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CURRENCY INCONVERTIBILITY, PORTFOLIO BALANCE AND RELATIVE PRICES

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Abstract

This paper analyzes regimes of currency inconvertibility in the framework of a simple general equilibrium model where an officially-traded good, a smuggled good and a non-traded good are produced and consumed by residents, who hold domestic and foreign currency in their portfolios. It is shown that stability requires the effect of relative prices on demand for traded and non-traded goods to dominate their effect on asset demands and that a once-and-for-all devaluation does not change the currency substitution ratio. To the extent that the monetary authorities wish to change the currency composition of private financial wealth, a crawling peg is therefore the appropriate instrument. The direction of change depends on the nature of expectations about relative asset returns. Under perfect foresight, an increase in the rate of crawl increases the currency substitution ratio whereas, if expectations are static, it reduces it.

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1. Introduction

The development of private international financial intermediation has led to an unprecedented growth of trade in assets taking place not only among industrialized countries but also spreading to semi-industrialized and even to some less developed countries. This has provided channels for individuals and organizations to build up foreign exchange balances, so that they can diversify their portfolios across assets denominated in different currencies. In response, many governments have attempted to recapture foreign exchange by offering special advantages to certain types of transactions, namely the ones where evasion of exchange controls would be easier, like tourist services and migrants' remittances. Nevertheless, in many countries with inconvertible currencies, "black" markets for foreign exchange have developed to the point that the relative price between domestic and foreign money that is determined in these markets may have a greater effect on private capital flows than the "official" rate, with obvious repercussions on the balance of payments problem of the country in question.

Restricting the ability to convert national currency into foreign exchange may involve a restriction on international trade, which would therefore tend to promote factor mobility and trade in assets. Also, even though the definition of convertibility in the Articles of Agreement of the IMF only refers to current account transactions, in a world of capital mobility and flexible exchange rates among major currencies, black markets for "convertible" currencies will develop when policies toward current account transactions are divorced from policies with respect to capital account transactions.

In this paper, the fruitfulness of the portfolio approach to the determination of the black market rate in situations of currency inconvertibility is suggested in Section 1 by a simple dynamic partial equilibrium portfolio model. Section 2 develops a portfolio balance model of a small open economy with three goods and two assets. One good is traded in the official foreign exchange market, another is smuggled through the black market, and the third is non-traded.[1] On the other hand, private financial wealth is composed of domestic and foreign money. Section 3 shows how the temporary equilibrium of the model determines the two relative prices, where the relative price of the two traded goods is also the black market premium. Section 4, in turn, shows how equilibrium determines the long-run equilibrium values of the asset stocks. After analyzing the perfect foresight path of a simplified version of the model in Section 5, the paper ends with a brief conclusion.

2. A dynamic partial equilibrium model of the black market for foreign exchange

Consider a country whose residents are not authorized to hold foreign assets, but who nevertheless allocate their financial wealth between domestic and foreign currency. If foreigners do not hold the inconvertible domestic currency in their portfolios, then, the domestic currency price of foreign currency in the black market will be such that the existing stock is willingly held. Furthermore, the only way for domestic residents to acquire foreign currency is through the underinvoicing or smuggling of exports and through the overinvoicing of imports. Under a fixed official exchange rate and in the absence of endogenous reported capital flows and errors and omissions, as well as interest payments on unrecorded capital flows, the two balances will give the change in the stock of foreign currency of the private sector and the central bank respectively. Assuming that the relevant "elasticities condition" holds, on the other hand, will imply that an increase in the black market (official) exchange rate will improve the unreported (reported) trade balance. If the shares traded through each market also respond to the black market premium, the black market (official) rate will also deteriorate the reported (unreported) trade balance, but by less.[2] Under these conditions, the difference between output and expenditure - which equals the sum of the two trade balances - will be lowered by the increase in either one of the exchange rates.

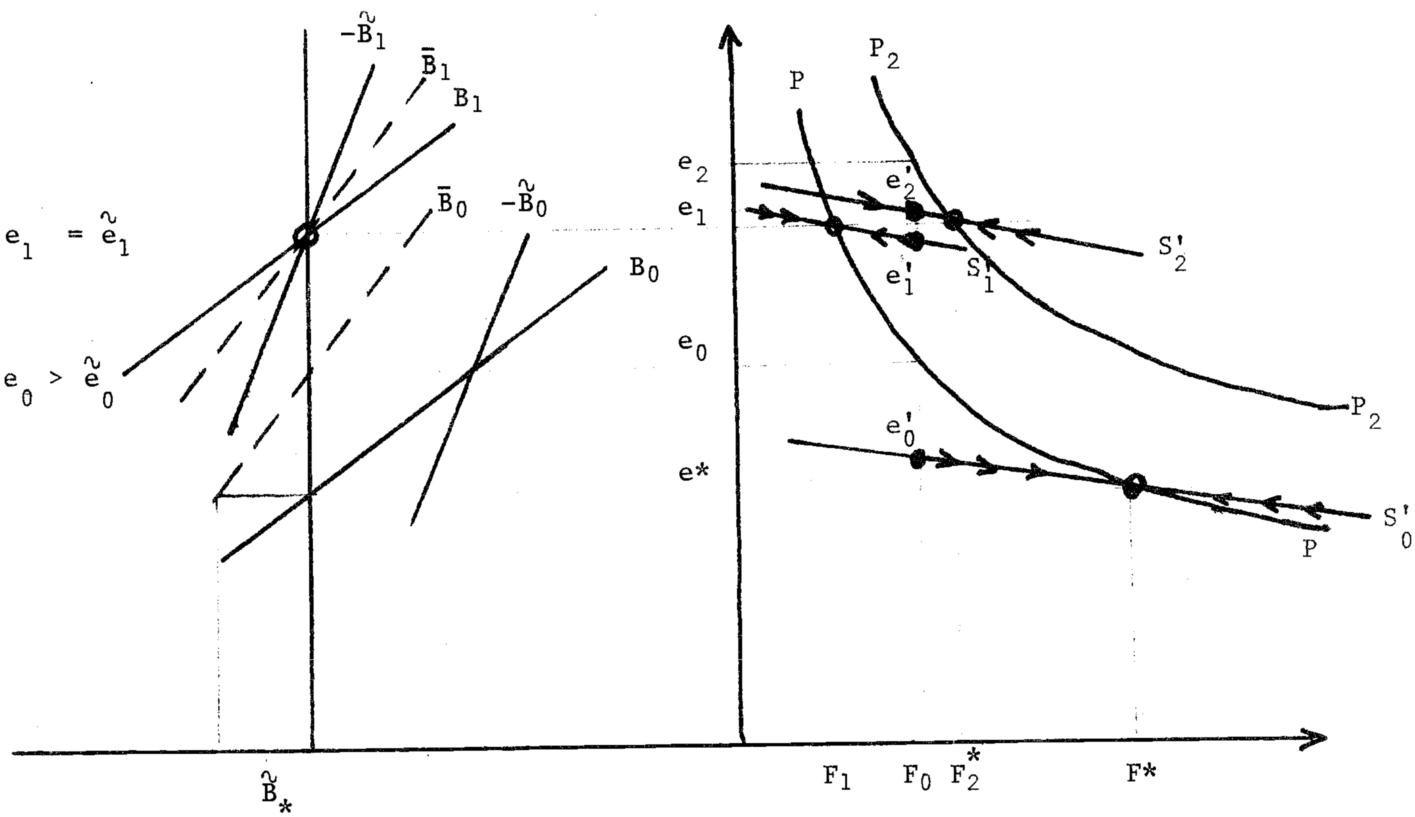
Ignoring the monetary consequences of the reported trade deficit, this system can be illustrated by a variant of the Kouri (1983) diagram, as in Figure 1. The left panel shows the instantaneous determination of the black market exchange rate, given demand for foreign currency and no expected change in relative returns, along a rectangular hyperbola PP. Under a perfect foresight, demand for foreign currency will increase when the black market rate depreciates, constraining its initial value for a given stock of foreign currency F_o to e_o', on the saddle path S_o' (below the corresponding initial value under static expectations, e_o). For a given official exchange rate, $\tilde{e}_o$ ($< e_o$), the left panel of Figure 1 plots the unreported trade surplus in foreign currency, B (which equals to the increase in F) as well as the exogenous reported trade deficit, $-\tilde{B}$. Assuming a stronger effect of the black market rate on B, the $-\tilde{B}$ locus will have a steeper slop and the total trade balance $\bar{B}$ will be upward sloping. For simplicity assume that $\bar{B}$ is zero initially but that the unreported trade balance equilibrium associated with the given official exchange rate is e*. If the associated reported deficit $\tilde{B}^*$ is corrected by a devaluation of the official exchange rate to its long-run equilibrium level, $\tilde{e}_1$, the three schedules will shift up and intersect on the vertical axis at a point where, by choice of units, $e_1 = \tilde{e}_1$. The implications of the offical devaluation differ depending on the expectations formation mechanism. If expectations are static, the black market exchange rate will gradually depreciate from e_o to e_1 as the stock of foreign assets decreases from F_o to F_1. If expectations are rational the black market rate jumps from e_o' to e_1' and then gradually depreciates to the same long-run value. If the official devaluation generates an increase in the demand for foreign assets from PP to P_2P_2, furthermore, the black market rate will overshoot to e_2(e_2' under perfect foresight) and gradually appreciate as the stock of foreign assets increases to F_2^*.

This analysis remains applicable when the official exchange rate, rather than being fixed, is set by the central bank according to the "acceleration hypothesis", so that it depreciates at a rate given by the rate of the reported trade balance over the stock of foreign assets of the central bank.[3] To analyze the effect of an exogenously given rate of crawl, however, the

Figure 1

Partial equilibrium analysis

of the black market rate



trade balances have to respond to the black market premium rather than to the levels of the two rates and this is best handled in the general equilibrium framework of the next section.

3. A three-good two-asset model of currency inconvertibility

While some basic features of the black market for foreign exchange can be derived from a straightforward extension of the Kouri (1983) partial equilibrium analysis, as done in the previous section, the interaction between the private and official valuation of foreign exchange, on the one hand, and the role of the black market premium as a relative price, on the other, call for a general equilibrium approach. In the model presented below, two relative prices are endogously determined, the black market premium and the relative price of non-traded goods. To sharpen the analysis, we will neglect the role of the black market premium emphasized in the previous section, namely determining whether a transaction will go through the official or the black foreign exchange market. Here there are exports and imports which either always traded through the official market, or always smuggled. Their prices in foreign currency are, in obvious notation, P^*_{XO}, P^*_{MO}, P^*_{XB}, and P^*_{MB}. Assuming further that the given factors of production are fully employed, we specify the five excess-supply functions for goods in terms of the vector of domestic currency prices, P, and financial wealth, W.

(1) $$X_i(P) - D_i\,(P,W) = ES_i\,(P,W) \quad i = XO,\ MO,\ XB,\ NT,$$

where $$P = (P_{XO}\ P_{MO}\ P_{XB}\ P_{MB}\ P_{NT})'$$

The elimination of the direct effect of asset returns on the demand for goods, as well as of the income effect is designed to simplify the analysis.[4]

For the same reason, demands for domestic money M and foreign exchange F depend on the expected rate of change of the black market exchange rate, ψ, as a measure of the real returns differential when nominal returns are zero,[5] on the vector of goods prices and on wealth.

(2) $eF = f(\psi, P, W)$

(3) $M = m(\qquad)$

(4) $W = M + eF$

The domestic money stock, in turn, is made up of domestic credit, C - interpreted as lump-sum transfer from the government — and the stock of foreign assets of the (government), central bank F^G, valued at the official exchange rate:

(5) $M = C + \tilde{e}F^G$

This formulation takes as given the money multiplier or, equivalently, assumes a stock demand for high-powered money.

Going back to the excess-supply functions for goods, and expressing them in terms of the price of the official export good $P_{XO} = \tilde{e}P^*_{XO}$,[6] we get, from zero homogeneity in prices and wealth:

(1') $X_i - D_i = ES_i\ (\rho, \hat{\tilde{W}})$

where $\rho = P/P_{XO} = (\rho_1\ \rho_2\ \rho_3\ \rho_4)'$

$\hat{\tilde{W}} = W/P_{XO}$

The four relative prices which are the components of the ρ vector can be expressed in terms of the terms-of-trade in the official and black markets, the black market premium and the relative price of non-traded goods:

$$t_i = P^*_{xi}/P^*_{mi} \quad i = O, B$$

$$p = (P_{XB}/P_{XO})(P^*_{XO}/P^*_{XB}) = e/\tilde{e}$$

$$q = P_{NT}/P_{XO}$$

If the terms of trade are given to the domestic economy, t_O and t_B as well as P^*_{XO}/P^*_{XB} are exogenous and can be set to one by choice of units.

Asset demands are linear homogenous in prices and wealth,[7] while the neglect of the effect of relative asset returns on the demand for goods implies that $f_\psi = -m_\psi$ in (2) and (3) above. We can therefore substitute for the demand for domestic money by using the wealth constraint and write portfolio balance as:

$$(2') \qquad pF = f(\psi, p, q, \tilde{M} + pF)$$

where $\tilde{M} = M/P_{XO}$

We also assume that the market for non-traded goods always clears:

$$(6) \qquad X_{NT}(p, q) = D(p, q, W)$$

Given asset stocks and the terms of trade, (2') and (6) determine the two endogenous relative prices p and q. To the extent that the official exchange rate is fixed, or that its rate of change is determined by the monetary authorities, furthermore, temporary equilibrium involves the adjustment of e and P_{NT}. In fact, given (6), it is clear that temporary equilibrium requires e and P_{NT} to move at the rate of crawl, so that p and q are constant.

Now excess supply for traded goods equals the accumlation of foreign currency by the central bank and the private sector through the reported and unreported trade balances respectively. We will allow for exogenous reported capital flows, whose amount in foreign currency is T, an outflow. Then we write foreign asset accumlation as:

(7) $\dot{F} = P^*_{XB}\ (ES_{XB} + ES_{MB}/t_B) = B(p,\ q,\ \tilde{W})$

(8) $\dot{F}^G + T = P^*_{XO}\ (ES_{XO} + ES_{MO}/t_O) = \tilde{B}(p,\ q,\ \tilde{W})$

It is clear from (7) that when F = 0 in steady-state equilibrium, B = 0 so that the excess-supply of smuggled goods is zero. The relationships of the reported trade balance to the change in domestic real money balances, $\tilde{M}$ still needs to be specified. Given (5), it will require assumptions about credit creation and the rate of crawl. Since we do not explicitly introduce a government sector, there is only the endogenous source of money creation, the excess supply of the official traded good. The exogenous components are the reported capital flows, domestic credit creation, which are both transfers, and capital gains on central bank reserves associated with changes in the official exchange rate, to the extent that a fraction γ thereof induces credit creation and $1 - \gamma$ goes into central bank net-worth forever. We can then write:

(9) $\dot{\tilde{M}} = \tilde{B} + \tilde{M} - T$

where $\delta = \frac{C}{M} - [\frac{\dot{C}}{M} + (1 - \gamma)\ \frac{F^G}{\tilde{M}}]\ \hat{\tilde{e}} - P^*_{XO}$

It is clear from (9) that if $\gamma = 0$, as usually assumed,[8] the expression in square brackets equals one and δ becomes the real increase in domestic credit creation as a proportion of the money stock. Notice also that if $\gamma = 1$ and P^*_{XO} is given, δ becomes inversely proportional to the real money stock, so that its level only enters through $\tilde{B}$.

Finally, we specify for the moment an adaptive expectation formation mechanism based on the level of the premium, so that

(10) $\psi = \psi(\bar{p})$

We will refer below to the perfect foreign assumption

(11) $\psi = \hat{e}$

Using (10) in (6), setting $P^*_{XO} = 1$ by choice of units and expressing (2') and (6) as implicit functions of asset stocks and relative prices, we can express the temporary equilibrium of the system as:

(12) $A(p, q, \tilde{M}, F, z) = pF - f = 0$

(13) $N(p, q, \tilde{M}, F, z) = ES_{NT} = 0$

where $z = (P^*_{XB}/P^*_{XO}\ t_B\ t_O)$ is a vector of exogenous relative prices.

4. <u>Temporary equilibrium</u>

By log differentiation of (12) and (13) we solve the system for the two endogenous relative prices, as a function of asset stocks and the vector of exogenous prices. Denoting the matrix of (positive) elasticities of the f function and semi-elasticities of the N function with respects to z by Π, we get,[9]

(14) $$\begin{bmatrix} 1 - \alpha\eta - \tilde{\varepsilon}_1 - \varepsilon_2 \\ -\alpha\omega_1 - \nu_2 \end{bmatrix} \begin{bmatrix} \hat{p} \\ \hat{q} \end{bmatrix} = \begin{bmatrix} (1-\alpha)\eta & -1 + \alpha\eta \\ (1-\alpha)\omega_1 & \alpha\omega_1 \end{bmatrix} \begin{pmatrix} \hat{\tilde{M}} \\ \hat{F} \end{pmatrix} + \Pi\hat{z}$$

where $\alpha = pF/W$ is the share of foreign assets in wealth;

$\eta = \frac{\partial f/\partial \tilde{W}}{\alpha}$ is the elasticity of demand for foreign assets with respect to real wealth;

π is the positive elasticity of expectations relative to the premium; $\varepsilon_1 (\varepsilon_2\ \varepsilon_3)$ is the positive elasticity of demand for foreign assets with respect to the premium (the relative price of non-traded goods, relative returns), e.g. $\varepsilon_2 = \frac{\partial f/f}{\partial \rho_4/\rho_4}$;

$\tilde{\varepsilon}_1 = \varepsilon_1 - \varepsilon_3 \pi$;

$\omega_1 = -\tilde{W}\partial N/\partial \tilde{W}$ is the positive semi-elasticity of the excess supply of non-traded goods with respect to real wealth;

$\nu_1 (\nu_2)$ is the positive semi-elasticity of the excess supply of non-traded goods with respect to the premium (the relative price of non-traded goods), e.g. $\nu_1 = -\rho_2 \partial N/\partial \rho_2 - \rho_3 \partial N/\partial \rho_3$.

It is clear from the left-hand-side of (14) that in q, q space the slope of the N schedule is unambiguously positive whereas the A schedule could slope downward if the valuation effect were weaker than the sum of the wealth effect and the relative price effect (net of the regressive expectation effect). Put another way, as the premium increases, demand of foreign assets increases by less. Since $\alpha\eta = \partial f/\partial \tilde{W}$ and $\tilde{\varepsilon}_1$ is the total elasticity of money demand with respect to the premium, we can write this condition as:

$$(15) \qquad \frac{\partial m}{\partial \tilde{W}} > \tilde{\varepsilon}_1$$

If condition (15) is met, the premium varies inversely with the currency ratio valued at the official rate, F/M, and with a coefficient less than one, so that if the premium increases, we know that the currency ratio valued at the black market rate, $pF/\tilde{M}$, will also increase. If marginal and average currency shares are the same ($\eta = 1$), furthermore, condition (15) puts the share of domestic money in wealth as an upper bound on the premium elasticity, net of expectations effects.

In a partial equilibrium framework where price and wealth effects are neglected, then, the A schedule would be horizontal. In general, to the extent that relative price effects are stronger on the flow demand for goods than on the stock demand for assets, the slope of the A schedule will be less than the slope of the N schedule. To the extent that $\varepsilon_2 < 1 - \alpha - \tilde{\varepsilon}_1$, furthermore, the slope will be less than the ray through the origin OR, as drawn in Figure 2, where temporary equilibrium obtains at T_o. It is clear from the Figure that an exogenous increase in ψ (not due to a decrease in p), by only moving up the A schedule, increases p by more than it increases q so that the new equilibrium is at T_ψ. This result holds independently of the slope of the A schedule.

The effects of changes in asset stocks are also straightforward. If the slopes are as drawn, then the determinant Δ of the Jacobian in (14) is positive and domestic monetary expansion will increase both relative prices while an increase in foreign money will decrease them. To see this solve (14) for given z and write:

$$(14') \quad \begin{bmatrix} \hat{p} \\ \hat{q} \end{bmatrix} = \begin{bmatrix} p_M & p_F \\ q_M & q_F \end{bmatrix} \begin{bmatrix} \hat{\tilde{M}} \\ \hat{F} \end{bmatrix}$$

where $p_M = (\nu_2 \quad \tilde{\omega}_1 \, \varepsilon_2) \; (1 - \alpha) \; \eta/\Delta;$

$p_F = (-\nu_2 + \tilde{\omega}_1 \varepsilon_2 h) \; (1 - \alpha\eta) \; /\Delta$

$q_M = [\nu_1 + \tilde{\omega}_1 \; (1 - \tilde{\varepsilon}_1)] \; (1 - \alpha) \; \eta/\Delta$

$q_F = -(\nu_1 + \tilde{\omega}_1 \tilde{\varepsilon}_1 h) \; (1 - \alpha\eta) \; /\Delta$

$\Delta = \nu_2 (1 - \alpha\eta - \tilde{\varepsilon}_1) - (\alpha\omega_1 + \nu_1)\varepsilon_2 > 0$

$h = \nu_2/1 - \alpha\eta$ is the marginal currency ratio

$\tilde{\omega}_1 = \omega_1/\eta$ is the ratio of the wealth semi-elasticity of excess supply of non-traded goods to the wealth elasticity of demand for foreign assets.

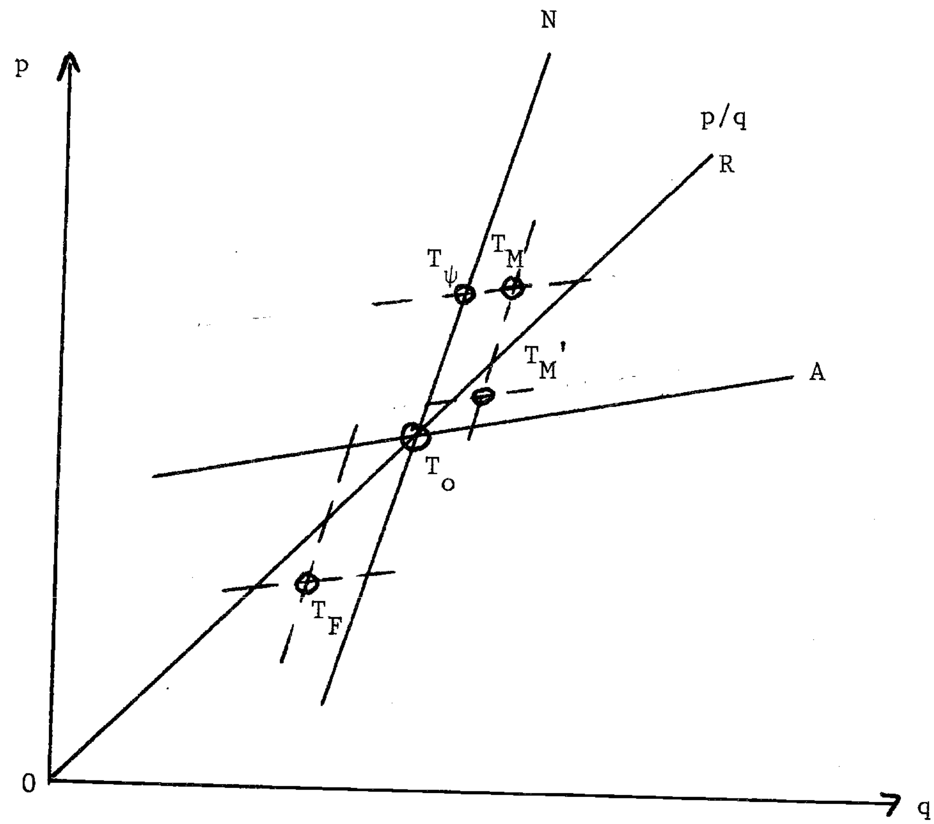
Note that $p_F < 0$ requires that the own price effect dominate the demand for non-traded goods or that the marginal currency ratio be sufficiently small. It is clear from Figure 2, though, that if A moves up and N to the right or conversely the new intersection might be the left or to the right of OR, depending on the relative sizes of p_M and q_M and p_F and q_F, respectively. Under our assumption that the slope of of the A schedule is less the p/q, the dominance of the "own" effect in the non-traded-goods market has to be sufficienty strong for

(16) $\nu_2 > \nu_1 + \tilde{\omega}_1 \; (1 - \tilde{\varepsilon}_1 + \varepsilon_2).$

If condition (16) holds, asset stocks will always have a stronger effect on p than on q, so that after an increase in domestic (foreign) money the system will move to T_M (T_F), to the left (right) of OR. If condition (16) does not hold, then $p_M < q_M$ (as in T_M') but it would still be the case that $|p_X| > |q_F|$ if:

Figure 2

Short-run effect of various disturbances under regressive expectations



(17) $\nu_2 > \nu_1 + (\tilde{\varepsilon}_1 + \varepsilon_2)\, \tilde{\omega}_1 h$

The effect of changes in the exogenous relative prices are easy to ascertain. The matrix Π is given by

(18) $$\Pi = \begin{bmatrix} \varepsilon_1 & -\varepsilon_B & -\varepsilon_0 \\ \nu_1 & -\nu_B & -\nu_0 \end{bmatrix}$$

where ε_B (ε_0) is the elasticity of the demand for foreign assets with respect to ρ_3 (ρ_1), respectively the inverse of the relevant terms of trade;

and ν_B (ν_0) is the semielasticity of N with respect to $\rho_3(\rho_1)$.

It is clear from the substitution of (18) into (14) and the use of Cramer's rule that an increase in P^*_{XB}/P^*_{XO} (which always appears multiplied by p) increase p and q while increase in t_B or t_O decrease p and q. The effect on p will be stronger if ν_2 is sufficiently large. The exact condition for the increase in the terms of trade t_i, i = B, O to lower p/q is stronger than (16) if ν_i/ε_i is large:

(19) $$\nu_2 > \alpha\omega_1 + \nu_1 + \frac{\nu_1}{\varepsilon_1}(1 - \alpha\eta - \tilde{\varepsilon}_1 - \varepsilon_2)$$

The condition for the increase in τ to increase p/q is weaker than (19):

(20) $$\nu_2 > \alpha\omega_1 + \frac{\nu_1}{\varepsilon_1}(1 - \alpha\eta + \varepsilon_3\pi - \varepsilon_2)$$

To consider the effect of a once-and-for-all devaluation of the official rate, notic that it increases the demand for foreign currency by $1 - \tilde{\varepsilon}_1 - \varepsilon_2$ and increases excess demand for non-traded goods by $\eta_2 - \nu_1 - \omega_1$, so that the black market exchange rate and the price of non-traded goods rise. It can, however, be shown that if $\Delta > 0$ they will necessarily rise by less, so that p and q fall. In fact, p/q will fall and the new equilibrium will be on or below OR if condition (16) holds. An official devaluation would thus bring the system from T_o to T_F in Figure 2, just like when the stock of foreign money increases. Note, finally, that an increase in the equilibrium ratio of foreign money to wealth lowers the slope of the N schedule and increases the slope of the A schedule.

5. Steady-state equilibrium

Still assuming regressive expectations about the change in the black market rate, we investigate the properties of steady-state equilibrium, defined by constant asset stocks $\tilde{M} = \tilde{M}^*$ and $F = F^*$. These are such that, aside from portfolio balance and equilibrium in the non-traded goods market, there is zero excess supply of the smuggled good, so that $B = 0$ and therefore, zero excess supply for the official traded good as well, $\tilde{B} = 0$. Asset accumulation can thus be represented by solving out for p and q in the excess supply function for traded goods. We express it as:

$$(21) \qquad \dot{\tilde{M}} = \tilde{B}\ (\tilde{M},\ F) + \delta\tilde{M} - T$$

$$(22) \qquad \dot{F} = B\ (m,\ F)$$

According to (21), when $\tilde{B} = 0$, domestic real money balances will not remain constant. Setting $\hat{P}^*_{XO} = 0$ in (9) above, we obtain a relationship between domestic credit creation, the reported capital outflow and the rate of crawl which has to be satisfied for the reported trade balance to imply a constant real domstic money stock. It can be written as:

$$(23) \qquad \hat{\tilde{e}} = (\dot{C} - \tilde{e}T)\ /\ (M - \gamma\tilde{e}F^G)$$

According to (23) the rate of crawl is determined by the difference between the exogenous domestic credit creation and reported capital outflow, over the money stock net of the fraction of central bank reserves which induces credit creation. Thus if, as usually assumed, $\gamma = 0$, the rate of crawl will be lowest. If, conversely all capital gains are monetized, the denominator reduces to C.

If (23) holds, then the only net source of increase in domestic real money balances is indeed the excess-supply of the officially traded good:

$$(21') \qquad \dot{\tilde{M}} = \tilde{B}$$

suppose now that both $\dot{C}$ and T are zero but that the rate of crawl is determined by the "acceleration hypothesis "described in Section 2 above. Then (21') does not hold and we have instead:

$$(21'') \qquad \dot{\tilde{M}} = (1 + \tilde{M}/F^{G})\tilde{B}$$

The two specifications have the same linear approximation. We will choose (21') for simplicity and will analyze the effect of an increase in domestic credit creation from a value which is exactly offset by the rate of crawl along the lines of (23). Under these conditions we can write the linear appoximation of the system around long run equilibrium in matrix form:

$$(25) \quad \begin{pmatrix} \dot{\tilde{M}} \\ \\ \dot{F} \end{pmatrix} = \begin{bmatrix} -\tilde{\pi}p_M-\tilde{\mu}q_M-\tilde{\omega}[1+\alpha(p_M-1)] & -\tilde{\pi}p_F-\tilde{\mu}q_F-\tilde{\omega}\alpha(p_F+1) \\ \\ \pi p_M-\mu q_M-\omega[1+\alpha(p_M-1)] & \pi p_F-\mu q_F-\omega\alpha(p_F+1) \end{bmatrix} \begin{bmatrix} \frac{\tilde{M}-\tilde{M}^*}{\tilde{M}^*} \\ \\ \frac{F-F^*}{F^*} \end{bmatrix}$$

where $\tilde{\pi}(\pi)$ is the positive semi-elasticity of the reported (unreported) current account with respect to the premium, e.g. $\tilde{\pi} = -p\partial\tilde{B}/\partial p$;

$\tilde{\mu}(\mu)$ is the positive semi-elasticity of the reported (unreported) current account with respect to the relative price of non-traded goods, e.g. $\mu = -q\partial B/\partial q$;

$\tilde{\omega}(\omega)$ is the positive semi-elasticity of the reported (unreported) current account with respect to real wealth, e.g. $\tilde{\omega} = -\tilde{W}\partial\tilde{B}/\partial W$;

and p_M , q_M , p_F , q_F are defined in (14').

The determinant of the Jacobian in (25) is given by:

(26) $$DET = \frac{1-\alpha}{\Delta}[\omega_1\Delta(\tilde{\pi}\mu+\pi\tilde{\mu}) + \nu_2(\tilde{\pi}\omega+\pi\tilde{\omega}) + (\nu_1+\alpha\omega_1)(\tilde{\mu}\omega-\mu\tilde{\omega})] \ .$$

It is clear from (26) that if the reported and unreported current account have the same response to wealth and the relative price of the non-traded good (so that $\omega = \tilde{\omega}$ and $\mu = \tilde{\mu}$) the determinant will be positive:

(26') $$DET = (1-\alpha)(\tilde{\pi}+\pi)[\mu\omega_1 + \nu_2\omega/\Delta] > 0$$

Under the same simplifying assumption, and if in addition $\eta = 1$, the trace will be given by:

(27) $$-TR = [\nu_2(1-\alpha) + \alpha\omega_1\varepsilon_2](\pi+\tilde{\pi}) + \omega - \omega_1[\varepsilon_2(\tilde{\pi}+\alpha\omega) - \mu(1-\alpha-\tilde{\varepsilon}_1)] \ .$$

Note that even when $\tilde{\varepsilon}_1$ is at the upper bound given by condition (15), and the term in μ vanishes, if ν_2 , π and $\tilde{\pi}$ are large relative to ε_2 and ω_1 , the expression in (27) will be positive and the system will be stable. The condition for the own effect on the unreported current account to be negative

($\partial B/\partial F < 0$) is of course stronger than (27). It requires that the direct premium effect dominate the effect of the relative price of non-traded goods and wealth:

$$(28) \qquad [\nu_2(1-\alpha) + \alpha\omega_1\varepsilon_2]\pi > \alpha\omega(\nu_2\tilde{\varepsilon}_1-\nu_1\varepsilon_2) + \mu[\nu_1(1-\alpha) + \alpha\omega_1\tilde{\varepsilon}_1] \quad .$$

On the cross effects in (25), the one on the reported current account ($\partial\tilde{B}/\partial F$) is positive because q_F and p_F are negative and $|p_F| > 1$. For $\partial B/\partial\tilde{M} > 0$, however we need

$$(29) \quad \pi(1-\alpha)[\nu_2-\varepsilon_2\omega_1] > \omega[\alpha(\nu_2\tilde{\varepsilon}_1-\nu_1\varepsilon_2) + 1] + \mu[(1-\alpha)\nu_1+\omega_1(1-\tilde{\varepsilon}_1)]$$

which is stronger than (28), but nevertheless plausible if ν_2 and π are large enough relative to ω and ε_2.

Under these conditions, the qualitative properties of steady-state equilibrium will basically be the same as in a similar model without non-traded goods and can be stated briefly.[10] It can be shown that — not surprisingly — an increase in the demand for foreign assets will increase h, the currency substitution ratio, while a once-and-for-all devaluation of the official rate will leave it unchanged. The effect of an increase in δ — which can be interpreted when $\gamma = 0$ and $T = 0$ either as an increase in domestic credit creation $\dot{C}/\tilde{e}$ or a decrease in the rate of crawl — is less immediate and hinges more crucially on expectations formation mechanisms to the extent that a change in the rate of crawl changes relative asset returns in the steady-state, when the black market premium is constant. Taking the total differential of the system from a situation where $\delta = 0$, we can express it as:

$$(30)\quad \begin{bmatrix} 1-\alpha\eta & \tilde{\varepsilon}_1 & -\varepsilon_2 & -(1-\alpha)\eta & 1-\alpha\eta \\ \gamma_1+\alpha\omega_1 & & -\nu_2 & (1-\alpha)\omega_1 & \alpha\omega_1 \\ \tilde{\pi}+\alpha\tilde{\omega} & & -\tilde{\mu} & (1-\alpha)\tilde{\omega} & \alpha\tilde{\omega} \\ -\tilde{\pi}+\alpha\tilde{\omega} & & -\mu & (1-\alpha)\omega & \alpha\omega \end{bmatrix} \begin{bmatrix} \hat{p} \\ \hat{q} \\ \tilde{M} \\ \hat{F} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \tilde{M}d\delta \\ 0 \end{bmatrix}$$

The determinant of the Jacobian in (30) simplifies to:

$$(31)\qquad DET = -(1-\alpha)\;[\nu_1\;(\tilde{\mu}\omega - \mu\tilde{\omega}) + \nu_2(\tilde{\pi}\omega + \pi\tilde{\omega}) - \omega_1(\pi\tilde{\mu} - \tilde{\pi}\mu)]$$

Assuming that the expression in square brackets in (31) is positive, and using Cramer's rule, we find that an increase in δ is likely to decrease p and q, decrease $\tilde{M}$ and increase F. The net effects on h and $p/q = \xi$ are given by:

$$(32)\qquad \frac{\Delta h}{\tilde{M}d\delta} = [1-\alpha(1+2\tilde{\varepsilon}_1)](\nu_2\omega - \mu\omega_1) - 2\varepsilon_2(\nu_1\omega + \pi\omega_1) - (\eta - 1)(\mu\nu_1 + \pi\nu_2)$$

$$(33)\qquad \frac{\Delta\hat{\xi}}{\tilde{M}d\delta} = (1-\alpha)[\omega(\nu_2 - \nu_1) - \omega_1(\pi + \mu)]$$

Under the assumption about the relative size of the parameters used so far, one would expect h to increase and ξ to decrease in response to an increase in δ. Thus an increase in domestic credit creation increases the currency substitution ratio and causes an appreciation of the real black markets rate (measured) in terms of non-traded goods). Conversely, an increase in the rate of crawl reduces h and increases ξ.[11] The former effect is reversed when the expected increase in the premium generates an increase in the demand for foreign assets, as shown in the next Section.

6. The implications of perfect foresight

The well-known portfolio models of flexible exchange rates with non-traded goods of Kouri (1975) and Calvo and Rodriguez (1977) assume that expectations about relative returns are continuously realized. Using (11) in (2') and inverting the portfolio balance equation, we obtain a differential equation for the black market rate:

$$\hat{e} = f^{-1}(\overset{+}{p}, \overset{-}{q}, \overset{-}{\tilde{M}}, \overset{+}{F}) \tag{34}$$

While several methods of solution have been tried, the simpliest in this case is to solve out for q, whose rate of change is given by the rate of crawl, and express the system in terms of p, $\tilde{M}$ and F. Taking a linear approximation around steady-state equilibrium, where p = 1 by choice of units, we write the system in matrix form as:

$$\begin{pmatrix} \dot{p} \\ \dot{\tilde{M}} \\ \dot{F} \end{pmatrix} = \begin{bmatrix} (1 - \alpha\zeta - \varepsilon_1 - \varepsilon_2\tilde{\nu}_1)/\varepsilon_3 & -(1-\alpha)\zeta/\varepsilon_3 & (1-\alpha\zeta)/\varepsilon_3 \\ -(\tilde{\pi} + \alpha\tilde{\Omega} + \tilde{\mu}\tilde{\nu}_1) & -(1-\alpha)\tilde{\Omega} & -\alpha\Omega \\ \pi - \alpha\Omega - \mu\tilde{\nu}_1 & -(1-\alpha)\Omega & -\alpha\Omega \end{bmatrix} \begin{bmatrix} p - 1 \\ \dfrac{\tilde{M} - \tilde{M}^*}{\tilde{M}^*} \\ \dfrac{F - F^*}{F} \end{bmatrix} \tag{35}$$

where

$$\zeta = \eta + \omega_1\varepsilon_2\nu_2$$

$$\tilde{\nu}_1 = \nu_1/\nu_2$$

$$\tilde{\Omega} = \tilde{\omega} + \omega_1\tilde{\mu}/\nu_2$$

$$\Omega = \omega + \omega_1\mu/\nu_2$$

The determinant of the matrix in (31) is unambiguously positive and equal to $(1 - \alpha)(\pi\tilde{\Omega} + \tilde{\pi}\Omega)/\varepsilon_3$, so that the system has the positive root associated with the black market premium and two negative roots, associated with the asset stocks. Equilibrium is thus a saddle-point. Suppose now that the rate of crawl increases. In steady-state the premium is constant, so that the rate of depreciation of the black market rate was to increase. This makes foreign currency more attractive and therefore raises the currency substitution ratio. Because of (24), the rate of change of domestic credit creation also increases.

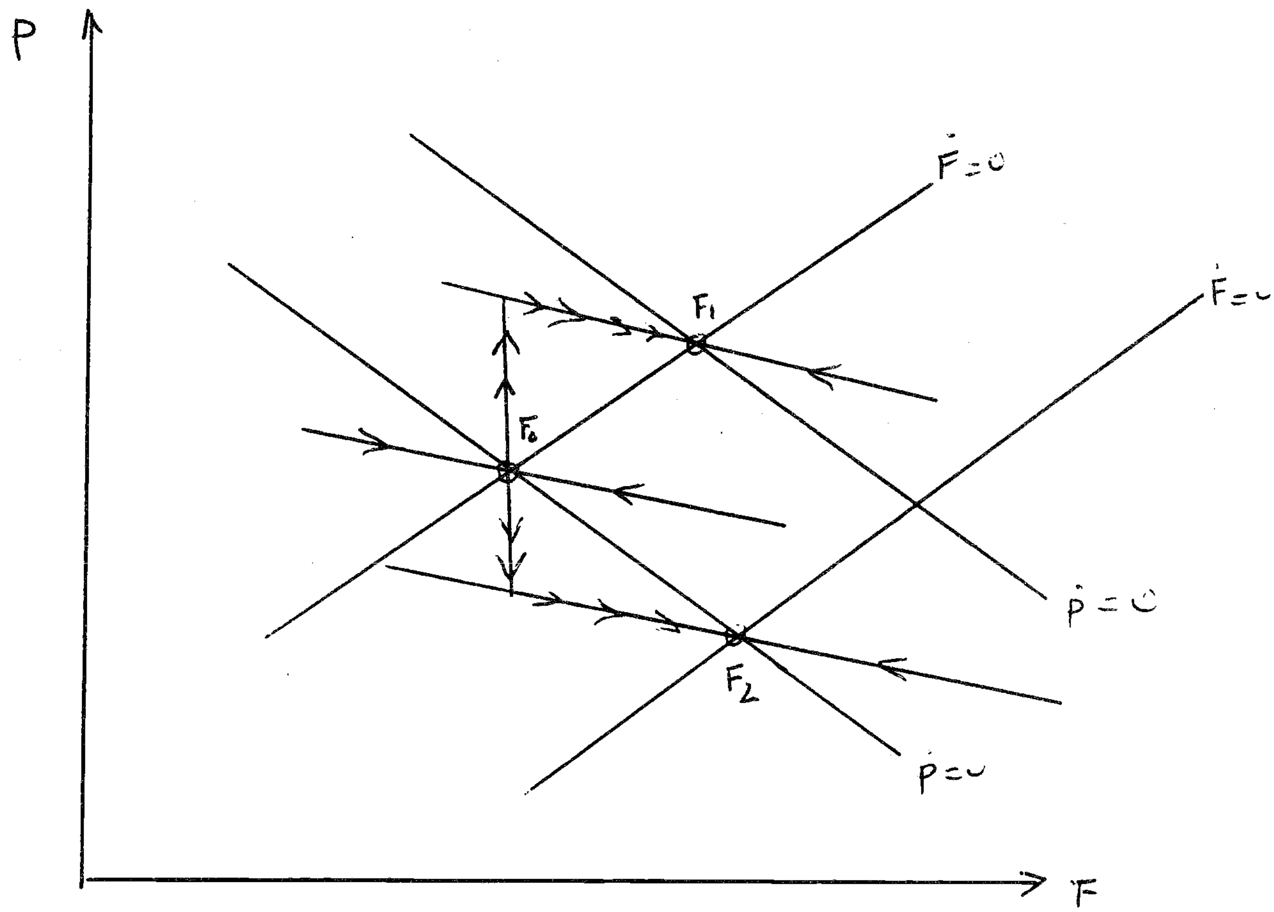
We can represent equilibrium in state-space in a similar fashion to the right panel of Figure 1 above by assuming that $\tilde{M} = \tilde{M}^*$ at all times, so that we plot the loci where $\dot{p} = 0$ and $\dot{F} = 0$ as in Figure 3.[12] Steady-state equilibrium obtains at F_0. Asssuming that $\tilde{e}$ is fixed, this differs from Figure 1 above because the reported current account is always in balance and because the unreported current account responds to wealth. As a consequence of the wealth effect, the locus where $\dot{F} = 0$ is upward sloping rather than horizontal. The effect of an increase in the demand for foreign assets is now a smaller jump in e, followed by a continuous appreciation of the black market rate along with the perfect foresight path to F_1. Conversely, the effect of an exogenous increase in the foreign demand for the smuggled good is a downward shift in the $\dot{F} = 0$ locus, which leads to a jump appreciation in e and a continuous appreciation along the new perfect foresight path to F_2.

Figure 3

Steady-state

effect of various disturbances

under perfect foresight



Conclusion

This paper has analyzed regimes of currency inconvertibility using the portfolio approach to exchange rate determination, summarized (in a partial context) in Section 1. We showed that the effect of relative prices on the demand for traded and non-traded goods had to dominate their effect on asset demands. It was also shown under what conditions the steady-state properties of the model were the same as the ones of a similar model without non-traded goods. In particular, the result that continuous changes in the official rate (a crawling peg) are necessary to change the currency composition of private financial wealth, continues to hold. The direction of the effect is different, however, under the assumption of perfect foresight, because the increase in the rate of crawl implies an increase in the expected rate of change of the black market rate, given that the premium is constant in the long-run.

NOTES

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1. Empirical applications to Portugal and Egypt are in Macedo (1982a, Essay III) and Macedo (1982b), respectively. Note that in Blejer (1978), the existence of a non-traded good is required for the black market rate to vary. A critical survey of his paper and of other black market literature is in Macedo (1981).

2. The exact conditions are worked out in Macedo (1982a, Essay II). On the portfolio approach to dual exchange rates see references ibid.

3. See Kouri (1983).

4. On the income effect in this model, see Macedo (1981, p. 35 note 13). In general, see Buiter and Eaton (1981).

5. This applies to relative real returns in the case of constant expenditure shares. See Macedo (1981, p. 36 note 14).

6. The results could also be presented in terms of a price index. See note 9 below.

7. This contrasts with the assumption in Macedo (1982b). See, however, Macedo (1981 and 1982a, p. 66 note 16 and p. 111).

8. See, however, the addendum to Johnson (1976) and Macedo (1981a, p. 67 note 18).

9. If a price index $\Pi P_i^{\beta i}$ was used the ε_i terms, call them $\bar{\varepsilon}_i$, would be subtracted by the weighted wealth effects $(1 - \alpha)\eta$ times $\beta_3 + \beta_4$ for ε_1 and β_5 for ε_2, e.g. $\bar{\varepsilon}_1 = \varepsilon_1 - (\beta_3 + \beta_4)(1 - \alpha)\eta$ and similarly for $\bar{\nu}_1$ and $\bar{\nu}_2$.

10. See the analysis in Macedo (1982a, p. 67-68).

11. The real appreciation is emphasized in Macedo (1982b).

12. This is somewhat arbitrary but finding the eigenvalues of the matrix in (35) did not prove immediate and three-dimensional phase diagrams are not easy to interpret either.

References

Blejer, M. (1978), Exchange Restrictions and the Monetary Approach to the Exchange Rate, in J. Frenkel and H. Johnson eds., The Economics of Exchange Rates, Adison-Wesley.

Buiter, W. and J. Eaton (1981), On the almost total adequacy of Keynesian Balance of Payments Models, American Economic Review, September.

Calvo, G. and C. Rodriguez (1977), A Model of Exchange Rate Determination Under Currency Substitution and Rational Expectations, Journal of Political Economy, June.

Johnson, H. (1976), The Monetary Approach to Balance of Payments Theory, J. Frenkel and H. Johnson, eds., The Monetary Approach to the Balance of Payments, London, Allen & Unwin.

Kouri, P. (1975), Essays in the Theory of Flexible Exchange Rates, Thesis, MIT, chapter II.

__________ (1983), Balance of Payments and the Foreign Exchange Market: A Dynamic Partial Equilibrium Model, in J. Bhandari and B. Putnam (eds.), The International Transmission of Disturbances under Flexible Exchange Rates, MIT Press.

Macedo, J. (1981), Currency Inconvertibility, Portfolio Balance and Relative Prices, Research Program in Development Studies, Woodrow Wilson School, Princeton University, Discussion Paper no. 98, August.

__________ (1982a), Portfolio Diversificaiton and Currency Inconvertibility: Three Essays in Interntional Monetary Economics, Lisbon (Portugal); Universidade Nova de Lisboa

__________ (1982b), Currency Diversification and Export Competitiveness: A Model of the "Dutch Disease: in Egypt, Journal of Development Economics, December.