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THE DYNAMICS OF INCOME, SCHOOLING, AND FERTILITY DISTRIBUTIONS OVER THE COURSE OF ECONOMIC DEVELOPMENT: A HUMAN CAPITAL PERSPECTIVE

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The Dynamics of Income, Schooling, and Fertility Distributions Over the Course of Economic Development: A Human Capital Perspective
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ABSTRACT

We develop a dynamic model of fertility and income distribution in which both are linked to the formation and distribution of human capital among families. Our model offers a dynamic version of Becker's (1967) model of income distribution within an endogenous growth framework. We view the population as consisting of heterogeneous families, which are subject to intra-family and inter-family interactions. Families determine fertility, human capital formation in children, and savings. We thus link income and fertility distributions over an entire development path, extending from a low-income, stagnant state to a self-sustaining growth regime. In this context, we also reexamine the "Kuznets hypothesis" concerning the relation between income inequality and income growth over a transitional development period. The paper offers new insights and supporting empirical evidence concerning the time-paths of distributional measures of fertility, educational attainments, and three income-related measures: family-income inequality, income-group inequality, and the Gini coefficient.

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I. INTRODUCTION

Kuznets (1955) observed that the relative distribution of income tends to move toward greater equality in more developed countries. In his follow-up 1963 paper, additional empirical evidence led him to propose that inequality first rises and then falls during the process of economic development. This is popularly known as the "Kuznets hypothesis". Following Kuznets, the literature developed in two basic directions. The first involved testing the hypothesis against empirical data from different countries.[1] The second attempted to construct theoretical models dealing with the development – inequality nexus as a causal relation going from either growth to inequality or vice versa.[2] Both sets of studies have offered conflicting conclusions about the competing hypotheses.

There are, however, a number of apparent empirical regularities concerning income distribution measures in both low- and high-income countries, which have received less attention in the literature. For example, the data we examine in section VI indicate that the Gini coefficient varies markedly **across** countries around the year 1995, from 60 in Brazil, to 27.2 in Belgium, 25.6 in Norway, and 23 in Finland, but remains almost constant **within** some poor and rich countries over time, as in India - around 32 for forty years - and Japan - around 35 for thirty years. Also, models of inequality and growth have not addressed corresponding movements in the level and relative variance of **fertility** and **educational attainments**, which appear to take place in countries experiencing a transition from low development stages to persistent growth regimes.

We attempt to provide an interpretation of this broader evidence, as well as of conflicting evidence concerning the Kuznets hypothesis and the causality issue, based on an OLG model of heterogeneous families, in which human capital is the engine of growth, and family choices affect its formation. Income growth and inequality are viewed as joint outcomes of a common set of underlying factors. In this context, our model offers a dynamic version of Becker's static model of income distribution (Becker, 1967) and a generalization of recent work by Zhong (1998) and Ehrlich and Yuen (2000). We show that the behavior of income inequality over the transitional development phase can vary within different countries, depending on the factors triggering the transition, and the specific income-inequality **measure** used. Moreover, we offer testable propositions about the association between income growth and two other variables as well: the level and distribution of both fertility and educational investments.

Our basic thesis is that the relationship between income growth and income distribution must be sorted out of three main forces: interactions between overlapping generations within families; heterogeneities in abilities, income endowments, and associated financing opportunities across families; and interactions among members of heterogeneous families in school or at the workplace. The first force determines the prospects of human capital formation in successive generations, while the second may increase or decrease the divergence across families, depending on the sources of family heterogeneity and the way these are correlated across families. The third force operates to contain either divergence or convergence tendencies through social interactions. Our formal structure thus attempts to account for the motivating forces operating within all families, major heterogeneity sources that separate them, and the role of social interactions that link them.[3]

Formally, we set up a model of endogenous growth with **finitely** lived individuals. Families optimize on investments in the quantity and quality of children, as well as on savings. We abstract, however, from modeling an explicit market for physical capital and identify human capital as the basic earnings-generating, as well as income-generating, asset. Unlike static models of human capital formation, which focus on investments individuals make themselves, in our model growth in human capital over time is enabled by parental investments. We allow these choices to be motivated by both altruistic rewards and material benefits in the form of old-age support or informal care old parents receive from adult children.

The model accounts for the evolution of income, schooling, and fertility distributions over three development phases: a stagnant steady state with low per-capita income and high fertility, a perpetual growth steady state with low fertility, and a transitional development stage linking the two. The transition takeoff can be triggered by discrete jumps in specific growth-enhancing parameters.

By this approach we are also able to provide new insights into the "Kuznets hypothesis". A basic insight is that the association between income level and income inequality depends on the comparative levels of the income and fertility inequality in the two steady states that frame the transitional phase. The structure is in a state of flux during the transition phase from stagnant to growth equilibrium depending on the way takeoff triggers affect different families. An inverted-U shape of the income inequality path can be derived as a special case of our general analysis, which can also rationalize, however, a rising and concave shape, as well as a falling and convex shape over the development phase. Some of the model's distinct implications are:

1. The relation between income growth (level and rate) and income inequality is associative, not causal. Their co-movement depends on the way specific parametric shocks affect families.
2. The dynamic evolution of income inequality depends also on the inequality measure used. Our model identifies 3 basic measures: family-income inequality, income-group inequality, and the Gini coefficient. The latter two reflect also the relative differences in size of families of different income levels. While all three measures generally move in tandem over the transitional development period, they can also move in different directions as a result of specific parametric shocks.
3. Fertility rates converge on equality at stable stagnant equilibrium (SE) or growth equilibrium (GE) steady states, and income growth **rates** must approach equality at the latter state. If heterogeneity across families is the result of differences in family-specific income-generating endowments,

abilities, and investment financing costs, then the fractions of full income spent on rearing and educating each child must also converge on equality at the SE and GE steady states.

4. The association between income levels and income inequality over the transitional development phase may assume a U shape, an inverted-U shape, or various combinations of the two, depending upon the way heterogeneity sources are correlated across families and how different takeoff-triggering parametric shocks affect different family groups. These also determine the comparative income inequality levels in the GE relative to the SE steady states.
5. Regardless of the behavior of income inequality measures along the development path, the relative inequality paths of both **fertility** and the fractions of full income devoted to **human capital investments** are expected to exhibit an inverted-U shape with flat tails.

In section II we introduce the model and its equilibrium solutions. In section III and IV we explore the model's dynamic properties and simulate alternative transitional development paths, and in section V we discuss some model extensions. Section VI presents new evidence on the dynamics of income, schooling, and fertility distributions based on international panel data over the period 1950-1998. The results are consistent with our testable propositions.

II. INEQUALITIES AND GROWTH WITH INTERDEPENDENT FAMILIES

A. The Economic Environment

To explore the pattern of income, educational investments, and fertility inequality along the development path, we extend the representative-family, OLG model of endogenous growth in Ehrlich and Lui [EL] (1991) to a heterogeneous-family model that recognizes relevant interactive forces within and across families all along the economic development path.

The Economy. The economy is comprised of a fixed distribution of heterogeneous family types, indexed by i (i = 1, 2, 3…I) where I is the number of family types in the economy.[4] Pursuing the distinctions made in Becker (1967), we emphasize three objective sources of heterogeneity: a. differences in ability (A^i); b. differences in income-producing "endowments", H^i_0, stemming from social status, political power or inherited wealth; c. differences in financing costs of educational investments (θ^i), largely due to capital market segmentation. All such differences are treated as inherited family endowments. We generally abstract from heterogeneity in other parameters that represent preferences or external production technologies, since these need not be related systematically to objective differences.

Each agent in this economy lives through three periods: childhood, adulthood, and old age. All family-based decisions are made during adulthood by parents. Agent (i,t) is thus one who is born into family i at period t-1 and becomes an adult (family decision-maker) at period t.

Goods Production and Income. The economy is competitive and human capital is the sole productive asset. The i-th young parent possesses a production capacity ($H^i_0+H^i_t$), composed of an inherited income-producing asset, e.g., social status, measured in units of human capital (H^i_0), and an acquired human capital component, (H^i_t), attained through parental inputs. Labor supply by each agent is fixed in any period. We also assume for convenience that all consumer goods, including educational services, can be purchased. Under a linear and strongly additive production technology for all goods, aggregate production and earnings equals aggregate employment of effective labor in each period, or Y = L, and the zero-profit condition for the representative competitive firm, $\pi=Y-\varpi L=0$, yields a time-invariant real wage rate $\varpi=1$, which also guarantees full employment, $Y=\Sigma_i(H^i_0+H^i_t)$. We initially abstract from any saving opportunities, so earnings are identical to income. In **Appendix A** and section V we extend our model to allow for savings opportunities, and show that the inferences we derive about earnings inequality extend to income inequality as well.

Human-capital production. The dynamic human-capital production rule is given by:

(1) $H^i_{t+1} = A^i h^i_t (H^i_0+H^i_t)^{1-\gamma}[(H^1_0+H^1_t)(N^1_t/N^i_t)]^\gamma \equiv A^i h^i_t (H^i_0+H^i_t)(S^i_t)^\gamma$,

where h^i_t ($\in [0,1]$) is the fraction of production capacity parents invest in educating each child, H^1_0 and H^1_t are the endowed and attained human capital stocks of family-group 1 – the one with the highest earning capacity – and N^i_t is the number of parents in family type i. S^i_t is defined below.

Equation (1) aims to capture two types of interactions within and across families: a. Human capital formation over time may be achieved only if the older generation of parents invests in the knowledge of the succeeding generation of children; b. Knowledge attained by agents with the highest production capacity (family-type 1 members) has a spillover effect on all others (i>1). Human capital formation is thus perceived to be a social, as well as private, process.

The intergenerational interaction in human capital production is captured by the relationship between H^i_{t+1} and H^i_t in equation (1). The intra-generational interaction is defined by the term $(S^i_t)^\gamma$ in equation (1), where $S^i_t \equiv [(H^1_0+H^1_t)/(H^i_0+H^i_t)][N^1/N^i] \equiv E^i_t P^i_t$. The ratio E^i reflects the production capacity of agents in group 1 relative to i, P^i reflects their odds of encounter (P^i), and $\gamma<1$ is a constant interaction term.[5] This specification captures a "social" effect operating in the course of knowledge formation when agents with lower earning capacity (knowledge and skill) learn from those of superior capacity at school or in job training. The spillover effect generated by agents in the top group is modeled as 'external' to all other agents, assuming that agents are unable to take advantage of their ultimate position a priori.[6] For the leading family group, $S^i_t=1$, and thus equation (1) becomes:

(1') $H^1_{t+1} = A^1 h^1_t (H^1_0+H^1_t)$.

Motivating Forces and Preferences. The model ascribes a critical role to the family in the growth process, as the family forms intimate intergenerational links. The major motive inducing parents to invest in children is altruism, which in the OLG framework translates to psychic benefits parents derive from the well being, or income-generating capacity, of adult children. Our formulation recognizes also a complementary motive: old-age insurance, modeled as in EL (1991, 1998). Children are dependent on parents for nurture and investment in productive capacity, and old parents can benefit from such investments through informal care and companionship provided by adult children during their old-age dependency phase. This motive would be operational regardless of whether parents can also save for their old age needs, because it is based on the inherent productivity of the family partnership in creating human capital. We henceforth refer to this formulation as our **benchmark case.** While the model's basic behavioral implications can be derived when parents are driven solely by altruism, we base our analysis on the benchmark case, because it assures the existence of interior solutions for fertility and human capital investments, and thus for all our inequality measures, all along the development path (see section III.A). The pure altruism case is discussed in **Appendix C.**

A general specification of the utility function of agent i at period t is given by

(2) $U(C^i_{1,t}, C^i_{2,t+1}, C^i_{3,t+1}) = [1/(1\text{-}\sigma)][C^i_{1,t}{}^{1-\sigma} - 1] + \delta\, [1/(1\text{-}\sigma)]\{[C^i_{2,t+1}{}^{1-\sigma} - 1] + [C^i_{3,t+1}{}^{1-\sigma} - 1]\}$,

where δ is an intertemporal discount factor, and σ the inverse of the elasticity of substitution in consumption. In equation (2), $C^i_{1,t}$ denotes consumption opportunities for young adults:

(3) $C^i_{1,t} = (H^i_0+H^i_t)[1 - v^i n^i_t - \theta^i h^i_t n^i_t] - w^i_t H^i_t$.

The control variable, n^i_t represents the number of children per young adult, treated as a continuous and certain variable, so $N^i_{t+1} = N^i_t\, n^i_t = N^i_0 \Pi^t_{s=0} n^i_s$, where N^i_0 stands for the initial number of families of type i in the population, and N^i_{t+1} represents its equilibrium value. The parameters v^i and θ^i represent unit costs of rearing and educating each child, with θ^i also reflecting differences in educational financing costs across families because of capital market imperfections, which favor richer families. Old age consumption per parent in equation (2) is given by

(4) $C^i_{2,t+1} = n^i_t w^i_{t+1} H^i_{t+1}$.

Since we abstract initially from savings, old-age consumption depends strictly on material transfers from children.[7] We assume that young parents form implicit contracts with their children that are enforceable and time consistent (see EL, 1991). By this contract, the old parent receives from each adult child an amount of support that is proportional to the stock of human capital accumulated by the child, $w^i_{t+1}H^i_{t+1}$.[8] For simplicity, we treat the compensation rate, or financial rate of return to parents on human capital investments, w^i_{t+1}, as exogenously determined by accepted social norms, although we can also treat it as endogenously determined by parents, acting to maximize children's welfare, without affecting our main propositions (see section V and **Appendix B**). The last term in equation (2),

(5) $C^i_{3,t+1} \equiv B^i(n^i_t)^\beta(H^i_0+H^i_{t+1})^\alpha$, with α=1 and β >1,

specifies an altruism motive in the context of our overlapping-generations model. It reflects emotional rewards parents receive vicariously from children's achievements and companionship. As we shall see, to secure interior solutions in both fertility and educational investments, it is necessary to restrict α =1 and β >1. (Growth equilibrium cannot be sustained if α>1.) To ensure the concavity of equation (2) we must further restrict $\beta(1-\sigma) < 1$. Note that, if we set $w^i_{t+1} = 0$ (the "pure altruism" case), equation (5) yields the same growth-equilibrium steady-state solutions in a representative-family framework as the altruism function in Becker et al. (1990) or EL (1991).

B. Basic Solutions

The objective function (2) is maximized by choosing $\{n^i_t$, and h^i_t or $H^i_{t+1}\}$, subject to (1), (3)-(5), taking $\{H^i_t, H^1_t, N^i_t, N^1_t, w^i_t, w^i_{t+1}\}$ as given. By substituting the constraints into (2), the maximization with respect to n^i_t, and h^i_t, yields the following pair of first-order conditions:

(6) $[C^i_{2,t+1}/ C^i_{1,t}]^\sigma \geq \delta R^i_{n,t} \equiv \delta A^i\, w^i_{t+1}(S^i_t)^\gamma(1+\beta M^{i\,*}_t)/[\theta^i +(v^i/h^i_t)]$, for $h^i_t \geq 0$,

(7) $[C^i_{2,t+1}/ C^i_{1,t}]^\sigma \geq \delta R^i_{h,t} \equiv \delta A^i\, w^i_{t+1}(S^i_t)^\gamma(1+\alpha M^{i\,*}_t)/\theta^i$, for $n^i_t \geq 0$,

where $M^{i\,*}_t \equiv (C^i_{3,t+1}/ C^i_{2,t+1})^{1-\sigma}$; R^i_n and R^i_h are expected rates of return to investment in n and h, and $S^i_t \equiv [(H^1_0+H^1_t)/(H^i_0+H^i_t)]\, [N^1_t /N^i_t] \equiv E^i_t P^i_t$, for i>1, is the source of the spillover effect running from family 1 to i in the human capital production function (1).

Equations (6) and (7) verify that in order for interior solutions for both fertility and human capital investment per child to exist, i.e., for $R^i_{n,t}$ and $R^i_{h,t}$ to equalize over all development phases, we must restrict $\beta>\alpha$, and $\alpha = 1$. This latter condition guarantees that altruism and material incentives remain in balance all along the development path, i.e., regardless of whether H^i_t is constant or perpetually rising. It also implies that

(8) $1 = [B^i(n^i_t)^{\beta-1}/ w^i_{t+1}]^{1-\sigma}[(\beta-1) (\theta^i h^i_t/v^i) - 1]$,

by the equality $R^i_{n,t} = R^i_{ht}$, with $\alpha =1$.

An interesting feature of these optimality conditions concerning the "control variables" n_t^i and h_t^i, is their dependence on the distributions of two underlying parameters: ability, A^i, and the financial unit cost of investment in human capital, θ^i. Equations (6) and (7) indicate that these parameters, as well as any exogenous parameters not entering equation (8) exert opposite effects on the optimal values of n_t^i and h_t^i, indicating a "quantity-quality" tradeoff in decisions affecting children. Moreover, the equilibrium solutions for fertility and the fraction of full income spent on educating each child, $\theta^i h^i_t$, depend strictly on "investment efficiency", or the **ratio** of ability to the unit financing-cost of investment, $e^i \equiv A^i/\theta^i$, as can be shown by rewriting equations (6) and (7) as the optimality conditions for n_t^i and $\theta^i h^i_t$. The inference is that only two independent sets of **objective** heterogeneity parameters generally affect our control variables solutions: investment efficiencies and family endowments.

C. Inequality measures:

Our model has direct bearing on three commonly used income inequality measures, which are linked to our key endogenous state variables: H^i and N^i:

a. $E^i_t \equiv (H^1_0+H^1_t)/(H^i_0+H^i_t)$ is a **family-income inequality** index: the ratio of the (full) income of an individual family in family-group 1 to that of a corresponding family in family-group i. An inequality measure related to E^i_t is inequality in attained human capital stocks, H^1_t/H^i_t, which can be captured empirically by the standard deviation of schooling attainments.

b. $S^i_t \equiv [(H^1_0+H^1_t)/(H^i_0+H^i_t)][N^1_t/N^i_t] \equiv E^i_t P^i_t$ is our **income-group inequality** index – a product of relative family-income levels and group sizes in group 1 relative to i>1, which also serves as the source of knowledge spillover effects in equation (1). It reflects the share of aggregate income of the top income class, relative to lower classes (e.g., quintiles) of families. Note that $P^i_t \equiv N^1_t / N^i_t$ is a related distributional measure – an **income-group-size inequality** index – but it is not independent of S^i_t and E^i_t, since, by definition, $P^i_t \equiv S^i_t/E^i_t$.

c. The **Gini coefficient**, $G^i_t \equiv (S^i_t - P^i_t)/[(1+S^i_t)(1+P^i_t)]$, is a non-linear function of, and recoverable from, the equilibrium solutions of S^i_t and P^i_t. Specifically, G^i_t is increasing in S^i_t, but decreasing in P^i_t (for a diagrammatic derivation of G^i_t in the two-family case, see **Figure 1**).

Aside from income inequality, our model offers new insights concerning the distributions of our key control variables: fertility and educational investments per child (n^i and h^i), which we measure empirically simply by their respective relative variance across families.

III. DYNAMIC IMPLICATIONS

A. Equilibrium regimes

Equations (6) and (7) represent complex second-order simultaneous difference equations, and generally, no explicit solutions exist for the basic endogenous variables of the model, n^i_t, h^i_t and S^i_t. Since the second order optimality conditions are satisfied, implicit solutions exist and can be obtained via numerical simulations. The simulations indicate that two stable steady states exist, corresponding to different parameter values: stagnant (s) and perpetual growth (g) equilibrium. The transitional development phase connecting the two is supported by the same parameter set that sustains the perpetual growth regime. Several propositions follow:

Proposition 1. In a stable steady state, fertility rates of different family groups must converge, $n^1_t = n^i_t$, under both stagnant and growth equilibria.

The rationale is that in a stable steady state, the size distribution of income classes $P^i_t \equiv N^1_t / N^i_t$ must be constant; otherwise the economy would be dominated by a single family-type. In turn, this proposition restricts certain parameter values to be identical across families. Given our assumed uniformity of preference and technical production parameters {B, α (=1), β, γ, δ, and σ} across families, we can allow for heterogeneity in initial endowments, H^i_0, abilities, A^i, and unit financing costs, θ^i, but in this case other parameters are not free to vary. In particular, a sufficient condition for stable stagnant and growth equilibrium steady states to exist is that the shares of earnings spent on both raising children, v^i, and supporting old parents, w^i, must be identical across families.[9]

Stagnant equilibrium (SE) steady state. Here all control and state variables other than population sizes, including human capital levels and all inequality measures, are stagnant over time. Multiple stagnant equilibria exist, but not all are stable. A stable SE steady state exists, and is locally stable, if: a. the evolution path of H^i_{t+1} as a function of H^i_t intersects the 45% degree line from above, i.e., the slope $a^i_t(s) \equiv dH^i_{t+1}/dH^i_t$ is then less than 1; b. The families' fertility rates, evaluated at the SE, do not rise with family-group sizes, or $dn^i_t/dN^i_t \leq 0$, so that $P^i_t \equiv N^1_t/N^i_t$ converges to a steady state value, $P^i(s)$. Our simulations consistently yield a single stable stagnant equilibrium solution.

If the only source of heterogeneity across families in the economy are differences in ability, income endowments, and investment-financing costs – we henceforth refer to this as our **heterogeneity condition** – and assuming that our SE steady state is unique, we can show using equations (1) and (1'), subject to (6) and (7) and the **stagnancy condition** $H^i_{t+1}=H^i_t$, that our control variables exhibit the following properties:

(9) $n^1(s) = n^i(s)$; $h^1(s)\theta^1 = h^i(s)\theta^i$; and $a^1 \equiv A^1h^1 = a^i \equiv A^ih^iS^i(s)^\gamma$.

Using (9), the SE values of our income inequality measures can be solved explicitly:

(10) $E^i(s) = H^1_t/H^i_t = H^1_0/H^i_0$, all i.

(11) $S^i(s) \equiv E^i(s)\, P^i(s) = [(A^1/\theta^1)/(A^i/\theta^i)]^{(1/\gamma)} \equiv (e^1/e^i)^{(1/\gamma)}$

Proposition 2. Under our heterogeneity condition, in a stable stagnant-equilibrium steady state, the fraction of full income devoted to educational investment per child, $\theta^i h^i_t$, and the marginal rate of change of human capital formation, a^i, are equalized across all families. Also, relative human capital attainments and families' full income $E^i(s)$ equal their relative income-generating endowments (equation 10). Income-group inequality, $S^i(s)$, in contrast, depends exclusively on the relative "investment efficiencies" of family 1 relative to i (equation 11). The Gini coefficient depends on both relative family endowments and investment efficiencies.

The explicit solution for the family-income inequality ratio, $E^i(s)$, follows from imposing the stagnancy condition $H^i_{t+1}=H^i_t$ on equations (1) and (1'), which can be shown to require that $H^1(s)/H^i(s) = H^1_0/H^i_0 = E^i(s)$. The solution for the equilibrium income-group inequality ratio, $S^i(s) \equiv E^i(s)P^i(s)$, then becomes independent of the endowments ratio, and strictly a function of relative investment efficiencies. The income-group-size inequality ratio, $P^i(s) \equiv N^1_t/N^i_t$, is simply the ratio of the two.[10]

A subtle point about proposition 2 is that it holds strictly under our benchmark case, which allows for intergenerational transfers benefiting old parents. If altruism is the sole operating motive for parents (i.e., w=0), it can be shown that the only stable SE steady state requires a corner solution in human capital investments, or $h^i=0$, for all family groups (see **Appendix C**). There are then no spillover effects that link family groups 1 and i>1. The family-income inequality is then automatically defined by the endowment ratio, $E^i(s)=H^1_0/H^i_0$, as is, in fact, the endogenous outcome in our benchmark case, but our income-group inequality index and the Gini coefficient are indeterminable. This corner solution can be avoided as soon as we introduce any material rate of return on educational investments for parents, w>0, which justifies at least **some** investment in the quality of children, and it is partly for this reason that we base our analysis on the benchmark case. The pure altruism case can be solved at the growth equilibrium steady state, however, and its behavioral implications are consistent with those we derive for our benchmark case.

Growth equilibrium (GE) steady state. Here the state variables H^i_t and possibly N^i_t grow without bound while the long-run values of $n^i(g)$, $h^i(g)$, and all inequality measures converge to fixed levels at the GE steady state (g). Local stability is assured if the slope of the evolution path of H^i_{t+1} as a function of H^i_t, $a^i_t(g) \equiv dH^i_{t+1}/dH^i_t$, exceeds 1 and the fertility rates cannot be increasing with group sizes, or $dn^i_t/dN^i_t \leq 0$.[11] The latter condition guarantees N^1_t/N^i_t to converge to a steady state value, $P^i(g)$.

Proposition 3. Suppose the model's parameters support a stable GE for all agents, but different family groups initially experience different marginal growth rates of human capital. The long-run growth rate of human capital in family group 1 converges on its steady-state level, $\lim_{t\to\infty} (a^1_t) \equiv a^1(g) = A^1h^1(g)$, where $a^1_t \equiv d(H^1_{t+1})/d(H^1_t)$. While initially, the marginal growth rates of all other groups, $(a^i_t) \equiv d(H^i_{t+1})/d(H^i_t)$, may be below or above that of group 1, they will rise or fall over time as relative income inequality expands or contracts, but will ultimately converge to the **same** steady-state value as that of group 1; i.e., $a^1(g) = A^1h^1 = \lim_{t\to\infty} (a^i_t) \equiv a^i(g) = A^ih^i\ S^i(g)^\gamma$, as our group-income inequality measure converges on its equilibrium value $S^i(g) = [A^1h^1(g)/A^ih^i(g)]^{(1/\gamma)}$.[12] The proof follows from the spillover effects running from agent type 1 to agent type i: Given the existence of a stable growth equilibrium solution, suppose that the steady-state growth rate of agent 1's income is higher than that of agent i, or $a^1(g)>a^i(g)$. In this case the inter-group earning inequality measures, both E^i_t and S^i_t, would be rising. Given the assumed role of the social-interaction, or earnings inequality term (S^i_t), however, this situation cannot persist: While the income growth rate of family 1, a^1_t, is independent of S^i_t, the rising earnings inequality would provide an impetus for agent i>1 to raise the rate of investment in human capital, thus a^i_t. This trend would persist until a^i_t became equal to $a^i(g)$ and S^i_t converged on the constant $S^i(g)$. The converse would take place if $a^1(g)<a^i(g)$.

Proposition 4. Given our heterogeneity condition, at the GE steady state, optimal fertility (n^i) and human capital investment cost per child ($\theta^i h^i$) ultimately become identical for all agents, and our income-group inequality index becomes exclusively dependent on the relative "investment efficiencies" of family 1 relative to i in the GE steady state, as is the case at the stable SE steady state.[13] Equation (11) thus holds for both state s and state g:

(12) $S^i(g) = S^i(s) = [(A^1/\theta^1)/(A^i/\theta^i)]^{(1/\gamma)} \equiv (e^1/e^i)^{(1/\gamma)}$, all i.

In contrast, the family-income inequality value, $E^i(g)$, is not **uniquely** determined at the growth steady state. This is essentially because in the stagnant steady state, $E^i(s)$ is determined strictly by the ratio of income-producing endowments across families by proposition 2. But the relative influence of these family-specific endowments vanishes under persistent growth of human capital attainments. The comparative levels of E^i in the stagnant- vs. growth-equilibrium steady states thus depend on the various factors determining the **evolution** of E^i_t along the transitional development path linking the two steady states, and the same holds for the income-group-size inequality index, P^i. These factors are elaborated on in the following sections.[14]

B. Comparative dynamic implications

Except for the case of log utility, no explicit analytical solutions generally exist for our control variables. We therefore resort to numerical simulations for insights about the roles of specific parameters over the development process. In the following analysis we maintain our heterogeneity condition. Without loss of generality, we also recognize just 2 family types (i=1, 2): the leading, high-income group 1, and all others. This dichotomy serves to emphasize implicitly the role of higher education in influencing the economy's growth and inequality dynamics.

1. ***Comparative dynamics under stagnant equilibria.*** By equation (12), the income-group inequality measure, $S^i(s)$, rises with the relative disparity in investment efficiencies (e^1/e^i) and falls with the size of the spillover effect (γ). Family-income inequality, $E^i(s)$, however, is dictated only by the relative family-specific endowments ($H^1{}_0/H^i{}_0$), while the **income-group-size** inequality is generally inversely related to $E^i(s)$, as $P^i(s) = S^i(s)/E^i(s)$. Changes in the assumed common values of all other basic parameters can affect optimal fertility, $n^i(s)$, and human capital investment, $h^i(s)$, but they cannot affect any of our income inequality measures. For example, higher altruistic preferences (B^1=B^i) reduce optimal h_i and raise n_i in all families. Higher unit costs of raising children (v^1=v^i) or old-age support rate (w^1=w^i), in contrast, yield just the opposite effects (see **Table 1 part 1**).

It is noteworthy that in a stagnant equilibrium, human capital attainments and income **levels**, while constant over time, do change with changes in basic parameters. For example, any skill-biased technological change or improved capital market financing opportunities that raise the efficiency level of the leading family, e^1, alone, or proportionally for all families, lead to higher human capital investment rates, and thus income levels, for **all** families. In contrast, an increase in the efficiency level e^i of family i>1 alone results in no change in income levels. In both of these cases, our family inequality measure, E^i, remains unchanged, as established by proposition 2.

What is also noteworthy, however, is that the income-group inequality measure, $S^i(s)$, and the Gini coefficient, **do** change in these cases. When A^1 alone rises, $S^i(s)$ increases because a higher A^1 raises **relative** fertility in family type 1, and thus the relative equilibrium group size, $P^i(s)$. Fertility levels fall in this case, while educational investment rates rise due to a quality-quantity tradeoff. The impact on the Gini coefficient, $G^i(s)$ is generally **ambiguous**, since it is an increasing function of $S^i(s)$, but a decreasing function of $P^i(s)$. In our simulations, however, the latter effect dominates, so $G^i(s)$ falls. Thus we see that parameter changes affecting income levels under a stagnant equilibrium may change different inequality measures in different directions, or not at all.

2. ***Takeoff triggers.*** Whether the economy is in stagnant or growth equilibrium depends on the magnitude of the basic parameters of the model. For example, an increase in A^i, v^i, w^i (treated as exogenous) or a decrease in θ^i, generally raises the expected rates of returns to the quantity and quality of children, R^i_{nt} and R^i_{ht}, relative to the marginal rate of substitution in consumption (see equations (6) and (7)). But whether a takeoff occurs depends on the way these changes affect the marginal human capital growth rate. Our simulations show that any sufficient change in these growth-enhancing parameters for both families, or even for the first family alone, can generate a takeoff for all families. These simulations produce another important feature of economic development – the "demographic transition"– whereby fertility levels generally decline, while investments in human capital rise, under any parameter shock that produces a takeoff from stagnant to growth equilibrium (see **Table 1 part 2**).

In the special logarithmic utility case, we can show **analytically,** using the explicit solutions for h^i in the SE or GE steady states (see footnotes 10 and 14), that a sufficient increase in A^i/θ^i or v^i, raising the marginal growth rate of H^i ($a^i = A^i h^i$) above unity, can also trigger a transition.

3. ***Comparative dynamics under growth equilibria.*** By equation (12), $S^i(g)$ is an increasing function of relative investment efficiency (e^1/e^i) and a decreasing function of the spillover coefficient γ, as was the case under our SE steady state. Similarly, here the long-term growth **rate** of all family incomes rises with parameter shocks that raise the leading family's investment efficiency, e^1, regardless of whether such shocks also raise investment efficiency in other families. This is because a skill-biased technical advance favoring the leading family raises its growth rate, and thus the growth rates in all other family groups as well. Changes in e^i ($i>1$) **alone**, in contrast, do not affect the economy's growth rate.

What about movements in income inequality? An exogenous rise in relative investment efficiency of family 1, e^1/e^2, because of a **skill-biased** technological advance, which by proposition 4 unambiguously raises $S^i(g) \equiv E^i(g)P^i(g)$, must raise either $E^i(g)$, or $P^i(g)$, or both. Whether $E^i(g)$ necessarily rises depends on whether the upward shift in A^1 raises the relative growth rate of human capital in family 1, $A^1h^1_t$, above that in family i over the **entire** adjustment period leading to a new steady state. This is actually the case if the utility function is logarithmic (σ=1), since in this case, changes in A^1 or γ have no effect on fertility, n^i, or human capital investments, h^i, and hence on relative group sizes, $P^i(g)$ (see footnote 14). The theoretical effect of a rise in e^1/e^2 on the Gini coefficient is generally ambiguous if both $S^i(g)$ and $P^i(g)$ increase as a result, since $G^i(g)$ rises with the former and falls with the latter. However, in all our simulations in part 3, the Gini coefficient moves in tandem with all other income inequality measures. Also, any exogenous changes that affect investment efficiency equally in all families do not affect any of our income inequality measures. Improved investment financing opportunities favoring lower-income families (i>1), in contrast, unambiguously lowers income-group inequality, $S^i(g)$ and, by our simulations, $E^i(g)$ and $G^i(g)$ as well.

Changes in common parameters other than (e^1/e^i) and γ, such as B, v, or w (when treated as exogenous) that maintain $S^i(g)$ constant, leave all other inequality measures constant by proposition 4. Such parameter changes can affect, however, the long-term income growth rate for all families through their impact on fertility and educational investments. For example a rise in the costs of bearing and rearing children lowers fertility and raises optimal educational investments, and thus the income growth rate in all families. The association between income-growth rate and income inequality thus depends largely on the parameter changes responsible for their co-movements: a skill-biased technical change favoring the higher-income group raises the growth rate for all families, but also all income inequality measures, generating a negative tradeoff between long-term income growth and income equality. Changes in other parameters, may leave either the uniform long-term growth rate or income inequality unchanged, as our simulations in **Table 1, part 3** illustrate.

IV. INEQUALITY PATHS OVER THE TRANSITIONAL DEVELOPMENT PHASE

The preceding analysis implies that the levels and distributions of fertility, human capital investment, and income along the development phase depend on the shocks that produce a takeoff from stagnant to growth equilibria. It is plausible to expect that a skill-biased technological progress will first reach the group with the highest ability, or will affect it proportionally more than the other groups. However, this group may not necessarily be the one with the highest income-generating endowment at the initial (stagnant) steady state – this depends on the correlation between ability and initial endowments across family groups. A similar argument applies if the parameter shock is a decline in investment-financing costs, since the ablest family need not be the richest. To contain the possible scenarios we consider three cases that are neither exhaustive nor necessarily of equal empirical plausibility:

a. Synchronous and uniform shocks: Shocks that affect family-specific investment efficiencies (e), and any other takeoff-triggering parameters (w, v) simultaneously and by the same proportion.

b. Shocks favorable to family 1: Such a shock affects family 1 either proportionally more than other families, or ahead of other families. Assume there is a positive correlation between income-generating endowments and efficiency at human capital investments, or COV (H^i_0, A^i/θ^i) > 0, all i, so that the higher-income family 1 is a leading family in both the stagnant and growth steady states. A family-1 friendly shock would be one that either raises the relative investment efficiency of family 1 over i, (e^1/e^i), or raises both (not necessarily equi-proportionally), but reaches family 1 ahead of i.

c. Shocks favorable to family i: A shock affects family i either proportionally more than family 1 or ahead of it. For example, suppose that ability and social privilege are negatively correlated across families, so that A^i exceeds A^1, but H^1_0 and $1/\theta^1$ are significantly larger than H^i_0 and $1/\theta^i$. In this case, family 1 is the leading family not because of superior ability, but because of superior social status or political power leading to larger income-generating endowments and lower investment financing costs. A family-i friendly shock could occur, e.g., when a reduction in capital market segmentation lowers the financing cost of education to all families, but especially to family i, thus lowering (e^1/e^i). Alternatively, such a shock may first affect members of family i, who could not initially finance private schooling.

A. Paths of Income Inequality Measures

'Case a' can be dubbed as "the separating equilibrium path"; we can show that

(13) $S^i(s) = S^i(g) = S^i_t = [(A^1/\theta^1)/(A^i/\theta^i)]^{(1/\gamma)}$, and $E^i(s) = E^i(g) = E^i_t = H^1_0/H^i_o$.

Put differently, our basic earnings inequality measures remain the same in both stagnant and growth equilibria, and all along the transition path, charting a horizontal inequality-income path. This is essentially because an equi-proportional increase in a takeoff-triggering parameter affects all optimality conditions symmetrically, leaving constant the spillover effect tying them. Since the Gini coefficient is a function of S^i and P^i, it also shows a flat transition path in this case.

In case b, if a takeoff-triggering technological advance reaches family 1 ahead of other families, the transitional development phase would be characterized by the co-existence of family groups in different stages of transition: Family 1 would become a "growth family", while other families remain "stagnant families". But the persistent growth in family 1's income will ultimately produce an economic takeoff for all families, and by propositions 3, all will ultimately grow at an equal rate. The time paths of our three income inequality measures (S^i, E^i, and G^i) will exhibit an **inverted-U shape**, consistent with the "Kuznets hypothesis" (see Figure 2). Whether the income inequality **level** at the growth equilibrium is higher or lower depends on whether the shock is ultimately uniform (equi-proportional) or non-uniform across families. A uniform shock will not affect the income-group inequality level, S^i, by equation (12), but will ultimately raise the family income inequality level, E^i, in a GE steady state. But a shock that is ultimately favorable to family i (because of progressive subsidization of higher education) may lower the inequality level at the GE steady state relative to the SE steady state. If a takeoff is triggered by a shock that raises A^i/θ^i simultaneously for all families, but proportionally more for family 1, the dynamic evolution of inequality would then be **monotonically increasing** over the development phase for all our three income inequality measures.

In case c the takeoff-triggering shocks will produce transition paths just opposite to those in case b for our three income-inequality measures. The time paths of all measures will assume a **U-shape** if family i experiences a takeoff shock ahead of family 1. Whether the inequality **level** rises or falls at the GE, relative to the SE, steady state depends on whether the non-synchronized shock ultimately becomes equi-proportional, in which case the income-group inequality, S^i, is constant but family-income inequality, E^i, falls, or if investment efficiency rises proportionally more for family i, in which case the income inequality level is **monotonously decreasing** (see Figure 3).[15]

Our simulations in parts b and c also reveal opposite associations between income **growth** rate and income **inequality** over the transitional development phase. In case b, income inequality and the per-capita income growth rate are positively associated, as Forbes (2000) finds, while in case c they are negatively associated at an early stage of the transition, while becoming positively associated at a later stage of development, which is what Barro (2000) finds. Our analysis thus shows that the dynamic association between income growth and income inequality can vary by the specific takeoff triggers, or at different stages of the transitional development phase.

B. Paths of Inequality in Fertility and Human Capital Investment

Since by propositions 1, 2, and 4 optimal fertility levels and the shares of income spent on educating each child are the same for all families at both the SE and the GE steady states, while they are different across families during the development phase connecting the two, except under the "separating equilibrium" case a, we have:

Proposition 5. Except in the "separating equilibrium" case, the transitional development path of inequality in completed fertility n will exhibit an **inverted-U shape**, but will tend toward equality in the two steady states framing the transitional phase. The same applies to the behavior of the transitional development path of the human capital investment, θh. The fertility-inequality time path will assume an inverted-U shape in all cases involving non-uniform or non-synchronous shocks. The inequality in the financial rate of human capital investment, θh, will also assume an inverted-U-shaped time path since optimal investment levels are identical for all families at both steady states under our heterogeneity condition. In the separating equilibrium case, the inequalities in n and θh assume a flat time path.

Furthermore, regardless of the type of shock generating the transitional development, all our simulations of this phase exhibit a "demographic transition" whereby the human capital investment rate continuously rises, while the fertility level generally falls over the development phase.

Note that when income inequality measures assume an inverted-U shape, as in case b of the preceding section, family 1's fertility level is lower than that in family i over the transitional development phase (see Figure 2b). This association between fertility rankings and income inequality is consistent with the findings in Kremer and Chen (2002) and La Croix and Doepke (2003). In contrast, when income inequality assumes a U shape, as in case c, family 1's fertility exceeds that of family i during the transitional phase. In our general equilibrium framework, however, such associations do not indicate causality, nor can they be persistent, since fertility differences must vanish in any steady state.

V. Model Extensions

Although our benchmark model abstracts from capital markets, we can incorporate in the model savings as "home production", where the return on savings is a function of the human capital attainments of the old parents, and the yield is subject to diminishing returns (see Appendix A). This is a natural assumption in the context of our closed-economy framework since both the rate of return on savings and the savings rate thus become endogenous variables. The extension allows us to recognize inequalities in labor earnings as well as in total income, incorporating property income as well. The two components of total income are shown to be operationally interchangeable in any steady state, however, since the optimal savings **rates** are shown to equalize across families in any stable steady state. Indeed, we can show that all of the propositions of previous sections pertaining to the behavior of our income inequality measures are maintained in this broader model, including the comparative dynamics predictions of Table 1. Also, the transitional development paths of our income inequality measures are found to be the same as in those derived for our benchmark model.

Over the transitional development phase, however, savings rate may differ across families. For example, when a takeoff occurs as a result of a skilled-bias technological advance reaching initially the higher income family group 1, our income inequality measures assume an inverted-U shape, and the savings rate of the higher-income family 1 initially falls below that of the other (stagnant) families. In the following stage, however, as family groups i>1 experience a takeoff because of the social-interaction effects coming from family-group 1, their savings rates fall below that of family 1. The aggregate savings rate then starts rising while income inequality is increasing. The resulting positive association between income inequality and the aggregate savings rate, however, is again not an indication of causality in our analysis (as in Keynes, 1920, or Kaldor, 1957), since it eventually reverses when income inequality starts falling, and it vanishes as all savings rates converge on equality.

Another extension involves treating the old-age support rate, w, or the financial rate of return to parents on their investments in the human capital of children, as an endogenous variable, rather than a constant. In **Appendix B**, w is treated as a choice variable for parents acting as agents of their children. Invoking our heterogeneity condition, we show that w will be **equalized** across family groups at both the stagnant and growth equilibria. We also find that optimal w falls within all family groups following takeoff-parameter shocks that cause the economy to takeoff from a stagnant to a growth equilibrium steady state. Moreover, comparative dynamic simulations of the case where w is treated as endogenous yield the same qualitative results for our inequality measures as those derived in Table 1.

VI. EMPIRICAL TESTS

We test empirically three theoretical implications of the model against an international panel data set:

a. By proposition 5, regardless of the behavior of income inequality over the transitional development phase, and independently of the specific takeoff-trigger, we expect **fertility inequality** to display an **inverted-U shape with flat tails** over the development phase, since all families' fertility levels converge on equality at the stagnant and growth steady states, while diverging during the transitional phase (except in the "separating equilibrium" case). To capture the development phase, we use real per-capita GDP, RGDP, which grows monotonically over this phase.

b. We similarly expect inequality in **human capital investments**, h^1/h^i, to display an inverted-U-shaped path over the transitional development phase. No reliable data on investment **flows** are available internationally. We resort instead to data on inequality in human capital **attainments** over the development path, H^i_t/ H^i_t. The time path of this measure, however, is expected to mimic, and converge on, that of family-income inequality, E^i, at the GE steady state.[16]

c. What we expect, then, is: regardless of the shape of the family-income inequality index, $E^i_t \equiv (H^1_0+H^1_t)/(H^i_0+H^i_t)$, the shape of the educational attainments path, H^i_t/ H^i_t would be **consistent** with that of E^i_t over the transition phase. By our analysis in section IV.A, this consistency should apply to all income-inequality measures: E^i, S^i, and G^i. No empirical data are available to approximate the family-income inequality measure. We approximate our income-group inequality, S^i, by an inter-quintile inequality measure, and G^i by the actually measured Gini coefficient.

B. Data and Variables Used

a. Fertility. Distributional data on the number of surviving children per woman, or TFR, are available from the World Fertility Surveys (WFS) and their successor - the Demographic and Health Surveys (DHS). The sample we construct is based on 72 surveys of 29 countries in various years between 1974 and 2000. From the individual-level micro data in each survey, we derive the distribution of surviving children of women age 40 and over. We restrict the data to women age 40 and over, to insure that the measures relate to women who completed childbearing. We then use the **standard deviation** of the distribution of surviving children per woman age 40 and over [SD-FERT] as our fertility inequality measure. But since the standard deviation is subject to a secular drift, we also enter the average **level** of TFR as a control variable, [AV-FERT].

b. Human capital. The source of educational attainments data (schooling years in the population age 15 and over) is Barro and Lee (2000).[17] We use the average number of years of schooling in the population age 15 and over as a proxy of our human capital **level**. As a proxy measure of inequality in educational attainments we use the standard deviation of the distribution of schooling years [SD-SCHYR] in the population age 15 and over. As in the fertility inequality regressions, we also add the mean schooling years as a control variable [AV-SCYR].

c. Income inequality. The income inequality data are from Dollar and Kraay (2001). These data cover 86 countries over the period 1950-1998. As empirical counterparts for our relative income inequality measures, we use two variables: the Gini coefficient [GINI], and the share of total income received by the top relative to the bottom quintile of families in the population [QUINT]. To be consistent with our model, we use only those observations that are calculated exclusively from **household income** data, excluding observations from personal income, personal expenditure or household expenditure data.

d. Regressors. Our basic regressor in all equations is the real per capita GDP level [RGDP], as reported in Heston, Summers, and Aten [HSA] (2001). This variable is entered to account for the economy's level of development, which, in turn, enables us to infer the behavior of our inequality measures over the entire development path in all countries in our samples. For robustness check, we also enter the time trend itself as a control variable. We use the government share of GDP [G] to account for the role of government spending in affecting our distributional variables. This variable is also taken from HSA. Summary statistics for all variables are reported in Appendix D.

C. Regression models

Our basic specification is an OLS, linear regression in which all variables are entered in natural form, but RGDP is entered in **cubic** form. This is for an important reason: we predict a **flattening** of the inequality – income path as the economy converges on steady-state growth equilibrium. Using an OLS estimation method is consistent with our model, since we expect the relation between our inequality measures and the level of development to be associative, not causal.

To examine the robustness of our results, however, we also run several modifications. The first employs simple OLS, the second uses an OLS, fixed-effects regression specifications, where we allow for varying constant terms for each country (models 2-4), and for each year as well (model 5). The country-specific fixed-effects regressions capture "within-countries" variability in the regressors, whereas the year-specific fixed-effects capture "within-calendar-year" variability. In models 3-5, we also test the effect of the government's share of GDP, G. In model 4, we include a time trend variable T to account for possible missing trended controls.

In the fertility regressions of Table 2, we employ country-specific **random-effects,** instead of fixed-effects, models to increase the regressions' degrees of freedom, because the number of observations per country is small (2.6 per country). The results from the fixed-effects specification are similar qualitatively, but the regression coefficients have larger standard errors.

To test for the possibility of serially correlated errors, we have applied an AR(1) serial correlation test to model 3 of each table. The Cochran-Orcutt test rejects the hypothesis in all cases.

D. Results

The fertility results are reported in Table 2. Model 1 estimates an inverted-U-shaped association between fertility inequality and real income, with inequality peaking at an RGDP level of $3,538 (67% of our sample's observations lie below this real GDP level and 33% above it). This estimated association is depicted in **Figure 4a**. This shape of the fertility-inequality path remains virtually the same when we apply a random-effects specification – with or without calendar-year dummy variables. As for the effect of other regressors, the standard deviation of the fertility distribution is monotonically related to the distribution's mean, as one would expect, since the standard deviation of any distribution is monotonically related to the distribution's mean. The share of government spending in GDP - a proxy for the average income tax - generally has an adverse, but insignificant effect on fertility inequality. The time trend variable, which we introduce as an additional correction for missing trended factors, is inversely related to fertility inequality.

Table 3 reports the results concerning inequality in educational attainments. Model 1 indicates an inverted-U-shaped association between educational attainment inequality and income, with the peak inequality level reached at RGDP=$4,712 (56% of our observations have income levels below this critical level and 44% above it.) This association is depicted in **Figure 4b**. The results based on models 2-5 show a similar pattern. Mean schooling expectedly raises the standard deviation of schooling.

Table 4 reports the results concerning the dynamic shape of the income inequality path, measured by the Gini coefficient. Model 1 indicates an inverted-U-shaped income-inequality association with income level, with the peak inequality reached at RGDP=$3,879 (16% of our sample's observations have income levels below this RGDP level and 84% above it). This relationship is also depicted in **Figure 4c**. All other model specifications produce a similar association. Note that although the income inequality at the highest income levels is lower than at lower ones (also see Appendix D), we cannot infer from this evidence if income inequality is higher or lower at the growth- v. stagnant steady states, since all countries in our sample may already be moving along their transitional development path toward growth equilibrium. The regressions with QUINT as the alternative dependent variable in Table 5 show very similar qualitative and quantitative results.

Tables 3-5 have special significance from our model's perspective: since the estimated associations they reflect between educational attainments and income level, and between our income inequality measures and income level take on an inverted-U shape, the results militate in favor of the Kuznets hypothesis. Note, however, that these results cannot be taken to support the Kuznets hypothesis as a general "law": our analysis indicates that the observed association can be affected by the specific **composition** of countries in our sample, in terms of the development stage they have achieved, as well as by the specific takeoff triggers operating in different countries.

Our results concerning the dynamic behavior of income inequality can be compared to those of Deininger and Squire [DS] (1998), who have derived a flat income-inequality – income-level curve using a fixed-effects regression specification. Although they use the same data and a similar regression model, they enter GDP via two variables, GDP and 1/GDP, while we use a cubic specification of GDP. The DS specification is similar to a quadratic specification, in that it does not allow for a tendency of the inequality measures to converge on a constant, steady state value, and thus flatten out as per-capita income reaches more advanced levels. Indeed, when we use the DS specification, we replicate their results. However, when we use our cubic GDP specification, we obtain an inverted-U-shaped income inequality – income curve for both of our Gini and inter-quintile income inequality measures.

VII. CONCLUDING REMARKS

The theoretical model we develop has a number of important limitations. First, we do not consider a market for physical capital. However, our basic propositions concerning earnings inequality apply to income inequality as well when we model savings as home production, where human capital also affects property income (see Appendix A). Second, the model is developed within a closed economy framework that does not allow for population migration. Third, space and data limitations do not allow us to consider the role of government's distributional fiscal policies. To partly deflect these omissions, in testing some of our hypotheses, we use fixed- and random-effects regressions to account for missing country-specific policy variables and common year effects for all countries.

These limitations notwithstanding, our model offers a dynamic extension of static models of income inequality based on the human capital approach. The model emphasizes the link between income inequality and inequalities in fertility and educational attainments. It identifies basic factors that explain the dynamic behavior of earnings and income inequalities over the development process. These include key variables identified in Becker's Woytinski lecture (1967): relative distributions of abilities, financing costs, or income-producing endowments, and the way these are correlated across families. It also identifies the set of factors that can trigger a transition from a low level of development into regimes of self-sustaining and persistent growth.

The most distinct empirical propositions we develop and partly test concern the dynamic behavior of relative inequalities in fertility and educational attainments, as well as in family earnings or income, over the process of development. Regardless of the dynamic paths assumed by any of our income inequality measures, we expect the relative inequalities in both fertility and educational investments across families to exhibit an inverted-U shape over the transitional development phase.

We also expect the dynamic pattern of inequality in educational attainments to mirror that of our family-income inequality index over the transitional development phase.

Concerning the dynamic correspondence between income growth and income inequality, our model offers several insights. First, the empirical association reflects the impact of underlying parameter changes that trigger movements in both income level or growth rate and income inequality, rather than any deterministic causal relation. We show that the observed association can go in similar or opposite directions depending on the specific parameter change generating it. Second, the association depends on the phase of development: whether the economy is in a stagnant-equilibrium, a growth-equilibrium steady state, or in a transitional development phase.

Third, the association partly depends on the inequality measure used. Theoretically, we distinguish three measures: family-income inequality (E^i_t), income-group (inter-quintile) inequality (S^i_t=E^i_t P^i_t), which also depends on a related income-group-size inequality measure (P^i_t), and the Gini coefficient (G^i_t), which is an increasing function of S^i_t but a decreasing function of P^i_t. Under our assumed homogeneity of preferences, we derive closed-from solutions for income-group inequality under both stagnant and growth steady states, $S^i(s)$ and $S^i(g)$, shown to be determined by relative investment efficiencies, and for family-income inequality under a stagnant steady state, $E^i(s)$, which is found to be strictly a function of unequal family-specific wealth endowments. No deterministic solutions are obtainable for the latter in a growth steady state, $E^i(g)$, and hence for the equilibrium values of $P^i(g)$, or for both $G^i(s)$ and $G^i(g)$. Since it is the Gini coefficient that is most often used empirically to capture income inequality, our analysis offers one important reason why findings concerning this measure may vary substantially in different studies: This is because of the **separate role** fertility differentials and income-group sizes play in determining the different income inequality measures. Indeed, the three income-inequality measures are shown to respond differently to parameter shocks at a low-income, stagnant steady state, although they generally move in tandem over the takeoff period leading to a self-sustaining growth-equilibrium.

Our analysis offers related inferences concerning the Kuznets hypothesis. We show that its validity depends on the way different families are affected by takeoff-triggering incentives. We can thus rationalize alternative shapes of income inequality paths over the transitional development phase, depending on the way technological shocks reach and affect families of different ability or financing opportunities. An inverted-U-shaped family-income inequality with inequality falling at advanced v. initial development phases can come about as a result of reduced capital market segmentation, first taken advantage of by more knowledgeable, higher-income families, but which lowers especially the financing-cost disadvantage of lower-income families.

The empirical results we obtain support our discriminating implications concerning the fertility-inequality time path. Although we lack a good empirical counterpart for human capital **investments**, we do have international panel data concerning **educational attainments**. Our model suggests that the behavior of educational attainments should mirror that of our family-income and the income-group inequality measures. The two measures of income distributions we analyze empirically (the inter-quintile distribution and the Gini coefficient), however, exhibit an inverted-U shape with a flattening upper tail, similar to the shape we estimate for the relative variance in educational attainments over the transition. Our estimated Kuznets-like shape of both educational inequalities and income inequalities may not be general, as we argue theoretically, but the similarity in the dynamic patterns of educational attainments and income is consistent with our model. The evidence developed in this paper thus lends support to the human capital approach to income distribution, as well as to the role of family choices, in explaining the dynamic behavior of both income growth and income distribution.

Appendix

A. We introduce a simple model of savings, treated as an input in "home production" of non-wage income at old age. Total savings is defined by $K_t \equiv (H^i_0+H^i_t)s^i_t$, where s^i_t is the fraction of productive capacity saved at adulthood, and K_t is assumed to fully depreciates within one generation. Income from savings is generated when old parents combine their accumulated assets, K_t, with their human capital inputs via the production function, $F= D(H^i_0+H^i_t)^{1-\kappa}[(H^i_0+H^i_t)s^i_t]^\kappa$, $0<\kappa<1$. This simplifying production function enables us to avoid modeling a distinct capital market, while capturing the idea that the equilibrium return from capital in a closed economy is subject to diminishing returns.

The consumption flows at adulthood and old age are now given by

(3′) $C^i_{1,t} = (H^i_0+H^i_t)[1 - v^i n^i_t - \theta^i h^i_t n^i_t - s^i_t] - w^i_t H^i_t,$

(4′) $C^i_{2,t+1} = n^i_t w^i_{t+1} H^i_{t+1} + D(H^i_0+H^i_t)^{1-\kappa}[(H^i_0+H^i_t)s^i_t]^\kappa.$

Income from savings is generated by $F= D(H^i_0+H^i_t)^{1-\kappa}[(H^i_0+H^i_t)s^i_t]^\kappa$, $0<\kappa<1$, in which old parents convert accumulated assets that fully depreciate within one generation to old-age consumption, and their human capital attainments play a productive role. This simplifying production function enables us to avoid modeling a distinct capital market, while capturing the idea that the equilibrium return from capital in a closed economy is subject to diminishing returns.

We can now distinguish income inequality from earnings inequality. The measures of total income inequality – by which we mean the pooled income of a family head: earnings as well as property income from savings – can be defined parallel to our earnings-inequality measures in section II.C, so that TS^i_t, e.g., corresponds to inequality in the **total** income (formally this means the wage income of adult parents plus the non-wage income of old parents) of group 1 relative to group i, and the same holds for family income inequality and the Gini coefficient:

$$TS^i_t \equiv [N^1_t (H^1_0+H^1_t) + N^1_{t-1} D(H^1_0+H^1_{t-1})(s^1_{t-1})^\kappa]/ [N^i_t (H^i_0+H^i_t) + N^i_{t-1} D(H^i_0+H^i_{t-1})(s^i_{t-1})^\kappa],$$

$$TE^i_t \equiv TS^i_t / TP^i_t;\ \ TP^i_t \equiv [(N^1_t + N^1_{t-1})/ (N^i_t + N^i_{t-1})], \text{ and}$$

$$TG^i_t \equiv [TS^i_t - (N^1_t + N^1_{t-1})/(N^i_t + N^i_{t-1})] /(1+ TS^i_t)/[1+(N^1_t + N^1_{t-1})/(N^i_t + N^i_{t-1})].$$

Under our heterogeneity condition, we can show that optimal savings (s^i) as well as fertility (n^i) and the cost-adjusted human capital investments ($\theta^i h^i$) are identical for all family groups, since the first-order optimality conditions indicate that these control variables will assume identical values in all family groups. Our total income inequality measures are therefore identical to the corresponding earnings-inequality measure at both the stagnant- and growth-equilibrium steady states. Moreover, we can show that the earnings, and hence total income inequalities in this extended model are in fact the same as those in the benchmark model without savings, given by equations (10), (11) and (12).

We can easily demonstrate that **all** of the propositions in sections III and IV are maintained in this extended model, as are the qualitative results of the comparative dynamics reported in Table 1 for both the SE and GE steady states. The time paths of the inequality measures considered in section IV are also shown to take the same pattern as in the model without savings.

Alternatively, we define inequality measures for non-wage income separately as follows:

$$SS^i_t \equiv [N^1_{t-1} D(H^1_0+H^1_{t-1})(s^1_{t-1})^\kappa] / [N^i_{t-1} D(H^i_0+H^i_{t-1})(s^i_{t-1})^\kappa],$$

$$SE^i_t = SS^i_t N^i_{t-1}/N^1_{t-1}, \text{ and}$$

$$SG^i_t \equiv [SS^i_t - N^1_{t-1}/N^i_{t-1}] /(1+ SS^i_t)/[1+ N^1_{t-1}/N^i_{t-1}].$$

We can again show that these alternative income inequality measures exhibit the same comparative dynamic implications, and the same dynamic paths over the transitional development period, as those analyzed for our earning inequality measures in Table 1 and in section IV. The reason for these results concerning the extended income inequality measures defined above is that, by our model, inequalities in both non-wage income and in labor earnings depend only on the relative human capital levels of the two families.

What would be the effect of changes in D or κ on our income inequality measures? As long as these changes are common to all families, they will affect only the **composition** of family income, but not the total income inequality measures, as our simulations of the extended model confirm.

B. In this appendix, we treat the old-age support rate, w, which is also the material rate of return to parents from investments in their kids' human capital, as an endogenous variable, rather than an exogenous constant. We follow EL (1991) in analyzing parents' choice of w^i_{t+1} as a time-consistent principal-agent problem, given that parents and (unborn) children cannot negotiate a Pareto-optimal bargaining solution for both n^i_t and h^i_t. Parents (acting as agents) select values of w^i_{t+1} that maximize equation (2) for children, taking as given the children's optimal choice of human capital investment and fertility. The resulting Stackelberg-equilibrium solution is thus inferred from:

$$dW^i(t+1)/dw^i_{t+1} = [\partial W^i(t+1)/\partial H^i_{t+1}]\,[\partial H^i_{t+1}/\partial w^i_{t+1}] + \partial W^i(t+1)/\partial w^i_{t+1}$$
$$= d^i_1(t+1)^{-\sigma}\, c^i_1(t+1)\, A^i\, (S^i_t)^{\gamma}\, (\partial h^i_t/\partial w^i_{t+1}) - d^i_1(t+1)^{-\sigma}\, A^i\, h^i_t\, (S^i_t)^{\gamma} = 0, \text{ where}$$

$d^i_1(t+1) \equiv (1 - v^i n^i_t - \theta^i h^i_t n^i_t - w^i_{t+1}\lambda^i_{t+1})$, $\lambda^i_{t+1} \equiv [H^i_{t+1} / (H^i_0 + H^i_{t+1})]$, and
$c^i_1(t+1) \equiv (1 - v^i n^i_t - \theta^i h^i_t n^i_t - w^i_{t+1})$. In a growth equilibrium steady state, $d^i_1(t+1) = c^i_1(t+1)$.

The optimal support rate, w^*, equates the marginal cost and benefit to grown-up children from rewarding their parents for the earning capacity they helped create, subject to the "reaction function" $\{h^i_t, w^i_{t+1}\}$ governing the parents' investment decision $(\partial h^i_t/\partial w^i_{t+1})$.

Under our heterogeneity condition, the optimal support rates w^{i*} become identical across family groups at both the stagnant and growth equilibrium. Consequently, the comparative dynamics simulations of a model with endogenous w become qualitatively identical to those reported in Table 1, where w treated as a fixed, but identical parameter across family groups.

Our simulations also show that the optimal value of w^* falls following any parametric shocks that produce takeoffs from stagnant- to growth-equilibrium steady states, essentially because the continuous growth in the level of offspring's human capital assets lowers the rate of return per unit asset demanded in compensation by altruistic parents. The simulations also indicate that w^* falls with A^1, 1/v, and B in the GE steady state. Similar results are obtained at the SE steady state, except that a higher A^1 raises w* in that state. Shifts in γ and H^i_0 have no effect on w^* in any steady state.

C. In the pure altruism case (i.e., w=0), we find two solutions that satisfy the first-order optimality conditions in a stagnant-equilibrium steady state: $h^i = 0$ and $h^i = \beta/[A^i\, S^i(s)^{\gamma}] - v^i/\theta^i > 0$, for all family groups. It can easily be proved that only the former solution is stable. This is essentially because under stagnant equilibrium, the rates of returns to h^i and n^i cannot be equalized if $h^i>0$, and this remains the case even if we allow for savings as well. The implication is that in a stable SE, a small increase in A^i (not sufficient to generate a takeoff) will not change income levels or the income inequality measures in our benchmark model. Also, since $h^i = 0$, our inter-family interaction term is not operative. Thus our income inequality measures $S^i(s)$, $P^i(s)$, and $G^i(s)$ are indeterminate, while the family-income inequality ratio is automatically defined by the endowment ratio, $E^i(s)=H^1_0/H^i_0$, as is the case in our benchmark model, where it is derived as an endogenous variable. As a result, we cannot pin down the time paths of S^i_t, P^i_t and G^i_t during the transitional development phase.

This corner solution in $h^i=0$ can be avoided if we introduce any positive old-age support rate w>0. This is because the marginal rate of returns to human capital investment h rises to infinity as h^i approaches zero, which indicates that there is always an interior solution for h^i in our model.

The pure altruism case can be solved, however, at the growth equilibrium steady state, and its behavioral implications are qualitatively the same as those we derive for our benchmark case.

D. Variables used and summary statistics

Variable	Description	Mean [Std. Dev.] All	Non-OECD	OECD
SD-FERT	Standard deviation of the distribution of surviving children per female ≥ 40	2.520 [0.306]	2.530 [0.283]	2.372 [0.535]
AV-FERT	Average of the distribution of surviving children per female ≥ 40	4.179 [1.161]	4.215 [1.180]	3.692 [0.787]
SD-SCHYR	Standard deviation of the distribution of schooling years in the population ≥ 15	3.684 [0.806]	3.646 [0.888]	3.782 [0.538]
AV-SCHYR	Average of the distribution of schooling years in the population ≥ 15	4.888 [2.755]	3.806 [2.107]	7.630 [2.259]
GINI*	Gini coefficient	37.76 [7.948]	45.26 [8.099]	34.51 [5.230]
QUINT*	Share of total income received by the top relative to the bottom quintile of families in the population	8.826 [5.259]	12.80 [7.027]	7.084 [2.887]
RGDP	Real per-capita income	6340 [5960]	3,667 [3,232]	13,114 [5,930]
G	GDP shares of government spending	19.53 [8.821]	20.70 [9.838]	16.56 [4.196]

* We calculate GINI and QUINT exclusively based on household income data reported in Dollar and Kraay (2001), excluding observations based on personal income, personal expenditures, or household expenditure data.

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ENDNOTES

[1]. Some empirical studies favor Kuznets' inverted-U: e.g., Kuznets (1963), Kravis (1960), Paukert (1973), Ahluwalia (1976), Lindert and Williamson (1985), and Barro (2000). Others reject it, or find no systematic relation, e.g., Anand and Kanbur (1993), Fields (1990), Fields and Jakubson (1994), and Deininger and Squire (1998). Studies of the relation between income's **rate of growth** and income inequality also report mixed results: Persson and Tabellini (1994), Alesina and Rodrik (1994), and Deininger and Squire (1998) show a negative relation; Forbes (2000) indicates a positive one; Barro (2000) reports that higher inequality lowers the growth rate in poor countries while encouraging it in rich countries while Banerjee and Duflo (2000) find an inverted-U relation between the two.

[2]. Models supporting Kuznets's causality direction are based on: structural shifts in a two-sector model (Kuznets 1955, 1963, Robinson, 1976, Anand and Kanbur 1993); trade effects (Wood and Ridao-Cano 1999); skill-biased technical progress (Eicher 1996, Aghion et al. 1999); and organizational changes (Kremer and Maskin 1996, Lindbeck and Snower 1997, Acemoglu 1999). Models favoring causality going from inequality to growth rely on: rising savings propensities as income grows (Keynes 1920, Kaldo, 1957); credit market imperfections (Loury1981, Galor and Zeira 1993, Banerjee and Newman 1993, Benabou 1996, Durlauf 1996, and Galor and Moav 2004); political economy changes (Venieris and Gupta 1986, Alesina and Perotti 1996, Benhabib and Rustichini 1996); and fertility changes by income (Kremer and Chen 2002, and La Croix and Doepke 2003).

[3]. Lucas (1988) also considers spillover effects in goods production, stemming from the average human capital level in a representative-agent model. Tamura (1991) applies a similar spillover-effects function in human capital production, but allows only for heterogeneity across agents in initial income endowments, and deals with the behavior of inequality exclusively under a dynamic growth regime. His model leads to full income-convergence. Zhong (1998), and Ehrlich and Yuen (2000) develop an analytical framework similar to ours, but abstract from the dynamic ramifications of fertility choices.

[4]. Positive assortative mating of agents of similar family types can justify this assumption. Becker (1973) and Burdett and Coles (1997) provide theoretical arguments and evidence supporting positive assortative mating in intelligence, education, and other characteristics. If heterogeneous families intermingle through negative sorting, and children inherit the average characteristics of parents, human capital production will eventually be identical in all families.

[5]. This specification is convenient because it assures the existence of a stagnant equilibrium steady state. Other specifications of this intergenerational interaction function, such as $S^i_t \equiv E^i_t$, or $S^i_t \equiv E^i_t [N^1_t/(N^1_t+N^i_t)]$, cannot prevent the ratio of family-group sizes from blowing up. Also, $S^i_t \equiv E^i_t P^i_t$ coincides with one of our income-group inequality index, as defined below.

[6]. Although spillover effects at work can be internalized to some extent by firms through an optimal wage policy, spillover effects during training are more difficult to price out.

[7]. For simplicity, we abstract here from uncertainty in the survival of children (but see EL, 1991).

[8]. We can alternatively specify the old-age support function as proportional to the children's full income, $w^i_{t+1}(H^i_0+H^i_{t+1})$, which can be shown to yield identical propositions. EL(1991) show, however, that the latter rule is suboptimal since the control variable h^i_t can be zero if H^i_0 is sufficiently large.

[9]. For example, in the log-utility case (σ =1), the share of income devoted to both raising children and supporting old parents, v^i and w^i, must satisfy the equality $(1- w^1)/v^1 = (1- w^i)/v^i$ to assure equal fertility rates $n^1=n^i$ (see fn 14). Moreover, if we treat w^1 and w^i as endogenous variables (see Appendix B), which would make them functions of v^1 and v^i, respectively, no equilibrium steady state may exist that also satisfies $n^1=n^i$, regardless of the value of σ. In contrast, if we set $v^1=v^i$, Appendix B shows that optimal $w^i = w^1$ **must** then be identical across all families.

[10]. In the log utility case, the SE steady state value of $h^i(s)$ has an explicit solution under our heterogeneity condition: $h^i(s) = \{\Omega- [\Omega^2 - 4(A^i/\theta^i)\, S^i(s)^\gamma\, v^i]^{1/2}\}/[4(A^i/\theta^i)\, S^i(s)^\gamma\, v^i]^{1/2}$; where $\Omega \equiv \beta - (A^i/\theta^i)\, S^i(s)^\gamma\, v^i$, and $S^i(s) \equiv [(A^1/\theta^1)/(A^i/\theta^i)]^{(1/\gamma)}$. This value is in fact one of two solution candidates that satisfy the optimality conditions, but other solution leads to an **unstable** equilibrium. The SE value $n^i(s)$ is given implicitly by $[v^i+\theta^i h^i(s)]n^i(s)\,/[\delta(1+\beta)] = 1 - [v^i+\theta^i h^i(s)]n^i(s) - w^i A^i h^i(s) S^i(s)^\gamma$.

[11]. Our model allows for the theoretical possibility of a "leadership switch" during the transition stage, in which case family 1 can be a different family type in a stagnant- v. a growth equilibrium.

[12]. Note that this proposition is not conditional on our heterogeneity condition, and holds as long as inter-group variations in fertility are compatible with a stable growth-equilibrium solution.

[13]. Proof: Imposing the GE condition $A^i h^i (S^i)^\gamma = A^1 h^1$ (proposition 3) on equations (6) and (7) for agent 1 and i>1, the first-order conditions with respect to n and θh become identical across family groups under our principle heterogeneity condition. Likewise in the SE steady state, the first-order optimality conditions with respect to n and θh become identical for agents 1 and i. The solutions for n^i and $\theta^i h^i$ are thus common to all agents in both steady states.

[14]. In the log utility case, the growth steady state values $h^i(g)$ and $n^i(g)$ have explicit solutions:
$h^i(g) = 2v^i/[\theta^i(\beta-1)]$; and $n^i(g) = \delta(\beta-1)(1-w^i)/[v^i + v^i\delta(\beta+1)]$.

[15]. If a reduction in θ^i affects family i many periods ahead of family 1, or by a sufficiently greater proportion, so that e^1/e^i actually falls, family i can overtake family 1, and become the "leading family" in terms of income-generating capacity. Income inequality will then reach a minimum at the point of overtaking, but will rise afterwards until it converges on its GE steady-state level. In this case the time path of income inequality will assume an S shape.

[16]. Our analysis is supported by De Georgorio and Lee (2002), who estimate a positive relationship between inequality in educational attainments and income inequality.

[17]. The Barro-Lee study reports average schooling years for four schooling levels in the population age 15 and up (zero, primary, secondary, and higher) and their population shares. We calculate the mean and standard deviation of this distribution for each country in all sample years.

Table 1: Simulating Comparative Dynamic Effects of Parameter Changes in a Two-agent Economy

Part 1. Stagnant Equilibrium

A^1/θ^1	A^2/θ^2	H^1_0	$B^1(B^2)$	$w^1(w^2)$	$v^1(v^2)$	γ	$n^1(n^2)$	$Y^1= H^1_0+H^1$	$Y^2= H^2_0+H^2$	E	S	$P=N^1/N^2$	Gini
2/1	1/1.01	50	.1	.01	.05	.4	6.958	54.905	1.0981	50	5.7993	.1160	.7490
3/1	1/1.01	50	.1	.01	.05	.4	1.879	453.67	9.0734	50	15.981	.3196	.6989
3/1	**1.5/1.01**	50	.1	.01	.05	.4	1.879	453.67	9.0734	50	5.7993	.1160	.7490
2/1	**1.5/1.01**	50	.1	.01	.05	.4	6.958	54.905	1.0981	50	2.1045	.0421	.6375
2/1	1/1.01	**60**	.1	.01	.05	.4	6.958	65.886	1.0981	60	5.7993	.0967	.7648
2/1	1/1.01	50	**.15**	.01	.05	.4	6.999	54.856	1.0971	50	5.7993	.1160	.7490
2/1	1/1.01	50	.1	**.015**	.05	.4	6.933	54.955	1.0991	50	5.7993	.1160	.7490
2/1	1/1.01	50	.1	.01	**.055**	.4	6.249	55.597	1.1119	50	5.7993	.1160	.7490
2/1	1/1.01	50	.1	.01	.05	**.45**	6.958	54.905	1.0981	50	4.7704	.0954	.7396

Part 2. Takeoff Triggers

		A^1	A^2	θ^1	θ^2	$w^1(w^2)$	$v^1(v^2)$	$n^1(n^2)$	h^1	h^2	E	S	$P=N^1/N^2$	Gini
(a)	(SE)	2	1	1	1.01	.01	.05	6.958	.0447	.0442	50	5.7993	.1160	.7490
	(GE)	**30**	**15**	1	1.01	.01	.05	1.227	.4886	.4837	50	5.7993	.1160	.7490
(b)	(SE)	2	1	1	1.01	.01	.05	6.958	.0447	.0442	50	5.7993	.1160	.7490
	(GE)	**30**	1	1	1.01	.01	.05	1.227	.4886	.4837	4.5E+7	5053.6	1.1E–4	.9997
(c)	(SE)	3	1	1	1.01	.01	.05	1.879	.2966	.2937	50	15.981	.3196	.6989
	(GE)	3	1	**0.5**	**0.505**	.01	.05	1.214	.9771	.9674	50	15.981	.3196	.6989
(d)	(SE)	3	1	1	1.01	.01	.05	1.879	.2966	.2937	50	15.981	.3196	.6989
	(GE)	3	1	**0.5**	**0.5**	.01	.05	1.214	.9771	.9771	48.5	15.588	.3211	.6967
(e)	(SE)	4.8	1	1	1.01	.01	.05	3.979	.1150	.1138	50	52.750	1.035	.4724
	(GE)	4.8	1	1	1.01	**.05**	.05	1.152	.4964	.4915	50	52.750	1.035	.4724
(f)	(SE)	4.8	1	1	1.01	.01	.05	3.979	.1150	.1138	50	52.750	1.035	.4724
	(GE)	4.8	1	1	1.01	.01	**.06**	1.009	.5865	.5807	50	52.750	1.035	.4724

Part 3. Growth Equilibrium

A^1/θ^1	A^2/θ^2	$B^1(B^2)$	$w^1(w^2)$	$v^1(v^2)$	γ	$n^1(n^2)$	h^1	h^2	$a^1=A^1h^1$	E	S	$P=N^1/N^2$	Gini
30/1	15/1.01	.1	.01	.05	.4	1.227	.4886	.4837	14.657	50	5.7993	.1160	.7490
40/1	15/1.01	.1	.01	.05	.4	1.229	.4885	.4836	19.542	113.9	11.905	.1045	.8279
40/1	**20/1.01**	.1	.01	.05	.4	1.229	.4885	.4836	19.542	50	5.7993	.1160	.7490
30/1	**20/1.01**	.1	.01	.05	.4	1.227	.4886	.4837	14.657	21.9	2.8251	.1290	.6243
30/1	15/1.01	**.15**	.01	.05	.4	1.233	.4866	.4818	14.598	50	5.7993	.1160	.7490
30/1	15/1.01	.1	**.015**	.05	.4	1.218	.4905	.4856	14.715	50	5.7993	.1160	.7490
30/1	15/1.01	.1	.01	**.055**	.4	1.115	.5375	.5321	16.125	50	5.7993	.1160	.7490
30/1	15/1.01	.1	.01	.05	**.45**	1.227	.4886	.4837	14.657	41.2	4.7704	.1158	.7229

Note: Parameters values that deviate from our benchmark values are presented in bold print.

Part 1. Comparative dynamics in the stagnant steady state are simulated by changing A^1/θ^1, A^2/θ^2, H^1_0, $B^1(=B^2)$, $w^1(=w^2)$, $v^1(=v^2)$, or γ, holding constant the values of all other parameters: $H^2_0 = 1$, $\sigma = 0.98$, $\delta = 0.9$, $\beta = 1.2$, $\alpha = 1$.

Part 2. Simulations show the impact of uniform (proportionate) and non-uniform changes in A^i and θ^i, as well as changes in the levels of $w^1=w^i$ and $v^1=v^i$ at both the SE and GE steady states in consecutive rows, holding constant the following parameters: $H^1_0 = 50$, $H^2_0 = 1$, $\sigma = 0.98$, $\delta = 0.9$, $\gamma = 0.4$, $\beta = 1.2$, $\alpha = 1$, $B^1 = B^2 = 0.1$.

Part 3. Comparative dynamics in the growth steady state are simulated by changing A^1/θ^1, A^2/θ^2, $B^1(=B^2)$, $w^1(=w^2)$, $v^1(=v^2)$, or γ, holding constant $\sigma = 0.98$, $\delta = 0.9$, $\beta = 1.2$, $\alpha = 1$.

Table 2 Fertility Inequality Regressions

Dependent Variable: SD_FERT

	Model 1	Model 2	Model 3	Model 4	Model 5
	OLS	Country Random Effects	Country Random Effects	Country Random Effects	Country RE & Year Dummies
Intercept	1.776852	2.087804	2.165747	2.181626	1.162350
	7.45	10.54	9.72	10.53	2.94
RGDP	0.000410	0.000249	0.000235	0.000270	0.000376
	2.23	1.59	1.49	1.83	2.27
$RGDP^2$	-8.49E-08	-5.75E-08	-5.62E-08	-5.59E-08	-8.13E-08
	-1.90	-1.60	-1.56	-1.66	-2.13
$RGDP^3$	5.08E-12	3.40E-12	3.40E-12	3.25E-12	5.07E-12
	1.64	1.44	1.44	1.47	2.00
AV_FERT	0.056786	0.042325	0.046133	0.134551	0.180190
	1.88	2.14	2.27	4.18	3.84
G			-0.003653	-0.004215	-0.005115
			-0.69	-0.87	-1.00
T				-0.013374	
				-3.46	
Adj. R^2	0.0854	0.1637	0.1959	0.2824	0.4373
N	72	72	72	72	72

Notes: The dependent variable is the standard deviation of the distribution of surviving children per woman age 40 and over. Data sources are the World Fertility Surveys and the Demographic and Health Surveys (various years). Rows show the estimated coefficients (β) and their t-statistics (β/S_β). This table's regressions employ a random effects specification to account for missing idiosyncratic variables, because the number of observations per country is small. No serial correlation correction is needed, since the Cochran-Orcutt test rejects the existence of an AR(1) serial correlation in Model 3.

Table 3. Education Attainment Inequality Regressions

Dependent Variable: SD_SCHYR

	Model 1	Model 2	Model 3	Model 4	Model 5
	OLS	Country Fixed Effects	Country Fixed Effects	Country Fixed Effects	Country& Year Fixed Effects
Intercept	2.634408 37.43	2.074857#	1.957999#	2.512515#	2.837973#
RGDP	9.29E-05 2.71	-3.79E-05 -1.02	-3.13E-05 -0.84	-4.36E-05 -1.32	-5.95E-05 -1.75
$RGDP^2$	-1.20E-08 -3.79	4.29E-09 1.48	4.01E-09 1.39	2.75E-09 1.07	3.34E-09 1.29
$RGDP^3$	3.03E-13 3.48	-1.59E-13 -2.21	-1.54E-13 -2.15	-1.19E-13 -1.87	-1.24E-13 -1.94
AV_SCHYR	0.205215 11.97	0.351542 18.40	0.342305 17.66	0.111584 4.50	0.115726 4.59
G			0.006915 2.51	0.000617 0.25	-0.000855 -0.33
T				0.029080 12.92	
Adj. R^2	0.3320	0.4891	0.4943	0.6022	0.6070
N	721	721	721	721	721

Notes: The dependent variable is the standard deviation in the distribution of schooling years attained in the population age 15 and over. The data source is Barro and Lee (2000). Rows show the estimated coefficients (β) and their t-statistics (β/S_β). # The Intercept coefficients represent mean values of all intercept terms. No serial correlation correction is needed, since data on the dependent variable are available every five years and the Cochran-Orcutt test rejects the existence of an AR(1) serial correlation in Model 3.

Table 4. Income Inequality Regressions: GINI

Dependent Variable: GINI

	Model 1	Model 2	Model 3	Model 4	Model 5
	OLS	Country Fixed Effects	Country Fixed Effects	Country Fixed Effects	Country& Year Fixed Effects
Intercept	41.948920 26.34	35.729530#	40.193390#	39.392580#	58.906320#
RGDP	0.001583 2.94	0.001095 2.62	0.000873 2.02	0.001081 2.03	0.001077 1.85
$RGDP^2$	-2.45E-07 -5.22	-1.08E-07 -3.54	-9.47E-08 -3.00	-1.04E-07 -3.00	-1.01E-07 -2.68
$RGDP^3$	7.04E-12 5.96	2.81E-12 3.99	2.51E-12 3.43	2.70E-12 3.43	2.61E-12 3.05
G			-0.181998 -2.19	-0.170141 -2.00	-0.216757 -2.22
T				-0.027090 -0.66	
Adj. R^2	0.4108	0.0691	0.0916	0.0932	0.2248
N	318	318	310	310	310

Notes: The dependent variable is the GINI coefficient, based on household income data. The data source is Dollar and Kraay (2001). Rows show the estimated coefficients (β) and their t-statistics (β/S_β). # Coefficient represents the mean value of the intercept term. No serial correlation correction is needed, since the Cochran-Orcutt test rejects AR(1) serial correlation in Model 3.

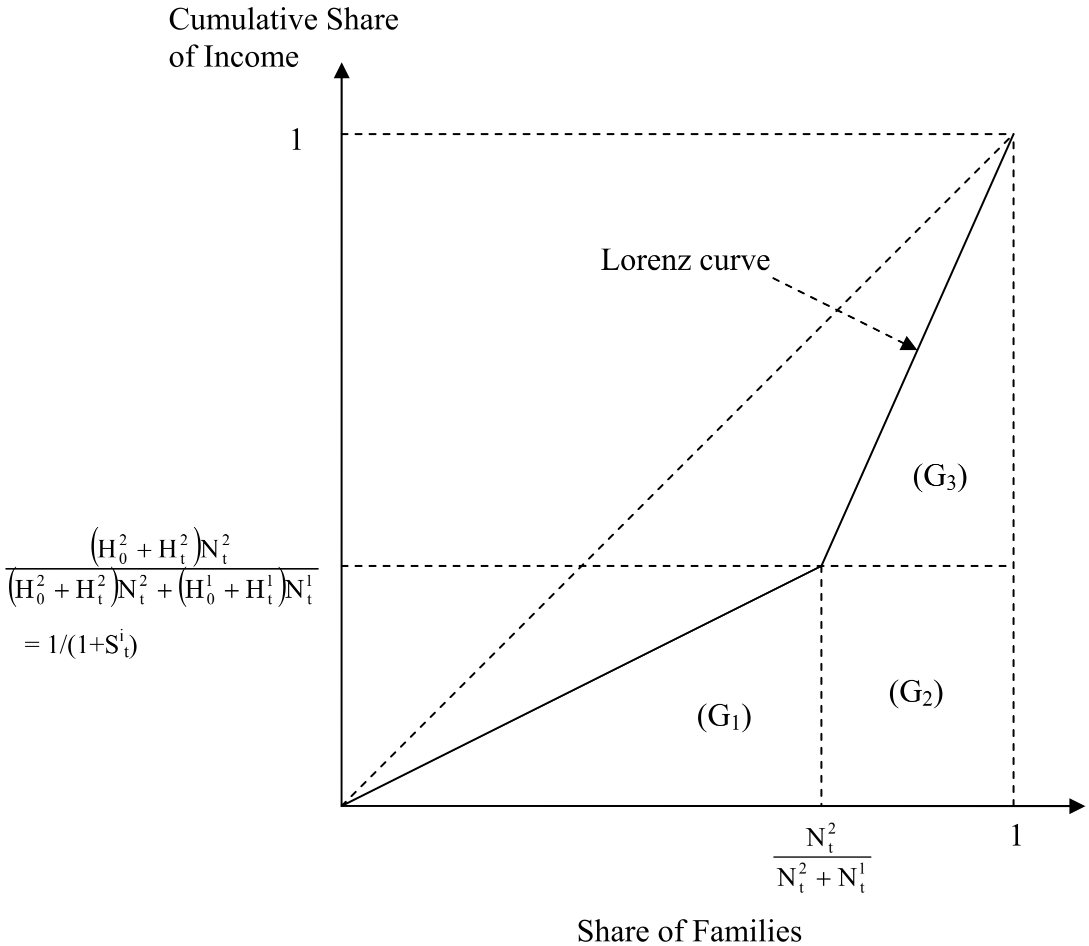
Table 5. Income Inequality Regressions: QUINT

Dependent Variable: QUINT

	Model 1	Model 2	Model 3	Model 4	Model 5
	OLS	Country Fixed Effects	Country Fixed Effects	Country Fixed Effects	Country & Year Fixed Effects
Intercept	9.110258	4.513832#	2.445769#	2.290842#	25.019580#
	7.52				
RGDP	0.001440	0.001294	0.001437	0.001478	0.001313
	3.47	3.82	4.10	3.37	2.76
$RGDP^2$	-1.80E-07	-1.00E-07	-1.13E-07	-1.15E-07	-1.07E-07
	-4.92	-3.98	-4.30	-3.95	-3.43
$RGDP^3$	4.97E-12	2.32E-12	2.65E-12	2.69E-12	2.55E-12
	5.34	3.96	4.30	4.02	3.55
G			0.101228	0.103520	0.077167
			1.51	1.50	0.99
T				-0.005237	
				-0.16	
Adj. R^2	0.2498	0.0663	0.0787	0.0788	0.2278
N	289	289	281	281	281

Notes: The dependent variable is the share of total income received by the top, relative to the bottom, quintile of families in the population. The data source is Dollar and Kraay (2001), and only household income data are used. Rows show the estimated coefficients (β) and their t-statistics (β/S_β). # Coefficient represents the mean value of the intercept terms. No serial correlation correction is needed, since the Cochran-Orcutt test rejects AR(1) serial correlation in Model 3.

Figure 1. The GINI Coefficient in the Two-family case



Area (G1) = $(1/2)\ [1/(1+S^i_t)]\ [N^2_t/(N^2_t+N^1_t)]$

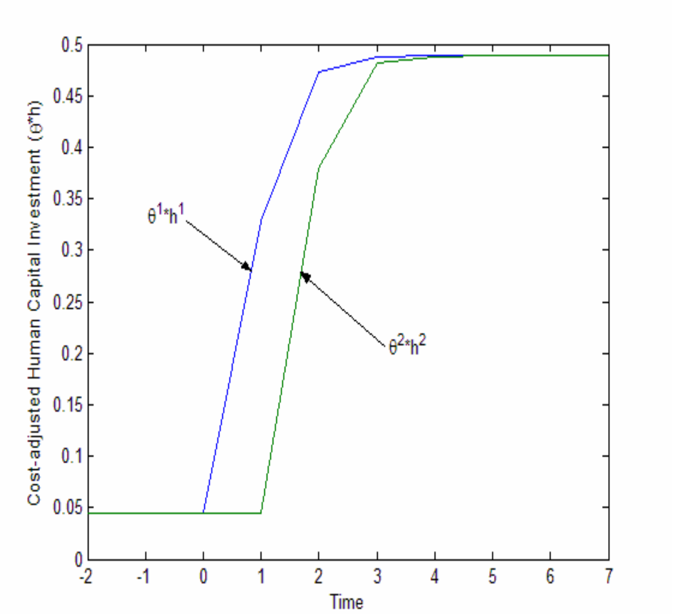
Area (G2) = $[1/(1+S^i_t)]\ [N^1_t/(N^2_t+N^1_t)]$

Area (G3) = $(1/2)\ [S^i_t/(1+S^i_t)]\ [N^1_t/(N^2_t+N^1_t)]$

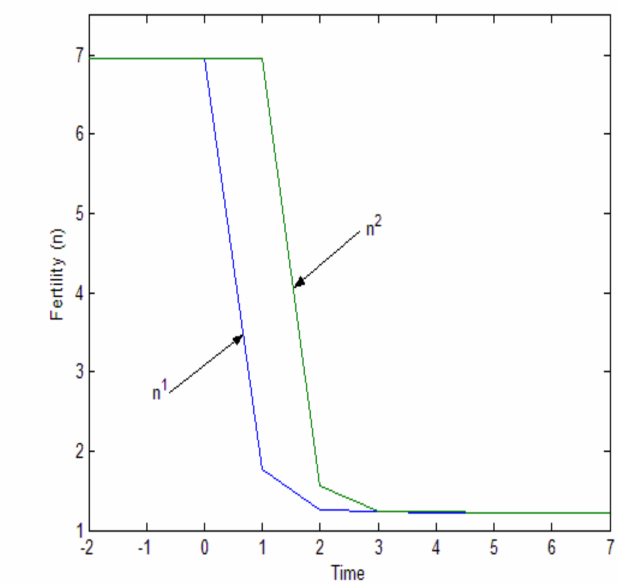
$$\begin{aligned} \text{GINI} \equiv G^i_t &= [1/2 - (G_1) - (G_2) - (G_3)] / (1/2) \\ &= [S^i_t - (N^1_t/ N^2_t)] / [(1+ S^i_t)\ (1+ (N^1_t/ N^2_t))]. \\ &= (S^i_t - P^i_t) / [(1+ S^i_t)\ (1+ P^i_t)]. \end{aligned}$$

Figure 2. Inverted U-shaped transition path

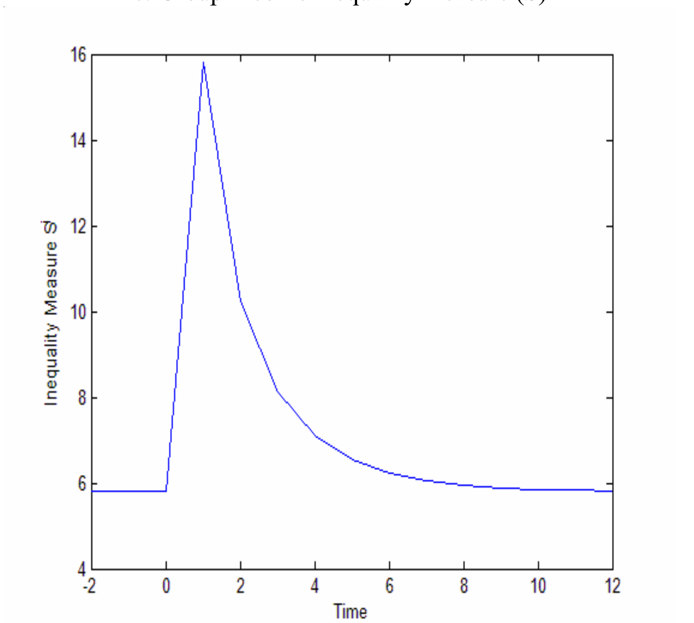
a. Cost-adjusted human capital investment (θh)



b. Level of fertility (n)



c. Group-income inequality measure (S)



d. Family-income inequality measure (E)

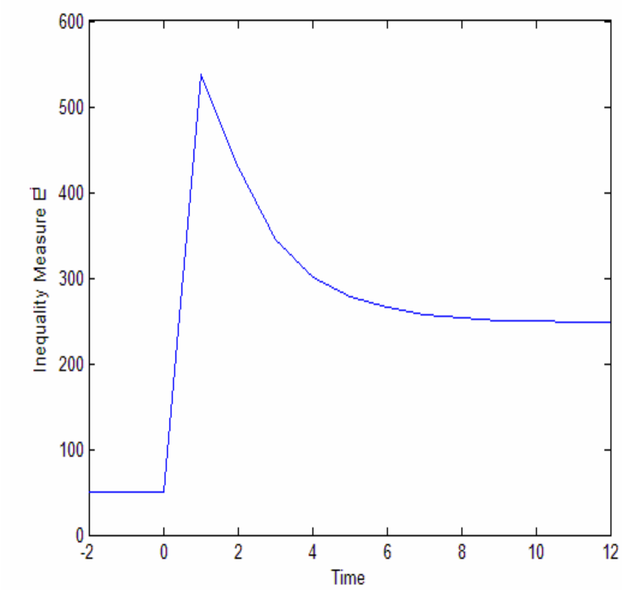
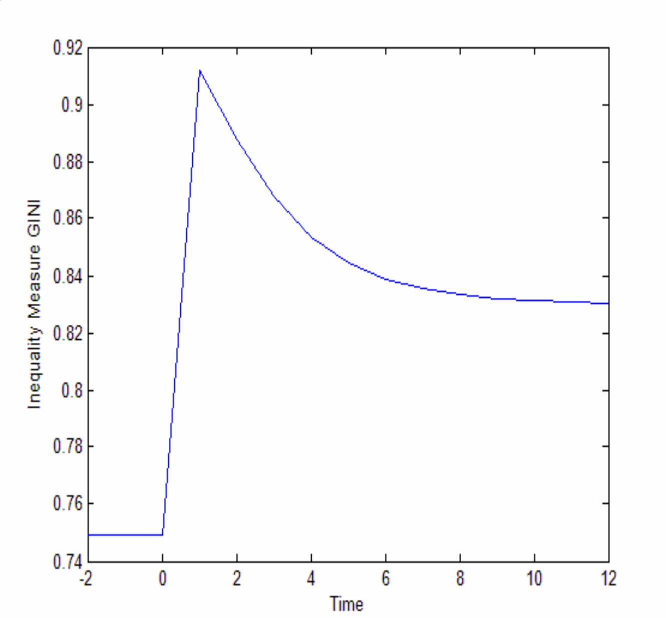
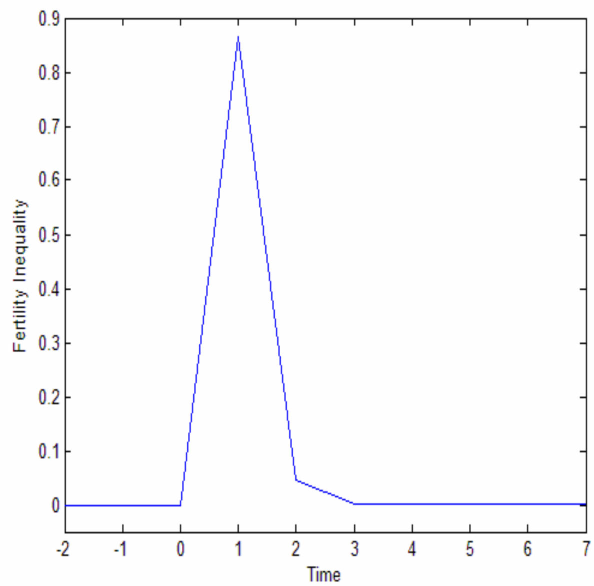


Figure 2. Inverted U-shaped transition path (continued)

e. Gini coefficient (G)

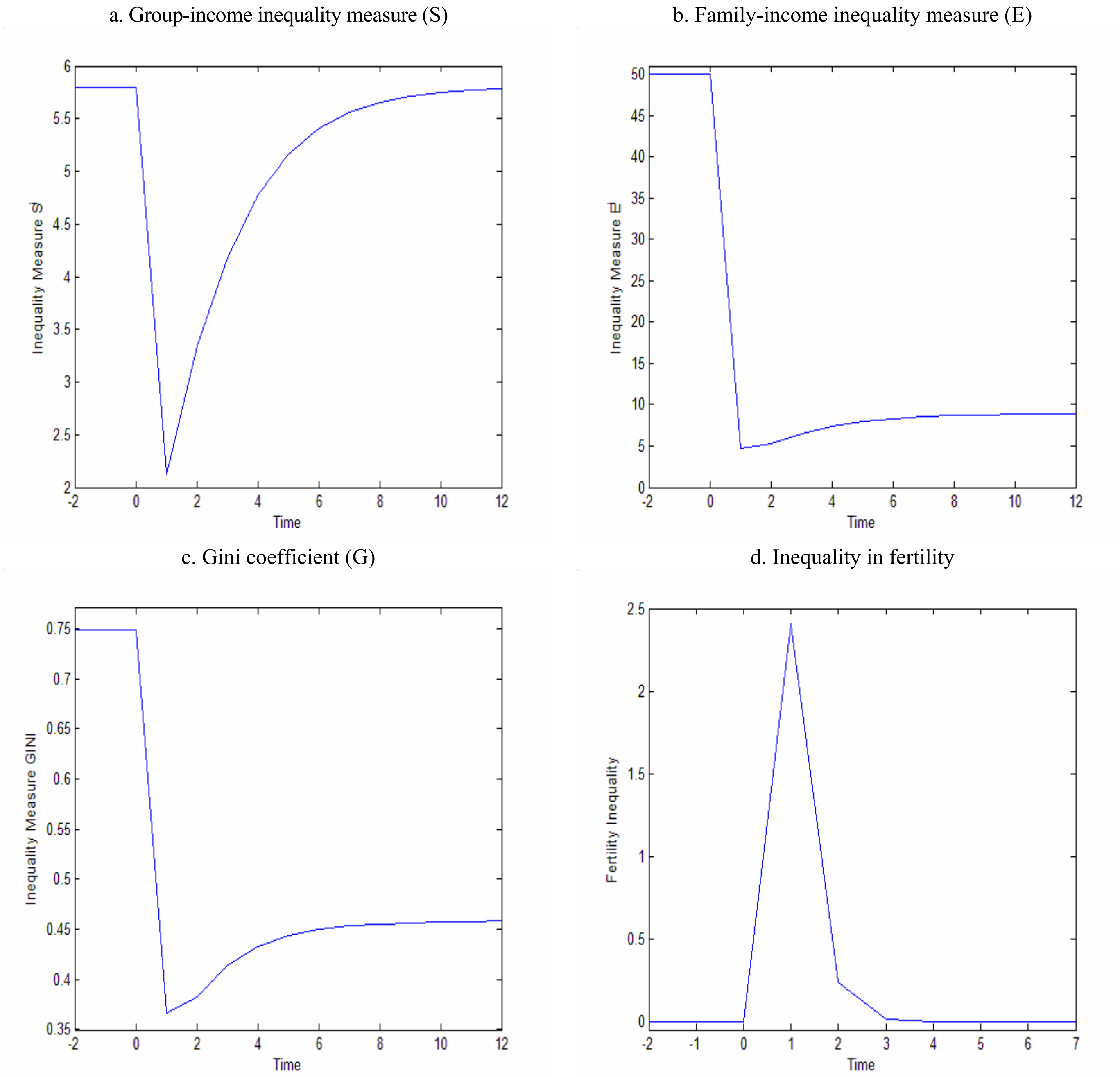


f. Inequality in fertility



Note: Parameter values used in these simulations are: θ^1=1, θ^2=1.01, H^1_0=50, H^2_0 = 1, B^1=B^2=0.1, w^1=w^2=0.01, v^1=v^2=0.05, γ = 0.4, σ = 0.98, δ = 0.9, β = 1.2, α = 1. Prior to period 1, the economy is in a stable SE steady state, with A^1=2 and A^2=1. In period 1, family 1 alone experiences a once-and-for-all permanent increase in A^1 to 30. In period 2, family 2 also experiences an equi-proportional increase in A^2 to 15.

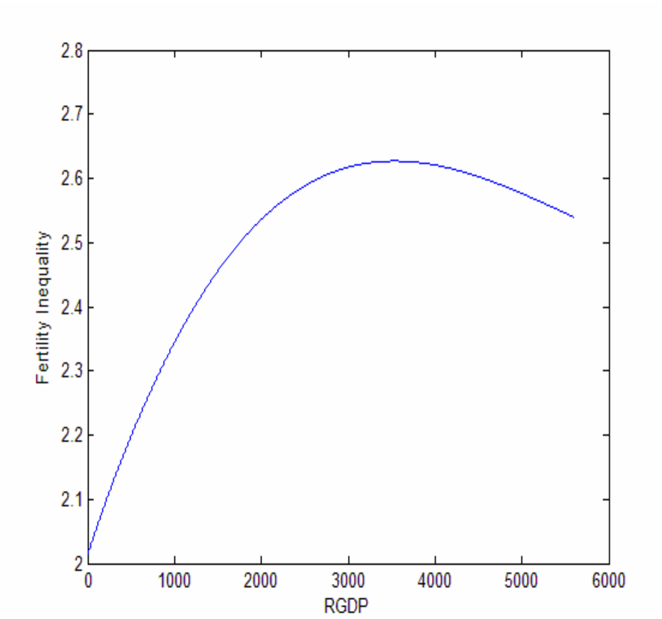
Figure 3. U-shaped transition path



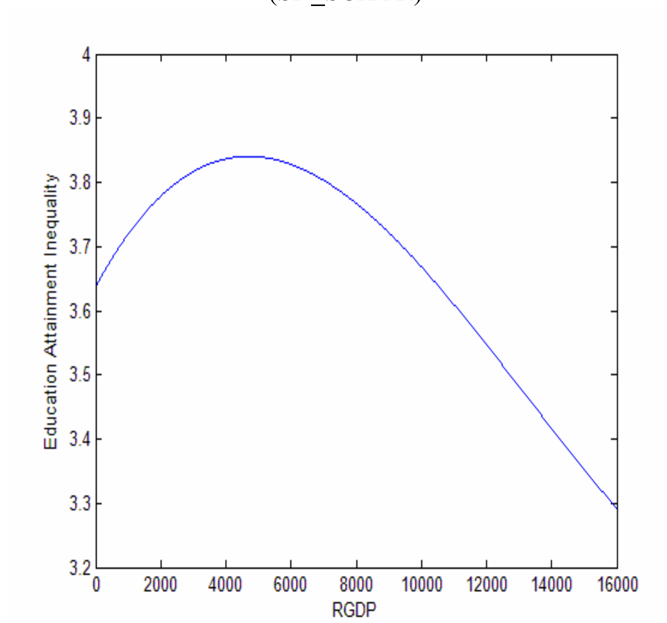
Note: Parameter values used in the simulations for Figure 2: $A^1=2$, $A^2=1$, $H^1_0=50$, $H^2_0 = 1$, $B^1=B^2=0.1$, $w^1=w^2=0.01$, $v^1=v^2=0.05$, $\gamma = 0.4$, $\sigma = 0.98$, $\delta = 0.9$, $\beta = 1.2$, $\alpha = 1$. Prior to period 1, the economy is in a stable SE steady state with $\theta^1=1$ and $\theta^2=1.01$. In period 1, family 2 alone experiences a once-and-for-all reduction in θ^2 to 1.01/15. In period 2, family 1 then experiences an equi-proportional reduction in θ^1 to 1/15.

Figure 4. Fitted Lines from the Regression Results

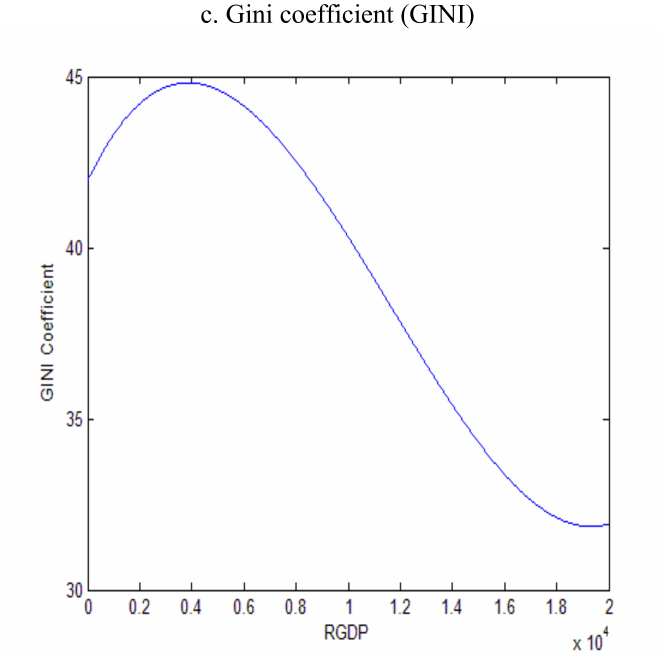
a. Mean-adjusted fertility inequality (SD_FERT)



b. Mean-adjusted educational attainment inequality (SD_SCHYR)



c. Gini coefficient (GINI)



Note: Diagram a, b, and c are based on the regression results of Model 1 in Table 2, 3, and 4, respectively The RGDP values on the x-axis cover 90% of the observations in each sample for the regressions.