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MODELING DEVIATIONS FROM PURCHASING POWER PARITY (PPP)

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ABSTRACT

The volatility of the exchange rate under floating rates can be interpreted in terms of approaches that allow for short term price rigidity as well as in terms of models that consider the magnification effect of new information. This paper combines the two approaches into a unified framework, where the degree to which prices are rigid is determined endogenously. It is shown that the variance of percentage deviations from ppp has an upper bound, and that the relationship between the variance of deviations from ppp and the aggregate variability is not monotonic. Allowing for a short-run Phillips curve with optimal indexation, it is also demonstrated that a higher price flexibility will reduce deviations from ppp and output volatility.

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I. Introduction

Recent experience with floating exchange rates has renewed interest in the doctrine of purchasing power parity (referred to as ppp henceforth). This doctrine hypothesizes that adjustment of the exchange rate should follow national inflation differentials closely if the law of one price is to hold.[1] To the degree that ppp holds, it provides a link between goods prices in different countries which, together with interest rate parity, enables us to understand the behavior of floating exchange rates by means of elegant modeling. Recently, however, a growing number of studies have provided ample evidence of deviations from ppp.[2] These findings cast doubt on the ability of models that rely on the doctrine of ppp to provide a satisfactory explanation for the behavior of exchange rates. As a partial solution, recent studies have added extra shock, which describes deviations from ppp generated by a random process. The purpose of this paper is to model deviations from ppp in a way that is more satisfactory and is consistent with the asset approach to exchange rate determination , as well as with sluggish adjustment in the goods market.

There are two different interpretations for deviations from ppp: first, as deviations from the law of one price; next, as an index number problem. In other words, even if the law of one price holds for each good, the difference in consumption basket across countries implies that changes in relative prices will result in deviations from ppp. Clearly, the first interpretation causes more headaches in explaining the international transmission. This paper will concentrate on this interpretation and, in order to simplify, will assume a one-good world. Hence ppp and the law of one price are synonyms.

In a recent paper Flood (1981) demonstrated that the volatility of the exchange rate can be interpreted in terms of models that allow for short term price rigidity (Dornbusch (1976)) as well as in terms of models that consider the magnification effect of new information (Frenkel (1976) and Mussa (1976)). This paper combines the two approaches into a unified framework, where the degree to which prices are rigid is determined endogenously. The benefit of such an approach is in deriving the dependence of the variance of deviations from ppp on the underlying structure of the economy. It is shown that the variance of percentage deviations from ppp has an upper bound, and that the relationship between the variance of deviation from ppp and the aggregate variability is not necessarily monotonic. These results seem to be applicable for deviations from parity conditions between any two related markets and not only for the problem of ppp. The analysis also demonstrates that deviations from ppp and volatility of exchange rates are closely related phenomena. They are explained by combining the two alternative approaches analyzed by Flood; thus, both approaches should be viewed as complementary and as reinforcing each other. Allowing for a short-run Phillips curve, it is also demonstrated that with optimal indexation a higher price flexibilty will reduce output volatility.

A study by Mussa (1981) suggests that the frequency of price adjustment should be related negatively to the cost of price changes. Comparing the price behavior of assets traded in a well organized market to the price behavior of goods, it is safe to claim that the cost of price adjustment for the former is lower than for the latter. The exchange rate functions simultaneously as the price of an asset (foreign currency) and as a component of the price of traded goods. The two prices are different because the behavior of the spot exchange rate resembles that of assets prices, whereas the behavior of the price of traded goods resembles that of goods prices. To capture this situation, let us model it in a discrete time framework where first we compare two polar cases. One is the situation where goods' prices are fully flexible; the other is where goods' prices are pre-set due to overly high transaction costs of last-minute goods price revisions.

The next step is to modify the framework, recognizing that there is limited price flexibility, which depends on the cost benefit of potential goods arbitrage.[3] This enables us to derive the more general case, where the nature of the deviations from ppp is closely related to the cost structure. Solving this case provides us with a measure of the variance of deviations from ppp, which in turn enables us to find boundary estimates for deviations from ppp, and to derive comparative statics results that seem to be in agreement with observed empirical regularities.

Section II describes the solution for the two polar cases of full price flexibility and pre-set prices. Section III generalizes the framework for the case of limited price flexibility due to transaction costs. Section IV adds the possibility of a short-run Phillips curve as a means of analyzing the effects of deviations from ppp on output fluctuations. Section V summarizes the findings. The appendix describes the solution for a specialized stochastic system.

II. Flexible versus Pre-Set Prices

The suggested treatment of deviations from ppp that is described in Section III can be applied to a variety of different models that use some version of ppp. To demonstrate, let us consider one possible approach to modeling exchange rates, by analyzing how the suggested methodology for treating deviations from ppp modifies the results. It should be kept in mind, however, that the suggested treatment is not specific to the approach considered subsequently.

Consider the case of a small, open economy under perfect capital mobility, in a world of one traded good. Suppose that the money market equilibrium is given in a log-linear form:

$$m_t - p_t = y_t - \alpha(i_t^* + E_t\, e_{t+1} - e_t) \qquad (1)$$

where m_t is the money supply at time t; p_t the price of the traded good; y_t real output; e_t the spot exchange rate at time t (the domestic price of foreign currency); and i_t^* the foreign interest rate. All variables, except i_t^*, are expressed in logarithms, and E_t is the conditional expectation operator, based on time t's information. Equation 1 embodies the interest rate parity condition, assuming the absence of a risk premium.

This paper focuses on the nature of the dependence of current prices (p_t) on the exchange rate (e_t). One polar case occurs when the price of the traded good is pre-set before period t. This occurs when last-minute price revision are extremely costly. In such a case ppp does not hold -- an outcome which corresponds to the situation where transaction costs [4] of goods arbitrage and price adjustment are high enough to prevent potential goods inflow (or outflow) that will equalize goods' prices within period t. To describe this possibility, suppose that the price p_t is set at the end of period t - 1 at its expected p.p.p. level, i.e.

$$p_t = E_{t-1}(e_t + p_t^*) \qquad (2)$$

For this case, eq. 1 can be simplified:

$$B_t = E_{t-1}\, e_t - \alpha E_t\, e_{t+1} + \alpha e_t \qquad (3)$$

where $B_t = m_t + \alpha i_t^* - y_t - E_{t-1}(p_t^*)$.

Let us assume that the current information set includes knowledge of the value of all current shocks, as well as of the structure of the model. Because the exchange rate is flexible, it adjusts so as to ensure money market equilibrium. Forward iteration provides us with the rational expectations solution for e_t [5]

$$e_t = \frac{1}{1+\alpha} \sum_{k=o}^{\infty} E_t\, B_{t+k}\left(\frac{\alpha}{1+\alpha}\right)^k + \frac{1}{\alpha}\left[\frac{1}{1+\alpha} \sum_{k=0}^{\infty}(E_t\, B_{t+k} - E_{t-1}\, B_{t+k})\left(\frac{\alpha}{1+\alpha}\right)^k\right] \qquad (4)$$

If ppp holds within each period, goods prices are flexible within each period, and $p_t = e_t + p_t^*$. This happens when transaction costs in the goods market are negligible, which is the other polar case. Let us denote the exchange rate and the good's price corresponding to this case by $\bar{e}_t$ and $\bar{p}_t$. Consequently, (see Mussa 1976):

$$\bar{e}_t = \frac{1}{1+\alpha}\left[\sum_{k=0}^{\infty} E_t\, B_{t+k}\left(\frac{\alpha}{1+\alpha}\right)^k\right] - \frac{p_t^* - E_{t-1}\, p_t^*}{1+\alpha}\,. \qquad (5)$$

Under flexible prices, $\bar{e}_t$ is a weighted average of expected B_{t+k}, minus a term corresponding to the unexpected change in foreign prices at time t. In the case of pre-set prices, e_t is equal to a weighted average of expected B_{t+k}, adjusted by $\frac{1}{\alpha}$ times the innovation in this weighted average. Notice that for both cases the expected value of the future exchange rate is the same:

$$E_t \bar{e}_{t+1} = E_t e_{t+1} \tag{6}$$

With the help of eq. 4 and 5 we get

$$e_t = \bar{e}_t + \frac{1}{\alpha}[\bar{e}_t + p_t^* - E_{t-1}(\bar{e}_t + p_t^*)] \tag{5'}$$

The expression in the bracket corresponds to the good's price adjustment to unexpected shocks in a flexible prices regime. When prices are pre-set, they cannot adjust to unexpected shocks. As a result, the interest rate (and the spot exchange rate) absorb the needed adjustment by depreciation of $\frac{1}{\alpha}[\bar{p}_t - E_{t-1}\ \bar{p}_t]$ above $\bar{e}_t$. Thus, good's price adjustment is substituted for interest rate adjustment at a rate of $\frac{1}{\alpha}$.

Equation 5 can also be presented as:

$$e_t - E_{t-1}e_t = \bar{e}_t - E_{t-1}\bar{e}_t + \frac{1}{\alpha}[\bar{p}_t - E_{t-1}\bar{p}_t] \tag{7}$$

The effect of pre-set prices is to magnify the unexpected depreciation by a factor of $\frac{1}{\alpha}$ the unexpected price adjustment under a flexible prices regime. Let us define by θ_t the percentage deviation from ppp under a pre-set prices regime. From eq. 7 we get:

$$\theta_t = e_t + p_t^* - p_t = (1 + \frac{1}{\alpha})\ [\bar{p}_t - E_{t-1}\bar{p}_t] \tag{8}$$

The effect of pre-set prices is to substitute price adjustment under a flexible prices regime with a deviation from ppp due to exchange rate adjustment, at a rate of $1 + \frac{1}{\alpha}$.

III. The case of limited price flexibility

The two polar cases analyzed in Section II correspond to the limiting case. In general, good arbitrage and price adjustment will occur if their benefits exceed the costs. To model this case, suppose that the real transaction costs associated with price adjustment (per unit of output) are given by C. C is a measure of the degree of goods' prices flexibility. The lower C is, the greater the degree of price flexibility, and C = o corresponds to the fully flexible prices case. We presume that the cost of making a price change (which is taken to be of the lump sum variety) must be balanced against the benefit of potential goods arbitrage (which is taken to depend on the extent of deviations from ppp).

Prices are then given by

$$p_t' = \begin{cases} E_{t-1}(e_t + p_t^*) & \text{if} \quad |\theta_t| < C \\ \bar{p}_t & \text{if} \quad |\theta_t| > C \end{cases} \tag{9}$$

The good's price is pre-set at the end of period t-1 at its expected level. If in period t the deviation from ppp that results from this pricing rule is below the threshold limit C, the price stays at its pre-set level. If, however, it exceeds the threshold C, the price differential is enough to compensate for the transaction cost, and arbitrage of the good will occur. To simplify, it is assumed that in this situation the arbitrage is forceful enough to equate the good's price with the international price, such that ppp holds. In the first case the price at time t corresponds to the pre-set condition; in the second to the flexible condition. In the first case the exchange rate at period t is e_t ; in the second it is $\bar{e}_t$.[6]

If Δ_t denotes deviation from ppp for the case analyzed in this section, we get:

$$\Delta_t = \begin{cases} \theta_t & \text{if} \quad |\theta_t| < C \\ 0 & \text{if} \quad |\theta_t| > C \end{cases} \qquad (10)$$

Suppose that θ_t follows normal distribution, $\theta_t \sim N(0, \sigma_\theta^2)$. (See Appendix A for an analysis of possible determinants of θ's distribution). Thus, Δ_t follows a truncated normal distribution. To study its nature, let us denote by Z the normalized value of C, $Z = \frac{C}{\sigma_\theta}$; and by $\Phi(Z)$ and $\phi(Z)$ the standard normal cumulative distribution and density function. Let us denote by V_x the variance of x. Solving for V_Δ we get that[7]

$$V_\Delta = V_\theta \cdot H(Z), \text{ where } H(Z) = 1 - 2\Phi(-Z) - 2\cdot Z\cdot\phi(Z) \qquad (11)$$

The variance of deviations from ppp corresponds to portion H(Z) of V_θ . H(Z) has a simple interpretation as the shaded area in Figure I. The variance of deviations from ppp under pre-set prices, V_θ , can be interpreted as a measure of relative price pressure, or as a possible measure of the variability of the various shocks affecting the economy (See Appendix A for a description of possible components of V_θ).

An alternative formulation for V_θ is

$$V_\Delta = C^2 \cdot H(Z)/Z^2 \qquad (12)$$

from which we derive that

$$\text{sign} \frac{\partial V_\Delta}{\partial V_\theta} = \text{sign} \left(H(Z) - Z^2(- \frac{d\phi}{dZ})\right) \qquad (13)$$

Higher variability of relative pressure has two opposing effects: First, it increases the probability of potential profitable arbitrage ($|\theta|>C$) , which tends to reduce deviations from ppp. Second, it increases the variability of

θ, including its variability when arbitrage does not occur. In terms of eq. 11, the first effect reduces H(Z), the second increases V_θ. To gain further insight, let us analyze eq. 13. $Z^2 \cdot (-\frac{d\phi}{dZ})$ has a simple interpretation as the area of triangle ABC in Figure 1. Thus, the sign of $\frac{\partial V_\Delta}{\partial V_\theta}$ depends on the difference between the shaded area and the triangle area in Figure 1. It is negative for small Z (like Z_1 in Figure 2) and positive for large enough Z (like Z_2 in Figure 2). By means of a numerical solution, we find that for $Z \simeq 1.36$ $\frac{\partial V_\Delta}{\partial V_\theta} = 0$, and that

$$\frac{\partial V_\Delta}{\partial V_\theta} \begin{array}{l} > 0 \\ < 0 \end{array} \quad \text{for } \sigma_\theta \begin{array}{l} < C/1.36 \\ > C/1.36 \end{array} \tag{14}$$

Starting from low relative variability ($\sigma_\theta \simeq 0$), an increase in σ_θ will increase deviations from ppp. Further increase in σ_θ will further increase deviations from ppp until σ_Δ reaches its upper limit. This occurs when $\sigma_\theta = C/1.36$. Above this point any further increase in σ_θ will reduce deviations from ppp. This is because an increase in σ_θ shifts the probability weight to the tails of the distribution. Thus, when deviations from ppp are large enough, they will induce forceful arbitrage, which tends to reduce the actual deviations. As can be seen from eq. 12, the upper boundary for the standard deviation from ppp is σ_Δ^*:

$$\sigma_\Delta^* = C \cdot \text{ Max } \sqrt{\frac{H(Z)}{Z^2}} = C \cdot 0.46 \qquad (\text{where } 0.46 = \frac{\sqrt{H(1.36)}}{1.36}) \tag{15}$$

and it is attained for $\sigma_\theta^* = C \cdot 0.735$.

The effect of lower C is to reduce deviations from ppp (for a given σ_θ), because it will increase the region where potential deviations from ppp induce arbitrage. In terms of Figure 1, it shifts Z_0 towards the origin reducing

H(Z). This is in agreement with the economic insight that deviations from the law of one price should be smaller in a well organized market, because in those markets C is lower. Polar examples for this might be the market for foreign exchange, where C is small, versus the market for non-traded goods, where C is large enough to nullify the law of one price.

IV. ppp and Output Fluctuations

The analysis has so far neglected the possibility of a short-run Phillips curve. In this section the previous analysis of deviations from ppp is integrated with a discussion of a frictional labor market, where unanticipated shocks result in an output response based on contracting elements. The analysis considers how deviations from ppp affect output variability. This is done by adding to the money market equilibrium condition (eq. 1) an aggregate supply function, which is a modified version of models used by Flood and Marion (1982) and Marston (1982). These authors consider the case of an economy where nominal wage contracts are negotiated in period t-1, before current prices are known, so as to equate expected labor demand to expected labor supply. But actual employment in period t is demand-determined, and depends on the realized real wage. These models also allow for partial indexation, which may be set according to some optimizing criteria, and assume built in asymmetry, where wages might be pre-set but goods' prices are flexible. The new aspect of the subsequent analysis is in allowing for a symmetric framework, where prices can exhibit short-run rigidity. The degree of this rigidity is endogenously determined, and is manifested in the deviations from ppp.

The model can be summarized by:

$$m_t - p_t = y_t - \alpha(i_t^* + E_t e_{t+1} - e_t) \tag{16}$$

$$y_t = d_o + d_1 \nu_t + d_2(p_t - E_{t-1} p_t) \tag{17}$$

Eq. 16 is the familiar money market equilibrium (eq.1). Eq. 17 describes the short-run Phillips curve where ν is a white noise supply shock. It can be justified in the following way.[8] Suppose that the labor supply is given by

$$l_t^s = \delta(w_t - p_t) \tag{18}$$

where w_t is (the logarithm of) money wage. The production function is given by

$$y_t = hl_t + \nu_t \ , \ 0 < h < 1 \tag{19}$$

where ν_t is a supply shock, and l_t the labor employed. The money wage in period t is given by

$$w_t = E_{t-1} w_t^* + b(p_t - E_{t-1} p_t). \tag{20}$$

$E_{t-1} w_t^*$ is the contract wage, which is equal to the expected money wage in a fully flexible regime (it is also equal to the wage that clears the labor market in the absence of uncertainty). Actual wage (w_t) is allowed to adjust in proportion b to the cost of living increase (b is the degree of wage indexation).

Assuming that in the short-run employment is demand determined, we get that

$$y_t = \frac{\nu_t}{1-h} + \frac{h}{1-h} \log h + \frac{h}{1-h} (1-b) \; (p_t - E_{t-1}p_t) - \frac{h}{1-h} E_{t-1}(w_t - p_t) \tag{21}$$

$$\tilde{y}_t = \nu_t \cdot \frac{1 + \delta}{1+(1-h)\delta} + E_{t-1}(w_t - p_t) \cdot h\delta \tag{22}$$

$$E_{t-1} \; w_t = \frac{\log h}{1+(1-h)\delta} + E_{t-1} \; p_t \; . \tag{23}$$

where $\tilde{y}_t$ is the full information, frictionless output that will result in a fully flexible economy.

Thus

$$d_o = \frac{h}{1-h} (\log h - E_{t-1}(w_t - p_t)); \; d_1 = \frac{1}{1-h} \; ; \; d_2 = \frac{h}{1-h} (1-b).$$

To concentrate on essentials, let us assume a simple stochastic framework, neglecting trends in the variables and assuming zero correlation between the random shocks:

$$m_t = \bar{m} + u_t \; ; \; p_t^* \sim N(0, \sigma_{p*}^2); \; i_t^* \sim N(0, \sigma_{i*}^2); \; u_t \sim N(0, \sigma_u^2) \tag{24}$$

$\nu_t \sim N(0,\sigma_\nu^2)$.

To simplify further, assume that the choice of units is such that $E_{t-1} \, e_t = 0 = E_{t-1} \, p_t$. Using the logic of the discussion in Section III, let us find the price behavior in the two polar cases. If prices of the goods are pre-set, $p_t = E_{t-1} \, p_t$, we get that:

$$y_t = d_o + d_1\nu_t$$

$$e_t = \frac{u_t - d_1\ \nu_t + \alpha i_t^*}{\alpha} \tag{25}$$

If prices are flexible we get:

$$y_t = d_o + d_1\nu_t + d_2(p_t^* + \bar{e}_t)$$

$$\bar{e}_t = \frac{u_t - d_1\nu_t - d_2 \cdot p_t^* - p_t^* + \alpha i_t^*}{\alpha + d_2 + 1} \tag{26}$$

$$\bar{p}_t = \frac{u_t + \alpha(i_t^* + p_t^*) - d_1\ \nu_t}{\alpha + d_2 + 1}$$

In the case of pre-set prices we find that deviations from ppp are given by

$$\theta_t = e_t + p_t^* - p_t = \frac{u_t - d_1\nu_t + \alpha(i_t^* + p_t^*)}{\alpha} \tag{27}$$

Notice that, similar to our results in Section II, we find that

$$e_t - E_{t-1}\ e_t = \bar{e}_t - E_{t-1}\ \bar{e}_t + \frac{d_2 + 1}{\alpha}(\bar{p}_t - E_{t-1}\ \bar{p}_t) \tag{7'}$$

$$\theta_t = (\bar{p}_t - E_{t-1}\ \bar{p}_t)\ (1 + \frac{d_2 + 1}{\alpha}) \tag{8'}$$

The effect of pre-set prices is to magnify the unexpected depreciation at a rate of $\frac{1 + d_2}{\alpha}$ times the unexpected price adjustment under a flexible price regime. As eq. 8' shows, the effect of pre-set prices is also to substitute price adjustment under a flexible prices regime with a deviation from ppp due to exchange rate adjustment, at a rate of $\frac{d_2 + 1}{\alpha}$.

Using the argument of Section III, for a given transaction costs associated with price adjustment (C) we get that if $|\theta_t| < C$ goods' prices at t are pre-set, and if $|\theta_t| > C$ they are flexible, adjusting to ensure ppp. Let us denote by p_t' actual prices, i.e.

$$p_t' = \begin{cases} 0 & \text{if } |\theta_t| < C \\ \bar{p}_t & \text{if } |\theta_t| > C \end{cases}$$

To study the effects of deviations from ppp on the relative stability of the economy, consider the following loss function:[9]

$$L = E(y_t - \tilde{y}_t)^2 \tag{28}$$

With the help of eq. 21-22 we get

$$y_t - \tilde{y}_t = d_2 p_t' + g\nu_t \tag{29}$$

where $g = \frac{h}{(1-h)\,(1 + (1-h)\delta)}$.

Using the properties of truncated multinormal distributions we get[10]

$$L = g^2 V_\nu + V_{p'}[(d_2)^2 + 2d_2 \cdot g \cdot \rho_{\nu,\bar{p}} \cdot \sigma_\nu/\sigma_{\bar{p}}] \tag{30}$$

where

$$V_{p'} = \left(\frac{\alpha}{\alpha + d_2 + 1}\right)^2 \cdot V_\theta[1 - H(z)] \tag{31}$$

$$\rho_{\nu,\bar{p}} = -\sqrt{\frac{V_\nu}{\frac{1}{(d_1^2)} V[u + \alpha(i^* + p^*)] + V_\nu}} \tag{32}$$

and H(z) was defined in eq. 11.

From the loss function (eq. 30) we derive the optimal degree of wage indexation $(\hat{b})$, given by[11]:

$$\hat{b} = 1 - \frac{1-h}{h} \cdot \frac{(\alpha+1) \cdot g \cdot (-\rho_{\nu,\bar{p}} \cdot \sigma_\nu/\sigma_{\bar{p}})}{[\alpha + 1 + g \cdot (-\rho_{\nu,\bar{p}} \cdot \sigma_\nu/\sigma_{\bar{p}})]} \qquad (33)$$

Notice that it is independent from the degree of deviations from ppp. The value of the loss function corresponding to $\hat{b}$ is:

$$\hat{L} = g^2[V_\nu - (1 - H(Z)) \cdot \rho^2_{\nu,\bar{p}} \cdot V_\theta \cdot a] \qquad (34)$$

$$\text{where} \quad a = \frac{\alpha^2 \cdot V_\nu/V_{\bar{p}}}{(g \cdot (-\rho_{\nu,\bar{p}}) \cdot \sigma_\nu/\sigma_{\bar{p}} + \alpha+1) \cdot (\alpha+1 + (1-\hat{b})\, h \,/\, (1-h))} \qquad (35)$$

Despite the fact that deviations from ppp do not affect the value of optimal wage indexation, they enter the loss function, affecting output volatility (relative to its desired level). In terms of eq. 30, deviations from ppp enter the loss function via their effect on price volatility $(V_{p'})$. A more inflexible goods price structure (dC > 0) will reduce price volatility (because $\frac{\partial H}{\partial C} > 0$). The effect of lower price volatility on output volatility is, however, tricky to assess. For an arbitrary choice of wage indexation, it is indeterminate. Assuming, however, optimal wage indexation (in terms of our loss function) lower price volatility turns out to be undesirable. This is because the optimal wage indexation $(\hat{b})$ takes advantage of the negative correlation between supply shocks and goods prices, by means of incomplete indexation. The extent of the beneficial effect of optimal indexation depends, however, on the internal flexibility of prices. Lower goods price flexibility reduces the importance of the above beneficial effect, implying a larger loss function ($\frac{\partial \hat{L}}{\partial \dot{C}} > 0$) and larger deviations from ppp. Notice that the causality runs in this case from a more inflexible price structure to higher deviations from ppp and a higher output variabililty (relative to desired output); and this argument is conditional on the optimal setting of the degree of wage indexation. Thus, this analysis suggests that the desirability of less volatile domestic prices (d C > 0) depends on the efficiency of the wage indexation scheme.

V. Concluding Remarks and Implications:

The model analyzed in this paper integrates deviations from ppp with an analysis of the determinants of the exchange rate, providing the links between the two. It suggests a number of testable implications that seem to be in agreement with recent empirical studies.

Deviations from ppp are closely related to the total variability in the economy by the structure of transaction costs in the goods market. For a given economy there is an upper boundary for deviations from ppp (as measured by the variance of percentage deviation from ppp). This implies that the link between variability in the economy and deviations from ppp differs between two types of economy. For rather stable economies, increases in the variability of the shocks that affect them tend to increase deviations from ppp. For highly unstable economies, increases in the variability tend to reduce deviations from ppp. For example, starting from a stable period (in relative terms) like the fifties and sixties, increases in variability due to oil shocks and changes in regimes to floating rates will increase deviations from ppp.[12] To the degree that higher inflation rates also come with more variable rates,[13] an increase in the inflation rate results in higher deviations from ppp. At some stage, however, higher inflation comes with lower deviations from ppp. This suggests that in periods of hyperinflation, the ppp doctrine might hold better than in periods of moderate inflation.[14] The doctrine of ppp should also hold better between neighboring countries, and between countries with larger potential trade, because of the lower transaction cost of trade in goods between such countries.

The above discussion was conducted for a simplified model, but its results have broader implications. Most of the discussion in the open macro economy literature is conducted for models where some version of the law of one price holds. This is correct even in models where domestic and foreign goods are imperfect substitutes, as long as the price of each class of goods in different locations is tied by the law of one price. The general question is to what degree the results obtained by those models for the optimal values of various endogenous behavioral parameters are robust to modification of the law of one price. The above discussion demonstrates that deviations from ppp and limited price flexibility can be added to the models in a way that leaves the value of optimal parameters intact, affecting only the volatility of various variables. It is also shown that the desirability of price flexibility depends on the efficiency of the wage indexation scheme. With optimal indexation a higher price flexibility will reduce output volatility.

A natural extention of the above analysis is to consider a multi-goods world with multi-periods overlapping contracts. The addition of these aspects should add continuity to the process of generating deviations from ppp, making the bound conditions for the variance of deviations from ppp dependent on the contracting technology and some average of the transaction costs of different goods.

Appendix A

The purpose of this appendix is to analyze the determinants of the measure of relative variability (V_θ) . This is done by specializing the model analyzed in the paper.

Consider the case where the processes generating m_t, y_t, p_t^* and i_t^* are

(A1) $$y_t = \bar{y} + \beta_t$$

(A2) $$i_t^* = i^*, \; p_t^* = \gamma_t$$

(A3) $$m_t = m_{t-1} - f(\delta)\cdot\varepsilon_{t-1} + f(\delta)\cdot\varepsilon_t + \delta, \text{ where } \frac{\partial f}{\partial \delta} > 0$$

β_t corresponds to a transitory output shock; δ to the rate of money growth, which is equal also to average inflation in a system without real growth; and $f(\delta)\varepsilon_t$ is the transitory increase in money supply. This specification embodies the notion that higher inflation rates are also more variable (See comment 13). ε_t, β_t, δ_t are assumed to be white noise, uncorrelated disturbances. In such a case we get

(A4) $$E_{t-1}e_t = m_{t-1} - \varepsilon_{t-1}\cdot f(\delta) + \delta - y + \alpha(i^* + \delta)$$

(A5) $$\bar{e}_t = E_{t-1}\bar{e}_t + \frac{f(\delta)\varepsilon_t - \beta_t - \gamma_t}{1 + \alpha}$$

(A6) $$\theta_t = (1 + \frac{1}{\alpha})\,(\gamma_t + \frac{f(\delta)\cdot\varepsilon_t - \beta_t - \gamma_t}{1 + \alpha}) = \frac{f(\delta)\varepsilon_t - \beta_t + \gamma_t\cdot\alpha}{\alpha}$$

Thus:

(A7) $$V_\theta = \frac{\alpha^2 \cdot V_\gamma + V_\beta + (f(\delta))^2 V_\varepsilon}{\alpha^2}$$

V_θ corresponds positively to the noise in the system. Anything that increases the variability of the shocks, including higher inflation rates (δ), increases V_θ.[15]

To analyze the effects of unstable rate of growth of the money supply, consider the case where (A3) is replaced with

(A3') $$m_t = m_{t-1} - \varepsilon_{t-1} + \varepsilon_t + \delta_t , \quad \text{where } \delta_t = \delta_{t-1} + w_t$$

ε_t is the transitory increase in the money supply, and δ_t is the rate of money growth, and w_t is a white noise term, affecting δ_t. In such a case we get

(A8) $$\bar{e}_t = m_{t-1} - \varepsilon_{t-1} + \delta_t(1+\alpha) + \alpha i^* - y + \frac{\varepsilon_t - \gamma_t - \beta_t}{1 + \alpha}$$

(A9) $$\theta_t = \frac{1}{\alpha} \left[w_t(1+\alpha)^2 + \varepsilon_t + \alpha \cdot \gamma_t - \beta_t\right]$$

$$V_\theta = (\frac{1}{\alpha} + 2 + \alpha) \quad V_w + \frac{V_\varepsilon + \alpha^2 \cdot V_\gamma + V_\beta}{\alpha^2}$$

Notice that unstable growth of the money supply (high V_w) might be a major component in explaining deviations from ppp, which is the essence of the magnification effect.

Appendix B

The purpose of this appendix is to add transportation costs to the analysis of deviations from ppp under limited price flexibility. In such a case, there are two costs to be considered in the price adjustment rule (eq. 9-10): the lump-sum type of cost of price adjustment (C) and transportation costs, taken to be $\bar{C}$ per unit. The existence of transportation costs differentiates domestic and foreign markets. Consider, for example, the case where $p_t < p_t^* + e_t$ (p_t are prices in the pre-set regime). Because of the existence of transportation costs $(\bar{C})$, the potential gains from goods arbitrage (export in this case) are measured by $(p_t^* + e_t - \bar{C}) - p_t$. Prices will adjust only if this expression exceeds the lump-sum cost of price changes (C). Thus, if $p_t < e_t + p_t^*$ we get

$$\text{(B1)} \quad \Delta_t = \begin{cases} \theta_t \text{ if } (p_t^* + e_t - \bar{C}) - p_t < C \\ \bar{C} \text{ if } (p_t^* + e_t - \bar{C}) - p_t > C \end{cases}$$

(where $\theta_t = p_t^* + e_t - p_t$)

Notice that price adjustment will not generate ppp because of the existence of transportation costs. Using the same argument we find that if $p_t > e_t + p_t^*$

$$\text{(B2)} \quad \Delta_t = \begin{cases} \theta_t \text{ if } p_t^* + e_t + \bar{C} - p_t > - C \\ - \bar{C} \text{ if } p_t^* + e_t + \bar{C} - p_t < - C \end{cases}$$

In general, deviations from ppp are given in such a case by

$$\text{(B3)} \quad \Delta_t = \begin{cases} \theta_t \text{ if } |\theta_t| < C + \bar{C} \\ \bar{C} \text{ if } \theta_t > C + \bar{C} \\ -\bar{C} \text{ if } \theta_t < -C-\bar{C} \end{cases}$$

Denoting by $\bar{Z}$ and Z' the normalized value of $\bar{C} + C$ and $\bar{C}$ (i.e. $\bar{Z} = (\bar{C} + C) / \sigma_\theta$ and $Z' = \bar{C}/\sigma_\theta$) we get that

(B4) $$V_\Delta = V_\theta \cdot \left[H(\bar{Z}) + 2(Z')^2 \cdot \Phi(-\bar{Z})\right]$$

where H is defined in eq. 11. Thus,

(B5) $$\text{sign} \frac{\partial V_\Delta}{\partial V_\theta} = \text{sign} \left[H(\bar{Z}) - \bar{Z}^2 \frac{d\phi(-\bar{Z})}{d\bar{Z}} (1-k^2)\right]$$

where $k = \bar{C} / (C + \bar{C})$.

Notice that the case analyzed in the paper corresponds to the possibility that $\bar{C} = 0$ $(k = 0)$. It can be shown that whenever $C > (\sqrt{3} - 1)\bar{C}$ (or $C > .73\bar{C}$), the results of section III hold. i.e. $\frac{\partial V_\Delta}{\partial V_\theta} > 0$ for small V_θ, and $\frac{\partial V_\Delta}{\partial V_\theta} < 0$ for high enough V_θ. The larger $C / \bar{C}$, the larger the region where $\frac{\partial V_\Delta}{\partial V_\theta} < 0$. If, however, $C < .73\bar{C}$ we find that $\frac{\partial V_\Delta}{\partial V_\theta} > 0$ for all V_θ. Thus, we can conclude that whenever the lump-sum cost element is significant relative to transortation costs, the dependence of V_Δ on V_θ will not be monotonic.

Comments

1. See Frenkel (1982), Kravis and Lipsey (1978), Officer (1976).

2. See Frenkel (1982), Isard (1977), Kravis and Lipsey (1978), Kravis, Heston and Summers (1981).

3. By good arbitrage we refer to those forces that tend to equate the goods' prices with international prices, like potential trade pressure. Those forces can be manifested in price adjustment and (or) in actual trade.

4. Those costs might include information costs and costs of price changes.

5. For convergence we assume that the relevant transversality condition holds. A related version of the two cases analyzed in this section can be found in Mussa (1976, 1982).

6. McCallum (1977) uses a similar pricing rule in the context of a closed economy. In general, we expect to observe more continuity in the arbitrage process, resulting from having a spectrum of goods with different transaction costs (C), and from the fact that the volume of potential arbitrage in each good might correspond in a more continuous manner to price differential (upper sloping arbitrage schedule). The benefit of the simplifying assumption used in this paper is in providing a tractable solution for deviations from ppp. Appendix B considers how the analysis is affected if we add the possibility of transportation costs. We find that whenever the lump-sum cost element is significant the results of Section III are unchanged.

7. To derive eq. 11 we use the fact that

$$(\sqrt{2\pi})^{-1}\int_{Z}^{Z} X^2 \exp(-X^2/2)\, dx = (\sqrt{2\pi})^{-1}\int_{Z}^{Z}\left[\exp(-X^2/2) - \left(X\exp(-X^2/2)\right)'\right] dx$$

$$= 1 - 2\Phi(-Z) - 2Z\phi(Z)$$

8. For further discussion of such a supply function see Marion (1982), Gray (1976) and Fischer (1977).

9. This is also the loss function used by Flood and Marion (1982).

10. We are using the fact that if x_2 is a normal variable, and x_1 is a trancated normal variable $E(x_1\ x_2) = E(x_1\ E(x_2/x_1)) = \rho E(x_1)^2 \cdot \sigma_{x_2}/\sigma_{x_1}$. (Assuming $E(x_2) = 0$).

11. Optimal indexation in a related framework is derived by Flood and Marion (1982).

12. This phenomenon is analyzed in Frenkel (1982).

13. For an analysis of this tendency see Logue and Willett.

14. For evidence of a tendency for ppp to hold in hyperinflation see Frenkel (1976, 1982).

15. Notice that within this framework fixed exchange rate is a consistent solution only if $\delta = 0$. In such a case $V_\theta = V_\gamma$.

Under fixed rate we get lower V_θ because instability in the money market is adjusted via the balance of payment mechanism instead of price adjustment. Thus, we expect to observe lower deviations from ppp under fixed rate (assuming $\sigma_\theta < C/1.36$).

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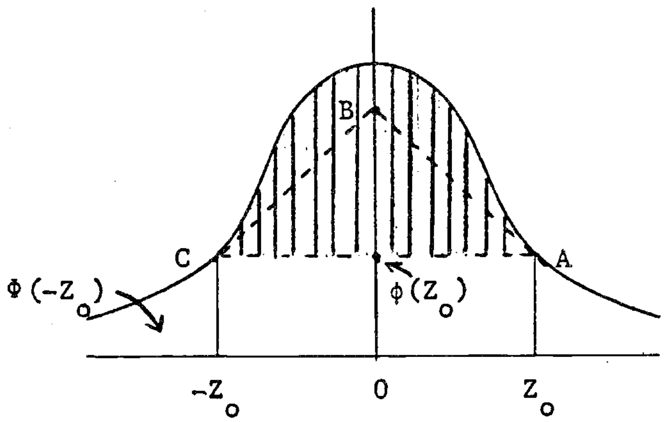

Figure 1

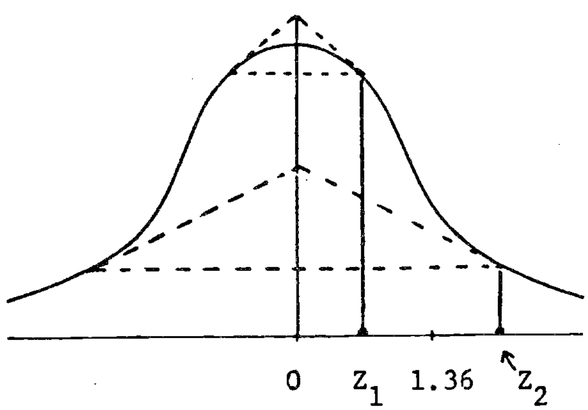

Figure 2