t \leq T$. When government spending increases there is a natural tendency for additional liquidity to be injected into the economy. Of course, in our example, the government is also cutting spending—on debt service. But debt service payments are made in foreign currency. So a natural experiment is one in which the government either accumulates reserves, with the savings from debt service, or accumulates new debt less quickly. At the same time the government allows the additional liquidity to finance its increased spending.[20] Offsetting this increase in liquidity is any increase in taxes. Whatever new liquidity is not removed in the form of new taxes represents new seigniorage revenue. The reason we call this passive monetary policy is that the government will almost naturally find itself in this position if it increases domestic spending while reaping savings on the foreign exchange part of its balance sheet.

In terms of our notation this means that the present value of new seigniorage revenue raised between periods 0 and T is given by

$$\int_0^T (\dot{M}_t/P_t - \pi m)e^{-rt}dt = \int_0^T (G_t - G)e^{-rt}dt - \int_0^T \omega(Y_t - Y)e^{-rt}dt. \qquad (2.16)$$

That is, new seigniorage through date T is equal to the difference between the present value of new spending and the present value of new taxes through date T. The right-hand side of (2.16) is determined completely by our previous results. The left-hand side of (2.16) is a nonlinear function of γ and $\bar{\gamma}$, so (2.16) represents one equation in our two unknowns, γ and $\bar{\gamma}$.

An interesting implication of passive monetary policy is that it creates a link between the amount of debt the government is left with at date T, and the amount of debt relief it receives under the initiative. To see this, we can roll the budget constraint, (1.1), forward from period 0 to period T to obtain

$$D_0 = e^{-rT}D_T + \int_0^T (\omega Y_t - G_t + A_t + \dot{M}_t/P_t)e^{-rt}dt. \qquad (2.17)$$

If we combine (2.16) and (2.17), and use the steady state condition (2.5), we obtain

$$D_T - D_0 = -e^{rT}\int_0^T (A_t - A)e^{-rt}dt = -e^{rT}\theta D_0 \qquad (2.18)$$

[20] We have characterized the increase in spending for poverty reduction as an increase in government purchases of domestic goods and services that causes an increase in domestic liquidity that the government can respond to passively or actively. To the extent that increased spending on poverty reduction is in the form of imports the differences between the two monetary policies will be less stark, because increased expenditures on imports will not inject liquidity into the domestic economy.

In other words, by the completion date of the initiative, the government's debt has been reduced by the capitalized value of the debt relief.

It is from this last result that we get our second equation to determine γ and $\bar{\gamma}$. Consider the government's lifetime budget constraint at time T:

$$D_T = \int_T^{\infty} (\Omega_t - G_t + A_t + \dot{M}_t/P_t) e^{-r(t-T)} dt. \tag{2.19}$$

Given (2.18) and (2.5), (2.19) can be rewritten as

$$-D_0\theta = \int_T^{\infty} \omega(Y_t - Y) e^{-rt} dt + \int_T^{\infty} (\dot{M}_t/P_t - \pi m) e^{-rt} dt. \tag{2.20}$$

This equation states that the present value (at date 0) of taxes and seigniorage raised after date T can decline by the value of the debt relief package. The left-hand side and the first term on the right-hand side of (2.20) are determined by our previous results. The seigniorage term in (2.20) is determined by $\bar{\gamma}$.

So our strategy for solving the model under passive monetary policy is to solve (2.20) numerically for $\bar{\gamma}$. Then we solve (2.16) numerically for γ. This procedure is described in more detail in the appendix.

In the next section of the paper we turn to a quantitative analysis of the model in which we compare the effects of the two monetary policies on the equilibrium paths of prices, inflation and debt. We show how the two policies, active and passive, offer a distinct choice to the policy maker. We should be clear, however, that the point we made in Section 1 pertains to all policy choices: given the conditionality of the initiative, the present value of *total* revenues that the government must raise from seigniorage and taxes will be unchanged relative to the initial steady state. If debt relief is effective in raising output, however, raising these revenues may be easier, in the sense that the government may be able to obtain the same revenue with lower tax rates and/or lower inflation. Here we have described a method for solving the model where the tax rate ω is held fixed. Thus, the indirect effects of debt relief, in our examples, all work through an increase in tax revenues, and an offsetting decline in seigniorage revenues.

3. Simulating the Model

Our main goals in this section are to (i) characterize the size of the indirect beneficial effect of debt relief on tax revenue, and (ii) determine the paths of the price level, the inflation rate, and the government's debt under different monetary policies. To achieve these goals we use a calibrated version of the model to explore its properties under the two policy regimes described above. We discuss the sensitivity of our results to our assumptions at the end of this section.

3.1. Calibration

Our first step is to calibrate some of our parameters. We set the real interest rate, $r = 0.05$. We set the interest elasticity of money demand to $\eta = 0.5$. This value seems broadly consistent with the estimates of interest elasticities reported by Easterly, Mauro, and Schmidt-Hebbel (1995) for developing countries. While our quantitative findings are sensitive to this choice, our qualitative findings are invariant to it. We normalize output as $Y = 1$. We set a, the constant in the Cagan money demand function, to be a value consistent with real balances being 25 percent of GDP in a zero inflation economy. Notice that our model predicts that in such an economy $M_t/(P_tY_t) = e^{a-\eta r}$, so that we set $a = \eta r + \ln(0.25) \approx -1.36$.

We set the initial level of government debt at $D = 0.7Y$, or 70 percent of GDP. We let steady state government purchases $G = 0.2Y$. We assume that the government runs a primary deficit in the absence of aid inflows. That is, we set $\omega = 0.15$, so that in the initial steady state $\Omega = \omega Y < G$. We assume that in the initial steady state, the government receives an aid inflow of 3 percent of GDP. I.e. $A = 0.03Y$.

Our assumptions regarding the initial values of D, G, Ω, and A determine the initial steady state inflation rate and the level of real balances. We have set these values to be such that the government's initial fiscal position is relatively weak—it requires a high inflation rate, and considerable amounts of seigniorage to close its budget constraint. In particular, given our parameter values, steady state seigniorage is given by

$$m\pi = rD + G - \Omega - A = 0.055Y$$

or 5.5 percent of GDP. The steady state inflation rate is about 25 percent.[21] Steady state real balances are $m \approx 0.22Y$.[22]

We assume that the HIPC initiative increases aid to our fictitious country over a 10 year period, i.e. $T = 10$. We set $\psi \approx 0.9$, so that the capitalized value of the forgiven debt service is equal to 50 percent of the country's initial stock of debt, i.e. we set $A\psi(e^{rT} - 1)/r = 0.5D_0 = 0.35Y$. Our assumptions about ψ means that the country receives a flow of debt relief of about 2.7 percent of GDP over the life of the initiative.

In choosing the parameters of the production technology described earlier, we seek a reasonable upper bound for the indirect effects of debt relief with HIPC conditionality. We set δ, the depreciation rate, equal to 0.1. We make relatively generous assumptions about the productivity of aid in generating additional output.[23] We assume that all aid is invested, i.e. $\alpha = 1$, and that the output-capital ratio is 0.5, i.e. $\rho = 0.5$. These seem like generous assumptions to us for the following reason. Suppose we find the value of λ such that

$$\int_0^\infty [Y_t - Y - (G_t - G)]e^{-\lambda t}dt = 0.$$

λ is the rate of return on the increased social spending, since it is the discount rate that renders the present value of the flow of investment equal to the present value of the flow of payoffs. Given our results from above we are looking for λ such that

$$\int_0^T [\left(1 - e^{-\delta t}\right) \frac{\rho\alpha}{\delta}\psi A - \psi A]e^{-\lambda t}dt + \int_T^\infty e^{-\delta t}(e^{\delta T} - 1)\frac{\rho\alpha}{\delta}\psi A e^{-\lambda t}dt = 0.$$

It is straightforward to show that the solution to this equation is

$$\lambda = \rho\alpha - \delta.$$

[21] That steady state inflation is relatively high is not crucial to our analysis. As we discussed in the previous section, a government in financial distress may not close the budget constraint by printing money. It may, instead, default or run arrears on its debt. How the government chooses to close the budget constraint in the pre-debt relief steady state does not affect our results concerning the post-debt relief paths of prices and debt.

[22] Having the real monetary base this large is probably counterfactual. However, it is not critical to our qualitative results. In fact, the smaller the monetary base in the steady state, the sharper our results would be: passive monetary policy would be more destabilizing.

[23] For results concerning the effectiveness of foreign aid, see Boone (1996) and Burnside and Dollar (2000).

So, our benchmark example is one in which $\lambda = 0.4$, i.e. the rate of return on social spending is 40 percent. We think this is likely an upper bound for what is plausible given the results in Burnside and Dollar (2000) and Easterly, Levine and Roodman (2003).

3.2. The Lifetime Budget Constraint: Quantitative Results

In Section 1 we emphasized a result, concerning the lifetime budget constraint, that was summarized by equation (1.6). This equation states that, with debt relief, the sum of lifetime seigniorage plus tax revenue will be the same as it would have been in the absence of debt relief. Because we have assumed that the tax rate, ω, is unchanged after debt relief, this means that

$$\int_0^\infty (\dot{M}_t/P_t - \pi m)e^{-rt}dt = -\int_0^\infty \omega(Y_t - Y)e^{-rt}dt \tag{3.1}$$

The government can reduce the seigniorage it collects by the amount of its increased tax receipts. We think of this reduction in seigniorage as the extent to which the government budget constraint is indirectly relaxed by debt relief. Using (2.12) we can see that (3.1) can be rewritten

$$\int_0^\infty (\dot{M}_t/P_t - \pi m)e^{-rt}dt = -\frac{\omega\rho\alpha}{\delta + r}\theta D_0. \tag{3.2}$$

In our calibrated examples $\omega\rho\alpha/(\delta + r)$ is equal to 0.5, so that the relaxation of the lifetime budget constraint represents 17.5 percent of GDP. This represents about 25 percent of the country's initial stock of debt, about 16 percent of lifetime seigniorage in the steady state, or about 5.8 percent of lifetime tax revenue. In this sense, the indirect effects of debt relief on the government's lifetime budget constraint are relatively modest, especially since we think our assumptions about ρ and α are generous.

3.3. Active Monetary Policy

In Section 2 we described an active monetary policy under which the government acts to maintain a constant inflation rate for $t \geq 0$. Figure 1 illustrates simulated paths of inflation, the money growth rate, real balances, government debt, foreign aid and seigniorage given the parameter values we selected above. Notice that, in the absence of the growth effect, the economy would never move away from its initial steady state, as indicated by the dashed lines in the figure.

The interesting dynamics in Figure 1 are generated by the growth effect. As we mentioned above, lifetime tax revenue rises by 17.5 percent of initial GDP. As a result, lifetime seigniorage can decline by about the same amount. This is reflected in the slightly lower inflation rate after debt relief: about 21 percent, versus 25 percent in the initial steady state. The decline in inflation is small because the increase in lifetime tax revenue only represents about one sixth of steady state lifetime seigniorage.

In the long-run the stock of debt is permanently reduced by the indirect effects of the debt relief initiative. By how much is debt reduced in the long-run? To answer this question we note that in the very long-run, output returns to its steady state level, Y, so that for large t, the flow budget constraint is approximately

$$\dot{D}_t = rD_t + G - \omega Y - A - \gamma\bar{m}$$

where $\bar{m} = e^{a-\eta(r+\bar{\gamma})}Y$. Thus, the stock of debt the economy converges to in the limit is

$$D_A = \frac{\omega Y - G + A + \bar{\gamma}\bar{m}}{r} = D_0 + \frac{\bar{\gamma}\bar{m} - \pi m}{r}.$$

Given our parameter values $D_A \approx 0.55Y$, as compared to the initial stock of debt $D_0 \approx 0.7Y$.

3.4. Passive Monetary Policy

Figure 2 illustrates simulated paths under passive monetary policy. Again, we show simulated paths with and without the indirect growth effect. Both paths indicate that there is a temporary rise in inflation during the period of increased aid and government spending on poverty reduction. As we stated above, the rise in inflation occurs because there is an instantaneous increase in liquidity that accompanies the increase in government spending. When there is no growth effect, all of this additional liquidity stays in the system and is reflected in higher seigniorage revenue and inflation. When there is a growth effect, some of the additional liquidity leaves the system in the form of increasing tax payments, so the short-run inflation effect is weaker. In our numerical examples, the inflation rate jumps from about 25 percent in the steady state to about 40 percent, if we ignore the growth effect. With the growth effect, inflation only rises to 33 percent in the short-run.

Of course, in the long-run the effect on inflation is reversed. This occurs because the short-run increase in seigniorage revenue leads to a rapid decline in debt. In fact, as we saw above, passive monetary policy naturally leads to a halving of the debt stock by time T. This means that once government spending goes back to its previous levels, the amount of seigniorage needed to close the government budget constraint is greatly reduced. If we ignore the growth effect, the long-run inflation rate is about 16 percent. With the growth effect, the long-run inflation rate is only 14 percent. Again, the growth effect reduces the inflation rate because the increase in tax revenue allows seigniorage revenue to be lower.

How much does debt decline in the long-run with passive monetary policy? For the same reasons given in the previous section, the long-run level of government debt is given by

$$D_P = D_0 + \frac{\bar{\gamma}\bar{m} - \pi m}{r}.$$

When there are no growth effects, $D_P = 0.35Y$. Debt is halved and the government reaches its new steady state level of indebtedness at time T. On the other hand, growth effects contribute to a further reduction in debt, to $D_P = 0.26Y$.

3.5. Summary Discussion

From these simulations we take away the message that debt relief that comes with conditionality requiring increased government spending is likely to have the following consequences:

1. At best, there would only be a modest relaxation of the government's lifetime budget constraint due to increased real activity and tax revenues.

2. Under passive monetary policy, debt and inflation could be substantially reduced in the long-run, but at the cost of significantly higher inflation during the period of increased spending.

3. Under active monetary policy intervention, inflation is kept stable, but there would only be modest long-run reductions in government debt and inflation.

How sensitive are our results to our calibration assumptions? We would like to emphasize that in our view our result concerning the lifetime budget constraint represents an upper bound on how much it will be relaxed by debt relief of a given magnitude. In our example, the government receives debt relief worth about 35 percent of GDP, and taxes output at a rate of 15 percent. Given our assumptions about the returns to aid, this leads to an increase in lifetime revenue equal to 17.5 percent of GDP. This figure depends proportionally on the tax rate, and on the amount of debt relief, but we think both of our assumptions in this regard are realistic. Furthermore, we think our assumptions about the mapping from increased spending on poverty reduction to real activity are generous, suggesting that our result represents an upper bound.

We would also like to de-emphasize the *quantitative* aspects of our results concerning the path of inflation, and put more emphasis on our *qualitative* findings. Under passive policy a substantial increase in government spending leads to an increase in inflation that is eventually reversed. We experimented with alternative parameter values to see if we could eliminate the short-run rise in the inflation rate in the example with passive monetary policy. We found that in order to do so, we would need to assume that aid had a 110 percent rate of return (almost 3 times the rate of return we assumed in our baseline example). Using active monetary policy, in contrast, the government can stabilize inflation. Here our results are simply the mirror image of Sargent and Wallace's (1981) *unpleasant monetarist arithmetic*: without fiscal reforms fighting inflation in the short-run implies inflation later. In our case, lower inflation in the future means higher inflation now. We think this qualitative result would survive significant modifications of our model.

Given the apparent limited revenue impact of increased spending on poverty reduction, our analysis points to the importance of reforms not directly linked to HIPC conditionality. In particular, our analysis suggests that if HIPC countries are to see significant improvements in their fiscal positions, they must undertake significant fiscal reforms under which they cut other forms of spending as they raise spending on poverty reduction. Returning to our numerical example, suppose that at time 0 the HIPC government implemented spending cuts symmetric to its increased spending on poverty reduction programs. How would this change our conclusions about the impact of debt relief on inflation and the stock of debt? The answer is straightforward: since the spending cuts would finance the increased spending on poverty reduction, there would be no net injection of liquidity into the economy. Figure 3 illustrates simulations of this scenario with either passive or active monetary policy.[24] With active monetary policy there is a sharp and permanent drop in inflation and debt is substantially reduced by the completion point (date $T = 10$). Even with passive monetary policy there would be a sizeable short-run decline in inflation, due to increased tax receipts, followed by a further decline in inflation after time T. Notice that the monetary policy issues we highlighted previously are now much less important: the paths of inflation are not dramatically different across the two scenarios. Furthermore, the long-run paths of debt are very similar. And, finally, the government's lifetime budget constraint is significantly relaxed. It can finance itself with 33 percent less seigniorage revenue.

Some HIPC countries have already undertaken steps towards the types of reforms we have indicated here. That is, they have not only increased spending on poverty reduction, but they have done this in the context of an overall fiscal reform. Our analysis suggests that they are the ones likely to successfully achieve long-run fiscal sustainability.

4. Optimal Policy Considerations

Our result, that the HIPC initiative provides little debt relief, may seem puzzling given that fiscal solvency is one of its primary stated objectives. In this section we address this puzzle by sketching a model of the behavior of recipient governments and

[24] In this example we imagine that $G_t = G$ for all t (because one category of spending rises while another falls). The path of A_t is as before. We assume that output rises as in our other examples, because the government is substituting growth-enhancing expenditure for consumption expenditure.

donors.[25] We think of recipient governments as having preferences over two types of purchases, those directed towards poverty reduction, which we denote by g_1, and those related to other government programs, which we denote by g_2. For simplicity, suppose the government has standard preferences over these two types of purchases: $U(g_1, g_2)$. To make things simple, abstract from seigniorage, and imagine a steady state budget constraint $pg_1 + g_2 + rb \leq \tau + a$, where p is the relative price of g_1, r is the effective interest rate on b, the government's stock of debt, τ is tax revenue and a is foreign aid, which we treat, as above, as a pure grant. We abstract, for the moment, from the government's decisions regarding b and τ, and treat these as given. We let $x = \tau + a - rb$ represent the government's net resources.

We assume that donors have different preferences than the local government, and that to make things simple, they have no taste for g_2.[26] We imagine that donors like g_1, but face costs—pecuniary and political—in providing foreign aid.

Imagine a game in which the donors lead, by setting a, and that recipient governments follow by setting g_1 and g_2, to maximize U, taking p, τ, a and rb as given. The recipient government will choose g_1 and g_2 consistent with $U_1/U_2 = p$ and $pg_1 + g_2 = x$. Hence, g_1 is the solution to

$$U_1(g_1, x - pg_1)/U_2(g_1, x - pg_1) = p. \tag{4.1}$$

The solution to the government's problem is characterized in Figure 4(a). The government chooses the point (g_1^*, g_2^*).

The donor, of course, in setting a, will take the recipient government's rule for choosing g_1 into account. If the recipient government's decision rule is $g_1 = \gamma(a)$, the donor will maximize $\gamma(a) - C(a)$, where $C(a)$ represents the donors' cost of providing aid. As long as $\gamma(a)$ is increasing and quasi-concave in a, and $C(a)$ is increasing and convex in a, the donor will choose $a = a^*$, where $\gamma'(a^*) = C'(a^*)$.

One interpretation of the HIPC initiative is that it is an attempt by the donors to increase g_1 by changing the nature of this strategic game. Notice that donors might perceive the recipient's preference for g_2 as the main barrier to increased spending

[25] Our sketched model resembles, in some respects, the model in Cordella and Dell'Ariccia (2003)

[26] We do not take a stand on whether the government's or the donor's preferences are more representative of the utility functions of households in the recipient country.

on poverty reduction.[27] If the recipient's marginal propensity to purchase g_1 out of additional resources, namely $\gamma'(a)$, were higher, the donors would be willing to provide more aid.

One way the donors could restructure the game would be to offer the recipient government a deal: if you increase pg_1 by at least Δ relative to your current level of spending pg_1^*, we will provide you with increased aid, or reduced debt service, in the amount Δ. This amounts to a change in the budget set of the recipient government like the one illustrated by the thick line in Figure 4(b). As long as the government's preferences are increasing in g_1 it will accept this deal, and if g_1 and g_2 are both normal goods, it will choose the corner solution $(g_1^* + \Delta/p, g_2^*)$. As long as the increase in g_1 pleases the donor more than it costs to increase aid, the donor will be happy to make the offer as described.

In our discussion to this point, the level of indebtedness has been notably absent. We have simply treated the level of debt, b, and the government's sources of revenue, τ and a, as fixed parameters. Imagine, now, that the resource envelope is redefined as $x = \tau + a + d - rb$, where d is arrears. We think it is quite reasonable to think of arrears as financing the government in steady state, since many HIPC governments have had problems servicing their debts for a very long time.

Notice that when arrears enter the picture in this way, we can see that implicit in Figures 4(a) and 4(b) is the assumption that the government's propensity to use arrears to close its budget constraint is unaffected by the donor's offer. To see this, imagine that governments choose their levels of taxation and arrears (a form of default) optimally given a trade-off between the costs of raising taxes and running arrears, and the benefits of government spending. That is, suppose we think of a government choosing g_1, g_2, τ and d, to maximize $U(g_1, g_2) - V(d, \tau)$ subject to $pg_1 + g_2 = \tau + a + d - rb$, where V is an increasing and convex function describing the costs of taxes and default.

[27] It is frequently argued, for example in Van Trotsenburg and MacArthur (1999), that debt itself is the primary barrier to increased spending on poverty reduction. However, if debt itself were the only problem, the donors could simply unconditionally cancel debt, or increase aid. Given that donors do not generally do this, we presume they are concerned with how these resources will be used.

At the point $(g_1^* + \Delta/p, g_2^*)$ the government's total arrears and taxes and, presumably, the costs associated with them, are the same as at the point (g_1^*, g_2^*), because the increase in g_1 is financed entirely with additional aid. If U is additively separable over g_1 and g_2, then the government has no incentive to reduce or increase its use of arrears or taxes.[28] So, the donor's strategy induces an increase in g_1 while leaving the recipient's propensity to default and/or use arrears as financing unchanged. It also has no effect on the government's choice of taxes nor on its choice of g_2.

Why would donors consider such a strategy? One possibility is that the donor does not view defaults, or the running of arrears, by recipient governments as very costly on the margin. This means that were the donor to provide more resources to the recipient government, the gain from doing so would be small. This does not seem plausible to us, given the large literature that explores the costs of high indebtedness in developing countries and given the emphasis that official HIPC initiative documents put on debt reduction.[29]

Another possibility is that donors recognize the welfare costs of high indebtedness, but perceive that these costs do not concern, or are not borne by, the recipient governments. In this situation moral hazard would make it very difficult to devise a strategy that would relieve debt and not, at the same time, simply lead to an increase in g_2 rather than a decrease in d. In this case, a strategy of tying increased resources to increased spending on g_1, one which, in a sense, provides no debt relief at all, could be preferred by the donors.[30]

Our point is not to argue that actual donor behavior is suboptimal. Rather we want to point out that implicit in the HIPC initiative's design is an objective function that puts little weight on reducing debt, per se. This is important when thinking

[28] The marginal utility of g_2 and the marginal costs of τ and d would be unchanged at the corner solution $(g_1^* + \Delta/p, g_2^*)$ under additive separability. Thus, the government would continue to consider its choices for these variables as optimal.

[29] On the costs of high indebtedness see, among many, the discussions in Greene (1989), Sachs (1989) and Agénor and Montiel (1996, Ch. 13). Many official documents relating to the HIPC initiative discuss the importance of debt reduction. Van Trotsenburg and MacArthur (1999) is an excellent example.

[30] Of course, moral hazard could also be the reason indebted countries became indebted in the first place. The evidence, for example from Easterly (2002) and Kraay and Nehru (2003), is mixed as to whether this is an important consideration.

about how to assess the HIPC initiative, *ex-post*. Our results would suggest that the initiative can only really be judged on its success in reducing poverty, in that on *a priori* grounds it should have little impact on fiscal solvency. Given the emphasis put on debt in official HIPC initiative documents, and in current policy debates about the initiative, we presume this point has been overlooked.

Our other results, concerning active versus passive monetary policy have a different flavor. Rather than focusing on the overall resource envelope, these results pertain more towards macroeconomic management. Little attention has been paid to the macroeconomic impact of the HIPC initiative. Our goal in discussing the possible inflationary impact of the initiative was simply to point out that stabilization policy becomes an issue when the government increases domestic spending. This does not imply any statement, on our part, that active policy should be preferred to passive policy. However, our results do suggest that, at a minimum, inflation outcomes will be different under the two regimes.

5. Concluding Remarks

In this paper, we have discussed the potential macroeconomic impact of the HIPC initiative. A central point of our paper is that the HIPC initiative provides only modest amounts of debt relief to heavily indebted poor countries.

Our analysis also suggests that short-run inflationary pressure might arise as a result of increased domestic spending for poverty reduction. Some recipient countries may be ill-equipped to deal with these pressures and could experience a substantial temporary increase in inflation. While our model does not capture the effect, this increase in inflation could impact negatively on growth. Our model suggests that recipient countries can use monetary policy to stabilize inflation, but, in doing so, they will tend to limit the longer run impact of the HIPC initiative on their debt levels. We have also shown that the impact on inflation will be lower if increased government spending produces favorable output effects.

The concerns we have raised will be less relevant when the recipient country has already achieved significant progress in macroeconomic stabilization. In those cases where the recipient government has already implemented fiscal reforms that would enable it to reduce its debt level over time, the risk of short-run inflation will be diminished and longer run fiscal sustainability will be ensured. But, in these sorts of countries, there is almost certainly less need, on the parts of donors, to attach conditions to debt relief.

A major shortcoming of our analysis is that our simple analytical framework does not allow us to explore more fully the impact of debt relief on real activity, the real exchange rate, and the external current account. In future research, we intend to extend our analysis to deal with these issues, using a general equilibrium small-open-economy model.

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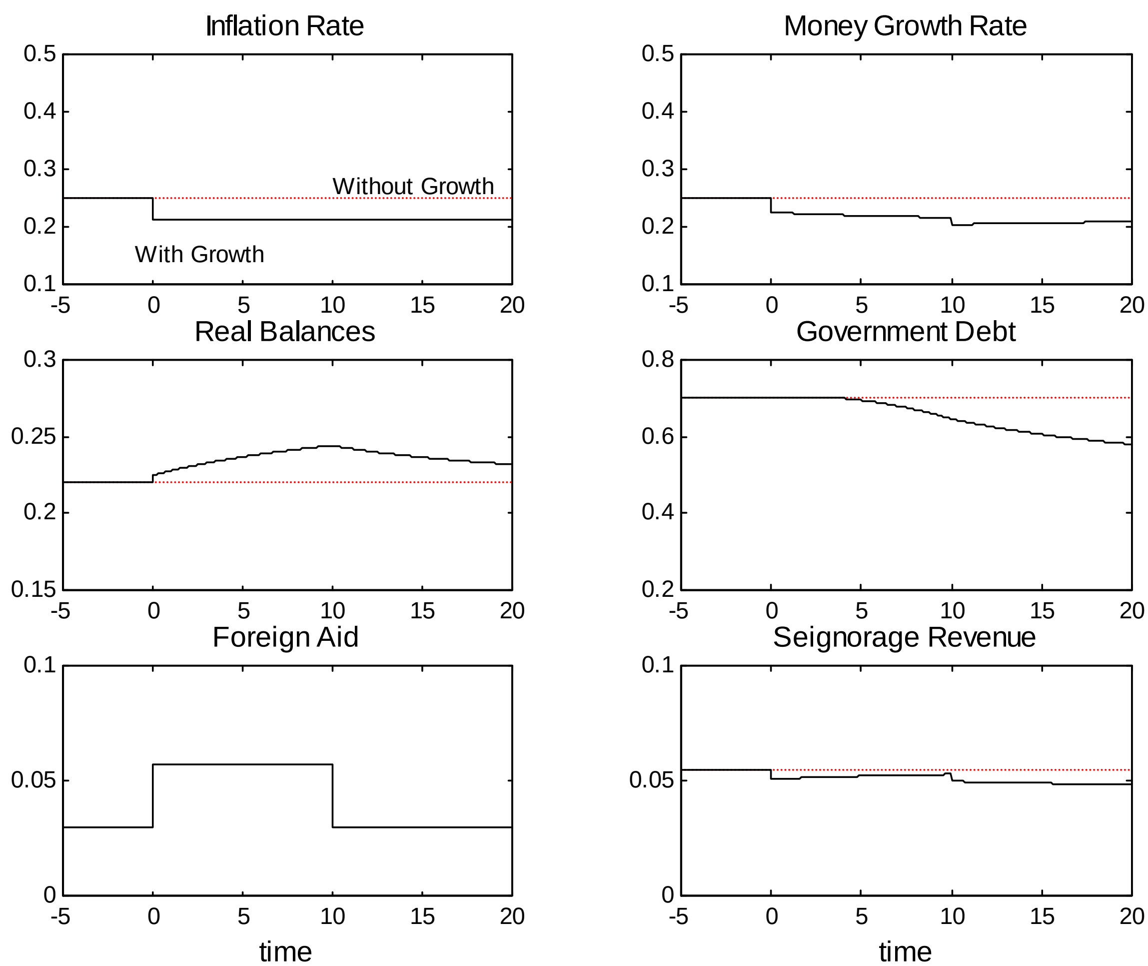
FIGURE 1

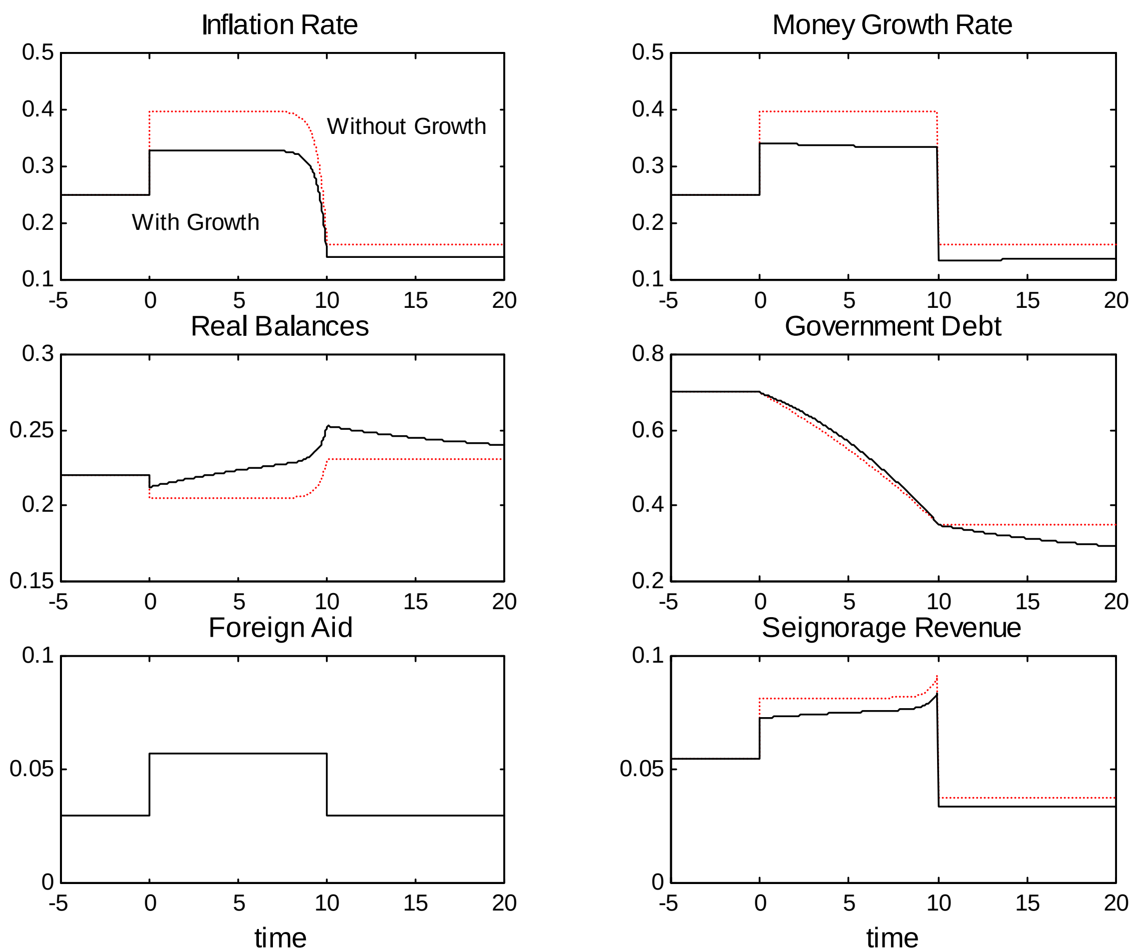
SIMULATIONS UNDER ACTIVE MONETARY POLICY



Notes: In these examples, the government actively smoothes inflation over time. The solid lines indicate the paths of each variable assuming that the increased spending on poverty reduction raises output. The dashed lines indicate what these paths would be in the absence of this additional growth. So the dashed lines indicate the direct effect of debt relief with HIPC conditionality, while the solid lines indicate the combination of the direct and indirect effects. The scales for real balances, debt, aid and seignorage can all be interpreted by noting that GDP = 1. The inflation and money growth rates are expressed in decimal form.

FIGURE 2

SIMULATIONS UNDER PASSIVE MONETARY POLICY

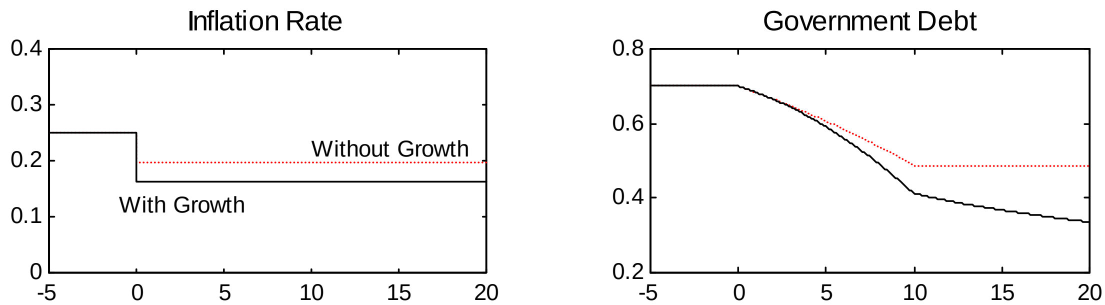


Notes: In these examples, the government does not actively intervene to smooth inflation. It allows the liquidity associated with the increase in government spending to stay in the economy to the extent that increased tax revenues do not remove it. The solid lines indicate the paths of each variable assuming that the increased spending on poverty reduction raises output. The dashed lines indicate what these paths would be in the absence of this additional growth. So the dashed lines indicate the direct effect of debt relief with HIPC conditionality, while the solid lines indicate the combination of the direct and indirect effects. The scales for real balances, debt, aid and seignorage can all be interpreted by noting that GDP = 1. The inflation and money growth rates are expressed in decimal form.

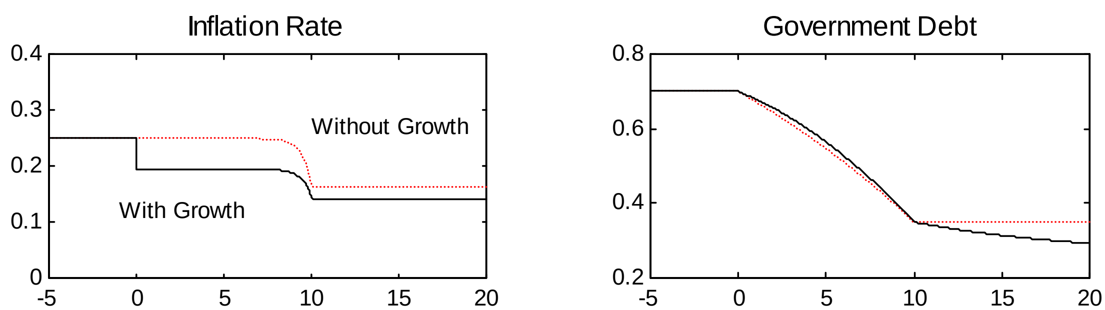
FIGURE 3

SIMULATIONS WITH ADDITIONAL FISCAL REFORMS

(a) Active Monetary Policy



(b) Passive Monetary Policy

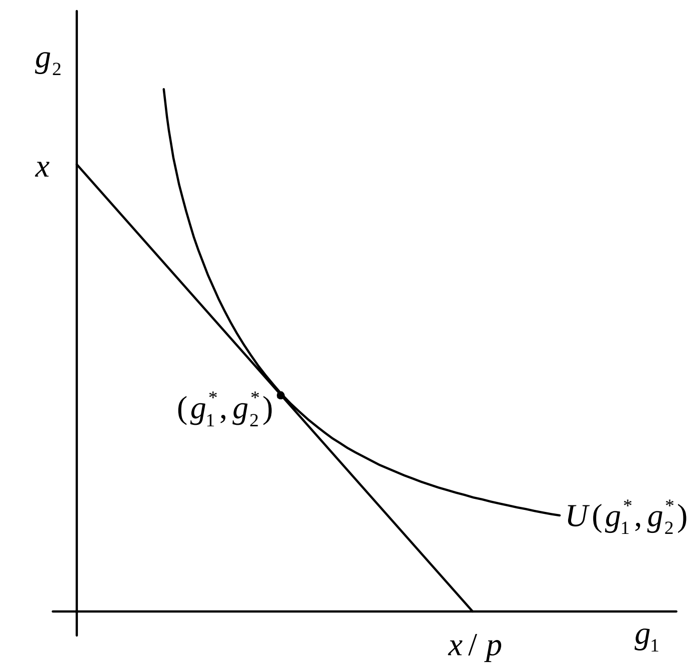


Notes: In these examples, the government implements spending cuts symmetric to its spending increase over the life of the debt relief initiative. The solid lines indicate the paths of each variable assuming that the increased spending on poverty reduction raises output. The dashed lines indicate what these paths would be in the absence of this additional growth. So the dashed lines indicate the direct effect of debt relief with HIPC conditionality, while the solid lines indicate the combination of the direct and indirect effects. The scales for real balances, debt, aid and seignorage can all be interpreted by noting that GDP = 1. The inflation and money growth rates are expressed in decimal form.

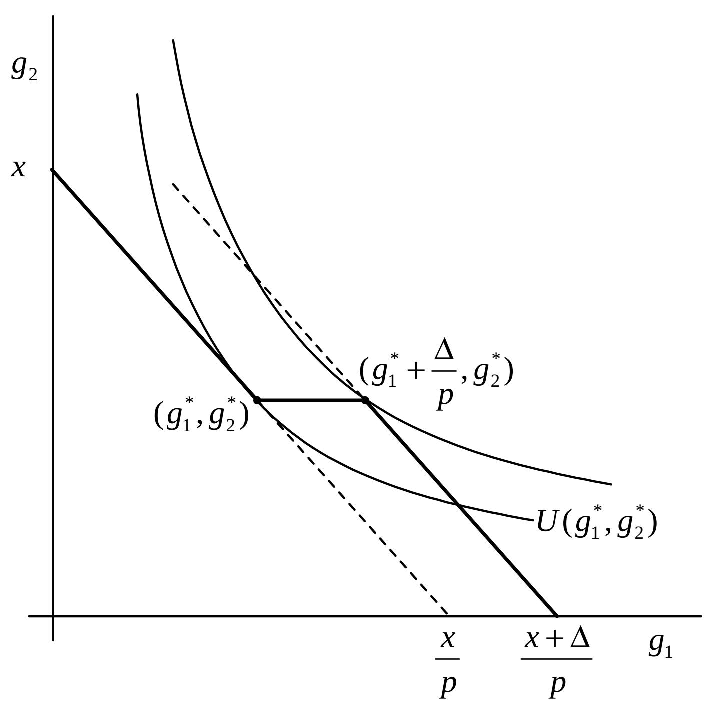
FIGURE 4

THE RECIPIENT GOVERNMENT'S OPTIMAL DECISIONS

(a) Prior to Debt Relief



(b) After Debt Relief with HIPC Conditionality



6. Technical Appendix

Two Useful Formulas Here we derive expressions for $\int_a^b e^{-(s-t)/\eta} ds$ and $\int_a^b s e^{-(s-t)/\eta} ds$. For any function $e^{\psi x}$

$$\int e^{\psi x} dx = C + e^{\psi x}/\psi \tag{6.1}$$

and

$$\int x e^{\psi x} dx = C + (x - 1/\psi)\left(e^{\psi x}/\psi\right) \tag{6.2}$$

where, in each case, C is some arbitrary constant of integration. Hence we can write

$$\begin{aligned} \int_a^b e^{-(s-t)/\eta} ds &= e^{t/\eta} \int_a^b e^{-s/\eta} ds = e^{t/\eta} (-\eta e^{-s/\eta})|_a^b \\ &= -\eta e^{t/\eta} (e^{-b/\eta} - e^{-a/\eta}) \end{aligned} \tag{6.3}$$

and

$$\begin{aligned} \int_a^b s e^{-(s-t)/\eta} ds &= e^{t/\eta} \int_a^b s e^{-s/\eta} ds = e^{t/\eta} [(s+\eta)\,(-\eta e^{-s/\eta})]|_a^b \\ &= -\eta e^{t/\eta} [(b+\eta)\, e^{-b/\eta} - (a+\eta)\, e^{-a/\eta}]. \end{aligned} \tag{6.4}$$

Solving the Cagan Model The solution to the Cagan model given in (2.3) is

$$p_t = \eta r - a + \frac{1}{\eta} \int_t^\infty e^{-(s-t)/\eta} \ln(M_s/Y_s) ds. \tag{6.5}$$

Given the monetary policy described in (2.13), for $0 \leq t \leq T$ we have:

$$\begin{aligned} p_t &= \eta r - a + \frac{1}{\eta} \int_t^T e^{-(s-t)/\eta} \ln[(M_0/Y_0) e^{\gamma s}] ds + \frac{1}{\eta} \int_T^\infty e^{-(s-t)/\eta} \ln[(M_T/Y_T) e^{\bar{\gamma}(s-T)}] ds \\ &= \eta r - a + \ln(M_0/Y_0) + \gamma(t+\eta) - \eta(\gamma - \bar{\gamma}) e^{(t-T)/\eta}. \end{aligned} \tag{6.6}$$

On the other hand, for $t > T$ we have

$$\begin{aligned} p_t &= \eta r - a + \frac{1}{\eta} \int_t^\infty e^{-(s-t)/\eta} \ln[(M_T/Y_T) e^{\bar{\gamma}(s-T)}] ds \\ &= \eta r - a + \ln(M_0/Y_0) + (\gamma - \bar{\gamma}) T + \bar{\gamma}\,(t+\eta)\,. \end{aligned} \tag{6.7}$$

Hence the inflation rate for $0 \leq t \leq T$ is

$$\pi_t = \dot{p}_t = \gamma - (\gamma - \bar{\gamma}) e^{(t-T)/\eta}$$

while for $t \geq T$ it is simply $\pi_t = \bar{\gamma}$.

Details of the Analysis in Section 2

Active Monetary Policy

We need to find a γ such that (2.15) is satisfied. From (2.12), we can rewrite (2.15) as

$$\frac{\omega\rho\alpha}{\delta+r}\theta D_0 = \int_0^\infty (\pi m - \dot{M}_t/P_t)e^{-rt}dt. \tag{6.8}$$

Given (2.13), (6.6) and (6.7) we know that $P_t = e^{-a+\eta(r+\gamma)}(M_0/Y_0)e^{\gamma t}$, and that $\dot{M}_t = (M_0/Y_0)e^{\gamma t}\dot{Y}_t + \gamma(M_0/Y_0)e^{\gamma t}Y_t$. Therefore, $\dot{M}_t/P_t = e^{a-\eta(r+\gamma)}(\dot{Y}_t + \gamma Y_t)$. Given our results about the path of output in Section 2, we can then rewrite (6.8) as

$$\begin{aligned}\frac{\omega\rho\alpha}{\delta+r}\theta D_0 &= \frac{\pi m}{r} - \frac{\gamma b Y}{r} - b\rho\alpha\psi A\left[\int_0^T e^{-(r+\delta)t}dt + (\gamma/\delta)\int_0^T \left(1-e^{-\delta t}\right)e^{-rt}dt\right] + \\ &\quad b\rho\alpha\psi A\left(1-\gamma/\delta\right)\left(e^{\delta T}-1\right)\int_T^\infty e^{-(r+\delta)t}dt,\end{aligned}$$

where $b = e^{a-\eta(r+\gamma)}$. This can be rewritten as

$$\frac{\omega\rho\alpha}{\delta+r}\theta D_0 = \frac{1}{r}\left\{\pi m - e^{a-\eta(r+\gamma)}\left[\gamma Y + \rho\alpha\psi A\frac{\left(1-e^{-rT}\right)(r+\gamma)}{(r+\delta)}\right]\right\} \tag{6.9}$$

We solve (6.9) numerically for γ.

Passive Monetary Policy

In this section we describe how we solve for γ and $\bar{\gamma}$ under passive monetary policy. The first step in our analysis is to solve (2.20) for $\bar{\gamma}$. Given (2.13) and (6.7) we have $P_t = e^{\eta(r+\bar{\gamma})-a}(M_0/Y_0)e^{(\gamma-\bar{\gamma})T+\bar{\gamma}t}$ and $\dot{M}_t = \dot{Y}_t(M_0/Y_0)e^{\gamma T+\bar{\gamma}(t-T)} + \bar{\gamma}Y_t(M_0/Y_0)e^{\gamma T+\bar{\gamma}(t-T)}$ for $t \geq T$. Hence, for $t > T$, $\dot{M}_t/P_t = e^{a-\eta(r+\bar{\gamma})}(\dot{Y}_t + \bar{\gamma}Y_t)$. This allows us to rewrite (2.20) as

$$-D_0\theta = \int_T^\infty \left\{(\omega+\bar{b}\bar{\gamma})(Y_t - Y) + \bar{b}(\bar{\gamma}Y + \dot{Y}_t)\right\}e^{-rt}dt - \int_T^\infty \pi m e^{-rt}dt,$$

where $\bar{b} = e^{a-\eta(r+\bar{\gamma})}$, or, given our results on the path of output in Section 2:

$$-D_0\theta = \rho\alpha\psi A\left(1-e^{-\delta T}\right)\frac{\omega + e^{a-\eta(r+\bar{\gamma})}(\bar{\gamma}-\delta)}{\delta(\delta+r)}e^{-rT} + [\bar{\gamma}e^{a-\eta(r+\bar{\gamma})}Y - \pi m]\frac{1}{r}e^{-rT} \tag{6.10}$$

We solve (6.10) numerically for $\bar{\gamma}$.

Next we solve (2.16) for γ. To do this we note that (2.13) and (6.6) imply $P_t = e^{\eta(r+\gamma)-a}(M_0/Y_0)\exp(\gamma t - \eta(\gamma-\bar{\gamma})e^{(t-T)/\eta})$ and $\dot{M}_t = (M_0/Y_0)e^{\gamma t}\dot{Y}_t + \gamma(M_0/Y_0)e^{\gamma t}Y_t$ for $0 \leq t \leq T$. Therefore, for $0 \leq t \leq T$,

$$\frac{\dot{M}_t}{P_t} = \frac{\gamma Y + \dot{Y}_t + \gamma(Y_t - Y)}{e^{\eta(r+\gamma)-a}\exp(-\eta(\gamma-\bar{\gamma})e^{(t-T)/\eta})}.$$

This allows us to write

$$\begin{aligned}\int_0^T \frac{\dot{M}_t}{P_t}e^{-rt}dt &= \gamma e^{a-\eta(r+\gamma)}\left(Y + \frac{\rho\alpha}{\delta}\psi A\right)\int_0^T \exp[-rt + \eta(\gamma-\bar{\gamma})e^{(t-T)/\eta}]dt + \\ &\left(1-\frac{\gamma}{\delta}\right)e^{a-\eta(r+\gamma)}\rho\alpha\psi A\int_0^T \exp[-(r+\delta)t + \eta(\gamma-\bar{\gamma})e^{(t-T)/\eta}]dt\end{aligned} \tag{6.11}$$

We can write

$$\int_0^T \pi m e^{-rt}dt = \pi m\frac{1-e^{-rT}}{r}. \tag{6.12}$$

The expressions on the right-hand side of (2.16) are

$$\int_0^T (G_t - G)e^{-rt}dt = \psi A\frac{1-e^{-rT}}{r} \tag{6.13}$$

and

$$\int_0^T \omega(Y_t - Y)e^{-rt}dt = \omega\frac{\rho\alpha}{\delta}\psi A\int_0^T\left(1-e^{-\delta t}\right)e^{-rt}dt = \omega\frac{\rho\alpha}{\delta}\psi A\frac{re^{-T(\delta+r)} - e^{-rT}(r+\delta)+\delta}{(\delta+r)\,r}. \tag{6.14}$$

Using (6.11)–(6.14) we can solve (2.16) numerically for γ given $\bar{\gamma}$.

Details of the Analysis in Section 3

Simulations in Figures 1 and 2

In the previous subsection of the appendix, we have shown how to obtain the equilibrium values of γ and $\bar{\gamma}$. Given the path of output, γ and $\bar{\gamma}$ completely determine the paths of M_t and P_t. In some of our simulations we also would like to have expressions for the stock of debt, D_t. We note that for $0 \leq t \leq T$, the stock of debt evolves according to

$$D_t = e^{rt}D_0 - e^{rt}\int_0^t (\omega Y_s - G_s + A_s + \dot{M}_s/S_s)e^{-rs}ds \tag{6.15}$$

We note that (2.5) lets us rewrite this as

$$D_t = D_0 - e^{rt}\left[\int_0^t \omega(Y_s - Y)e^{-rs}ds + \int_0^t (\dot{M}_s/S_s - \pi m)e^{-rs}ds\right]. \tag{6.16}$$

We can write the first integral in (6.16) as

$$\begin{aligned}\int_0^t \omega(Y_s - Y)e^{-rs}ds &= \omega\frac{\rho\alpha}{\delta}\psi A\int_0^t \left(1 - e^{-\delta s}\right)e^{-rs}ds \\ &= \omega\frac{\rho\alpha}{\delta}\psi A\frac{re^{-t(\delta+r)} - e^{-rt}(r+\delta)+\delta}{(\delta+r)\,r} \end{aligned} \tag{6.17}$$

Given our results above, we can write the second integral in (6.16) as

$$\begin{aligned}\int_0^t (\frac{\dot{M}_s}{P_s} - \pi m)e^{-rs}ds &= b\Big\{\gamma\left(Y + \frac{\rho\alpha}{\delta}\psi A\right)\int_0^t \exp[-rs + \eta(\gamma-\bar{\gamma})e^{(s-T)/\eta}]ds + \\ &\left(1 - \frac{\gamma}{\delta}\right)\rho\alpha\psi A\int_0^t \exp[-(r+\delta)s + \eta(\gamma-\bar{\gamma})e^{(s-T)/\eta}]ds\Big\} - \\ &\pi m\frac{1-e^{-rt}}{r}. \end{aligned} \tag{6.18}$$

Under active monetary policy this reduces to

$$\int_0^t (\frac{\dot{M}_s}{P_s} - \pi m)e^{-rs}ds = (\gamma bY - \pi m)\frac{1-e^{-rt}}{r} + b\rho\alpha\psi A\left[\frac{\gamma}{\delta}\frac{1-e^{-rt}}{r} + \left(1-\frac{\gamma}{\delta}\right)\frac{1-e^{-(r+\delta)t}}{\delta+r}\right]. \tag{6.19}$$

For $t > T$ we note that (6.16) still holds. The first integral on the right-hand side of (6.16) is

$$\begin{aligned}\int_0^t \omega(Y_s - Y)e^{-rs}ds &= \omega\frac{\rho\alpha}{\delta}\psi A\left[\int_0^T \left(1 - e^{-\delta s}\right)e^{-rs}ds + \left(e^{\delta T} - 1\right)\int_T^t e^{-(r+\delta)s}ds\right] \\ &= \omega\frac{\rho\alpha}{\delta}\psi A\frac{\delta(1-e^{-rT}) + re^{-t(\delta+r)}(1-e^{\delta T})}{(\delta+r)\,r}. \end{aligned} \tag{6.20}$$

For $t > T$, the second integral on the right-hand side of (6.16) becomes

$$\int_0^t (\frac{\dot{M}_s}{P_s} - \pi m)e^{-rs}ds = \int_0^T (\frac{\dot{M}_s}{P_s} - \pi m)e^{-rs}ds + \int_T^t (\frac{\dot{M}_s}{P_s} - \pi m)e^{-rs}ds. \tag{6.21}$$

The first part of the right-hand side of (6.21) can be evaluated using (6.18) (at $t = T$). The second part is given by

$$\int_T^t (\frac{\dot{M}_s}{P_s} - \pi m)e^{-rs}ds = \int_T^t \bar{b}[\bar{\gamma}Y + (\bar{\gamma}-\delta)e^{-\delta s}\left(e^{\delta T} - 1\right)\frac{\rho\alpha}{\delta}\psi A]e^{-rs}ds - \pi m\frac{e^{-rT} - e^{-rt}}{r},$$

or

$$\int_T^t (\frac{\dot{M}_s}{P_s} - \pi m) e^{-rs} ds = (\bar{\gamma}\bar{b}Y - \pi m)\frac{e^{-rT} - e^{-rt}}{r} + \bar{b}(\bar{\gamma} - \delta)\left(e^{\delta T} - 1\right)\frac{\rho\alpha}{\delta}\psi A \frac{e^{-T(\delta+r)} - e^{-t(\delta+r)}}{r+\delta}. \quad (6.22)$$

Simulations in Figure 3

In this section we imagine that $G_t = G$, for $t \geq 0$. The paths of A_t and Y_t are the same as in our other examples. We now show how to solve the model under active and passive monetary policy.

Under active monetary policy the government chooses the single parameter of monetary policy, $\gamma = \bar{\gamma}$, to satisfy its lifetime budget constraint, (1.2). With our new assumptions this becomes

$$\int_0^\infty \left[\omega(Y_t - Y) + (A_t - A) + (\dot{M}_t/P_t - \pi m)\right] e^{-rt} dt = 0 \quad (6.23)$$

The first part of this expression is given by (2.12). The second part is given by (2.8). The negative of the third part appears on the right-hand side of (6.9). Combining these terms we have

$$\left(\frac{\omega\rho\alpha}{\delta + r} + 1\right)\frac{1 - e^{-rT}}{r}\psi A = \frac{1}{r}\left\{\pi m - e^{a - \eta(r+\gamma)}\left[\gamma Y + \rho\alpha\psi A\frac{\left(1 - e^{-rT}\right)(r+\gamma)}{(r+\delta)}\right]\right\}$$

which we solve for γ.

Under passive monetary policy we need to replace (2.16) by

$$\int_0^T (\dot{M}_t/P_t - \pi m) e^{-rt} dt = -\int_0^T \omega(Y_t - Y) e^{-rt} dt \quad (6.24)$$

since there is no natural injection of liquidity. If we combine (6.24) and (2.17), and use the steady state condition (2.5), we again obtain (2.18). We also obtain (2.20) as in Section 2. This immediately implies that the solution for $\bar{\gamma}$ is the same as before. But the solution for γ is different. To solve for γ, using (6.24) we note that the expressions in (6.11), (6.12) and (6.14) can be used.

To simulate the path of debt under either scenario we note that the previous formulas apply with one exception. The path of debt for $0 \leq t \leq T$ follows

$$D_t = D_0 - e^{rt}\left[\int_0^t \omega(Y_s - Y)e^{-rs} ds + \int_0^t (A_t - A)e^{-rs} ds + \int_0^t (\dot{M}_s/S_s - \pi m)e^{-rs} ds\right]$$

where $\int_0^t (A_t - A)e^{-rs} ds = \psi A(1 - e^{-rt})/r$.