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DO ASSET-DEMAND FUNCTIONS OPTIMIZE OVER THE MEAN AND VARIANCE OF REAL RETURNS? A SIX-CURRENCY TEST

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ABSTRACT

International asset demands are functions of expected returns. Optimal portfolio theory tells us that the coefficients in this relationship depend on the variance-covariance matrix of real returns. But previous estimates of the optimal portfolio (1) assume expected returns constant and (2) are not set up to test the hypothesis of mean-variance optimization. We use maximum likelihood estimation to impose a constraint between the coefficients and the error variance-covariance matrix. For a portfolio of six currencies, we are able statistically to reject the constraint. Evidently investors are either not sophisticated enough to maximize a function of the mean and variance of end-of-period wealth, or else are too sophisticated to do so.

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1. INTRODUCTION

Much progress has been made lately in the application of finance theory to the problem of developing international asset-demand functions that are useful in macroeconomics. Investors balance their portfolios among the assets of various countries as a function of the expected rates of return. The contribution of finance theory is to show what the parameters in these functions are. Under the assumption that investors optimize with respect to the mean and variance of end-of-period wealth, the parameters are seen to depend in a simple way on the variance-covariance matrix of returns and on the degree of risk aversion.[1]

We believe it is fair to say that the empirical literature in this area has lagged behind the theoretical literature. Solnik (1977, p. 511) concludes, "international asset pricing seems to be a very fruitful area for theoretical research, not empirical." A number of studies have taken the techniques for estimating the Capital Asset Pricing Model (CAPM) that have been developed for other financial markets and have extended them to foreign currencies.[2] But, as noted by Dumas (1982), few of these studies are set up as explicit tests of the hypothesis that actual asset-demand functions are in fact of the mean-variance optimizing form. Indeed, many of the studies would not hold up under such testing. Kouri and de Macedo (1978, p. 129), for example, find that the optimal portfolio would assign negative shares to assets denominated in French francs and yen; yet we know that there are positive net supplies of these assets held in the world market (the cumulated government debts of France and Japan, corrected for foreign exchange intervention).

By making use of data on net asset supplies, the present paper is able to test explicitly the hypothesis that asset-demand functions are of the mean-variance optimizing form.

Another problem with most previous empirical finance studies is that they make the assumption that the expected returns perceived by investors are constant over time.[3] This assumption is made, often implicitly, in order to be able to estimate the expected returns from the unconditional ex post sample mean. In the case of the variances and covariances, the stationarity assumption is appropriate. It is necessary if the parameters of the asset-demand functions are to be considered unchanging over time. But in the case of the expected returns, the stationarity assumption is not appropriate for a macro model. It would imply that the <u>arguments</u> of the asset-demand functions, as opposed to the parameters of the functions, are constant over time. It is an essential element of most macro models that expected returns be allowed to vary. This problem with the previous studies is related to the fact that they make no use of asset supply data: fluctuations in asset supplies (the values of the functions) are what allows fluctuations in the rates of return (the arguments).

In the present paper the rates of return are related to the asset supplies by an equation in which the error term is identified as the market's expectational error.[4] The hypothesis that the functions are optimizing can be implemented by imposing the constraint that the coefficient matrix is proportionate to the variance-covariance matrix of the error term, and estimating by maximum likelihood (MLE). If the optimizing hypothesis were true, the constrained MLE estimates would be the most efficient estimates of the parameters. Moreover, one can test the hypothesis by comparing the likelihood when the parameters are estimated subject to this constraint to the likelihood when they are estimated unconstrained. Our finding is that a likelihood ratio test *rejects* the constraint of mean-variance optimization. This evidence suggests that market agents are either not sophisticated enough to maximize their end-of-period wealth with regard to mean and variance, or else perhaps are *more* sophisticated than this, maximizing instead a more complicated intertemporal function.

This paper continues past work by the authors. There are two important new features. First, we extend the test of mean-variance optimization to a portfolio of six nominal assets: marks, pounds, yen, French francs, Canadian dollars, and U.S. dollars. Dumas (1982, p. 5) and many other authors, have emphasized the importance of looking at "a reasonably complete list of individual assets available across the world." Of course it would be desirable to include equities and all other assets, but data difficulties inevitably put a limit on the number of assets we can consider.

Second, we use price data to measure real returns explicitly, thus allowing for inflation risk, rather than treating the exchange rate as the only stochastic variable.[5] As Kouri and de Macedo (1978, p. 118) have emphasized, "rational lenders and borrowers are presumably concerned with the real values of their assets and liabilities, and hence the purchasing power of a currency over goods and services available in the world economy is the appropriate standard of its value." The price for allowing stochastic inflation rates is that we are not able to allow consumption preferences to differ among investors residing in different countries. We assume, rather, that all investors have the same preferences and thus can be aggregated together.[6]

Section 2 of this paper shows how asset-demand functions can be estimated, without imposing the constraint of mean-variance optimization. Section 3 derives theoretically the optimizing form of the functions. Section 4 estimates the asset-demand functions subject to the constraint that they are indeed of this form, and does the likelihood ratio test. Section 5 draws conclusions. Details of the data calculations are available in an appendix to Frankel (1982b).

2. ESTIMATION OF UNCONSTRAINED ASSET-DEMAND FUNCTIONS

In this paper we assume that investors allocate their portfolio among assets denominated in six currencies. We define a column vector of five portfolio shares:

$$x_t' \equiv [x_t^{DM}\ x_t^{£}\ x_t^{¥}\ x_t^{F}\ x_t^{C\$}].$$

The residual is the share allocated to U.S. dollars: $(1 - x_t'\iota)$, where ι is a column vector of five ones. The asset-demand function gives us the demands as a function of the expected rates of return on the assets relative to the numeraire asset, the dollar:

$$x_t = \alpha + \beta\left(Er_{t+1} - \iota Er^{\$}_{t+1}\right), \tag{1}$$

where $Er^{\$}_{t+1}$ is the expected real return on dollar assets, Er_{t+1} is a column vector of the expected real returns on the other five assets, β is a matrix of coefficients, and α is a vector of intercepts. We will show in the next section that the linear form (1) is correct if agents are mean-variance optimizing. But the important point is that at this stage we are not constraining the parameters in α and β to be anything in particular. They could be based on investors' arbitrary "tastes" for assets as easily as on mean-variance optimization. Of course we have already restricted the function somewhat; for example many macroeconomic models include real income levels, representing a transactions demand for the assets.

In the past, the stumbling block in econometric estimation of portfolio-balance equations has been the measurement of expected returns. The solution adopted here is to invert equation (1), so that expected returns are viewed as depending on asset supplies:

$$Er_{t+1} - \iota Er^{\$}_{t+1} = -\beta^{-1}\alpha + \beta^{-1}x_t. \tag{2}$$

To deal with the unobservability of expectations, we make the assumption that investors form them rationally. The ex post relative return $\left(r_{t+1} - \iota r^{\$}_{t+1}\right)$, which <u>is</u> observable, is assumed equal to the expected return plus a random error term ε_{t+1}. By "random," we mean uncorrelated with all information available at the beginning of the period over which the return is measured:

$$r_{t+1} - \iota r_{t+1}^{\$} = Er_{t+1} - \iota Er_{t+1}^{\$} + \varepsilon_{t+1}, \qquad E(\varepsilon_{t+1}|I_t) = 0. \tag{3}$$

Substituting (2) into (3),

$$r_{t+1} - \iota r_{t+1}^{\$} = -\beta^{-1}\alpha + \beta^{-1}x_t + \varepsilon_{t+1}. \tag{4}$$

The parameters of equation (4) can now be estimated by regression. The regression error is simply the expectational error ε_{t+1}, which we know to be uncorrelated with the right-hand-side variables by the assumption of rational expectations.[7]

Table 1 reports regressions of the system of equations (4). The low R^2s are not in themselves a problem; there is widespread agreement that the market is able to forecast no more than a small proportion of the changes in exchange rates.[8] But the high sums of squared residuals extend to high standard errors of the parameter estimates. Only one or two in each equation are significantly different from zero. Of those, the two diagonal elements, which are the only ones on which we have a priori information, are of the incorrect sign: an increase in the supply of Canadian dollars or marks appears to induce a decline, rather than an increase, in the expected future returns on those two assets. On the other hand, we are able to reject with a likelihood ratio test the constraint that all coefficients are zero. The log likelihood for the five unconstrained equations taken together is 1086.49, whereas the constrained log likelihood is only 1057.11. (Twice the difference is distributed χ^2 with 25 degrees of freedom.)

TABLE 1

UNCONSTRAINED ASSET-DEMAND FUNCTIONS
OLS

DEPENDENT VARIABLE: $r_{t+1} - r^{\$}_{t+1}$, Real Rate of Return on National Currency Relative to the Dollar, Computed from Eurocurrency Interest Rates and Exchange Rates and a Price Index that is a Weighted Average of the Six Countries' CPIs, with Weights Determined by GNPs.
INDEPENDENT VARIABLES: x_t, Shares of Asset Supplies in the World Portfolio, with Total Wealth Computed as the Sum of the Asset Supplies each Evaluated at its Respective Exchange Rate.

SAMPLE: JUNE 1973-AUGUST 1980 (87 Observations)

National Currency	$-\beta^{-1}\alpha$, Constants	β^{-1}, Coefficients on $x^{C\$}_t$	x^{F}_t	x^{DM}_t	$x^{¥}_t$	$x^{£}_t$	log lik.	R^2	D.W.	S.E.R.	$F_{(5,81)}$
Canadian dollar	.125* (.060)	-1.466* (.692)	-.020 (.322)	.384 (.243)	-.120 (.082)	.150 (.087)	251.87	.09	2.04	.01338	1.63
French franc	.014 (.138)	1.770 (1.584)	-1.132 (.737)	-.710 (.557)	.311 (.188)	-.159 (.199)	179.87	.08	2.37	.03061	1.37
Deutsche mark	.153 (.145)	1.324 (1.669)	-.818 (.776)	-1.773* (.587)	.361 (.198)	-.211 (.210)	175.30	.13	2.20	.03226	2.48*
Japanese yen	.289* (.130)	.319 (1.494)	-1.309 (.695)	-2.213* (.525)	.271 (.177)	-.141 (.188)	184.98	.21	2.04	.02887	2.04
British pound	.028 (.121)	1.772 (1.389)	-.938 (.646)	-.993* (.488)	.419* (.165)	-.182 (.175)	191.27	.15	2.02	.02685	2.86*

*Significantly different from zero at the 95 percent level.
(Standard errors reported in parentheses.)
Overall log likelihood = 1086.485.

One assumption that we have already made is borne out. The absence of serial correlation in the error term supports the hypothesis of rational expectations.

The main lesson to be drawn from table 1 is the low degree of precision that plagues estimation of general portfolio-balance equations, and the need to bring additional information to bear. This provides the motivation for considering the constraints placed on the parameters by the hypothesis, developed in the following section, that they are derived from mean-variance optimization by investors. If one believes this hypothesis, then the resulting estimates will be more precise.

3. DERIVATION OF ASSET-DEMAND FUNCTIONS FROM MEAN-VARIANCE OPTIMIZATION

In this section we derive the correct form for the asset-demands of an investor who maximizes a function of the mean and variance of his end-of-period real wealth.[9] The reader familiar with Kouri (1977) or Dornbusch (1982), or with the general approach, which is standard in the CAPM literature, is urged to skip to the next section.

Let W_t be real wealth. The investor must choose the vector of portfolio shares x_t that he wishes to allocate to the various assets. End-of-period real wealth will be given by:

$$\begin{aligned} W_{t+1} &= W_t + W_t\, x_t'\, r_{t+1} + W_t\left(1-x_t'\iota\right)r_{t+1}^{\$} \\ &= W_t\left[x_t'\, z_{t+1} + 1 + r_{t+1}^{\$}\right], \end{aligned} \tag{5}$$

where we have defined the vector of returns on the five assets relative to the dollar: $z_{t+1} \equiv r_{t+1} - \iota r^{\$}_{t+1}$.

The expected value and variance of end-of-period wealth (5), conditional on current information, are as follows:

$$EW_{t+1} = W_t[x_t' \, Ez_{t+1} + 1 + Er^{\$}_{t+1}]$$

$$VW_{t+1} = W_t^2[x_t' \, \Omega x_t + Vr^{\$}_{t+1} + 2x_t' \, \text{Cov}(z_{t+1}, r^{\$}_{t+1})],$$

where we have defined the variance-covariance matrix of relative returns:

$$\Omega \equiv E(z_{t+1} - Ez_{t+1})(z_{t+1} - Ez_{t+1})'.$$

The hypothesis is that investors maximize a function of the expected value and variance:

$$F[E(W_{t+1}), V(W_{t+1})].$$

We differentiate with respect to x_t:

$$\frac{dF}{dx_t} = F_1 \frac{dEW_{t+1}}{dx_t} + F_2 \frac{dVW_{t+1}}{dx_t} = 0.$$

$$F_1W_t[Ez_{t+1}] + F_2W_t^{\,2}[2\Omega x_t + 2 \, \text{Cov}(z_{t+1}, r^{\$}_{t+1})] = 0.$$

We define the coefficient of relative risk-aversion $\rho \equiv -W_t2F_2/F_1$, which is assumed constant.[10] Then we have our result:

$$Ez_{t+1} = \rho \text{ Cov}(z_{t+1}, r^{\$}_{t+1}) + \rho\Omega x_t. \tag{6}$$

This expression for the expected relative return is analogous to the unconstrained system (2), which was estimated in the previous section, with $\beta^{-1} \equiv \rho\Omega$ and $\beta^{-1}\alpha \equiv \rho \text{ Cov}(z_{t+1}, r^{\$}_{t+1})$. For economic intuition, we invert (6) to solve for the portfolio shares, the form analogous to (1):

$$x_t = -\Omega^{-1} \text{ Cov}(z_{t+1}, r^{\$}_{t+1}) + (\rho\Omega)^{-1}Ez_{t+1}. \tag{7}$$

The asset demands consist of two parts. The first term represents the "minimum-variance" portfolio. If an investor is extremely risk-averse ($\rho = \infty$), the investor will hold the minimum-variance portfolio. For example, suppose he views the dollar as a safe asset, which requires not only that he consume only U.S. goods but also that U.S. prices are non-stochastic when expressed in terms of dollars. Then his minimum-variance portfolio is zero in each of the other five assets. The second term represents the "speculative" portfolio. A higher expected return on a given asset induces investors to hold more of that asset than is in the minimum-variance portfolio, to an extent limited only by the degree of risk-aversion and the uncertainty of the return.

4. ESTIMATION OF ASSET-DEMAND FUNCTIONS CONSTRAINED TO BE OPTIMIZING

In this section we estimate the system (4) subject to the constraint that we found in the last section to be an implication of mean-variance optimization:[11, 12] $\beta^{-1} = \rho\Omega$. The key insight is that Ω is precisely the variance-covariance matrix of the error term: $\Omega \equiv E\varepsilon_{t+1}\varepsilon_{t+1}'$. Imposition of a constraint between coefficients and variances, as opposed to a constraint among coefficients, is unusual in econometrics, and requires maximum likelihood estimation.[13] The Appendix derives the first-order conditions for the maximization of the likelihood function and describes the program used.

If the aim, under the a priori constraint of mean-variance optimization, is to use the information to get the most efficient possible estimates of the parameters, then one might wish to impose not only the constraint that the coefficient matrix is proportional to the variance-covariance matrix Ω, but to impose as well an a priori value for the constant of proportionality, which is the constant of relative risk aversion ρ. De Macedo (1980) and Krugman (1981) refer to the "Samuelson presumption" that $\rho = 2.0$. Table 2 reports the estimated parameters for the case $\rho = 2.0$. The results look quite different from those in table 1. If one believes in the constraints, then the difference is simply the result of more efficient estimates. One would have to invert the coefficient matrix in order to recover the original β matrix and see which assets are close substitutes for which other assets.

But we have chosen in this paper to emphasize the use of our technique to test the hypothesis of mean-variance optimization, rather than the use of the technique to impose the hypothesis. The log likelihood for the estimates in table 2 is 1057.05, a decrease from the unconstrained log likelihood 1086.49. In other words, the fit has

TABLE 2

CONSTRAINED ASSET-DEMAND FUNCTIONS
MLE

DEPENDENT VARIABLE: $r_{t+1} - r^{\$}_{t+1}$, REAL RATE OF RETURN ON NATIONAL CURRENCY RELATIVE TO THE DOLLAR

SAMPLE: JUNE 1973–AUGUST 1980 (87 OBSERVATIONS)

National Currency	Constants	β^{-1} Constrained to $\rho\Omega$, with $\rho = 2.0$				
		$x_t^{C\$}$	x_t^{F}	x_t^{DM}	$x_t^{¥}$	$x_t^{£}$
Canadian dollar	−.00103	.00037	.00010	.00021	.00002	.00009
French franc	.00140	.00010	.00188	.00169	.00099	.00100
Deutsche mark	.00050	.00021	.00169	.00223	.00106	.00107
Japanese yen	.00193	.00002	.00099	.00106	.00196	.00081
Pound sterling	.00211	.00009	.00100	.00107	.00081	.00158

Log likelihood = 1057.05.

See table 1 for definitions of variables.

worsened. Twice the difference is 59.0, which is above the 5 percent critical level of 37.7. This constitutes a clear rejection of the optimization hypothesis.

Perhaps 2.0 is not the correct value for the constant of risk-aversion ρ. We used the MLE program to find simultaneously the values of ρ and Ω that maximize the likelihood. The log likelihood at this point is 1057.96. (The MLE estimate of ρ is -67.0!) The value of ρ makes almost no difference; we are still able to reject the hypothesis easily.

5. CONCLUSIONS

The theory of expected utility maximization, and in particular the simple framework of mean-variance optimization, is a very attractive way to bring more structure to the problem of asset-demand functions. The reader who is a priori inclined to accept that framework can view the numbers reported in table 2 as efficient estimates of the parameters in an international asset-demand function. The estimates are efficient because they use the information that, if investors are indeed optimizing, the coefficient matrix should be proportional to the error variance-covariance matrix. At the same time, the estimates can be argued to be superior to those in previous studies of the optimal portfolio because they use data on asset supplies and thus allow expected real returns to change from period to period.

However the primary aim of this paper is to test explicitly the validity of the hypothesis of mean-variance optimization. The likelihood ratio test rejects the constraints imposed by the hypothesis. Thus, if we are to believe these results, the unconstrained parameter estimates reported in table 1, as imprecise as they are, are the best we can do.

How could investors fail to optimize with respect to the mean and variance of their real wealth? It is possible that they are simply not sophisticated enough. The literature on equity markets, for example, cannot be said to have found good empirical evidence for the CAPM theory.[14] Of course it is possible that agents are rational, but are optimizing subject to constraints such as imperfect capital markets. A corporation may use as its measure of risk the variance of its own dollar profits, as opposed to the covariance with the market portfolio that the finance theory says it should use; and yet this may be rational if the corporation finances its projects internally and has to pay a penalty whenever an unexpected fall in earnings forces it to borrow externally. The same could be true of an individual.[15]

On the other hand, investors may be <u>too</u> sophisticated to optimize (merely) with respect to the mean and variance of their real end-of-period wealth. Stulz (1981), Hodrick (1981), and Hansen and Hodrick (1982) argue that investors maximize a more complicated intertemporal utility function. Unfortunately their theoretical results are not as conducive to empirical testing as is the one-period mean-variance framework.[16]

The theory tested in this paper is one commonly discussed in the literature. The theory requires many assumptions: one-period expected utility maximization, a normal distribution for underlying returns, a constant variance-covariance matrix, constant relative risk-aversion, homogenous investors, rational expectations, asset supplies that are properly measured from variables like government debt and foreign exchange intervention, and perfect capital markets. The failure of any one of these assumptions would explain the test result, the rejection of the theory.

APPENDIX

The parameters of the model were estimated by a maximum likelihood routine based on Berndt, Hall, Hall and Hausman's (1974) maximizing algorithm for nonlinear models. The program makes use of the likelihood function, and its first derivatives. The log of the likelihood, under the normality assumption, is

$$L = -\frac{GT}{2}\log(2\pi) - \frac{T}{2}\log|\Omega| - \frac{1}{2}\sum_{t=1}^{T} \varepsilon'_{t+1}\Omega^{-1}\varepsilon_{t+1}$$

where

$$\begin{aligned}\varepsilon_{t+1} &= z_{t+1} - Ez_{t+1}\\ &= z_{t+1} - c - \rho\Omega x_t\end{aligned}$$

and G is the number of equations (five) and T is the number of observations (eighty-seven).

In standard econometric problems the symmetry of the variance-covariance matrix Ω can safely be ignored in deriving the first-order conditions, because the ij^{th} element and the ji^{th} element of Ω enter the likelihood function symmetrically. In our problem, this is not true because of the restriction that Ω be proportional to the coefficient matrix, so care must be taken to properly allow for the symmetry. First, we derive $\partial L/\partial\Omega$ for an arbitrary (nonsymmetric) Ω.

$$\partial L/\partial\Omega = -\frac{T}{2}\frac{\partial \log|\Omega|}{\partial\Omega} - \frac{1}{2}\sum_{t=1}^{T}\frac{\partial\varepsilon'_{t+1}\Omega^{-1}\varepsilon_{t+1}}{\partial\Omega}$$

$$= -\frac{T}{2}\Omega'^{-1} + \frac{1}{2}\sum_{t=1}^{T}[\Omega'^{-1}\varepsilon_{t+1}\varepsilon'_{t+1}\Omega'^{-1} + \rho(\Omega^{-1}\varepsilon_{t+1} + \Omega'^{-1}\varepsilon_{t+1})x'_t].$$

Now, imposing symmetry, we let

$$Q = [q_{ij}] = -\frac{T}{2}\Omega^{-1} + \frac{1}{2}\sum_{t=1}^{T}[\Omega^{-1}\varepsilon_{t+1}\varepsilon'_{t+1}\Omega^{-1} + 2\rho\Omega^{-1}\varepsilon_{t+1}x'_t].$$

Then, if ω_{ij} is the ij^{th} element of Ω,

$$\partial L/\partial\omega_{ii} = q_{ii}$$

and

$$\partial L/\partial\omega_{ij} = q_{ij} + q_{ji}, \quad i \neq j.$$

We also have

$$\partial L/\partial\rho = \sum_{t=1}^{T}\varepsilon'_{t+1}x_t$$

and

$$\partial L/\partial c = \sum_{t=1}^{T}\Omega^{-1}\varepsilon_{t+1}$$

where Ω has been assumed symmetric.

FOOTNOTES

[1]The field was pioneered by Kouri (1976, 1977), Solnik (1973), and Grauer, Litzenberger, and Stehle (1976). The results have recently been reformulated in a manner simple enough for direct use in macroeconomic models by Dornbusch (1982), with an amendment by Krugman (1981). Other recent contributions include Adler and Dumas (1976, 1981), Frankel (1979), Fama and Farber (1979), Garman and Kohlhagen (1980), Stulz (1981), and Hodrick (1981).

[2]Examples are Roll and Solnik (1977), Cornell and Dietrich (1978), Kouri and de Macedo (1978), de Macedo (1980), and Dornbusch (1980).

[3]Two recent studies of the optimal portfolio, von Furstenberg (1981) and de Macedo, Goldstein, and Meerschwam (1982), do allow expected returns to vary over time, but only gradually: they are estimated from the time series of actual returns as in the technique of "rolling regressions." The present study allows expected returns to change from period to period, for example in response to new information not contained in the past time series of the returns themselves or even of other obvious macroeconomic variables.

[4]Examples of studies that attempt to relate returns to asset supplies without imposing all constraints of mean-variance optimization are Dooley and Isard (1979), Frankel (1982a), and Kasliwal (1982). The last does impose symmetry on the coefficient matrix.

[5]Frankel (1981) and Frankel (1982b) treat only the exchange rates as stochastic. The former paper differs further from the present one by restricting the assets to two: marks and dollars. The latter paper differs from the present one by imposing the optimization hypothesis, and thus obtaining more efficient estimates of the parameters, rather than testing the hypothesis. Among previous studies of the optimal portfolio, Kouri and de Macedo (1978), de Macedo (1980), and de Macedo, Goldstein, and Meerschwam (1982) have allowed for stochastic price levels. Among joint tests of market efficiency and risk-neutrality, Frenkel and Razin (1980) and Engel (1982) have allowed for stochastic price levels.

[6]Among theoretical models, some like Grauer, Litzenberger, and Stehle (1976), Fama and Farber (1979), and Frankel (1979), assume that all investors consume a common basket of goods; others like Solnik (1973) assume that investors of each country consume only their own goods; while still others like Kouri (1976) and Dornbusch (1982) allow investors of each country to consume baskets that include foreign goods but that are more heavily weighted toward their own goods. The last framework is adopted in Frankel (1981, 1982b). Since data on asset supplies are available only in aggregate form, not broken down by holder, differing asset-demand functions have to be aggregated before they can be estimated. When all investors share the same source of uncertainty, the exchange rate, this can be done, using data on the distribution of wealth, as in those two papers. When prices of national goods are stochastic as well, as in the present paper, the aggregation is not possible.

[7]The validity of the technique depends on the assumption that the asset-demand function (1) holds exactly and that asset supplies are correctly measured. As always, omitted variables or measurement errors would render the estimates biased and inconsistent. These considerations justify, at a minimum, special care in the calculation of the asset supply variables, described in the data appendix available in Frankel (1982b). Very briefly, the net supply of assets denominated in a country's currency is calculated as the cumulation of that country's government debt, corrected for three factors: (1) debt issued in foreign currency, (2) foreign exchange intervention by the country's central bank (inferred from data on international reserve holdings by correcting for valuation changes), and (3) foreign exchange intervention in the domestic currency by <u>other</u> countries' central banks (a factor often neglected in empirical studies). It might seem that standard CAPM tests have fewer measurement error problems since only data on rates of returns are used. But, we argue, the assumption that expected returns can be measured by sample means may present a far greater measurement problem.

[8]See, for example, Mussa (1979).

[9]The assumption that returns are normally distributed is sufficient to imply that investors look only at the mean and variance. The normality assumption might be justified by an appeal to a continuous-time diffusion process observed at discrete intervals, and is necessary for the maximum likelihood estimation in any case.

[10]The Arrow-Pratt measure of risk aversion is defined as $\rho \equiv -U''W/U'$, where $U(W)$ is the utility function, the expectation of which is to be maximized. One can take a Taylor-series approximation to $EU(W)$ and differentiate it with respect to $E(W)$ and $V(W)$ to show that the two definitions of ρ are equivalent.

The utility function will have a constant coefficient of relative risk-aversion if it is exponential in form:

$$U(W) = \frac{1}{\gamma} W^{\gamma}, \quad \text{where} \quad \rho = 1 - \gamma.$$

(The solution to the one-period maximization problem considered here will be the correct solution to the general intertemporal maximization problem, if the utility function is further restricted to the logarithmic form, the limiting case as γ goes to zero, which implies $\rho = 1$, or if events occurring during the period are independent of the expected returns that prevail in the following period. See Merton (1973, pp. 877-78) or Fama (1970.))

[11]We do not impose the other constraint,

$$\beta^{-1}\alpha = \rho \operatorname{Cov}\left(z_{t+1}, r_{t+1}^{\$}\right),$$

because the absolute expected return on dollars $Er_{t+1}^{\$}$ does not fall out of the regression as the relative expected returns Ez_{t+1} do. The constraint offers only five overidentifying restrictions anyway, whereas we already have twenty-five from our constraint on the coefficient matrix.

[12]An alternative approach would be to derive the maximizing system in terms of six absolute returns, rather than five relative returns:

$$\begin{bmatrix} r_{t+1} \\ \cdots\cdots \\ r^{\$}_{t+1} \end{bmatrix} = r^0_t \begin{bmatrix} \iota \\ \cdots\cdots \\ 1 \end{bmatrix} + \rho\Sigma \begin{bmatrix} x_t \\ \cdots\cdots \\ 1-x_t \end{bmatrix} + u_{t+1}$$

where Σ is the variance-covariance matrix of the errors u_{t+1} made in predicting the absolute returns r_{t+1}, as opposed to the errors ε_{t+1} made in predicting the relative returns z_{t+1}. The advantage would be that because Σ is six-by-six, we would have more overidentifying restrictions. The disadvantage is that the intercept term $(r^0 \equiv \lambda W/F_1$, where λ is the Lagrangian shadow-price of wealth), though constant across equations, is not constant across time. A separate value of r^0_t could be estimated for each point in time, but the large-sample properties of such an estimator are unclear. Subtracting the last row from each of the others eliminates r^0_t, and restores us to equation (6). The lost row of Σ seems a small price to pay.

[13]The idea of estimating asset-demand equations by drawing the link between the matrix of coefficients of the expected returns and the variance-covariance matrix of the actual returns is not entirely new. See, for example, Parkin (1970) and Wills (1979).

[14]See, for example, Roll (1977) and the references cited there.

[15]If optimizing residents of different countries consume different baskets of goods, then they will use the variances of different quantities to measure risk, and the aggregation in this paper will be invalid. As in Frankel (1982b), we can disaggregate according to seven

areas of residence: the six countries whose currencies are used here, and the rest of the world. Residents of each area are assumed to evaluate returns in terms of a weighted average of the six countries' prices, with prices assumed nonstochastic when denominated in the currency of the producing country and weights determined by that area's consumption shares. A likelihood ratio test then again rejects the constraint of mean-variance optimization. The likelihoods are 1043 unconstrained and 987 constrained with $\rho = 2.0$. We are indebted to Tony Rodrigues for these calculations.

[16]However, if the coefficient of risk-aversion ρ is close to 1.0, then the intertemporal complications vanish, as mentioned in footnote 10. As a further bonus, the need to distinguish among investors according to their consumption basket, discussed in footnotes 6 and 15, also vanishes. (See Adler and Dumas (1981) or Krugman (1981).) Our likelihood ratio test, of course, also rejects the mean-variance optimization hypothesis when ρ is constrained to 1.0.

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