PRODUCTIVITY, TRADABILITY,
AND THE LONG-RUN PRICE PUZZLE

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ABSTRACT

Long-run cross-country price data exhibit a puzzle. Today, richer countries exhibit higher price levels than poorer countries, a stylized fact usually attributed to the "Balassa- Samuelson" effect. But looking back fifty years, or more, this effect virtually disappears from the data. What is often assumed to be a universal property is actually quite specific to recent times. What might explain this historical pattern? We adopt a framework where goods are differentiated by tradability and productivity. A model with monopolistic competition, a continuum-of-goods, and endogenous tradability allows for theory and history to be consistent for a wide range of underlying productivity shocks.

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1. Introduction and Conventional Wisdom

It is conventional wisdom today that richer countries have higher price levels than poorer countries. Figure 1a illustrates this idea, displaying the association of 1995 log price levels and log per capita incomes based on Penn World Table (PWT) data. That this is the consensus view is clear since similar charts appear in most textbook discussions of these phenomena (e.g., Krugman and Obstfeld, 2003, Figure 15.4). While a number of potential explanations have been proposed, the standard story appeals to the Balassa-Samuelson (BS) theory, based on the divergence of productivity levels in a world of traded and nontraded goods. Having languished from time to time, these ideas are now enjoying a renaissance and are being incorporated into many new open-economy macroeconomic models.

Of course, the apparent robustness of this story has proved to be of considerable relevance for many derivative conclusions in the theoretical and empirical literature. From work on sophisticated mathematical models of real exchange rates to the serious applied problem of judging differences in international living standards, the presumed correlation has had important economic and political ramifications. Since many of our PPP-based real income estimates, past and present, often rely on extrapolations from the PWT based on this kind of relationship, and since such estimates are then used for such diverse tasks as evaluating long-run growth performance or allocating foreign aid, it is important that the patterns in the data be judged stable and predictable.

This paper raises some challenges to this comfortable consensus on the sources of covariance in international prices and incomes. The first challenge is empirical. Whilst correlations such as those seen in Figure 1a are indisputably present in today's data, one need only look back into the past to find evidence of weak or zero correlations between national price levels and incomes per capita. After examining postwar data in detail we conclude that the price-income correlation was not really very strong until the last three or four decades. The search to explain this empirical finding leads us to propose a new model of real exchange rates that poses a theoretical challenge to a building block of the traditional Balassa-Samuelson model, the exogenous distinction between traded and nontraded sectors. In particular, while standard Balassa-Samuelson theory must assume that productivity gains are concentrated by coincidence in the existing traded goods sector, our model accounts for how productivity gains in the production of particular goods can in turn lead to those goods becoming traded endogenously. It therefore offers insight into how the trading pattern underlying the Balassa-Samuelson relationship can evolve over time.

The finding that the BS coefficient has emerged gradually in the data has a number of implications. While it reaffirms the potential for macroeconomic fundamentals to play a role in driving the real exchange rate in the long run, it also warns that the relationships modelers need to capture may be subtle and changing over time. And it suggests that empirical work on real exchange rates and deviations from PPP likewise

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should consider allowing for these relationships to change in the time series (Taylor and Taylor 2004).

Our focus on the Balassa-Samuelson theory reflects a recently renewed appreciation of the role nontraded good prices play in helping drive real exchange rates (see Burstein, Eichenbaum and Rebelo, 2005; Betts and Kehoe, 2005). Our theoretical approach to incorporate trade costs to understand real exchange rates is related to recent work by Ghironi and Melitz (2004) and Fitzgerald (2003). In contrast to Fitzgerald, our shipping technology has fixed costs while hers are iceberg only. Instead, her fixed costs appear in production, leading to intra-industry trade and endogenous specialization through a scale-related mechanism *à la* Krugman. Her paper is not directly related to the Balassa-Samuelson channel, which is the focus of our work. Ghironi and Melitz (2004), like us, propose a theory of an "endogenous Balassa-Samuelson effect." However the mechanism at work in their model is very different in spirit from that proposed by Balassa and Samuelson, as it arises out of entry of new firms into domestic production, and it is not dependent upon productivity differences between traded and nontraded goods. In contrast, this paper may be viewed as an update and defense of the original Balassa-Samuelson logic, where we extend the basic theory with some new trade features to understand how and why the empirical effect has emerged over time.

2. Stylized "Facts" and a Simple Model

In theories of the real exchange rate built around tradable and nontradable goods the central stylized fact to be "explained" is the effect noted by many scholars over the years, but highlighted by Bela Balassa and Paul Samuelson in their seminal papers from the year 1964: the tendency for poorer countries to have lower overall price levels than rich countries.[1] Figure 1a shows a scatter plot of log relative price levels versus log relative income per capita for a cross section of 142 countries for the year 1995. We performed an OLS regression on these data, of the form

$$\ln(P_i / P_{US}) = \boldsymbol{a} + \boldsymbol{b}\ln(y_i / y_{US}) + \boldsymbol{e}_i, \tag{1}$$

and the fitted values are shown as a straight line in the figure.[2] We follow standard terminology and use the term *Balassa-Samuelson effect,* or BS effect, as shorthand for a

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[1] In Samuelson's (1994) paper it is so central as to be labeled the "BASIC FACT" (uppercase in the original). In that paper it is also explained, as we shall see, why "explained" might belong in quotes.
[2] Note that in our diagrams the abscissa is measured in PPP-adjusted *real* international dollars and so too is the independent variable in the econometrics that follow. The BS effect could, equivalently, be measured by the slope of a plot of relative prices on the ordinate versus *nominal* incomes converted at exchange rates on the abscissa (e.g., Balassa 1964, Figure 1). In this case, the slope would be slightly different (lower) and equal to $\boldsymbol{b}/(1+\boldsymbol{b})$. This nonlinear transformation of the coefficient, naturally, should make no material difference to the analysis, but in small samples, or with noisy PPP estimates, the effect might be to blur a borderline slope, which might explain the conflicting results of Clague and Tanzi (1972) using actual versus PPP exchange rates. Officer (1982, 216) considers PPP-adjusted exchange rates the correct choice.

positive (and statistically significant) slope estimate b. In our example, the slope is 0.41 with a standard error of 0.04.

The theory that Balassa and Samuelson constructed to explain this phenomenon is also now textbook material, and the simplest version runs as follows.[3] Consider two countries, home and foreign, where foreign variables are denoted with an asterisk (*). Let there be two goods, traded (T) and nontraded (N), produced competitively in each country using only homogeneous labor as an input, with wages W and W^* in each country. Let the labor productivity in each sector be A_T and A_N at home, and A_T^* and A_N^* in the foreign country. Trade is costless for the traded goods, so their prices are equalized in the two countries and this pins down the relative wage levels in the two countries, since $W/A_T = P_T = P_T^* = W^*/A_T^*$. The wage levels, in turn, pin down the nontraded goods prices with $W/A_N = P_N$ and $W^*/A_N^* = P_N^*$. With an arbitrary choice of numéraire, say $P_T^* = 1$, one can easily solve the six equations in the six unknowns for four prices and two wage levels. Now construct a simple price index, say, Cobb-Douglas, where the share of nontraded goods in consumption is q in both countries. Then the relationship between the price levels of the two countries is given by

$$\frac{P}{P^*} = \frac{(P_N)^q (P_T)^{1-q}}{(P_N^*)^q (P_T^*)^{1-q}} = \left(\frac{P_N}{P_N^*}\right)^q = \left(\frac{A_T/A_N}{A_T^*/A_N^*}\right)^q. \tag{2}$$

The relevance of equation (2) for explaining the typical pattern in regression (1) depends on the assumed sources of economic growth. Though rarely tested directly, the conventional auxiliary assumption asserts that differential modern economic growth has been achieved as a result of the rapid productivity advance (in rich countries) of traded goods industries (for example, textiles, manufacturing, hi-tech). In the meantime, the nontraded goods sectors are assumed to have been relatively quiescent, and their productivities to have changed relatively little (the well worn example is the haircut). For notational convenience, let $A_T = ab$, $A_N = a$, $A_T^* = a^*b^*$, $A_N^* = a^*$. The a and a^* terms capture *balanced productivity growth,* which affects both sectors, and the b and b^* terms capture *biased growth* (or "BS growth") that only affects traded goods. Equation (2) can then be rewritten

$$\frac{P}{P^*} = \left(\frac{b}{b^*}\right)^q. \tag{3}$$

This expression is independent of a and a^* because *balanced productivity growth does not affect any relative prices.* We can also find a simple expression for real national income, which is given by real wage income, hence

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[3] A two-factor variant with internationally mobile capital is presented by Froot and Rogoff (1995). The results are similar.

$$\frac{W/P}{W^*/P^*} = \left(\frac{A_T P_T}{A_T^* P_T^*}\right) \Big/ \left(\frac{P_N}{P_N^*}\right)^q = \left(\frac{a}{a^*}\right)\left(\frac{b}{b^*}\right)^{1-q}. \tag{4}$$

Thus, *assuming* fixed a and a^* but varying b and b^*, the BS effect is present and the elasticity of the relative price level with respect to real income would be $q/(1-q)$. This elasticity will be zero when the range of nontraded goods vanishes, but is otherwise positive and can be arbitrarily large when the nontraded share approaches unity.

However, it is also clear from the above that the BS effect is *not* guaranteed to exist. The BS effect will be zero if the sources of growth are spread across the two sectors evenly, as when a/a^* changes but b/b^* does not. And, since $q > 0$, it is only biased technological change favoring the traded sector that can generate the BS effect. And there is even the possibility of an anti-BS effect: if technological change were biased toward nontraded goods (here equivalent to a rise in a and a fall in b such that ab remains constant) then the price level could *fall* as a country got richer.

3. A Puzzle

Returning to the long-run data, Figure 1b shows that in the 1950 PWT data for 53 countries the BS effect was much weaker, with a slope of only 0.08 and a standard error of 0.07. Table 1 confirms that, in a sequence of annual PWT cross sections every fifth year from 1950 to 1995, the BS effect has gradually strengthened, with the slope estimate roughly quadrupling in size over half a century. In their 1964 contribution, Balassa and Samuelson were certainly very timely, if not quite ahead of their time, since the 1960 cross section supports their hypothesis, but from a statistical standpoint there was not much evidence when they wrote, and the overwhelming support in large samples has accumulated ever since.

We have examined the data in other ways to check the robustness of our finding. Running a cross section regression on these data in *every year* produced the slope estimates (and 95% confidence intervals) shown in Figures 2a. Figure 2b shows that the PWT result is not an artifact of changing sample size, since the effect is just as large when we restrict attention to a balanced panel of 53 countries from 1950 to 1998. Thus, even outside the benchmark dates in Table 1, our basic story is reinforced. The PWT data show an almost monotonic upward trend in the slope estimate.

Our evidence hints at a new stylized fact: perhaps the BS effect has *not* been a universal feature of modern economic development. If indeed the BS effect has strengthened markedly in the last 50 years, this could reconcile the mixed findings in a wider literature that has from time to time examined the BS effect in postwar data. Older studies that focused on the 1950s and 1960s tended to find weak evidence, such as Officer (1982).[4] More recent studies have sometimes found stronger results, but,

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[4] Our slope coefficients can be compared with those of Officer (1982, Chapter 16, especially Table 2). He examined the OECD economies in successive annual cross-sections from 1950 to 1973 and found no BS

according to a recent survey, "[o]verall, the empirical evidence on the Harrod-Balassa-Samuelson effect is quite mixed" (Sarno and Taylor 2002, 82).[5]

The data seem to be sending a consistent message. The BS effect has not always been a fact of economic life, and appears to be a phenomenon of only the postwar period, growing in strength steadily over time. This new stylized fact raises interesting questions: Which models can most plausibly explain such an historical pattern in the data? And what do the theory and the empirics tell us about the mechanics of economic development?

4. A More General Model

The simple model presented above potentially could be rigged to produce the actual historical changes seen, essentially by imposing convenient changes in exogenous variables. There are two ways for the BS result to emerge over time in that model. Either the biasedness of productivity growth would have to increase, meaning more growth via b than via a; or, the nontraded share q would have to rise over time to steepen the slope for a given b.

But addressing the puzzle in terms of such purely exogenous changes in the parameters of the model is not a very satisfying explanation, and it offers little to no insight into the underlying causes of the phenomenon or the mechanisms that propagate it. However, this particular shortcoming of the simple BS model can be addressed readily by extending it with recent advances in trade theory.

4.1 Model Motivation

Recent theories in trade offer some interesting suggestions for why the result above might arise endogenously. It has been documented in recent work on trade that there is a good deal of heterogeneity in terms of productivity among firms, even within the same sector. It is only a relatively small number of firms that participate in international trade, and these firms systematically tend to be large and have high levels of productivity. Trade theory has offered models to replicate this pattern, by assuming a continuum of firms with heterogeneous draws from a productivity distribution, who must pay a fixed cost to engage in international trade (Melitz 2003). The result is that only those firms with the highest productivity draws find it profitable to engage in trade.

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effect in (1). The slope was often close to zero in the 1950s and 1960s, though rising to around 0.25 circa 1970. Our results are in broad agreement: we find that most of the BS action has arrived in the years since Officer's analysis ended, although even in the 1960s, a small effect *was* there, but was only detectable in a large sample that included more low income countries. Thus, Officer's weak results might be explained by a small sample (*N*=15 at most). Alternatively, a focus on countries at similar levels of development might rob the regression of variance or introduce countervailing effects (Bhagwati 1984; Fitzgerald 2003).
[5] See, *inter alia*, Hsieh (1982); Marston (1990); Micossi and Milesi-Ferreti (1994); De Gregorio, Giovannini, and Wolf (1994); De Gregorio and Wolf (1994); Chinn and Johnston (1996); Ito, Isard, and Symansky (1999); Chinn (2000); Lothian and Taylor (2004). The newer studies and their estimates are not directly comparable to ours, since they address some of the internal workings of the BS hypothesis, e.g., using manufacturing and/or services employment to attempt to pin down the underlying sectoral productivity shocks.

The purpose of this section is to adapt this logic to help explain the macroeconomic stylized fact of rising relative prices over time. First, this logic offers a clear way to endogenize the BS precondition, that productivity shocks be biased toward traded goods. This coincidence need not be by chance; firms or industries endogenously become traded in direct response to the fact they happen to benefit from a positive sequence of productivity shocks. Further, one might hope that a model where the BS effect arises endogenously would be better equipped to explain why this effect could continue to grow over time. As productivity shocks accumulate over time, they could become a more dominant determinant of what goods are traded, so that the correlation of productivity and tradability rises over time as well.

Is there any empirical motivation for thinking that the set of traded goods can actually change and evolve much over time, as the story above would suggest? Again the trade literature is helpful. Using a panel of U.S. manufacturing plants from 1987 to 1997, Bernard and Jensen (2001) find that there is significant movement of firms between traded and nontraded status on a year-to-year basis: on average 13.9% of non-exporters begin to export in any given year during the sample, and 12.6% of exporters stop. In addition, trade literature has noted that a fair fraction of the growth in trade observed over time has been at the extensive margin, with the entry of goods previously nontraded.[6]

To see if there is any evidence at the macro level for our story, we obtained data on real productivity for U.S. industries at 2, 3, and 4-digit industry levels for 1958 to 1994, and corresponding data on the share each sector represents in U.S. exports. The productivity data were drawn from the NBER-CES manufacturing industry database. Nominal productivity was defined as value added per employee (Vadd divided by emp) based on 1972 SIC codes; the industry shipments price deflator (Piship) was used for the conversion to real terms.[7] The export data were drawn from the NBER database on "U.S. Trade by 1972-SIC Category." When the industries were ranked in terms of productivity and tradability, we found that there was a fairly large increase in the Spearman correlation between productivity ranking and tradability ranking at all levels of disaggregation. At the 2-digit industry level the correlation rose from 0.34 in 1958 to 0.53 in 1994; at the 3-digit level the increase is from 0.29 to 0.47, and at the 4-digit level from 0.31 to 0.40. An increase in correlations is also apparent when we compute "raw" correlations of the actual (log) levels of exports and productivity. See Table 2 for full results.

The U.S. export data above can also be used to see if there was any obvious change in the concentration of trade over time. While there was not a clear pattern at more aggregated levels, there is evidence of increasing concentration at the 4-digit SIC level for U.S. manufactures. Figure 3 shows the number of 4-digit industries (out of a total of 450) needed to account for 50% of U.S. manufacturing exports for each year from 1958 to 1994. (The data are converted into real terms using the shipments price deflator.) There was a decline over the period from 25 industries to 15, mainly in the latter half of

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[6] See Kehoe and Ruhl (2002) for a discussion.
[7] For further information see Bartelsman and Gray (1996). Both databases are at http://www.nber.org/data/.

the time period. This indicates, first, that there is a pretty high degree of concentration of trade in a small share of goods categories, with the majority of trade accounted for by around only 5 percent of industry categories. But it also offers suggestive evidence that this degree of concentration may have risen over time.

This evidence offers some guidance for our theoretical research. It is noteworthy that the basic precondition for the BS theory finds support in the disaggregated data: high productivity levels do seem to be more concentrated in more tradable goods. This indicates that the basic mechanism of the BS theory may be a useful approach to maintain in our current analysis.[8] Further, we find some support for the Balassa-Samuelson-based explanation, as presented in section 2 above, for the finding that the relative price effect has risen over time. If the precondition for the simple BS model to explain this fact is that productivity shocks have become progressively more skewed toward the tradable goods sector, the counterpart for a world of many goods with varying degrees of tradability would be that the correlation between productivity and tradability has increased over time. The model developed below will show one way of incorporating these recent advances in trade theory into a macro context, as a way of extending the BS model.

4.2 Model Specification

The model here differs from the simple model of section 2 in several key respects. It specifies a continuum of goods that differ in terms of their productivity levels, rather than just two sectors with differing productivity levels. Tradability of each good is endogenized as the decision of a firm, which must pay a fixed cost to trade its good. Firms are here monopolistically competitive and pay the fixed cost out of their resulting profits.

In the model there are two countries, home and foreign (the latter denoted *), and each country's output consists of a distinct continuum of differentiated goods (labels H and F will be used here).[9] Each country's goods are indexed on the unit interval, and for each classification of goods there is a continuum of brands.[10] These are produced by monopolistically competitive firms using labor as the sole input in a linear (constant returns to scale) technology. In principle, any good can be exported but there are fixed costs of exporting f_X for any good, which are borne by the exporting firm.[11] Traded

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[8] This contrasts with the theory of Ghironi and Melitz (2004), which does not rely upon the usual BS precondition; instead it generates the relative price effect mainly on the basis of new firm entrants into the domestic market driving up the demand for labor and hence the wage.

[9] Note that by assuming different goods in each country, the LOP basis of PPP is essentially discarded. This model can generalize simply to an *N*-country world with a distinct range of goods in each country. A different approach to the Ricardian model is to allow a common set of goods to be potentially produced in all countries, and then have only the lowest-cost supplier actually export the good to each market, as in the probabilistic model of Eaton and Kortum (2002). An interesting avenue for future research would be to explore the implications of endogenous tradability and the BS effect in that type of setting.

[10] The only reason to refer here to brands of goods is to allow a separate elasticity in the markup decision of the firms from that used by consumers when substituting between home and foreign goods. We wished to avoid implausibly high markups, despite evidence of low substitutability between broader classes of goods.

[11] Bergin and Glick (2003, 2004) consider variable (iceberg) trade costs also. The general intuition developed in this paper would generalize to this case, although the computational difficulty would, in

sector aggregates are labeled *T* and nontraded *N*. Agents preferences are standard CES and apply to home goods and importables. Key assumptions are that each country specializes in a range of goods that are unique to that country (albeit goods from different countries are substitutes) and that the continua of goods are exogenously given.

Details of the model are laid out in the Appendix, but the essential building blocks are as follows. Home and foreign residents consume CES aggregates (C, C^*) of their own goods (C_H, C_F^*) and the other country's traded (export) goods (C_{FT}, C_{HT}^*). Let n and n^* denote the (endogenous) share of nontraded goods in each country, where goods are ordered such that $[0, n]$ and $[0, n^*]$ are the nontraded goods. Then the top-level aggregations are

$$C = \left(\left(q[n^*] \right)^{\frac{1}{f}} \left(C_H \right)^{\frac{f-1}{f}} + \left(1 - q[n^*] \right)^{\frac{1}{f}} \left(C_{FT} \right)^{\frac{f-1}{f}} \right)^{\frac{f}{f-1}}$$

$$C^* = \left(\left(q[n] \right)^{\frac{1}{f}} \left(C_F^* \right)^{\frac{f-1}{f}} + \left(1 - q[n] \right)^{\frac{1}{f}} \left(C_{HT}^* \right)^{\frac{f-1}{f}} \right)^{\frac{f}{f-1}}$$

(5)

where $f > 1$ is the elasticity of substitution between domestic and foreign goods aggregates, and $q[n^*]$ is the domestic residents' own-goods bias coefficient that depends (endogenously) on the number of imported varieties. As n^* rises and fewer varieties are imported by domestic residents, the relative weight in the consumption basket placed on home goods rises and on imported goods falls, i.e., $q' > 0$. The own-bias coefficient for foreign residents, $q[n]$, is interpreted analogously.[12] The indices for domestically produced goods, and their subindices, are in turn defined by

$$\left(C_H \right)^{(f-1)/f} = \int_0^n \left(c_{Hi} \right)^{(f-1)/f} di + \int_n^1 \left(c_{Hi} \right)^{(f-1)/f} di$$

$$= n \left(\frac{C_{HN}}{n} \right)^{(f-1)/f} + (1-n) \left(\frac{C_{HT}}{1-n} \right)^{(f-1)/f}$$

(6)

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general, be greater (for reasons to be explained later). To facilitate intuition, we study the fixed cost case where the greater range of closed form solutions obtain. It suffices to note that the model studied here does include the case of uniform iceberg costs on all traded goods. Problems arise when the iceberg cost varies by good, as noted below.

[12] In the appendix we show formally that the bias weights on own goods $q[n^*]$, $q[n]$ depends on the mass of own goods (normalized to 1 in each country) relative to the total mass of (own plus imported) varieties available for own consumption ($1+1-n^*$ in the domestic country, $1+1-n$ in the foreign country).

$$\left(C_F^*\right)^{(f-1)/f} = \int_0^{n^*} \left(c_{Fi}^*\right)^{(f-1)/f} di + \int_{n^*}^1 \left(c_{Fi}^*\right)^{(f-1)/f} di$$

$$= n^* \left(\frac{C_{FN}^*}{n^*}\right)^{(f-1)/f} + (1-n^*)\left(\frac{C_{FT}^*}{1-n^*}\right)^{(f-1)/f}$$

where the elasticity of substitution among home goods and among foreign goods is also assumed equal to f. At home, the *HN* goods occupy $[0, n]$ and *HT* goods $[n, 1]$; on a separate foreign interval, the *FN* goods occupy $[0, n^*]$ and *FT* goods $[n^*, 1]$. Using duality, and noting symmetry, it is equivalent to proceed using the corresponding price indices for all of the above aggregates: $P, P^*, P_{HN}, P_{HT}, P_{FT}, P_{FN}^*, P_{FT}^*, P_{HT}^*$.

The production side is simple. Each home firm employs l_{Hi} workers and pays them the wage W, and similarly for the foreign firms. On each interval, the output of good i is given by

$$y_{Hi} = A_i l_{Hi}, \qquad\qquad y_{Fi} = A_i^* l_{Fi} \qquad\qquad (7)$$

where A_i, A_i^* are productivity coefficients for each individual good i. Profit maximization by the monopolists leads to the standard cost-markup pricing. In the presence of iceberg shipping costs, pricing would differ across markets; but this is not crucial for intuition and is omitted in the case considered here:[13]

$$p_{Hi} = p_{Hi}^* = \frac{j}{j-1}\frac{W}{A_i}, \quad p_{Fi} = p_{Fi}^* = \frac{j}{j-1}\frac{W^*}{A_i^*} \qquad\qquad (8)$$

where $j/(1-j)$, the markup factor, is assumed to depend on an elasticity of substitution parameter j that differs from that across varieties of goods f.[14]

The crux of the model is the distribution of productivities across goods in each country. Firms in the domestic (foreign) country have a distribution of productivity levels $F[A_i]$ ($F[A_i^*]$). Among these firms, $1-n = F[A_n]$ ($1-n^* = F[A_n^*]$) are exporters. We define weighted productivity averages for home goods $\tilde{A}$, nontraded home goods $\tilde{A}_N$, traded home goods $\tilde{A}_T$, and their foreign analogues

$$\left(\tilde{A}\right)^{f-1} \equiv \int_0^1 A_i^{f-1} di, \qquad \left(\tilde{A}^*\right)^{f-1} \equiv \int_0^1 \left(A_i^*\right)^{f-1} di$$

$$\left(\tilde{A}_N[n]\right)^{f-1} \equiv \frac{1}{n}\int_0^n \left(A_i\right)^{f-1} di, \qquad \left(\tilde{A}_N^*[n^*]\right)^{f-1} \equiv \left(\frac{1}{n^*}\right)\int_0^{n^*} \left(A_i^*\right)^{f-1} di \qquad (9)$$

$$\left(\tilde{A}_T[n]\right)^{f-1} \equiv \left(\frac{1}{1-n}\right)\int_n^1 A_i^{f-1} di, \quad \left(\tilde{A}_T^*[n^*]\right)^{f-1} \equiv \left(\frac{1}{1-n^*}\right)\int_{n^*}^1 \left(A_i^*\right)^{f-1} di$$

While $\tilde{A}$ is independent of n, $\tilde{A}_N$ and $\tilde{A}_T$ are not. If goods are ordered with increasing productivity, then $\partial\tilde{A}_T/\partial n > 0$, $\partial\tilde{A}_N/\partial n < 0$, i.e., average productivity rises (falls) in the traded sector (nontraded) sector with increasing n. Intuitively, as the share of nontraded goods in the economy rises, goods at the low productivity end of the traded goods sector become nontraded, and the average level of productivity of all remaining traded goods rises.[15]

4.3. Solving for Tradedness and Relative Prices

The model solves conditionally as follows. Given n and n^*, consider the set of 24 endogenous variables, consisting of C, C_H, C_{HN}, C_{HT}, C_{FT}, P, P_H, P_{HN}, P_{HT}, P_{FT}, and W, plus the foreign counterpart variables (denoted *). Production markups link prices to wages. Standard CES demand conditions link prices to consumption quantities and, thence, via technology, to derived labor demand. Market clearing (plus balanced trade) and a numéraire choice complete the solution.

It is possible to derive an equilibrium for trade, given n and n^*, as shown in the Appendix. But what determines these nontraded shares? On the margin, the producers of the borderline traded-nontraded good must be indifferent between home and foreign sales, and, given the fixed costs of shipping, this entails two simple extra conditions that pin down n and n^* in equilibrium:

$$\left(p_{Hn}^* - \frac{W}{A_n}\right)\left(\frac{1}{P}\right)c_{Hn}^* = \left(\frac{W}{P}\right)f_X, \qquad \left(p_{Fn} - \frac{W^*}{A_n^*}\right)\left(\frac{1}{P^*}\right)c_{Fn} = \left(\frac{W^*}{P^*}\right)f_X^* \qquad (10)$$

Consider the first of these equations. The left hand side represents the real profit per unit of home good sold in the foreign market multiplied by sales volume; the right hand side is the fixed cost of exporting, where W/P represents the effective real wage rate of the labor employed to cover these costs. For the borderline good n the two sides are equal,

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[15] We will show that the solution to our model can be simply expressed as a function of the weighted productivities. Setting aside the endogeneity of n, the relationship of our model to the basic, textbook BS model is then clear, since the model depends only on "average" productivity levels in the traded and nontraded parts of the economy each viewed as a whole. A similar result has been derived independently in another model of endogenous tradability by Melitz (2003).

and profits from export are zero. The same is true in the foreign country. Taking sales as given, the profit term (like the A term) is increasing in n, so all goods on the interval to the left of n are nontraded and all to the right are traded, confirming the maintained assumption.

In this framework, endogenous tradability arises naturally, because only goods with high enough productivity are able to surmount the export cost barrier. [16] Moreover, goods which are a priori nontraded ($i < n$) but experience productivity gains relative to other goods may become traded over time as their operating profits exceed the fixed costs of exporting. Correspondingly, goods which are initially traded may subsequently become nontraded as their profits are reduced below export costs by the wage increases generated by the goods with more rapid productivity growth.

The equilibrium conditions can be combined to gain some analytical results and insights into the effects of productivity on the tradability of goods and on relative aggregate price levels, i.e., the real exchange rate. We can also show that an increase in the dispersion of productivities across goods will tend to concentrate trade in a smaller number of goods. As derived in the appendix, the export profitability conditions can be rearranged to obtain:

$$\frac{1}{\boldsymbol{j}-1}\left(\frac{A_n}{\tilde{A}_T[n]}\right)^{\boldsymbol{f}}\frac{C_{HT}^*}{1-n}=f_X\,,\qquad\qquad\frac{1}{\boldsymbol{j}-1}\left(\frac{A_n^*}{\tilde{A}_T^*[n^*]}\right)^{\boldsymbol{f}-1}\frac{C_{FT}}{1-n^*}=f_X^*\qquad(11)$$

Observe that in this form export profits depend on the productivity of the marginally traded nth good A_n, relative to the average productivity of all exported goods, $\tilde{A}_T[n]$. For given aggregate exports C_{HT}^*, a productivity shock biased towards traded goods will lower the productivity of the initially marginal traded good relative to the average, since the productivity of all intramarginally traded goods is higher. The resulting decline in $A_n/\tilde{A}_T[n]$ implies export operating profits fall below the fixed costs of exporting, inducing some firms to cease exporting. As the equilibrium n rises and trade becomes concentrated in a smaller set of firms, the profitability of the marginal exported good rises as its share of aggregate exports ($C_{HT}^*/(1-n)$) rises until these profits increase to just cover the fixed costs of entering into the foreign export market. Note that a balanced change in productivity for all home goods, leaving $A_n/\tilde{A}_T$ unchanged, does not affect the marginal profitability of exports. So if technology accumulates in a progressively more skewed distribution across goods, this tends to raise the equilibrium value of n and concentrate trade in a smaller set of goods over time.

As also derived in the appendix, the relative national price level may be written

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[16] This insight has been more extensively developed in the trade and policy literature and is not well known in the macroeconomics literature. See for example, Richardson and Rindal (1996) and Bernard and Jensen (1999). For a related equilibrium analysis, see especially Helpman, Melitz, and Yeaple (2004).

$$\left(\frac{P}{P^*}\right)^{f-1} = \left(\frac{q[n]\left(\frac{W/\tilde{A}}{W^*/\tilde{A}^*}\right)^{f-1} + (1-q[n])\left(\frac{\tilde{A}_T[n]}{\tilde{A}}\right)^{f-1}}{q[n^*] + (1-q[n^*])\left(\frac{\tilde{A}_T^*[n^*]}{\tilde{A}^*}\cdot\frac{W/\tilde{A}}{W^*/\tilde{A}^*}\right)^{f-1}} \right) \tag{12}$$

where the notation indicates the dependence of q, $\tilde{A}_T, \tilde{A}_T^*$ on n^*, n. Observe that the domestic country experiences a real appreciation (P/P^* rises) in response to: (i) a ceteris paribus increase in the relative cross-country domestic wage rate, adjusted by economy-wide average productivity levels ($(W/\tilde{A})/(W^*/\tilde{A}^*)$); or (ii) a ceteris paribus increase in home productivity biased towards tradables ($\tilde{A}_T/\tilde{A}$).[17] These effects are analogous to those of the standard BS model.[18] Clearly, the fact that the set of traded goods is determined endogenously in this model raises the likelihood of generating BS effects on prices. If large productivity shocks accumulate in a good, regardless of which good this happens to be, the good will tend to become traded and hence it will raise the $\tilde{A}_T/\tilde{A}$ term in the relative price expression (12).

However, expression (12) also indicates additional effects on relative prices operating through endogenous changes in n (or n^*) associated with the changing tradability of goods. These effects operate though two channels: (i) an effect on the relative productivity of tradables $\tilde{A}_T/\tilde{A}$; and (ii) an effect through the home bias weight q. With the first channel, if the share of nontraded goods in the economy n rises, goods at the low productivity end of the traded goods sector become nontraded, and the average level of productivity of all remaining traded goods rises. This amplifies the rise in $\tilde{A}_T/\tilde{A}$ and increase in relative prices. With the second channel, if fewer home varieties are traded, the weight placed on imported goods in the foreign price index falls, i.e., $1-q$ falls and q rises. Inspection of (12) indicates that this raises the impact of the relative cross-country domestic wage rate ($(W/\tilde{A})/(W^*/\tilde{A}^*)$), but lowers the impact of relative productivity in the traded sector ($\tilde{A}_T/\tilde{A}$). Given these multiple and potentially offsetting channels, it is not immediately obvious on the basis of analytical results whether the endogenous movement in the share n will make BS effects more or less likely. Simulations will address this ambiguity.

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[17] The first result presumes own goods bias in each country : $q[n] \geq 1/2, q[n^*] \geq 1/2$.

[18] In the special case of no export costs at all: $f_X = f_X^* = 0$, implying all goods are traded ($n = n^* = 0$), then

$q = 1/2$, $\tilde{A} = \tilde{A}_T$, $\tilde{A}^* = \tilde{A}_T^*$, so that $\left(\frac{P}{P^*}\right)^{f-1} = \frac{\left(\tilde{A}^*/W^*\right)^{f-1}+\left(\tilde{A}/W\right)^{f-1}}{\left(\tilde{A}/W\right)^{f-1}+\left(\tilde{A}^*/W^*\right)^{f-1}} = 1$, i.e., consumption price levels

are equalized across countries.

5. Numerical Simulations

The purpose of this section is to provide an illustration of how the model can generate a BS effect on national relative prices that arises endogenously and grows over time. Although we view the exercise mainly as an illustration of principles rather than a formal quantitative investigation, we nonetheless connect model and distribution parameters with data where possible to enhance the relevance of the experiment. To implement the model numerically, the continuum of goods is made discrete, with ten industries. The experiment allows the productivity of the ten industries in the home country, which we interpret as the U.S., to grow in response to a sequence of random productivity shocks that differ by industry, and it allows the tradability of the industries to respond as productivity accumulates in this heterogeneous manner. This experiment is run 1,000 times, and the range of paths traced by the implied BS coefficient b among the runs is summarized (where, in the two-country experiment, b is simply the log price ratio divided by the log income ratio, as in previous sections).

For the starting cross-sectional distribution of productivity among these ten industries at the beginning of each experiment run, we simply take the distribution for the first year of the U.S. productivity data set used in section 4 above. Thus, the levels of the ten industries in the home country are set equal to the ten decile levels of productivity in the U.S. data for 1958. For each of the 1,000 runs, a sequence of 40 random productivity shocks is drawn for each industry from a given Pareto probability distribution, representing 40 years of technological advancement. In any period, each firm uses its best draw to date; in this setting there is no technological regress (as with Gaussian shocks to productivity). The Pareto distribution was chosen here following the convention in the trade literature, which largely comes from the fact this distribution seems to characterize the highly skewed cross-sectional distribution of productivities and sizes among firms.

We also follow the literature in taking the draws to represent a change in the set of new ideas available to the industry. With each new draw, the sector has the choice of using the new productivity level or keeping the old one, whichever is higher.[19] The goal is to have a distribution for the home country at the end of 40 years of random draws that on average resembles the distribution for the last complete year in our data on the U.S. productivity distribution, 1994. In particular, the data show a large number of industries with low levels of productivity gain, with a small number of industries with very high productivity gains. With suitably chosen parameters, the stochastic specification here is well-suited to reflect-this feature.[20]

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[19] This "extreme value" approach has convenient properties, such as converging in the limit to a Frechet distribution, which likewise seems to characterize the highly skewed shape of the cross-sectional distribution of productivities across firms.

[20] Here we provide further detail regarding the mechanics of the stochastic draws. For each period random draws are taken for 1,000 industries, and the deciles of this distribution are assigned in random order to the 10 industries of the model. We do this rather than simply taking random draws in isolation for each of the 10 industries, since in this case there tended to be occasional very extreme draws occurring somewhere in the experiment which prevented model solution. This problem is prevented by taking deciles from a larger set of draws. The idea is to reflect what an experiment would be like if we had 1,000 distinct industries rather than just 10. We should note that the tenth decile is actually the 9.8 decile of the distribution, to

Figure 4 shows that this specification of stochastic productivity accumulation does lead to a cross-sectional distribution of productivity levels that resembles what we see in the data. The figure superimposes the distribution for the actual 1994 U.S. productivity data (over 448 industries), and the 5%–95% confidence band for the 1,000 draws of the terminal home productivity distribution in our benchmark simulation (over the 10 industry deciles). Regarding the foreign country, the initial productivity distribution is taken to be the same as for the home country, and it is simply held constant at this level throughout the experiment. So the experiment focuses on how the relative national price level evolves as the productivity distribution of the home country shifts up relative to that of the foreign country.

Model parameters are calibrated as follows. The elasticity of substitution between different industries (f) is 2, and the elasticity between brands within an industry (j) is 6. The markup is derived from the latter value, and follows Rotemberg and Woodford (1992) and the convention in the open economy macro literature in implying a price markup of 20% over marginal cost.[21] The fixed cost (f_X) is calibrated at 0.10, so that the number of nontraded industries (n) for the initial distribution of productivity is 5 out of the 10 industries.

Table 3 reports the mean values for various summary statistics over the 1,000 runs after 10, 20, 30, and 40 years of technological accumulation. Note that the relative income of the home country rises over time, as does the relative price level, though the latter accelerates relative to the income level. The key result is that the BS coefficient indeed rises over time. This is confirmed in Figure 5, which plots the 5% and 95% bands for the BS coefficient among the 1,000 runs. Note that the bands for the initial period are wide and centered around zero, but that they narrow and rise over time. By the end of the 40 years, both bands are above the zero line, indicating that by the end of the period almost all of the 1,000 random experiments predicted a positive BS effect.[22]

A more detailed comparison of the time paths of the BS coefficients shows more nuance. Both the simulation and the empirical estimation indicate initial values for the BS coefficient that are insignificantly different from zero, and from each other. While both show a rise in the BS coefficient over time, the rate of increase in the simulation falls progressively in later periods, while that in the historical estimation is more irregular, with an acceleration at the end of the sample. Finally, the terminal value of the simulated

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further limit the likelihood of very extreme draws that prevent model solution. To compensate for this truncation of the upper tail, we use more extreme Pareto distribution parameter values. The Pareto "scale" parameter is 0.001 and the "shape" parameter is 0.8. The 40 years of accumulated productivity in the experiment is combined additively with the initial productivity distribution.

[21] Since exports involve a fixed cost, the markup of export prices over average cost will be lower.

[22] There is debate as to whether it is reasonable to allow the definition of the price index to reflect changes in spending patterns implied by models of endogenous trade. Given the long time horizon we consider in the present experiment, updating of the price index weights is not unreasonable. Nonetheless, it is worth noting that the result derived here does not rely upon such updating. If we hold constant the weighting of foreign and home goods in the price index, we still find that the BS effect rises over time and is strongly positive. In particular, the BS coefficients corresponding to those reported for our first experiment for years 10 through 40 are: 0.31, 0.75, 1.09, and 1.31.

BS coefficient is definitely below that observed in the historical estimation, with the confidence bands for each not quite intersecting with each other. We conclude that while the model can generate the general upward trend observed in the BS coefficient, it remains too stylized to replicate some of the more detailed features of the time path.

Which feature of the model is most responsible for generating this rising BS coefficient in the simulation? Of distinct importance is the fact that the ordering of the industries in terms of their tradability evolved over the 40 years of the experiment. Consider one set of draws, where productivity gains happen to be concentrated in the industries that had low starting productivity levels and hence were the a-priori nontraded industries. As productivity accumulates in these industries, this raises the relative productivity of the nontraded sector to be more similar to that of the traded, which generates a strong negative BS coefficient. But over time, these industries one by one surpass some of the a-priori traded goods in terms of productivity and thereby become traded. The rise in wages generated by the productivity gains in these industries makes some of the a-priori traded goods unprofitable enough so as not to cover the fixed cost of exporting, and these previously-traded industries then become nontraded. As a result, productivity ends up accumulating among industries that are traded ex-post, which means the precondition for a positive BS effect is satisfied endogenously.

Table 3 also indicates a second effect at work generating the large positive BS coefficient. The last two columns show that in addition to changes in the relative tradability of industries, a progressively larger set of home industries are becoming nontraded over time. This again results from the fact that when the highly skewed Pareto distribution asserts itself over time, the small number of highly productive firms at the upper end of the distribution generates a significant rise in home wages. This forces out from trade a number of marginal industries. By the end of the 40 years, only 1 or 2 of the 10 industries engage in trade. This does not mean that the volume of trade is small, but just that the trade is highly concentrated in the most productive firms. As discussed in section 4.3, this rise in the share of nontraded goods can amplify the positive BS effect.

To demonstrate the role of endogenous tradability in generating the rising BS effect, the next experiment closes off this effect. The set of industries that are permitted to trade is exogenously held constant throughout the experiment, rather than being allowed to evolve in response to the incidence of productivity gains. Table 4 and Figure 6 show that a positive BS coefficient does not emerge here and does not grow over time. The mean BS coefficient is strongly negative and the 5%–95% bands are wide, including a zero BS effect for all periods. Thus, endogenous tradability is necessary for our result.

Recall that there are two features of endogenous tradability at work to generate the BS effect in the benchmark experiment: the endogenous ranking of tradability and the endogenous share of nontradeds. To distinguish the separate contributions of these two features, the final experiment here allows for the former but not the latter. The total number of traded industries is fixed exogenously, but the identity of the particular traded industries is determined endogenously in the model. More precisely, the ranking in terms of tradability of the 10 industries evolves endogenously in response to shocks, but only the 5 most tradable industries are allowed to trade in each period. Table 5 and Figure 7

show that this case does generate a rising BS coefficient, but it is limited in how high a positive value it can achieve. The mean value of the 1,000 runs is only 0.04, much smaller than the benchmark experiment, but the 5%–95% band does clear the 0 line by the end of the 40 years, indicating the positive sign of the coefficient, though small, is meaningful. The conclusion arising from these three experiments is that both the endogenous ordering of tradability and the endogenous concentration of traded goods are important contributors in replicating a rising BS coefficient.

6. Conclusion

This paper has investigated the empirical basis of, and theoretical rationale for, the venerable Balassa-Samuelson effect over the very long run. The first surprise is that there is little to no support for this effect as of the mid 20th century, contrary to general conception. The second surprise is that this effect has grown steadily over time to rather large values in the most recent years. These observations invite new thinking, with implications for international economists, macroeconomists, and economic historians.

We present a new model that supplies a set of tools for thinking about these results and we think it will be of interest to theoreticians in various fields. International economists will find here a new application of endogenous tradability that is complementary to several new directions in the literature. Macroeconomists and economic historians interested in long-run growth in more then one sector will discover not only an important set of stylized facts but also some conjectures on how these should be modeled in terms of sectoral growth behavior.

The picture offered by our theoretical model is that the BS effect may be the product of a gradual evolutionary process in a growing economy. Productivity shocks that are heterogeneous among goods not only induce a response in relative prices, as usually conceived in a standard BS model, but they can also induce a response in the relative tradability of these goods as well. The new insight offered by the endogenous trade approach here, is that the Balassa-Samuelson effect need not be viewed as the chance outcome of shocks that happen to be biased toward an a-priori traded sector, but rather it can be viewed as the natural, even likely, outcome of an evolutionary process that does not need to depend on the incidence of the shocks. But while the BS effect may be a likely outcome, this story implies that it can take time for the structure of the economy to adapt in response to the shocks and gradually express the BS effect.

We remain agnostic as to whether the most significant reordering of tradability occurs at the firm level or more aggregated industry level. Trade literature recently has provided evidence that significant shifts in tradability take place at the individual firm level. In this paper we have offered some suggestive evidence that reordering also has taken place at a more aggregated level between industries. But endogenous tradability at either level could usefully contribute to the mechanisms identified in our model which generate an endogenous and growing Balassa-Samuelson effect.

The finding here also has implications for the future of the Balassa-Samuelson theory. It has been observed that productivity gains in the service sector have been

important for recent growth in many developed countries. Since services are commonly classified as part of the nontraded sector, this might be thought to imply a period of limited real exchange rate appreciations and shrinking BS coefficients. However, our research indicates this need not be the case if the dividing line between traded and nontraded sectors evolves in response to these technological developments. Indeed, it appears that some service industries are becoming increasingly traded internationally, leading some commentators to argue that the world is becoming "flatter" (Friedman 2005). This rise in tradability likely is not alone a response to a rise in productivity in these industries, but also a response to lower trade costs for services like telecommunications. Nonetheless, it underscores the importance of allowing for evolution in the traded-nontraded distinction when we endeavor to interpret the long-run behavior of the real exchange rate.

Appendix

Consumption

Our model consists of two countries, with each specializing in a range of goods that are uniquely produced in that country, denoted by *H* and *F*, respectively. Residents in each country consume a CES-weighted basket of locally-produced goods and tradable goods imported from the other country. The continuum of goods produced in each country is indexed by i on the interval $[0,1]$.

Let n and n^* denote the (endogenous) share of goods in each country that are nontraded, where goods are ordered such that $i \in [0, n]$, $[0, n^*]$ are nontraded and $i \in [n,1]$, $[n^*, 1]$ are traded in the home and foreign country, respectively.

   To derive expression (5) in the text for aggregate consumption by domestic residents *C* as a CES index of domestic consumption (C_H) and imports of the foreign country's traded (export) good (C_{FT}), we denote consumption by domestic residents of the individual varieties i of the home good and foreign goods as c_{Hi}, $i \in [0,1]$, and c_{FTi}, $i \in [0, 1-n^*]$, respectively. Indexing all goods available for consumption in the domestic country by j on the interval $[0, 2-n^*]$ and ordering consumption as $c_j = c_{Hi}$ for $j = i \in [0,1]$, and $c_j = c_{Fi}$ for $j = i+1$, $j \in [1, 2-n^*]$ implies[23]

$$C^{\frac{f-1}{f}} = \left(\frac{1}{2-n^*}\right)^{\frac{1}{f}} \left[\int_0^1 (c_j)^{\frac{f-1}{f}} dj + \int_1^{2-n^*} (c_j)^{\frac{f-1}{f}} dj\right]$$

$$= \left(\frac{1}{2-n^*}\right)^{\frac{1}{f}} \left(\frac{1}{1}\right)^{\frac{1}{f}} \int_0^1 (c_j)^{\frac{f-1}{f}} dj + \left(\frac{1-n^*}{2-n^*}\right)^{\frac{1}{f}} \left(\frac{1}{1-n^*}\right)^{\frac{1}{f}} \int_1^{2-n^*} (c_j)^{\frac{f-1}{f}} dj$$

$$= \left(\frac{1}{2-n^*}\right)^{\frac{1}{f}} (C_H)^{\frac{f-1}{f}} + \left(\frac{1-n^*}{2-n^*}\right)^{\frac{1}{f}} (C_{FT})^{\frac{f-1}{f}}$$

$$= (q[n^*])^{\frac{1}{f}} (C_H)^{\frac{f-1}{f}} + (1-q[n^*])^{\frac{1}{f}} (C_{FT})^{\frac{f-1}{f}}$$

where $(C_H)^{\frac{f-1}{f}} \equiv \int_0^1 (c_{Hi})^{\frac{f-1}{f}}$, $C_{FT} \equiv \left[\left(\frac{1}{1-n^*}\right)^{\frac{1}{f}} \int_{n^*}^1 (c_{Fi})^{\frac{f-1}{f}} di\right]^{\frac{f}{f-1}}$,   (A1)

$f > 1$ is the elasticity of substitution between all varieties, and $q[n^*]$ is the own-goods bias coefficient that depends (endogenously) on the number of varieties produced locally relative to the total number of varieties available to the domestic country:

$$q[n^*] \equiv \frac{1}{2-n^*}, \qquad 1-q[n^*] \equiv \frac{1-n^*}{2-n^*}, \qquad 0 \le q[n^*] \le 1$$

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[23] With the total mass of goods produced in each country normalized to 1, $1-n$ and $1-n^*$ also represent the number of goods in each country that are traded. Consequently, the total mass of varieties of goods available for consumption in the home country is the sum of the mass of domestic varieties and of the goods exported by the foreign country, i.e., $1+(1-n^*) = 2-n^*$; analogously, the total mass of goods available in the foreign country is $2-n$.

The derivation of the foreign consumption aggregate C^* is analogous. Observe that if $n = n^* = 0$ and all home and foreign varieties are traded, then there is no goods bias, i.e., $q[n] = q[n^*] = 1/2$. As n, n^* rise and fewer varieties are imported by each country, the relative weight placed on own goods rises and on imported goods falls, i.e., $q' > 0$.[24]

Consumption of each country's own-good (C_H, C_F^*) is in turn defined as a CES consumption index of its nontraded (C_{HN}, C_{FN}^*) and traded own goods (C_{HT}, C_{FT}^*), as given by (6) in the text, where

$$C_{HN} \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{f}} \int_0^n \left(c_{Hi} \right)^{\frac{f-1}{f}} di \right]^{\frac{f}{f-1}}, C_{HT} \equiv \left[\left(\frac{1}{1-n} \right)^{\frac{1}{f}} \int_n^1 \left(c_{Hi} \right)^{\frac{f-1}{f}} di \right]^{\frac{f}{f-1}} \tag{A2}$$

$$C_{FN}^* \equiv \left[\left(\frac{1}{n^*} \right)^{\frac{1}{f}} \int_0^{n^*} \left(c_{Fi}^* \right)^{\frac{f-1}{f}} di \right]^{\frac{f}{f-1}}, C_{FT}^* \equiv \left[\left(\frac{1}{1-n^*} \right)^{\frac{1}{f}} \int_{n^*}^1 \left(c_{Fi}^* \right)^{\frac{f-1}{f}} di \right]^{\frac{f}{f-1}} \tag{A3}$$

Note that the elasticity of substitution f is assumed constant within and across countries.[25]

Prices and Relative Demands

Price indexes are defined as usual for each category of goods, in correspondence to the consumption indexes above:

$$P = \left(q[n^*] \left(P_H \right)^{1-f} + \left(1 - q[n^*] \right) \left(P_{FT} \right)^{1-f} \right)^{\frac{1}{1-f}} \tag{A4}$$

where $P_H^{1-f} = \int_0^n \left(p_{Hi} \right)^{1-f} di + \int_n^1 \left(p_{Hi} \right)^{1-f} di = n \left(P_{HN} \right)^{1-f} + (1-n) \left(P_{HT} \right)^{1-f} \tag{A5}$

$$P_{HN} \equiv \left(\left(\frac{1}{n} \right) \int_0^n p_{Hi}^{1-f} di \right)^{\frac{1}{1-f}}, \qquad P_{HT} \equiv \left(\left(\frac{1}{1-n} \right) \int_n^1 p_{Hi}^{1-f} di \right)^{\frac{1}{1-f}} \tag{A6}$$

$$P_{FT} \equiv \left(\left(\frac{1}{1-n^*} \right) \int_{n^*}^1 p_{Fi}^{1-f} di \right)^{\frac{1}{1-f}} \tag{A7}$$

with P denoting the aggregate price level, P_H the price index of all home goods, P_{HN} the price index of nontraded home goods, P_{HT} the price index of traded home goods, and P_{FT} the price (to domestic residents) of imported foreign goods. The price indices for foreign goods and imported home goods (by foreign residents) are analogous.

Note that the consumption and price indices imply the following relative demand functions for domestic residents:[26]

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[24] Since the mass of home (foreign) goods is normalized to 1, the bias coefficient of domestic (foreign) residents depends on n^* (n), the number of varieties of the foreign (domestic) good that are imported.

[25] That is, $c_{Hi} / c_{Hj} = \left(p_{Hi} / p_{Hj} \right)^{-f}$, $c_{Fi}^* / c_{Fj}^* = \left(p_{Fi}^* / p_{Fj}^* \right)^{-f}$ for any two goods i and j.

[26] Also note that the CES specification implies for individual goods variety i

$c_{Hi} / C_H = \left(p_{Hi} / P_H \right)^{-f}$, $c_{Hi} / C_{HT} = (1-n)^{-1} \left(p_{Hi} / P_{HT} \right)^{-f}$, $c_{Fi} / C_{FT} = \left(1-n^* \right)^{-1} \left(p_{Fi} / P_{FT} \right)^{-f}$

and $c_{Fi}^* / C_F^* = \left(p_{Fi}^* / P_F^* \right)^{-f}$, $c_{Fi}^* / C_{FT}^* = \left(1-n^* \right)^{-1} \left(p_{Fi}^* / P_{FT}^* \right)^{-f}$, $c_{Hi}^* / C_{HT}^* = (1-n)^{-1} \left(p_{Hi}^* / P_{HT}^* \right)^{-f}$

$$C_H/C = \left(\boldsymbol{q}[n^*]\right)\left(P_H/P\right)^{-f}, \qquad C_{FT}/C = \left(1-\boldsymbol{q}[n^*]\right)\left(P_{FT}/P\right)^{-f} \tag{A8}$$

$$C_{HN}/C_H = n\left(P_{HN}/P_H\right)^{-f}, \qquad C_{HT}/C_H = \left(1-n\right)\left(P_{HT}/P_H\right)^{-f} \tag{A9}$$

Again, the conditions for foreign residents are analogous.

Production and Productivity

The production sector in each country consists of constant-returns-to-scale technologies for the output of each differentiated good:, as given by (7) in the text. Profit maximization under monopolistic competition implies pricing is determined by the standard cost markup rules, given by (8) in the text.

It is straightforward to express the price index for nontraded and traded home goods in terms of the productivity averages by using (8) to substitute for p_{Hi}, p_{Hi}^* and (9) in turn to substitute with $\tilde{A}, \tilde{A}_T, \tilde{A}_N$ in (A6):

$$P_{HN} = \left(\left(\frac{1}{n}\right)\left[\int_0^n \left(\frac{\boldsymbol{j}}{\boldsymbol{j}-1}\frac{W}{A_i}\right)^{1-f}di\right]\right)^{\frac{1}{1-f}} = \frac{\boldsymbol{j}}{\boldsymbol{j}-1}\left(\frac{W}{\tilde{A}_N[n]}\right) \tag{A10}$$

$$P_{HT} = P_{HT}^* = \left(\left(\frac{1}{1-n}\right)\left[\int_n^1 \left(\frac{\boldsymbol{j}}{\boldsymbol{j}-1}\frac{W}{A_i}\right)^{1-f}di\right]\right)^{\frac{1}{1-f}} = \left(\frac{\boldsymbol{j}}{\boldsymbol{j}-1}\right)\frac{W}{\tilde{A}_T[n]} \tag{A11}$$

with (A5) implying

$$P_H = \frac{\boldsymbol{j}}{\boldsymbol{j}-1}\frac{W}{\tilde{A}} \tag{A12}$$

Observe nontraded and traded goods depend on the share of nontraded goods n. which itself is an endogenous variable that will be solved as part of the general equilibrium system. Since $\tilde{A}$ is independent of n, the nontraded vs. traded goods composition of the economy affects P_H only through its effect on the average wage level in the economy.[27] The foreign counterparts are analogous.

Marginal Trading Condition

The equilibrium shares of nontraded goods n, n^* are determined by the condition that at the margin the (real) operating profits from exporting the nth home good equals the (real) fixed cost of exporting f_X, given by (10).

Labor Market Equilibrium

Labor market equilibrium in the domestic country requires that labor employed in production of nontraded and traded home goods *plus* labor employed to cover the fixed costs of exporting equal the (exogenous) domestic labor supply L_H :

$$\int_0^n l_{Hi}di + \int_n^1 l_{Hi}di + (1-n)f_X = L_H \tag{A13}$$

Substituting for l_{Hi} with the production function (7):

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[27] In the absence of any transport costs at all, all goods are traded ($n=0$), implying $\tilde{A} = \tilde{A}_T$, and $P_H = P_{HT} = P_{HT}^*$.

$$\int_0^n \frac{y_{Hi}}{A_i} di + \int_n^1 \frac{y_{Hi}}{A_i} di + (1-n)f_X = L_H \text{ or}$$

$$\int_0^n \frac{c_{Hi}}{A_i} di + \int_n^1 \frac{c_{Hi} + c_{Hi}^*}{A_i} di + (1-n)f_X = L_H$$

since $y_{Hi} = c_{Hi}$ for $i \in [0,n]$, $y_{Hi} = c_{Hi} + c_{Hi}^*$ for $i \in [n,1]$. Substituting with

$c_{Hi}/C_H = \left(p_{Hi}/P_H\right)^{-f}$ and $c_{Hi}^*/C_{HT}^* = (1-n)^{-1}\left(p_{Hi}^*/P_{HT}^*\right)^{-f}$ gives

$$\int_0^n \frac{C_H}{A_i}\left(\frac{p_{Hi}}{P_H}\right)^{-f} di + \int_n^1 \left(\frac{1}{A_i}\right)\left[C_H\left(\frac{p_{Hi}}{P_H}\right)^{-f} + \left(\frac{1}{1-n}\right)C_{HT}^*\left(\frac{p_{Hi}^*}{P_{HT}^*}\right)^{-f}\right] di + (1-n)f_X = L_H$$

Using (8) to substitute for p_{Hi}, p_{Hi}^* and (9) to substitute with $\tilde{A}$, $\tilde{A}_T[n]$ gives

$$W = \left(L_H - (1-n)f_X\right)^{-1/f}\left(\frac{j-1}{j}\right)\left[\left(\frac{P_H C_H}{(P_H)^{1-f}}\right)(\tilde{A})^{f-1} + \left(\frac{P_{HT}^* C_{HT}^*}{(P_{HT}^*)^{1-f}}\right)(\tilde{A}_T[n])^{f-1}\right]^{\frac{1}{f}}$$

Substituting for P_H, P_{HT}^* with (A12), (A11) and canceling terms gives

$$WL_H - (1-n)f_X W = \left(\frac{j-1}{j}\right)\left[P_H C_H + P_{HT}^* C_{HT}^*\right] \tag{A14}$$

i.e., the domestic wage bill -- net of wages paid for workers employed in covering fixed costs, $W(1-n)f_X$ -- is proportional to the value of home goods consumed domestically or exported, with the proportionality constant equal to 1 minus the profit rate $1/j$. The foreign counterpart expression is analogous.

Closing the Model
We close the model with the balanced trade condition that the value of exports equals the value of imports

$$P_{HT}^* C_{HT}^* = P_{FT} C_{FT} \tag{A15}$$

and the normalization condition

$$P^* = 1. \tag{A16}$$

Equilibrium determines the 24 variables $C, C_H, C_{HN}, C_{HT}, C_{FT} \; P, P_H, P_{HN}, P_{HT}, P_{FT}, W$, and n and their foreign counterparts (denoted by *) by solving the system of 24 equations (5), (6), (A1)-(A5), (A10)-(A11), (11), and (A14) plus their foreign counterparts, together with (A15) and (A16).

Tradability Condition and the Real Exchange Rate
Further insights may be obtained by developing the marginal export condition and a relative price expression in terms of productivity averages. Since the condition

$c_{Hi}^*/C_{HT}^* = (1-n)^{-1}\left(p_{Hi}^*/P_{HT}^*\right)^{-f}$ holds for all goods i in the range $[n,1]$ (see footnote 27), it can

be used to substitute for c_{Hn}^* in (10). Canceling the variable P from both sides, substituting for

p_{Hn}^* with (8), multiplying and dividing by P_{HT}^*, and lastly substituting with (A11) for P_{HT}^* in the denominator on the lefthand side gives (11). Expression (12) for the real exchange rate is obtained by using (A12), (A11), and their foreign analogues to substitute for P_H, P_{HT}^*, P_F^*, P_{FT}, respectively, in the definitions of P, P^* given by (A4) and its foreign counterpart.

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Table 1: The Balassa-Samuelson Effect 1950–95 in Cross Section
Cross-section OLS regression of the log relative price level on log relative real income per capita

Year	N	b	s.e.	
1950	53	0.08	(0.07)	
1955	69	0.11	(0.07)	
1960	108	0.19	(0.05)	***
1965	109	0.20	(0.05)	***
1970	111	0.20	(0.05)	***
1975	112	0.20	(0.05)	***
1980	118	0.20	(0.05)	***
1985	118	0.27	(0.04)	***
1990	128	0.36	(0.04)	***
1995	142	0.41	(0.04)	***

Significance levels: * 10%, **5%, *** 1%.
Source: Penn World Table 6.

Table 2. Correlations between Productivity and Tradability
in U.S. Manufacturing Data

SIC industry level	Spearman rank correlations		Level correlations (in logs)	
	1958	1994	1958	1994
2 digit	0.34	0.53	0.21	0.35
3 digit	0.29	0.47	0.25	0.45
4 digit	0.31	0.40	0.27	0.38

Source: productivity data from the NBER-CES manufacturing industry database,
export data from the NBER database on "U.S. Trade by 1972-SIC Category"

Table 3. Simulation Results: Endogenous Set of Nontraded Goods

Simulated Year	Income Ratio (y/y^*)	Price Ratio (p/p^*)	BS coefficient $\left(\dfrac{\log(p/p^*)}{\log(y/y^*)}\right)$	Home nontraded share [1]	Foreign nontraded share [1]
0	1.00	1.00	--	0.50	0.50
10	1.11	1.00	0.024	0.59	0.50
20	1.28	1.04	0.147	0.71	0.48
30	1.44	1.08	0.213	0.78	0.43
40	1.61	1.12	0.233	0.84	0.40

Values reported are means over 1,000 stochastic simulations with calibration: $f = 2$, $j = 6$, $f_X = 0.10$, Pareto shape parameter = 0.8, Pareto scale parameter = 0.001.

[1] Nontraded share is the fraction of the ten industries not engaging in trade ($n/10$ or $n^*/10$).

Table 4. Simulation Results: Exogenous Set of Nontraded Goods

Simulated Year	Income Ratio (y/y^*)	Price Ratio (p/p^*)	BS coefficient $\left(\dfrac{\log(p/p^*)}{\log(y/y^*)}\right)$	Home nontraded share [1]	Foreign nontraded share [1]
0	1.00	1.00	--	0.50	0.50
10	1.11	0.95	-0.695	0.50	0.50
20	1.27	0.90	-0.686	0.50	0.50
30	1.42	0.86	-0.674	0.50	0.50
40	1.57	0.83	-0.659	0.50	0.50

Values reported are means over 1,000 stochastic simulations with calibration: $f = 2$, $j = 6$, $f_X = 0.10$, Pareto shape parameter = 0.8, Pareto scale parameter = 0.001.

[1] Nontraded share is the fraction of the ten industries not engaging in trade ($n/10$ or $n^*/10$).

Table 5. Simulation Results: Fixed Share of Nontraded Goods ($n=n^*=5$)

Simulated Year	Income Ratio (y/y^*)	Price Ratio (p/p^*)	BS coefficient $\left(\dfrac{\log(p/p^*)}{\log(y/y^*)}\right)$	Home nontraded share [1]	Foreign nontraded share [1]
0	1.00	1.00	--	0.50	0.50
10	1.14	1.00	-0.031	0.50	0.50
20	1.35	1.01	0.028	0.50	0.50
30	1.55	1.02	0.040	0.50	0.50
40	1.75	1.02	0.043	0.50	0.50

Values reported are means over 1,000 stochastic simulations with calibration: $f = 2$, $j = 6$, $f_X = 0.10$, Pareto shape parameter = 0.8, Pareto scale parameter = 0.001.

[1] Nontraded share is the fraction of the ten industries not engaging in trade ($n/10$ or $n^*/10$).

Figure 1a
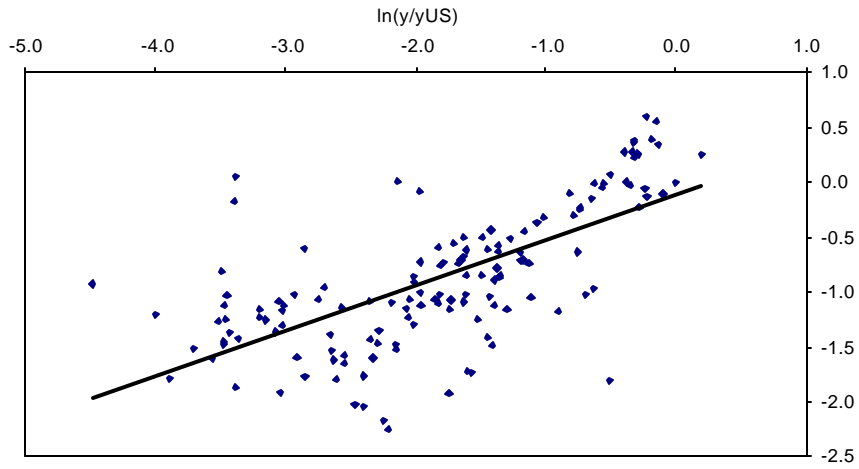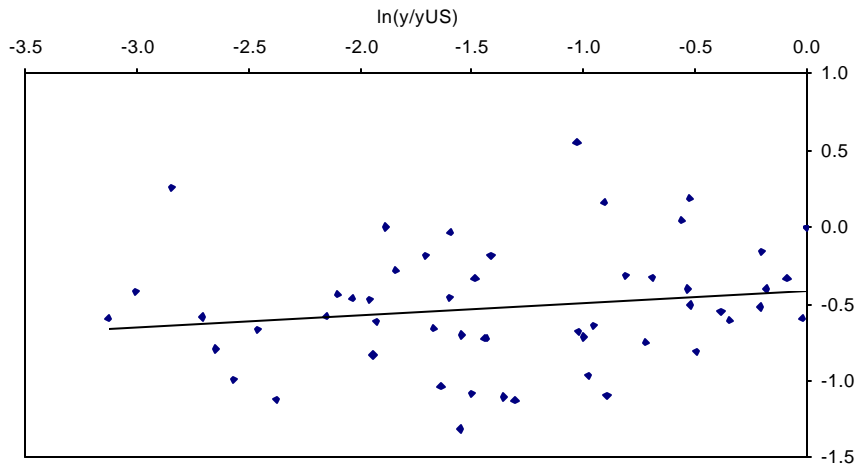**Log price level versus log per capita income, 199**
US=0, PWT sample (N=142)

ln(y/yUS)

Figure 1b
Log price level versus log per capita income, 195
US=0, PWT sample (N=53)

ln(y/yUS)

Notes and Sources: Scatter plots of log price level versus log per capita income. Various samples. See text.

Figure 2a
The Balassa-Samuelson Effect in Cross Sections
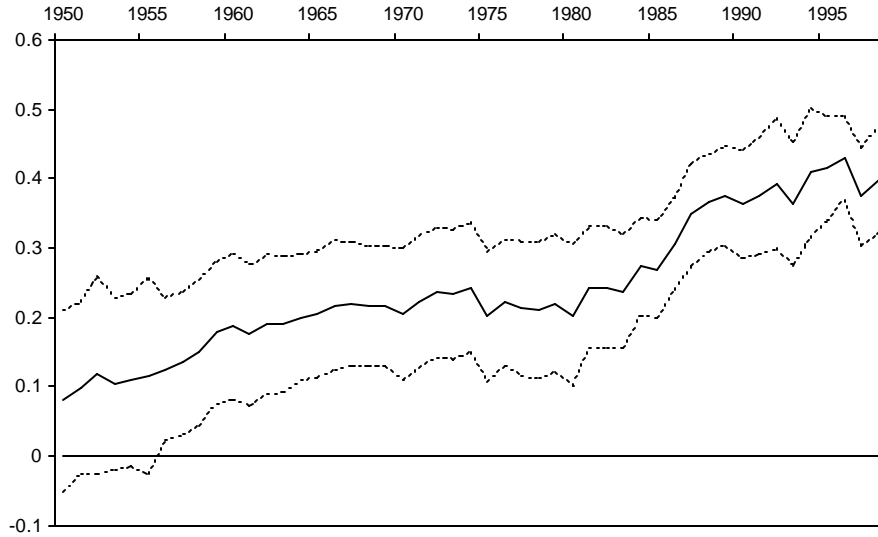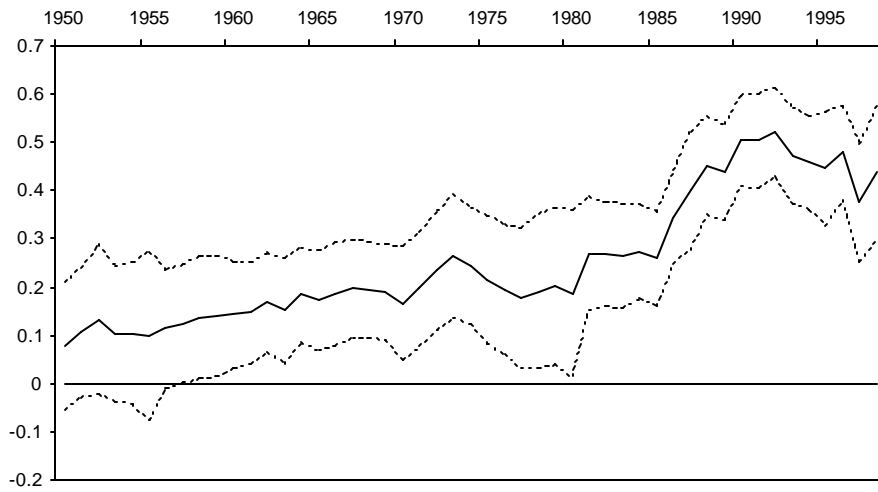PWT data 1950G1998, full sample (53²N²165)

Figure 2b
The Balassa-Samuelson Effect in Cross Sections
PWT data 1950G1998, 1950 sample of countries (N=53

Notes and Sources: Coefficient from cross-country regression of log price level on log per capita income (solid line) with 95% confidence interval (dashed lines). Various samples. See text.
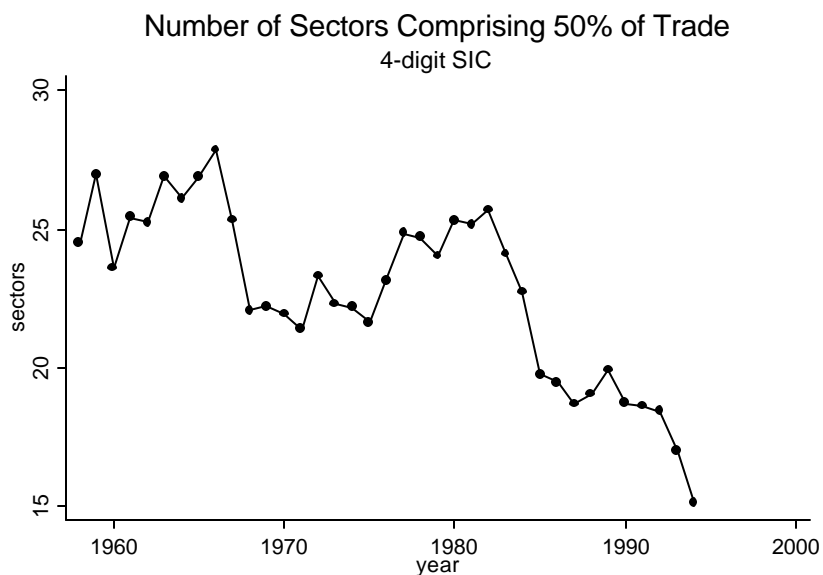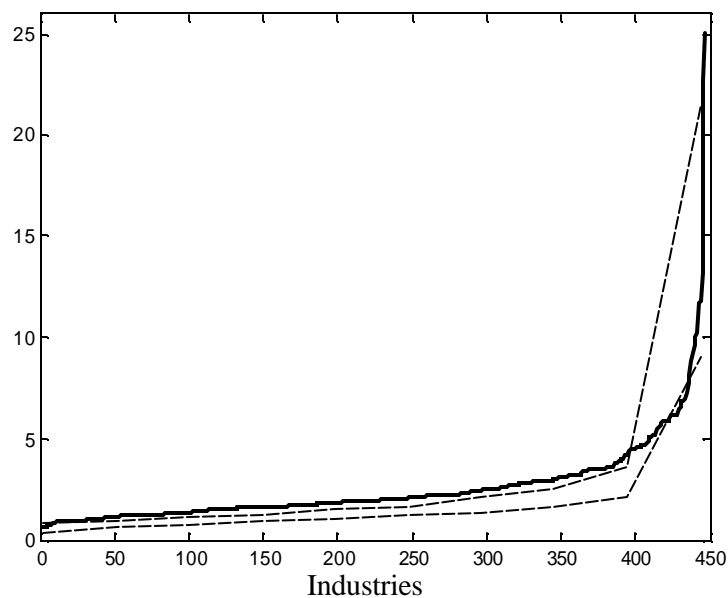
Figure 3. Concentration of Trade in U.S. Manufacturing

Number of Sectors Comprising 50% of Trade
4-digit SIC

**Figure 4. Final Productivity Distribution in Data and Simulations**

Note: Distribution for actual 1994 data (solid line) and the 5–95%
coverage band after 40 years of stochastic draws for the 10 deciles used
for the endogenous *n* case (exogenous *n* case is similar)

**Figure 5. Balassa Samuelson Coefficient for
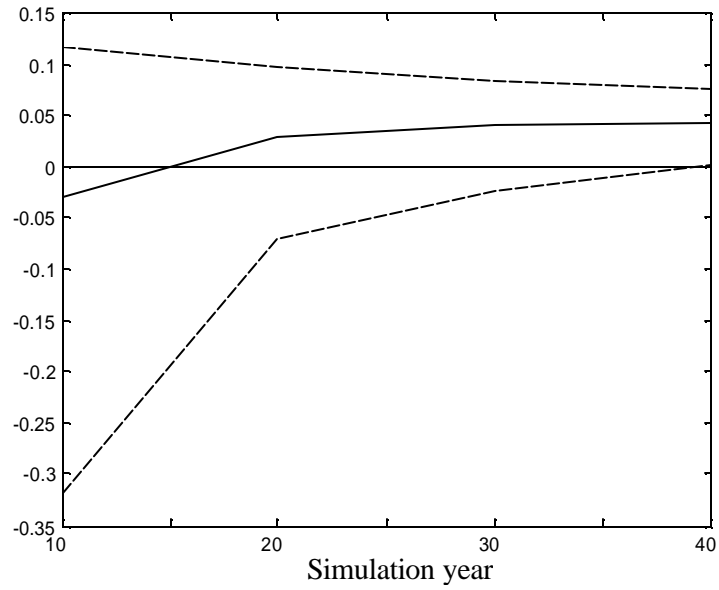Stochastic Simulation with Endogenous Nontraded Goods**
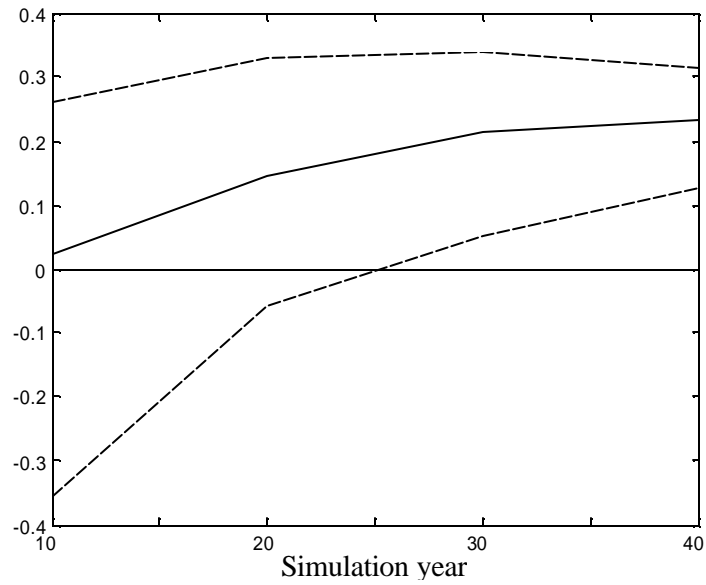
Note: ratio of log price level over log income (solid line) with 5%−95% coverage band (dashed lines)

**Figure 6. Balassa Samuelson Coefficient for
Stochastic Simulation with Exogenous set of Nontraded Goods**
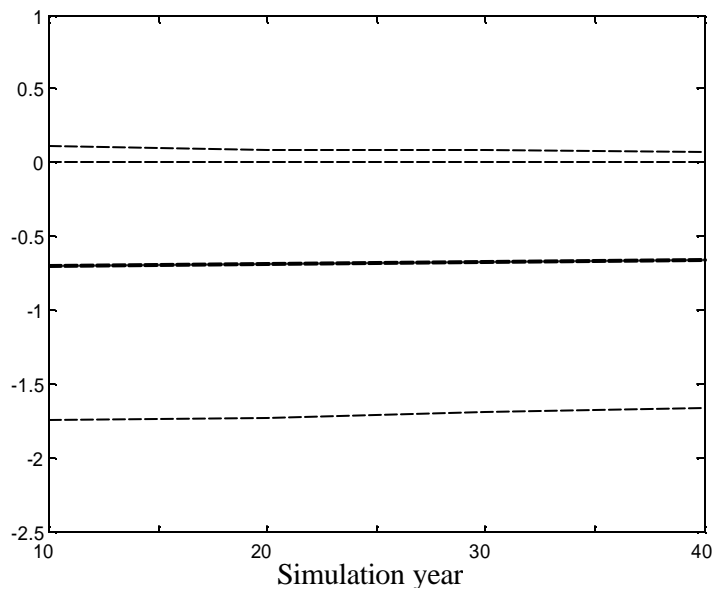
Note: ratio of log price level over log income (solid line) with 5%−95% coverage band (dashed lines)

**Figure 7. Balassa Samuelson Coefficient for
Stochastic Simulation with Fixed Nontraded Share, *n***

Note: ratio of log price level over log income (solid line) with 5%–95%
coverage band (dashed lines)