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DEFAULTABLE DEBT, INTEREST RATES AND THE CURRENT ACCOUNT

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ABSTRACT

World capital markets have experienced large scale sovereign defaults on a number of occasions, the most recent being Argentina's default in 2002. In this paper we develop a quantitative model of debt and default in a small open economy. We use this model to match four empirical regularities regarding emerging markets: defaults occur in equilibrium, interest rates are countercyclical, net exports are countercyclical, and interest rates and the current account are positively correlated. That is, emerging markets on average borrow more in good times and at lower interest rates as compared to slumps. Our ability to match these facts within the framework of an otherwise standard business cycle model with endogenous default relies on the importance of a stochastic trend in emerging markets.

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1 Introduction

World capital markets have experienced large scale sovereign defaults on a number of occasions, the most recent being Argentina's default in 2002. This latest crisis is the fifth Argentine default or restructuring episode in the last 180 years.[1] While Argentina may be an extreme case, sovereign defaults occur with some frequency in emerging markets. A second set of facts about emerging markets relates to the behavior of the interest rates at which these economies borrow from the rest of the world and their current accounts. Interest rates and the current account are strongly countercyclical and positively correlated to each other. That is, emerging markets tend to borrow more in good times and at lower interest rates as compared to slumps. These features contrast with those observed in developed small open economies.

In this paper we develop a quantitative model of debt and default in a small open economy, which we use to match the above facts. Our approach follows the classic framework of Eaton and Gersovitz (1981) in which risk sharing is limited to one period bonds and repayment is enforced by the threat of financial autarky. In all other respects the model is a standard small open economy model where the only source of shocks are domestic productivity shocks. In this framework, we show that the model's ability to match the facts in the data improve substantially when the productivity process is characterized by a volatile stochastic trend as opposed to transitory fluctuations around a stable trend. In a previous paper (Aguiar and Gopinath (2004)), we document empirically that emerging markets are indeed more appropriately characterized as having a volatile trend. The fraction of variance at business cycle frequencies explained by permanent shocks is shown to be around 50% in a small developed economy (Canada) and more than 80% in an emerging market (Mexico). This characterization captures the frequent switches in regimes these markets endure, often associated with clearly defined changes in government policy, including dramatic changes in monetary, fiscal, and trade policies.[2]

To isolate the importance of trend volatility in explaining default, we first consider a

[1] See Reinhart, Rogoff and Savastana (2003).

[2] There is a large literature on the political economy of emerging markets in general, and the tensions behind the sporadic appearances of pro-growth regimes in particular, that supports our emphasis on trend volatility (see, for example, Dornbusch and Edwards(1992)).

standard business cycle model in which shocks represent transitory deviations around a stable trend. We find that default occurs extremely rarely – roughly two defaults every 2,500 years. The intuition for this is described in detail in Section 3. The weakness of the standard model begins with the fact that autarky is not a severe punishment, even adjusting for the relatively large income volatility observed in emerging markets. The welfare gain of smoothing transitory shocks to consumption around a stable trend is small. This in turn prevents lenders from extending debt, which we demonstrate through a simple calculation a la Lucas (1987). We can support a higher level of debt in equilibrium by assuming an additional loss of output in autarky. However, in a model of purely transitory shocks, this does not lead to default at a rate that resembles those observed in many economies over the last 150 years.

To see the intuition behind why default occurs so rarely in a model with transitory shocks and a stable trend, consider that the decision to default rests on the difference between the present value of utility (value function) in autarky versus that of financial integration. Quantitatively, the level of default that arises in equilibrium depends on the *relative* sensitivity of the two value functions to shocks to productivity. To see why transitory shocks imply infrequent default, consider when productivity is close to a random walk. While a persistent shock has a large impact on the present value of expected utility, the impact of such a shock is similar across the two value functions. That is, with a nearly random walk income process, there is limited need to save out of additional endowment, leaving little difference between financial autarky and a good credit history, regardless of the realization of income. At the other extreme, if the transitory shock is *iid* over time, then there is an incentive to borrow and lend, making integration much more valuable than autarky. However, an *iid* shock has limited impact on the entire present discounted value of utility, and so the difference between integration and autarky is not sensitive to the particularly realization of the *iid* shock. At either extreme, therefore, the decision to default is not sensitive to the realization of the shock. Consequently, when shocks are transitory, the level of outstanding debt – and not the realization of the stochastic shock –is the primary determinant of default. This is reflected in financial markets by an interest rate schedule that is extremely sensitive to quantity borrowed. Borrowers internalize the steepness of the "loan supply curve" and recognize that an additional unit of debt at the margin will have a large effect on the cost of debt. Agents therefore typically do not borrow to the point where default is probable.

On the other hand, a shock to trend growth has a large impact on the two value functions (because of the shock's persistence) and on the *difference* between the two value functions. The latter effect arises because a positive shock to trend implies that income is higher today, but even higher tomorrow, placing a premium on the ability to access capital markets to bring forward anticipated income. In this context, the decision to default is relatively more sensitive to the particular realization of the shock and less sensitive to the amount of debt. Correspondingly, the interest rate is less sensitive to the amount of debt held. For a given probability of default, the cost of an additional unit of debt is therefore lower in a model with trend shocks (and in which agents internalize the interest rate schedule). Agents are consequently willing to borrow to the point that default is relatively likely. This theme is developed in Section 4.

Note that this intuition stresses that the marginal cost of borrowing is lower in the presence of trend shocks. It is also the case that the marginal benefit to borrowing is higher as well. The option to default provides insurance against repayment in bad states. With trend shocks, a given shock has a magnified effect on permanent income and thus on consumption. This additional consumption volatility increases the value of insurance. However, we find that quantitatively the demand for insurance varies little across our two specifications once we constrain the models to produce the same volatility of income at business cycle frequencies. Rather, we show that the important difference between an economy with trend rather than transitory shocks lies in the equilibrium price of insurance. However, reflecting the role of default in providing insurance, we show that as we ceterus paribas double the volatility of either trend or transitory shocks, the rate of default roughly doubles as well.

The next set of facts concerns the phenomenon of countercyclical current accounts and interest rates. In the current framework where all interest rate movements are driven by changes in the default rate, the steepness of the interest rate schedule makes it challenging to even qualitatively match the positive correlation between interest rates and the current account. This is because, on the one hand, an increase in borrowing in good states (countercyclical current account) will, all else equal, imply a movement along the heuristic "loan supply curve" and a sharp rise in the interest rate. On the other hand, if the good state is expected to persist, this lowers the expected probability of default and is associated with a favorable shift in the interest rate schedule. To generate a positive correlation between the current account and interest rates we need the effect of the shift of the curve to dominate the movement along the curve. A stochastic trend is again useful in matching this fact since the interest rate function tends to be less steeply sloped and trend shocks have a significant effects on the probability of default. Accordingly, in our benchmark simulations, a model with trend shocks matches qualitatively the empirical feature of countercyclical current accounts, countercyclical interest rates and a positive correlation between the two processes. The model with transitory shocks however fails to match these facts. The prediction for which both models perform poorly is in matching the volatility of the interest rate process. This might not be surprising since we only rely on default rates as an explanation for movements in interest rates. The role of risk premia and external shocks have been set aside.

The model with shocks to trend generates default roughly once every 125 years. This matches the rate observed in many emerging markets. However, it falls short of the extreme rates seen in Latin America. For example, Argentina defaulted or rescheduled debt five times and Venezuela nine times in a 180 year period (Reinhart, Rogoff, Savastana (2003)). We bring the default rate closer to that observed empirically for Latin America by introducing third-party bail-outs. Realistic bailouts raise the rate of default dramatically – bailouts up to 18% of GDP lead to defaults once every 27 years. This model performs well on most dimensions save one. The subsidy implied by bailouts breaks the tight linkage between default probability and the interest rate. Interest rate volatility is therefore an order of magnitude below that observed empirically.

The business cycle behavior of markets in which agents can choose to default has received increasing attention in the literature. Kehoe and Perri (2002) examine optimal debt contracts subject to a participation constraints. Default does not arise explicitly in equilibrium because of the ability to write incentive compatible contracts with state contingent payments. Moreover, the participation constraint binds strongest in good states of nature, i.e. the states in which the optimal contract calls for payments by the agent. A business cycle implication of the model that is at odds with the data is the procyclicality of net exports.[3]

The approach we adopt here is a dynamic stochastic general equilibrium version of Eaton and Gersovitz (1981) and is similar to the formulations in Chatterjee et al (2002) on household default and Arellano (2003) on emerging market default. In these models, default can arise in relatively bad states of the world. Our point of distinction from the previous literature is the emphasis we place on the role of the stochastic trend in driving the income process in emerging markets. We find that the presence of trend shocks substantially improves the ability of the model to generate empirically relevant levels of default. Moreover, we obtain the coincidence of countercyclical net exports, countercyclical interest rates and the positive correlation between interest rates and current account observed in the data. This is distinct from what is obtained in Arellano (2003) and Kehoe and Perri (2002). A point to note is that we are able to generate positive comovement between interest rates and the current account without recourse to any external shocks. It is sometimes conjectured that the reason borrowing is higher in good times (counter to risk sharing predictions) is because these are times when, for reasons that have nothing to do with domestic conditions, the rest of the world is willing to lend to emerging markets at favorable terms. We show that this does not necessarily have to be the case.

In the next section we describe empirical facts regarding default and business cycle moments. Section 2 describes the model environment, parameterization and solution method. Section 3 describes the model with a stable trend and its predictions. Section 4 describes the model with a stochastic trend and performs sensitivity analysis. Section 5 examines the effect of third party bailouts on the default rate and Section 6 concludes.

1.1 Empirical Facts

Reinhart et al (2003) document that among emerging markets with at least one default or restructuring episode between 1824 and 1999, the average country experienced roughly 3

[3] This arises because in response to a positive shock, insurance calls for a payment from the booming country. Moreover, capital flows into the booming country for additional investment is restricted to limit the attractiveness of autarky.

crises every 100 years.[4] The same study documents that the external debt to GDP ratio at the time of default or restructuring averaged 71%. A goal of any quantitative model of emerging market default is to generate a fairly high frequency of default coinciding with an equilibrium that sustains a large debt to GDP ratio.

Table 1 documents business cycle features for Argentina over the period 1983.1 to 2000.2 using an HP filter with a smoothing parameter of 1600 for quarterly frequencies. Argentina is used as a benchmark because a long time series on interest rates is available. A striking feature of the business cycle is the strong countercyclicality of net exports (-0.89) and interest rates spreads(-0.59). Interest rates and the current account are also strongly positively correlated (0.68). These features regarding interest rates, in addition to the high level of volatility of these rates, have been documented by Neumeyer and Perri (2004) to be true for several other emerging market economies and to contrast with the business cycle features of Canada, a developed small open economy. Aguiar and Gopinath (2004) document evidence of the stronger countercyclicality of the current account for emerging markets relative to developed small open economies. A model that endogenizes interest rates therefore must predict the coincidence of higher borrowings and lower interest rates in booms and the reverse in slumps.

In the model we describe below we emphasize the distinction between shocks to the stochastic trend and transitory shocks. This is motivated by previous research (Aguiar and Gopinath (2004)) that documents that emerging markets are subject to more volatile shifts in stochastic trend as compared to a developed small open economy. In Figure 1 we plot the level of output and the stochastic trend for Canada relative to Mexico. As suggested by the picture the trend is far less stable in the case of Argentina as compared to Canada.[5]

[4] Conditioning on at least one default ignores the many emerging markets that never defaulted. However, countries that have defaulted appear to differ at some fundamental level from those that have never defaulted. For example, Reinhart et al (2003) document that previous default is a leading predictor of future default. Therefore, it may be misleading to consider economies that have and have not defaulted as draws from the same distribution.

[5] The trend is constructed by isolating permanent shocks to income using long run restrictions in a VAR. To calculate the depicted trend, we suppress all fluctuation due to transitory shocks and feed only permanent shocks through the system. For details see Aguair and Gopinath (2004).

2 Model Environment

In this paper we provide a quantitative model that attempts to match and explain the above facts. In doing so we emphasize the distinction between the type of shocks driving the income process in these economies. Specifically we describe at length the ability of a model with purely transitory shocks to income around a stable trend versus a model with persistent growth shocks to quantitatively match the facts in the data.

To model default we adopt the classic framework of Eaton and Gersovitz (1981). Specifically, we assume that international assets are limited to one period bonds. If the economy refuses to pay any part of the debt that comes due, we say the economy is in default. Once in default, the economy is forced into financial autarky for a period of time as punishment. This is similar to the framework adopted in Chatterjee et al. (2002) and Arellano (2003). An alternative is to allow a state contingent contract that is written subject to a participation constraint. This is the approach pioneered by Kehoe and Levine (1993). Our approach ties into a long literature and at least nominally reflects the fact that most international capital flows take the form of bonds, defaults occur in equilibrium, and economies have difficulty gaining access to international financial markets for some period after defaulting. An advantage of the alternative approach is that it captures the fact that there may be ex post renegotiation and debt rescheduling, something we rule out a priori.[6]

We begin our analysis with a standard model of a small open economy that receives a stochastic endowment stream, y_t. (We discuss a production economy in Section 4.1.). The economy trades a single good and single asset, a one period bond, with the rest of the world. The representative agent has CRRA preferences over consumption of the good:

$$u = \frac{c^{1-\gamma}}{1-\gamma}. \tag{1}$$

The endowment y_t is composed of a transitory component z_t and a trend Γ_t:

$$y_t = e^{z_t}\Gamma_t. \tag{2}$$

[6] Kletzer (2000) describes a model of sovereign default where the debt contract is renegotiation proof.

The transitory shock, z_t, follows an $AR(1)$ around a long run mean μ_z

$$z_t = \mu_z(1-\rho_z) + \rho_z z_{t-1} + \varepsilon_t^z \tag{3}$$

$|\rho_z| < 1$, $\varepsilon_t^z \sim N(0, \sigma_z^2)$,and the trend follows

$$\Gamma_t = g_t \Gamma_{t-1} \tag{4}$$

$$\ln(g_t) = (1-\rho_g)(\ln(\mu_g) - c) + \rho_g \ln(g_{t-1}) + \varepsilon_t^g \tag{5}$$

$|\rho_g| < 1$, $\varepsilon_t^g \sim N(0, \sigma_g^2)$, and $c = \frac{1}{2}\frac{\sigma_g^2}{1-\rho_g^2}$.

We denote the *growth rate* of trend income as g_t, which has a long run mean μ_g. The log growth rate follows an $AR(1)$ process with AR coefficient $|\rho_g| < 1$. Note that a positive shock ε^g implies a permanently higher level of output, and to the extent that $\rho_g > 0$, a positive shock today implies that the growth of output will continue to be higher beyond the current period. We assume $\beta(\mu_g)^{1-\gamma} < 1$ to ensure a well defined problem, where $0 < \beta < 1$ denotes the agent's discount rate.

Let a_t denote the net foreign assets of the agent at time t. Each bond delivers one unit of the good next period for a price of q this period. We will see below that in equilibrium q depends on a_t and the state of the economy. We denote the value function of an economy with assets a_t and access to international credit as $V(a_t, z_t, g_t)$ (see the Appendix for a formal derivation with more complete notation). At the start of the period, the agent decides whether to default or not. Let V^B denote the value function of the agent once it defaults. The superscript B refers to the fact that the economy has a bad credit history and therefore cannot transact with international capital markets (i.e. reverts to financial autarky). Let V^G denote the value function given that the agent decides to maintain a good credit history this period. The value of being in good credit standing at the start of period t with net assets a_t can then be defined as $V(a_t, z_t, g_t) = \max \langle V_t^G, V_t^B \rangle$. At the start of period t, an economy in good credit standing and net assets a_t will default only if $V^B(z_t, g_t) > V^G(a_t, z_t, g_t)$.

An economy with a bad credit rating must consume its endowment. However, with probability λ it will be "redeemed" and start the next period with a good credit rating and renewed access to capital markets. Gelos *et al* (2003) estimate the average number of years a country is excluded from foreign borrowing to be 3 years for countries that defaulted during the period 1980-1999.[7] If redeemed, all past debt is forgiven and the economy starts off with zero net assets. We also add a parameter δ that governs the additional loss of output in autarky. Rose (2002) finds evidence of a significant and sizeable (8% a year) decline in bilateral trade flows following the initiation of debt renegotiation by a country in a sample covering 200 trading partners over the period 1948-97. This cost δ will be shown to be necessary to sustain quantitatively reasonable levels of debt in equilibrium.

In recursive form, we therefore have:

$$V^B(z_t, g_t) = u((1-\delta)y_t) + \lambda\beta E_t V(0, z_{t+1}, g_{t+1}) + (1-\lambda)\beta E_t V^B(z_{t+1}, g_{t+1}) \qquad (6)$$

where E_t is expectation over next period's endowment and we have used the fact that λ is independent of realizations of y. If the economy does not default, we have:

$$\begin{aligned} V^G(a_t, z_t, g_t) &= \max_{c_t} \{u(c_t) + \beta E_t V(a_{t+1}, z_{t+1}, g_{t+1})\} \qquad (7) \\ s.t.\ c_t &= y_t + a_t - q_t a_{t+1} \end{aligned}$$

The Appendix derives some key properties of the value functions.

The international capital market consists of risk neutral investors that are willing to borrow or lend at an expected return of r^*, the prevailing world risk free rate. Klingen *et al* (2004) present evidence that long-run ex post risk premia have been close to zero for emerging markets. From 1970-2000 they find that returns averaged 9% per annum which is about the same as the return on a 10 year U.S. Treasury Bond. Of course, it is difficult to estimate ex ante returns from a relatively short time series of ex post returns. Nevertheless, our formulation of international capital markets is a useful benchmark and consistent with the available evidence.

The default function $D(a_t, z_t, g_t) = 1$ if $V^B(z_t, g_t) > V^G(a_t, z_t, g_t)$ and zero otherwise. Then equilibrium in the capital market implies

$$q(a_{t+1}, z_t, g_t) = \frac{E_t\{(1 - D_{t+1})\}}{1 + r^*}. \qquad (8)$$

[7] In Gelos et. al. (2003) the year of default is defined as the year in which the sovereign defaulted on foreign currency debt. Market access is defined to be resumed when there is evidence of issuance of public or publicly guaranteed bonds or syndicated loans.

The higher the expected probability of default the lower the price of the bond. Equilibrium is the fixed point q that satisfies (8) evaluated at the default function D, where D in turn solves the representative agent's optimization problem given q.

To gain insight into the model, we explore the agent's first order conditions. For expositional purposes, we use notation that assumes the interest rate function q and value functions are differentiable. However, we do not claim or prove differentiability and our quantitative results do not depend on differentiability. This will be clear when we describe the methodology used to solve the model. Given continuity, which is established in the Appendix, we can view our differentiable functions as at worst a close approximation that greatly simplifies exposition. For a country in good credit standing, the Euler Equation for consumption absent a decision to default can be expressed as

$$E_t\left\{\beta\frac{u'(c_{t+1})}{u'(c_t)}(1-D_{t+1})\right\}=q_t+a_{t+1}\frac{\partial q_t}{\partial a_{t+1}} \tag{9}$$

The $(1-D_{t+1})$ term reflects the fact that at the margin, additional borrowing/saving today affects future consumption only in the states in which the agent does not default in the following period. As long as the economy is a net saver, we have the standard Euler Equation, as $D_{t+1}=0$ for all realizations of z and g, $q=1/(1+r^*)$, and $q'\equiv\partial q/\partial a=0$. However, if the economy is indebted to the point it may default next period, then $q<1/(1+r^*)$, $q'>0$, and $a<0$. The agent sets the expected marginal rate of substitution (conditional on not defaulting next period) equal to the (inverse) interest rate (q) plus an additional term, aq'. This latter term arises because the representative agent internalizes the fact that additional borrowing leads to a higher interest rate. If the debt in question is sovereign debt then it is unreasonable to assume that the agent is "small" relative to the loan supply function for the country. Even if borrowing were undertaken at a disaggregated level, the use of loan ratings and credit scores for individual borrowers suggest that each agent faces an idiosyncratic interest rate that at the margin varies with the agent's idiosyncratic probability of default. The importance of this term in the first order condition will be discussed at length below.

To emphasize the distinction between the role of transitory and permanent shocks we present two extreme cases of the model described above. Model I will correspond to the case when the only shock is the transitory shock z_t and Model II to the case when the only shock has permanent effects, g_t.[8] Since few results can be analytically derived we discuss at the outset the calibration and solution method employed.

2.1 Calibration and Model Solution

Benchmark parameters that are common to all models are reported in Table 2A. Each period refers to a quarter. The coefficient of relative risk aversion of 2 is standard. We set the quarterly risk free world interest rate at 1%. The probability of redemption λ is set equal to 0.1, which implies that the economy is denied market access for 2.5 years on average. This is similar to the three years observed in the data (Gelos et. al. (2003)). The additional loss of output in autarky is set at 2%. We will see in our sensitivity analysis (Section 4.1) that high impatience is necessary for generating reasonable default in equilibrium. Correspondingly, our benchmark calibration sets $\beta = 0.8$. Authors such as Arellano (2003) and Chatterjee et. al. also employ similarly low values of β to generate default. The mean quarterly growth rate is calibrated to 0.6% to match the number for Argentina, implying $\mu_g = 1.006$.

The remaining parameters characterize the underlying income process and therefore vary across models (Table 2B). To focus on the nature of the shocks, we ensure that the HP filtered income volatility derived in simulations of both models approximately match the same observed volatility in the data. In Model I, output follows an $AR(1)$ process with stable trend and an autocorrelation coefficient of $\rho_z = 0.9$, which is similar to the values used in many business cycle models and $\sigma_z = 3.4\%$. We set the mean of log output equal to $-1/2\sigma_z^2$ so that average detrended output in levels is standardized to one. In Model II, $\sigma_z = 0$, $\sigma_g = 3\%$ and $\rho_g = 0.17$.

To solve the model numerically we use the discrete state-space method. We first recast the Bellman equations in detrended form and then discretize the state space. We approximate the continuous $AR(1)$ process for income with a discrete Markov chain using

[8] One of the reasons we consider the two extremes is to minimize the dimensionality of the problem, which we solve employing discrete state space methods. Using insufficient grids of the state space can generate extremely unreliable results in this set up.

25 equally spaced grids[9] of the original processes steady state distribution. We then integrate the underlying normal density over each interval to compute the values of the Markov transition matrix. This follows the procedure of Hussey and Tauchen (1991).

The asset space is discretized into 400 possible values.[10] We ensured that the limits of our asset space never bind along the simulated equilibrium paths. The solution algorithm involves the following,

(i) Assume an initial price function $q^0(a, z, g)$. Our initial guess is the risk free rate.

(ii) Use this q^0 and an initial guess for $V^{B,0}$ and $V^{G,0}$ to iterate on the Bellman equations (6) and (7) to solve for the optimal value functions V^B, V^G, $V = \max \left\langle V^G, V^B \right\rangle$ and the optimal policy functions.

(iii) For the initial guess q^0, we now have an estimate of the default function $D^0(a, z, g)$. Next, we update the price function as $q^1 = \frac{E_t\{(1-D_{t+1})\}}{1+r^*}$ and using this q^1 repeat steps (ii) and (iii) until $|q^{i+1} - q^i| < \varepsilon$, where i represents the number of the iteration and ε is a very small number.[11]

3 Model I: Stable Trend

Model I assumes a deterministic trend and the process for z_t is given by (3)

$$\Gamma_t = (\mu_g)^t \tag{10}$$

[9] It is important to span the stationary distribution sufficiently so as to include large negative deviations from the average even if these are extremely rare events because default is more likely to occur in these states.

[10] It is important to have a very fine partition of the asset space in order to make financial integration as attractive as possible to the representative agent.

[11] Note that we have not ruled out multiple equilibria. That is, a q schedule that implies an extremely high interest rate for borrowing may lead the agent to discount the benefit of financial integration and result in a high propensity to default, validating the high interest rate. Conversely, the same economy may support a low interest rate equilibrium that implies a corresponding unwillingness to default. We search for a fixed point by starting with a q that discourages default (maximizes the benefits of integration), namely $q = 1/(1+r^*)$ in all states and all debt levels. Given this q, the agent's opportunity set is as large as possible and therefore the value of integration is greatest. Therefore, states in which the agent defaults given this initial q will also be default states at higher interest rates. We then update q accordingly, and iterate until convergence.

where $\mu_g > 1$.

For a given realization of z, clearly the agent is more likely to default at lower values of a. Since V^B refers to financial autarky, its value is invariant to a. Conversely, V^G is strictly increasing in assets. This follows straightforwardly from the budget constraint and strict monotonicty of utility. For each z, there is a unique point of intersection, say $\overline{a}(z)$, and the agent will default if foreign assets lie below $\overline{a}$.

The default decision as a function of z is less clear-cut. In the case when $\lambda = 0$, it must be the case that V^G should be steeper than V^B at the indifference point, for a given a.To see this, consider the value of an additional unit of endowment at the indifference point. The agent in autarky must consume this additional income. The agent with a good credit standing can consume it or save it. The larger opportunity set implies that the value of the additional endowment is greater for the agent in good credit standing. Hence, the slope of V^G relative to z must be greater than the slope of V^B relative to z, at the indifference point. However, if $\lambda > 0$, then the agent in autarky has an outcome not available to the one in good credit standing, i.e. redemption with debt forgiveness. In this case, the previous argument's premise does not hold.

In Figure 2A, we plot the difference between the value function with a good credit rating (V^G) and that of autarky (V^A) as a function of z for our calibration. The agent defaults when output is relatively low. The top panel of Figure 3 plots the region of default in (z, a) space. The line that separates the darkly shaded from the lightly shaded region represents combinations of z and a along which the agent is indifferent between defaulting and not defaulting. The darkly shaded region represents combinations of low productivity and negative foreign assets for which it is optimal to default.

The fact that financial autarky is relatively attractive in bad states of the world is not a feature shared by alternative models based on Kehoe and Levine (1993). In that model, optimal "debt" contracts are structured so that the agent never chooses autarky. However, the participation constraint binds strongest in good states of nature, i.e. the states in which the optimal contract calls for payments by the agent. The difference stems from the fact that in a simple defaultable bond framework insurance is extremely limited. In particular, a sequence of negative shocks leads to increasingly higher levels of debt. It may then transpire that the agent must repay even in a bad state of nature, when interest payments exceed available new borrowing (since the amount of debt is limited by the possible endowment stream) and the agent is forced to pay out regardless of the shock. The burden of any repayment is largest in the small endowment states and therefore that is where default will occur. In the optimal contract setting absent other imperfections, there is no reason to demand repayment in a bad state of nature, regardless of the history of shocks. This allows more efficient insurance and ensures that bad draws are never associated with repayments.[12]

3.1 Debt and Default Implications in Model I

A simple calculation quickly reveals that it is difficult to sustain a quantitatively realistic level of debt in a standard framework without recourse to additional punishment. To demonstrate this, we borrow the methodology Lucas (1987) used to describe the relatively small welfare costs of business cycles. Indeed, the fact that cycles have limited welfare costs is precisely why it is difficult to support a large amount of debt in equilibrium.[13]

Consider our endowment economy in which the standard deviation of shocks to detrended output are roughly 4%. For this calculation, we stack the deck against autarky by assuming no domestic savings (capital or storage technology), that shocks are *iid*, and that autarky lasts forever. We stack the deck in favor of financial integration by supposing that integration implies a constant consumption stream (perfect insurance). In order to maintain perfect consumption insurance, we suppose that the agent must make interest payments of rB each period. We now solve for how large rB can be before the agent prefers autarky. We then interpret B as the amount of sustainable debt when interest payments are equal to r.

[12] Of course with additional frictions, there are optimal contracting environments in which insurance also fails to the extent that repayment is called for in the worst states of nature. For example, in models of moral hazard (such as Atkeson (1991)), a sequence of bad shocks may reduce ex post payments to preserve strong ex ante incentives.

[13] Bulow and Rogoff (1989) present a theoretical argument for why borrowing is unsustainable in an open economy that can continue to save globally. We point out here that in a standard endowment model with purely transitory shocks to income, even if punishment takes the form of autarky, quantitatively very little debt is sustainable in equilbrium

Specifically, let

$$Y_t = \overline{Y} e^{z_t} e^{-(\frac{1}{2})\sigma_z^2} \tag{11}$$

where $z \sim N(0, \sigma_z^2)$ and *iid* over time. We ensure that $EY_t = \overline{Y}$ regardless of the volatility of the shocks. Then,

$$V^B = E \sum_t \beta^t \frac{Y_t^{1-\gamma}}{1-\gamma} = \frac{(\overline{Y} e^{-(\frac{1}{2})\gamma\sigma_z^2})^{1-\gamma}}{(1-\gamma)(1-\beta)}. \tag{12}$$

Assuming that financial integration results in perfect consumption insurance,

$$V^G = E \sum_t \beta^t \frac{c_t^{1-\gamma}}{1-\gamma} = \frac{(\overline{Y} - rB)^{1-\gamma}}{(1-\gamma)(1-\beta)}. \tag{13}$$

The economy will not default as long as $V^G \geq V^B$, or $\frac{rB}{\overline{Y}} \leq 1 - \exp(-(\frac{1}{2})\gamma\sigma_z^2)$. The volatility of detrended output for Argentina is 4.08% (i.e. $\sigma_z^2 = 0.0408^2 = 0.0017$). For a coefficient of relative risk aversion of 2, this implies the maximum debt payments as a percentage of GDP is 0.17%. Or, at a quarterly interest rate of 2%, debt cannot exceed 8.32% of output.[14] In a model that allows for capital accumulation, the value of financial integration need not be higher as economies can self insure by accumulating domestic capital. Gourinchas and Jeanne (2004) calibrate the small welfare gains from financial integration for countries with low levels of capital to be equivalent to a 1% rise in permanent consumption.

Our simulated model will be shown to support higher debt levels because we impose an additional loss of δ percent of output during autarky. Introducing such a loss into the above calculation implies a debt cutoff of $\frac{rB}{\overline{Y}} \leq 1 - (1-\delta)\exp(-(\frac{1}{2})\gamma\sigma_z^2)$. If $\delta = 0.02$, we can support debt *payments* of 20% of GDP, which implies a potentially large debt to GDP ratio. It is clear that to sustain any reasonable amount of debt in equilibrium in a standard model, we need to incorporate punishments beyond the inability to self-insure, particularly since in reality financial integration does not involve full insurance and autarky does not imply complete exclusion from markets.

A second implication of the model is that default rarely occurs in equilibrium. This rests on a more subtle argument that has to do with the shape of the q schedule. Note

[14] At a relatively large γ of 5, we still have that debt payments cannot exceed 0.42% of output, or a maximum debt to GDP ratio of 21% at a 2% interest rate.

that the fact that default rarely occurs does not contradict that autarky may be relatively painless. In equilibrium, interest rates respond to the high incentive to default.

We begin with the Euler Equation for consumption (9), which we repeat here:

$$E_t\left\{\beta\frac{u'(c_{t+1})}{u'(c_t)}(1-D_{t+1})\right\}=q_t+a_{t+1}q'$$

Now suppose that the current endowment shock is below average. Absent any borrowing, this implies a positively sloped consumption profile and a marginal rate of substitution significantly less than the risk free bond price. To satisfy the first order condition, the agent then borrows. As it reduces a, the marginal rate of substitution begins to rise and q begins to fall (i.e., the interest rate increases) as the probability of default rises. The reason that this higher probability of default does not arise quantitatively is due to the presence of q' in the Euler Equation. As the agent borrows, q' increases (i.e., q is concave over the relevant range). It turns out that quantitatively, this second term dominates the first. The agent is willing to maintain a steep consumption profile at a low interest rate because it internalizes the effect of additional borrowing on the interest rate it must pay. We will next discuss the intuition behind the large response of q'.

Figure 4A plots the q schedule as a function of assets for the highest and lowest realizations of z. Over asset regions for which agents never default, the implied interest rate is the risk free rate ($q=\frac{1}{1+r^*}$). However, the schedule is extremely steep over the range of assets for which default is possible. The intuition for this can be built from Figure 3. Let $\overline{z}(a)$ denote the threshold endowment below which the agent defaults for the given asset level. That is, $\overline{z}$ is the line separating the shaded region from the unshaded region in Figure 3. For a given a_{t+1}, we can then express the probability of default at time $t+1$ as $\Pr(z_{t+1}<\overline{z}(a_{t+1})|z_t)$, and correspondingly $q_t(a_{t+1})=\frac{(1-\Pr(z_{t+1}<\overline{z}(a_{t+1})|z_t))}{(1+r^*)}=\frac{(1-F_t(\overline{z}(a_{t+1}))}{(1+r^*)}$, where F is the cdf of a normal random variable with mean $\mu_z(1-\rho_z)+\rho_z z_t$ and variance σ_z^2. The slope of the interest rate schedule is then

$$q'(a)=-\frac{f(\overline{z}(a_{t+1}))}{1+r^*}\frac{d\overline{z}}{da}\tag{14}$$

where $f=F'(z)$. From Figure 3, we see that quantitatively, $\frac{d\overline{z}}{da}$ is extremely large. The steepness of the $\overline{z}(a)$ translates directly to the steepness of the q schedule.

The last step of the reasoning involves explaining why $\overline{z}(a)$ is so steep. Recall that $\overline{z}(a)$ represents combinations of a and z for which the agent is indifferent to default, i.e., $V^G = V^B$. Then from the implicit function theorem,

$$\frac{d\overline{z}}{da} = \frac{-\frac{\partial V^G}{\partial a}}{\left(\frac{\partial V^G}{\partial z} - \frac{\partial V^B}{\partial z}\right)}. \tag{15}$$

In the case of Model I, $(\frac{\partial V^G}{\partial z} - \frac{\partial V^B}{\partial z})$ tends to be a very small number. This can be seen from Figure 2A that plots the difference between V^G and V^B across z (for a given a). This implies that the slopes of V^G and V^B with respect to z are not that different. Why $\frac{\partial V^G}{\partial z}$ is not much larger than $\frac{\partial V^B}{\partial z}$ results from the underlying process for z. Suppose that z is a random walk. In this case, a shock to z today is expected to persist indefinitely and will have a large impact on expected lifetime utility. However, with a random walk income process there is limited need to save out of additional endowment (the only reason would be precautionary savings and the insurance provided by the option to default). This implies an additional unit of endowment will be consumed, leaving little difference between financial autarky and a good credit history. This issue arises for strictly transitory shock processes as well. Consider the other extreme and suppose that z is *iid* over time. Then there is a stronger incentive to borrow and lend. However, the lack of persistence implies the impact of an additional unit of endowment today is limited to its effect on current endowment, resulting in a limited impact on the entire present discounted value of utility. That is, both $\frac{\partial V^G}{\partial z}$ and $\frac{\partial V^B}{\partial z}$ are relatively small and therefore so is the difference. Therefore, whether a shock follows a random walk or is *iid*, a given shock realization has little impact on the difference between V^G and V^B. Consequently, $\frac{d\overline{z}}{da}$ is very large.

The key point is that in our benchmark Model I, the q schedule is extremely steep over the relevant range of borrowing. Moreover, from equation (14), it will also be strongly concave over values of a for which default probabilities are reasonably small.[15] We can now explore explicitly how the shape of the q schedule prevents default in equilibrium. Recall that the expected marginal rate of substitution (MRS) must equal $q + aq'$. For

[15] To see this, note from figure 2 that $\overline{z}(a)$ is approximately linear. Therefore, $q''(a)$ will depend on the slope of f (the normal pdf) at $\overline{z}(a)$ multiplied by $\frac{d\overline{z}}{da}$. Therefore, q'' will also reflect the steepness of the $\overline{z}(a)$ schedule. An alternative intuition is that q is flat in the nondefault range of a,but very steep for lower values of a. This dramatic change in slope implies q' is very sensitive to a over the relevant range.

various values of a, we can calculate how much of the movement in MRS is matched by movements in q and how much is matched by aq'. The dashed line in Figure 5 plots q against $MRS = q + aq'$ for a particular realization of z. We see that large movements in the marginal rate of substitution are accommodated by very small movements in q itself. Therefore, a large implicit demand for borrowing (low MRS) does not result in additional borrowing and a high probability of default. Instead, it generates minimal borrowing and a large movement in the slope of the interest rate function. The flip side of this can be seen from the fact that net exports are extremely stable.

3.2 Business Cycles Implications in Model I

Table 3, column 3A, reports key business cycle moments from Model I.[16] Default is a rare event as it occurs on average only two times in 10,000 periods (i.e., once every 2,500 years). The discussion in the previous section explains why this is the case for models with purely transitory shocks. Net export and interest volatility is much lower than in the data. The lack of interest rate volatility is an immediate consequence of the fact that default rarely occurs in equilibrium. The model supports a maximum debt to GDP ratio of 26%.

A typical feature of these models is that the current account and the interest rate tend to be negatively correlated, a counterfactual implication. This follows from the steepness of the interest rate function. That is, if the agent borrows more in good states of the world (countercyclical current account) then one effect is for the interest rate to rise as the agent moves up the "loan supply curve". The countering effect is that a persistent good state can imply a lower probability of default and therefore a shift in the q schedule. To generate the empirical fact that countries borrow more in good times at lower interest rates we need the second effect to dominate the first. However, the steepness of the q schedule in Model I makes this a less likely outcome. Consequently, in our parameterization of Model I, we obtain a countercyclical current account as the data suggests, however the interest rate process is now procyclical.

[16] To analyze the model economy's stationary distribution we simulate the model for 10,000 periods and extract the last 500 observations to rule out any effect of initial conditions. We then log and HP filter the simulated series in the same way as the empirical data, using a HP filter smoothing parameter of 1600. The reported numbers are averages over 500 such simulations.

We have adopted a relatively persistent process for income that generates a countercyclical current account, but at the cost of procyclical interest rates. The reason the current account can be countercyclical even in an endowment economy with purely transitory shocks is because of the endogenous response of interest rates. When the income shock is temporarily high, there is the typical incentive to save to smooth consumption. In addition there is the interest rate effect that works through shifts in the interest rate function. That is, all else equal, the expected probability of default is lower when the current income state is high and expected to persist. This was seen in Figure 4A where the price function shifts in for high z. What is less visible, but is also the case, is that the slope of the interest rate function also is reduced. While q is countercyclical in equilibrium, we can calculate the marginal cost of borrowing an additional unit along the equilibrium path and this is found to be countercyclical. This is then consistent with households wanting to borrow more when income is temporarily high.

There are alternative parameterizations of Model I that produces a countercyclical interest rate process, as called for by the data. However, this occurs only when the current account is procyclical, contradicting the strong countercyclicality of net exports observed in emerging markets. We will see in the next section that introducing shocks to trend can generate the positive comovement between interest rates and the current account.

Before moving on to an alternative, we summarize how a standard model with transitory shocks has difficulty matching key empirical facts. The rate of default in equilibrium is orders of magnitude too small. This in turn leads to a counterfactually stable interest rate process. Moreover, the cyclicality of the interest rate is the opposite of the cyclicality of net exports, while in the data both are countercyclical and positively correlated in emerging markets.

4 Model II: Stochastic Trend

In Model II: $y_t = \Gamma_t$ where we let Γ_t vary stochastically as in (4) and (5). The behavior of this model is captured by the simulation results reported in Table 3B. One important distinction between the model with stochastic trend and Model I is the rate of default in equilibrium. Specifically, the rate of default increases by a factor of ten. The reason for this can be seen by contrasting the behavior of the difference between V^B and V^G between Figure 2A (Model I) and Figure 2B (Model II). High and low states of the growth shock will have substantially different effects on life time utility. Moreover, with persistent shocks, the value of financial integration will be high. Consequently, (V^B-V^G) has a greater slope with respect to the g shocks (Figure 2B) as compared to z shocks (Figure 2A). By the logic introduced in the context of Model I, this in turn suggests $\frac{d\overline{z}}{da}$ has a smaller slope. This is seen in Figure 3. Compared to the figure in the top panel, the region of default is larger and the slope of the line of indifference is smaller in the case of growth shocks. This then translates into a less steep q function as seen in Figure 4B. Moreover, it also suggests that default will occur in equilibrium with more frequency. Recall that a given marginal rate of substitution equals $q + aq'$. In figure 4, we plot how much a movement in the marginal rate of substitution is reflected in a movement in q. In contrast to Model I, we now see that more of the movement is reflected in q, which in turn implies that more of the movement is reflected in a higher probability of default.

The results of the simulation of the model are reported in Table 3. Some improvements over Model I are immediately apparent. Both the current account and interest rates are countercyclical and positively correlated (though the magnitudes are lower than in the data). A positive shock increases output today, but increases output tomorrow even more (due to the persistence of the growth rate). This induces agents to borrow in good times. In Figure 4, we plot the q schedule as a function of (detrended) a for a low and high value of g. We see that a positive shock lowers the interest rate (raises q) for all levels of debt. In the reported parameterization, the shift in the q schedule dominates the movement along the schedule induced by additional borrowing. Correspondingly, the volatility of interest rates increases by a factor of 2.5. Net export volatility rises by a factor of 5 to match more closely the volatility in the data. However, interest rate volatility still remains well below that observed empirically for Latin America.

The fact that Model II features shocks to trend –and therefore shocks to permanent income– implies that consumption volatility is higher than was the case in Model I. This consumption volatility would suggest a greater demand for the insurance provided by a defaultable bond. The marginal benefit of insurance is captured by the Euler Equation.

To explore this quantitatively, we calculate $\beta\frac{u'(c_{t+1})}{u'(c_t)}(1-D_{t+1})$ at each point along the simulated path (restricting ourselves to periods of financial integration). The mean of this series is 0.7917 for Model II, compared to 0.7924 for Model I. Qualitatively, it is the case that trend shocks produce an increased willingness to borrow. That is, the lower mean marginal rate of substitution will be associated with a higher marginal cost of borrowing. However, quantitatively, the difference is miniscule. To put the difference in context, suppose that q' were always zero so that the interest rate reflected the full cost of debt at the margin. In this case, the difference in marginal rates of substitution would equal the difference in the price of bonds q. Recalling that $q = (1 - \Pr(Default))/(1 + r^*)$, a difference in q of $0.7924 - 0.7917 = 0.0007$ translates into a difference in average probability of default of $(1 + r^*)0.0007 = 0.00071$, where $r^* = 0.01$. That is, the difference in marginal rates of substitution would imply that Model II experiences 7 more defaults per $10,000$ quarters. The actual difference is closer to twenty. Put in the context of Figure 4, the difference between Model I and Model II is not in the marginal rate of substitution, but rather how a given MRS is matched by q versus q'.

4.1 Sensitivity Analysis

In this section we perform sensitivity tests within the framework of Model II. First, we consider a case with endogenous labor supply. The production function now takes the form

$$y_t = e^{z_t}\Gamma_t L_t^{\alpha}.$$

We set $\alpha = 0.68$. The utility function takes the GHH form (from Greenwood et al 1988)

$$u_t = \frac{\left(c_t - \frac{1}{\omega}\mu_g\Gamma_{t-1}L_t^{\omega}\right)^{1-\gamma}}{1-\gamma}, \tag{16}$$

The ω parameter is calibrated to 1.455 implying an elasticity of labor supply of 2.2. This is the value employed in previous studies (Mendoza(1991), Neumeyer and Perri (2004)).The results for the simulation are reported in Table 4. The number of defaults are lower compared to the endowment model, however the business cycle moments line up as in the data.

The importance of a high level of impatience in generating reasonable levels of default is also seen in Table 4 where we report the results for higher levels of β. As β increases the number of defaults drop from 23 to 11 per 2500 years. The need for impatience stems from the fact that the agent's marginal rate of substitution is set equal to the inverse interest rate (q) *plus* the marginal effect of additional borrowing on the interest rate (aq'). Therefore, to obtain small deviations between the equilibrium interest rate and the world interest rate, the agent's MRS must be considerably larger than the inverse of the interest rate. This in turn requires a rate of time preference that exceeds the world interest rate. While changes in β influence the rate of default, we see from Table 4 that the signs of the key correlations remain unchanged.

The Euler Equation (9) reflects that default has an insurance (i.e. state contingent) component absent from a risk-free bond. The agent only repays in the good (nondefault) states of nature. While the agent cannot explicitly move resources across states of nature in the next period, she can move resources from good states next period forward to today, leaving resources available in bad/default states next period unaffected. Given that default provides insurance, one would expect that the value of this insurance would increase with uncertainty. Table 5 explores this conjecture formally. For each model, we doubled the innovation variance relative to the case reported in Table 3. We see that the rate of default roughly doubles relative to the benchmark case for both models. It is interesting to note that despite the increased rate of default, the higher volatility economies do not hold more debt in equilibrium, reflecting that these economies face a different interest rate function.

Finally, we explore the sensitivity of our results to our cost of autarky parameter δ. The last two columns of Table 5 report simulations of the two model economies where we have set $\delta = 0.005$ rather than 0.02. As expected, the lower cost of autarky results in very low debt levels observed in equilibrium, where instead of roughly 25% debt to income ratios, we now have ratios closer to 5%. This lack of borrowing is also reflected in the lower volatility of net exports and interest rates. Nevertheless, defaults occur with roughly the same frequency as was the case in the benchmark models.

5 Third Party Bailouts

Incorporating shocks to trend has improved the model's performance on a number of dimensions. In particular, it generates more default in equilibrium and allows us to match the fact that both net exports and interest rates are countercyclical in emerging markets. However, the default rate of once every 125 years remains low, at least compared to the track record of many Latin American countries. In this section, we try to improve on Model II by augmenting the model with a phenomenon observed in many default episodes – bailouts. For example, Argentina received a $40 billion bailout in 2001 from the IMF, an amount nearly 15% of Argentine GDP in 2001. Such massive bailouts must influence the equilibrium debt market studied in the previous two sections.

We model bailouts as a transfer from an (unmodelled) third party to creditors in the case of default. While in practice bailouts often tend to nominally take the form of loans, we assume that bailouts are grants. To the extent that such loans in practice are extended at below market interest rates, they incorporate a transfer to the defaulting country. We also assume that creditors view bailouts as pure transfers. Again, it may be the case in practice that creditors ultimately underwrite the bailouts through tax payments. A reasonable assumption is that creditors do not internalize this aspect of bailouts.

Specifically, bailouts take the following form. Creditors are reimbursed the amount of outstanding debt up to some limit a^*. Any unpaid debt beyond a^* is a loss to the creditor. From the creditors viewpoint, therefore, every dollar lent up to a^* is guaranteed. Any lending beyond that has an expected return determined by the probability of default. Specifically, the break even price of debt solves

$$q_t = \frac{1}{1+r^*}\left\{\min\left\langle 1, \frac{a^*}{a}\right\rangle + E_t\{1-D_{t+1}\}\max\left\langle 1-\frac{a^*}{a}, 0\right\rangle\right\}.$$

The presence of bailouts obviously implies that debt up to a^* carries a risk free interest rate. Moreover, the probability of default is used to discount only that fraction of debt that exceeds the limit. The net result is to shift up and flatten the q schedule. To consider why the q schedule is flatter, consider the case without bailouts, i.e. $a^* = 0$. Each additional dollar of debt raises the probability of default. As default implies zero repayment, this lowers the return on all debt. However, with bailouts, an increase in the probability of default affects only the return on debt beyond a^*. While this may have a large impact on the return of the marginal dollar, the sensitivity of the average return is mitigated by the fact that part of the debt is guaranteed.

From the agent's perspective, bailouts subsidize default. Mechanically, this results in an interest rate that is not only lower but also less sensitive to additional borrowing. Therefore, a given marginal rate of substitution will be associated with a higher probability of default. The increase in the rate of default is not surprising given that bailouts are a pure transfer from a third party.

This intuition is confirmed in our simulation results reported in Column 3C of Table 3. We calibrate a^* so that the maximum bailout is 18% of (mean detrended) output and we now set the time preference rate at 0.95 so that impatience rates are not as high as in the previous benchmark simulations. We see that the agent now defaults roughly once every 27 years. This would be close to the Argentine default rate. The increased rate of default however does not raise interest rate volatility. This reflects the fact that bailouts insulate interest rates from changes in the probability of default. As before, net exports are still countercyclical, but now have a volatility that is closer to that observed empirically. The shallowness of the q function allows agents to borrow and lend more freely.

In sum, allowing for bailouts of fairly modest levels compared to those observed in practice enables our model to match the extreme rates of default observed in many Latin American economies. The model also matches the countercyclicality of net exports and interest rates. However, by breaking the tight link between default and interest rates, the model fails to produce reasonable volatility in the interest rate.

6 Conclusion

We present a model of endogenous default that emphasizes the role of switches in growth regimes in matching important business cycle features of Emerging Markets and in generating default levels that are closer to the frequency observed in the data. The reason why a model with growth shocks performs better is that in such an environment a given probability of default is associated with a smaller borrowing cost at the margin. This in turn rests on the fact that trend shocks have a greater impact on the propensity to default than do standard transitory shocks, making interest rates relatively less sensitive to the amount borrowed and relatively more sensitive to the realization of the shock.

Further, to match the business cycle features of the interest rate and current account, the model should predict that agents borrow more at lower interest rates during booms and the reverse during slumps. Since the interest rate schedule tends to be very steep in these models, the typical prediction is for the interest rate and current account to be negatively correlated. In the model with growth shocks, however, we find that for plausible parameterizations the shift of the interest rate schedule in good states, in anticipation of lower default probabilities, dominates the increase in interest rates that arise because of additional borrowing. While the predictions still remain short of matching quantitatively the magnitudes obtained in the data, the predicted sign of the correlations of income, net exports, and the interest rate are in line with empirical facts.

7 Appendix

This appendix characterizes and derives properties of the representative agent's problem. The agent enters period t with assets a_t, a credit standing $h_t \in \{B, G\} \equiv H$, and faces an income process governed by $y_t = e^{z_t}\Gamma_t$. The variables z and g are drawn from $\overline{z} = [z_1, ..., z_{N_z}]$ and $\overline{g} = [g_1, ..., g_{N_g}]$, respectively, where $\overline{z}$ and $\overline{g}$ are strictly positive. We assume that the transition probability functions for z and g satisfy monotonicity (i.e., if f is nondecreasing in (z, g), then so is $E(f(z', g')|z, g)$). This is satisfied by the discretized model presented and solved in section 2.

Let a tilde denote detrended variables: $\widetilde{x}_t \equiv \frac{x_t}{\mu_g \Gamma_{t-1}}$, $x = \{c, y, a\}$. Let s denote the sate, $s = (\widetilde{a}, z, g, h)$, which takes values in $S = \overline{a} \times \overline{z} \times \overline{g} \times H$, where $\overline{a}$ represents the range of possible values of $\widetilde{a}$. Preferences are given by $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. The agent faces an interest rate schedule (vector) $q : S \to [0, 1/(1 + r^*)]$, which we assume is continuous in $(\widetilde{a}, z, g)$. Let Q denote the set of possible q. We will show below that defining q over detrended variables is equivalent to defining the interest rate function over the non-detrended state (a, z, Γ, h). By construction, $q = 0$ if $h = B$.

In recursive form, we can express the agent's problem as:

$$V^*(a_t, z_t, \Gamma_t, h_t|q) =$$

if $h_t = B$:

$$u((1-\delta)y_t) + \lambda\beta E_t V^*(0, z_{t+1}, \Gamma_{t+1}, G|q) + (1-\lambda)\beta E_t V^*(z_{t+1}, \Gamma_{t+1}, B|q)$$

if $h_t = G$:

$$\max \left\langle \begin{array}{c} u((1-\delta)y_t) + \lambda\beta E_t V^*(0, z_{t+1}, \Gamma_{t+1}, G|q) + (1-\lambda)\beta E_t V^*(z_{t+1}, \Gamma_{t+1}, B|q), \\ \max_{c_t} \{u(c_t) + \beta E_t V^*(a_{t+1}, z_{t+1}, \Gamma_{t+1}, G|q)\} \\ s.t. \ c_t = y_t + a_t - q_t a_{t+1} \\ \widetilde{a}_{t+1} = a_{t+1}/(\mu_g \Gamma_t) \in \overline{a} \end{array} \right\rangle$$

Define $\widetilde{V} \equiv \frac{V^*}{(\mu_g \Gamma_{t-1})^{1-\gamma}}$. If V^* solves the above Bellman equation, then $\widetilde{V}$ solves:

$$\widetilde{V}(\widetilde{a}_t, z_t, g_t, h_t|q) = \tag{17}$$

if $h_t = B$:

$$u((1-\delta)\widetilde{y}_t) + \lambda\beta g_t^{1-\gamma} E_t \widetilde{V}(0, z_{t+1}, g_{t+1}, G|q) + (1-\lambda)\beta g_t^{1-\gamma} E_t \widetilde{V}(z_{t+1}, g_{t+1}, B|q)$$

if $h_t = G$:

$$\max \left\langle \begin{array}{c} u((1-\delta)\widetilde{y}_t) + \lambda\beta g_t^{1-\gamma} E_t \widetilde{V}(0, z_{t+1}, g_{t+1}, G|q) + (1-\lambda)\beta g_t^{1-\gamma} E_t \widetilde{V}(z_{t+1}, g_{t+1}, B|q), \\ \max_{\widetilde{c}_t} \left\{ u(\widetilde{c}_t) + \beta g_t^{1-\gamma} E_t \widetilde{V}(\widetilde{a}_{t+1}, z_{t+1}, g_{t+1}, G|q) \right\} \\ s.t. \ \widetilde{c}_t = \widetilde{y}_t + \widetilde{a}_t - g_t q_t \widetilde{a}_{t+1} \\ \widetilde{a}_{t+1} \in \overline{a} \end{array} \right\rangle$$

where $q_t = q(\widetilde{a}_t, z_t, g_t, G)$. In the text, we have used V^B to denote $\widetilde{V}(\widetilde{a}_t, z_t, g_t, B|q)$, V^G to denote $\max_{\widetilde{c}_t} \left\{ u(\widetilde{c}_t) + \beta g_t^{1-\gamma} E_t \widetilde{V}(\widetilde{a}_{t+1}, z_{t+1}, g_{t+1}, G|q) \right\}$ and V to denote $\widetilde{V}(\widetilde{a}_t, z_t, g_t, G|q)$.

The function $\widetilde{V}$ maps $X \equiv S \times Q$ into $\mathcal{R}$. Let T be the operator associated with the detrended Bellman equation (17), where T maps $B(X)$, the space of bounded functions of X, into $B(X)$.[17] T satisfies Blackwell's monotonicity and discounting conditions for a contraction (using the sup norm). This implies there is a well-defined, unique

[17] Note that the option to default and the lower bound on endowments insures the agent against having zero consumption.

solution to (17) that can be computed through iteration. Moreover, note that if $v \in B(X)$ is weakly increasing in $(\widetilde{a}, z, g, q)$, then so is Tv. The main issue to establish continuity in $(\widetilde{a}, z, g, q)$ concerns the option to default. However, there can be no discontinuity at the point the agent elects to default. To see this, note that if v is continuous, then $u((1-\delta)\widetilde{y}_t) + \lambda\beta E_t v(0, z_{t+1}, g_{t+1}, G|q) + (1-\lambda)\beta g_t^{1-\gamma} E_t v(z_{t+1}, g_{t+1}, B|q)$ and $\max_{\widetilde{c}_t} \left\{ u(\widetilde{c}_t) + \beta g_t^{1-\gamma} E_t v(\widetilde{a}_{t+1}, z_{t+1}, g_{t+1}, G|q) \right\}$ are both continuous (where we have used the discrete nature of (z, g) to maintain continuity through the expectation operator).[18] The max of two continuous functions is also continuous. Therefore, T preserves continuity in $(\widetilde{a}, z, g, q)$. Given that the set of weakly increasing, continuous bounded functions is a closed subset of $B(X)$, and T maps this set into itself, then $\widetilde{V}$ is weakly increasing and continuous in $(\widetilde{a}, z, g, q)$.

To close the model, we define equilibrium as an interest rate schedule that provides creditor with a return of r^* conditional on the actions of the representative agent consistent with q. Specifically, the equilibrium q satisfies

$$q = \Pr\{\widetilde{V}(\widetilde{a}_{t+1}, z_{t+1}, g_{t+1}, G|q) > \widetilde{V}(\widetilde{a}_{t+1}, z_{t+1}, g_{t+1}, B|q)|\widetilde{a}_{t+1}, z_t, g_t)\}/(1 + r^*).$$

Note that if $\widetilde{V}(\widetilde{a}_{t+1}, z_{t+1}, g_{t+1}, G|q) > \widetilde{V}(\widetilde{a}_{t+1}, z_{t+1}, g_{t+1}, B|q)$ then $V^*(\widetilde{a}_{t+1}, z_{t+1}, \Gamma_{t+1}, G|q) > V^*(a_{t+1}, z_{t+1}, \Gamma_{t+1}, B|q)$, and vice versa. Therefore, we can define q over the detrended variables.

[18] The discrete nature of the shocks, while important for computation, is not necessary for the present analysis. If $\overline{z}$ and $\overline{g}$ represent intervals, we add the assumption that the expectation operator associated with the transition probabilities preserve continuity (i.e., the Feller property).

References

[1] Aguiar, Mark and Gita Gopinath (2004), "Emerging Market Business Cycles: The Cycle is the Trend", working paper.

[2] Atkeson, Andrew (1991), "International Lending with Moral Hazard and Risk of Repudiation", *Econometrica,* vol 59, pp 1069-1089.

[3] Arellano, Christina (2003), "Default Risk, the Real Exchange Rate and Income Fluctuations in Emerging Economies", working paper.

[4] Bulow, Jeremy and Kenneth S. Rogoff (1989), "Sovereign Debt: Is to Forgive to Forget", *American Economic Review* 79, March, 43-50.

[5] Chatterjee, Satyajit, Corbae Dean, Nakajimo Makoto and Jose Victor Rios Rull (2002), "A Quantitative theory of Unsecured Consumer Credit with the Risk of Default", working paper.

[6] Dornbusch, Rudiger and Sebastian Edwards (1992), "Macroeconomics of Populism in Latin America", *NBER Conference Volume.*

[7] Eaton, Jonathan and Mark Gersovitz (1981), "Debt with Potential Repudiation: Theoretical and Empirical Analysis", *Review of Economic Studies* (48), 289-309.

[8] Gelos, Gaston R., Ratna Sahay, Guido Sandleris (2003), "Sovereign Borrowing by Developing Countries: What Determines Market Access?", *IMF Working Paper.*

[9] Gourinchas, Pierre-Olivier and Olivier Jeanne (2003), "The Elusive Gains from International Financial Integration", *NBER working paper 9684.*

[10] Kehoe, Timothy and David K. Levine (1993), "Debt Constrained Asset Markets", *Review of Economic Studies. 60, 865-88.*

[11] Kehoe, Patrick and Fabrizio Perri (2002), "International Business Cycles with Endogenous Incomplete Markets", *Econometrica* 70 (3), 907-28.

[12] Kletzer, Kenneth M. and Brian D. Wright (2000), "Sovereign Debt as Intertemporal Barter", American Economic Review, Vol. 90, No. 3, 621-639.

[13] Klingen, Christoph A, Beatrice Weder and Jeromin Zettelmeyer (2004), "How Private Creditors fared in Emerging Debt Markets, 1970-2000", *IMF Working Paper 04/13.*

[14] Perri, Fabrizio and Andy Neumeyer (2004), "Emerging Market Business Cycles: The Role of Interest Rates", *NBER Working Paper* 10387.

[15] Reinhart, Carmen M., Kenneth S. Rogoff and Miguel A. Savastano (2003), "Debt Intolerance", in William Brainard and George Perry (eds.), *Brookings Papers on Economic Activity*, 1-74.

[16] Rose, Andrew (2003), "One Reason Countries Pay Their Debts: Renegotiation and International Trade", *NBER Working Paper 8853.*

[17] Tauchen, George and Robert Hussey (1991), "Quadrature based methods for obtaining approximate solutions to non linear asset pricing models", *Econometrica*, Vol. 59, No. 2, pp. 371-396.

Table 1: Argentina Business Cycle Statistics (1983.1-2000.2)

Data	HP	SE
$\sigma(Y)$	4.08	(0.52)
$\sigma(R_s)$	3.17	(0.54)
$\sigma(TB/Y)$	1.36	(0.24)
$\sigma(C)/\sigma(Y)$	1.19	(0.04)
$\rho(Y)$	0.85	(0.08)
$\rho(R_s, Y)$	-0.59	(0.11)
$\rho(TB/Y, Y)$	-0.89	(0.10)
$\rho(R_s, TB/Y)$	0.68	(0.13)
$\rho(C, Y)$	0.96	(0.01)

The series were deseasonalized if a significant seasonal component was identified. We log the income, consumption and investment series and compute the ratio of the trade balance (TB) to GDP (Y) and the interest rate spread (R_s). R_s refers to the difference between Argentina dollar interest rates and US 3 month treasury bond rate (annualized numbers). All series were then HP filtered with a smoothing parameter of 1600. GMM estimated standard errors are reported in parenthesis under column SE. The standard deviations (Y, R_s , TB/Y) are reported in percentage terms.

Table 2A: Common Benchmark Parameter Values

Risk Aversion	γ	2
World Interest Rate	r^*	1%
Loss of Output in Autarky	δ	2%
Probability of Redemption	λ	10%
Mean (Log) Transitory Productivity	μ_z	$-\frac{1}{2}\sigma_z^2$
Mean Growth Rate	μ_g	1.006

Table 2B: Model Specific Benchmark Parameter Values

	Model I: Transitory Shocks	*Model II: Growth Shocks*	*Model II with Bail Outs*
σ_z	3.4%	0	0
ρ_z	0.90	NA	NA
σ_g	0	3%	3%
ρ_g	NA	0.17	0.17
β	0.8	0.8	0.95
Bail Out Limit	NA	NA	18%

Table 3: Simulation Results

	Data	Model I (3A)	Model II (3B)	Model II with Bail Outs (3C)
$\sigma(y)$	4.08	4.32	4.45	4.43
$\sigma(c)$	4.85	4.37	4.71	4.68
$\sigma(TB/Y)$	1.36	0.17	0.95	1.10
$\sigma(R_s)$	3.17	0.04	0.32	0.12
$\rho(C,Y)$	0.96	0.99	0.98	0.97
$\rho(TB/Y,Y)$	-0.89	-0.33	-0.19	-0.12
$\rho(R_s,Y)$	-0.59	0.51	-0.03	-0.02
$\rho(R_s,TB/Y)$	0.68	-0.21	0.11	0.38
Rate of Default (per 10,000 quarters)	75	2	23	92
Mean Debt Output Ratio (%)		27	19	18
Maximum R_s *(basis points)*		23	151	57

Note: Simulation results reported are averages over 500 simulations each of length 500 (drawn from a stationary distribution). The simulated data is treated in an identical manner to the empirical data. Standard deviations are reported in percentages.

Table 4: Sensitivity Analysis

	Endogenous Labor Supply	$\beta = 0.85$	$\beta = 0.9$
$\sigma(y)$	4.44	4.43	4.43
$\sigma(c)$	4.85	4.67	4.66
$\sigma(TB/Y)$	0.79	0.85	0.76
$\sigma(R_s)$	0.24	0.24	0.16
$\sigma(H)$	2.88	NA	NA
$\rho(C,Y)$	0.99	0.98	0.99
$\rho(H,Y)$	0.73	NA	NA
$\rho(TB/Y,Y)$	-0.49	-0.20	-0.23
$\rho(R_s,Y)$	-0.23	-0.002	-0.05
$\rho(R_s,TB/Y)$	0.26	0.01	0.11
Rate of Default (per 10,000 quarters)	16	16	11
Mean Debt Output Ratio (%)	15	19	19
Maximum R_s *(basis points)*	191	120	80
$\sigma(g)$	2.3	3.0	3.0

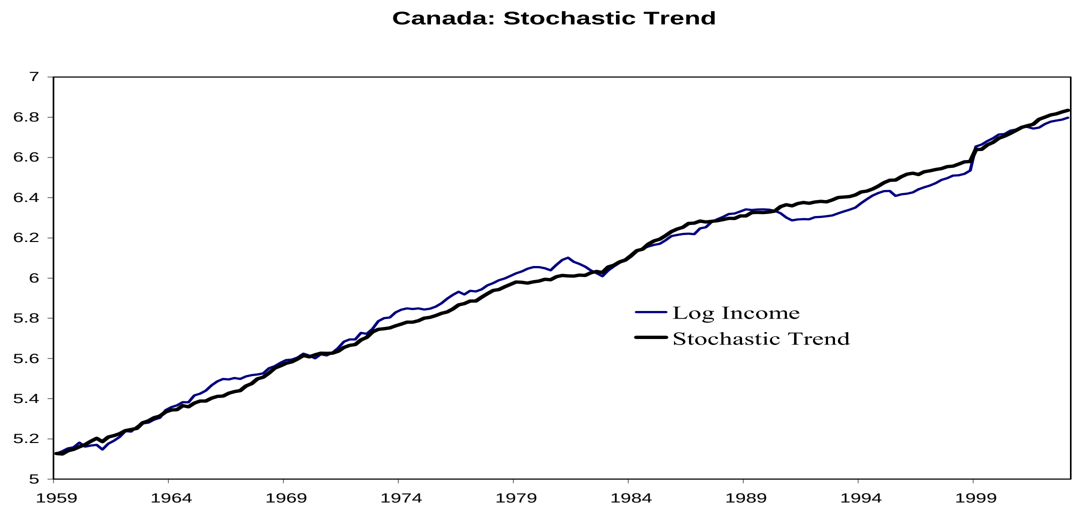
Note: Parameter values used in the simulation results reported above are as in Table 2A and 2B for Model II, except that β is allowed to vary across columns 3, 4, and 5 and in column 2 we consider the case of endogenous labor supply where the elasticity of labor supply is taken to be 2.2 implying an ω of 1.455. Simulation results reported are averages over 500 simulations each of length 500 (drawn from a stationary distribution). The simulated data is treated in an identical manner to the empirical data. Standard deviations are reported in percentages

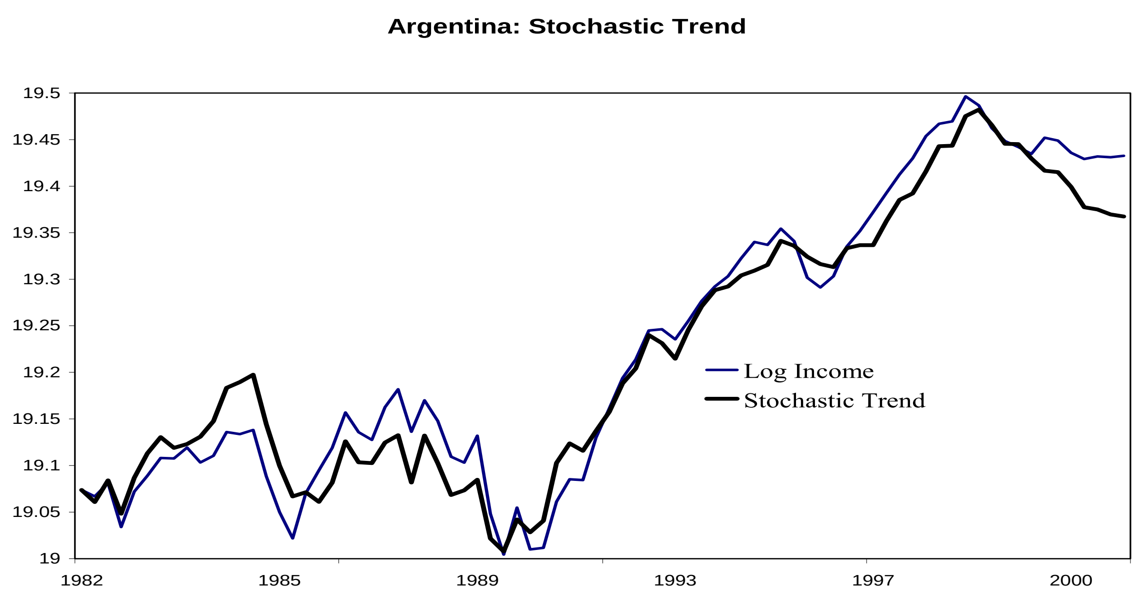
Table 5: Additional Comparative Statics

	Innovation Variance		Cost of Autarky	
	Model I (σ_z=6.4%)	Model II (σ_g=6%)	Model I (δ=0.5%)	Model II (δ=0.5%)
$\sigma(y)$	8.60	8.25	4.32	4.42
$\sigma(c)$	8.68	8.56	4.33	4.46
$\sigma(TB/Y)$	0.36	1.28	0.07	0.25
$\sigma(R_s)$	0.16	0.64	0.08	0.32
$\rho(C,Y)$	0.99	0.99	0.99	0.99
$\rho(TB/Y,Y)$	-0.29	-0.18	-0.31	-0.17
$\rho(R_s,Y)$	0.30	0.07	0.33	-0.03
$\rho(R_s,TB/Y)$	-0.17	-0.05	-0.15	0.05
Rate of Default (per 10,000 quarters)	4	47	3	23
Mean Debt Output Ratio (%)	25	15	6	5
Maximum R_s *(basis points)*	52	301	40	185

Note: Simulation results reported are averages over 500 simulations each of length 500 (drawn from a stationary distribution). Standard deviations are reported in percentages.

Figure 1: Stochastic Trends estimated using the KPSW(1991) methodolgy





Note: This figure is replicated from Aguiar and Gopinath (2004) "Emerging Market Business Cycles: The Cycle is the Trend". See the paper for details.

Figure 2A: Model I

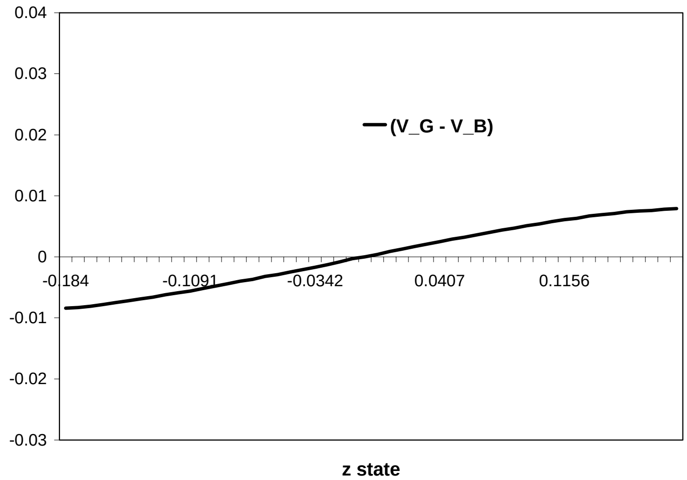
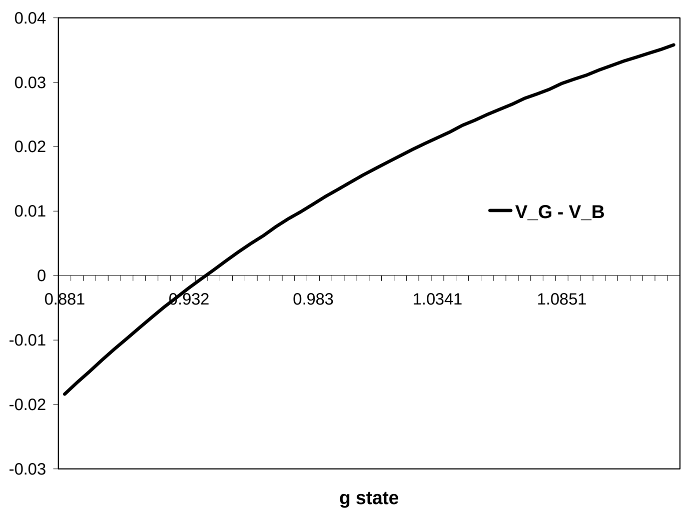
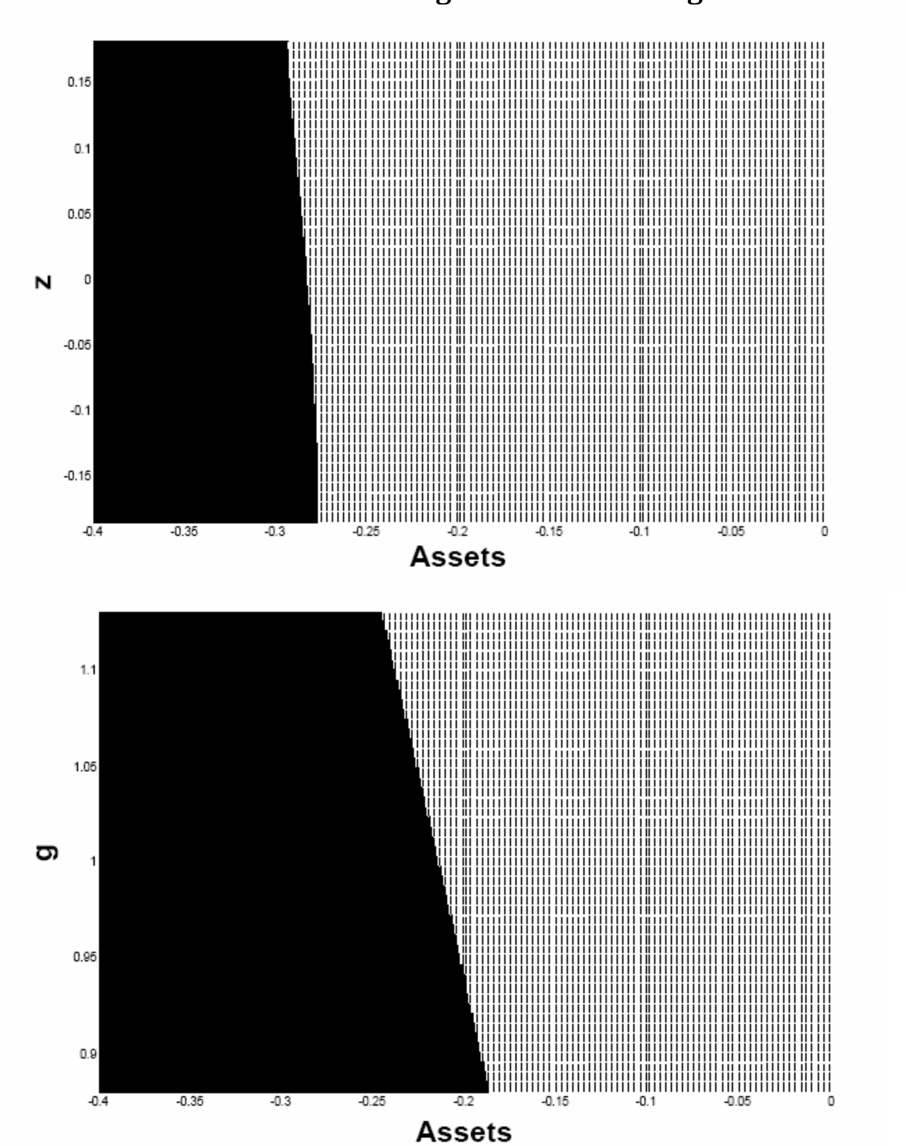


Figure 2B: Model II



Note: V_G represents the value function when the agent chooses to repay and is in good credit standing and V_B is the value function when the agent chooses to default. We have plotted here the difference between the two value functions for a given level of assets, across different productivity states. Figure 2A corresponds to the case when z varies and Figure 2B to the case when g varies. The (V_G-V_B) line is more steeply sloped in the case of g shocks.

Figure 3: Default Region



Note: The darkly shaded region represents combinations of the productivity state and assets for which the economy will prefer default. The lightly shaded region accordingly is the nondefault region. The vertical axis represents the realization of the productivity shock. The horizontal axis represents assets normalized by (mean) trend income. In both pictures, the agent is more likely to default when holding larger amounts of debt (negative assets) and when in worse productivity states. The line of indifference is less steeply sloped in the case of g shocks.

Figure 4A: Model I

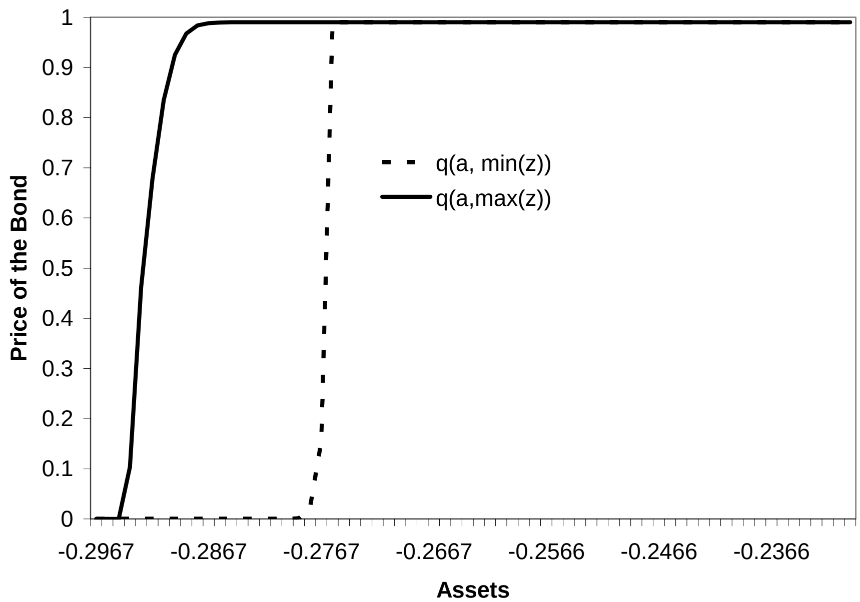
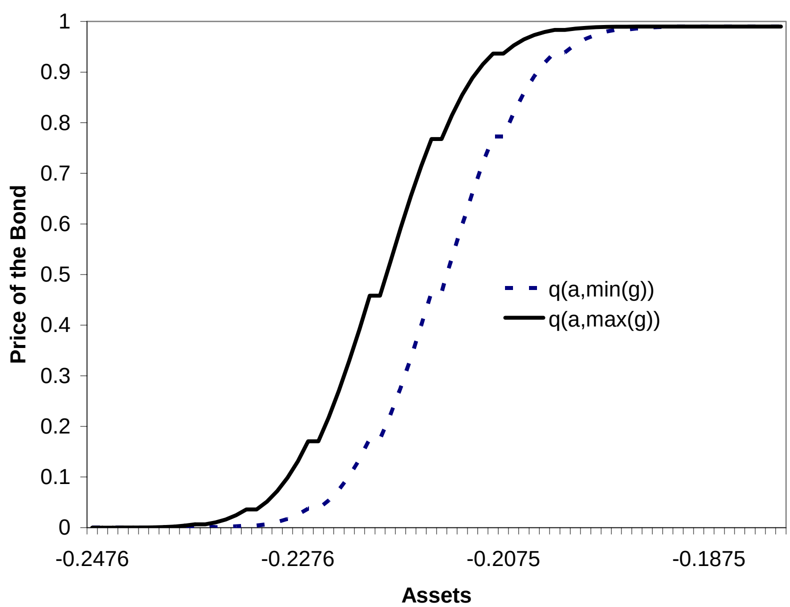
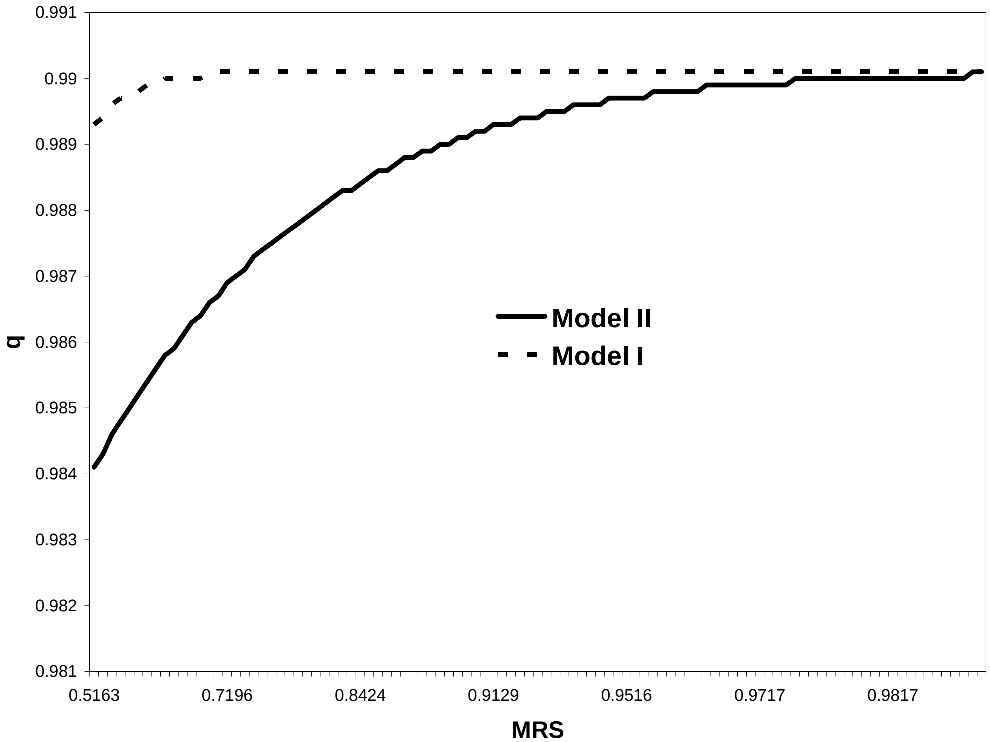


Figure 4B: Model II



Note: Figures 4A and 4B plot the Price of the Bond (inverse of one plus the interest rate) as a function of assets for the highest and lowest values of z in the case of Fig. 4A and the same for growth shocks in Fig. 4B. The price function is less sensitive to changes in borrowing in the case of g shocks (Fig. 4B).

Figure 5: Marginal Rate of Substitution and the Interest Rate



Note: The lines above plot the relation between the expected (conditional) marginal rate of substitution (MRS) and q, for given value of z in the case of the dotted line and g in the case of the solid line. The idea is to calculate, for various values of a, how much of the movement in MRS is matched by movements in q and how much by aq´. We see that large movements in the marginal rate of substitution are accommodated by very small movements in q itself in the model with only z shocks. Therefore, a large implicit demand for borrowing (low MRS) does not result in additional borrowing and a high probability of default. Instead, it generates minimal borrowing and a large movement in the slope of the interest rate function. This is less so for the case of growth shocks