OPTIMAL OPERATIONAL MONETARY POLICY
IN THE CHRISTIANO-EICHENBAUM-EVANS MODEL
OF THE U.S. BUSINESS CYCLE

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ABSTRACT

This paper identifies optimal interest-rate rules within a rich, dynamic, general equilibrium model that has been shown to account well for observed aggregate dynamics in the postwar United States. We perform policy evaluations based on second-order accurate approximations to conditional and unconditional expected welfare. We require that interest-rate rules be operational, in the sense that they include as arguments only a few readily observable macroeconomic indicators and respect the zero bound on nominal interest rates. We find that the optimal operational monetary policy is a real-interest-rate targeting rule. That is, an interest-rate feedback rule featuring a unit inflation coefficient, a mute response to output, and no interest-rate smoothing. Contrary to existing studies, we find a significant degree of optimal inflation volatility. A key factor driving this result is the assumption of indexation to past inflation. Under indexation to long-run inflation the optimal inflation volatility is close to zero. Finally, we show that initial conditions matter for welfare rankings of policies.

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1   Introduction

Optimal monetary policy is the subject of a large and fast growing body of research in macroeconomics. A central characteristic of all existing studies is that optimal monetary policy is derived in highly stylized environments. Typically, optimal monetary policy is characterized for economies with a single or a very small number of deviations from the frictionless neoclassical paradigm. A case in point are the numerous recent studies concerned with optimal policy within the context of the two-equation, one-friction, neo-Keynesian model without capital accumulation.[1] Other cases in which the optimal monetary policy design problem is studied within theoretical frameworks featuring a small number of rigidities include models with distorting income taxes (Lucas and Stokey, 1983; Schmitt-Grohé and Uribe, 2004a), and models with sticky product and factor prices (Erceg, et al., 2000). An advantage of this stylized approach is that it facilitates understanding the ways in which monetary policy should respond to mitigate the distortionary effects of a particular friction in isolation.

An important drawback of studying optimal monetary policy one distortion at a time is that highly simplified models are unlikely to provide a satisfactory account of cyclical movements for more than just a few macroeconomic variables of interest. For this reason, the usefulness of this strategy to produce policy advice for the real world is necessarily limited.

The approach to optimal monetary policy that we propose in this paper departs from the literature extant in that it is based on a rich theoretical framework capable of explaining observed business cycle fluctuations for a wide range of nominal and real variables. Following the lead of Kimball (1995), the model emphasizes the importance of combining nominal as well as real rigidities in explaining the propagation of macroeconomic shocks. Specifically, the model features four nominal frictions, sticky prices, sticky wages, money in the utility function, and a cash-in-advance constraint on the wage bill of firms, and four sources of real rigidities, investment adjustment costs, variable capacity utilization, habit formation, and imperfect competition in product and factor markets. Aggregate fluctuations are driven by supply shocks, which take the form of stochastic variations in total factor productivity, and demand shocks stemming from exogenous innovations to the level of government purchases. Altig et al. (2003) and Christiano, Eichenbaum, and Evans (2003) argue that the model economy for which we seek to design optimal monetary policy can indeed explain the observed responses of inflation, real wages, nominal interest rates, money growth, output, investment,

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[1]Examples of this line of research include Ireland (1997), Rotemberg and Woodford (1997), Woodford (2003), and Clarida, Galí, and Gertler (2000), among many others.

consumption, labor productivity, and real profits to productivity and monetary shocks in the postwar United States. In this respect, the present paper aspires to be a step ahead in the research program of generating monetary policy evaluation that is of relevance for the actual practice of central banking.

In our quest for the optimal monetary policy scheme we restrict attention to what we call operational interest rate rules. By an operational interest-rate rule we mean an interest-rate rule that satisfies three requirements. First, it prescribes that the nominal interest rate is set as a function of a few readily observable macroeconomic variables. In the tradition of Taylor (1993), we focus on rules whereby the nominal interest rate depends on measures of inflation, aggregate activity, and possibly its own lag. Second, the operational rule must induce an equilibrium satisfying the zero lower bound on nominal interest rates. And third, operational rules must render the rational expectations equilibrium unique. This last restriction closes the door to expectations driven aggregate fluctuations.

The object that monetary policy aims to maximize in our study is the expectation of lifetime utility of the representative household conditional on a particular initial state of the economy. Our focus on a conditional welfare measure represents a fundamental departure from most existing normative evaluations of monetary policy, which rank policies based upon unconditional expectations of utility.[2] As Kim et al. (2003) point out, unconditional welfare measures ignore the welfare effects of transitioning from a particular initial state to the stochastic steady state induced by the policy under consideration. Indeed, we document that under plausible initial conditions, conditional welfare measures can result in different rankings of policies than the more commonly used unconditional measure. This finding highlights the fact that transitional dynamics matter for policy evaluation.

In our welfare evaluations, we depart from the widespread practice in the neo-Keynesian literature on optimal monetary policy of limiting attention to models in which the nonstochastic steady state is undistorted. Most often, this approach involves assuming the existence of a battery of subsidies to production and employment aimed at eliminating the long-run distortions originating from monopolistic competition in factor and product markets. The efficiency of the deterministic steady-state allocation is assumed for purely computational reasons. For it allows the use of first-order approximation techniques to evaluate welfare accurately up to second order.[3] This practice has two potential shortcomings. First, the instruments necessary to bring about an undistorted steady state (e.g., labor and output subsidies financed by lump-sum taxation) are empirically uncompelling. Second, it is ex ante not clear whether a policy that is optimal for an economy with an efficient steady state

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[2]Exceptions are Kollmann (2003) and Schmitt-Grohé and Uribe (2004b).

[3]This simplification was pioneered by Rotemberg and Woodford (1997).

will also be so for an economy where the instruments necessary to engineer the nondistorted steady state are unavailable. For these reasons, we refrain from making the efficient-steady-state assumption and instead work with a model whose steady state is distorted.

Departing from a model whose steady state is Pareto efficient has a number of important ramifications. One is that to obtain a second-order accurate measure of welfare it no longer suffices to approximate the equilibrium of the model up to first order. Instead, we obtain a second-order accurate approximation to welfare by solving the equilibrium of the model up to second order. Specifically, we use the methodology and computer code developed in Schmitt-Grohé and Uribe (2004c) to compute higher-order approximations to policy functions of dynamic, stochastic models. One advantage of this numerical strategy is that because it is based on perturbation arguments, it is particularly well suited to handle economies with a large number of state variables like the one studied in this paper.

In the neo-Keynesian literature, a recursive form for the equilibrium version of the price-setting equation of firms (and workers if wages are sticky) is obtained by resorting to linear approximations at various stages of its derivation. The resulting expression is often referred to as the expectations augmented Phillips curve. For the reasons given above, such a linear expression is of no use for evaluating welfare in distorted economies. Therefore, in this paper we derive a recursive representation of the exact (nonlinear) Phillips curves for price and wage dynamics. A third consequence of working with an economy with long-run distortions and thus not being able to work with linearized versions of the equilibrium conditions of the model is the need to track the evolution of price and wage dispersion over time. This task gives rise to additional state variables summarizing the degree of relative price dispersion in the economy. We provide a recursive representation for the equilibrium law of motion of these state variables.

The results from our numerical work suggest that in the model economy we study, the optimal operational interest-rate rule takes the form of a real-interest-rate targeting rule. For it features an inflation coefficient close to unity, a mute response to output, no interest-rate smoothing, and is forward looking. The optimal rule satisfies the Taylor principle because the inflation coefficient is greater than unity albeit very close to 1. Optimal monetary policy calls for significant inflation volatility. This result stands in contrast with those obtained in the related literature. The main element of the model driving the desirability of inflation volatility is indexation of nominal factor and product prices to 1-period lagged inflation. Under the alternative assumption of indexation to long-run inflation, the conventional result of the optimality of inflation stability reemerges. All of the above results are robust to a number of variations of the baseline model, including cashless economies, no habit formation, and economies with high costs of varying capacity utilization.

The remainder of the paper is organized in six sections. Section 2 presents the theoretical economy and derives nonlinear recursive representations for the price and wage Phillips curves as well as for the state variables summarizing the degree of wage and price dispersion. Section 3 describes the calibration of the model and discusses the solution method. Section 4 provides intuition for the workings of the model by analyzing its dynamic response to supply and demand shocks under a simple Taylor-type interest-rate feedback rule. In section 5 we derive a second-order accurate measure of welfare conditional upon the current state of the economy. There, we also derive a second-order accurate consumption-based metric for the welfare differences between two alternative monetary policies. The core of the paper is contained in section 6. It computes the optimal operational interest-rate rule and performs an extensive sensitivity analysis. Section 7 provides concluding remarks.

2   The Model

The skeleton of the model economy that we use for policy evaluation is the standard neo-classical growth model driven by productivity and government spending shocks. In addition the economy features four sources of nominal frictions and five real rigidities. The nominal frictions include price and wage stickiness à la Calvo (1983) and Yun (1996) with indexation to past inflation, and money demands by households and firms. The real rigidities originate from internal habit formation in consumption, monopolistic competition in factor and product markets, investment adjustment costs, and variable costs of adjusting capacity utilization.

To perform monetary policy evaluation, we are forced to approximate the equilibrium conditions of the economy to an order higher than linear. To this end, we derive the exact nonlinear recursive representation of the complete set of equilibrium conditions. Of particular interest is the recursive nonlinear representation of the equilibrium Phillips curves for prices and wages. These representations depart from most of the existing literature, which restricts attention to linear approximations to these functions. Another byproduct of deriving the exact nonlinear set of equilibrium conditions is the emergence of two state variables measuring the degree of price and wage dispersion in the economy induced by the sluggishness in the adjustment of nominal product and factor prices. We present a recursive representation of these state variables and track their dynamic behavior.

2.1 Households

The economy is populated by a large representative family with a continuum of members. Each member is a worker. Consumption and hours worked are identical across family members. The household's preferences are defined over per capita consumption, c_t, per capita labor effort, h_t, and per capita holdings of real money balances, m_t^h, and are described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - bc_{t-1}, h_t, m_t^h), \tag{1}$$

where E_t denotes the mathematical expectations operator conditional on information available at time t, $\beta \in (0,1)$ represents a subjective discount factor, and U is a period utility index assumed to be strictly increasing in its first and third arguments, strictly decreasing in its second argument, and strictly concave. Preferences display internal habit formation, measured by the parameter $b \in [0,1)$. The consumption good is assumed to be a composite made of a continuum of differentiated goods, c_{it} indexed by $i \in [0,1]$ via the aggregator

$$c_t = \left[\int_0^1 c_{it}^{1-1/\eta} di \right]^{1/(1-1/\eta)}, \tag{2}$$

where the parameter $\eta > 1$ denotes the intratemporal elasticity of substitution across different varieties of consumption goods.

For any given level of consumption of the composite good, purchases of each individual variety of goods $i \in [0,1]$ in period t must solve the dual problem of minimizing total expenditure, $\int_0^1 P_{it} c_{it} di$, subject to the aggregation constraint (2), where P_{it} denotes the nominal price of a good of variety i at time t. The demand for goods of variety i is then given by

$$c_{it} = \left(\frac{P_{it}}{P_t} \right)^{-\eta} c_t, \tag{3}$$

where P_t is a nominal price index given by

$$P_t \equiv \left[\int_0^1 P_{it}^{1-\eta} di \right]^{\frac{1}{1-\eta}}. \tag{4}$$

This price index has the property that the minimum cost of a bundle of intermediate goods yielding c_t units of the composite good is given by $P_t c_t$.

Labor decisions are made by a central authority within the household, a union, which supplies labor monopolistically to a continuum of labor markets of measure 1 indexed by $j \in$

$[0, 1]$.[4] In each labor market j, the union faces a demand for labor given by $\left(W_t^j/W_t\right)^{-\tilde{\eta}} h_t^d$. Here W_t^j denotes the nominal wage charged by the union in labor market j at time t, W_t is an index of nominal wages prevailing in the economy, and h_t^d is a measure of aggregate labor demand by firms. We postpone a formal derivation of this labor demand function until we consider the firm's problem. In each particular labor market, the union takes W_t and h_t^d as exogenous. The case in which the union takes aggregate labor variables as endogenous can be interpreted as an environment with highly centralized labor unions. Higher-level labor organizations play an important role in some European and Latin American countries, but are less prominent in the United States. Given the wage charged in each labor market $j \in [0, 1]$, the union is assumed to supply enough labor, h_t^j, to satisfy demand. That is,

$$h_t^j = \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}} h_t^d, \tag{5}$$

where $w_t^j \equiv W_t^j/P_t$ and $w_t \equiv W_t/P_t$. In addition, the total number of hours allocated to the different labor markets must satisfy the resource constraint $h_t = \int_0^1 h_t^j dj$. Combining this restriction with equation (5), we obtain

$$h_t = h_t^d \int_0^1 \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}} dj. \tag{6}$$

The household is assumed to own physical capital, k_t, which accumulates according to the following law of motion

$$k_{t+1} = (1 - \delta)k_t + i_t \left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}}\right)\right], \tag{7}$$

where i_t denotes gross investment and δ is a parameter denoting the rate of depreciation of physical capital. The function $\mathcal{S}$ introduces investment adjustment costs and is assumed to satisfy $\mathcal{S}(1) = \mathcal{S}'(1) = 0$ and $\mathcal{S}''(1) > 0$. These assumptions imply the absence of adjustment costs up to first order in the vicinity of the deterministic steady state. Owners of physical capital can control the intensity at which this factor is utilized. Formally, we let u_t measure capacity utilization in period t. We assume that using the stock of capital with intensity u_t entails a cost of $a(u_t)k_t$ units of the composite final good. The function a is assumed to satisfy $a(1) = 0$, and $a'(1), a''(1) > 0$. Both the specification of capital adjustment costs

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[4]This setup departs slightly from most existing expositions of models with nominal wage inertia. It avoids the need to assume separability of preferences in leisure and consumption to ensure homogeneity of consumption across households.

and capacity utilization costs are somewhat peculiar. More standard formulations assume that adjustment costs depend on the level of investment rather than on its growth rate, as is assumed here. Also, costs of capacity utilization typically take the form of a higher rate of depreciation of physical capital. The modeling choice here is guided by the need to fit the response of investment and capacity utilization to a monetary shock in the US economy. For further discussion of this point, see Christiano, Eichenbaum, and Evans (2003, section 6.1).

Households rent the capital stock to firms at the real rental rate r_t^k per effective unit of capital. Thus, total income stemming from the rental of capital is given by $r_t^k u_t k_t$. The investment good is assumed to be a composite good made with the aggregator function (2). Thus, the demand for each intermediate good $i \in [0, 1]$ for investment purposes, i_{it}, is then given by $i_{it} = i_t (P_{it}/P_t)^{-\eta}$.

Households have access to a complete set of nominal state-contingent assets. Specifically, each period $t \geq 0$, consumers can purchase any desired state-contingent nominal payment X_{t+1}^h in period $t + 1$ at the dollar cost $E_t r_{t,t+1} X_{t+1}^h$. The variable $r_{t,t+1}$ denotes a stochastic nominal discount factor between periods t and $t + 1$. The household's period-by-period budget constraint is given by:

$$E_t r_{t,t+1} x_{t+1}^h + c_t + i_t + m_t^h + a(u_t)k_t = \frac{x_t^h}{\pi_t} + h_t^d \int_0^1 w_t^j \left(\frac{w_t^j}{w_t}\right)^{-\tilde{\eta}} dj + r_t^k u_t k_t + \phi_t - \tau_t + \frac{m_{t-1}^h}{\pi_t}. \quad (8)$$

The variable $x_t^h/\pi_t \equiv X_t^h/P_t$ denotes the real payoff in period t of nominal state-contingent assets purchased in period $t-1$. The variable ϕ_t denotes profits received from the ownership of firms, τ_t denotes lump-sum taxes, and $\pi_t \equiv P_t/P_{t-1}$ denotes the gross rate of consumer-price inflation.

We introduce wage stickiness in the model by assuming that each period the household (or union) cannot set the nominal wage optimally in a fraction $\tilde{\alpha} \in [0, 1)$ of randomly chosen labor markets. In these markets, the wage rate is indexed to the previous period's consumer-price inflation according to the rule $W_t^j = W_{t-1}^j \pi_{t-1}^{\tilde{\chi}}$, where $\tilde{\chi}$ is a parameter measuring the degree of wage indexation. When $\tilde{\chi}$ equals 0, there is no wage indexation. When $\tilde{\chi}$ equals 1, there is full wage indexation to past consumer price inflation. In general, $\tilde{\chi}$ can take any value between 0 and 1.

The household chooses processes for c_t, h_t, x_{t+1}^h, w_t^j, k_{t+1}, i_t, m_t^h, and u_t so as to maximize the utility function (1) subject to (6)-(8), the wage stickiness friction, and a no-Ponzi-game constraint, taking as given the processes w_t, r_t^k, h_t^d, $r_{t,t+1}$, π_t, ϕ_t, and τ_t and the initial conditions x_0^h, k_0, and m_{-1}^h. Of course, the household's optimal plan must satisfy constraints (6)-(8). In addition, letting $\beta^t \lambda_t w_t/\tilde{\mu}_t$, $\beta^t q_t \lambda_t$, and $\beta^t \lambda_t$ denote Lagrange multipliers

associated with constraints (6), (7), and (8), respectively, the first-order conditions with respect to c_t, x_{t+1}^h, h_t, k_{t+1}, i_t, u_t, m_t^h, and w_t^j, in that order, are given by

$$U_c(c_t - bc_{t-1}, h_t, m_t^h) - b\beta E_t U_c(c_{t+1} - bc_t, h_{t+1}, m_{t+1}^h) = \lambda_t, \tag{9}$$

$$\lambda_t r_{t,t+1} = \beta \lambda_{t+1} \frac{P_t}{P_{t+1}} \tag{10}$$

$$-U_h(c_t - bc_{t-1}, h_t, m_t^h) = \frac{\lambda_t w_t}{\tilde{\mu}_t}, \tag{11}$$

$$\lambda_t q_t = \beta E_t \lambda_{t+1} \left[r_{t+1}^k u_{t+1} - a(u_{t+1}) + q_{t+1}(1 - \delta) \right], \tag{12}$$

$$\lambda_t = \lambda_t q_t \left[1 - \mathcal{S}\left(\frac{i_t}{i_{t-1}}\right) - \left(\frac{i_t}{i_{t-1}}\right) \mathcal{S}'\left(\frac{i_t}{i_{t-1}}\right) \right] + \beta E_t \lambda_{t+1} q_{t+1} \left(\frac{i_{t+1}}{i_t}\right)^2 \mathcal{S}'\left(\frac{i_{t+1}}{i_t}\right) \tag{13}$$

$$r_t^k = a'(u_t) \tag{14}$$

$$\lambda_t = U_m(c_t - bc_{t-1}, h_t, m_t^h) + \beta E_t \frac{\lambda_{t+1}}{\pi_{t+1}}. \tag{15}$$

$$w_t^j = \begin{cases} \tilde{w}_t & \text{if } w_t^j \text{ is set optimally in } t \\ w_{t-1}^j \pi_{t-1}^{\tilde{\chi}}/\pi_t & \text{otherwise} \end{cases}$$

where $\tilde{w}_t$ denotes the real wage prevailing in the $1 - \tilde{\alpha}$ labor markets in which the union can set wages optimally in period t. Note that because the labor demand curve faced by the union is identical across all labor markets, and because the cost of supplying labor is the same for all markets, one can assume that wage rates and employment will be identical across all labor markets updating wages in a given period. The real wage $\tilde{w}_t$ must satisfy

$$0 = E_t \sum_{s=0}^{\infty} (\beta \tilde{\alpha})^s \lambda_{t+s} \left(\frac{\tilde{w}_t}{w_{t+s}}\right)^{-\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)^{\tilde{\eta}} \left[\frac{\tilde{\eta} - 1}{\tilde{\eta}} \frac{\tilde{w}_t}{\prod_{k=1}^{s}\left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}}\right)} - \frac{w_{t+s}}{\tilde{\mu}_{t+s}} \right].$$

This expression states that in labor markets in which the wage rate is reoptimized in period t, the real wage is set so as to equate the union's future expected average marginal revenue to the average marginal cost of supplying labor. The union's marginal revenue s periods after its last wage reoptimization is given by $\frac{\tilde{\eta}-1}{\tilde{\eta}} \tilde{w}_t \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\tilde{\chi}}}{\pi_{t+k}}\right)$. Here, $\tilde{\eta}/(\tilde{\eta} - 1)$ represents the markup of wages over marginal cost of labor that would prevail in the absence of wage stickiness. The third factor in the expression for marginal revenue reflects the fact that as time goes by without a chance to reoptimize, the real wage declines as the price level increases when wages are imperfectly indexed to past inflation. In turn, the marginal cost

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of supplying labor is given by the marginal rate of substitution between consumption and leisure, or $\frac{-U_{ht+s}}{\lambda_{t+s}} = \frac{w_{t+s}}{\tilde{\mu}_{t+s}}$. The variable $\tilde{\mu}_t$ is a wedge between the disutility of labor and the average real wage prevailing in the economy. Thus, $\tilde{\mu}_t$ can be interpreted as the average markup that unions impose on the labor market. The weights used to compute averages are decreasing in time and increasing in the amount of labor supplied to the market.

We wish to write the wage-setting equation in recursive form. To this end, define

$$f_t^1 = E_t \sum_{s=0}^{\infty} (\beta \tilde{\alpha})^s \lambda_{t+s} \left(\frac{w_{t+s}}{\tilde{w}_t} \right)^{\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}} \right)^{\tilde{\eta}-1}$$

and

$$f_t^2 = \tilde{w}_t^{-\tilde{\eta}} E_t \sum_{s=0}^{\infty} (\beta \tilde{\alpha})^s \frac{\lambda_{t+s}}{\tilde{\mu}_{t+s}} w_{t+s}^{1+\tilde{\eta}} h_{t+s}^d \prod_{k=1}^{s} \left(\frac{\pi_{t+k}}{\pi_{t+k-1}^{\tilde{\chi}}} \right)^{\tilde{\eta}}.$$

One can express f_t^1 and f_t^2 recursively as

$$f_t^1 = \lambda_t \left(\frac{w_t}{\tilde{w}_t} \right)^{\tilde{\eta}} h_t^d + \tilde{\alpha} \beta E_t \left(\frac{\pi_{t+1}}{\pi_t^{\tilde{\chi}}} \right)^{\tilde{\eta}-1} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t} \right)^{\tilde{\eta}} f_{t+1}^1, \tag{16}$$

$$f_t^2 = \frac{\lambda_t}{\tilde{\mu}_t} w_t \left(\frac{w_t}{\tilde{w}_t} \right)^{\tilde{\eta}} h_t^d + \tilde{\alpha} \beta E_t \left(\frac{\pi_{t+1}}{\pi_t^{\tilde{\chi}}} \right)^{\tilde{\eta}} \left(\frac{\tilde{w}_{t+1}}{\tilde{w}_t} \right)^{\tilde{\eta}} f_{t+1}^2. \tag{17}$$

With these definitions at hand, the wage-setting equation becomes

$$0 = (1 - \tilde{\eta}) \tilde{w}_t f_t^1 + \tilde{\eta} f_t^2. \tag{18}$$

The household's optimality conditions imply a liquidity function featuring a negative relation between real balances and the short-term nominal interest rate. To see this, we first note that the absence of arbitrage opportunities in financial markets requires that R_t be equal to the reciprocal of the price in period t of a nominal security that pays one unit of currency in every state of period $t+1$. Formally, $1/R_t = E_t r_{t,t+1}$. This relation together with the household's optimality condition (10) implies that

$$\lambda_t = \beta R_t E_t \frac{\lambda_{t+1}}{\pi_{t+1}}, \tag{19}$$

which is a standard Euler equation for pricing nominally risk-free assets. Now combining this expression with equations (9) and (15) we obtain

$$\frac{U_m(c_t - bc_{t-1}, h_t, m_t^h)}{U_c(c_t - bc_{t-1}, h_t, m_t^h) - b\beta E_t U_c(c_{t+1} - bc_t, h_{t+1}, m_{t+1}^h)} = 1 - \frac{1}{R_t}.$$

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The right-hand side of this expression represents the opportunity cost of holding money, which is an increasing function of the nominal interest rate. The left-hand side is the marginal rate of substitution between consumption and real money balances. Note that the presence of internal habit formation implies that the demand for money depends not only on current but also past and future expected consumption. In addition, if preferences are nonseparable in leisure and either consumption or real balances, then hours worked will also feature in the liquidity preference function.

2.2 Firms

Each variety of final goods is produced by a single firm in a monopolistically competitive environment. Each firm $i \in [0, 1]$ produces output using as factor inputs capital services, k_{it}, and labor services, h_{it}. The production technology is given by

$$z_t F(k_{it}, h_{it}) - \psi,$$

where the function F is assumed to be homogenous of degree one, concave, and strictly increasing in both arguments. The variable z_t denotes an aggregate, exogenous, and stochastic productivity shock. The parameter $\psi > 0$ introduces fixed costs of operating a firm in each period. It implies that the production function exhibits increasing returns to scale. We model fixed costs to ensure a realistic profit-to-output ratio in steady state.

Aggregate demand for good i, which we denote by y_{it}, is given by

$$y_{it} = (P_{it}/P_t)^{-\eta} y_t,$$

where

$$y_t \equiv c_t + i_t + g_t + a(u_t) k_t, \tag{20}$$

denotes aggregate absorption. The variable g_t denotes government consumption of the composite good in period t.

We rationalize a demand for money by firms by imposing that wage payments are subject to a cash-in-advance constraint of the form:

$$m_{it}^f \geq \nu w_t h_{it}, \tag{21}$$

where m_{it}^f denotes the demand for real money balances by firm i in period t and $\nu \geq 0$ is a parameter indicating the fraction of the wage bill that must be backed with monetary assets.

The period-by-period budget constraint of firm i is given by

$$E_t r_{t,t+1} x^f_{it+1} + m^f_{it} + r^k_t k_{it} + w_t h_{it} + \phi_{it} = \frac{x^f_{it} + m^f_{it-1}}{\pi_t} + \left(\frac{P_{it}}{P_t}\right)^{1-\eta} y_t,$$

where $E_t r_{t,t+1} x^f_{it+1}$ denotes the total real cost of one-period state-contingent assets that the firm purchases in period t in terms of the composite good. The variable ϕ_{it} denotes real dividend payments that firm i makes to households in period t. Implicit in this specification of the firm's budget constraint is the assumption that firms rent capital services from a centralized market.[5] We assume that the firm must satisfy demand at the posted price. Formally, we impose

$$z_t F(k_{it}, h_{it}) - \psi \geq \left(\frac{P_{it}}{P_t}\right)^{-\eta} y_t. \tag{22}$$

The objective of the firm is to choose contingent plans for P_{it}, h_{it}, k_{it}, x^f_{it+1}, and m^f_{it} so as to maximize the present discounted value of dividend payments, given by

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \phi_{it+s},$$

where $r_{t,t+s} \equiv \prod_{k=1}^{s} r_{t+k-1,t+k}$, for $s \geq 1$, denotes the stochastic nominal discount factor between t and $t+s$, and $r_{t,t} \equiv 1$. Firms are assumed to be subject to a borrowing constraint that prevents them from engaging in Ponzi games.

Throughout our analysis, we will focus on equilibria featuring a strictly positive nominal interest rate. This implies that the cash-in-advance constraint (21) will always bind. Then, letting $r_{t,t+s} P_{t+s} \text{mc}_{it+s}$ be the Lagrange multiplier associated with constraint (22), the first-order conditions of the firm's maximization problem with respect to capital and labor services are, respectively,

$$\text{mc}_{it} z_t F_h(k_{it}, h_{it}) = w_t \left[1 + \nu \frac{R_t - 1}{R_t}\right] \tag{23}$$

and

$$\text{mc}_{it} z_t F_k(k_{it}, h_{it}) = r^k_t. \tag{24}$$

It is clear from these optimality conditions that the presence of a working-capital requirement introduces a financial cost of labor that is increasing in the nominal interest rate. We note

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[5]This is a common assumption in the related literature (e.g., Christiano et al., 2003; Kollmann, 2003; Carlstrom and Fuerst, 2003; and Rotemberg and Woodford, 1992). A polar assumption is that capital is sector specific, as in Woodford (2003, chapter 5.3) and Sveen and Weinke (2003). Both assumptions are clearly extreme. A more realistic treatment of investment dynamics would incorporate a mix of firm-specific and homogeneous capital.

also that because all firms face the same factor prices and because they all have access to the same production technology with the function F being linearly homogeneous, marginal costs, mc_{it}, are identical across firms. Indeed, because the above first-order conditions hold for all firms independently of whether they are allowed to reset prices optimally or not, marginal costs are identical across all firms in the economy.

Clearly, because $r_{t,t+s}$ represents both the firm's stochastic discount factor and the market pricing kernel for financial assets, and because the firm's objective function is linear in asset holdings, it follows that any asset accumulation plan of the firm satisfying the no-Ponzi constraint is optimal. Suppose, without loss of generality, that the firm manages its portfolio so as to ensure that its financial position at the beginning of each period is nil. Formally, assume that $x_{it+1}^f + m_{it}^f = 0$ at all dates and states. Note that this financial strategy makes x_{it+1}^f state-noncontingent. In this case, distributed dividends take the form

$$\phi_{it} = \left(\frac{P_{it}}{P_t}\right)^{1-\eta} y_t - r_t^k k_{it} - w_t h_{it} - (1 - R_t^{-1}) m_{it}^f. \tag{25}$$

For this expression to hold in period zero, we impose the initial condition $x_{i0}^f + m_{i-1}^f = 0$. The last term on the right-hand side of this expression represents the firm's financial costs associated with the cash-in-advance constraint on wages. This financial cost is increasing in the opportunity cost of holding money, $1 - R_t^{-1}$, which is an increasing function of the short-term nominal interest rate R_t.

Prices are assumed to be sticky à la Calvo (1983) and Yun (1996). Specifically, each period $t \geq 0$ a fraction $\alpha \in [0, 1)$ of randomly picked firms is not allowed to optimally set the nominal price of the good they produce. Instead, these firms index their prices to past inflation according to the rule $P_{it} = P_{it-1} \pi_{t-1}^\chi$. The interpretation of the parameter χ is the same as that of its wage counterpart $\tilde{\chi}$. The remaining $1 - \alpha$ firms choose prices optimally. Consider the price-setting problem faced by a firm that gets to reoptimize the price in period t. This price, which we denote by $\tilde{P}_t$, is set so as to maximize the expected present discounted value of profits. That is, $\tilde{P}_t$ maximizes the following Lagrangian:

$$
\begin{aligned}
\mathcal{L} &= E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \alpha^s \left\{ \left(\frac{\tilde{P}_t}{P_t}\right)^{1-\eta} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^\chi}{\pi_{t+k}}\right)^{1-\eta} y_{t+s} - r_{t+s}^k k_{it+s} - w_{t+s} h_{it+s}[1 + \nu(1 - R_{t+s}^{-1})] \right. \\
&\quad \left. + \mathrm{mc}_{it+s} \left[z_{t+s} F(k_{it+s}, h_{it+s}) - \psi - \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^\chi}{\pi_{t+k}}\right)^{-\eta} y_{t+s} \right] \right\}.
\end{aligned}
$$

The first-order condition with respect to $\tilde{P}_t$ is

$$E_t \sum_{s=0}^{\infty} r_{t,t+s} P_{t+s} \alpha^s \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} \prod_{k=1}^{s} \left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right)^{-\eta} y_{t+s} \left[\frac{\eta-1}{\eta}\left(\frac{\tilde{P}_t}{P_t}\right) \prod_{k=1}^{s}\left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}}\right) - mc_{it+s}\right] = 0.$$

(26)

According to this expression, optimizing firms set nominal prices so as to equate average future expected marginal revenues to average future expected marginal costs. The weights used in calculating these averages are decreasing with time and increasing in the size of the demand for the good produce by the firm. Under flexible prices ($\alpha = 0$), the above optimality condition reduces to a static relation equating marginal costs to marginal revenues period by period.

It will prove useful to express this first-order condition recursively. To that end, let

$$x_t^1 \equiv E_t \sum_{s=0}^{\infty} r_{t,t+s} \alpha^s y_{t+s} mc_{it+s} \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta-1} \prod_{k=1}^{s}\left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}^{(1+\eta)/\eta}}\right)^{-\eta}$$

and

$$x_t^2 \equiv E_t \sum_{s=0}^{\infty} r_{t,t+s} \alpha^s y_{t+s} \left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} \prod_{k=1}^{s}\left(\frac{\pi_{t+k-1}^{\chi}}{\pi_{t+k}^{\eta/(\eta-1)}}\right)^{1-\eta}.$$

Express x_t^1 and x_t^2 recursively as

$$x_t^1 = y_t mc_t \tilde{p}_t^{-\eta-1} + \alpha\beta E_t \frac{\lambda_{t+1}}{\lambda_t}(\tilde{p}_t/\tilde{p}_{t+1})^{-\eta-1}\left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{-\eta} x_{t+1}^1,$$

(27)

$$x_t^2 = y_t \tilde{p}_t^{-\eta} + \alpha\beta E_t \frac{\lambda_{t+1}}{\lambda_t}\left(\frac{\pi_t^{\chi}}{\pi_{t+1}}\right)^{1-\eta}\left(\frac{\tilde{p}_t}{\tilde{p}_{t+1}}\right)^{-\eta} x_{t+1}^2.$$

(28)

Then we can write the first-order condition with respect to $\tilde{P}_t$ as

$$\eta x_t^1 = (\eta - 1)x_t^2.$$

(29)

The labor input used by firm $i \in [0,1]$, denoted h_{it}, is assumed to be a composite made of a continuum of differentiated labor services, h_{it}^j indexed by $j \in [0,1]$. Formally,

$$h_{it} = \left[\int_0^1 h_{it}^{j\,1-1/\tilde{\eta}} dj\right]^{1/(1-1/\tilde{\eta})},$$

(30)

where the parameter $\tilde{\eta} > 1$ denotes the intratemporal elasticity of substitution across dif-

ferent types of activities. For any given level of h_{it}, the demand for each variety of labor $j \in [0, 1]$ in period t must solve the dual problem of minimizing total labor cost, $\int_0^1 W_t^j h_{it}^j dj$, subject to the aggregation constraint (30), where W_t^j denotes the nominal wage rate paid to labor of variety j at time t. The optimal demand for labor of type j is then given by

$$h_{it}^j = \left(\frac{W_t^j}{W_t} \right)^{-\tilde{\eta}} h_{it}, \tag{31}$$

where W_t is a nominal wage index given by

$$W_t \equiv \left[\int_0^1 W_t^{j 1-\tilde{\eta}} dj \right]^{\frac{1}{1-\tilde{\eta}}}. \tag{32}$$

This wage index has the property that the minimum cost of a bundle of intermediate labor inputs yielding h_{it} units of the composite labor input is given by $W_t h_{it}$.

2.3 The Government

Government expenditure is financed via lump-sum taxes and seignorage

$$g_t = \tau_t + m_t - \frac{m_{t-1}}{\pi_t},$$

where $m_t = m_t^h + \int_0^1 m_{it}^f di$ denotes real money balances and g_t denotes per capita government spending on a composite good produced via the aggregator function given in equation (2). We assume that the government minimizes the cost of producing g_t. Thus, we have that the public demand for a particular variety $i \in [0, 1]$ of differentiated goods, g_{it}, is given by $g_{it} = (P_{it}/P_t)^{-\eta} g_t$.

Monetary policy is given by an interest-rate feedback rule of the form

$$\hat{R}_t = \alpha_\pi E_t \hat{\pi}_{t-i} + \alpha_y E_t \hat{y}_{t-i} + \alpha_R \hat{R}_{t-1}, \tag{33}$$

where a hat over a variable denotes the log-deviation of that variable from its deterministic steady-state level (i.e., for any x_t, $\hat{x}_t \equiv \ln(x_t/x)$, where x denotes the non-stochastic steady-state value of x_t). The parameter i allows for backward-looking rules ($i = 1$), current-looking rules ($i = 0$), and forward-looking rules ($i = -1$). We assume that in period 0 the government chooses the policy quadruple (α_π, α_y, α_R, i) so as to maximize the lifetime utility of the representative household. The government is assumed to be endowed with a commitment technology that allows it to maintain throughout time the policy decision it makes in period

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0. As a result, the announced policy enjoys full credibility on the part of the private sector.

The family of interest rate feedback rules defined by (33) has the property of being easily implementable. For all of the variables featured in the rule are generally available macroeconomic indicators. We note that the type of monetary policy rules that are typically analyzed in the related literature require no less information on the part of the policymaker than the feedback rule given in equation (33). In effect, the rules most commonly studied feature an output gap measure defined as deviations of output from the level that would obtain in the absence of nominal rigidities. Computing the flexible-price level of aggregate activity requires the policymaker to know not just the deterministic steady state of the economy, but also the joint distribution of all the shocks driving the economy and the current realizations of such shocks.

2.4   Aggregation and Equilibrium

We limit attention to a symmetric equilibrium in which all firms that get to change their price optimally at a given time indeed choose the same price. It then follows from (4) that the aggregate price index can be written as $P_t^{1-\eta} = \alpha(P_{t-1}\pi_{t-1}^\chi)^{1-\eta} + (1-\alpha)\tilde{P}_t^{1-\eta}$. Dividing this expression through by $P_t^{1-\eta}$ one obtains

$$1 = \alpha\pi_t^{\eta-1}\pi_{t-1}^{\chi(1-\eta)} + (1-\alpha)\tilde{p}_t^{1-\eta}. \tag{34}$$

2.4.1   Market Clearing in the Final Goods Market

Naturally, the set of equilibrium conditions includes a resource constraint. Such a restriction is typically of the type $z_t F(k_t, h_t) - \psi = c_t + i_t + g_t + a(u_t)k_t$. In the present model, however, this restriction is not valid. This is because the model implies relative price dispersion across varieties. This price dispersion, which is induced by the assumed nature of price stickiness, is inefficient and entails output loss. To see this, consider the following expression stating that supply must equal demand at the firm level:

$$z_t F(k_{it}, h_{it}) - \psi = (c_t + i_t + g_t + a(u_t)k_t)\left(\frac{P_{it}}{P_t}\right)^{-\eta}.$$

Integrating over all firms and taking into account that (a) the capital-labor ratio is common across firms, (b) that the aggregate demand for the composite labor input, h_t^d, satisfies

$$h_t^d = \int_0^1 h_{it}di,$$

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and that (c) the aggregate effective level of capital, $u_t k_t$ satisfies

$$u_t k_t = \int_0^1 k_{it} di,$$

we obtain

$$h_t^d z_t F\left(\frac{u_t k_t}{h_t^d}, 1\right) - \psi = (c_t + i_t + g_t + a(u_t)k_t) \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di.$$

Let $s_t \equiv \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di$. Then we have

$$
\begin{aligned}
s_t &= \int_0^1 \left(\frac{P_{it}}{P_t}\right)^{-\eta} di \\
&= (1-\alpha)\left(\frac{\tilde{P}_t}{P_t}\right)^{-\eta} + (1-\alpha)\alpha\left(\frac{\tilde{P}_{t-1}\pi_{t-1}^\chi}{P_t}\right)^{-\eta} + (1-\alpha)\alpha^2\left(\frac{\tilde{P}_{t-2}\pi_{t-1}^\chi \pi_{t-2}^\chi}{P_t}\right)^{-\eta} + \dots \\
&= (1-\alpha)\sum_{j=0}^\infty \alpha^j \left(\frac{\tilde{P}_{t-j}\prod_{s=1}^j \pi_{t-j-1+s}^\chi}{P_t}\right)^{-\eta} \\
&= (1-\alpha)\tilde{p}_t^{-\eta} + \alpha\left(\frac{\pi_t}{\pi_{t-1}^\chi}\right)^\eta s_{t-1}.
\end{aligned}
$$

Summarizing, the resource constraint in the present model is given by the following two expressions

$$z_t F(u_t k_t, h_t^d) - \psi = (c_t + i_t + g_t + a(u_t)k_t)s_t \tag{35}$$

$$s_t = (1-\alpha)\tilde{p}_t^{-\eta} + \alpha\left(\frac{\pi_t}{\pi_{t-1}^\chi}\right)^\eta s_{t-1}, \tag{36}$$

with s_{-1} given. The state variable s_t summarizes the resource costs induced by the inefficient price dispersion featured in the Calvo model in equilibrium. Three observations are in order about the price dispersion measure s_t. First, s_t is bounded below by 1. That is, price dispersion is always a costly distortion in this model. To see that s_t is bounded below by 1, let $v_{it} \equiv (P_{it}/P_t)^{1-\eta}$. It follows from the definition of the price index given in equation (4) that $\left[\int_0^1 v_{it}\right]^{\eta/(\eta-1)} = 1$. Also, by definition we have $s_t = \int_0^1 v_{it}^{\eta/(\eta-1)}$. Then, taking into account that $\eta/(\eta-1) > 1$, Jensen's inequality implies that $1 = \left[\int_0^1 v_{it}\right]^{\eta/(\eta-1)} \leq \int_0^1 v_{it}^{\eta/(\eta-1)} = s_t$. Second, in an economy where the non-stochastic level of inflation is nil (i.e., when $\pi = 1$) or where prices are fully indexed to any variable ω_t with the property that its deterministic steady-state value equals the deterministic steady-state value of inflation (i.e., $\omega = \pi$), the variable s_t follows, up to first order, the univariate autoregressive process $\hat{s}_t = \alpha \hat{s}_{t-1}$. In these cases, the price dispersion measure s_t has no first-order real consequences for the stationary

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distribution of any endogenous variable of the model. This means that studies that restrict attention to linear approximations to the equilibrium conditions are justified to ignore the variable s_t if the model features no price dispersion in the deterministic steady state. But s_t matters up to first order when the deterministic steady state features movements in relative prices across goods varieties. More importantly, the price dispersion variable s_t must be taken into account if one is interested in higher-order approximations to the equilibrium conditions even if relative prices are stable in the deterministic steady state. Omitting s_t in higher-order expansions would amount to leaving out certain higher-order terms while including others. Finally, when prices are fully flexible, $\alpha = 0$, we have that $\tilde{p}_t = 1$ and thus $s_t = 1$. (Obviously, in a flexible-price equilibrium there is no price dispersion across varieties.)

As discussed above, equilibrium marginal costs and capital-labor ratios are identical across firms. Therefore, one can aggregate the firm's optimality conditions with respect to labor and capital, equations (23) and (24), as

$$\mathrm{mc}_t z_t F_h(u_t k_t, h_t^d) = w_t \left[1 + \nu \frac{R_t - 1}{R_t} \right] \tag{37}$$

and

$$\mathrm{mc}_t z_t F_k(u_t k_t, h_t^d) = r_t^k. \tag{38}$$

2.4.2 Market Clearing in the Labor Market

It follows from equation (31) that the aggregate demand for labor of type $j \in [0, 1]$, which we denote by $h_t^j \equiv \int_0^1 h_{it}^j di$, is given by

$$h_t^j = \left(\frac{W_t^j}{W_t} \right)^{-\tilde{\eta}} h_t^d, \tag{39}$$

where $h_t^d \equiv \int_0^1 h_{it} di$ denotes the aggregate demand for the composite labor input. Taking into account that at any point in time the nominal wage rate is identical across all labor markets at which wages are allowed to change optimally, we have that labor demand in each of those markets is

$$\tilde{h}_t = \left(\frac{\tilde{w}_t}{w_t} \right)^{-\tilde{\eta}} h_t^d. \tag{40}$$

Combining this expression with equation (39), describing the demand for labor of type $j \in [0,1]$, and with the time constraint (6), which must hold with equality, we can write

$$h_t = (1 - \tilde{\alpha}) h_t^d \sum_{s=0}^{\infty} \tilde{\alpha}^s \left(\frac{\tilde{W}_{t-s} \prod_{k=1}^{s} \pi_{t+k-s-1}^{\tilde{\chi}}}{W_t} \right)^{-\tilde{\eta}}$$

Let $\tilde{s}_t \equiv (1 - \tilde{\alpha}) \sum_{s=0}^{\infty} \tilde{\alpha}^s \left(\frac{\tilde{W}_{t-s} \prod_{k=1}^{s} \pi_{t+k-s-1}^{\tilde{\chi}}}{W_t} \right)^{-\tilde{\eta}}$. The variable $\tilde{s}_t$ measures the degree of wage dispersion across different types of labor. The above expression can be written as

$$h_t = \tilde{s}_t h_t^d. \tag{41}$$

The state variable $\tilde{s}_t$ evolves over time according to

$$\tilde{s}_t = (1 - \tilde{\alpha}) \left(\frac{\tilde{w}_t}{w_t} \right)^{-\tilde{\eta}} + \tilde{\alpha} \left(\frac{w_{t-1}}{w_t} \right)^{-\tilde{\eta}} \left(\frac{\pi_t}{\pi_{t-1}^{\tilde{\chi}}} \right)^{\tilde{\eta}} \tilde{s}_{t-1}. \tag{42}$$

We note that because all job varieties are ex-ante identical, any wage dispersion is inefficient. This is reflected in the fact that $\tilde{s}_t$ is bounded below by 1. The proof of this statement is identical to that offered earlier for s_t. To see this, note that $\tilde{s}_t$ can be written as $\tilde{s}_t = \int_0^1 \left(\frac{W_{it}}{W_t} \right)^{-\tilde{\eta}} di$. This inefficiency introduces a wedge that makes the number of hours supplied to the market, h_t, larger than the number of productive units of labor input, h_t^d. In an environment without long-run wage dispersion, the dead-weight loss created by wage dispersion is nil up to first order. Formally, a first-order approximation of the law of motion of $\tilde{s}_t$ yields a univariate autoregressive process of the form $\hat{\tilde{s}}_t = \tilde{\alpha} \hat{\tilde{s}}_{t-1}$, as long as there is no wage dispersion in the deterministic steady state. When wages are fully flexible, $\tilde{\alpha} = 0$, wage dispersion disappears, and thus $\tilde{s}_t$ equals 1.

It follows from our definition of the wage index given in equation (32) that in equilibrium the real wage rate must satisfy

$$w_t^{1-\tilde{\eta}} = (1 - \tilde{\alpha}) \tilde{w}_t^{1-\tilde{\eta}} + \tilde{\alpha} w_{t-1}^{1-\tilde{\eta}} \left(\frac{\pi_{t-1}^{\tilde{\chi}}}{\pi_t} \right)^{1-\tilde{\eta}}. \tag{43}$$

A stationary competitive equilibrium is a set of stationary processes c_t, h_t, i_t, k_{t+1}, m_t^h, λ_t, R_t, π_t, w_t, $\tilde{\mu}_t$, q_t, r_t^k, u_t, f_t^1, f_t^2, $\tilde{w}_t$, h_t^d, $\tilde{h}_t$, y_t, mc_t, x_t^1, x_t^2, $\tilde{p}_t$, s_t, $\tilde{s}_t$ satisfying (7), (9), (11)-(20), (27)-(29), (33)-(38), and (40)-(43), given exogenous stochastic processes $\{g_t, z_t\}_{t=0}^{\infty}$ and initial conditions c_{-1}, w_{-1}, s_{-1}, $\tilde{s}_{-1}$, π_{-1}, k_0, i_{-1}, R_{-1}, and, in the case of backward-looking interest-rate rules, y_{-1}.

3 Solution Method, Functional Forms, and Calibration

The above set of equilibrium conditions can be written as

$$E_t f(y_{t+1}, y_t, x_{t+1}, x_t) = 0, \tag{44}$$

where the vector x_t contains the predetermined, or state variables and is of size n_x, and the vector y_t contains the nonpredetermined variables of the model and is of size n_y. Let $n = n_x + n_y$. The state vector x_t can be partitioned as $x_t = [x_t^1; \ x_t^2]'$. The vector x_t^1 consists of endogenous predetermined state variables and the vector x_t^2 consists of exogenous state variables. Specifically, we assume that x_t^2 follows the exogenous stochastic process given by

$$x_{t+1}^2 = \Lambda x_t^2 + \tilde{\eta}_\epsilon \sigma \epsilon_{t+1}, \tag{45}$$

where both the vector x_t^2 and the innovation ϵ_t are of order $n_\epsilon \times 1$. The vector ϵ_t is assumed to have a bounded support and to be independently and identically distributed, with mean zero and variance/covariance matrix I. The scalar $\sigma \geq 0$ and the $n_\epsilon \times n_\epsilon$ matrix $\tilde{\eta}_\epsilon$ are known parameters. All eigenvalues of the matrix Λ are assumed to have modulus less than one. The solution to the model given in equation (44) is of the form:

$$y_t = g(x_t, \sigma)$$

and

$$x_{t+1} = h(x_t, \sigma) + \eta_\epsilon \sigma \epsilon_{t+1},$$

where g maps $R^{n_x} \times R^+$ into R^{n_y} and h maps $R^{n_x} \times R^+$ into R^{n_x}. The matrix η_ϵ is of order $n_x \times n_\epsilon$ and is given by

$$\eta_\epsilon = \begin{bmatrix} \emptyset \\ \tilde{\eta}_\epsilon \end{bmatrix}.$$

We use the methodology and computer code developed in Schmitt-Grohé and Uribe (2004c) to compute a second-order approximation of the functions g and h around the non-stochastic steady state, $x_t = x$ and $\sigma = 0$.[6] The deterministic steady state of the model, (x, y), is the solution to the system $f(y, y, x, x) = 0$.

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[6]Matlab code that implements the second-order approximation technique proposed in Schmitt-Grohé and Uribe (2004c) is available on line at `www.econ.duke.edu/~uribe`.

3.1 Functional Forms

We take from Christiano, Eichenbaum, and Evans (2003) the following functional forms for the period utility function, the production function, the investment adjustment cost function, and the map relating output loss to the level of capacity utilization:

$$U = \ln(c_t - bc_{t-1}) - \frac{\phi_0}{2}h_t^2 + \phi_1 \frac{m_t^{h^{1-\sigma_m}}}{1 - \sigma_m},$$

$$F(k, h) = k^\theta h^{1-\theta},$$

$$S\left(\frac{i_t}{i_{t-1}}\right) = \frac{\kappa}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2,$$

and

$$a(u) = \gamma_1(u - 1) + \frac{\gamma_2}{2}(u - 1)^2.$$

We note that because the function $a(u)$ is assumed to be quadratic, the second-order expansion of the right-hand side of the optimality condition (14)—i.e., the second-order expansion of $a'(u_t)$—is linear in u. Thus, potential second-order effects stemming from the curvature of $a'(u)$ are left out. We also experimented with the alternative specification $a(u) = \frac{\gamma_1^2}{\gamma_2}[e^{\gamma_2/\gamma_1(u-1)} - 1]$, which allows for curvature in $a'(u)$. As in the case of the benchmark specification, however, we picked a two-parameter functional form because the calibration of Christiano et al. upon which we draw heavily, provides only independent information about the first and second derivatives of the capacity utilization cost function. Evaluated at the steady-state value of the degree of capacity utilization ($u = 1$), this function as well as its first and second derivatives take the same values as the benchmark quadratic specification. Moreover, this functional form has the property that all of its derivatives have a constant elasticity at the steady state equal to γ_2/γ_1. For the benchmark values assigned to γ_1 and γ_2, the welfare consequences of monetary policy reported below are identical under both specifications of the capacity utilization cost function.

3.2 Calibration

The time unit is meant to be one quarter. The calibration follows Christiano, Eichenbaum, and Evans (2003) and is summarized in table 1. We restrict the steady-state labor input, h^d, and the steady-state capacity utilization rate, u, to unity and steady-state profits to zero. The discount factor takes the value of 3 percent per annum. The share of capital in value added is assumed to be 36 percent. The depreciation rate of capital is set at 10 percent per year. The fraction of the wage bill that is subject to a working-capital constraint is

Table 1: Calibration

Parameter	Value	Description
β	0.9926	Subjective discount factor
θ	0.36	Share of capital in value added
ψ	0.5827	Fixed cost parameter
δ	0.025	Depreciation rate
ν	1	Fraction of wage bill subject to a CIA constraint
η	6	Price-elasticity of demand for a specific good variety
$\tilde{\eta}$	21	Wage-elasticity of demand for a specific labor variety
$1 - \alpha$	0.4	Fraction of firms setting prices optimally each quarter
$1 - \tilde{\alpha}$	0.36	Fraction of labor markets setting wages optimally each quarter
b	0.65	Degree of habit persistence
ϕ_0	1.1196	Preference parameter
ϕ_1	0.5393	Preference parameter
σ_m	10.62	Preference parameter
κ	2.48	Parameter governing investment adjustment costs
χ	1	Degree of price indexation
$\tilde{\chi}$	1	Degree of wage indexation
γ_1	0.0324	Parameter governing capacity adjustment costs
γ_2	0.000324	Parameter governing capacity adjustment costs
π^*	1.0103	Gross inflation target
z	1	Steady-state value of technology shock
λ_z	0.979	Serial correlation of the log of the technology shock
η_z	0.0072	Coefficient multiplying the innovation to the log of technology
g	0.5244	Steady-state value of government spending
λ_g	0.96	Serial correlation of the log of government spending
η_g	0.02	Coefficient multiplying the innovation to the log of gov. spending
σ	0.18	Parameter scaling the std. deviations of exogenous shocks

assumed to be 100 percent. Households are assumed to hold 44 percent of the aggregate money supply in the steady state. The remaining 56 percent of money balances is held by firms. We set the long-run inflation rate to 4.2 percent per year which is consistent with the average growth rate of the U.S. GDP deflator over the period 1960-1998. We assume full indexation in prices and wages to the previous period's price inflation. That is, we set $\chi = \tilde{\chi} = 1$. The degree of nominal rigidity is calibrated so that product prices change on average every 2.5 quarters and nominal wages every 2.8 quarters. Steady state markups are assumed to be 20 percent in product markets and 5 percent in labor markets. The degree of habit formation, defined by the parameter b, is set at 0.65. The elasticity of the subutility function that depends on real balances is assigned a value of -9.62, that is, $\sigma_m = 10.62$. The steady-state elasticity of the marginal capacity utilization cost, $a''(1)/a'(1)$, is calibrated to be 0.01. The parameter κ determining the degree of investment adjustment costs takes the value 2.48.

Following Christiano and Eichenbaum (1992), the steady-state share of government purchases in value added is assumed to be 18 percent. The processes describing the evolution of the productivity shock and public spending are assumed to be univariate and first-order autoregressive in logs. Formally,

$$\ln z_{t+1} = \lambda_z \ln z_t + \sigma \eta_z \epsilon_{t+1}^z$$

and

$$\ln(g_{t+1}/g) = \lambda_g \ln(g_t/g) + \sigma \eta_g \epsilon_{t+1}^g.$$

The innovations ϵ_t^z and ϵ_t^g are assumed to be i.i.d. with mean zero and standard deviation equal to one. In the notation of equation (45), we have that $\Lambda = [\lambda_g \, 0; 0 \, \lambda_z]$ and $\tilde{\eta}_\epsilon = [\eta_g \, 0; 0 \, \eta_z]$. King and Rebelo (1999) calibrate the productivity shock process by estimating an AR(1) representation for the Solow residual. We set λ_z to 0.979, which corresponds to their estimate of the first-order serial correlation of the Solow residual. As in Christiano and Eichenbaum (1992), we set λ_g equal to 0.96. We calibrate the parameter η_z to 0.0072 and the parameter η_g to 0.02. These values correspond to the volatility of the innovation to the Solow residual process estimated by King and Rebelo (1999) and to the volatility of the innovation to the government purchases process estimated by Christiano and Eichenbaum (1992), respectively.

To complete the calibration of the exogenous driving force process, it remains to pick a value for the parameter σ, which scales the volatility of the disturbances ϵ_t^z and ϵ_t^g. Following the calibrations of King and Rebelo (1999) and Christiano and Eichenbaum (1992) would imply setting σ to unity. However, for this value of the scaling parameter the Christiano-

Eichenbaum-Evans model studied here implies an output volatility equal to 10 percent when monetary policy is assumed to take the form of a simple Taylor rule, $\hat{R}_t = 1.5\hat{\pi}_t$. Cooley and Prescott (1995) estimate that the volatility of the cyclical component of U.S. output in the postwar era is 1.72 percent, which is about six times smaller than the value implied by the model. One reason why the Christiano-Eichenbaum-Evans model implies such a high output volatility is that the assumed costs of varying the capital capacity utilization are modest, which allows firms to adjust the intensity at which they utilize the existing stock of capital fairly easily over the business cycle. We therefore set σ to 0.18, which brings the volatility of output down to 1.72 percent under a simple Taylor rule. We note however, that because the deviations of our welfare measure from steady state are proportional to σ^2, the welfare rankings of alternative monetary policies, which is at the core of the present study, is independent of the size of σ^2, provided the implied path for the nominal interest rate does not violate the zero lower bound.

4　Dynamics Under a Taylor Rule

To gather intuition about the dynamic behavior of the model economy, it is useful to analyze the equilibrium response of a number of macroeconomic variables of interest to supply and demand shocks. Before conducting such an exercise, it is necessary to specify a monetary regime. Here we assume that monetary policy takes the form of a simple interest-rate feedback rule whereby the short-term interest rate is set as an increasing function of deviations of inflation from a target, with a coefficient of 1.5. Formally, we assume that
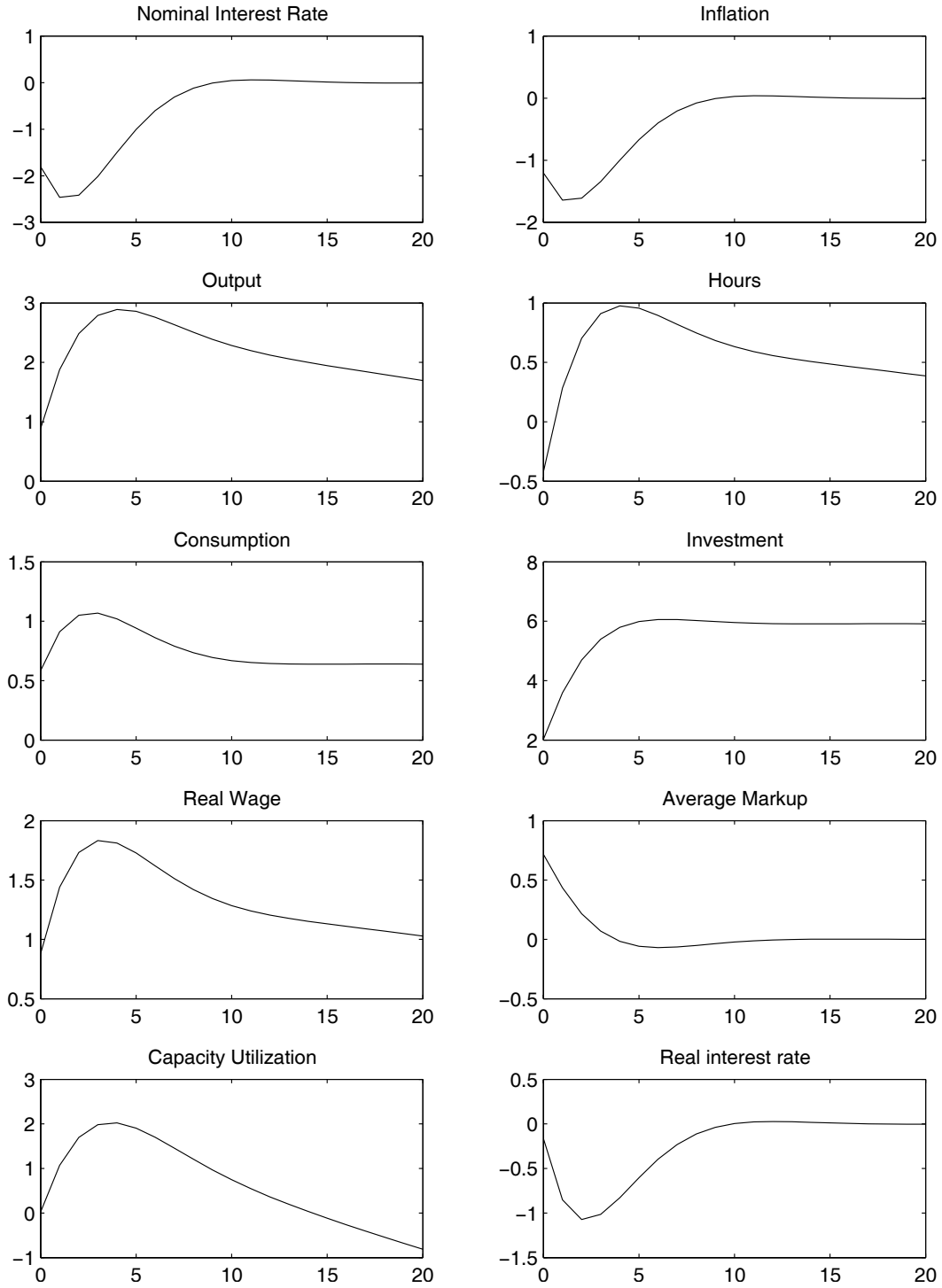
$$\hat{R}_t = 1.5\hat{\pi}_t.$$

According to the available empirical evidence (e.g., Taylor, 1993; and Clarida, Galí, and Gertler, 1998), this is a realistic policy benchmark for the U.S. economy and other industrialized countries over the past two decades. Arguably, a more realistic formulation of the interest-rate rule would feature the cyclical component of output and the lagged nominal interest rate as additional arguments of the interest-rate rule (Sack, 1998). However, such a rich feedback rule complicates the task of building intuition about the workings of the model. We will analyze richer rules later on when we perform policy evaluations.

Figure 1 displays the economy's response to a 1 percent increase in productivity (z_t). In response to this shock, inflation and the nominal interest rate fall, while the average markup of prices over marginal cost increases.[7] To understand why inflation falls in response to a

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[7]We define the average markup in the economy as the sales-weighted average over markups charged by

Figure 1: Response To A Unit Innovation In Technology Under A Simple Taylor Rule

positive technology shock, it is crucial to analyze the behavior of the real interest rate. To see this, note that up to first order (and leaving aside uncertainty) the Taylor rule, $\hat{R}_t = 1.5\hat{\pi}_t$, together with the Fisher relation, $\hat{R}_t = \hat{\pi}_{t+1} + \hat{r}_t$, where $\hat{r}_t$ denotes the real interest rate, imply that inflation follows the first-order process $\hat{\pi}_{t+1} = 1.5\hat{\pi}_t - \hat{r}_t$. It is immediate from this difference equation that, because along the entire transition the real interest rate is below its steady-state level (bottom right panel of figure 1), in order for the inflation rate not to explode, it must be the case that inflation is below its steady-state level during the adjustment process.

The question, therefore, is why a positive productivity shock depresses real interest rates. The answer to this question has a lot to do with the magnitude of investment adjustment costs. In the absence of investment adjustment costs ($\kappa = 0$), the model studied here predicts that the real interest rate actually increases on impact and then falls gradually. This is because in the absence of investment adjustment costs, investment displays a large increase on impact, delaying the positive response of consumption. Indeed, in this case (habit-adjusted) consumption is hump-shaped. In order for consumers to be induced to undertake a hump-shaped habit-adjusted consumption profile, the real interest rate must be above its long-run level during the initial phase of the transition. By contrast, in the presence of investment adjustment costs (κ positive), which is the case under the baseline calibration of the model studied here, the real interest rate falls on impact. This decline occurs because in this case investment is slow to react to the productivity shock making consumption absorb much of the initial increase in output. As a result, habit-adjusted consumption increases on impact and then falls. To induce households to adopt such a declining spending path, the real interest rate must be below its steady-state value. It follows from this analysis, that when monetary policy takes the form of a simple Taylor Rule, an increase in productivity is inflationary in the absence of investment adjustment costs and is deflationary in their presence.

Having established why inflation falls in response to a positive productivity shock, one can also understand why markups are above average along the transition. Those firms that get to set prices optimally when the productivity shock occurs will tend to keep their markup close to its long-run mean. Note that equation (26) implies that in their price setting behavior these firms penalize more heavily deviations of markups from $\eta/(\eta-1)$ in the short run because demand is highest during this part of the transition. In addition, the fact that inflation falls on impact requires that firms that reoptimize prices in that period actually

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individual firms. Formally, $\mu_t = \left[\int_0^1 \mu_{it}p_{it}y_{it}di\right] / \left[\int_0^1 p_{it}y_{it}di\right]$. Recalling that $y_{it} = p_{it}^{-\eta}y_t$, that $\mu_{it} = p_{it}/mc_t$, and that $s_t = \int_0^1 p_{it}^{-\eta}di$, and defining $\bar{s}_t = \int_0^1 p_{it}^{2-\eta}di$, we can write $\mu_t = \bar{s}_t/(s_t mc_t)$. Finally, note that $\bar{s}_t$ can be written recursively as $\bar{s}_t = (1-\alpha)\tilde{p}_t^{2-\eta} + \alpha\bar{s}_{t-1}(\pi_t/\pi_{t-1}^\chi)^{\eta-2}$, with $\bar{s}_{-1}$ given.

lower nominal prices. This implies that the relative price of firms that get to change prices optimally, $\tilde{P}_t/P_t$, falls. Since markups for these firms are little changed, we deduce that real marginal costs fall. Now, since real marginal costs are common across all firms, the markup of firms who do not have the chance to reset prices optimally must go up. It follows that the average markup in the economy increases, as indicated by row 4 column 2 of figure 1. The increase in markups produces an inefficient macroeconomic adjustment in response to the productivity shock.

Recently, there has been a lively debate about the effect of technology shocks on hours worked. At the heart of the controversy is whether in post-Volker U.S. data hours increase or fall in response to a positive innovation in productivity. It is of interest to note that in the Christiano-Eichenbaum-Evans model hours fall on impact in response to a positive shock to labor productivity when monetary policy takes the form of a simple Taylor rule. If one believes that a simple interest-rate feedback rule whereby the short-term nominal interest rate depends only on consumer price inflation with a response coefficient of 1.5 is a reasonable description of U.S. monetary policy over the past two decades, then the Christiano-Eichenbaum-Evans model is in line with those suggesting that the empirical evidence points to a decline in hours in response to a positive technology shock. It is fair to mention that in the empirical debate technology shocks are typically identified as permanent changes in total factor productivity, whereas in our study productivity shocks, although highly persistent, are assumed to follow a stationary autoregressive process.

One factor that contributes to the contractionary effect of productivity shocks on aggregate employment in the model is the presence of price stickiness in product markets. When the number of firms that do not get to reoptimize (reduce) prices after a positive technology shock is sufficiently large (i.e., when prices are sufficiently sticky), then the fact that employment at those firms falls drives the behavior of aggregate employment dominating the increase in employment at firms that get to reoptimize prices when the shock occurs.

Figure 2 displays the economy's response to a 1 percent increase in government spending. Because the increase in unproductive public spending makes households poorer, hours worked increase. To smooth out the necessary adjustment in consumption and leisure, the private sector reduces investment in physical capital and uses the existing capital stock more intensively. The resulting response of consumption is negative but negligible, reflecting the fact that households are fairly successful in smoothing out consumption expenditure. The flat time path of consumption implies a modest reaction in real interest rates. In turn, recalling that inflation is linked to real interest rates via the difference equation $\hat{\pi}_{t+1} = 1.5\hat{\pi}_t - \hat{r}_t$, the small response in real interest rates translates into an equally small response in inflation. The Taylor rule then implies that nominal interest rates are also little changed.

Figure 2: Response To A Unit Innovation In Government Spending Under A Simple Taylor Rule
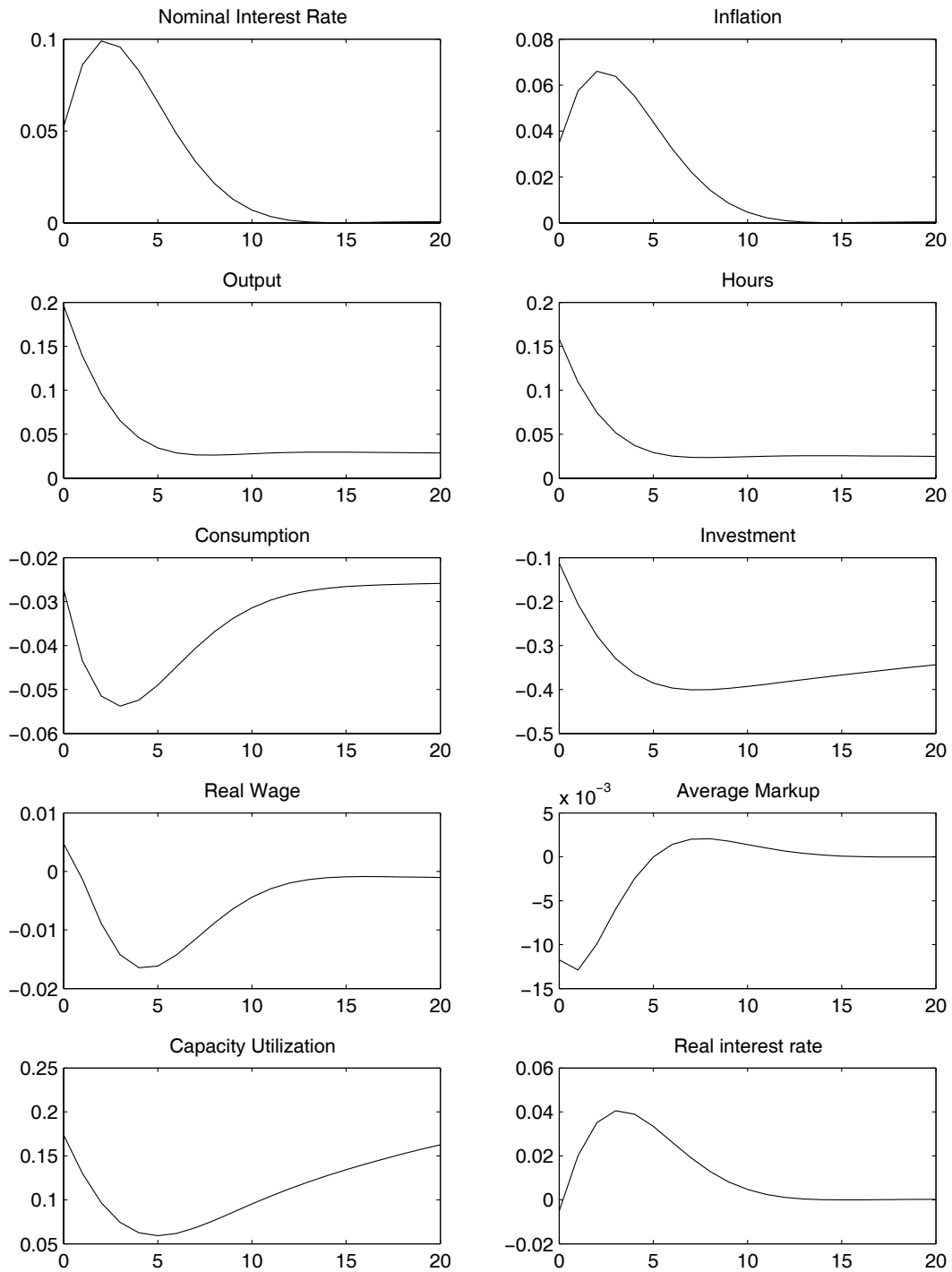
5 The Welfare Measure

We measure the level of utility associated with a particular monetary policy specification as follows. Let the equilibrium processes for consumption, money holdings, and hours associated with a particular monetary regime be denoted by $\{c_t, m_t^h, h_t\}$. Then we measure welfare as the conditional expectation of lifetime utility as of time zero evaluated at $\{c_t, m_t^h, h_t\}$. We denote this welfare measure by V_0. Formally, V_0 is given by

$$V_0 \equiv E_0 \sum_{t=0}^{\infty} \beta^t U(c_t - bc_{t-1}, m_t^h, h_t). \tag{46}$$

In addition, we assume that at time zero all state variables of the economy, including c_{-1}, equal their respective steady-state values. We depart from the usual practice of identifying the welfare measure with the unconditional expectation of lifetime utility because using unconditional expectations of welfare amounts to not taking into account the transitional dynamics leading to the stochastic steady state. Because the deterministic steady state is the same across all policy regimes we consider, our choice of computing expected welfare conditional on the initial state being the nonstochastic steady state ensures that the economy begins from the same initial point under all possible policies. Therefore, our strategy will deliver the constrained optimal monetary rule associated with a particular initial state of the economy. It is of interest to investigate the robustness of our results with respect to alternative initial conditions. For, in principle, the welfare ranking of the alternative polices will depend upon the assumed value for (or distribution of) the initial state vector. For further discussion of this issue, see Kim et al., 2003.

We compute the welfare cost of a particular monetary regime relative to a reference rule as follows. Consider two policy regimes, a reference policy regime denoted by r and an alternative policy regime denoted by a. Then we define the welfare associated with policy regime r as

$$V_0^r = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^r - bc_{t-1}^r, m_t^{hr}, h_t^r),$$

where c_t^r, m_t^{hr}, and h_t^r denote the contingent plans for consumption, real money balances, and hours under policy regime r. Similarly, define the welfare associated with policy regime a as

$$V_0^a = E_0 \sum_{t=0}^{\infty} \beta^t U(c_t^a - bc_{t-1}^a, m_t^{ha}, h_t^a).$$

Let λ denote the welfare cost of adopting policy regime a instead of the reference policy regime r. We measure λ as the fraction of regime r's consumption process that a household

would be willing to give up to be as well off under regime a as under regime r. That is, λ is implicitly defined by

$$V_0^a = U(c_0^r(1-\lambda) - bc_{-1}, m_0^{hr}, h_0^r) + E_0 \sum_{t=1}^{\infty} \beta^t U((c_t^r - bc_{t-1}^r)(1-\lambda), m_t^{hr}, h_t^r). \qquad (47)$$

Note that we do not apply the factor $1 - \lambda$ to c_{-1}, because this variable is predetermined at the time of the policy evaluation. Using the particular functional form assumed for the period utility function, equation (49) can be written as

$$V_0^a - V_0^r = \ln[(1-\lambda)c_0^r - bc_{-1}] + \frac{\beta}{1-\beta} \ln(1-\lambda) - \ln(c_0^r - bc_{-1}).$$

Resorting to the notation introduced in section 3, let $V_0^a = g^{va}(x_0, \sigma)$, $V_0^r = g^{vr}(x_0, \sigma)$, and $\ln c_0 = g^{cr}(x_0, \sigma)$, where x_0 is the value of the state vector in period zero and σ is a parameter scaling the standard deviation of the exogenous shocks. Then we can rewrite the above expression as

$$g^{va}(x_0,\sigma) - g^{vr}(x_0,\sigma) = \ln[(1-\lambda)e^{g^{cr}(x_0,\sigma)} - bc_{-1}] + \frac{\beta}{1-\beta}\ln(1-\lambda) - \ln(e^{g^{cr}(x_0,\sigma)} - bc_{-1}). \quad (48)$$

It is clear from this expression that λ is a function of x_0 and σ, which we write as

$$\lambda = \Lambda(x_0, \sigma).$$

Consider a second-order approximation of the function Λ around the point $x_0 = x$ and $\sigma = 0$, where x denotes the deterministic steady-state of the state vector. Because we wish to characterize welfare conditional upon the initial state being the deterministic steady state, that is, $x_0 = x$, in performing the second-order expansion of λ only its first and second derivatives with respect to σ have to be considered. Formally, we have

$$\lambda \approx \Lambda(x, 0) + \Lambda_\sigma(x, 0)\sigma + \frac{\Lambda_{\sigma\sigma}(x, 0)}{2}\sigma^2.$$

Because the deterministic steady-state level of welfare is the same across all monetary policies belonging to the class defined in equation (33), it follows that λ vanishes at the point $x_0 = x$ and $\sigma = 0$. Formally,

$$\Lambda(x, 0) = 0.$$

Totally differentiating equation (48) with respect to σ and evaluating the result at $(x_0, \sigma) =$

$(x, 0)$ one obtains

$$g_\sigma^{va}(x,0) - g_\sigma^{vr}(x,0) = \frac{1}{1-b}[g_\sigma^{cr}(x,0) - \Lambda_\sigma(x,0)] + \frac{\beta}{1-\beta}\Lambda_\sigma(x,0) - \frac{1}{1-b}g_\sigma^{cr}(x,0)$$

Schmitt-Grohé and Uribe (2004c) show that the first derivatives of the policy functions with respect to σ evaluated at $x_t = x$ and $\sigma = 0$ are nil (so $g_\sigma^{va} = g_\sigma^{vr} = g_\sigma^{cr} = 0$). It follows immediately from the above expression that

$$\Lambda_\sigma(x, 0) = 0.$$

Now totally differentiating (48) twice with respect to σ and evaluating the result at $(x_0, \sigma) = (x, 0)$ yields

$$\Lambda_{\sigma\sigma}(x,0) = -\frac{g_{\sigma\sigma}^{va}(x,0) - g_{\sigma\sigma}^{vr}(x,0)}{\frac{\beta}{1-\beta} + \frac{1}{1-b}}.$$

Thus, our welfare cost measure, in percentage terms, is given by

$$\text{welfare cost} = -\frac{g_{\sigma\sigma}^{va}(x,0) - g_{\sigma\sigma}^{vr}(x,0)}{\frac{\beta}{1-\beta} + \frac{1}{1-b}} \times \frac{\sigma^2}{2} \times 100. \tag{49}$$

As mentioned above, in deriving this measure of welfare costs we do not apply the factor $1 - \lambda$ to the level of consumption in period -1. Alternatively, one could define the welfare cost index applying the factor $1 - \lambda$ to the entire argument of the first term of the initial period's utility function, $c_0 - bc_{-1}$, rather than just to c_0. In this case, λ is implicitly given by $V_0^a = E_0 \sum_{t=0}^{\infty} \beta^t U((c_t^r - bc_{t-1}^r)(1-\lambda), m_t^{hr}, h_t^r)$. A second-order-accurate measure of welfare costs is then given by $-(1 - \beta)[g_{\sigma\sigma}^{va}(x,0) - g_{\sigma\sigma}^{vr}(x,0)] \times (\sigma^2/2) \times 100$. Clearly, because this welfare measures is proportional to the one we adopt, both measure will deliver identical welfare rankings. Moreover, under the baseline calibration adopted in this paper the ratio of the welfare cost measure we adopt to the alternative discussed here is 0.9865. Therefore, both criteria deliver almost identical welfare cost numbers.[8]

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[8]The reason why λ is always bigger in absolute value under the alternative welfare cost measure is quite intuitive. Suppose that the reference policy (r) yields less welfare than the alternative policy (a). In this case, λ is negative, because households living under the reference policy must be compensated by an increase in consumption. However, if the increase in consumption also applies to the initial stock of habit c_{-1}, as is the case under the welfare cost criterion not adopted in this paper, then the compensated initial stock of habit is higher making the agent worse off. As a result λ must be bigger.

6 Policy Evaluation

In our search for the optimal monetary rule, we limit attention to what we call operational rules. To be operational, an interest-rate rule must set the nominal interest rate as a function of a few readily observable macroeconomic variables. We study rules where the nominal interest rate responds to its own lag, inflation, and output. In addition, an operational rule is required to posses the following two properties: First, it must induce a locally unique rational expectations equilibrium.[9] Second, operational rules must satisfy the nonnegativity constraint on the nominal interest rate. For technical reasons, we are unable to impose this constraint directly. Specifically, the second-order perturbation approach we apply in our numerical work is ill equipped to handle occasionally binding constraints.[10] Instead, we require that under an operational rule the target value for the nominal interest rate be greater than two times the standard deviation of the nominal interest rate. Formally, we require that $\ln(R^*) \geq 2\sigma_{\hat{R}_t}$. If the equilibrium nominal interest rate was normally distributed around its target value, then this constraint would ensure a positive nominal interest rate 98 percent of the time. A possible alternative would be to require that the unconditional mean of the nominal interest rate, rather than its target value, be greater than 2 standard deviations of the nominal interest rate, that is, $E \ln R_t > 2\sigma_{\hat{R}_t}$. One reason why we do not pursue this alternative is that it is computationally more costly. This is because obtaining a second-order accurate approximation to the unconditional mean of the nominal interest rate requires approximating the equilibrium up to second order, whereas obtaining a second-order accurate approximation to the variance of the nominal interest rate requires approximating the equilibrium conditions only up to first order. Admittedly, as long as $E \ln R_t < \ln R^*$, the criterion $\ln R^* > 2\sigma_{\hat{R}_t}$ is less restrictive than the criterion $E \ln R_t > 2\sigma_{\hat{R}_t}$. Indeed, in our economy, there is a tendency for optimal policy to induce a mean interest rate below the target R^*. This is because the demand for money in combination with indexation creates incentives for policy to gravitate toward the Friedman rule. As we show later, however, although the operational rule that we identify as optimal implies that $E \ln R_t < \ln R^*$, it satisfies the tighter criterion $E \ln R_t > 2\sigma_{\hat{R}_t}$.

An operational rule is judged to be optimal when it yields a higher level of welfare to the representative agent than any other operational rule.

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[9]An alternative, not pursued in this study, is to enlarge the set of operational monetary rules by allowing for policy regimes that render the equilibrium locally indeterminate. This approach requires introducing plausible assumptions about the expectations coordination mechanism, which is outside the scope of our investigation.

[10]Other authors have also faced the need to circumvent the problem of not being able to directly impose the zero-bound constraint on nominal interest rates. See, for instance, Rotemberg and Woodford (1999, p. 75).
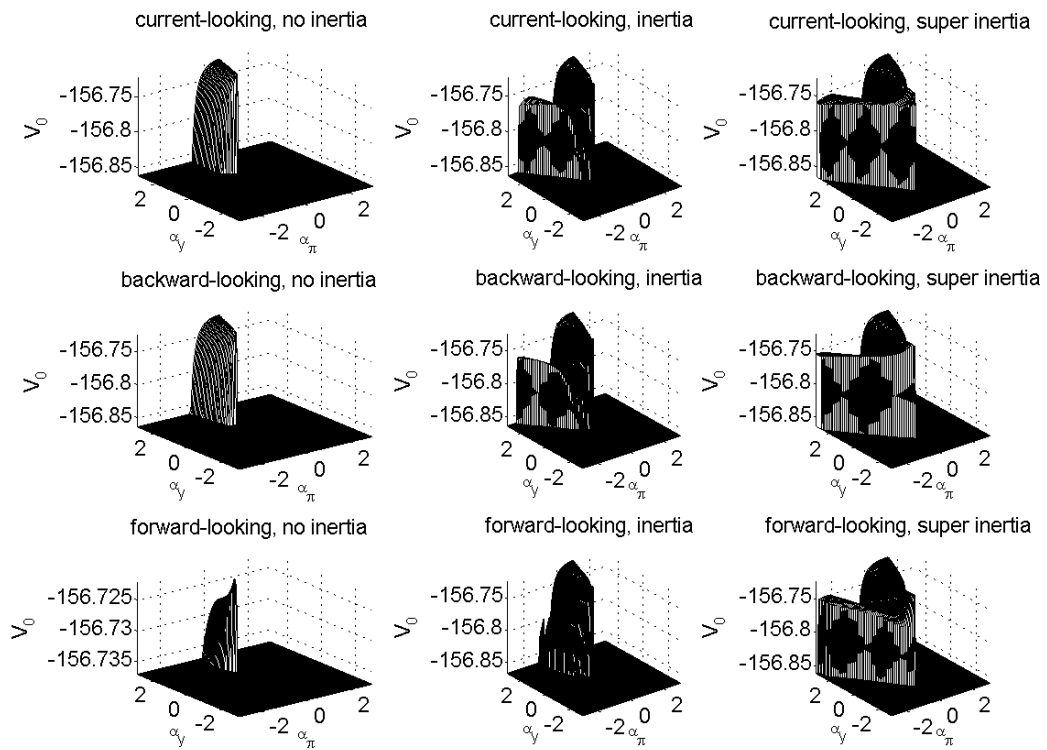
The family of interest-rate rules presented in equation (33) can be fully characterized by four parameters, namely, α_π, measuring the responsiveness of interest rates to deviations of inflation from the central bank's target, α_y, the output-gap coefficient, α_R, measuring the degree of policy inertia, and i, measuring the backward-, current-, or forward-looking nature of policy. We further restrict the policy quadruple $(\alpha_\pi, \alpha_y, \alpha_R, i)$ to lie in the following space: The parameters α_π, measuring the responsiveness of interest rates to deviations of inflation from the central bank's target, and α_y, the output gap coefficient, are confined to take values in the interval -3 and 3. For both α_π and α_y, we consider a grid with a step size of 0.0625. The degree of policy inertia, measured by the parameter α_R, can take 3 possible values, 0, 1, and 2. When α_R equals zero we say that policy lacks inertia. When α_R equals unity, policy is referred to as inertial. And, following the terminology of Rotemberg and Woodford (1999), when α_R equals two, we say that policy is superinertial. Finally, we consider three possible values for the parameter i, 1, 0, and -1. When $i = 1$, the central bank is backward-looking in the sense that it sets interest rates based on past information regarding inflation and output. When i equals zero, the central bank responds to contemporaneous realizations of inflation and the output gap. And when $i = -1$, the monetary authority is forward-looking, as the current instrument setting is a function of expectations of future price and output movements. Summarizing, our discretized policy parameter space contains 97 possible values for α_π, 97 possible values for α_y, 3 possible values for α_R, and 3 possible values for i. This implies that we evaluate a total of 84,681 different monetary policy regimes.

An alternative to our approach of discretizing the policy space would be to rely on a numerical optimization routine to find the optimal interest-rate rule coefficients α_π, α_y, and α_R for each value of the policy parameter i. The reason why we choose not to adopt this numerical strategy is that it identifies local extrema. And, as will become clear shortly, in our model economy, welfare is a nonmonotonic function of the policy parameters, with several local maxima.

6.1   Welfare Surfaces

Figure 3 shows the level of welfare associated with the different policy rules as functions of the policy parameters α_π and α_y holding constant the degree of policy inertia α_R and the parameter i indicating whether the rule is backward-, current-, or forward-looking. The welfare measure that is plotted corresponds to a second-order approximation to the expression given in equation (46). Only policies that imply a welfare cost of at most 0.1 percent of the lifetime consumption stream under the best operational rule are shown. The welfare surfaces appear to be piecewise smooth in α_π and α_y. However, the welfare graph displays

Figure 3: Welfare as a Function of α_π and α_y

current-looking, no inertia

current-looking, inertia

current-looking, super inertia

backward-looking, no inertia

backward-looking, inertia

backward-looking, super inertia

forward-looking, no inertia

forward-looking, inertia

forward-looking, super inertia

multiple local maxima. This feature motivates our choice of finding the optimal interest rate rule using a grid search over the policy parameters rather than relying on local optimization routines.

Figure 4 depicts in black the pairs α_π and α_y for which the associated equilibrium is determinate but the zero-bound constraint is violated, with grey the pair for which the equilibrium is determinate and the zero-bound constraint is satisfied, and with white the pairs for which the equilibrium fails to be locally determinate, given values for the policy parameters α_R and i. A cross marks the optimal operational rule given α_R and i. Each row of graphs reveals a tight connection between the degree of interest-rate smoothing, α_R, and the threshold value for the inflation sensitivity parameter α_π above which an operational rule exists. In the absence of inertia ($\alpha_R = 0$, left column), operational rules exist only for inflation coefficients greater than unity ($\alpha_\pi > 1$). This stability condition also appears in simpler monetary models. When policy is inertial ($\alpha_R = 1$, middle column), operational rules exist for values of α_π greater than zero, and when policy is superinertial ($\alpha_R = 2$, right column), operational rules exist for $\alpha_\pi > -1$. This relation suggests a pattern according to which operational rules exist only for values of α_π greater than $1 - \alpha_R$. The area of operational rules is roughly symmetric around acyclical rules ($\alpha_y = 0$). As a result, the more cyclical is the rule (whether pro- or countercyclical) the less likely it is to be operational. For the subclass of acyclical rules, the set of operational rules is the same as the set of rules that induce a locally determinate equilibrium.

## 6.2   The Optimal Operational Rule

Table 2 displays the inflation and output coefficients of the operational rules that maximize welfare for each of the 3 values of the degree of policy inertia considered ($\alpha_R = 0, 1, 2$) and for each of the 3 values taken by the parameter i measuring the current-, backward-, or forward-looking nature of policy. The table also shows the level of welfare attained by each optimized rule and the associated welfare cost relative to the best operational rule using the welfare cost measure given in equation (49).

The optimal operational rule takes the form

$$\hat{R}_t = 1.125 E_t \hat{\pi}_{t+1} - 0.0625 E_t \hat{y}_{t+1}.$$

This rule can be interpreted as a real-interest-rate targeting rule, for it is forward looking, has an inflation coefficient close to unity, has an output coefficient of about zero, and displays no inertia. Importantly, the inflation coefficient, although close to unity, is in accordance with the Taylor principle, prescribing a more than one-to-one response of interest rates to

Figure 4: Determinacy of Equilibrium and Operational Rules
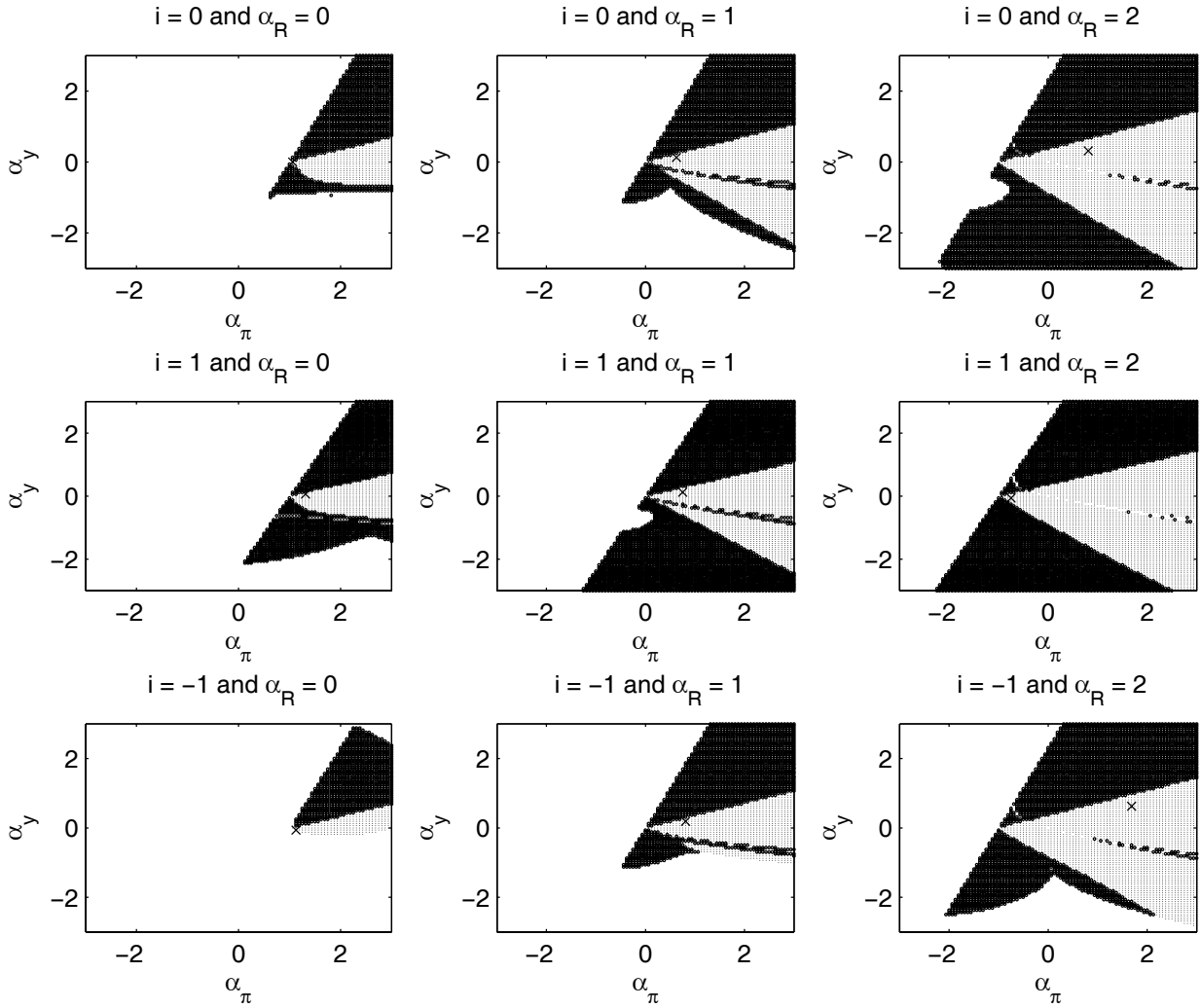
Note: White region: Equilibrium is not locally determinate. Black region: Equilibrium is locally determinate but zero-bound constraint is violated. Gray region: Equilibrium is locally determinate and zero-bound condition is satisfied. Cross: Optimal operational rule for given α_R and i.

Table 2: Optimal Operational Interest-Rate Rules

Optimized Interest-Rate Rules $\hat{R}_t = \alpha_\pi E_t \hat{\pi}_{t-i} + \alpha_y E_t \hat{y}_{t-i} + \alpha_R \hat{R}_{t-1}$				
	α_π	α_y	Welfare	Welfare Cost
Current-Looking Rules $(i = 0)$				
No Inertia $(\alpha_R = 0)$	1.0625	0	-156.7227	0.0005
Inertia $(\alpha_R = 1)$	0.625	0.1250	-156.7237	0.0012
Super Inertia $(\alpha_R = 2)$	0.8125	0.3125	-156.7248	0.0020
Backward-Looking Rules $(i = 1)$				
No Inertia $(\alpha_R = 0)$	1.3125	0.0625	-156.7233	0.0009
Inertia $(\alpha_R = 1)$	0.75	0.1250	-156.7243	0.0017
Super Inertia $(\alpha_R = 2)$	-0.75	-0.0625	-156.7250	0.0022
Forward-Looking Rules $(i = -1)$				
No Inertia $(\alpha_R = 0)$	1.1250	-0.0625	-156.7220	0
Inertia $(\alpha_R = 1)$	0.8125	0.1875	-156.7232	0.0008
Super Inertia $(\alpha_R = 2)$	1.6875	0.6250	-156.7237	0.0012
Non-Optimized Rules				
Inflation Targeting $(\hat{\pi}_t = 0)$	–	–	Indeterminate	
Taylor Rule	1.5	0.5	Zero-bound violated	
Simple Taylor Rule	1.5	–	-156.7261	0.0029

Note: (1) R_t denotes the gross nominal interest rate, $\pi_t \equiv P_t/P_{t-1}$ denotes the gross inflation rate, and y_t denotes output. (2) For any variable x_t, its non-stochastic steady-state value is denoted by x, and its log-deviation from steady state by $\hat{x}_t \equiv \ln(x_t/x)$. (3) In all cases, the parameter α_π is restricted to lie in the interval $[-3, 3]$ while α_y is restricted to be in the interval $[-3, 3]$. (4) Welfare is computed as a second-order approximation to the expression in equation (46), conditional on that period's state being the non-stochastic steady state. (5) The welfare cost is measured relative to the optimal rule ($\alpha_\pi = 1.125$, $\alpha_y = -0.0625$, $\alpha_R = 0$, and $i = -1$), and is defined in equation (49).

deviations of inflation from target.

Real-interest-rate targeting appears to be a robust feature of optimal operational policy. In effect, even if one limits attention to either contemporaneous- or backward-looking rules, the optimal rule lacks inertia, is acyclical in the sense that the output coefficient is nil, and features an inflation coefficient close to 1. The finding that real-interest-rate targeting is optimal even when policy is restricted to being contemporaneous or backward looking is important because some may argue that a forward-looking rule is not truly operational. For forward-looking rules require knowledge on the part of the central bank of the private sector's expectations of future output and inflation, which is not readily observable information. Moreover, some may argue that in practice current realizations of output and inflation are not in the information set of the monetary authority at the time it makes policy decisions, thus disqualifying current-looking rules from the set of operational rules. For these critics, a rule is operational only when it is based on past observations of macroeconomic indicators. Table 2 shows that the welfare costs of restricting policy to be operational in this tighter sense are negligible. Specifically, the welfare cost of the best backward-looking rule vis-à-vis the optimal rule is only 0.0009 percent of consumption per period.

Table 2 also reports welfare evaluations of some commonly prescribed non-optimized monetary policy rules. A monetary rule whereby the inflation rate is at its target in and out of equilibrium, labeled inflation targeting in the bottom panel of table 2, induces indeterminacy of the rational expectations equilibrium. For this reason, we are unable to evaluate the welfare consequences of such a policy. A standard Taylor rule with an inflation coefficient of 1.5 and an output coefficient of 0.5 fails to be operational, as it implies that $2\sigma_{\hat{R}_t} > \ln R^*$ so that the associated equilibrium frequently violates the zero bound on nominal interest rates. A simpler Taylor rule, with an inflation coefficient of 1.5 and an output coefficient of 0 does satisfy all of the requirements of an operational rule. It entails a welfare cost of 0.0029 percent of the lifetime consumption associated with the optimal rule. This welfare cost number, although small in absolute terms, is greater than those associated with any of the optimized rules shown in table 2.

6.3 Inertia, Cyclicality, and the Taylor Principle

The three main characteristics of the optimal operational rule are lack of inertia in interest rates ($\alpha_R = 0$), acyclicality (α_y close to zero), and a mildly active response to inflation (i.e., an inflation coefficient greater than and close to unity). These three features of optimal policy survive a large number of fundamental modifications to the baseline model.

Table 4 presents the optimal operational rule for 7 variations of the baseline model: (1)

indexation to long-run inflation (i.e., non-reoptimized prices/wages are adjusted by the factor π^* each period); (2) flexible wages ($\tilde{\alpha} = 0.01$); (3) flexible prices ($\alpha = 0.01$); (4) Higher costs of varying capacity utilization ($\gamma_2/\gamma_1 = 2$); (5) No habit formation ($b = 0$); (6) No money held by households ($\phi_1 = 0$); and (7) no money at all ($\phi_1 = \nu = 0$).

Under all model variations considered, optimal operational policy lacks inertia in interest rates. An exception is the case in which prices are indexed to long-run inflation. Interestingly, indexation to long-run inflation is implicitly or explicitly assumed in related studies that identify inertia as a feature of optimal monetary policy (e.g., Rotemberg and Woodford, 1999). This result underlines the importance of the exact indexation modality in shaping optimal policy.

The Taylor principle prescribing interest rate rules with an inflation coefficient greater than one is present in 5 of the 7 perturbations to the baseline model we perform. The largest inflation coefficient obtains under flexible wages, where $\alpha_\pi = 3$. This number indeed hits the upper bound imposed in our numerical computations. Intuitively, when wages are flexible, the monetary authority has a larger incentive to stabilize inflation to ameliorate the consequences of the sticky-price distortion. In this case, the central bank stabilizes inflation by setting the inflation coefficient in the interest-rate rule as high as it can. As a result, the equilibrium under flexible wages features a standard deviation of inflation of only 0.5 percent, 4 times smaller than in the baseline model.

Another robust feature of optimal operational rules appears to be a muted response of interest rates to output. This result cannot be immediately related to the findings in the related literature. The reason is that in most existing studies the central bank is assumed to respond to a sophisticated output gap measure given by the difference between actual output and the level of output that would obtain in a world without nominal rigidities. As we mentioned earlier, this measure of the output gap is not readily observable and thus does not satisfy our requirement of operationality.

6.4   Optimal Macroeconomic Adjustment

Figure 5 compares the dynamic response of the model economy induced by a 1-percent technology shock under the optimal operational rule (solid line) to that arising under a simple Taylor rule (broken line). The response of the real side of the economy is remarkably similar under both rules. Under the optimal rule, however, the productivity shock produces smaller though more persistent fluctuations in all real variables. In particular, the real interest rate displays a much smoother response under the optimal rule than under the simple Taylor rule. On the nominal side, both rules call for an immediate easing in monetary policy to

Figure 5: Comparing the Optimal Rule with a Simple Taylor Rule: Response to a Unit Innovation in Technology

_____ Optimal rule   - - - - Simple Taylor rule   -o-o- No nominal frictions
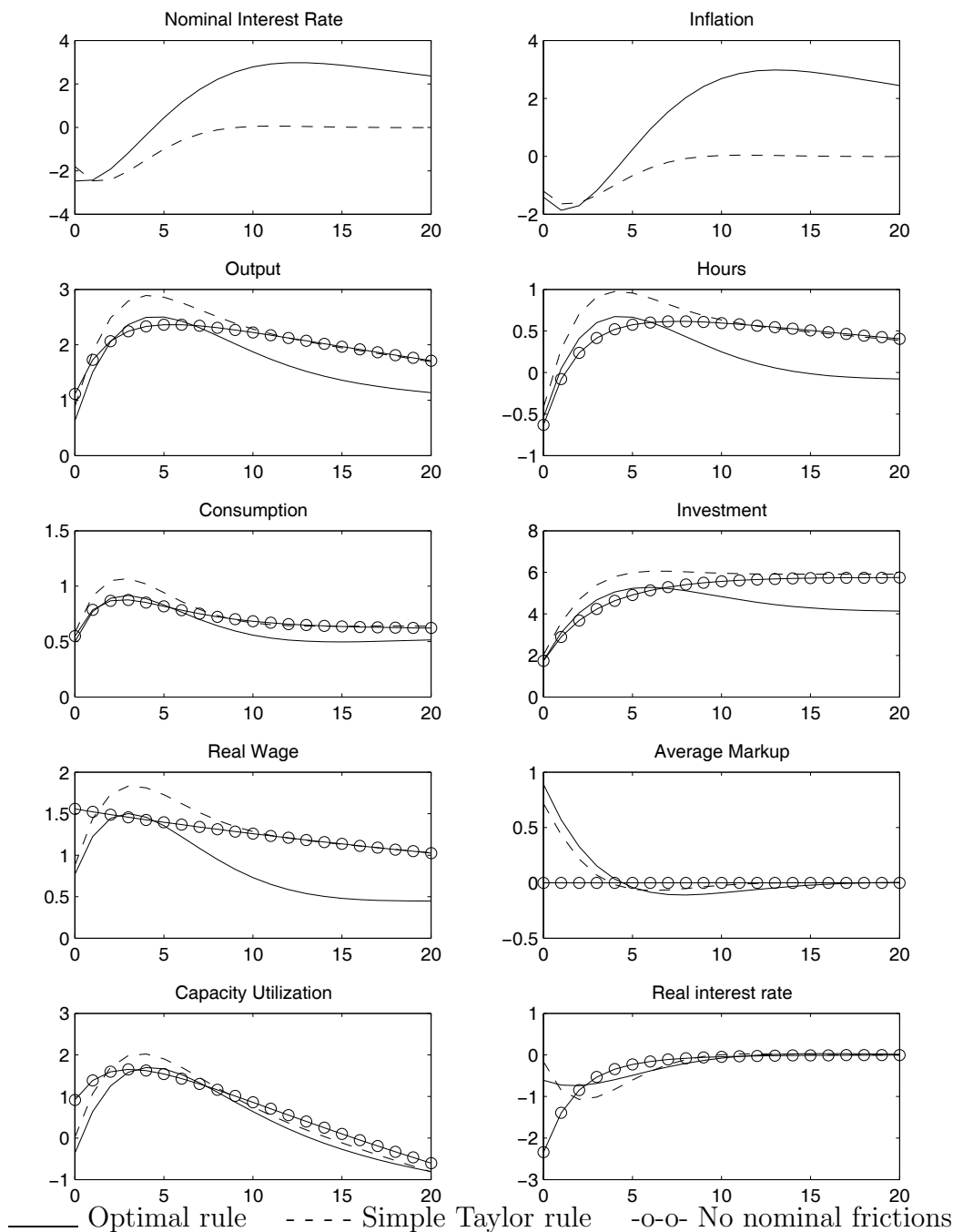
Notes: (1) The optimal rule takes the form $\hat{R}_t = 1.125 E_t \hat{\pi}_{t+1} - 0.0625 E_t \hat{y}_{t+1}$. The simple Taylor rule takes the form $\hat{R}_t = 1.5 \hat{\pi}_t$. The economy without nominal frictions features $\nu = \phi_1 = \alpha = \tilde{\alpha} = 0$. (2) The impulse is a 1 percent increase in the productivity shock z_t. All real variables except for the real interest rate are expressed in percent deviations from their respective steady-state values. The real interest rate, the nominal interest rate, and inflation are expressed in annual percent-point deviations from steady state.

accommodate the expansion caused by the improvement in productivity. However, under the optimal rule the initial reduction in nominal interest rates is accompanied by a commitment to tighten money market conditions in the future. By contrast, under the simple Taylor rule the nominal interest rule remains below target along the entire transition. As a consequence, the two rules induce quite different inflationary dynamics. Under the best operational rule an initial decline in inflation is accompanied by a large future increase in prices, whereas under the simple Taylor rule this overshooting of inflation is absent.

As a point of reference, figure 5 also shows, with a circled line, the dynamics of an economy without any of the 4 nominal frictions that impede efficient adjustment in the baseline model. That is, an economy with flexible prices and wages ($\alpha = \tilde{\alpha} = 0$), and without a demand for money by households or firms ($\phi_1 = \nu = 0$).[11] The optimal operational rule does a remarkably good job at replicating the dynamics of the efficient economy during the initial phase of the transition. On the other hand, for the later part of the transition the Taylor rule keeps the real allocation closer to that associated with the economy without nominal frictions. Our welfare evaluation suggests that the welfare gains of mimicking the initial transition outweigh the benefits of replicating the later part of the adjustment. Recall, however, that the welfare differences between the best operational rule and the simple Taylor rule are minimal.

6.5   Is Inflation Stabilization Desirable?

Much of the existing literature on optimal monetary policy in the neo-Keynesian tradition has stressed the importance of inflation stabilization as a central goal of monetary policy. A general result that emerges from this body of work is that as Woodford (2004) puts it in his recent survey of the inflation targeting literature, "it is not a bad first approximation to say that the goal of monetary policy should be price stability." This conclusion emerges from models that are highly stylized in the sense that they include a small number of nominal and real frictions.

Price stability ceases to dominate the monetary policy prescription in the richer theoretical environment studied in this paper. Specifically, table 3 shows that under the optimal operational rule price inflation is significantly volatile, with a standard deviation of 2 percent per annum, about half the U.S. mean inflation in the past 4 decades. The main force driving the optimality of relatively high inflation volatility is the assumed indexation of wages and prices to the lagged rate of inflation. As shown on line 2 of table 4, when the assumption of indexation to lagged inflation is replaced by the assumption of indexation to long-run

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[11]Because in this economy monetary policy plays no role in the determination of real variables, figure 5 does not show the response of inflation and the nominal interest rate in this economy.

inflation, then the optimal volatility of inflation falls tenfold to 0.2 percent per annum. A reason why the role of the precise nature of the indexation scheme in determining the degree of optimal inflation volatility did not emerge earlier in the related literature is that by focusing exclusively in the case of no indexation and zero long-run inflation, existing studies in effect concentrated in a special case of indexation to long-run inflation. The intuition for why the indexation scheme is related to the optimal degree of inflation volatility is relatively straightforward. Under indexation to lagged inflation, the distorting price dispersion induced by price stickiness is smaller than under indexation to long-run inflation. This is because when firms get the chance to reoptimize prices they need not fully incorporate expected future deviations of inflation from trend in their current markups, as they know that they will be allowed to keep up with aggregate price changes through indexation. On the other hand, when indexation is based on long-run inflation, it is crucial for firms to incorporate in their optimized prices the expected future deviations of inflation from trend. An economy with indexation to lagged inflation is in this sense closer to a flexible-price economy than an economy with indexation to long-run inflation. An obvious implication of our finding is that investigating what assumptions about indexation is empirically most compelling should be a high priority for the applied monetary research agenda.

Previous authors, notably Erceg, Henderson, and Levin (2000) have argued that the optimality of low inflation volatility is not robust to the introduction of sluggish factor price adjustment. Specifically, these authors extend the synthetic new Keynesian model to allow for nominal wage stickiness and find that full price stabilization is suboptimal. We find that this result generalizes to the richer economic environment studied here. Specifically, line 3 of table 4 shows the standard deviation associated with the best operational rule falls from 2 percent to 0.5 percent per annum when wage stickiness is removed.

6.6 The Role of Money

Optimal monetary policy induces a surprising positive relationship between the size of money demand and the optimal degree of interest rate and inflation volatility. In effect, in the baseline economy inflation and the nominal interest rate display a volatility of about 2 percent (see table 3). If one eliminates the demand for money by households by setting $\phi_1 = 0$, the volatilities of inflation and the nominal interest rate drop to 0.8 (see table 4). This effect appears at first as counterintuitive. For one would expect that the larger is the demand for money, the stronger is the incentive for optimal policy to smooth the opportunity cost of holding money, given by the nominal interest rate. This incentive is present in our model economy. But there is also a payoff for increasing interest rate volatility as the importance of

Table 3: Aggregate Volatility Induced by Alternative Monetary Regimes

Optimized Rules $\hat{R}_t = \alpha_\pi E_t \hat{\pi}_{t-i} + \alpha_y E_t \hat{y}_{t-i} + \alpha_R \hat{R}_{t-1}$						
	σ_c	σ_h	σ_{m^h}	σ_y	σ_π	σ_R
Current-Looking Rules ($i = 0$)						
No Inertia ($\alpha_R = 0$)	1.2	0.8	1.1	1.7	0.7	0.8
Inertia ($\alpha_R = 1$)	1.2	1.1	2.2	2.0	1.6	1.6
Super Inertia ($\alpha_R = 2$)	1.2	1.1	1.8	2.0	1.4	1.4
Backward-Looking Rules ($i = 1$)						
No Inertia ($\alpha_R = 0$)	1.2	1.1	2.1	2.0	1.6	1.6
Inertia ($\alpha_R = 1$)	1.2	1.1	1.8	2.0	1.3	1.3
Super Inertial ($\alpha_R = 2$)	1.1	0.6	1.9	1.4	1.5	1.4
Forward-Looking Rules ($i = -1$)						
No Inertia ($\alpha_R = 0$)	1.1	0.6	2.7	1.3	2.0	2.1
Inertia ($\alpha_R = 1$)	1.3	1.2	2.6	2.1	2.0	2.0
Super Inertia ($\alpha_R = 2$)	1.3	1.2	2.7	2.1	2.0	2.0
Simple Taylor Rule						
($\alpha_R = \alpha_y = 0$, $\alpha_\pi = 1.5$)	1.2	0.9	0.9	1.7	0.4	0.6

Note: For any variable x_t, σ_x denotes its unconditional standard deviation. The variables c_t, h_t, m_t^h, and y_t are measured as percentage deviations from their respective deterministic steady-state levels; π_t and R_t are in percent per annum.

Table 4: Optimal Operational Rules: Sensitivity Analysis

Optimized Interest-Rate Rules $\hat{R}_t = \alpha_\pi E_t \hat{\pi}_{t-i} + \alpha_y E_t \hat{y}_{t-i} + \alpha_R \hat{R}_{t-1}$							
	i	α_π	α_y	α_R	σ_R	σ_π	σ_y
Baseline	-1	1.13	-0.06	0	2.1	2.0	1.3
Indexation to Long-Run Inflation	-1	0.19	0	1	0.1	0.2	1.6
Flexible Wages ($\tilde{\alpha} = 0.01$)	-1	3	-0.19	0	0.6	0.5	1.5
Flexible Prices ($\alpha = 0.01$)	1	-0.69	-0.31	2	3.5	3.7	0.9
Higher Capital Utilization Costs ($\gamma_2/\gamma_1 = 2$)	0	1.44	0.25	0	3.4	3.3	1.4
No Habit Formation ($b = 0$)	-1	1.13	0	0	1.5	1.3	1.8
No Money Held by Households ($\phi_1 = 0$)	0	1.06	0	0	0.8	0.7	1.7
No Money At All ($\phi_1 = \nu = 0$)	-1	1.75	0.31	0	2.8	2.8	1.6
One Nominal Friction at a Time							
Sticky Prices Only ($\phi_1 = \nu = \tilde{\alpha} = 0$)	0	3	0	1	0.2	0.0	1.6
Sticky Wages Only ($\phi_1 = \nu = \alpha = 0$)	0	1.5	0.25	0	3.5	3.4	1.6
Household Money Only ($\nu = \alpha = \tilde{\alpha} = 0$)	1	0.44	-0.25	1	3.5	4.0	1.6
Firm Money Only ($\phi_1 = \alpha = \tilde{\alpha} = 0$)	1	-3	-0.25	1	1.0	0.7	1.8

Notes: See tables 2 and 3.

money holdings increases. Specifically, the benefit is that higher interest rate volatility may achieve a lower average nominal interest rate, thus increasing average money holdings, which is welfare increasing, since money held by households enters into the utility function. The policy maker resorts to this cumbersome way of reducing the opportunity cost of holding money because by assumption he must work with an exogenously given target for inflation and the nominal interest rate. Comparing the means and standard deviations of the nominal interest rate in the benchmark economy with those of the economy in which households do not demand money, suggest that the policy maker resolves this mean-volatility tradeoff in favor of lower mean nominal interest rates at the cost of higher interest rate volatility.[12]

6.7 Transitional Dynamics Matter

The existing literature on macroeconomic stabilization says little about the role of initial conditions in shaping optimal policy. Typically, policy-evaluation studies are conducted on the basis of unconditional measures of welfare, which by design 'integrate away' the role of the initial state. We find that transitional dynamics are indeed an important factor

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[12]The unconditional mean of the nominal interest rate in the models with and without money held by households are, respectively, 6.9 and 7.1.

Table 5: Unconditional Means Induced by Alternative Monetary Regimes

Optimized Interest-Rate Rules $\hat{R}_t = \alpha_\pi E_t \hat{\pi}_{t-i} + \alpha_y E_t \hat{y}_{t-i} + \alpha_R \hat{R}_{t-1}$							
	c_t	h_t	m_t^h	y_t	π_t	R_t	V_t
Current-Looking Rules ($i = 0$)							
No Inertia ($\alpha_R = 0$)	0.1025	-0.1018	0.0953	-0.1248	4.1101	7.0657	-156.4310
Inertia ($\alpha_R = 1$)	0.1115	-0.1437	0.0853	-0.1749	4.2541	7.2099	-156.3668
Super Inertia ($\alpha_R = 2$)	0.1092	-0.1365	0.0559	-0.1671	4.2296	7.1852	-156.3789
Backward-Looking Rules ($i = 1$)							
No Inertia ($\alpha_R = 0$)	0.1088	-0.14	0.0658	-0.1702	4.2503	7.2061	-156.3752
Inertia ($\alpha_R = 1$)	0.1084	-0.1344	0.363	-0.1643	4.2237	7.1795	-156.3826
Super Inertial ($\alpha_R = 2$)	0.0759	-0.06	0.3181	-0.0776	4.0372	6.9930	-156.5230
Forward-Looking Rules ($i = -1$)							
No Inertia ($\alpha_R = 0$)	0.0817	-0.0498	0.6323	-0.0635	3.9821	6.9373	-156.5276
Inertia ($\alpha_R = 1$)	0.1155	0.1543	0.1606	-0.1873	4.2871	7.2429	-156.3480
Super Inertia ($\alpha_R = 2$)	0.1157	-0.1554	0.1619	-0.1888	4.2899	7.2457	-156.3464
Simple Taylor Rule							
($\alpha_R = \alpha_y = 0$, $\alpha_\pi = 1.5$)	0.1002	-0.1019	0.1242	-0.1270	4.1124	7.0673	-156.4342

Notes: c_t, h_t, and y_t are measured as percentage deviations from their deterministic steady-state levels; π_t and R_t are in percent per annum; and $V_t \equiv E_t \sum_{j=0}^{\infty} \beta^j U(c_{t+j} - bc_{t-1+j}, m_{t+j}^h, h_{t+j})$ is expressed in levels.

determining the choice of the optimal policy rule. This can be seen by comparing the conditional welfare numbers reported in table 2 with the unconditional expectation of lifetime utility, which are shown in the last column of table 5. Based on an unconditional measure of welfare, the optimal operational rule is in fact the worst of all rules shown in the table. By construction, the unconditional welfare measure is a weighted average of the conditional welfare levels associated with all possible values of the state vector with weights given by their unconditional probabilities. It follows that for some initial conditions the conditional welfare ranking reported in table 2 should be reversed as well. This result is indicative of the fact that the optimal operational rule lacks time consistency. In other words, if the monetary authority was given the chance to reoptimized the parameters defining the interest-rate feedback rule after period 0, it would in general not choose the the policy quadruple that was optimal from the point of view of period 0.

The fact that welfare rankings appears not to be independent of the initial state of the economy suggests that it is important to consider relevant initial conditions. Here we explore the initial condition given by the unconditional expectation of the state vector in an economy in which monetary policy takes the form of a simple Taylor rule ($\hat{R}_t = 1.5\hat{\pi}_t$). The reason

Table 6: Comparing Three Alternative Welfare Measures

Interest-Rate Rule	α_π	α_y	Unconditional Welfare	Conditional Welfare $x_0 = x$	$x_0 = Ex_t^{STR}$
Current-Looking Rules ($i = 0$)					
No Inertia ($\alpha_R = 0$)	1.0625	0	-156.4310	-156.7227	-156.5049
Inertia ($\alpha_R = 1$)	0.625	0.1250	-156.3668	-156.7237	-156.5132
Super Inertia ($\alpha_R = 2$)	0.8125	0.3125	-156.3789	-156.7248	-156.5133
Backward-Looking Rules ($i = 1$)					
No Inertia ($\alpha_R = 0$)	1.3125	0.0625	-156.3752	-156.7233	-156.5124
Inertia ($\alpha_R = 1$)	0.75	0.1250	-156.3826	-156.7243	-156.5126
Super Inertia ($\alpha_R = 2$)	-0.75	-0.0625	-156.5230	-156.7250	-156.5025
Forward-Looking Rules ($i = -1$)					
No Inertia ($\alpha_R = 0$)	1.1250	-0.0625	-156.5276	-156.7220	-156.4940
Inertia ($\alpha_R = 1$)	0.8125	0.1875	-156.3480	-156.7232	-156.5138
Super Inertia ($\alpha_R = 2$)	1.6875	0.6250	-156.3464	-156.7237	-156.5143
Simple Taylor Rule	1.5	$-$	-156.4342	-156.7261	-156.5095

Notes: (1) See notes to table 2. (2) x_t denotes the state vector in period t, x denotes the deterministic steady-state value of x_t, and Ex_t^{STR} denotes the unconditional expectation of x_t when the monetary policy takes the form of a simple Taylor rule (STR) of the form $\hat{R}_t = 1.5\hat{\pi}_t$.

we consider this initial condition is that the policy rule that induces it can be interpreted as a rough characterization of actual U.S. monetary policy post 1980.

Table 6 shows three alternative welfare measures for the 10 operational policies presented in table 2: namely, unconditional expected welfare, expected welfare conditional on the initial state being the deterministic steady state, and expected welfare conditional on the initial state being the mean of the state vector in an economy that follows a simple Taylor rule. An important result conveyed by table 6 is that changing the initial condition of the economy from the deterministic steady state to an arguably more relevant one given by the mean of the distribution of the state vector under a simple Taylor rule does not alter the fact that the real-interest-rate targeting rule $\hat{R}_t = 1.125E_t\hat{\pi}_{t+1} - 0.0625E_t\hat{y}_{t+1}$ welfare dominates all other 9 rules considered in table 2. This result suggests that a monetary authority that has being following a simple Taylor rule in the past could find it beneficial to switch policy to a real interest-rate targeting rule like the optimal operational rule if the economy is in its average state.

6.8   Optimal Policy One Nominal Friction at a Time

It is well known from the work of Goodfriend and King (1997) among many others that in a model where sluggish product price adjustment is the only nominal friction, optimal policy stabilizes inflation completely. That is, the optimal monetary policy calls for setting the variance of π_t equal to zero. It is also well known that one way to implement this policy is by means of an interest-rate feedback rule with a very large inflation coefficient (Woodford, 2001). The existing literature containing this result is based on simple theoretical frameworks. Here we show that the same conclusions hold is the much richer and empirically plausible model studied in this paper. In effect, table 4 shows that if one shuts down all of the nominal rigidities present in our model but price stickiness by setting ϕ_1, ν, and $\tilde{\alpha}$ equal to zero—and thereby engineering a cashless economy with flexible wages—then the best interest-rate rule calls for a short-run inflation coefficient α_π equal to the upper bound of 3 imposed by our grid, and a long-run inflation coefficient, $\alpha_\pi/(1 - \alpha_R)$, of infinity by making the rule inertial. Moreover, the optimal interest-rate rule is welfare dominated by a strict inflation targeting rule whereby the inflation rate is kept at its target level at all times, $\hat{\pi}_t = 0$.

Not surprisingly, in a world where the only nominal friction stems from wage stickiness, inflation stabilization is no longer optimal. Table 4 shows that in this case the optimal level of inflation volatility is 3.5 percent per annum. Also as expected, in the economy with sticky wages only the volatility of wage inflation is smaller than the volatility of price inflation (0.8 versus 3.5 percent per year) whereas in the economy with sticky prices only it is the other way around (0.1 versus 0). What is somewhat surprising is that the presence of wage stickiness does not call for stabilizing wage inflation. In the sticky-wage-only economy, the volatility of wage inflation under the optimal interest-rate rule is 0.8 percent per year whereas in the sticky-price-only economy it is only 0.1 percent per year. Interestingly, in the model we study in this paper, the presence of wage stickiness does not appear to introduce a meaningful inflation-output volatility tradeoff, as found in simpler setups (e.g., Erceg, Henderson, Levin, 2000). Specifically, under the optimized rule the volatility of output is the same, 1.6 percent, in the economies with sticky prices only, sticky wages only, and sticky wages and sticky prices only (i.e., no money).

In the economy where money demand by household represents the only nominal friction, optimal monetary policy is geared toward reducing the average opportunity cost of holding money to increase mean real money holdings, which enter the welfare index directly. Specifically, the mean of the nominal interest rate is 6.8 percent, compared to 7.2 in the economy without money. As a result, real balances in the economy with money by households only are on average 2.4 percent above their steady-state level. The reduction in the average nominal

interest rate is achieved at the costs of relatively high interest-rate volatility. As mentioned earlier, this awkward taste for nominal interest-rate volatility on the part of the central bank is induced by the exogeneity of the inflation target. If the inflation target was endogenous, the monetary authority should choose an inflation level in line with the Friedman rule, and, in the case of money being the only nominal friction, no interest-rate volatility. A different pattern emerges when the only nominal friction in the economy is a demand for money by firms. In this case, the mean-volatility tradeoff for the nominal interest rate is resolved in favor of interest-rate stability.

7 Conclusion

The contribution of this paper is to perform second-order accurate welfare evaluations of operational interest rate feedback rules in a large-scale, empirically compelling model of the U.S. business cycle. The stabilization policy problem in this economy is complicated by the presence of a multitude of nominal and real frictions including sticky wages and prices, a demand for money by households and firms, monopolistic competition, investment adjustment costs, habit formation, and variable costs of adjusting the degree of capacity utilization.

The key characteristic of the optimal operational rule is that it stabilizes the real interest rate. The presence of nominal rigidities play a significant role in shaping optimal monetary policy. This is reflected in the fact that when either one of the nominal rigidities is eliminated or when the price or wage indexation scheme is altered, then the form of the optimal operational rule changes significantly.

The optimal operational rule induces an equilibrium with a non-negligible degree of inflation volatility. This implication of optimal policy is sensitive to the particular indexation scheme assumed for prices and wages. In particular, when the baseline assumption of indexation to lagged price inflation is replaced by the assumption of indexation to the long-run level of inflation, then inflation volatility under the optimal operational rule drops significantly.

This paper can be extended in a variety of ways. One is to investigate how optimal policy changes as the sources of uncertainty driving business cycles is expanded. One alternative is to incorporate the rich set of shocks identified in econometric estimations of the model considered in this paper (e.g., Smets and Wouters, 2004).

The class of operational rules studied in this paper is clearly not exhaustive. It would be of interest to investigate whether the inclusion of macroeconomic indicators other than those considered here would improve the policy makers ability to stabilize the economy. In particular, the related literature has emphasized the use of measures of the output gap that

are different from that used in the present study. Additionally, it has been argued that in models with nominal wage and price rigidities the optimal policy should target an average of wage and price inflation as opposed to only price inflation, which is the case analyzed in this paper.

The optimal policy problem analyzed in this paper takes the central bank's inflation target as exogenously given. A natural extension is to endogenize this variable. However, in the theoretical framework studied here, the optimal inflation target is the one associated with the Friedman rule. This is because the assumption of full indexation to past inflation implies the absence of inefficient price and wage dispersion in the long run. Thus the only remaining nominal friction is the demand for money by households and firms. The elimination of these frictions calls for driving the opportunity costs of holding money to zero in the long run. In other words, the zero bound on nominal interest rate binds in the non-stochastic steady state. Unfortunately, the perturbation technique employed in this paper is ill suited to handle this case. Therefore, analyzing the case of an endogenous inflation target entails either changing the model so that the Friedman rule is no longer optimal in the long-run or adopting alternative numerical techniques for computing welfare accurately up to second-order or higher.

A finding of this paper is that the initial state of the economy plays a role in determining the parameters defining the optimal interest-rate rule. This finding suggests that the optimal operational rule identified here is time inconsistent. Throughout the paper we assume that the government is able to commit to the policy announcements made at time 0. It would be of interest to compute optimal operational rules in an environment without commitment.

Finally, the current study limits attention to the special case of passive fiscal policy, taking the form of a balanced-budget rule with lump-sum taxation. It is well known that the set of operational monetary rules depends on the stance of fiscal policy. For instance, the determinacy properties of the rational expectations equilibrium associated with a particular monetary rule can change as fiscal policy is altered. Therefore, it would be of interest to introduce operational fiscal rules as an additional policy instrument.

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