NBER WORKING PAPER SERIES

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Working Paper 10820
http://www.nber.org/papers/w10820

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
September 2004

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NBER Working Paper No. 10820
September 2004
JEL No. G11, G12

ABSTRACT

Traditional mean-variance efficient portfolios do not capture the potential wealth creation opportunities provided by predictability of asset returns. We propose a simple method for constructing optimally managed portfolios that exploits the possibility that asset returns are predictable. We implement these portfolios in both single and multi-period horizon settings. We compare alternative portfolio strategies which include both buy-and-hold and fixed weight portfolios. We find that managed portfolios can significantly improve the mean-variance trade-off, in particular, for investors with investment horizons of three to five years. Also, in contrast to popular advice, we show that the buy-and-hold strategy should be avoided.

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1 Introduction

The choice of the optimal investment strategy is a core concern in financial economics. Typical investment advice is to 'invest for the long run,' which usually means buy and hold assets for a given horizon. What are the implications of such an investment strategy? There are four important dimensions for the portfolio choice problem: the asset menu, the investment horizon, the current state of the economy, and the investor's risk aversion. In this paper, we evaluate the impact of all these dimensions.

The traditional implementation of the portfolio choice problem involves a mean-variance optimization over a set of asset expected returns, variances and covariances. When these moments are measured as sample averages based on historical data, the efficient portfolio weights are constant. That is, the investment weights are unaffected by economic conditions and the only action required is to rebalance the assets each period to achieve fixed weights. The possibility that asset returns are predictable creates an opportunity to construct dynamic trading strategies which offer superior risk and expected return trade-offs relative to standard portfolios. These managed portfolios are impacted by economic conditions. We augment the asset menu to include the dynamic trades of the primitive assets based on variables that predict returns. As a result, in our optimal portfolio, the investment weights change through time. We are able to immediately compare the performance of the traditional strategy (fixed weights) to the strategy that uses dynamic strategies (time-varying weights).

The idea of incorporating dynamic strategies into portfolio choice originates in Hansen and Richard (1987). Recent research using these insights includes Bansal, Hsieh, and Viswanathan (1993), Bansal and Harvey (1996), Ferson and Siegel (2001), Cochrane (2001), and Brandt and Santa-Clara (2004). The basic idea is as follows. There are two ways to solve the mean-variance portfolio choice problem when there is conditioning information. The first is to specify the joint conditional distribution of the asset returns and to solve for the optimal portfolio. An alternative method, proposed in Hansen and Richard (1987), is to add dynamic strategies to the menu of assets, and solve the much easier unconditional mean-variance problem. One limitation of previous research is that it has mostly focused on one-period portfolio problems.

The role of time-varying investment weights is most natural in the context of multi-horizon asset allocation problems. A common piece of asset management advice is to follow the buy-and-hold strategy. This strategy is equivalent to increasing the weight in the winner stocks and decreasing the weight in the losing stocks. An alternative strategy is the fixed-weight strategy. In this case, the investor always holds a fixed fraction of his wealth in a particular asset. The rebalancing necessary to maintain fixed weights has a contrarian flavor (i.e., selling the winners and buying the losers). The key advantage to both these strategies is that they are easy to implement. However, both of these strategies ignore information about the current state of the economy. Our research goal is to contrast the performance of these strategies with a portfolio choice method that incorporates dynamic trading strategies.

Our research makes three contributions. First, following Bansal and Harvey (1996), we present a simplified method of solving the mean-variance portfolio problem that incorporates conditioning information. We do this for both a single period and a long-horizon portfolio problem. Second, we characterize the degree of predictability in asset returns by comparing the performance of portfolio strategies that include or exclude conditioning information. Third, we compare and contrast dynamic strategies with both the fixed weights and buy-and-hold strategies over four popular portfolio choice problems.

To highlight the importance of predictability in dynamic asset allocation, Campbell and Viceira (1999) use a vector autoregression (VAR) approach. They use this structure to address issues about optimal dynamic asset allocation for an infinite lived agent who can rebalance each time period. Liu (2002) considers more parametric solutions (derived numerically) to derive optimal portfolio weights. Our long-lived investors, in contrast, design an optimal asset allocation plan at date t by trying to maximize the utility over wealth for date $t + h$. They understand that returns are predictable hence put fixed date t weights also on actively managed funds (i.e., dynamic trades) which incorporate conditioning information and let their portfolio evolve until date $t + h$. Our approach answers the question: at date t, what fraction of wealth should be put in different assets given, (i) a horizon h, (ii) quadratic preferences for the terminal date, (iii) access to actively managed funds, and (iv) the agent chooses not to actively change the allocation after date t, though the actively managed funds are allowed to do so.

In other words, our investor is a passive investor who invests in actively managed funds. The dynamic trading is done by the active managers—not directly by our investor. In this sense, our strategy is static from the perspective of the investor. We suspect that this environment captures the opportunity set and behavior for a sizable group of investors and hence is of considerable practical interest. The earlier paper by Bekaert and Liu (2004) considers one-period ahead asset allocation with conditioning information, but does not entertain, as in our paper, multi-horizon asset allocation problems.

Our results can be summarized as follows. First, investment in actively managed (market timing) funds relative to traditional fixed weight strategy produces considerable economic gains. Hence, active funds can be of great benefit to investors. Second, the traditional adage of buy-and-hold for long horizons is poor advice even compared to the traditional fixed-weight investment strategy which incorporates no conditioning information. Our evidence is in line with Samuelson (1994) who also argues that buy-and-hold may not necessarily be a good strategy. The strategy that incorporates conditioning information performs the best. Third, we show that the economic gains from using conditioning information increase with horizon of the portfolio allocation decision. Fourth, we present an simplified approach to think about multi-horizon portfolio choice which incorporates conditioning information.

The paper is organized as follows. In the second section we show how to construct the unconditional mean-variance frontier which incorporates dynamic trading strategies. We characterize the mean-variance frontier, discuss the portfolio choice problem of a mean-variance investor, and consider restricted trading strategies. This section also outlines a multi-horizon portfolio choice problem. The third section describes the data and present evidence on predictive regressions. In the fourth section, we evaluate different portfolio choice problems and measure the gains from dynamic trades. Some conclusions are offered in the final section.

2 Dynamic Trading Strategies

2.1 Characterizing Mean-Variance Frontiers

The traditional approach to mean-variance portfolio choice solves for fixed portfolio weights of returns, and ignores the fact that an investor can potentially rebalance their portfolio in an dynamic manner based on information available at date t. However, it makes sense that portfolio weights might change through time as the changes in the economic environment can alter the distribution of returns. The traditional approach can fail to exploit the potential benefits of predictability of returns (and the second moment matrix) in forming portfolios.

Predictability of returns and/or of their second moments creates new investment opportunities for the investor. These new opportunities are based on trading strategies that exploit the predictability of the returns. For example, if a rise in the dividend-yield suggests that future equity returns will be higher, then buying more equity as the dividend yield rises can create superior portfolio performance relative to the traditional approach. That is, by exploiting this predictability one can potentially have a "bigger" unconditional (i.e., average) investment opportunity set relative to the one that ignores this predictability. One may also want to evaluate the conditional opportunity set: the mean variance opportunity set for a given value for the dividend-yield and other predictive variables that determine the distribution of returns. Of course, if returns are i.i.d. then the incorporation of conditioning information variables will not provide a bigger investment opportunity set.

How can one characterize the conditional and the unconditional mean-variance opportunities? Hansen and Richard (1987) provide a simple characterization of these mean-variance frontiers. Both these frontiers exploit time-variation in the mean and second moment matrix of returns. The unconditional frontier includes both basis assets and dynamic strategies. This unconditional frontier is the optimal frontier in the sense that it provides the best risk-return trade off among the set of all dynamic trades that are feasible for an investor. Further, the unconditional frontier and the conditional frontier for a given collection of returns can be characterized by two specific portfolios discussed below. These two portfolios exploit the predictability in the mean and the variance-covariance matrix of returns that is important for a mean-variance investor. The ability to characterize the mean variance set, both conditionally and unconditionally, with the same set of two specific portfolios makes their approach attractive to use.

To describe the Hansen and Richard (1987) formulation of the conditional and unconditional frontiers, consider the following vectors. Let $\mathbf{R}_{t+1}$ denote a $K+1$ vector of gross returns between t and $t+1$. The first element in $\mathbf{R}_{t+1}$ is $R_{0,t+1}$, and other elements in $\mathbf{R}_{t+1}$ are $R_{i,t+1}$ ($i \neq 0$). Next, let $\mathbf{X}_{t+1}$ denote a K vector of excess returns with a typical element being $X_{i,t+1} = R_{i,t+1} - R_{0,t+1}$. The normalizing asset with return $R_{0,t+1}$ can be any of the $K+1$ assets. Further, we will refer to $\mathbf{R}$ as the entire set of gross returns and $\mathbf{X}$ as the set of all potential excess returns.

Hansen and Richard (1987) characterize the investment opportunity conditionally (that is, the relevant opportunity set at a given point in time, or for a given state), and also characterize it on average (that is, unconditionally, or on average across states). An important feature is that the conditional and unconditional frontiers are characterized by the same two dynamic trade-based payoffs $R^*_{t+1} \in \mathbf{R}$ and $X^*_{t+1} \in \mathbf{X}$.

The unit-cost payoff R^*_{t+1} is on the conditional mean variance frontier, and has the unique property that it is has the smallest noncentral conditional second moment (i.e., global minimum second noncentral moment return). It can be derived as the solutions to the following problem:

$$\min_{\mathbf{\Lambda}_t} \mathrm{E}_t \left(R^2_{t+1} \right), \tag{1}$$

where E_t denotes expectations conditional on information at date t and $\mathbf{\Lambda}_t$ is a vector of weights on the basis assets. The optimal portfolio R^*_{t+1} can be obtained by exploiting the first-order conditions associated with problem (1):

$$\mathrm{E}_t \left(R^*_{t+1} \mathbf{X}_{t+1} \right) = \mathbf{0}, \tag{2}$$

which implies that the dynamic portfolio weights satisfy

$$\mathbf{\Lambda}^{R^*}_t = -\mathrm{E}_t \left(\mathbf{X}_{t+1} \mathbf{X}'_{t+1} \right)^{-1} \mathrm{E}_t \left(\mathbf{X}_{t+1} R_{0,t+1} \right). \tag{3}$$

The zero-cost payoff X^*_{t+1} has the unique property that it has the largest conditional mean to conditional standard deviation ratio (i.e., conditional Sharpe ratio), and Hansen and Richard (1987) show that X^*_{t+1} can be obtained by exploiting

$$\mathrm{E}_t\left(X^*_{t+1}\mathbf{X}_{t+1}\right) = \mathrm{E}_t\left(\mathbf{X}_{t+1}\right), \tag{4}$$

which implies that the dynamic portfolio weights, $\mathbf{\Lambda}^{X*}_t$ satisfy

$$\mathbf{\Lambda}^{X^*}_t = \mathrm{E}_t\left(\mathbf{X}_{t+1}\mathbf{X}'_{t+1}\right)^{-1}\mathrm{E}_t\left(\mathbf{X}_{t+1}\right). \tag{5}$$

Note that all the portfolio weights are dynamic and depend on the first two conditional moments of the returns.

In section 2.2, we will explore the mean-variance portfolio choice problem. However, it is important to note that while the expressions for $\mathbf{\Lambda}^{R^*}_t$ and $\mathbf{\Lambda}^{X^*}_t$ are straightforward, the solutions are complicated because it is necessary to specify both the conditional first and second moments of the asset returns. In section 2.3, we propose a simple method to avoid specifying these moments.

All returns can be decomposed into convenient pieces, for example return $R_{i,t+1}$, can be stated as

$$R_{i,t+1} = R^*_{t+1} + R_{i,t+1} - R^*_{t+1}. \tag{6}$$

To characterize the mean-variance frontier, consider the following decomposition of the excess return

$$R_{i,t+1} - R^*_{t+1} = w_{i,t}X^*_{t+1} + \eta_{i,t+1}, \tag{7}$$

where $w_{i,t}$ is the conditional linear projection coefficient. Exploiting the orthogonality of excess returns with R^*_{t+1}, and that $\mathrm{Var}_t\left(X^*_{t+1}\right) = \mathrm{E}_t\left(X^*_{t+1}\right)\left[1 - \mathrm{E}_t\left(X^*_{t+1}\right)\right]$, the coefficient $w_{i,t}$ can be further simplified,

$$w_{i,t} = \frac{\mathrm{Cov}_t\left(X^*_{t+1}, R_{i,t+1} - R^*_{t+1}\right)}{\mathrm{Var}_t\left(X^*_{t+1}\right)} = \frac{\mathrm{E}_t\left(R_{i,t+1} - R^*_{t+1}\right)}{\mathrm{E}_t\left(X^*_{t+1}\right)}. \tag{8}$$

This property ensures that $\mathrm{E}_t\left(X^*_{t+1}\eta_{i,t+1}\right) = 0$ and $\mathrm{E}_t\left(\eta_{i,t+1}\right) = 0$.

Any return $R_{i,t+1}$ can be written as

$$R_{i,t+1} = R^*_{t+1} + w_{i,t}X^*_{t+1} + \eta_{i,t+1}. \tag{9}$$

The three pieces in the above decomposition are orthogonal to each other and the term $\eta_{i,t+1}$ has a zero mean. Consequently, the conditional mean-variance frontier is simply characterized by R^*_{t+1} and X^*_{t+1}, and entails no investment in the residual term $\eta_{i,t+1}$. The collection of all returns with a conditional mean of c_t are described by

$$R^*_{t+1} + \frac{c_t - \mathrm{E}_t\left(R^*_{t+1}\right)}{\mathrm{E}_t\left(X^*_{t+1}\right)} X^*_{t+1} + \eta_{t+1}, \tag{10}$$

where η_{t+1} is an error term orthogonal to R^*_{t+1} and X^*_{t+1}. The minimum-variance unit-cost payoff with a conditional mean of c_t is then

$$R^*_{t+1} + \frac{c_t - \mathrm{E}_t\left(R^*_{t+1}\right)}{\mathrm{E}_t\left(X^*_{t+1}\right)} X^*_{t+1}. \tag{11}$$

This follows from recognizing that η_{t+1} is a zero-cost payoff that adds nothing to the mean, but adds to the variance of the return with a target mean of c_t.

2.2 Mean-Variance Portfolio Choice

Consider an investor that maximizes conditional mean-variance utility with risk aversion parameter b:

$$\mathrm{E}_t\left(R_{t+1}\right) - \frac{b}{2}\mathrm{Var}_t\left(R_{t+1}\right), \tag{12}$$

subject to the constraint that this portfolio costs one dollar. All returns can be decomposed into R^*_{t+1}, X^*_{t+1}, and η_{t+1}. As η_{t+1} is a zero-cost portfolio that adds nothing to the mean but adds to the variance, it follows that a mean-variance agent will invest only in R^*_{t+1} and X^*_{t+1}. In particular, the investor's choice is the solution to the problem

$$\max_{w_t} \mathrm{E}_t\left(R^*_{t+1} + w_t X^*_{t+1}\right) - \frac{b}{2}\left[\mathrm{Var}_t\left(R^*_{t+1}\right) + w_t 2\mathrm{Var}_t\left(X^*_{t+1}\right) + 2w_t \mathrm{Cov}_t\left(R^*_{t+1}, X^*_{t+1}\right)\right]. \tag{13}$$

The investor's optimal portfolio weight then satisfies

$$w_t = \frac{\mathrm{E}_t\left(X^*_{t+1}\right) - b\mathrm{Cov}_t\left(R^*_{t+1}, X^*_{t+1}\right)}{b\mathrm{Var}_t\left(X^*_{t+1}\right)} = \frac{\left[1 + b\mathrm{E}_t\left(R^*_{t+1}\right)\right]\mathrm{E}_t\left(X^*_{t+1}\right)}{b\mathrm{Var}_t\left(X^*_{t+1}\right)}. \tag{14}$$

The key insight of Hansen and Richard (1987) is that the unconditional mean-variance frontier (which includes both basis assets and managed portfolios) can also be characterized by R^*_{t+1} and X^*_{t+1}. They show that the conditional moment restrictions (2) and (4), via the law of iterated expectations, also hold unconditionally. The same investor choosing a portfolio weight unconditionally, will hence choose w given by

$$w = \frac{\mathrm{E}\left(X^*_{t+1}\right) - b\mathrm{Cov}\left(R^*_{t+1}, X^*_{t+1}\right)}{b\mathrm{Var}\left(X^*_{t+1}\right)} = \frac{\left[1 + b\mathrm{E}\left(R^*_{t+1}\right)\right]\mathrm{E}\left(X^*_{t+1}\right)}{b\mathrm{Var}\left(X^*_{t+1}\right)}. \tag{15}$$

Even unconditionally, the investor's optimal portfolio choice involves dynamic trading as X^*_{t+1} and R^*_{t+1} are payoffs constructed from dynamic trades and the solutions to X^*_{t+1} and R^*_{t+1} involve conditional expectations. The optimal portfolio weight between X^*_{t+1} and R^*_{t+1} is fixed, but the weights given to individual securities in the portfolio are dynamically changing. That is, even though w is fixed, $\mathbf{\Lambda}^{R^*}_t$ and $\mathbf{\Lambda}^{X^*}_t$ are time-varying.

As a practical matter, even the optimal dynamic trades are hard to implement if the set of random returns are large as it is hard to credibly model the first two conditional moments of a large cross-section of assets.

2.3 Restricted Dynamic Trading Strategies

We now consider the practical solution to portfolio choice, by restricting the set of dynamic trading in particular ways. A consequence of restricting the dynamic trades is that the implied dynamic portfolio weights will not be exploiting all the information required to characterize the first two conditional moments. On the other hand, it seems reasonable to believe that adding the restricted set of dynamic trading strategies should bring us close to risk-expected return opportunities that obtain from exploiting all information needed for the first two conditional moments.

Define the conditioning information as $\mathbf{Z}_t$, which is an L vector of variables known at date t that help predict future returns. One element in $\mathbf{Z}_t$ is a constant. Now let $\mathbf{Y}_{t+1}$ denote the stacked KL vector of dynamic trading based payoffs, with a typical element being $X_{i,t+1}Z_{j,t}$. Note that each element of $\mathbf{Y}_{t+1}$ is a zero-cost payoff.[1] The set of portfolio holdings that are permitted in the restricted payoff structure are simply, $R_{0,t+1} + \mathbf{\Lambda}'\mathbf{Y}_{t+1}$, where $\mathbf{\Lambda}$ is a KL vector of constants that determines the portfolio weights. While these weights are fixed, as the conditioning information changes the weights on the K basis assets will change.

[1]Alternatively, it is possible to include squares of $\mathbf{Z}_t$ or even cross-products of $\mathbf{Z}_t$.

Relative to the traditional mean-variance problem, the inclusion of the dynamic trading based payoffs (or scaled excess returns) yield dynamic trades. The scaling variables can include non-parametric functions of past returns (e.g., squared returns motivated by ARCH and GARCH volatility models) allowing the investor to fully exploit all the predictability in the mean and the variance-covariance of returns. Intelligent choice of the scaling variables can allow the investor to significantly improve the unconditional investment opportunity set with a few scaling variables. One can also interpret the implementation our "restricted approach" as a traditional mean variance approach but conducted over the augmented asset menu that includes the basis assets and the scaled excess returns. We implement the characterization of the mean-variance frontiers using the Hansen and Richard approach as it is easy to implement and affords a simple link between the unconditional and conditional frontiers.

We construct the analogs of R^*_{t+1} and X^*_{t+1}, using the restricted trading strategies. The restricted benchmark payoffs corresponding to R^*_{t+1} can be derived by exploiting the equation analogous to equation (2):

$$\mathrm{E}((R_{0,t+1} + \mathbf{\Lambda}'\mathbf{Y}_{t+1})\,\mathbf{Y}_{t+1}) = \mathbf{0}. \tag{16}$$

It follows that the optimal parameter vector has to equal

$$\mathbf{\Lambda}^{R^*} = -\mathrm{E}\left(\mathbf{Y}_{t+1}\mathbf{Y}'_{t+1}\right)^{-1}\mathrm{E}(\mathbf{Y}_{t+1}R_{0,t+1}). \tag{17}$$

Next we can use equation (4), to solve for the portfolio weights for X^*_{t+1}.

$$\mathrm{E}\left(X^*_{t+1}\mathbf{Y}_{t+1}\right) = \mathrm{E}\left(\mathbf{Y}_{t+1}\right). \tag{18}$$

Hence,

$$\mathbf{\Lambda}^{X^*} = \mathrm{E}\left(\mathbf{Y}_{t+1}\mathbf{Y}'_{t+1}\right)^{-1}\mathrm{E}\left(\mathbf{Y}_{t+1}\right). \tag{19}$$

To characterize the unconditional mean-variance frontier, we now use the parameter vectors $\mathbf{\Lambda}^{R^*}$ and $\mathbf{\Lambda}^{X^*}$ to construct the restricted analogs of R^*_{t+1} and X^*_{t+1}. There is a critical difference between $\mathbf{\Lambda}^{R^*}_t$ in (2) and $\mathbf{\Lambda}^{X^*}_t$ in (4) and the above $\mathbf{\Lambda}^{R^*}$ and $\mathbf{\Lambda}^{X^*}$—the time subscripts on the weights.

In the original problem with the K basis assets, the weights that determine R^*_t and X^*_t change through time as a function of the conditional means, variances and covariances of the basis assets. In the above problem, the weights on the KL investments (which include both the basis assets and the basis assets multiplied by the conditioning information) are constant. The key insight is that as a result of time variation in the conditioning information the effective weights on the basis assets also change in this setup. It is straightforward to also solve for the weights of a mean-variance investor with a risk aversion parameter b as in the previous sub-section.

2.4 Multi-Horizon Portfolio Choice

2.4.1 Dynamic Portfolio Choice with Conditioning Information

The above analysis can be extended to consider mean-variance opportunity sets for horizons longer than one period. For expositional ease we show our results for a two period case; the case with longer horizons is a simple extension of the two period analysis.

Consider the one-period dynamic trade $R_{d,t+1} = R_{0,t+1} + Z_{j,t}(R_{i,t+1} - R_{0,t+1})$. The two period excess return if one were to invest γ dollars at date t in the portfolio is $\gamma\left(R_{d,t+1}R_{d,t+2} - R_{0,t+1}R_{0,t+2}\right)$. This payoff costs zero dollars at date t and the excess return is equal to

$$\gamma Z_{j,t}R_{0,t+2}(R_{i,t+1} - R_{0,t+1}) + \gamma Z_{j,t+1}R_{0,t+1}(R_{i,t+2} - R_{0,t+2}), \tag{20}$$

where we have ignored the small cross product term $\gamma^2 Z_{j,t}(R_{i,t+1} - R_{0,t+1})Z_{j,t+1}(R_{i,t+2} - R_{0,t+2})$. In other words, the two period excess return trade is essentially a dynamic trade on a series of one period excess returns. This simple description helps interpret our construct of the multi-horizon mean variance frontier discussed below.

The set of portfolio holdings that are permitted in the restricted payoff structure are simply, $R_{0,t+1}R_{0,t+2} + \mathbf{\Lambda}_2'\mathbf{Y}_{t,t+2}$, where $\mathbf{\Lambda}_2$ is a KL vector of constants, and a typical element of $\mathbf{Y}_{t,t+2}$ is $R_{d,t+1}R_{d,t+2} - R_{0,t+1}R_{0,t+2}$, as described above. Then we can construct the global minimum second (noncentral) moment portfolio, $R^*_{t,t+2}$ by solving:

$$\mathrm{E}((R_{0,t+1}R_{0,t+2} + \mathbf{\Lambda}_2'\mathbf{Y}_{t,t+2})\,\mathbf{Y}_{t,t+2}) = \mathbf{0}, \tag{21}$$

with

$$\mathbf{\Lambda}_2^{R^*} = -\mathrm{E}\left(\mathbf{Y}_{t,t+2}\mathbf{Y}'_{t,t+2}\right)^{-1}\mathrm{E}(\mathbf{Y}_{t,t+2}R_{0,t+1}R_{0,t+2}), \tag{22}$$

where the sub-script underscores that these parameters depend on the horizon under construction. Analogously, to determine the zero cost payoff with the maximal Sharpe ratio, we first set $X^*_{t,t+2} = \mathbf{\Lambda}'_2\mathbf{Y}_{t,t+2}$, and the solution for $\mathbf{\Lambda}_2$ must satisfy

$$\mathrm{E}\left(\left(\mathbf{\Lambda}'_{\mathbf{2}}\mathbf{Y}_{t,t+2}\right)\mathbf{Y}_{t,t+2}\right) = \mathrm{E}\left(\mathbf{Y}_{t,t+2}\right). \tag{23}$$

Hence,

$$\mathbf{\Lambda}_2^{X^*} = \mathrm{E}\left(\mathbf{Y}_{t,t+2}\mathbf{Y}'_{t,t+2}\right)^{-1}\mathrm{E}\left(\mathbf{Y}_{t,t+2}\right). \tag{24}$$

The above analysis can be extended to any horizon h by constructing

$$R_{0,t+1}R_{0,t+2}\cdots R_{0,t+h}$$

and $\mathbf{Y}_{t,t+h}$ with a typical element

$$R_{d,t+1}R_{d,t+2}\cdots R_{d,t+h} - R_{0,t+1}R_{0,t+2}\cdots R_{0,t+h}.$$

The solutions for the analogs of R^*_{t+h} and X^*_{t+h} can be constructed by solving for $\mathbf{\Lambda}_h^{R^*}$ and $\mathbf{\Lambda}_h^{X^*}$. Note that when h is one, then the problem coincides with the one period problem. Further, given $\mathbf{\Lambda}_h^{R^*}$ and $\mathbf{\Lambda}_h^{X^*}$, the mean-variance portfolio choice for a given horizon follows the same logic as discussed in the one period case in section 2.2.

2.4.2 Alternative Long Horizon Trading Strategies

In addition to the above dynamic trades, we also consider two other trading strategies. One is the classic buy-and-hold strategy. In this case, the agent invests at the initial date in different assets and then holds this position to the end of the horizon. Hence, the portfolio weights $w_{i,t}$ on a given asset evolves as

$$w_{i,t+1} = w_{i,t}\frac{R_{i,t+1}}{R_{p,t+1}}. \tag{25}$$

In a buy and hold strategy, the investor invests more in assets which have high gains relative to the overall portfolio return. This feature of the buy-and-hold also implies that at long horizons the portfolio becomes less balanced and may impose undue risks on the investor due to lack of diversification. In computing the optimal buy-and-hold strategy the choice variable is the initial portfolio weights—given this the evolution of weights across time is governed by equation (25).

The third long run strategy is the fixed weight strategy in the basis assets. This can be seen as a special case of the time-varying weight case where $\mathbf{Z}_t$ only contains a constant and no information variables. In this strategy, the investor's rebalancing resembles contrarian-like strategy. The investor buys more, in dollar terms, of the stocks that perform relatively poorly and sells the winning stocks.[2] This rebalancing ensures that the portfolio weights are constant across time. To estimate the portfolio weights, we need to estimate the starting weights and hold them fixed for the entire sample. In contrast to the buy-and-hold, this strategy involves rebalancing every period. However, both the fixed-weight and the buy-and-hold strategies have the common feature that they ignore all conditioning information.

3 Portfolio Choice Applications

3.1 Four Asset Allocation Problems

We consider four asset allocation problems which are detailed in Table 1. Problem I (labeled U.S. Equity/Fixed Income) considers the choice between Treasury Bills, Treasury Bonds, and a U.S. equity portfolio represented by the S&P 500. Problem II (World Equity) considers allocation to a measure of a world market portfolio (exclusive the U.S.) from MSCI in addition to Treasury Bills and the S&P 500. Problem III (U.S. Growth and Value) considers the allocation between portfolios of growth and value stocks in addition to Treasury Bills. Problem IV (U.S. Bonds) considers the allocation between Treasury Bills, Treasury Bonds, and portfolios of corporate bonds and mortgages. All problems consider real returns (nominal returns are adjusted for inflation). Note that

[2] This is only true if there is no leverage. For example, consider the case with one stock and one riskless asset. Assuming that the fixed weight of the stock is 200%. If the initial wealth is \$100. The investor will borrow \$100 to buy \$200 of the stock. Assuming that the gross riskless return is 1 and the stock value goes to \$300. The total wealth of the investor before rebalance is \$200. For the stock weight to be 200%, the investor needs to borrow another \$100 to buy additional \$100 of the stock.

all asset returns (including Treasury Bills) are risky in real terms as there is inflation uncertainty. The sample periods are determined by data availability as shown in the table, and vary across the four problems.

Dynamic trading strategies are formed by conditional information variables. These variables together with the actual returns on the above asset classes are described next. In our set up, we solve for the optimal portfolio weights for investment in actively managed funds. As such, we make no allowances for transactions costs. In practice, the historical performance of the active managers would already reflect the transactions costs and our method could be directly applied. However, in the following examples, we create example active strategies which are presumed to be executed on futures with low transactions costs.[3]

3.2 Summary Statistics and Predictability

Some summary statistics on the eight asset classes used for portfolio choice problems are presented in Panel A of Table 2. The information variables that will be used to form dynamic trading strategies are: the S&P 500 dividend yield (labeled Dividend Yield), the Baa-Aaa yield spread (Spread), the difference of a 10-year Treasury and a 90-day Treasury bond yield (Slope), and the yield on the 90-day Treasury bill (Level). The information variables are all lagged one month. The choice of the conditioning variables is motivated by the research of Keim and Stambaugh (1986), Campbell (1987), Fama and French (1988), and Harvey (1989). Information on data sources are given in the table.

The average real returns on the S&P 500 and the MSCI World portfolios have been 70bp and 62bp per month, or about 8% per year, during their sample periods. The average return on the T-bill portfolio has been about 12bp per month. The average returns on the bond portfolios have been between 29bp and 40bp per month. The average return on the portfolio of Value stocks has been about 90bp per month, 21bp higher than the return on the portfolio of Growth stocks.

[3]Balduzzi and Lynch (1999) and Lynch and Balduzzi (2000) show, in a different setting, that predictability of asset returns has large effects on rebalancing even in the presence of transaction costs.

The standard deviations of the equity portfolios have been between 3% and 4% per month, or between 10% and 14% per year. The standard deviations of the bond portfolios have been about half of the equity levels, that is, about 5% to 7% per year.

The lowest monthly returns on the equity portfolios (about -15%) were realized in October 1987. The lowest and highest realized return on the bond portfolios were during the turbulent period from 1979 to 1982 when the Federal Reserve experimented with their monetary policy.

Panel B of Table 2 reports the correlations of the real returns and the conditional information variables. The portfolios of corporate bonds and mortgages have a return correlation of 86%. The correlation between the portfolios of growth and value stocks is about 80%, lower than each of the two portfolios versus the S&P 500. The correlations between the U.S. portfolios (equity and bonds) and the MSCI world excluding U.S. portfolio are, except for the S&P 500, less than 60%. The correlations between the equity and bond portfolios are about 20% to 30%. Among the information variables, it is only the correlation between Level and Slope that is higher than 60%. Hence, the information variables seem to measure different aspects of the state of the economy.

To gauge the potential gains from dynamic trades, we project the excess returns (the returns on seven of the asset class portfolios over and above the return on the T-bill) on a constant and each of the four information variables. [Recall that there are gains from dynamic trading strategies if the information variables contain information on the first two moments of the primitive assets. It is straightforward to show that predictability of the excess returns is a sufficient condition for dynamic trading to be important for an investor with mean-variance preferences.] Regression adjusted R-squares are reported in Panel A of Table 2.

The adjusted R-squares are about 2% to 3%. The variables are jointly significant for almost all asset classes (the p-values from a Wald test of jointly significant coefficients are effectively 0%). Only the test on the Mortgage portfolio is marginally significant (a p-value of 6%). Note that the Mortgage portfolio is the portfolio with fewest number of observations. Overall, we find significant coefficients for the Dividend Yield, Spread, and Level variables, especially in the predictability regression for the U.S. equity portfolios. Further, the Slope variable is highly significant for the bond portfolios.

4 Evaluating Trading Strategies

4.1 One-Period Portfolio Choice

Table 3 highlights the importance of conditioning information in the context of static portfolio choice. There is a substantial increase in the maximal Sharpe Ratio that one can obtain by using the predictive variables. Column two provides the Sharpe ratio based on the traditional mean variance analysis (fixed weights). The incorporation of the conditioning information increases the Sharpe ratio by about 30% to 70% relative to the traditional Sharpe ratios. For example, the monthly Sharpe ratio is 0.18 for the U.S. Equity/Fixed Income problem where the weights are held constant and 0.27 when dynamic trading strategies are introduced.

For various risk aversion levels there is substantial rise in utility associated with using conditioning information. The gain in utility is reported in the last column of the table – the certainty equivalent (CE) loss. For example, with a risk aversion parameter of 10 for the U.S Equity/Fixed Income allocation problem the increase in utility corresponds to 20bp per month with a bootstrapped standard error of 11bp. Stated differently, in order to achieve the same utility as the time-varying (dynamic) weight investor, the fixed weight investor must have a 2.4% increment in annual return.

The standard errors in Table 3, and standard errors and confidence bands in subsequent tables and figures, are generated in a parametric bootstrap. Returns and predictive variables are modeled as a VAR(1) where the residuals are re-sampled. The portfolio choice is carried out using these generated data and used to construct the statistic of interest. 3,000 replications were used in constructing standard errors and (bias-adjusted) confidence bands.

The substantial increase in the mean-variance trade-off can also be seen in Figure 1, which depicts 95% confidence bands for the unconditional mean –standard deviation frontiers for fixed weight and time-varying (dynamic) weight strategies.

4.2 Multi-Horizon Portfolio Choice

The multi-horizon portfolio choice results are presented in Table 4. We evaluate the importance of conditioning information in a multi-horizon context by comparing the Sharpe ratios of the fixed weight strategy to the dynamic strategies for different holding periods.

The results in Table 4 show, in general, that conditioning information is more important in longer horizon portfolio problems. That is for a given horizon, the relative ratio (ratio of dynamic strategy's Sharpe ratio to the fixed weight strategy's Sharpe ratio) is larger at longer horizons for most of the portfolio allocation problems.

We also report the confidence interval for long-horizon risk return trade-offs (i.e., for the Sharpe ratio of X^*). In almost all cases, the 95% confidence interval lie above the Sharpe ratio of the fixed weight strategy, indicating that increases in Sharpe ratios due to dynamic strategies are statistically significant.

A comparison of the various portfolio choices also shows that the biggest impact of conditioning information is in the context of the U.S. Bonds allocation. The addition of conditioning information more than doubles the amount of expected return for a given level of risk in each of the multiperiod scenarios. In the five-year horizon, the Sharpe ratio of the dynamic strategy is almost four times greater than the fixed-weight Sharpe ratio.

To further evaluate the importance of conditioning information we also report the utility level for different horizons, risk aversion, and alternative portfolio choices in Table 5. The utility level from the restricted dynamic multi-horizon portfolio choice dominates the traditional fixed weight allocation prescriptions in almost all cases. In many cases, the gain in utility is more pronounced at longer horizons (i.e., 3-5 years). Consistent with Table 4, the largest differences are found with the U.S. Bonds choice problem.

The bootstrap standard errors reveal that the utility gains are most significant for domestic equity portfolios. For example, in the U.S. Equity/Fixed Income and the Growth/Value portfolios, the gains are more than two standard errors from zero in 29 of 30 cases for the 2–4 year holding horizons. The significance levels are the weakest for the international equity portfolios – which also has a much shorter sample than the domestic equity portfolios. The utility gains for the bond portfolios also suffer from a shorter sample. The most significant utility gains are for shorter horizon portfolios (4 of 5 cases for the one-year horizon). In the 2-4 year holding periods, the utility gains are always more than one standard error from zero.

4.3 Fixed weight and buy-and-hold

In Table 6, we compare two alternative long-horizon portfolio choices; the fixed weight portfolio at different horizons and a buy and hold strategy. We restrict our attention to the U.S. Equity/Fixed Income allocation problem.[4] Portfolio weights for an asset class are restricted to be between 0% and 100%.[5]

Table 5 showed that dynamic strategies are superior to the fixed weight strategies. Table 6 shows that fixed weight strategies do better than buy-and-hold strategies. However, the difference between fixed weight and buy and hold is small. Note that the utility differences reported in Table 6 are total differences over the investment horizon whereas Table 5 reported monthly gains. Nevertheless, for the portfolios in the 2–5 year horizon, the fixed weight strategy often produces significantly more utility than the buy-and-hold strategy. In this comparison, the largest and most significant differences are found for the investor with low risk aversion and with the longest horizon.

For a given risk aversion and horizon, Table 6 shows that the utility from fixed weights is higher than that of buy and hold for all portfolio problems. The intuition for this is simply that buy and hold leads to lack of diversification across time. Longer the horizon, the greater is the relative loss in utility—this again, is due to the poor diversification properties of the buy-and-hold strategy which gets worse with horizon. Our estimation (not reported) shows that the initial weight for the buy and hold strategy is lower than the weight in the fixed-weight strategy. Given the structure of the evolution of the portfolio weights for buy-and-hold, the weight to the highest mean return asset increases with time.

[4]The results for the other portfolio choice problems are available on request.
[5]The qualitative results are not sensitive to the imposed short sales restrictions.

5 Conclusions

Portfolio choice with conditioning information usually involves specifying the joint conditional distribution of asset returns, extracting the conditional means, covariances and variances and optimizing every period. Following Hansen and Richard (1987), we offer a simple alternative. By adding the conditioning information directly to the asset menu (by scaling returns by lagged information variables), one can solve the unconditional mean-variance problem and obtain a single set of weights. As the information variable change through time, the effective weights on the basis assets change. The addition of these scaled returns, or dynamic trading strategies, makes conditional portfolio selection straightforward.

Our paper solves the portfolio problem of the investor who wants the benefits associated with incorporating conditioning information, but would prefer to delegate the dynamic trading to active managers.

In addition to operationalizing the conditional portfolio selection problem, our paper implements the selection problem in both single and multi-period investment horizons. We are able to compare three different strategies: the dynamic strategy resulting from adding scaled returns to the selection problem, fixed weight traditional mean-variance selection, and the popular buy and hold strategy. We study four popular portfolio choice problems.

Our results are as follows. Conditioning information is important. Even with low levels of predictability, there is a substantial loss in opportunity when fixed-weight strategies (which assume no predictability) are implemented relative to the dynamic portfolio strategies that incorporate conditioning information. The popular buy-and-hold strategy performs even worse than the fixed-weight strategy. The buy and hold is a particularly poor strategy for longer-horizon investment decisions. The intuition is that the buy and hold leads to substantially undiversified portfolios in the long-term. Contrary to popular advice, the buy-and-hold strategy should be avoided.

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Table 1: Asset Allocation Problems

Asset Allocation Problem	Asset Classes Included	Sample Period
I. U.S. Equity/Fixed Income	T-Bills, T-Bonds, and S&P 500	1953–2003
II. World Equity	T-Bills, S&P 500, and MSCI World (exclusive U.S.)	1970–2003
III. U.S. Growth/Value	T-Bills, Growth Stocks, and Value Stocks	1953–2003
IV. U.S. Bonds	T-Bills, T-Bonds, Corporate Bonds, and Mortgage Bonds	1976–2003

This table shows four asset allocation problems. The actual data used are described in Table 2.

Table 2: Summary Statistics of Real Returns and Conditional Information Variables

	Inclusion Date	Mean	Standard Deviation	Adjusted R-square	T	Correlations 1.	2.	3.	4.	5.	6.	7.	8.
Panel A. Real Returns													
1. T-Bills	53-05	0.12	0.24	—	603	1.00							
2. T-Bonds	53-05	0.29	2.24	1.98	603	0.36	1.00						
3. S&P 500	53-05	0.70	3.45	2.70	603	0.22	0.21	1.00					
4. MSCI World	70-01	0.62	4.00	2.87	403	0.33	0.25	0.61	1.00				
5. Growth Stocks	53-05	0.69	3.76	2.87	603	0.21	0.19	0.97	0.59	1.00			
6. Value Stocks	53-05	0.90	3.76	2.56	603	0.21	0.16	0.87	0.53	0.79	1.00		
7. Corporate Bonds	53-05	0.30	1.86	2.82	603	0.34	0.90	0.28	0.24	0.25	0.25	1.00	
8. Mortgage Bonds	76-01	0.40	1.55	3.56	331	0.41	0.81	0.26	0.22	0.23	0.23	0.86	1.00
Panel B. Conditional Information Variables													
1. Dividend Yield	53-05	3.48	1.17	—	603	1.00							
2. Spread	53-05	0.95	0.42	—	603	0.40	1.00						
3. Slope	53-05	1.03	1.30	—	603	-0.18	0.11	1.00					
4. Level	53-05	5.61	3.11	—	603	0.43	0.58	-0.49	1.00				

This table presents summary statistics of monthly real dollar returns for eight asset classes from inclusion date to July 2003 (Panel A), and conditioning information variables from May 1953 to July 2003 (Panel B). The S&P 500 and MSCI World (exclusive U.S.) portfolios are taken from Datastream. The Value and Growth portfolios up to December 2002 are collected from the web site of Kenneth French. Observations on the Value and Growth portfolios in 2003 are from Wilshire. T-Bill, T-Bond, and Corporate Bond portfolios are from Ibbotson Associates. The Mortgage portfolio is from Merrill Lynch. All portfolio returns are originally in nominal terms, but they are adjusted for inflation (taken from Ibbotson). The Dividend Yield variable is the dividend yield on the S&P 500 in Datastream. The Spread variable is the difference between the yields on Baa-rated bonds and Aaa-rated bonds. The Slope variable is the difference between the yields on U.S. 10-year Treasuries and 90-day Treasury bills. The Level variable is the yield on a 90-day Treasury. The Spread, Slope, and Level data constructed from Federal Reserve data. The means and standard deviations for the real returns are expressed in % per month. The inclusion date (year-month) is the first month with observations. The column labeled Adjusted R-squares shows the coefficient of determination expressed in % in predictability regressions of portfolio excess returns on the lagged conditioning information variables. The excess returns are real portfolio returns minus the real return on the T-bill portfolio. T refers to the number of observations. The right-hand side of the table shows correlations between returns on the asset classes (in Panel A) and between the conditional information variables (in Panel B).

Table 3: Fixed Weight versus Time-Varying Weight Strategies

	Fixed Weights			Time-Varying Weights				
Risk Aversion	Mean	Standard Deviation	Ratio for X^*	Mean	Standard Deviation	Ratio for X^*	Certainty Equivalent	
I. U.S. Equity/Fixed Income								
5	0.73	3.52	0.18	1.51	5.27	0.27	0.39	(0.21)
10	0.42	1.76	0.18	0.80	2.62	0.27	0.20	(0.11)
15	0.32	1.17	0.18	0.57	1.74	0.27	0.13	(0.07)
20	0.26	0.89	0.18	0.45	1.31	0.27	0.10	(0.05)
25	0.23	0.72	0.18	0.38	1.05	0.27	0.08	(0.04)
II. World Equity								
5	0.57	2.97	0.15	1.42	5.08	0.26	0.43	(0.22)
10	0.34	1.49	0.15	0.76	2.52	0.26	0.21	(0.11)
15	0.27	1.00	0.15	0.54	1.67	0.26	0.14	(0.07)
20	0.23	0.76	0.15	0.43	1.25	0.26	0.11	(0.05)
25	0.21	0.62	0.15	0.37	1.00	0.26	0.08	(0.04)
III. U.S. Growth/Value								
5	0.99	4.18	0.21	1.65	5.53	0.28	0.33	(0.17)
10	0.55	2.08	0.21	0.87	2.75	0.28	0.16	(0.08)
15	0.40	1.39	0.21	0.62	1.83	0.28	0.11	(0.06)
20	0.33	1.05	0.21	0.49	1.37	0.28	0.08	(0.04)
25	0.28	0.84	0.21	0.41	1.10	0.28	0.06	(0.03)
IV. U.S. Bonds								
5	0.52	3.34	0.17	1.86	5.80	0.29	0.58	(0.41)
10	0.35	1.66	0.17	0.99	2.86	0.29	0.29	(0.20)
15	0.29	1.11	0.17	0.71	1.89	0.29	0.20	(0.14)
20	0.26	0.84	0.17	0.56	1.41	0.29	0.15	(0.10)
25	0.24	0.68	0.17	0.48	1.12	0.29	0.12	(0.08)

This table characterizes the portfolio choice for mean-variance investors with different risk aversion parameters (b = 5, 10, 15, 20, and 25) for the four asset allocation problems in Table 1. The asset allocation is based monthly data that covers 1953-2003, 1970-2003, 1953-2003, and 1976-2003 for the four asset allocation problems, respectively. The fixed weight strategy is the result of the traditional mean-variance problem. A time-varying (dynamic) weight strategy refers to the strategy that accommodates the predictability of excess returns. The $\mathbf{Z}_t$ vector used to form the strategy contains a constant and the conditional information variables Dividend Yield, Slope, Spread, and Level. Mean and Standard Deviation refer to resulted mean return and standard deviation given an investor's risk aversion. Ratio for X^* refers to the mean to standard deviation ratio of the zero-cost portfolio X^* (same for all risk aversion levels); it is the highest Sharpe Ratio (on a monthly basis). Certainty equivalent refers to the difference in utility of the investor for the fixed weight and time-varying weight strategies, which is a measure of the potential improvement of considering dynamic trades. A bootstrap standard error is reported next to the certainty equivalent measure and within parenthesis. Means, standard deviations, and certainty equivalent are expressed in % per month. The Ratio of X^* is expressed in per month.

Table 4: Characterizing Multi-Horizon Strategies

Horizon	Fixed Weights: Ratio for X^*	Fixed Weights: Confidence Band	Time-Varying Weights: Ratio for X^*	Time-Varying Weights: Confidence Band	Relative Ratio
I. U.S. Equity/Fixed Income					
12	0.15	[0.12 : 0.18]	0.21	[0.16 : 0.27]	1.40
24	0.14	[0.11 : 0.18]	0.25	[0.19 : 0.32]	1.78
36	0.15	[0.11 : 0.20]	0.26	[0.20 : 0.35]	1.78
48	0.15	[0.12 : 0.21]	0.26	[0.19 : 0.38]	1.70
60	0.15	[0.11 : 0.21]	0.25	[0.17 : 0.38]	1.72
II. World Equity					
12	0.14	[0.11 : 0.17]	0.25	[0.19 : 0.33]	1.77
24	0.14	[0.10 : 0.18]	0.23	[0.16 : 0.33]	1.67
36	0.14	[0.10 : 0.20]	0.21	[0.13 : 0.32]	1.44
48	0.15	[0.10 : 0.21]	0.23	[0.13 : 0.36]	1.54
60	0.15	[0.10 : 0.22]	0.23	[0.12 : 0.39]	1.54
III. U.S. Growth/Value					
12	0.17	[0.15 : 0.20]	0.20	[0.15 : 0.25]	1.13
24	0.18	[0.15 : 0.22]	0.23	[0.18 : 0.30]	1.31
36	0.20	[0.16 : 0.25]	0.27	[0.21 : 0.35]	1.35
48	0.22	[0.18 : 0.28]	0.30	[0.23 : 0.39]	1.34
60	0.21	[0.17 : 0.27]	0.31	[0.24 : 0.43]	1.48
IV. U.S. Bonds					
12	0.14	[0.11 : 0.19]	0.30	[0.21 : 0.42]	2.12
24	0.14	[0.09 : 0.20]	0.34	[0.22 : 0.51]	2.48
36	0.15	[0.10 : 0.22]	0.35	[0.19 : 0.58]	2.41
48	0.15	[0.09 : 0.25]	0.42	[0.21 : 0.69]	2.84
60	0.15	[0.07 : 0.26]	0.56	[0.33 : 0.88]	3.84

This table presents results of portfolio choice problems for different horizons (h = 1, 12, 24, 36, 48, and 60 months) for the four asset allocation problems in Table 1. The asset allocation is based monthly data that covers 1953-2003, 1970-2003, 1953-2003, and 1976-2003 for the four asset allocation problems, respectively. The fixed weight strategy is the result of the traditional mean-variance problem. A time-varying (dynamic) weight strategy refers to the strategy that accommodates the predictability of excess returns. The $\mathbf{Z}_t$ vector used to form the strategy contains a constant and the conditional information variables Dividend Yield, Slope, Spread, and Level. Ratio for X^* refers to the mean to standard deviation ratio of the zero-cost portfolio X^* (same for all risk aversion levels); it is the highest Sharpe Ratio (on a monthly basis) for various horizons. The ratio is divided by the square root of the horizon to be expressed on a monthly basis. Relative Ratio is the ratio for X^* in a time-varying weight strategy over the ratio for X^* in the fixed weight strategy. Confidence Band refers to 2.5% and 97.5% percentiles of bootstrapped distributions of the Ratio for X^*.

Table 5: Certainty Equivalent: Time-Varying Weights versus Fixed Weights

Risk Aversion	Horizon 12		24		36		48		60	
I. U.S. Equity/Fixed Income										
5	0.23	(0.13)	0.44	(0.15)	0.46	(0.17)	0.43	(0.21)	0.40	(0.25)
10	0.12	(0.07)	0.22	(0.07)	0.23	(0.09)	0.21	(0.11)	0.19	(0.13)
15	0.09	(0.04)	0.15	(0.05)	0.16	(0.06)	0.14	(0.07)	0.12	(0.09)
20	0.07	(0.03)	0.12	(0.04)	0.12	(0.04)	0.10	(0.05)	0.08	(0.06)
25	0.06	(0.03)	0.10	(0.03)	0.10	(0.04)	0.08	(0.04)	0.06	(0.05)
II. World Equity										
5	0.41	(0.16)	0.31	(0.19)	0.18	(0.23)	0.27	(0.29)	0.28	(0.37)
10	0.20	(0.08)	0.14	(0.09)	0.08	(0.11)	0.13	(0.14)	0.14	(0.19)
15	0.14	(0.05)	0.08	(0.06)	0.05	(0.08)	0.09	(0.10)	0.09	(0.13)
20	0.10	(0.04)	0.06	(0.05)	0.04	(0.06)	0.07	(0.07)	0.07	(0.10)
25	0.08	(0.03)	0.04	(0.04)	0.03	(0.05)	0.06	(0.06)	0.06	(0.08)
III. U.S. Growth/Value										
5	0.08	(0.09)	0.22	(0.10)	0.32	(0.12)	0.39	(0.15)	0.50	(0.18)
10	0.04	(0.05)	0.11	(0.05)	0.16	(0.06)	0.20	(0.07)	0.25	(0.09)
15	0.03	(0.03)	0.08	(0.03)	0.12	(0.04)	0.14	(0.05)	0.17	(0.06)
20	0.02	(0.02)	0.06	(0.03)	0.09	(0.03)	0.11	(0.04)	0.14	(0.04)
25	0.02	(0.02)	0.06	(0.02)	0.08	(0.02)	0.10	(0.03)	0.12	(0.04)
IV. U.S. Bonds										
5	0.74	(0.36)	0.95	(0.57)	1.01	(0.89)	1.53	(1.20)	2.89	(1.67)
10	0.38	(0.18)	0.47	(0.29)	0.50	(0.45)	0.77	(0.60)	1.45	(0.84)
15	0.26	(0.12)	0.31	(0.19)	0.33	(0.30)	0.51	(0.40)	0.97	(0.56)
20	0.20	(0.09)	0.23	(0.15)	0.25	(0.22)	0.39	(0.30)	0.73	(0.42)
25	0.17	(0.07)	0.19	(0.12)	0.20	(0.18)	0.31	(0.24)	0.59	(0.34)

This table presents the certainty equivalent between a time-varying weight strategy and a fixed weight strategy for mean-variance investors with different risk aversion parameters (b = 5, 10, 15, 20, and 25) and over different horizons (h = 12, 24, 36, 48, and 60 months) for the four asset allocation problems in Table 1. The asset allocation is based monthly data that covers 1953-2003, 1970-2003, 1953-2003, and 1976-2003 for the four asset allocation problems, respectively. The certainty equivalent is multiplied by 100 and divided by the horizon to be expressed in % and on a monthly basis. A bootstrap standard error is reported next to the certainty equivalent measure and within parenthesis. The fixed weight strategy is the result of the traditional mean-variance problem. A time-varying (dynamic) weight strategy refers to the strategy that accommodates the predictability of excess returns. The $\mathbf{Z}_t$ vector used to form the strategy contains a constant and the conditional information variables Dividend Yield, Slope, Spread, and Level.

Table 6: Certainty Equivalent: Fixed Weights versus Buy-and-Hold Weights

Risk Aversion	Horizon 12		24		36		48		60	
5	0.03	(0.03)	0.35	(0.12)	0.86	(0.26)	1.69	(0.45)	2.86	(0.84)
10	0.05	(0.03)	0.34	(0.11)	0.97	(0.27)	2.02	(0.58)	3.07	(1.05)
15	0.04	(0.03)	0.23	(0.09)	0.70	(0.28)	1.41	(0.62)	1.70	(1.10)
20	0.03	(0.02)	0.16	(0.08)	0.47	(0.27)	0.80	(0.61)	0.77	(1.06)
25	0.03	(0.02)	0.10	(0.07)	0.30	(0.26)	0.39	(0.58)	0.24	(1.01)

This table presents the certainty equivalent between a fixed weight strategy and a buy-and-hold weight strategy for mean-variance investors with different risk aversion parameters (b = 5, 10, 15, 20, and 25) and over different horizons (h = 12, 24, 36, 48, and 60 months) for the U.S. Equity/Fixed Income asset allocation problem in Table 1. The sample period is 1953-2003. The certainty equivalent is in percent over the entire investment horizon. A bootstrap standard error is reported next to the certainty equivalent measure and within parenthesis. The fixed weight strategy is the result of keeping fixed weights over the horizon. The buy-and-hold weight strategy refers to the strategy where initial weights are chosen and future weights reflect the initial weights updated according to past performance. Weights for each asset class are restricted to be between 0% and 100% for the fixed-weight and buy-and-hold strategies.

Figure 1: Mean – Standard Deviation Frontiers: Fixed and Time-Varying Weights

The figure shows 95% confidence bands for unconditional mean – standard deviation frontiers for fixed weight (solid curve) and time-varying weight (dashed curve) strategies for the four asset allocation problems in Table 1. The asset allocation is based monthly data that covers 1953-2003, 1970-2003, 1953-2003, and 1976-2003 for the four asset allocation problems, respectively. The curves represent the lower (2.5%) and upper (97.5%) frontiers for the strategies. The $\mathbf{Z}_t$ vector used to form the time-varying weight strategy contains a constant and the conditional information variables Dividend Yield, Slope, Spread, and Level. The filled squares show the averages and standard deviations of the real returns of the base assets in each asset allocation problem.