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UNBALANCED GROWTH

Kala Krishna
Cesar A. Perez

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ABSTRACT

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Kala Krishna
Department of Economics
523 Kern Graduate Building
The Pennsylvania State University
University Park, PA 16802
and NBER
kmk4@psu.edu

Cesar A. Perez
The Pennsylvania State University

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Kala Krishna*
Pennsylvania State University and NBER

César A. Pérez
Pennsylvania State University

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Abstract

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1 Introduction

In the post World War II decades, much attention was directed to the problem of development. Why, it was asked, did some countries develop and others, which looked quite similar, did not? The obvious answer seemed to be that there were alternative routes any given economy could take. Two distinct schools of thought emerged.

Rosenstein-Rodan [10], along with Nurkse [9], Scitovsky [11], and Fleming [3], argued that there was a vicious circle present. Firms did not industrialize because there was no market for their goods and there was no market for their goods because income was low and income was low because firms did not industrialize. This kind of low-level equilibrium, it was argued, could be broken

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by the simultaneous industrialization of a large part of the economy. Failure to industrialize is seen in essence as a coordination problem. This has come to be called the "*big push*" or "*balanced growth*" doctrine.

On the other hand, the "*unbalanced growth*" camp led by Albert O. Hirschman [4], while agreeing on the existence of a vicious circle, argued that industrialization of certain "*leading*" sectors would pull along the rest of the economy. Hirschman's discussion of "*backward*" and "*forward*" linkages was an integral part of this analysis. These refer to the effects of one investment on the profitability of investment at earlier and later stages of production. Investment by a firm can, through *forward* linkages, motivate investment by another firm that uses the first firm's output as an input. Similarly, through *backward* linkages, one firm's investment can motivate another firm, which provides inputs to the first firm, to invest. Instead of industrializing a large number of sectors, he argued that what was needed was the industrialization of the "leading" sectors. Then, through backward and forward linkages these sectors would spark the industrialization of the rest of the economy. Thus, growth is unbalanced as it does not occur everywhere, only in certain sectors, which then pull others along.

These ideas, although highly influential in the 1950s, lost much of their appeal in later decades. Krugman [6] argues that this fall from grace stemmed from the fact that the ideas lacked formalization. He says:

"Like it or not, however, the influence of ideas that have not been embalmed in models soon decays. And this was the fate of high development theory. Myrdal's effective presentation of the idea of circular and cumulative processes, or Hirschman's evocation of linkages, were stimulating and immensely influential in the 1950s and early 1960s. By the 1970s..., they had come to seem not so much wrong as meaningless. What were these guys talking about? Where were their models? And so high development theory was not so much rejected as simply bypassed."

It was only in the late 80s and early 90s, beginning with the work of Murphy, Shleifer, and Vishny [8] (MSV from here on), that formal models which address some of these issues were introduced. MSV's paper delineated the conditions under which the balanced growth doctrine might apply. They show that an industrializing firm must be able to affect aggregate income through more than just its profits if the balanced growth approach is to be warranted. The idea is very simple. Aggregate income is composed of wage income and profits. A firm can acquire a better technology (an act we call industrialization) by paying a fixed cost. Initially aggregate income is too low, thereby demand is too low, making industrialization unprofitable. In their basic model there is no role for government to spark off self-sustaining industrialization by a "big push." The reason is that if industrialization is not privately profitable, it must reduce income in the aggregate when undertaken. This then reduces demand for other firms thereby reducing their motivation to industrialize.

For government intervention to be desirable, a firm's industrialization must increase aggregate income, even though its profits are negative. In this event it is possible that after a number of sectors have industrialized, aggregate demand may have increased enough to make industrialization profitable for other sectors. If profits are the only way a firm's industrialization affects aggregate income, then this is not possible. In their paper MSV look at three ways in which a firm's industrialization may affect aggregate income: (1) wage differentials between sectors, (2) an inter-temporal model that shifts income between periods, and (3) reductions in cost through investment in infrastructure.

In the literature, the focus has been on *balanced* rather than *unbalanced* growth. Models along the lines of those developed by MSV are not suited to exploring the ideas that lie behind *unbalanced* growth. In their models, all firms are *alike* and the only issue is whether it is possible for industrialization by one firm to create *spillovers* in *demand* for other firms through an increase in *aggregate* income.[1] In the words of Streeten [13]:

"Insofar as unbalance does create desirable attitudes, the crucial question is not whether to create unbalance, but *what* is the *optimum* degree of unbalance, *where* to unbalance and *how much*, in order to accelerate growth; which are the "growing points," where should the spearheads be thrust, on which slope would snowballs grow into avalanches?"

In the same vein Bardhan [1] says:

"From the policy point of view the new literature on learning and strategic complementarities, like the earlier development literature on externalities, underestimates the difficulty of identifying the few sectors and locations where the spillover effects may be large ... ."

In contrast to MSV, our model is structured to shed light on these key questions. We consider an economy with a hierarchy of goods. Only the good at the top of the hierarchy is consumed. Each good uses the good below it in the hierarchy, as well as labor, as inputs. The vertical structure of the model implies that a cost reduction at any stage reduces the price of the final good and increases its demand. Consequently, the derived demand for all intermediates rises and this increase in their demand shifts the change in profits from industrialization at any stage upwards. Linkages are related to the extent by which industrialization by a sector affects the profitability of industrialization of sectors above and below it. If forward linkages and backward ones are equally strong, then a parallel shift occurs. If forward linkages are stronger than backward linkages, the change in profits from industrialization shift up more for goods downstream

[1] To be fair, the third model analyzed by MSV does consider the role of infrastructure, an input, on profitability of investment by other identical firms. However, it is not formulated to answer the key questions regarding unbalanced growth.

from the sector and less for firms upstream from it.[2]

The model presented here fits squarely into the complementarities and cumulative processes literature. The acquisition, by a sector, of better technology translates into a lower price for the consumption good and a corresponding increase in demand for other sectors' output. This makes their potential investments more profitable. Because of fixed costs associated with the new technology, no firm acting independently finds it profitable to invest. Investment complementarities are not taken advantage of, and the vicious circle of low demand-no investment-low demand becomes an underdevelopment trap. But, as Matsuyama [7] in his excellent survey of the complementarities literature[3] points out:

"...the circularity does not always imply a vicious circle. If the economy acquires more than a critical mass of support industries, the very fact that the relation is circular generates a virtuous circle."

The unbalanced growth approach is based on the idea that industrializing the "leading" sectors is the most efficient way to reach this critical mass and turn a vicious circle into a virtuous one.

The structure of our model resembles that of MSV and is a combination of a vertical structure similar to the vertical complementarities model in Matsuyama [7] and the investment with start-up costs model in Shleifer [12]. However, our model is designed to address a different set of questions.

We define the leading sector to be the one whose investment has the largest

[2] The issue of where linkages are the strongest is where the consumer good-oriented growth camp and the heavy industry-oriented growth camp differ. However, linkages may well have been strongest upstream for a large, relatively closed economy, like India in the 60's, which produced many of the intermediates it used, while they may well have been strongest downstream for a smaller, more open economy. Hence, both camps could have been correct, but for different countries!

[3] His survey is strictly about complementarities in models with monopolistic competition. Although our model does not involve monopolistic competition the complementarities are exactly those his paper surveys.

positive effect on other sectors' decision to invest. Since the change in the incentive to invest comes from the effect on the final goods cost, this boils down to identifying the sector whose industrialization reduces the cost of the final good the most. We find that when sectors use each other's outputs as inputs relatively intensively (our intermediate input parameter $\gamma > 1$) then the leading sector tends to be the sector farthest upstream. Conversely, low intensity use of other's output implies the leading sector is downstream.[4]

The paper is organized as follows. In the next section we lay out the basic model. Section 3 deals with the effects of industrialization as a function of where it occurs in the hierarchy. This identifies the leading sectors depending on the extent of the linkages between sectors. Section 4 asks whether subsidization can create a virtuous circle where initial subsidization sparks self-sustaining industrialization. While industrialization by any sector always helps other sectors, small positive effects need not spark off industrialization of the entire economy. We find that the extent of cost savings together with the extent of linkages determine whether self-sustaining industrialization can occur. Then we ask how subsidization can improve welfare and when a case can be made for subsidization. Section 5 provides some concluding remarks.

2 The Basic Model

We will first set out the model in the absence of any decisions about investment and consider these issues in the following section. On the demand side, we have a representative agent setup. The utility function of this agent is given by:

$$U(T, Z) = T^{\varepsilon} Z^{1-\varepsilon}$$

[4] In a more general model where the input output relations are more diffuse other factors like the share in cost of an intermediate good in the cost of final goods will come into play.

where Z is produced by using a unit of labor alone. The production process of T is more complicated. The final good T is the culmination of the hierarchical production process discussed above. We assume that there are $T + 1$ steps involved in making T. A unit of T requires γ units of the good directly below it in the hierarchy, indexed by $T - 1$, as well as a unit of labor. Thus, T is produced in the $T + 1$ stage of the process. The intermediate good indexed by 0 is at the bottom of the hierarchy and just requires a unit of labor for its production.

As we assume perfect competition, price equals cost. Taking Z as the numeraire good, since the price of Z equals its cost:

$$p(Z) = 1 = w. \tag{1}$$

Throughout we will use $p(j)$ and $c(j)$ to denote the price and cost of good j. We also have that the price of intermediate good 0 equals its cost:

$$p(0) = c(0) = w = 1. \tag{2}$$

From the technology assumed:

$$c(t) = \gamma c(t-1) + w \tag{3}$$

for $0 < t \leq T$. By expanding (3) we get:

$$c(t) = w\left(\frac{\gamma^{t+1} - 1}{\gamma - 1}\right) \tag{4}$$

for $\gamma \neq 1$, and:

$$c(t) = wt + w \tag{5}$$

for $\gamma = 1$. Note that $c(t)$ must increase in t since goods higher up in the hierarchy are necessarily more expensive to produce and must cost more. $c(t)$ is linear in t if $\gamma = 1$, lies above the $\gamma = 1$ cost function when $\gamma > 1$, and below it when $\gamma < 1$. Intuitively, c rises faster with t as the linkage between the goods in the hierarchy, γ, rises. $c(t)$ is depicted in Figure 1.[5] For simplicity we draw all functions as continuous on t.

Setting $w = 1$:

$$p(t) = c(t) = \frac{\gamma^{t+1} - 1}{\gamma - 1}.$$

Demand for Z is given by:

$$\begin{aligned} D_z(p(Z)) &= \frac{(1-\varepsilon)\,Y}{p(Z)} \\ &= (1-\varepsilon)\,L, \end{aligned} \tag{6}$$

where L denotes the labor endowment and Y national income. Of course, $Y = wL = L$, since there are no profits. Demand for T is given by:

$$\begin{aligned} D_T(p(T)) &= \frac{\varepsilon Y}{p(T)} \\ &= \frac{\varepsilon L}{c(T)} \end{aligned} \tag{7}$$

as price equals cost. Similarly,

$$\begin{aligned} D_t &= \gamma^{T-t} D_T(p(T)) \\ &= \frac{\gamma^{T-t}\varepsilon L}{c(T)}. \end{aligned} \tag{8}$$

D_t depends on the price of the final good alone because of the Leontieff technology. Note that D_t increases in t for $\gamma < 1$ and decreases in t for $\gamma > 1$. The demand for all the inputs stems ultimately from the demand for the final good at the top of the hierarchy. From (8) it is clear that if $\gamma > 1$, then market size falls as t rises. If $\gamma < 1$, market size rises as t rises. When $\gamma = 1$ market size is fixed. Figure 2 depicts D_t.[6]

[5] The reader may prefer to think of (3) as a first order difference equation. Shocks are propagated along the sectoral structure of the economy according to γ^{T-t}. They are amplified forward if $\gamma > 1$ and backwards if $\gamma < 1$.

[6] One might suspect that the market size effect is driving the results. However, adjusting market size to be equal at all stages does not alter our results. See [5] for details.

3 Industrialization

We assume that consumers cannot save and hence spend all their income in each period. The future, containing an infinite number of periods, is discounted by all agents according to the discount factor δ. For each good/sector t there exists a technology that reduces marginal cost by reducing the direct labor input requirement from 1 to $1/\theta$ where $\theta > 1$. It also requires F units of labor as a setup cost. We assume that this technology is non-appropriable by others for one time period. The act of adopting this new technology is called industrialization.

We assume that in each period, industrialization can occur in at most one sector. If more than one sector is willing to industrialize, then the sector with the most to gain is assumed to industrialize. If more than one sector innovated in a time period, or if profits persisted for more than a single period, expectations about other firms' behavior would become important for each potential innovator's profits. The behavior of other firms would affect $p(T)$, and hence the demand for each input during a time frame relevant for the profits of firms considering industrialization. In such a setting, multiple equilibria based on expectations about other firms' behavior could occur. Such multiplicity is well understood, see for example, Shleifer [12], and the excellent survey by Matsuyama [7], and is deliberately ruled out in this paper.

There are a number of firms who compete in price in each sector with each firm selling a homogeneous product. At most one firm in a sector will choose to industrialize. The industrializing firm then chooses to price at the cost of non-industrializing firms as the elasticity of the demand curve it is facing is less than unity.[7] In the following period, other firms can freely access the technology and

[7] $D_t(p(t)) = \gamma^{T-t} D_T(p(T))$ so that the elasticity of demand for good t equals the elasticity of demand for the final good times the ratio of the percentage increase in $p(T)$ relative to the percentage increase in $p(t)$. The former is unity as utility takes a Cobb Douglas form, and the latter is less than unity as the final good price increases less than proportionately relative to that of any intermediate input since any intermediate input has a cost share less than unity.

competition dissipates all profits. However, although profits go to zero after the first period of innovation, the effects of the innovation remain and are perceived by other firms through an increase in demand as a result of a lower price for good T.

We will distinguish between three things: 1. The sector's incentive to industrialize which is related to its profits from doing so. 2. The effects a sector's industrialization has on sectors above and below it in the hierarchy of production, i.e., the extent of its forward and backward linkages. 3. The identity of the leading sector. This is the sector whose industrialization has the largest effect on the industrialization choice of other sectors. Since these effects operate via the price of the final good, this is the sector whose industrialization has the largest effect on the cost of the final good.

3.1 The Gains from Industrialization.

We need an expression for the change in profits of a firm, indexed by t, if it decided to industrialize when no other firm had industrialized. We will assume that these profits are below zero to start with so that no firm chooses to industrialize when no industrialization has occurred. Following that we examine how t's profits from industrialization are affected by industrialization of sectors above and below it in the hierarchy.

Given our model, a firm will choose to industrialize if its current variable profits exceed F. Define $c(t|B)$ to be the cost of producing a unit of t when all the sectors in the set B have industrialized but no one in sector t has industrialized. Similarly, $c^I(t|B)$ is the cost of producing a unit of t when t has industrialized as well as all sectors in the set B. We will denote by $c(t)$ and $c^I(t)$ the case where $B = \phi$, the empty set. $p^I(t|B)$ and $p(t|B)$ can be similarly defined with $p(t)$ and $p^I(t)$ again denoting the case where B is the empty set.

If a firm in sector t is the first to industrialize, it has marginal costs given by:

$$c^I(t) = \gamma c(t-1) + \frac{1}{\theta} \tag{9}$$

where $c(t-1)$ is defined by (3). It also incurs a fixed cost given by F. Since it chooses to price as high as it can, as argued earlier, it prices at its competitors' cost, $c(t)$. Its profit from industrializing (in the period in which its industrialization takes place) when no one else has, denoted by $\pi^I(t)$ is given by:

$$\pi^I(t) = \frac{a\gamma^{T-t}\varepsilon\left(L + \pi^I(t)\right)}{p(T)} - F$$

where $a = 1 - 1/\theta$. Solving for $\pi^I(t)$ in order to incorporate the effects of profits on income gives:[8]

$$\pi^I(t) = \frac{\frac{a\gamma^{T-t}\varepsilon L}{p(T)} - F}{\left[1 - \frac{a\gamma^{T-t}\varepsilon}{p(T)}\right]}. \tag{10}$$

Similarly:[9]

$$\pi^I(t|B) = \frac{\frac{a\gamma^{T-t}\varepsilon L}{p(T|B)} - F}{\left[1 - \frac{a\gamma^{T-t}\varepsilon}{p(T|B)}\right]}. \tag{11}$$

Proposition 1 If $\gamma < 1$, then the sector with the greatest incentive to industrialize is the one furthest downstream. If $\gamma > 1$, it is the one furthest upstream. If $\gamma = 1$, all sectors have the same incentive to industrialize.

Proof From equations (10) and (11) it follows that products with larger demands have a greater incentive to industrialize as the fixed cost can be spread over a larger number of units. Figure 3 depicts $\pi^I(t)$ for these three cases.

The gains from industrialization initially lie strictly below zero as we assume no industrialization occurs without intervention of some kind.

[8] Assume that firms do not take into account the effect of their profits on aggregate income would not affect the spirit of the results that follow.

[9] Note that we assume that $\frac{a\gamma^{T-t}\varepsilon L}{p(T)} - F < 0$ which implies $1 - \frac{a\gamma^{T-t}\varepsilon}{p(T)} > 0$ since $L > F$.

3.2 Industrialization and the Gains from Industrialization.

Here, we consider the effects of industrialization of a group of sectors on the change in profits from industrialization of the remaining sectors. We need to distinguish two distinct effects. When a firm industrializes it makes profits (positive or negative) in the period in which it industrializes. This change in profits affects income, and hence the potential change in profits of other firm's in that same period. This effect we call the *impact* effect. Note that like in MSV, if it is not profitable for a sector to industrialize on its own, then its industrialization must have a negative impact effect. In the period of industrialization, there is no price change, and if profits are negative, income is reduced by industrialization.

In the next period the rest of the sector acquires the technology and price falls. This reduction in price, as a result of better technology, increases demand and therefore increases the potential change in profits for other firms not yet industrialized. We call this the *long-run* effect. While impact effects are just as in MSV, the long-run effects are new.

Recall equation (11) and note that industrialization by any firm t reduces the price of good t in the following period. This, in turn, reduces the cost, and price, of the final good and raises the quantity demanded. This feeds back to the demand for all the intermediate goods in the hierarchy. The "leading sector" is that sector which has the *largest* effect on the gains from industrialization of other sectors. From equation (11) it is obvious that the leading sector must be the sector that lowers $p(T|B)$ the most. This will depend on the value of γ.

3.2.1 $\gamma = 1$

In this case, industrialization of any firm anywhere in the hierarchy shifts all gains from industrialization up equally. Because $\gamma = 1$ there is neither dilution nor magnification of this effect as we move upstream or downstream. In essence, a unit of the final good requires a unit of each of the intermediate goods. Thus, industrialization of any sector has the same effect on the change in profits of any un-industrialized firm. This implies that in this case there is no such thing as a leading sector.

More formally, when $\gamma = 1$:

$$\pi^I(t|B) = \frac{\frac{a\varepsilon L}{p(T|B)} - F}{\left[1 - \frac{a\varepsilon}{p(T|B)}\right]}.$$

Also, $p(T) = T + 1$ and if any n of $T + 1$ rungs in the hierarchy industrialize the price of the final good is given by:

$$p(T|n) = T + 1 - na.$$

Therefore, $p(T)$ depends only on the *extent* of industrialization and not the location of the industrializing firms in the hierarchy. This case is depicted in Figure 4. $\pi^I(t|n)$ is a horizontal line which shifts up proportional to the extent of industrialization as shown. Once a certain threshold amount of industrialization has occurred, all remaining industrialization takes place automatically, i.e., it is self sustaining. This threshold is the smallest n for which $\pi^I(t|n)$ exceeds zero. When $\gamma = 1$ all sectors are equally good candidates for leading sectors. Note that, it is possible to have self-sustaining industrialization once a threshold level is crossed yet no industrialization occurs in the absence of intervention.

There is a strong similarity between this case and MSV. Here, every sector is identical to any other sector in the sense that they all have the same effect on the gains from industrialization of other firms. We need enough of them to industrialize to reduce the price enough so that demand increases enough to make industrialization worthwhile. In MSV, enough sectors need to industrialize to increase aggregate income enough so that demand increases enough to make industrialization worthwhile. In either case only the number of sectors that industrialize matters.

One important difference between MSV and our model is that here we *do not* need something like a wage premium to make self-sustaining industrialization possible. The vertical linkages provide us with a way for one firm's industrialization to positively affect the profits of other firms.

3.2.2 $\gamma \neq 1$

We now turn to the slightly more complex case where $\gamma \neq 1$. Again $p(T|B) < p(T)$, with $p(T|B)$ now given by:

$$p(T|B) = w\left(\frac{\gamma^{T+1}-1}{\gamma-1}\right) - wa\sum_{j=0}^{T}\left(\gamma^{T-j}\iota\right) \tag{12}$$

where ι is an indicator function equal to one for $j \in B$ and equal to zero otherwise. Now due to the market size effect, $\pi^I(t|B)$is decreasing (increasing) in t if γ is more (less) than unity so that firms highest (lowest) in the hierarchy have the greatest incentive to industrialize.

Industrialization of the sectors in the set B shifts up the gains from industrialization of the remaining sectors. Consider what happens in the context of linkages. Linkages are related to the extent by which industrialization by a sector affects the profitability of industrialization of sectors above and below it.[10]

Proposition 2 When $\gamma < 1$, backward linkages are stronger than forward linkages and profits from industrialization shift up more for low t than high t. When $\gamma > 1$, forward linkages are stronger than backward linkages and profits from industrialization shift up more for high t than low t. When

[10] If we did away with differences in market size we would still be left with the question of what determines the identity of the leading sector.

$\gamma = 1$, both are equally strong and profits from industrialization shift up in a parallel manner.

Proof When t industrializes, demand increases for things it uses as inputs, i.e., for $k < t$ demand rises from

$$\gamma^{T-k}\left(\frac{\varepsilon L}{\frac{\gamma^{T+1}-1}{\gamma-1}}\right)$$

to

$$\gamma^{T-k}\left(\frac{\varepsilon L}{\frac{\gamma^{T+1}-1}{\gamma-1}-a\gamma^{T-t}}\right).$$

These are backward linkages. At the same time demand also increases for sectors who use $t's$ output as inputs, i.e., for $s > t$ demand rises from

$$\gamma^{T-s}\left(\frac{\varepsilon L}{\frac{\gamma^{T+1}-1}{\gamma-1}}\right)$$

to

$$\gamma^{T-s}\left(\frac{\varepsilon L}{\frac{\gamma^{T+1}-1}{\gamma-1}-a\gamma^{T-t}}\right).$$

These are forward linkages. When $\gamma = 1$, both linkages are equal in strength and a parallel shift occurs as depicted in Figure 4. If $\gamma > 1$, then $\gamma^{T-k} > \gamma^{T-s}$ and backward linkages are stronger than forward linkages. Consequently, profits from industrialization shift up more for low t than high t. If $\gamma < 1$, then $\gamma^{T-k} < \gamma^{T-s}$ and forward linkages are stronger than backward linkages, so that profits from industrialization shift up more for high t than low t. The case where $\gamma > 1$ is depicted in Figures 5 and 6. The analogous diagrams can be drawn for $\gamma < 1$ and this is left to the reader.

The leading sector is the one that reduces $p(T|B)$ the most. When $\gamma \neq 1$, the magnitude of the reduction in $p(T|B)$ depends *not only* on the number of sectors industrialized but also on *which* sectors have industrialized.

Proposition 3 When $\gamma > 1$ the sector with the lowest index which has not already industrialized is the leading sector. When $\gamma < 1$, the sector with the highest index which has not yet industrialized, is the leading sector. When $\gamma = 1$, all sectors are leading sectors.

Proof From equation (12) it is easy to see that the fall in the price of the final good due to industrialization of Sector t is proportional to $a\gamma^{T-t}$. When $\gamma > 1$, $a\gamma^{T-t}$ is largest for the lowest t, while for $\gamma < 1$, $a\gamma^{T-t}$ is largest for $t = T$. When $\gamma = 1$, $a\gamma^{T-t}$ is constant. When $\gamma > 1$ the amplification of a cost reduction is forward, to higher t, so that the leading sector being the one with the smallest index makes sense. When $\gamma < 1$ the amplification of a cost reduction is backward, to lower t, so that the leading sector being the one with the largest index again makes sense.

4 The Case for Subsidizing Industrialization

Why does the market not ensure that the optimal extent of industrialization occurs? There are two distortions in our model. First, producers maximize profits, rather than welfare that includes both producer surplus and consumer surplus. In our model we set things up so that in the first period of industrialization, there are no consumer surplus gains and the producer appropriates all the gains. From the second period onward, there are no producer surplus gains, only consumer surplus ones. Note that if it is not profitable to industrialize, then the first period losses must be made up by later gains in consumer surplus for industrialization to be worth subsidizing through this channel alone.

The second and less standard distortion has to do with the fact that there are positive spillovers from industrialization across sectors which industrializing firms cannot appropriate. Since price of the final good falls in the period after any sector industrializes, derived demand, and hence, the profits from industrialization shift up for all un-industrialized sectors. This makes for too little industrialization.

What can we say about optimal policy? We can begin by noting that if there is a case for subsidizing industrialization, then the optimal way of doing so must involve subsidizing the leading sector among the ones that are not industrialized. Why? By definition, the leading sector has the greatest positive externality on the profits of other sectors so the benefits of its industrializing are highest. Moreover, as it is the most profitable, the subsidy needed to induce industrialization is at its lowest.

So suppose one proceeded to subsidize in this fashion, targeting the leading sector that is not industrialized. Could such a policy ever stimulate self-sustaining industrialization? If so, then the government need just subsidize to this point, confident that complete industrialization would follow automatically.

We focus on the case where $\gamma > 1$. The case where $\gamma < 1$ is analogous. Let $\pi^I(n|[0,n))$ denote the profits obtained by firm n from industrializing when all sectors from 0 up to but not including n have industrialized. As n rises to n_1 and further to n_2 more and more industrialization occurs as depicted in Figure 5. This reduces the price of the final good and this channel shifts $\pi^I(n|[0,n))$ upward. But, as n rises we are also moving along a downward sloping curve which reduces $\pi^I(n|[0,n))$. In Figure 5 the shift in the curve overcomes the movement along the curve, i.e.,

$$\frac{d\pi^I(n|[0,n))}{dn} > 0. \tag{13}$$

In the Appendix we show that for (13) to be true, we need $\theta > \gamma^{T+1}$. In other words, θ must be large, i.e., industrialization must result in substantial cost savings. If $\pi^I(n|(n,T])$ becomes positive for $n < T$, then if a government subsidizes industrialization up to this point, it will become self sustaining and proceed without further government help. The initial losses from industrialization could be covered by later profits. Figure 6 depicts the case where $\theta < \gamma^{T+1}$ and the movement along the curve dominates. As a result, it is not possible for initial losses from industrialization to be covered by later profits.

We leave it to the reader to draw the figures analogous to Figure 5 and 6 when $\gamma < 1$. When $\gamma < 1$, then the leading sector is sector T. Industrialization should start downstream. Profits shift up and are higher downstream. For the shift up in profits to dominate the fact that the leading sector is moving up stream it must be that

$$\frac{\partial \pi^I(n|(n,T])}{\partial n} < 0. \tag{14}$$

As shown in the Appendix, for (14) to hold we need

$$\theta > \frac{1}{\gamma^{T+1}}.$$

Assume that industrialization is not profitable to begin with and construct the welfare that obtains from subsidizing n sectors in the optimal manner. This involves targeting the leading sector that has not yet industrialized in each of n consecutive periods. Since $\gamma > 1$, we would wish to start with sector 0.[11]

Welfare is defined to be the welfare of the aggregate consumer. Let $V(1, P, I)$ denote the indirect utility function of this consumer as a function of the price of the final good Z, which is unity as it takes one unit of labor to make it, the final good, T, denoted by P, and his income, I.[12] Suppose, for example, that B industries have industrialized and t is industrializing. Then $P = P(T/B)$ and $I = L + \pi^I(t/B)$, the sum of labor income and the profits of industrializing firms excluding subsidies. Subsidies to firms are just transfers and cancel out in

[11] Once again, the arguments for $\gamma < 1$ are analogous and left to the reader.
[12] Recall that there are two goods, Z, and T, and the price of Z is unity.

calculating national income. Hence, welfare is merely[13]

$$V(1, P(T/B), L + \pi^I(t/B)).$$

There are two possibilities where welfare is slightly different. Either there is no possibility of self-sustaining industrialization, i.e., $\gamma^{T+1} > \theta > 1$ or there is, i.e., $\theta > \gamma^{T+1} > 1$. Consider the former case where self-sustaining industrialization is not possible. Suppose the government subsidizes the leading sector for n periods, i.e., it subsidizes the first n sectors in sequence for n periods. In any of these periods, indirect utility may be higher or lower than without industrialization. Income may fall below L if profits are negative, which pulls indirect utility down. However, as the price of the final good is lower, indirect utility is pulled up. From $t = n$ onwards, prices are constant at $P(T/B = \{s : s < n\})$ which is less than $P(T)$ while income is L. Hence, indirect utility is higher than it would have been without industrialization.

Now what needs to be added if self-sustaining industrialization is possible? Not much! If self-sustaining industrialization occurs once sectors up to k have been industrialized, then if $n < k$, there is nothing to be added. If $n \geq k$ then we just need to realize that industrialization will keep on going. The outcome is exactly that with $n = T + 1$, i.e., when the government subsidizes all sectors.

Thus, if $\gamma^{T+1} > \theta$, or $\theta > \gamma^{T+1}$ and $n < k$, then welfare when n sectors industrialize starting from the one furthest upstream is

$$\begin{aligned} W(n) &= \sum_{t=0}^{n-1} \left[\delta^t V\left(1, P(T/B = \{s : s < t\}), L + \pi^I(t/B = \{s : s < t\}\right)\right] \quad (15) \\ &+ \left(\frac{\delta^n}{1-\delta}\right) V(1, V\left(1, P(T/B = \{s : s < n-1\}), L\right). \end{aligned}$$

[13] It is clear that income is just labor income, L, plus any net profits since subsidies cancel out.

If $\theta > \gamma^{T+1}$ and $n \geq k$, then

$$
\begin{aligned}
W(n) &= W(T+1) \qquad (16) \\
&= \sum_{t=0}^{T} \left[\delta^t V\left(1, P(T/B = \{s : s < t\}), L + \pi^I(t/B = \{s : s < t\}\right)\right] \qquad (17) \\
&+ \left(\frac{\delta^{T+1}}{1-\delta}\right) V(1, V\left(1, P(T/B = \{s : s < T\}), L\right).
\end{aligned}
$$

In general there is no reason to expect $W(n)$ to be concave or for a simple policy to be optimal. However, there are a few things we can say about welfare.

Proposition 4 If δ is close to unity, then subsidizing industrialization is welfare improving even when it is not privately profitable. If δ is close to zero, then it is not.

Proof As δ approaches unity, the second term in (15) or (16) goes to infinity. As δ approaches zero, the losses in current income from industrialization can never be made up.

It is hard to say what the optimal extent of subsidization is. However, it is clear that subsidizing n leading sectors in sequence must raise welfare if δ is large enough.

Proposition 5 For each n, there exists a $\delta(n)$, such that for $\delta > \delta(n)$, it is better to subsidize n leading sectors in sequence than to do nothing.

Proof This follows from the fact that the first term in (15) or (16) can be negative while the second term must be positive and raising δ raises the (unbounded above) weight put on the second term.

Thus, one can make a case for industrialization when it is not privately profitable without relying solely on the welfare gains emanating from lower prices of the final good post industrialization.

5 Concluding Remarks

Our aim in this paper has been to formalize in a simple way the concept of unbalanced growth. Models like the simple one given here can help cast light on the old debate about paths to industrialization. It suggests that in a closed economy, when the future is not discounted too highly, there is a case for subsidizing industrialization. Other things equal, the sector whose industrialization has the largest impact on the cost of making the final good is the sector to target.

We hope in future work to look at more general settings and at issues like the role of trade. Through trade some of the benefits to industrialization of a sector accrue to the rest of the world suggesting that there may be a case for coordinating industrialization world wide and for the subsidization of industrialization at an international level.

6 Appendix: Conditions for Self-Reinforcing Industrialization

Recall that from equation (11)

$$\pi^I(n|[0,n)) = \frac{\frac{a\gamma^{T-n}\varepsilon L}{p(T|[0,n))} - F}{\left[1 - \frac{a\gamma^{T-n}\varepsilon}{p(T|[0,n))}\right]} \tag{18}$$

where

$$\begin{aligned} p(T|[0,n)) &= \left(\frac{\gamma}{\theta}\left[\frac{\gamma^n - 1}{\gamma - 1}\right] + 1\right)\gamma^{T-n} + \frac{\gamma^{T-n} - 1}{\gamma - 1} \\ &= \frac{\gamma^{T+1} - \theta - (1-\theta)\gamma^{T-n+1}}{\theta(\gamma - 1)}. \end{aligned} \tag{19}$$

Due to industrialization, the cost of the inputs up stream of n falls to $\frac{1}{\theta}\left[\frac{\gamma^n-1}{\gamma-1}\right]$, so that the cost of production of the n^{th} good becomes $p(n|[0,n)) = \frac{\gamma}{\theta}\left[\frac{\gamma^n-1}{\gamma-1}\right] + 1$. This in turn allows us to write the price of the final good as the cost of the implicit requirement of the n^{th} good, $\gamma^{T-n}p(n|[0,n))$, plus the labor requirement from then on, $\frac{\gamma^{T-n}-1}{\gamma-1}$. This is what gives us (19).

We also know that

$$\pi^I(n|[0,n)) = \frac{\frac{a\gamma^{T-n}\varepsilon L}{p(T|[0,n))} - F}{\left[1 - \frac{a\gamma^{T-n}\varepsilon}{p(T|[0,n))}\right]} \tag{20}$$

If $\frac{\gamma^{T-n}\varepsilon}{p(T|[0,n))}$ rises, then the numerator rises while the denominator falls so that $\pi^I(n|[0,n))$ must rise. Hence, we need only consider the derivative of $\frac{\gamma^{T-n}}{p(T|[0,n))}$. Differentiating gives

$$\frac{d\left[\frac{\gamma^{T-n}\varepsilon}{p(T|[0,n))}\right]}{dn} = \frac{-p(T|[0,n))\gamma^{T-n}\ln\gamma - \gamma^{T-n}\frac{dp(T|[0,n))}{dn}}{\left[p(T|[0,n))\right]^2}. \tag{21}$$

Since

$$p(T|[0,n)) = \frac{\gamma^{T+1} - \theta - (1-\theta)\gamma^{T-n+1}}{\theta\,(\gamma-1)}. \tag{22}$$

$$\frac{dp(T|[0,n))}{dn} = \frac{(1-\theta)\,\gamma^{T-n+1}\ln\gamma}{\theta\,(\gamma-1)} < 0$$

as $\theta > 1$. The sign of (21) is the same as the sign of its numerator or

$$-\left[\gamma^{T+1} - \theta\right].$$

If $-\left[\gamma^{T+1} - \theta\right] > 0$, i.e., $\theta > \gamma^{T+1}$, then $\pi^I(n|[0,n))$ rises with n.

As n rises more industrialization occurs. This reduces the price of the final good and this channel raises the profits from industrializing at n. Also, as n rises γ^{T-n} falls for $\gamma > 1$. This channel reduces $\pi^I(n|[0,n))$. With $\gamma > 1$, this movement along the $\pi^I(t|[0,n))$ curve reduces profits as $\pi^I(t|[0,n))$ is falling in t. From this it follows that increasing industrialization from below when $\gamma > 1$ entails conflicting forces. If $\theta > \gamma^{T+1}$, then the former dominates so that $\pi^I(t|[0,n))$ rises with n.

Similarly, when industrialization starts at the top of the hierarchy:

$$\pi^I(n|(n,T]) = \frac{\frac{a\gamma^{T-n}\varepsilon L}{p(T|(n,T])} - F}{\left[1 - \frac{a\gamma^{T-n}\varepsilon}{p(T|(n,T])}\right]} \tag{23}$$

where

$$\begin{aligned} p(T|(n,T]) &= \left(\frac{\gamma^{n+1}-1}{\gamma-1}\right)\gamma^{T-n} + \frac{1}{\theta}\left(\frac{\gamma^{T-n}-1}{\gamma-1}\right) \\ &= \frac{\theta\gamma^{T+1} + (1-\theta)\gamma^{T-n} - 1}{\theta\left(\gamma-1\right)}. \end{aligned} \tag{24}$$

Due to industrialization good T can be thought of as using γ^{T-n} units of n which costs $\frac{\gamma^{n+1}-1}{\gamma-1}$ and $\frac{1}{\theta}\left(\gamma^{T-n-1} + ... + 1\right)$ units of labor. This gives us the above expression.

Differentiating $\pi^I(n|[0,n))$ with respect to n gives:

$$\frac{\left[1 - \frac{a\gamma^{T-n}\varepsilon}{p(T|(n,T])}\right] L - \left[\frac{a\gamma^{T-n}\varepsilon L}{p(T|(n,T])} - F\right] a\gamma^{T-n}\varepsilon}{\left[1 - \frac{a\gamma^{T-t}\varepsilon}{p(T|(n,T])}\right]^2 p(T|(n,T])^2} \left[p(T|(n,T])\left(-\ln\gamma\right) - \left(\frac{\partial p(T|(n,T])}{\partial n}\right)\right]$$

which is negative when

$$\theta > \frac{1}{\gamma^{T+1}}.$$

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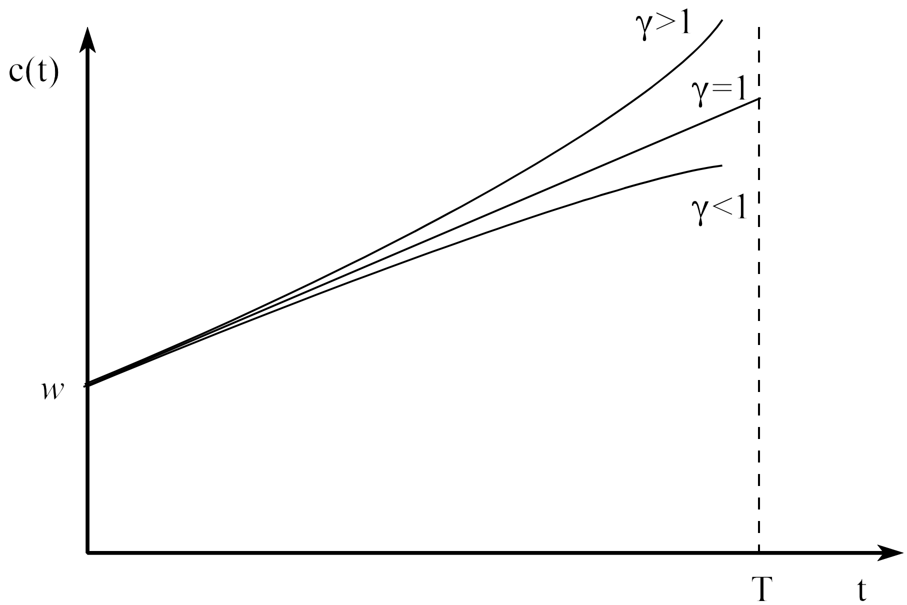

Figure 1

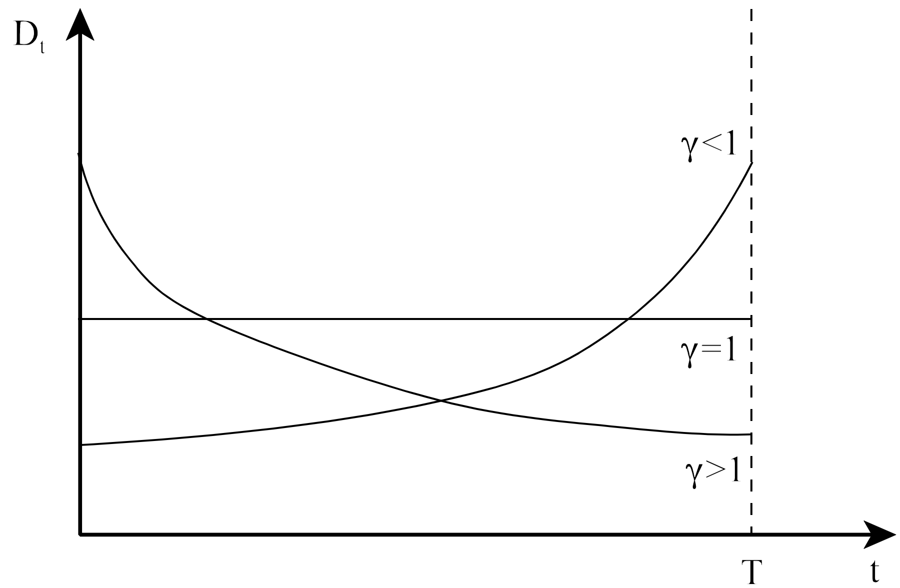

Figure 2

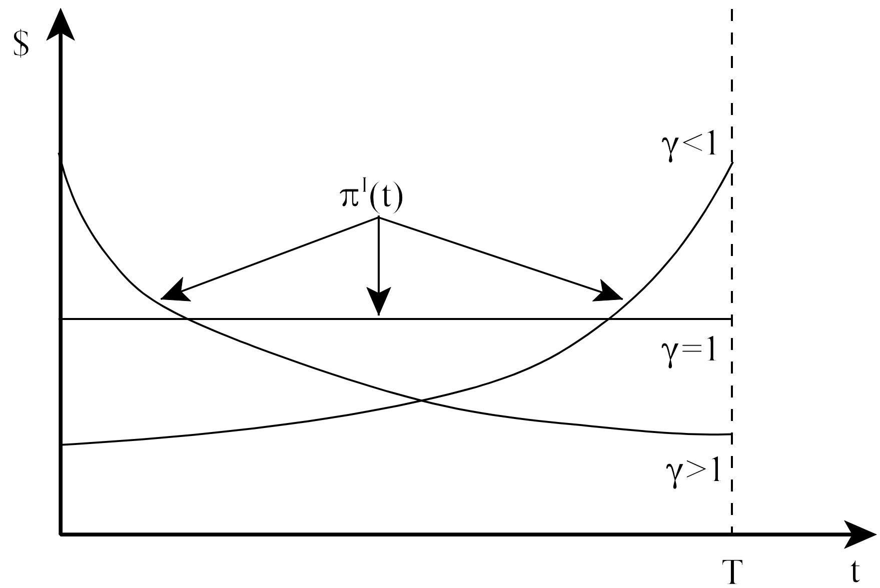

Figure 3

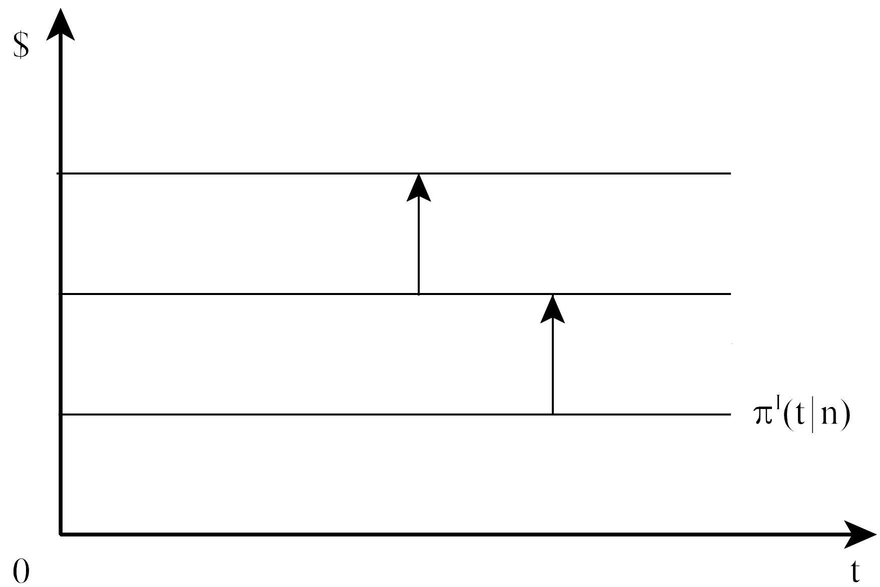

Figure 4

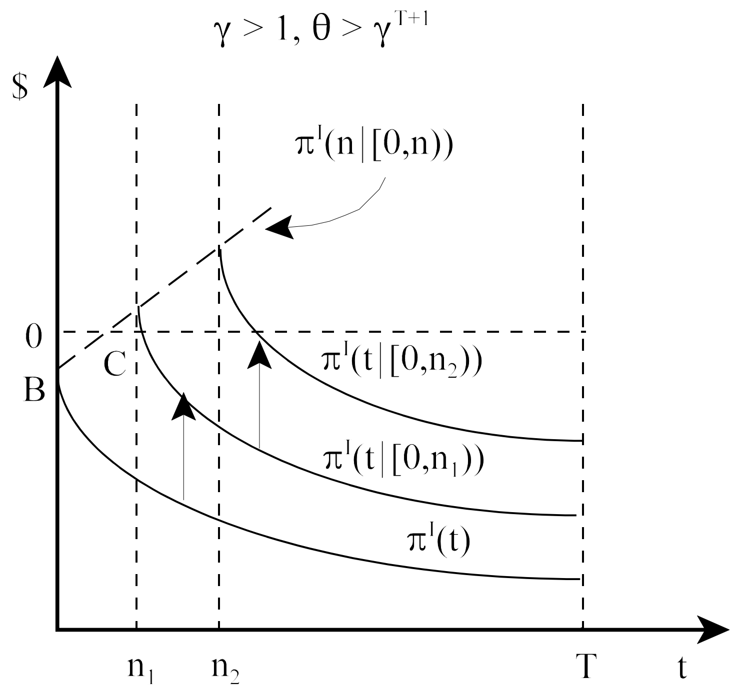

Figure 5

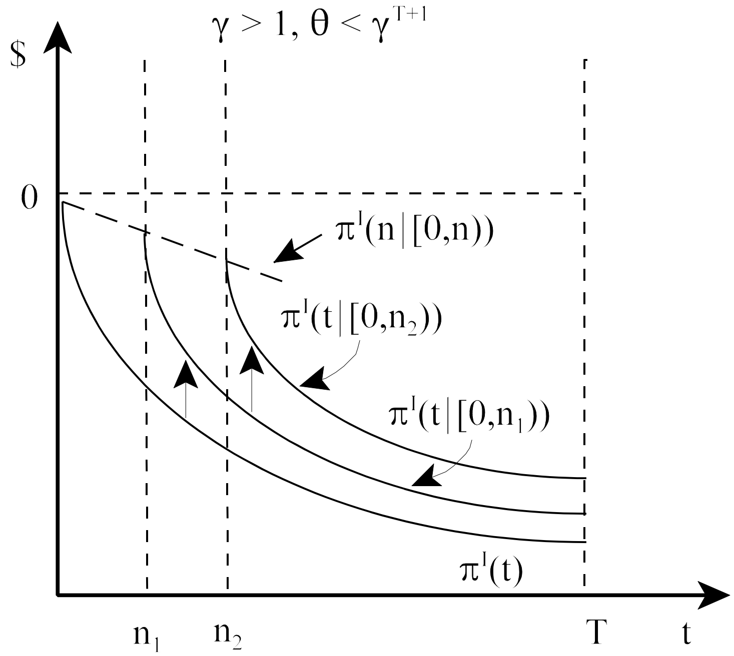

Figure 6

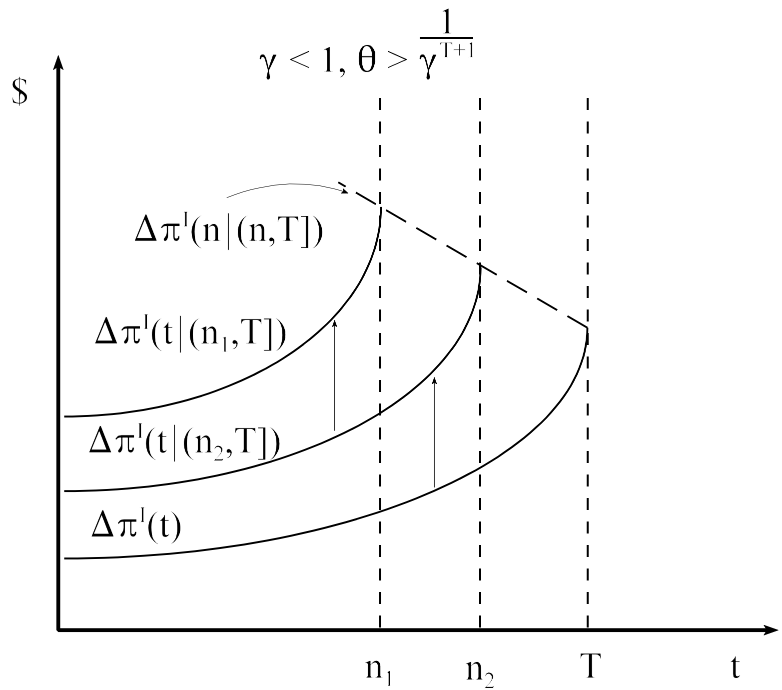

Figure 7

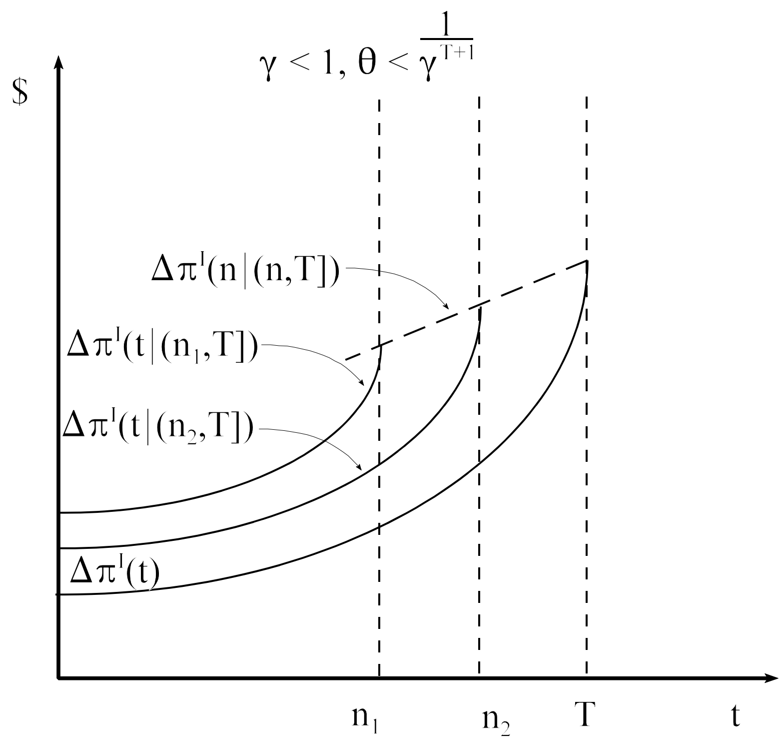
$\gamma < 1, \theta < \frac{1}{\gamma^{T+1}}$
$
$\Delta\pi^I(n|(n,T])$
$\Delta\pi^I(t|(n_1,T])$
$\Delta\pi^I(t|(n_2,T])$
$\Delta\pi^I(t)$
n_1
n_2
T
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Figure 8