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ABSTRACT

In this paper, we analyze the determinants of international movements of physical capital in a model with uncertainty and international trade in goods and securities.

In our model, the world allocation of capital is governed, to some extent, by the asset preferences of risk averse consumer-investors. In a one-good variant in the spirit of the MacDougall model, we find that relative factor abundance, relative labor force size and relative production riskiness have separate but interrelated influences on the direction of equilibrium capital movements. These same factors remain important in a two-good version with Heckscher-Ohlin production structure. In this case, the direction of physical capital flow is determinate (unlike in a world of certainty), and may hinge on the identity of the factor which is used intensively in the industry with random technology.

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I. Introduction

The theory of international trade has been extended in recent years to incorporate uncertain trading environments. The early writers in this area (e.g. Kemp and Liviatan (1973), Turnovsky (1974) and Batra (1975)) argued that the introduction of randomness into the standard deterministic models had proven to be very damaging to many orthodox results, including those concerning the pattern of trade. However, as Helpman and Razin (1978a, 1978b) later showed, many of the negative findings were due to the implicity assumed absence of markets for international risk sharing in those early models. When international trade in equities is admitted as a possibility, a number of the familiar theorems are restored. Under such conditions, if uncertainty takes the form of industry-specific (but not country-specific) multiplicative technological shift factors, and if certain restrictions are placed on agents' utility functions, then, as Anderson (1981) has shown, the usual comparative cost considerations re-emerge as the determinants of the pattern of trade in securities.[1] Trade in commodities may also be predicted by the cost-based (i.e. Ricardian and Heckscher-Ohlin) theorems, at least in an "on average" sense.

International movements of physical capital, even more so than the flow of goods, may be influenced by the existence of technological uncertainty. The worldwide allocation of capital takes place largely before the resolution of uncertainty, and is motivated by, among other factors, the desire on the part of risk-averse agents to hedge against risk. Capital flows not only into those sectors and countries where its expected marginal product is high, but also into those which singly or together provide investors with a relatively stable pattern of income across states of nature.

In this paper, we study the interrelationship between international capital movements and international trade in securities under conditions of technological uncertainty. The problem takes on greatest interest under the assumption that random disturbances in an industry are not perfectly correlated across countries. Interestingly, the existence of such uncertainty introduces some fundamentally new elements into the determination of the direction and level of capital movements. Essentially, the general equilibrium supply functions for real equities derive from the familiar supply relationships. But the demands for equities also have an important qualitative effect on the equilibrium allocation of resources, even when all individuals in both countries have identical and homethetic tastes for goods and assets.

We begin, in the next section, with a model in the spirit of MacDougall (1960). In addition to the usual influence of the autarky factor-endowment ratios, we find an important role for the relative sizes of the labor forces and for the distributions of the random technology variables, in the determination of the volume and direction of capital movements. Indeed, capital may flow to the relatively capital-abundant country, even if the risks in the two countries are entirely symmetrical.

In Section III, we extend the model to include an internationally-traded safe asset, i.e. a traded bond. Under a restriction on the utility functions that is analogous to that needed to prove the Heckscher-Ohlin theorem (i.e. internationally identical and homothetic tastes for goods and assets), the introduction of the bond market does not alter the conclusions of Section II.

Finally, in Section IV, we investigate a two-good variant, adopting the framework of the Heckscher-Ohlin model. Whereas the nonrandom model is characterized by perfect substitution between commodity trade and factor movements (see

Mundell, 1957), and therefore by an indeterminacy in the level and direction of goods trade and capital movements, the equilibrium conditions under uncertainty determine nontrivially the volumes of all these flows (as well as trade in securities).[2] Under the assumption that technology in one of the industries in each country is nonstochastic, we are able to identify the separate roles of the relative sizes of the two labor forces and the relative factor intensities of the two industries in the determination of the direction of physical capital movements.

Our results are summarized in a concluding section.

II. The MacDougall Model with Uncertainty

The simplest model in which capital movements can be analyzed is one with two countries, one good and two factors. MacDougall (1960) developed such a model to study the welfare implications of capital movements in a deterministic world under a variety of assumptions about technology, the behavior of labor, market structure and tax policy.

In this paper, we are interested only in the positive implications of the simplest variant of the MacDougall formulation (i.e. constant-returns-to-scale production functions, fixed labor supplies, perfect competition and laissez-faire). In this case, equilibrium is characterized by equalization of the marginal products of capital in the two countries. Such an equilibrium is illustrated in Figure 1, where the horizontal dimension of the box represents the fixed world endowment of capital, and the marginal product of capital as a function of the capital allocated to the home (foreign) country is plotted with respect to the origin at the left (right) of the figure. The equilibrium allocation is at E, while A_1 and A_2 are two possible autarky allocations. At A_1, the marginal product of capital at home exceeds that in the foreign country, whereas the opposite is true at A_2.

Suppose production functions are the same in the two countries. Then A_1 must be characterized by a higher capital-to-labor ratio abroad than at home, and A_2 by the reverse situation. Evidently, if technologies are the same and nonrandom, capital flows to the country which, in autarky, has a greater relative abundance of labor.

We begin our analysis of capital movements under uncertainty by introducing technological randomness into each of the countries of the MacDougall model, and allowing for international trade in equities in the manner of Helpman and Razin.

Let the S possible states of nature be indexed by α, $\alpha = 1,2,\ldots,S$. Each state of nature is defined by the realization of two country-specific random variables, $\theta(\alpha)$ and $\theta^*(\alpha)$, corresponding to the state of technology in the home and foreign industries, respectively.[3] The output of the j^{th} home-country firm in state α is

$$X_j(\alpha) = \theta(\alpha)F(L_j, K_j) \qquad \text{for } \alpha = 1,2,\ldots,S; \; j = 1,2,\ldots,J$$

where F is a standard, constant-returns-to-scale production function (the same for all firms), L_j is the firm's labor input and K_j is its input of physical capital. Similarly the output of the j^{th} foreign firm in state α is given by

$$X_j^*(\alpha) = \theta^*(\alpha)F(L_j^*, K_j^*) \qquad \text{for } \alpha = 1,2,\ldots,S; \; j = 1,2,\ldots,J^*.$$

The production function for foreign firms is identical to that for home firms, but the multiplicative uncertainty term, which is the same for all firms within each country, is not necessarily the same for firms located in different countries. With these assumptions, firms in each country can be aggregated to the industry level, so that henceforth we omit the j subscripts.

Firms in each country choose their inputs prior to the resolution of uncertainty so as to maximize their net stock market values (i.e. gross values less factor payments). Let q and q^* be the prices of a unit of real equity in a representative home and foreign firm, respectively, with $q \equiv 1$ by choice of numeraire. A unit of real equity in a home firm pays $\theta(\alpha)$ units of the (single) consumption good if state α is realized. Similarly, $\theta^*(\alpha)$ is the return to a unit of the foreign equity. Home and foreign firms produce $Z = F(L, K)$ and $Z^* = F(L^*, K^*)$ units of real equities, respectively, which have gross stock market values of $F(L, K)$ and $q^*F(L^*, K^*)$. Thus, the home country industry chooses L and K to maximize $F(L, K) - wL - rK$, where w is the home wage rate and r the home rental rate for capital, both expressed in terms of home equities. The first-order conditions for maximization are

$$F_L(L, K) = w \tag{1}$$

$$F_K(L, K) = r \tag{2}$$

The foreign industry seeks to maximize $q^*F(L^*, K^*) - w^*L^* - r^*K^*$, and thus chooses L^* and K^* to satisfy

$$q^*F_L(L^*, K^*) = w^* \tag{3}$$

$$q^*F_K(L^*, K^*) = r^* \tag{4}$$

Capital and labor endowments in the home and foreign countries are $\bar{K}$ and $\bar{K}^*$ respectively, and labor endowments are $\bar{L}$ and $\bar{L}^*$. Labor is internationally immobile, so that the labor markets must clear separately in each country. In equilibrium, we have:

$$L = \overline{L} \tag{5}$$

$$L^* = \overline{L}^* \tag{6}$$

Capital movements are costless and unrestricted, which implies the existence of a unified, world, physical-capital market. The conditions for equilibrium in this market are:

$$K + K^* = \overline{K} + \overline{K}^* \tag{7}$$

$$r = r^* \tag{8}$$

We turn finally to consumer behavior. Consumer-investors in each country are endowed with physical capital, labor and shares of ownership in firms. Prior to the resolution of uncertainty, each individual sells his factor endowments, bears his fraction of each firm's factor costs in accordance with his initial ownership, and buys and sells shares of stock in the various firms.

Let $V^i[I^i(\alpha)]$ be the concave, von Neumann - Morgenstern (indirect) utility function for individual i, where $I^i(\alpha)$ is the individual's income in state α. Suppose the individual were to hold in his ultimate portfolio z^i shares of stock in home firms and z^{*i} shares in foreign firms. Then his income in state α would be

$$I^i(\alpha) = \theta(\alpha)z^i + \theta^*(\alpha)z^{*i}$$

The portfolio choice problem of this individual is to maximize expected utility, $EV^i[I^i(\alpha)]$, given (common) subjective beliefs about the probability distribution for the states of nature, and subject to the budget constraint that the cost of his portfolio not exceed the value of his initial endowment. The portfolio allocation which maximizes expected utility must satisfy

$$\frac{E\theta^*(\alpha)V_I^i[\theta(\alpha)z^i + \theta^*(\alpha)z^{*i}]}{E\theta(\alpha)V_I^i[\theta(\alpha)z^i + \theta^*(\alpha)z^{*i}]} = q^* \tag{9}$$

where $V_I^i(\cdot)$ is the marginal utility of income.

The model is closed by the world market-clearing conditions for the real equities of firms located in each of the countries, i.e.

$$\sum_i z^i = Z \tag{10}$$

$$\sum_i z^{*i} = Z^* \tag{11}$$

where the summation is over all individuals in the world.

Before proceeding to an investigation of the properties of the cum-factor movements equilibrium, we choose to place a restriction on the form of the utility functions that is analagous to the one often invoked in nonstochastic trade models for proofs of theorems on the determinants of commodity trade. In the present context, the assumption is that all consumers, worldwide, have identical and homothetic preferences over equities. The purpose of this assumption is to neutralize any bias in the pattern of trade in securities or in the direction of capital movements introduced on the demand side by differences in tastes or by income distributional considerations.[4]

For consumers' preferences over securities to be identical and homothetic, it is sufficient that they all have utility functions that exhibit identical and constant relative aversion to income risk; i.e., that their utility functions be of the form $V(\cdot) = (I^i)^{1-\gamma}/(1 - \gamma)$, for some $\gamma \neq 1$, or of the form $V(\cdot) = \log(I^i)$. If the utility function takes one of these forms, (9) can be rewritten as

$$\frac{E\theta^*(\alpha)V_I[\theta(\alpha) + \theta^*(\alpha)\tilde{z}^i]}{E\theta(\alpha)V_I[\theta(\alpha) + \theta^*(\alpha)\tilde{z}^i]} = q^* \qquad (9')$$

(where $\tilde{z}^i \equiv z^{*i}/z^i$), from which it is clear that the relative holdings of the two stocks in any investor's portfolio is independent of his nationality or level of wealth.

Identical, homothetic preferences have the property of being aggregable. That is, world demand for assets can be consistently represented by a set of community asset indifferences curves of the form $EV[\theta(\alpha)z + \theta^*(\alpha)z^*] = \overline{V}$. These also represent demands in each country taken separately. Utility is a quasi-concave function of asset holdings, and is strictly so, if individuals are risk averse (i.e. $V_{II} < 0$), and if $\theta(\alpha)$ and $\theta^*(\alpha)$ are less-than-perfectly correlated.

The nature of the world equilibrium with free capital movements is best understood with the aid of Figure 2. For illustrative purposes, we depict a situation in which $\theta(\alpha)$ and $\theta^*(\alpha)$ have symmetrical distributions.[5] In quadrants II and IV, we draw the production functions for real equities in each country, as a function of the amount of capital located there. The sum of the allocations of capital to the two countries is constrained by equation (7) to be equal to the exogenous world supply. This constraint is represented by a straight line with slope of negative one, in quadrant III. Together, these constraints trace out, in quadrant IV, a world transformation locus, TT', relating the feasible supplies of the two real equities. Each point on the frontier corresponds to a particular division of capital between the two countries. The slope of the transformation curve at any point is given by $-F_K(\overline{L}^*, K^*)/F_K(\overline{L}, K)$.

A representative of the family of homothetic asset indifference curves is depicted in quadrant IV by VV'. The slope of VV' is given by (the negative of) the marginal rate of substitution of assets, i.e. by the left hand side of equation (9). Under the assumption that $\theta(\alpha)$ and $\theta^*(\alpha)$ are symmetrically distributed, the slope of VV' must be a negative one where $z = z^*$.

World equilibrium occurs at E, the point of tangency of an asset indifference curve and the world transformation locus. This is because consumers set (the negative of) the marginal rate of substitution between assets (MRSA) equal to the relative price of securities, in equation (9), while competition in the world market for physical capital leads to equality between (the negative of) the marginal rate of transformation (MRTA) and the relative price of securities, by equations (2), (4) and (8).

In the diagram, the deterministic equilibrium or "MacDougall point" is labelled M. As we have noted, the deterministic equilibrium is characterized by $F_K(\overline{L}, K) = F_K(\overline{L}^*, K^*)$, so the marginal rate of transformation at M is negative one. By the concavity of the asset indifference curve, and the fact that the latter has a slope of negative one at point D, it follows that equilibrium with uncertainty must lie (weakly) between the MacDougall point and the 45^o line; i.e., it must exhibit more diversification. If agents are risk neutral, or if θ and θ^* are perfectly correlated, then the asset indifference curves are straight lines with slopes of negative one, and the two equilibria coincide at M. Alternatively, if $\overline{L} = \overline{L}^*$, the MRTA is negative one at the 45^o line, and again the equilibria coincide. In all other cases, the equilibrium under uncertainty differs from that for the case of no uncertainty by an amount that depends upon the degree of relative risk aversion and the correlation between disturbances in the two countries. A higher degree of relative risk aversion and a less positive (or more negative) correlation between $\theta(\alpha)$ and $\theta^*(\alpha)$ tend to render the asset indifference curves more concave, and thus contribute to a larger distance between points M and E in the figure.

The introduction of uncertainty may alter the nature of the capital-movements equilibrium either quantitatively or qualitatively. Consider the three potential autarky points at A_1, A_2 and A_3. (Autarky production is represented by the point along TT' that corresponds to the exogenously given initial endowment, at, for example, a_1, a_2 or a_3). If autarky production is at A_1, then in both the deterministic model and the model with uncertainty the home country imports capital, but <u>more</u> capital movement takes place in the latter case. With autarky at A_2, capital flows out of the home country in both situations, but now the introduction of uncertainty <u>lessens</u> the extent of capital movement. Finally, if autarky is at A_3, the introduction of uncertainty <u>reverses the direction</u> of capital movement, relative to the outcome in a deterministic world.

What can be said in general about the direction of capital movement in a one-good world with uncertainty and international trade in securities? In order to isolate the separate influences of the relative size of the labor forces, the relative factor abundances, and the relative riskiness of the two countries, we consider, in turn, initial situations that deviate from complete symmetry along only one of these dimentions. Our findings are summarized in three propositions.

<u>Proposition 1</u>: If $\theta(\alpha)$ and $\theta^*(\alpha)$ have symmetric distributions, and $\overline{L} = \overline{L}^*$, then physical capital moves toward the country with the smaller autarky endowment of capital.

Proof: The symmetry of the distributions of $\theta(\alpha)$ and $\theta^*(\alpha)$ implies that the MRSA is negative one for $z = z^*$ The equality of labor forces implies that the MRTA is negative one for $Z = Z^*$. Thus, an equal division of the world's capital stock across countries $(K = K^*)$ satisfies all the conditions for equilibrium. Also, equilibrium is unique, so capital must flow from the country in which it is initially abundant toward the less-well-endowed country.

Proposition 2: If $\theta(\alpha)$ and $\theta^*(\alpha)$ have symmetric distributions, and $\bar{K}/\bar{L} = \bar{K}^*/\bar{L}^*$, then physical capital moves toward the country with the smaller labor force.

Proof: If $\bar{L}$ is greater (less) then $\bar{L}^*$, the autarky point lies along TT' above (below) the point where $Z = Z^*$. The MRTA is negative one at the autarky point (recall that MRTA $= - F_K(\bar{L}^*, K^*)/F_K(\bar{L}, K)$, and that F is homogeneous of degree one) and decreases monotonically for movements downward along TT'. The slope of the asset indifference curve that intersects TT' is negative one at the point where $Z = Z^*$, and increases monotonically for movements downward along the transformation locus. It follows that equilibrium must lie on TT' between the autarky point and the point where $Z = Z^*$. In equilibrium, there is more production of the real equity of the initially smaller country than there is in autarky, i.e. capital moves toward the country with the smaller labor force.

Proposition 3: If $\bar{K} = \bar{K}^*$ and $\bar{L} = \bar{L}^*$, and the distribution of the random variable in one country is riskier than the distribution in the other country (in the sense of a mean-preserving spread), then capital moves toward the less risky country.

Proof: For definiteness, suppose that the distribution of $\theta^*(\alpha)$ is riskier than the distribution of $\theta(\alpha)$. Then, $\theta^*(\alpha) = \tilde{\theta}^*(\alpha) + \varepsilon(\alpha)$, where $\theta(\alpha)$ and $\tilde{\theta}^*(\alpha)$ have symmetric distributions, and $E[\varepsilon(\alpha)|\tilde{\theta}^*(\alpha)] = E[\varepsilon(\alpha)|\theta(\alpha)] = 0.$[6] The asset indifference curve which intersects TT' at the point where $Z = Z^*$ has slope given in (9') by

$$-\frac{E(\tilde{\theta}^*(\alpha) + \varepsilon(\alpha))V_I[\theta(\alpha) + \tilde{\theta}^*(\alpha) + \varepsilon(\alpha)]}{E\ \theta(\alpha)V_I[\theta(\alpha) + \tilde{\theta}^*(\alpha) + \varepsilon(\alpha)]} = -\left\{1 + \frac{E\ \varepsilon(\alpha)V_I[\theta(\alpha) + \tilde{\theta}^*(\alpha) + \varepsilon(\alpha)]}{E\ \theta(\alpha)V_I[\theta(\alpha) + \tilde{\theta}^*(\alpha) + \varepsilon(\alpha)]}\right\}$$

where this equality follows from that fact that $\theta(\alpha)$ and $\tilde{\theta}^*(\alpha)$ are symmetrically distributed, and each is independent of $\varepsilon(\alpha)$. The second term in the brackets is negative, since $\varepsilon(\alpha)$ and $V_I(\cdot)$ have negative covariance. Thus, the asset indifference curve has slope greater than negative one at $z = z^*$, while the slope of TT' is equal to negative one there. It follows that equilibrium is at a point above the 45^o line, i.e. that capital moves to the less risky country.

In general situations, the direction of capital movement is determined by the interaction of the separate influences of relative country size, relative factor abundance and relative country riskiness. However, the nature of this interaction can be quite complex. It is not true, for example, that an increase in the riskiness of one country (in a mean-preserving spread sense) will always cause capital to flow out of that country. If the country that becomes more risky also has a smaller labor force, and if the disturbances in the two countries are negatively correlated, then the increase in riskiness makes the real equity of that country a more attractive asset. Intuitively, the extra income the asset provides when the marginal utility of income is high outweighs the utility cost of the income forgone when marginal utility is low. One general statement that can be made is that *ceteris paribus*, more capital will flow into a country the smaller is its labor force. The desire for diversification on the part of consumer-investors implies a tendency for real equity supplies to be equalized.

III. The MacDougall Model with a Traded Bond

In the previous section we studied capital movements in a one-good model with uncertainty, where the only assets available to consumer-investors were risky real equities. In the present section, we extend the analysis to incorporate a market for a safe asset (i.e., an internationally traded bond), while maintaining all of our earlier assumptions, including especially the one restricting agents' asset preferences to be identical and homothetic. We will show that this extension does not alter any of the conclusions of the previous section.

Let b^i be the holding of an internationally traded bond, with price q_b, by the i^{th} individual. This asset pays a return of one unit of the consumption good in all states of nature. The consumer-investor must allocate his *ex ante* wealth over three assets, the two real equities, and the bond. The first-order conditions for expected utility maximization imply, in place of equation (9), the following equations:

$$\frac{E\ \theta^*(\alpha)V_I[\theta(\alpha)z^i + \theta^*(\alpha)z^{*i} + b^i]}{E\ \theta(\alpha)V_I[\theta(\alpha)z^i + \theta^*(\alpha)z^{*i} + b^i]} = q^* \tag{12}$$

$$\frac{E\ V_I[\theta(\alpha)z^i + \theta^*(\alpha)z^{*i} + b^i]}{E\ \theta(\alpha)V_I[\theta(\alpha)z_i + \theta^*(\alpha)z^{*i} + b^i]} = q_b \tag{13}$$

The bond-market clearing condition is

$$\sum_i b^i = 0 \tag{14}$$

All of the remaining equilibrium conditions of the earlier set-up continue to apply.

The fact that all individuals have identical and homethetic demands for assets implies that, in equilibrium, each will allocate the same fraction of his wealth to any given asset. If the equilibrium bond-holding of one individual is either strictly positive or strictly negative, such would also be true for every other individual. In either case, equation (14) could not be satisfied. It follows that in equilibrium, $b^i = 0$, for all i. Once this fact is recognized, it is clear that the asset holdings that satisfy equation (9) will also satisfy equation (12). Indeed, all the conditions of the capital-movements equilibrium in the absence of bond trading are also consistent with equilibrium when a bond market is assumed to exist. Equation (13), then, serves to determine the price of bonds such that in equilibrium all agents choose to take a net position of zero in the market for this safe asset. We summarize this finding in the following proposition.

Proposition 4: When all agents have identical, homothetic, assets utility functions, the equilibrium allocation of resources with free capital movements and free trade in equities and an "inside" bond is identical to the equilibrium allocation when the bond market does not exist.

IV. The Heckscher-Ohlin-Mundell Model with Uncertainty

In a deterministic world with two goods and two factors, international trade in goods and international factor movements are perfect substitutes, provided that both countries are incompletely specialized in the ultimate equilibrium (see Mundell, 1957). In other words, the factor price equalization theorem implies that if capital movements are allowed, starting from an equilibrium with free trade in goods, no movements are actually needed to maintain equilibrium.[7]

In the present section we extend the model of Section II to incorporate a second consumption good, and therefore a second security, in each country. The resulting model is a cum-uncertainty analogue of the deterministic Heckscher-Ohlin model with free capital movements. In contrast to the results of Mundell for the nonrandom case, we find that goods trade is not a perfect substitute for factor movements, and that equilibrium generally involves transactions in both international markets in determinate amounts. The reason for this difference is that without uncertainty, the location of production of a good is immaterial; with uncertainty, the location of production is economically relevant when equities are imperfect substitutes. We assume that two goods are produced in each country with capital and labor, and adopt completely the Heckscher-Ohlin production structure with regard to real equity outputs. As before, the output of one of the goods (good 1) is stochastic and is given by $X = \theta F(L_x, K_x)$ in the home country and $X^* = \theta^* F(L_x^*, K_x^*)$ in the foreign country. The outputs of real equities in this industry are once again denoted Z and Z^*, with equity prices $q_x (\equiv 1)$ and q_x^*.

For simplicity, assume that the technology for the second good is nonstochastic. Then the outputs of real equities in this industry are identically equal to the outputs of good 2, and are related to factor inputs by $Y = G(L_y, K_y)$ and $Y^* = G(L_y^*, K_y^*)$. Shares of stock in firms in industry 2 are perfect substitutes, irrespective of country of location. Let q_y denote the common equity price for shares of firms in this industry, measured relative to the price of the home-country real equities of industry 1. Finally, let $p(\alpha)$ be the relative price of good 1 in terms of good 2 in state of nature α.

As is known from the works of Helpman and Razin, the conditions for production equilibrium are the same as those in the standard Heckscher-Ohlin model, except that real equity prices substitute here for commodity prices. Of course, with capital internationally mobile, there is the additional condition that the rental rates for capital be equalized across countries. As in the deterministic model, an equilibrium with incomplete specialization in each country will turn out to be possible, so long as the ultimate factor endowment ratios are not "too" disparate. Since capital is freely mobile, this implies only a condition on the comparative sizes of the two labor forces. We will concentrate our attention on those situations where an equilibrium with incomplete specialization is possible.

Consider now the optimization problem of the typical consumer-investor. This individual wishes to maximize the mathematical expectation of his ex post utility, defined over his levels of consumption of the two goods, $c_x^i(\alpha)$ and $c_y^i(\alpha)$. Let $U[c_x^i(\alpha), c_y^i(\alpha)]$ represent his ex post utility function. Anderson (1981) has shown that, to ensure that all individuals have identical homethetic commodity and equity preferences, it is necessary and sufficient to restrict the form of $U(\cdot,\cdot)$ to be any linear transform of a positive power function (of the same degree for all individuals) of a homogeneous, quasi-concave function (also identical for all individuals).

The individual's maximization can be thought of as consisting of two stages. After the uncertainty is resolved, the individual qua consumer who holds a portfolio of z^i shares of firms in the home industry 1, z^{*i} shares firms in the foreign industry 1, and y^i shares of firms in industry 2 in either country has income [8]

$$I^i(\alpha) = \theta(\alpha)p(\alpha)z^i + \theta^*(\alpha)p(\alpha)z^{*i} + y^i.$$

He allocates this income to maximize ex post utility according to the familiar condition

$$\frac{U_{c_x}[c_x^i(\alpha),\ c_y^i(\alpha)]}{U_{c_y}[c_x^i(\alpha),\ c_y^i(\alpha)]} = p(\alpha) \tag{14}$$

The solution to this second stage problem can be used to define an indirect utility function, $V[p(\alpha),\ I^i(\alpha)]$.

In the first stage (i.e. prior to the resolution of uncertainty), the individual qua investor sells his initial endowment and allocates the proceeds among the three assets to maximize $EV[p(\alpha),\ I^i(\alpha)]$. We assume that price expectations are formed rationally at this stage. Let $\tilde{z}^i \equiv z^{*i}/z^i$ and $\tilde{y}^i \equiv y^i/z^i$. Then the first-order conditions for expected-utility maximization imply.

$$\frac{E\{\theta^*(\alpha)p(\alpha)V_I[p(\alpha),\ \theta(\alpha)p(\alpha) + \theta^*(\alpha)p(\alpha)\tilde{z} + \tilde{y}]\}}{E\{\theta(\alpha)p(\alpha)V_I[p(\alpha),\ \theta(\alpha)p(\alpha) + \theta^*(\alpha)p(\alpha)\tilde{z} + \tilde{y}]\}} = q_x^* \tag{15}$$

$$\frac{E\{V_I[p(\alpha),\ \theta(\alpha)p(\alpha) + \theta^*(\alpha)p(\alpha)\tilde{z} + \tilde{y}]\}}{E\{\theta(\alpha)p(\alpha)V_I[p(\alpha),\ \theta(\alpha)p(\alpha) + \theta^*(\alpha)p(\alpha)\tilde{z} + \tilde{y}]\}} = q_y \tag{16}$$

where the fact that asset perferences are identical and homothetic allows us to drop the i superscripts.

The model is closed, as before, by market-clearing conditions for assets (e.g., $\sum_i y^i = Y + Y^*$, $\sum_i z_i = Z$, etc.). These are augmented now by ex post market-clearing conditions state-by-state for each of the goods (i.e., $\sum_i c_x^i(\alpha) = X(\alpha) + X^*(\alpha)$ and $\sum_i c_y^i(\alpha) = Y + Y^*$).

We will henceforth restrict our investigation to situations where the distributions of the country-specific random variables are symmetric (though not perfectly correlated). Our strategy will be to construct a candidate equilibrium based on the assumption that production of real equities is incompletely specialized in each country, and then to show that, as long as $\bar{L}$ and $\bar{L}^*$ are not too disparate, no feasibility conditions are violated by this candidate equilibrium. Provided that agents are risk averse, that $\theta(\alpha)$ and $\theta^*(\alpha)$ are not perfectly correlated, and that an equilibrium with incomplete specialization exists, the constructed equilibrium is the only possible equilibrium.[9]

Consider first the <u>ex post</u> relative commodity price that is realized in state of nature α. By the homotheticity of aggregate demands, this market-clearing price is a function only of the relative outputs of the two goods, i.e. $p(\alpha) = \Pi[(\theta(\alpha)Z + \theta^*(\alpha)Z^*)/(Y + Y^*)]$.

In the <u>ex ante</u> equilibrium, incomplete specialization in each country and rental rate equalization implies that the relative security-price ratio that governs resource allocation in the home country, q_y, must equal the relative security-price ratio in the foreign country, q_y/q_x^*. Evidently, in an equilibrium with incomplete specialization, $q_x^* = 1$.

Next consider equation (15). In equilibrium, $\tilde{z} = Z^*/Z$. Substituting equilibrium values for $p(\alpha)$, $\tilde{z}$ and q_x^*, it should be clear that (15) is satisfied if (and only if) $Z = Z^*$. This is because, when $Z = Z^*$, $\Pi(\cdot)$ is symmetric in $\theta(\alpha)$ and $\theta^*(\alpha)$, as is $V_I(\cdot)$. Since, by assumption, $\theta(\alpha)$ and $\theta^*(\alpha)$ have symmetric distributions, the marginal rate of asset substitution between the two industry 1 securities is equal to one. The construction of equilibrium is completed by the choice of a $\tilde{y} = 2(Y + Y^*)/(Z + Z^*)$ and a q_y that are feasible, and that simultaneously satisfy (16) and the net-stock-market-value maximization (i.e. asset supply) conditions.[10]

It remains only to show that such a choice is possible. Suppose first that $\overline{L} = \overline{L}^*$. Then an equal allocation of capital across countries will ensure that $Z = Z^*$ for any choice of q_y. It is easy then to check that, if both goods are essential, there must exist some choice of q_y that clears the world equity markets. Now, the fact that $U(\cdot,\cdot)$ is strictly concave implies that the endogenous variables of the model are all continuous functions of the exogenous parameters. In particular, for a sufficiently small change in either $\overline{L}$ or $\overline{L}^*$, holding the other constant, the world equilibrium must continue to be characterized by incomplete specialization in each country.

The key property of an incomplete-specialization equilibrium that we will exploit in our investigation of the determinants of the direction of capital movements is the one requiring equalization of absolute levels of the output of the real equity of industry 1 across countries. There is, of course, no such requirement for outputs of good 1 in the two countries of a deterministic model. The difference arises because, whereas the outputs of the first industry (i.e. good one) are perfect substitutes in a nonstochastic world, they no longer are so when what firms produce are real equities in a world of uncertainty. As will become evident in the course of our discussion, this difference explains why factor movements do not of necessity occur under certainty, given free trade in goods, but do generally occur with uncertainty.

As before, let us proceed by considering in turn cases which deviate from perfect symmetry between countries along only one dimension. First, we have:

Proposition 5: If $\theta(\alpha)$ and $\theta^*(\alpha)$ are symmetrically distributed and $\overline{L} = \overline{L}^*$, capital moves toward the country with the smaller endowment of capital. That country has positive expected exports of both goods. The ratio of its expected exports is equal to the ratio of its expected consumption levels.

Proof: With $\overline{L} = \overline{L}^*$, the equilibrium conditions $Z = Z^*$ and $q_x^* = 1$ (which hold for an incomplete-specialization equilibrium) can only be satisfied if $K_x + K_y = K_x^* + K_y^*$, i.e. if the world's capital stock is equally allocated across countries. Thus, capital must move toward the country which has the smaller endowment.

Consider now the pattern of ex post goods trade. Let $c_h(\alpha)$ and $c_h^*(\alpha)$ be the aggregate consumption levels of good h $(h = X, Y)$ in the home and foreign country, in state α. For definiteness, assume that $\overline{K} > \overline{K}^*$, and define $\lambda \equiv [(w\overline{L} + r\overline{K})/w\overline{L}^* + r\overline{K}^*] - 1$. λ is the percent excess of home endowment wealth over foreign endowment wealth.[11] The homotheticity of commodity preferences implies that $c_h(\alpha) = (1 + \lambda)c_h^*(\alpha)$. Since ex post factor allocations are equal, $Y = Y^* = [Ec_y(\alpha) + Ec_y^*(\alpha)]/2$ and $EX(\alpha) = EX^*(\alpha) = [E_x(\alpha) + Fc_x^*(\alpha)]/2$. Expected exports of good 2 by the foreign country are[12]

$$Y^* - Ec_y^*(\alpha) = \frac{\lambda E c_y^*(\alpha)}{2} > 0.$$

Its expected exports of good 1 are

$$EX^*(\alpha) - Ec_x^*(\alpha) = \frac{\lambda E c_x^*(\alpha)}{2} > 0.$$

The surplus on trade account is balanced by a deficit on service account.

Ex post trade flows will vary across states of nature. The expected trade pattern is, however, neutral, and clearly is not determined by initial relative factor abundance. This is similar to the result reported by Svensson (1982), who found in a different context that trade in goods might not be explainable by autarky relative factor abundance when factors are internationally mobile.

Proposition 6: Suppose $\theta(\alpha)$ and $\theta^*(\alpha)$ are symmetrically distributed and $\overline{K}/\overline{L} = \overline{K}^*/\overline{L}^*$. (i) If the good whose technology is uncertain is relatively capital intensive, then capital moves toward the smaller country, the smaller country has positive expected exports of both goods, and has greater relative expected exports of the good whose technology is uncertain than it has relative expected consumption of that good. (ii) If the good whose technology is uncertain is relatively labor intensive, then capital moves toward the larger country, the smaller country has positive expected exports of the good whose technology is uncertain and has positive expected imports of the other good.

Proof: The proof makes use of Figure 3, and the fact that, in an incomplete-specialization equilibrium, $Z = Z^*$. (Assume, for definiteness, that the home country is larger.) In each panel of the figure, we illustrate the real-equity production possibility frontiers (EPPF) for the two countries under autarky. Since relative factor endowments are equal, the EPPF for the larger country is a radial expansion of the EPPF for the smaller country.

Consider the hypothetical real-equity production that would take place in each country at the relative equity prices that prevail in the equilibrium with free capital movements, were actual capital movements to be zero. These points are marked B and B^* in each panel of the figure. Clearly, the absence of capital movements is inconsistent with equilibrium, since at these points, $Z > Z^*$.

Now suppose that the industry with uncertain technology is relatively capital intensive (see panel a). Imagine a transfer of capital from the home to the foreign country, holding the relative equity prices constant at their equilibrium values. This would cause the two countries to move in opposite directions along their respective capital-Rybczynski lines, BR and B^*R^*.[13] These Rybczynski lines are parallel, because the equality of relative equity prices in equilibrium implies that factor prices are equal for this hypothetical transfer. Eventually, points E and E^* are reached, such that $Z = Z^*$. These real-equity production points are consistent with full equilibrium.

In panel _b_, the industry with uncertain technology is assumed to be labor intensive. Then, equalization across countries of the real-equity outputs of industry 1 requires a transfer of capital from the small country to the large country. The equilibrium production points are at F and F^*. It is perhaps counterintuitive that capital moves in this case toward the larger country, even though factor endowment ratios are equal and country risks are symmetric. This direction of capital movement is dictated by the strong implications of the Rybczynski theorem for the output effects of endowment changes.

Consider, finally, the pattern of (expected) _ex post_ commodity trade. Expected exports by the foreign country of the product of the product of the uncertain industry are

$$EX^*(\alpha) - Ec_x^*(\alpha) = \lambda c_x^*(\alpha)/2 > 0.$$

The foreign country has exports of good 2 given by

$$Y^* - c_y^* = \frac{(1 + \lambda)Y^* - Y}{2 + \lambda} .$$

Referring once again to Figure 3, note that in each panel the output of the equity of industry 2 at point B is $1 + \lambda$ times as large as the output of this equity at point B^*. It follows that, in equilibrium, the smaller country has positive exports of good 2 if the production of this good is relatively labor intensive and positive imports otherwise. When the small country is an expected exporter of both goods, its relative expected export levels can be compared to its relative expected consumption levels. In this case,

$$Y^* - c_y^* = [\lambda c_y^* + Y^* - Y]/2$$

$$< \lambda c_y/2$$

which implies that

$$\frac{Y^* - c_y^*}{EX^*(\alpha) - Ec_x^*(\alpha)} < \frac{c_y^*}{Ec_x^*(\alpha)}$$

Once again, the pattern of (expected) commodity trade is not predicted by initial relative factor abundance.

V. Conclusions

When production is characterized by technological uncertainty that is, at least to some extent, country specific, the international allocation of mobile factors is influenced by the asset preferences of risk averse consumer-investors. In this paper, we have studied the determinants of the direction of international capital movements in a model of trade in commodities and real equities, under the assumption that preferences over commodities and assets are identical and homothetic worldwide. In a one-good variant of our model which is in the spirit of MacDougall (1960), we found that physical capital flows in an uncertain world are subject to the combined influences of relative factor abundance, relative size of labor force and relative country riskiness. When deviation from complete symmetry is along only one of these dimensions, capital moves toward the relatively labor abundant country, the smaller country and the less risky country, respectively. However, in more general situations, the interaction between these effects can be quite complex.

Many of the lessons from the MacDougall model remain applicable when the model is extended to incorporate a second good that is produced in each country with a nonstochastic technology. In contrast to the deterministic two-good, two-factor model with potential capital mobility, equilibrium under uncertainty generally requires some movement of capital even when goods are freely traded. Relative factor abundance, relative size of labor force and relative country riskiness still play important roles in the determination of the direction of factor movements, as well as in the determination of the expected pattern of commodity trade. An additional insight gained from the two-good variant is that the direction of physical capital flow may hinge on the identity of the factor which is used intensively in the industry with random technology.

Footnotes

1. Helpman and Razin (1978a, 1978b) have asserted that comparative labor costs explain the pattern of trade in securities in a Ricardian-type model with uncertainty. However, their argument is incomplete, and requires for its validity certain restrictions on the distribution of the random variables and on utility functions, of the sort imposed by Anderson (1981) in his proofs of the Heckscher-Ohlin and Travis-Vanek theorems under uncertainty.

2. Note that Helpman and Razin (1978b, p. 248) were mistaken when they claimed that, generally, "trade in goods and securities does substitute for factor movements". The issue is clarified in their later discussion (1978a), where they recognized that factor price equalization in their model requires that the industry-specific shocks be perfectly correlated across countries.

3. We use asterisks to refer to variables for the foreign country.

4. Anderson (1981) was the first to recognize the relevance of this assumption in the context of the Helpman-Razin model of trade in goods and securities. When, in Section IV, we introduce a second good, our assumption will be that preferences over goods are identical and homethetic as well.

5. In other words, we suppose for the sake of the diagram, that the joint density function for the two random variables, $\Psi(\cdot,\cdot)$ satisfies $\Psi(\theta,\theta^*) = \Psi(\theta^*,\theta)$.

6. See Rothchild and Stiglitz (1970).

7. Similarly, goods trade need not emerge if the initial situation is one of free capital movements. As Mundell has shown, this implies that any tariff on goods will be prohibitive when capital is internationally mobile, provided that a situation of incomplete specialization is consistent with equilibrium.

8. Note that the return to a unit of real equity is $\theta(\alpha)p(\alpha)$ for a firm in the home industry 1, and is $\theta^*(\alpha)p(\alpha)$ for a firm in the foreign industry 1. A unit of real equity for a firm in industry 2 irrespective of location, pays a return of one unit of the numeraire good.

9. The uniqueness of equilibrium can be demonstrated as follows. Recall that with identical homethetic asset and commodity preferences all agents demands are aggregable. Let $\bar{c}_x(\alpha)$ and $\bar{c}_y(\alpha)$ be the aggregate world consumption levels. The <u>laissez-faire</u> market equilibrium has, in this case, the property that it is the solution to a social planner's problem of the form

$$\max \ EU[\bar{c}_x(\alpha), \bar{c}_y(\alpha)]$$

$$\text{subject to } \bar{c}_x(\alpha) \leq \theta(\alpha)F(L_x, K_x) + \theta^*(\alpha)F(L_x^*, K_x^*);$$

$$\bar{c}_y(\alpha) \leq \theta^*(\alpha)G(\bar{L}_x, K_y) + \theta^*(\alpha)G(\bar{L}^* - L_x^*, K_y^*); \quad \text{and}$$

$$K_x + K_x^* + K_y + K_y^* \leq \bar{K} + \bar{K}^*$$

9. (continued)

The constraint set for this problem is weakly convex (i.e. it has flat sections for the reasons identified by Mundell). However, if $U(\cdot,\cdot)$ exhibits risk aversion, and $\theta(\alpha)$ and $\theta^*(\alpha)$ are less-than-perfectly correlated, the objective function is strictly concave, and thus the solution must be unique.

10. The condition $\tilde{y} = 2(Y + Y^*)/(Z + Z^*)$ is implied by asset-market clearance, given that $Z = Z^*$.

11. Note that the existence of free capital movement and free trade in equities implies, in an incomplete-specialization equilibrium, that wage rates are equalized. Also, in equilibrium, the value of initial stock holdings is zero (see Helpman and Razin, 1978a, Ch. 4).

12. Rearrangement of the *ex post* market clearing condition for good 2 gives: $Y - c_y^*(\alpha) = [(1 + \lambda)Y^* - Y]/(2 + \lambda)$. Thus, the volume of trade in good 2 is actually state-independent.

13. Helpman and Razin (1978a, 1978b) have shown that the Rybczynski theorem is applicable to real equity outputs when securities are internationally traded.

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Figure 1

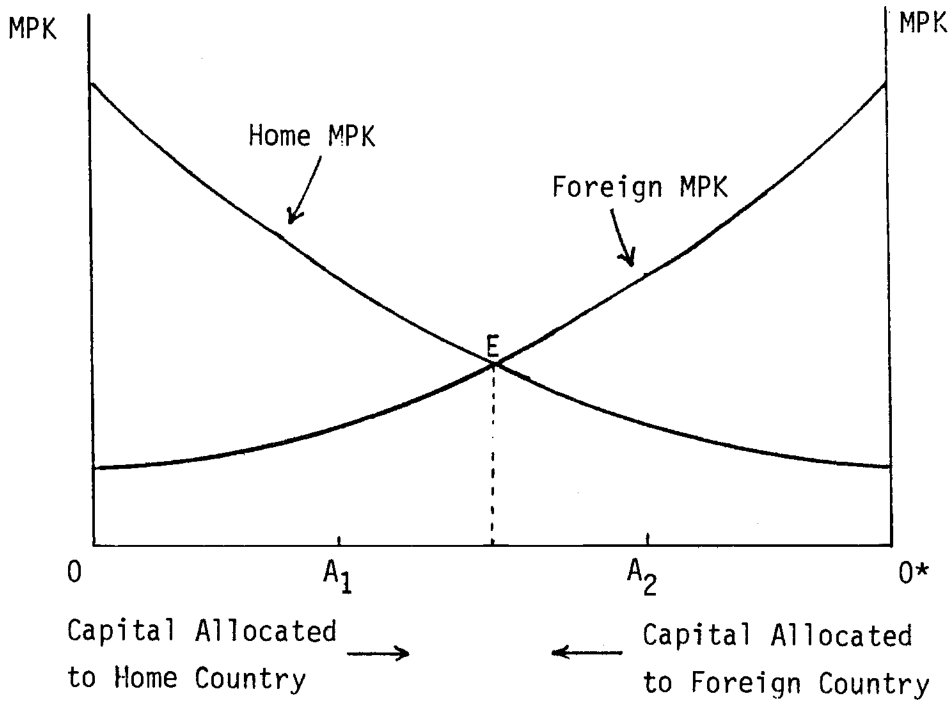
MPK
MPK
Home MPK
Foreign MPK
E
0
A1
A2
0*
Capital Allocated to Home Country
Capital Allocated to Foreign Country

Figure 2

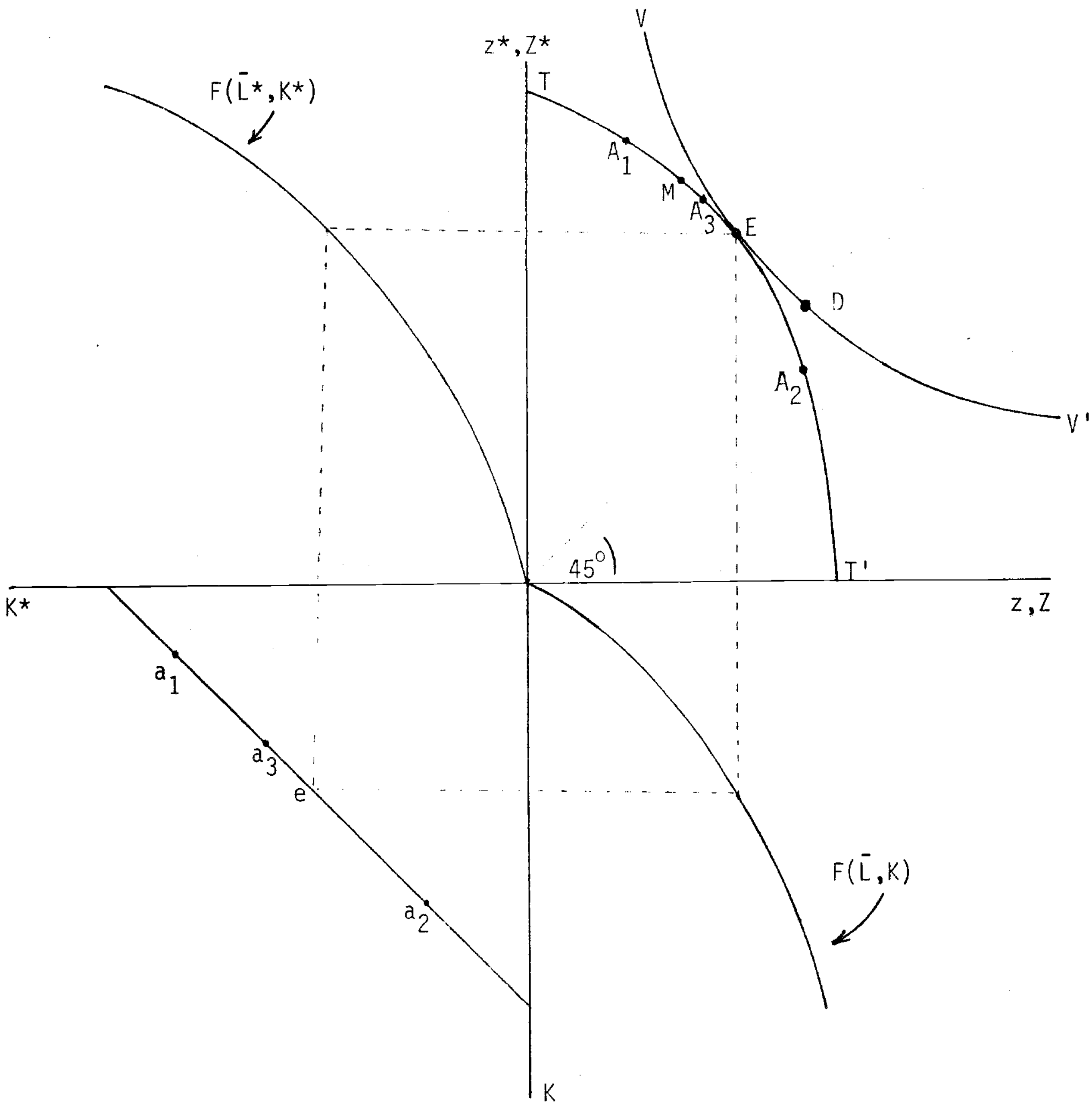

Figure 3