NBER WORKING PAPER SERIES

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Working Paper 10379
http://www.nber.org/papers/w10379

NATIONAL BUREAU OF ECONOMIC RESEARCH
1050 Massachusetts Avenue
Cambridge, MA 02138
March 2004

We thank the following for valuable comments: Philippe Bacchetta, Gene Kandel, Nelson Mark, Michael Moore, Andrew Rose, Eric van Wincoop, and seminar participants at the University of Chicago, U.C. Berkeley, the University of Virginia, Princeton, the IMF, the conference on Micro Analysis in International Macro (April 2003), the NBER IFM meeting (July 2003), and the Econometric Society meetings (Jan. 2004). Both authors thank the National Science Foundation for financial support, which includes funding for a clearinghouse for micro-based research on exchange rates. The views expressed herein are those of the authors and not necessarily those of the National Bureau of Economic Research.

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JEL No. F3, F4, G1

ABSTRACT

We address the exchange rate determination puzzle by examining how information is aggregated in a dynamic general equilibrium (DGE) setting. Unlike other DGE macro models, which enrich either preference structures or production structures, our model enriches the information structure. The model departs from microstructure-style modeling by identifying the real activities where dispersed information originates, as well as the technology by which information is subsequently aggregated and impounded. Results relevant to the determination puzzle include: (1) persistent gaps between exchange rates and macro fundamentals, (2) excess volatility relative to macro fundamentals, (3) exchange rate movements without macro news, (4) little or no exchange rate movement when macro news occurs, and (5) a structural-economic rationale for why transaction flows perform well in accounting for monthly exchange rate changes, whereas macro variables perform poorly. Though past micro analysis has made progress on results (1) through (3), results (4) and (5) are new. Excess volatility arises in our model for a new reason: rational exchange rate errors feed back into the fundamentals that the exchange rate is trying to track.

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Introduction

Two micro-founded approaches to exchange rates emerged in the 1990s, but there remains a distressing disconnect between them. This paper addresses whether connecting them can resolve the most researched puzzle in international macroeconomics: the fact that macro fundamentals do not explain monthly exchange rate changes (the determination puzzle; see Meese and Rogoff 1983). The two approaches that emerged are the dynamic general equilibrium (DGE) approach and the microstructure approach.[2] DGE modeling is increasingly rich in its preference structures (tastes) and production structures (technology), but has not yet ventured beyond common-knowledge information structures (e.g., toward information that originates in a dispersed form). The microstructure approach, in contrast, is focused on richer information structures, at the cost of relying on rather stylized, partial equilibrium analysis (e.g., informative signals are introduced without ever specifying what the deep economic fundamentals are, and without ever considering that the fundamentals themselves are determined in part by the signals received). The "new micro" approach we propose here connects the two.[3] The model embeds a micro process of information aggregation into a macro DGE setting.

In this paper we adopt stochastic productivity as the driving force behind the exchange rate. By going this route, we have chosen to anchor exchange rate determination with a real factor, though this is not necessary. (The information approach is flexible enough to accommodate, for example, shocks to money demands, or shocks to risk preferences.) Anchoring exchange rates with a real factor shows that our information dynamics are not special to financial transactions and the associated nominal variables. The essential ingredient is that individuals' currency trades are more correlated with (unobserved) shocks to home-country productivity than with shocks to foreign productivity. We have in mind an economy in which bits of information about realized productivity are present initially at the micro level, i.e., at the level of individual firms. No one of these firms considers itself to have superior information. But if the currency trades of these individual firms are correlated with their own micro-level productivities, then *aggregated* trades initiated by home agents convey incremental information about the home shock. This information structure differentiates the macro side of our model from the DGE macro literature. Beyond this, the macro features of our model are quite standard, in fact rather streamlined. There is a continuum

[2] DGE examples include Backus, Kehoe, and Kydland (1994), Obstfeld and Rogoff (1995,1998), and Chari, Kehoe, and McGrattan (2002), among many others. Use of signed transaction quantities, the hallmark of microstructure analysis, includes Lyons (1995), Rime (2001), Evans (2002), Evans and Lyons (2002a), Bjonnes and Rime (2003), and Payne (2003), among many others.

[3] Though "new micro" analysis relies heavily on the theory of microstructure finance, it draws only from the information branch of microstructure modeling and addresses different questions, hence the need for a label other than microstructure. (New micro is oriented toward macro questions, whereas microstructure finance is oriented toward micro questions such as institution design, regulation, individual behavior, and partial-equilibrium price determination.)

of agents in each of the two countries, with utility defined over consumption of a home and a foreign good. Agents have access to two financial assets, home- and foreign-currency deposits, which pay interest monthly.

The micro features of the model are closely related to microeconomic models of asset trade. In these models, financial intermediaries act as marketmakers who provide two-way prices. We introduce liquidity provision of this type by assuming that all agents engage in both consumption and marketmaking.[4] This consolidates the activities of households with that of financial institutions in a way similar to the "yeoman farmer" consolidation of consumption and production decisions in the new-macro branch of DGE models. The consolidation greatly facilitates integration of elements from the microstructure approach into a DGE setting.[5] In particular, it ensures that the objectives of financial-market participants are exactly aligned with those of consumers. All trading is therefore consistent with expected utility maximization; noise traders, behavioral traders, and other non-rational agent types are absent.

The model shows that richer—and more realistic—information structures produce exchange rate behavior that aligns closely with empirical facts. With respect to the determination puzzle in particular, relevant results include: (1) persistent gaps between exchange rates and macro fundamentals, (2) excess volatility relative to macro fundamentals, (3) exchange rate movements without macro news, and (4) little or no exchange rate movement when macro news occurs. Intuition for these results is as follows. Persistent gaps between exchange rates and fundamentals arise because the underlying state of fundamentals—which corresponds to the union of all information sets—is revealed only gradually. Though exchange rates fully reflect all public information, they never reflect all information. Volatility in excess of fundamentals occurs because real allocations are distorted by rational exchange rate errors—an "embedding effect"; these distorted real allocations induce additional volatility in exchange rates.[6] (Note that past micro models of exchange rates cannot produce excess volatility from this source since they do not permit feedback from information and exchange rates back to fundamentals.) Exchange rates move without macro news because microeconomic actions—in particular, trades—convey information, even when public macro news is not present. On the flipside, there may be no impact on exchange rates from macro news if prior microeconomic aggregation of information renders that news redundant (this being another result

[4] Note the emphasis here on liquidity provision that is private, in contrast to the public provision of liquidity (in the form of central banks) at the center of the monetary approach to exchange rates.

[5] To non-macro readers this type of consolidation is surely unfamiliar. The assumption facilitates general equilibrium analysis because the agent population remains defined over a single continuum, and differences along that continuum arise as parsimoniously as possible to capture the model's essential features.

[6] For further intuition on embedding, recognize that the exchange rate, as an asset price, is free to jump, whereas real variables (like total output) are not. Suppose home agents over-estimate real ouput and consume too much today (resulting in part from an overvalued real exchange rate). The following period the exchange rate must depreciate from its over-valued level, not only enough to reduce consumption to reflect lower-than-expected output, but also to compensate for the distorted consumption decision.

that is new to micro modeling).

This paper belongs to a recent theoretical literature that emerged to address why exchange rates are so well explained by signed transaction flows (e.g., 40 to 80 percent of daily changes explained, for a host of major currencies; see Evans and Lyons 2002a,b). Our model shows why signed transaction flows *should* have better explanatory power than macro variables. The basic idea is that, in a setting of dispersed information, aggregate transaction flows provide a stronger signal of changing macro fundamentals. The model of Hau and Rey (2002) goes a different route in addressing the empirical significance of transaction flows by introducing two elements: a central role for cross-border equity flows and a private-sector supply of foreign exchange that is price elastic. The latter means that cross-border equity flows affect exchange rates via induced currency transactions. In a nutshell, their focus for understanding currency movements is on shocks in equity markets, a substantial departure from the traditional asset approach which emphasizes instead the importance of bond markets. Their focus is not on information aggregation as ours is here (no information aggregation takes place in their model). A second paper along this theoretical line is Bacchetta and van Wincoop (2003), which does explicitly address how transaction flows relate to information aggregation. Their model is a rational-expectations model of trade (in the spirit of Grossman and Stiglitz 1980). An important finding in that paper is that greater dispersion of information across agents can lead to greater price impact from non-fundamental trades (resulting from rational confusion of non-fundamental trades for fundamental trades). Our modeling departs from theirs in two main ways. First, our DGE setting extends "upstream" in the information process in that it specifies the structural source of the information that currency markets need to aggregate (i.e., the underlying economic activities that produce it.) Second, marketmaking in our model aligns closely with actual institutions, so empirical implications are readily implementable with existing data. A third recent paper, Devereux and Engel (2002), shares both our DGE approach and a role for marketmakers, but nevertheless maintains a common-knowledge information structure. Marketmakers in their model are explicitly non-rational.[7]

The dispersed information we address here is qualitatively different than concentrated information, where one or a few "insiders" have large information advantages (and know it).[8] It is dispersed information that characterizes most variables at the center of exchange rate modeling, such as output, money demand, inflation, consumption preferences, and risk preferences. These variables are not realized at the macro level, but rather as aggregations of underlying micro realizations. For

[7]In a risk-neutral setting, the trades of marketmakers in the Devereux and Engel (2002) model would not affect price: they convey no cash-flow information and, with risk neutrality, they cannot affect risk premia (also true of the non-rational traders in Jeanne and Rose 2002). In contrast, trades of marketmakers in our model would still affect price under risk neutrality because they do so by affecting expected cash flows.

[8]In this way, the model abstracts from strategic behavior at the individual level. Strategic interaction is important for understanding collapsing fixed exchange rates (see, e.g., Corsetti et al. 2001), but less important for the everyday functioning of major floating-rate currencies.

some of these measures, such as risk preferences and money demands, official aggregations of the underlying micro-level shocks do not exist, leaving the full task of aggregation to markets. For other variables, official aggregations exist, but publication trails underlying realizations by 1-4 months, leaving much room for market-based aggregation in advance of publication. (Even after publication there is room for market-based aggregation: initial publication is often noisy, as evidenced by large subsequent revisions.) Existing macro models of exchange rates do not admit information that requires aggregation by markets. Instead, relevant information is either symmetric economy-wide, or, in some models, asymmetrically assigned to a single agent—the central bank. This is at odds with empirical evidence, which shows that dispersed information is indeed being impounded by markets.[9] The challenge is modeling the mechanics of this information process and how markets implement it.

Methodologically, the DGE environment we study has a number of novel features. First, financial markets in our model are incomplete, which, among other things, makes room for the exchange rate to be determined from more than just the marginal rate of substitution between home and foreign consumption goods (see Duarte and Stockman 2001). In particular, the exchange rate is pinned down by expectations via a present-value relation in a manner familiar to the asset approach. Second, the model embeds social learning: agents learn from the equilibrium actions of others. Third, the presence of social learning means that we need to solve each agent's decision problem and inference problem jointly. More concretely, the solution begins with a conjecture about each agent's information set, and concludes with verification that these conjectured information sets line up with information provided by market outcomes. Fourth, our solution accounts for agent risk aversion. Risks associated with incomplete knowledge about the economy's state influence consumption and trading decisions (which, in turn, affect the inferences agents draw from market outcomes). To our knowledge, this is the first paper to solve a DGE model with this combination of risk-averse decision-making, heterogeneous information, and social learning.

Our DGE methodology highlights several implications of dispersed information that are not evident from partial equilibrium microstructure analysis. First, though the timing of information receipt is exogenous, the timing of impounding in price is endogenous. This is because the market signals that lead to that impounding are themselves endogenous. Second, DGE modeling of price discovery shows that real decisions are affected, with the degree depending on the pace of (endogenous) revelation. Accordingly, in a DGE setting such as this, one can address questions such as, What is the welfare-optimal pace of revelation? (It is well known that fast revelation may not be

[9] This evidence is from both micro studies of individual price setters and macro studies of price setting marketwide. See, e.g., the papers noted in footnote 2, as well as Covrig and Melvin (2002), Froot and Ramadorai (2002), and Evans and Lyons (2002b). The basic idea here is that demand is playing two distinct roles: the traditional market-clearing role and a non-traditional information-communication role.

optimal because, for example, it can impede risk sharing.) Third, the information structure of the DGE model provides needed clarity on why transaction effects on exchange rates should persist and, importantly, whether that persistence applies to real exchange rates or only to nominal rates.[10] Persistence will apply to real exchange rates if, for example, signed transaction flow is conveying information about underlying real shocks that are themselves permanent.

The remainder of the paper is organized as follows. Section 1 presents some over-arching characteristics of the model. Details of the model are laid out formally in Section 2. Section 3 describes the steps involved in solving for equilibrium. Section 4 studies the equilibrium, with particular focus on pricing dynamics at both high frequencies and low. Propositions there address the pace of revelation, the embedding of mistakes in fundamentals, and excess volatility. In Section 5, we address various other implications of interest (e.g., announcement effects, trading volume, and the information content of different flow measures). Section 6 concludes. An appendix presents the paper's analytical detail.

1 Theoretical Overview

Our intention is to lay out a new genre of information model, one that identifies primitive shocks and their propagation in ways that partial-equilibrium models cannot. This new genre of model will have three essential ingredients: (i) it needs to specify an endowment process (or production technology) for dispersed information, (ii) it needs to specify the information available for financial pricing, and (iii) it needs a solution methodology that maps individual information sets into equilibrium actions that, once observed, support those individual information sets. The model below is but one set of choices in these three dimensions. In its presentation, we touch on other options so that those with modeling preferences different from ours will have a sense of the wider possibilities.

The first of these essential ingredients—specification of dispersed information—is a qualitative departure from existing DGE work in macro. Rather than enriching the preference structure (tastes) or production structure (technology), this genre of model enriches the information structure. In departing from common knowledge, the focus is on price effects from information that persist, not on "microstructure effects" (where by the latter we mean transitory price effects from marketmaker risk management, or from bouncing between bid and ask prices); from a macro perspective, these microstructure effects are second order. The fundamental driver of the model we present is real

[10]With respect to the information conveyed by flows, it is important to distinguish order flows from portfolio flows. Order flows—by tracking the initiating side of transactions—are a theoretically sound way to distinguish *shifts* in demand curves from *movements along* demand curves. Informationally, the two are different: there is news in curve shifts, but no news in price-induced movements along known curves (the latter representing a type of feedback trading). For portfolio flows, theory provides little guidance on which flows in the aggregate mix reflect the news, i.e., the demand-curve shifts. We return to this when discussing implications of the model in section 5.

productivity. But as noted, we could just as easily set up the model with another real shock as the fundamental driver, or with a nominal shock as the fundamental driver (e.g., assuming that individuals' trades are correlated with unobserved shocks to home money demand). Finally, for those interested in integrating sticky goods prices and imperfectly competitive firms, these key features of open-macro modeling could be introduced in the usual way. We chose the most streamlined structure possible to highlight the new information dimension.

The second essential ingredient—specification of the information available for financial pricing—really embeds two elements. The first is that financial pricing needs to be grounded in well-specified decision problems. Our model makes this explicit. Second, and more important, any explicit model of liquidity provision needs to take a stand on information sets: what information do agents have at their disposal for setting transactable prices? To get traction on this front, the genre of models we work with here relaxes (realistically) the common but extreme assumption that information aggregation takes place instantaneously (i.e., before any transactions take place; see, e.g., Grossman and Stiglitz 1980). When information aggregation takes place instantaneously, the resulting transaction information conveys no information that is not already embedded in the transaction price. There is no learning from order flow ex post, and indeed, no information value to transaction flows whatsoever (both radically counterfactual implications).[11] In the model below, we choose instead a "simultaneous trade" design (see, e.g., Lyons 1997). The simultaneous trade design specifies simultaneous actions, in the sense that trading at any point in time occurs simultaneously throughout the economy (in the spirit of simultaneous-move games in game theory). In essence, this imposes a constraint on the information available for making trading decisions because simultaneous moves cannot be conditioned on one another. More concretely, one cannot condition on the actual trading intentions of all other agents in the economy at the time one chooses to trade (save doing one's best to forecast them). We find this an inherently realistic assumption. Though a convenient way to relax the extreme assumption of instantaneous aggregation, it is certainly not the only possibility for the genre. For example, one could also take an intermediate road and assume that financial transactions at any "point" in time are executed sequentially (a la Glosten and Milgrom 1985). In this case, the earlier the trade in the sequence, the more limited the conditioning information (because early trades cannot condition on information conveyed by later trades). This alternative produces the same qualitative constraint on information sets used for pricing that we employ here.

The third essential ingredient of this modeling genre is its solution methodology, which needs to

[11] Another unfortunate feature of Walrasian mechanisms is that agents generally do not take positions that they intend in the future to liquidate (because all trades are conditioned on all concurrent trading information). Among other things, this produces counterfactual predictions about how liquidity is provided in financial markets: transitory position-taking is a deep property of liquidity provision, and is important for understanding how trade quantities (i.e., realized order flow) map into price changes.

map individual information sets into equilibrium actions, and then back to information sets. Here we adopt a guess-and-verify method with the following 5 steps, the first and last of which sharply distinguish our information-theoretic approach from past DGE modeling. In step 1, we make a conjecture about the information available to agents at each point in time. This involves specifying agents' information endowments as well as what they learn by observing trading outcomes. Based on this information structure, in step 2 we guess the form of the equilibrium pricing rules for spot rates and interest rates. In step 3, we solve for optimal consumption and portfolio allocations (based on analytic approximation methods in Campbell and Viceira 2002). Step 4 verifies that agent choices for consumption, investment, and currency holdings clear markets. In step 5, we verify that the conjectured information structure (from step 1) can be supported by an inference problem based on endowment information and information from trading (the latter includes both prices and order flows).

A fourth ingredient of our model below, though not an essential ingredient, is consolidation of consumers and financial liquidity providers. Whereas new-macro DGE models focus on richer micro-foundations on the economy's supply side, hence their consolidation of consumers with producers, our focus is instead on richer micro-foundations in the area of financial price determination. This consolidation serves three main purposes. First, it consolidates budget constraints across the two sets of activities, which simplifies the analytics. Second, it ensures that messy incentive misalignments do not arise (e.g., there are no agency problems). Third, it ensures that the preferences of liquidity providers are in no sense special, as is often the case in partial-equilibrium microstructure modeling. We recognize that for some questions it will be necessary to drop this non-essential ingredient.

2 The Model

2.1 Environment

2.1.1 Preferences

The world is populated by a continuum of infinitely-lived agents indexed by $z \in [0,1]$ who are evenly split between the home country (i.e., for $z \in [0,1/2)$) and foreign country ($z \in [1/2,1]$). For concreteness, we shall refer to the home country as the US and the foreign country as the UK. Preferences for the z'th agent are given by:

$$\mathsf{U}_{t,z} = \mathsf{E}_{t,z} \sum_{i=0}^{\infty} \beta^i U(C_{t+i,z}, \hat{C}_{t+i,z}) \tag{1}$$

where $0 < \beta < 1$ is the subjective discount factor, and $U(.)$ is a concave sub-utility function, which we specialize to log (which exhibits constant relative risk aversion, CRRA):

$$U(\hat{C}_{t,z}, C_{t,z}) = \tfrac{1}{2}\ln \hat{C}_{t,z} + \tfrac{1}{2}\ln C_{t,z}.$$

All agents have identical preferences over the consumption of US goods $C_{t,z}$ and UK goods $\hat{C}_{t,z}$. $\mathsf{E}_{t,z}$ denotes expectations conditioned on agent $z's$ information set at time t, $\Omega_{t,z}$. Expectations conditioned on a common time t information set (i.e., $\Omega_t \equiv \cap_{z\in[0,1]}\Omega_{t,z}$) will be denoted by E_t.

2.1.2 Timing

Decision-making in the model takes place at two frequencies. Consumption-savings decisions take place at a lower frequency than financial decision-making (where the latter includes determination of asset prices and reallocation of portfolios via trading). To implement this idea we split each "month" t into four periods (see Figure 1). Consumption-savings decisions are made "monthly," while financial decisions are made periodically within the month. As will become clear, the use of the term "month" is nothing more than a convenient label: the economic intuition developed by the model is exactly the same if we replaced "month" t by some other consumption-relevant period. That said, let us now describe the structure of the model by considering the "monthly" sequence of four events.

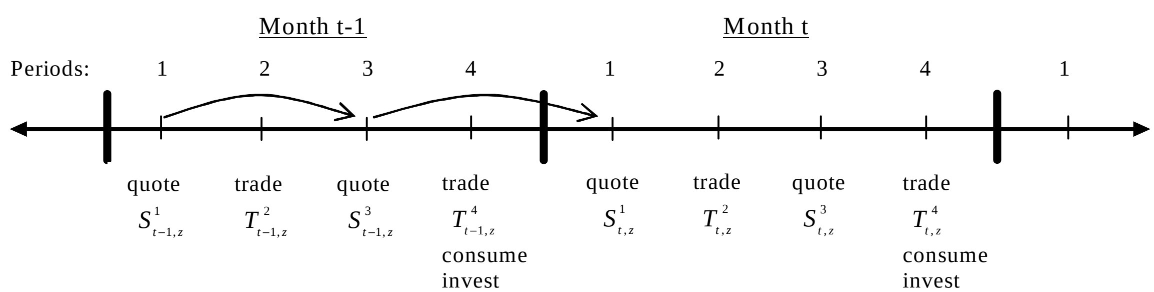

Figure 1: Model Timing

Period 1 (Quoting): Agents begin the month with holdings in three assets: dollar deposits, $B^1_{t,z}$, pound sterling deposits $\hat{B}^1_{t,z}$, and domestic capital ($K_{t,z}$ for US agents and $\hat{K}_{t,z}$ for UK agents).[12] Each agent then quotes a spot price $S^1_{t,z}$ (\$/£) at which he is willing to buy or sell foreign currency

[12]Though agents do not hold foreign real capital, even if they did, financial markets here would still be incomplete: the deeper source of incompleteness in our model is that dispersed information precludes a full set of state-contingent claims. We address this more fully below in section 5.

(£s). These quotes are observable to all agents.[13]

Period 2 (Trading): Each agent z chooses the amount of foreign currency, $T^2_{t,z}$, he wishes to purchase (negative values for sales) by initiating a trade with other agents. The sum of these signed trade quantities is what we shall refer to as the period's order flow. Trading is simultaneous, trading with multiple partners is feasible, and trades are divided equally among agents offering the same quote. (That trades are divided equally is important: in equilibrium it will imply that all agents receive the same incoming order-flow realization.) Once these transactions have taken place, agent z's deposits at the start of period 3 are given by:

$$\begin{aligned} B^3_{t,z} &= B^1_{t,z} + S^1_t T^2_{t,z*} - S^1_t T^2_{t,z}, \\ \hat{B}^3_{t,z} &= \hat{B}^1_{t,z} + T^2_{t,z} - T^2_{t,z*}, \end{aligned}$$

where $T^2_{t,z*}$ denotes the *incoming* foreign currency orders from other agents trading at z's quoted price. S^1_t is the period-1 spot rate quote at which z purchases pounds. In equilibrium, this will be the spot rate quoted by all agents (i.e., $S^1_t = S^1_{t,z}$) for reasons we explain below. Notice that period-3 currency holdings depend not only on the transactions initiated by z, (i.e., $T^2_{t,z}$) but also on the transactions initiated by other agents $T^2_{t,z*}$. An important assumption of our model is that the choice of $T^2_{t,z}$ by agent z cannot be conditioned on the incoming orders $T^2_{t,z*}$ because period-2 trading takes place *simultaneously*. Consequently, though agents target their desired allocation across dollar and pound assets, resulting allocations include a stochastic component from the arrival of unexpected orders from others.

Period 3 (Quoting): All agents again quote a spot price and also a pair of one-month interest rates for dollar and pound deposits.[14] The spot quote, $S^3_{t,z}$, is good for a purchase or sale of pounds, while the interest rates, $R_{t,z}$ and $\hat{R}_{t,z}$ indicate the rates at which the agent is willing to borrow or lend one-month in dollars and pounds, respectively. As in period 1, all quotes are publicly observable.

[13] It will be clear below that consumers in this model have both speculative and non-speculative motives for trading (the non-speculative motive arising from the need to facilitate periodic consumption and investment). That these motives are not purely speculative obviates concern about so-called "no trade" results (i.e., the theorem proposed by Milgrom and Stokey 1982, that if I know that your only motive for trade with me is superior information, then I would never want to trade with you at any price at which you want to trade).

[14] Deposit rates are not set in every period because interest is assumed to accrue at the monthly frequency only. As a qualitative matter, abstracting from intra-month interest misses little in the context of the world's major currencies, all of which are generally characterized by relatively low inflation and low nominal interest rates.

Period 4 (Trading and Real Decisions): In period 4, agents choose a second round of foreign currency purchases (if there remain motives for further intra-month trade).[15] They also choose their real allocations: consumption of US and UK goods and real investment expenditures. After US agents z have chosen their consumption of US and UK goods, $C_{t,z}$ and $\hat{C}_{t,z}$, their foreign currency purchases $T^4_{t,z}$, and their real investment $I_{t,z}$, the resulting deposit holdings in period 1 of month $t+1$ are:

$$B^1_{t+1,z} = R_t(B^3_{t,z} - S^3_t T^4_{t,z} - I_{t,z} + S^3_t T^4_{t,z*} + C_{t,z*}), \quad (2)$$
$$\hat{B}^1_{t+1,z} = \hat{R}_t(\hat{B}^3_{t,z} + T^4_{t,z} - \hat{C}_{t,z} - T^4_{t,z*}) \quad (3)$$

where R_t and $\hat{R}_t$ are the dollar and pound interest rates (gross) that are quoted by all agents in period 3 of month t (in equilibrium, $R_{t,z} = R_t$ and $\hat{R}_{t,z} = \hat{R}_t$ for all z, as shown below). As in period 2 trading, actual deposit holdings following period-4 trading also depend on the actions of other agents. In particular, incoming orders for foreign currency $T^4_{t,z*}$ and incoming orders for US goods $C_{t,z*}$ affect the deposit levels in the first period of the following month. Notice, for example, that $B^1_{t+1,z}$ is augmented by $C_{t,z*}$: these are deposits received in exchange for exports of US goods. For UK agents, the dynamics of deposit holdings are similarly determined by:

$$B^1_{t+1,z} = R_t(B^3_{t,z} - S^3_t T^4_{t,z} - C_{t,z} + S^3_t T^4_{t,z*}), \quad (4)$$
$$\hat{B}^1_{t+1,z} = \hat{R}_t(\hat{B}^3_{t,z} + T^4_{t,z} - \hat{I}_{t,z} - T^4_{t,z*} + \hat{C}_{t,z*}) \quad (5)$$

Finally, we turn to the dynamics of the capital stocks. The production of US and UK goods at the start of month $t+1$, $Y_{t+1,z}$ and $\hat{Y}_{t+1,z}$, is given by:

$$Y_{t+1,z} = A_{t+1}\left(K_{t,z} - C_{t,z} - C_{t,z*} + I_{t,z}\right),$$
$$\hat{Y}_{t+1,z} = \hat{A}_{t+1}(\hat{K}_{t,z} - \hat{C}_{t,z} - \hat{C}_{t,z*} + \hat{I}_{t,z}),$$

where A_{t+1} and $\hat{A}_{t+1}$ are shocks to US and UK productivity. The capital stock at the end of period 4 trading is denoted by the term in parenthesis in each equation. These production functions lead to the following capital accumulation equations:

$$K_{t+1,z} = R^k_{t+1}\left(K_{t,z} - C_{t,z} - C_{t,z*} + I_{t,z}\right), \quad (6)$$
$$\hat{K}_{t+1,z} = \hat{R}^k_{t+1}\left(\hat{K}_{t,z} - \hat{C}_{t,z} - \hat{C}_{t,z*} + \hat{I}_{t,z}\right), \quad (7)$$

[15] That motives for further currency trade within the month will indeed remain is one of the model's important properties. It addresses the question of why agents would want to trade at such high frequencies.

where $R^k_{t+1} \equiv 1 + A_{t+1}$, and $\hat{R}^k_{t+1} = 1 + \hat{A}_{t+1}$ denote the one month returns on US and UK capital. (Depreciation is zero in both countries.) Equation (6) shows how US consumers' holdings of capital evolve, while the dynamics of UK consumers' holdings follow (7).

2.1.3 Productivity and the Information Structure

Our model becomes explicitly "international" with the specification of relative productivity, the driving force behind the exchange rate. The key feature that differentiates US from UK agents is that each agent type is better informed about the productivity of home firms than foreign firms. (This could result, for example, through direct observation of the productivity realization for one's own firm.) As a result, agents in different countries do not share the same expectation about current and future returns to real capital. Below we examine how this dispersed information is impounded in exchange rates and interest rates via trading. Our focus is thus on the process of information transmission, not so much on the specific type of underlying information. The analysis can be extended to include dispersed information about multiple underlying information types.

The exogenous productivity processes are expressed here in terms of log returns on real capital. Though we specify these separately for the US and UK, as we shall see, only relative productivity will matter for exchange rate determination:

$$\ln R^k_t \equiv r^k_t = r + u_t + e_t + \theta(e_{t-1} - \hat{e}_{t-1}), \tag{8a}$$

$$\ln \hat{R}^k_t \equiv \hat{r}^k_t = r + \hat{u}_t + \hat{e}_t + \theta(\hat{e}_{t-1} - e_{t-1}). \tag{8b}$$

We assume that the u_t, $\hat{u}_t$, e_t, and $\hat{e}_t$ are normally distributed mean zero shocks. The u_t and $\hat{u}_t$ shocks have a common variance σ^2_u and the e_t and $\hat{e}_t$ shocks have a common variance σ^2_e. We allow for the possibility of non-zero covariance between the u_t and $\hat{u}_t$ shocks, which we denote $\rho\sigma^2_u$, but for tractability we assume that the e_t and $\hat{e}_t$ shocks are independently distributed.

Our specification for log capital returns includes two random components beyond the constant r: a transitory component u_t ($\hat{u}_t$) and a persistent component e_t ($\hat{e}_t$). The transitory component u_t ($\hat{u}_t$) is a one-month effect on US (UK) returns with cross-country correlation ρ. Unlike u_t ($\hat{u}_t$), the random variable e_t ($\hat{e}_t$) is contemporaneously independent across countries, but gives rise to an intertemporal impact that depends on this component's cross-country differential from the previous period. It should be clear from these two productivity processes that the differential, i.e., $r^k_t - \hat{r}^k_t$, follows a simple MA(1) process. This greatly facilitates analysis of the differential as a driving force (richer processes for this differential get technically difficult quickly). Though not intended as precise empirical representations, we consider it uncontroversial that capital returns should include both transitory and persistent components.

For the analysis below, we examine information structures in which each month t all US agents observe in period 1 their home shocks $\{u_t, e_t\}$, whereas all UK agents observe their home shocks $\{\hat{u}_t, \hat{e}_t\}$.[16] Dispersed information thus exists inter-nationally but not intra-nationally. (One can think of intra-national information as having been aggregated "in the background"; treating these respective information sets as signals by adding idiosyncratic noise is a straightforward extension.) For our purposes here, the specifications in (8) highlight the theoretical consequences of dispersed information in the simplest possible way.

2.2 Decision-Making

Agents make two types of decisions: consumption-savings decisions and financial pricing (quoting) decisions. The former are familiar from standard macro models, but the latter are new. By quoting spot prices and interest rates at which they stand ready to trade, agents are taking on the liquidity-providing role of financial intermediaries. Specifically, the quote problem facing agents in periods 1 and 3 is identical to that facing a marketmaker in a simultaneous trading model (see, for example, Lyons 1997, Rime 2001, Evans and Lyons 2002a). We therefore draw on this literature to determine how quotes are set.

Equilibrium quotes are derived as a Nash equilibrium with the following two properties: (i) they are consistent with market clearing, and (ii) they are a function of public information only. Though the latter property is not necessary for the information transmission role of transaction flows, it is important for this role, so let us address it more fully. With this property, the information in unanticipated flow can only be impounded into price after it is realized and publicly observed. This lies at the opposite pole of the information assumptions underlying Walrasian mechanisms (Grossman and Stiglitz 1980 being an example) in which the market price at a given time impounds information in *every* trade occurring at that time. The Walrasian mechanism is akin to assuming that all trades are conditioned on one another. This is obviously counter-factual in most markets, and certainly so in FX.[17] As noted in the previous section, what is really necessary for the transmission role of transaction flow is that market prices do not yet impound all information in executed transactions. That equilibrium quotes are conditioned only on public information in our model insures this, and goes a bit further to simplify the analytics. This aspect of the model can be viewed as taking seriously the information constraints that price-setters actually face.

We should stress, though, that quotes being conditioned only on public information is not an

[16] This is not the same as assuming two representative consumers: two consumers would interact strategically (a rather implausible notion here), whereas in our continuum consumers are perfectly competitive.

[17] Even if the FX market were organized as a centralized auction with full transparency, this would not be not suffcient for Walrasian-type aggregation: it would also have to be true that in equilibrium all agents would actually choose to trade simultaneously (so that each could condition on the price effects of others' trades). In any case, actual FX markets are not centralized auctions, but rather decentralized dealer markets with trade transparency that is is relatively low.

assumption, but a result. Put differently, we make other assumptions that are sufficient for this outcome (drawing from the simultaneous-trade references above). Those assumptions are (1) that actions within any given quoting or trading period are simultaneous, (2) that quotes are a single price good for any size, and (3) that trading with multiple marketmakers is feasible.[18] The resulting solution to the quote problem facing agent z in periods $j = \{1, 3\}$ will be a quote $S^j_{t,z} = S^j_t$, where S^j_t is a function of public information Ω^j_t (determined below). Similarly, the period-3 interest rate quotes are given by $R_{t,z} = R_t$ and $\hat{R}_{t,z} = \hat{R}_t$, where R_t and $\hat{R}_t$ are functions of Ω^3_t. To understand why these quotes represent a Nash equilibrium, consider a marketmaker who is pondering whether to depart from this public-information price by quoting a weighted average of public information and his own individual information. Any price that deviates from other prices would attract pure arbitrage trade flows, and therefore could not possibly represent an equilibrium. Instead, it is optimal for marketmakers to quote the same price as others (which means the price is necessarily conditioned on public information), and then exploit their individual information by initiating trades at other marketmakers' prices. (In some models, marketmakers can only establish desired positions by setting price to attract incoming trades, which is not the case here since they always have the option of initiating outgoing trades.)

Next we turn to the consumption and portfolio choices made in periods 2 and 4. Let $W^j_{t,z}$ denote the wealth of individual z at the beginning of period j in month t. This comprises the value of home and foreign deposit holdings and domestic capital:

$$\begin{aligned} W^2_{t,z} &\equiv B^1_{t,z} + S^1_t \hat{B}^1_{t,z} + K_{t,z} + S^1_t \hat{K}_{t,z} \\ W^4_{t,z} &\equiv B^3_{t,z} + S^3_t \hat{B}^3_{t,z} + K_{t,z} + S^3_t \hat{K}_{t,z} \end{aligned}$$

Notice that wealth is valued in dollars using the equilibrium spot rate quoted in the period before trading takes place.[19]

In period 2 agents initiate transactions (i.e., choose $T^2_{t,z}$) to allocate wealth optimally between dollar and pound assets. Because trading takes place simultaneously, however, the choice of $T^2_{t,z}$ cannot be conditioned on the orders they simultaneously receive from others, $T^2_{t,z*}$. Of course, in choosing $T^2_{t,z}$ agents do their best to forecast $T^2_{t,z*}$, but they cannot condition on its realization. We denote this forecast of the incoming order as $\mathsf{E}^2_{t,z} T^2_{t,z*}$. (Hereafter we use $\mathsf{E}^j_{t,z}$ to denote expectations conditioned on information available to individual z at the *beginning* of period j in month t.)

[18] The assumption of no spreads is not necessary, though it greatly facilitates the analytics. Specifically, each trader-consumer's quote could be a schedule of prices, one for each incoming order quantity from minus infinity to plus infinity, as long as that schedule is conditioned only on the incoming order, as opposed to the realization of all other orders in the market (i.e., the quoting trader would in this way be able to protect against adverse selection in the single incoming trade).

[19] No single agent can hold both $K_{t,z}$ and $\hat{K}_{t,z}$ since agents hold domestic real capital only; thus, depending on whether $z \lesseqgtr 1/2$, one of these two terms in each equation will equal zero.

Let $J_z^2(W_{t,z}^2)$ and $J_z^4(W_{t,z}^4)$ denote the value functions for agent z at the beginning of periods 2 and 4. $T_{t,z}^2$ is determined as the solution to the following dynamic programming problem:

$$J_z^2(W_{t,z}^2) = \max_{\lambda_{t,z}} \mathsf{E}_{t,z}^2 \left[J_z^4(W_{t,z}^4) \right], \tag{9}$$

s.t.

$$W_{t,z}^4 = H_{t,z}^3 W_{t,z}^2, \tag{10}$$

where

$$\begin{aligned}
H_{t,z}^3 &\equiv \left(1 + \left(\frac{S_t^3}{S_t^1} - 1\right)(\lambda_{t,z} - \xi_t)\right), \\
\lambda_{t,z} &\equiv \frac{S_t^1\left(\hat{B}_{t,z}^1 + \hat{K}_{t,z} + T_{t,z}^2 - \mathsf{E}_{t,z}^2 T_{t,z*}^2\right)}{W_{t,z}^2}, \\
\xi_{t,z} &\equiv \frac{S_t^1(T_{t,z*}^2 - \mathsf{E}_{t,z}^2 T_{t,z*}^2)}{W_{t,z}^2}.
\end{aligned}$$

The choice variable $\lambda_{t,z}$ is key. It identifies the target fraction of wealth agents wish to hold in pounds, given their expectations about incoming orders they will receive during trading, $\mathsf{E}_{t,z}^2 T_{t,z*}^2$. (Outgoing orders $T_{t,z}^2$ are determined from the optimal choice of $\lambda_{t,z}$ given $\mathsf{E}_{t,z}^2 T_{t,z*}^2$, $\hat{B}_{t,z}^1 + \hat{Y}_{t,z} + \hat{K}_{t,z}$, and $W_{t,z}^2$.) $H_{t,z}^3$ identifies the within-month return on wealth (i.e., between periods 1 and 3). This depends on the rate of appreciation in the pound and the actual faction of wealth held in foreign deposits at the end of period-2 trading. The latter term is $\lambda_{t,z} - \xi_{t,z}$, where $\xi_{t,z}$ represents the position-effect of unexpected incoming pound orders from other agents (a shock). This means that the return on wealth, $H_{t,z}^3$, is subject to two sources of uncertainty: uncertainty about the future spot rate S_t^3, and uncertainty about order flow in the form of trades initiated by other agents $T_{t,z*}^2$.

In period 4, agents choose consumption of US and UK goods, foreign currency orders, and investment expenditures. Let $\alpha_{t,z}$ and $\gamma_{t,z}$ denote the desired fractions of wealth held in pounds and domestic capital respectively:

$$\alpha_{t,z} \equiv \frac{S_t^3 \hat{K}_{t,z} + S_t^3 \hat{B}_{t,z}^3 + S_t^3\left(T_{t,z}^4 - \mathsf{E}_{t,z}^4 T_{t,z*}^4\right) - S_t^3 \hat{C}_{t,z}}{W_{t,z}^4},$$

$$\gamma_{t,z} \equiv \begin{cases} \dfrac{K_{t,z} + I_{t,z} - C_{t,z} - \mathsf{E}_{t,z}^4 C_{t,z*}}{W_{t,z}^4} & z < 1/2, \\[2ex] \dfrac{\hat{K}_{t,z} + \hat{I}_{t,z} - \hat{C}_{t,z} - \mathsf{E}_{t,z}^4 \hat{C}_{t,z*}}{W_{t,z}^4} & z \geq 1/2. \end{cases}$$

The period-4 problem can now be written as:

$$J_z^4(W_{t,z}^4) = \max_{\{C_{t,z},\hat{C}_{t,z},\alpha_{t,z},\gamma_{t,z}\}} \left\{ U(\hat{C}_{t,z}, C_{t,z}) + \beta \mathsf{E}_{t,z}^4 \left[J_z^2(W_{t+1,z}^2) \right] \right\}, \tag{11}$$

s.t.

$$W_{t+1,z}^2 = R_t H_{t+1,z}^1 W_{t,z}^4 - R_t \left(C_{t,z} + S_t^3 \hat{C}_{t,z} \right), \tag{12}$$

where:

$$H_{t+1,z}^1 = \begin{cases} 1 + \left(\frac{S_{t+1}^1 \hat{R}_t}{S_t^3 R_t} - 1 \right) (\alpha_{t,z} - \varsigma_{t,z}) + \left(\frac{R_{t+1}^k}{R_t} - 1 \right) (\gamma_{t,z} - \zeta_{t,z}) & z < 1/2 \\ \\ 1 + \left(\frac{S_{t+1}^1 \hat{R}_t}{S_t^3 R_t} - 1 \right) (\alpha_{t,z} - \varsigma_{t,z}) + \left(\frac{S_{t+1}^1 \hat{R}_{t+1}^k}{S_t^3 R_t} - \frac{S_{t+1}^1 \hat{R}_t}{S_t^3 R_t} \right) \left(\gamma_{t,z} - \hat{\zeta}_{t,z} \right) & z \geq 1/2 \end{cases}.$$

with $R_{t+1}^k \equiv 1 + A_{t+1}$ and $\hat{R}_{t+1}^k \equiv 1 + \hat{A}_{t+1}$.

$H_{t+1,z}^1$ is the excess return on wealth (measured relative to the dollar one-month interest rate R_t). As above, realized returns depend on the actual faction of wealth held in pounds $\alpha_{t,z} - \varsigma_{t,z}$, where $\varsigma_{t,z} \equiv S_t^3(T_{t,z*}^4 - \mathsf{E}_{t,z}^4 T_{t,z*}^4)/W_{t,z}^4$ represents the effects of unexpected currency orders that arise from period-4 trading. Monthly returns also depend on the fraction of wealth held in real capital. For the US case this is given by $\gamma_{t,z} - \zeta_{t,z}$, where $\zeta_{t,z} \equiv \left(C_{t,z*} - \mathsf{E}_{t,z}^4 C_{t,z*} \right) / W_{t,z}^4$ identifies the effects of unexpected demand for US goods (i.e. US exports).[20] In the UK case, the fraction is $\gamma_{t,z} - \hat{\zeta}_{t,z}$, where $\hat{\zeta}_{t,z} \equiv \left(\hat{C}_{t,z*} - \mathsf{E}_{t,z}^4 \hat{C}_{t,z*} \right) / W_{t,z}^4$. Monthly returns are therefore subject to four sources of uncertainty: uncertainty about future spot rates (i.e., S_{t+1}^1, which affects deposit returns); uncertainty about future productivity (which affects real capital returns); uncertainty about incoming currency orders; and uncertainty about export demand.

The first-order conditions governing consumption and portfolio choice (i.e., $C_{t,z}, \hat{C}_{t,z}, \lambda_{t,z}, \alpha_{t,z}$) take the same form for both US and UK agents:

$$\hat{C}_{t,z} \quad : \quad U_{\hat{c}}(\hat{C}_{t,z}, C_{t,z}) = \beta R_t S_t^3 \mathsf{E}_{t,z}^4 \left[V_{t+1,z} H_{t+1,z}^3 \right], \tag{13}$$

$$C_{t,z} \quad : \quad U_c(\hat{C}_{t,z}, C_{t,z}) = \beta R_t \mathsf{E}_{t,z}^4 \left[V_{t+1,z} H_{t+1,z}^3 \right], \tag{14}$$

$$\lambda_{t,z} \quad : \quad 0 = \mathsf{E}_{t,z}^2 \left[V_{t,z} \left(\frac{S_t^3}{S_t^1} - 1 \right) \right], \tag{15}$$

$$\alpha_{t,z} \quad : \quad 0 = \mathsf{E}_{t,z}^4 \left[V_{t+1,z} H_{t+1,z}^3 \left(\frac{S_{t+1}^1 R}{S_t^3 R_t} - 1 \right) \right], \tag{16}$$

where $V_{t,z} \equiv dJ_z^4(W_{t,z}^4)/dW_{t,z}^4$ is the marginal utility of wealth. The first-order conditions governing

[20]When superior information about home-country income is not symmetrized by month's end, one manifestation of the residual uncertainty is a shock to export demand.

real investment (i.e. $\gamma_{t,z}$) differ between US and UK agents and are given by:

$$\gamma_{t,z<1/2} \quad : \quad 0 = \mathsf{E}^4_{t,z}\left[V_{t+1,z}H^3_{t+1,z}\left(\frac{R^k_{t+1}}{R_t}-1\right)\right], \tag{17}$$

$$\gamma_{t,z\geq 1/2} \quad : \quad 0 = \mathsf{E}^4_{t,z}\left[V_{t+1,z}H^3_{t+1,z}\left(\frac{S^1_{t+1}\hat{R}^k_{t+1}}{S^3_t R_t}-1\right)\right]. \tag{18}$$

To further characterize the form of optimal consumption, portfolio and investment decisions, we need to identify the marginal utility of wealth (which we denote $V_{t,z}$). This is implicitly defined by the recursion:

$$V_{t,z} = \beta R_t \mathsf{E}^4_{t,z}\left[V_{t+1,z}H^3_{t+1,z}H^1_{t+1,z}\right]. \tag{19}$$

In a standard macro model where agents provide no liquidity provision, equations (15) - (19) together imply that $V_{t,z} = U_c(\hat{C}_{t,z}, C_{t,z})$. The first-order conditions can then be rewritten in familiar form using the marginal rate of substitution. This is not generally the case in our model. As we shall show, the marginal utility of wealth $V_{t,z}$ can diverge from the marginal utility of consumption because unexpected currency and export orders affect portfolio returns.

2.3 Market Clearing

Market clearing in the currency market requires that the dollar value of pound orders initiated equals the dollar value of pound orders received:

$$\int T^j_{t,z}dz = \int T^j_{t,z^*}dz^*,$$

for $j = \{2, 4\}$.

We assume that dollar and pound deposits are in zero net supply so that aggregate deposit holdings at the start of periods 1 and 3 are given by:

$$\int B^1_{t,z}dz = 0, \qquad \int \hat{B}^1_{t,z}dz = 0, \tag{20}$$

$$\int B^3_{t,z}dz = 0, \qquad \int \hat{B}^3_{t,z}dz = 0. \tag{21}$$

Combining these conditions with the budget constraints for dollar and pound deposits implies that both US and UK real investment expenditures $I_{t,z}$ and $\hat{I}_{t,z}$ must equal zero if the deposit and goods markets are to clear.[21] The reason is that both currency and goods market transactions only affect the distribution of deposits, not their aggregate level. This means that $I_{t,z}$ and $\hat{I}_{t,z}$ must be

[21] Though this feature of the model appears rather special, it is not driving our results.

financed by an increase aggregate deposit holdings, an implication that is inconsistent with market clearing. As a consequence, the capital available for production after period-4 trading is complete is $K_{t,z} - \int C_{t,z}dz$ in the US and $\hat{K}_{t,z} - \int \hat{C}_{t,z}dz$ in the UK. Each capital stock is augmented by production that takes place between month t and $t+1$, so that the stock of US and UK capital in period-1 of month $t+1$ are given by

$$K_{t+1,z} = R^k_{t+1}\left(K_{t,z} - \int C_{t,z}dz\right), \tag{22}$$

$$\hat{K}_{t+1,z} = \hat{R}^k_{t+1}\left(\hat{K}_{t,z} - \int \hat{C}_{t,z}dz\right). \tag{23}$$

There equations summarize the implications of market clearing for the dynamics of capital.

3 Solving for Equilibrium

An equilibrium in this model is described by: (i) a set of quote functions that clear markets given the consumption, investment, and portfolio choices of agents; and (ii) a set of consumption, investment, and portfolio rules that maximize expected utility given spot rates, interest rates, and exogenous productivity. In this section we describe how the equilibrium is constructed.

3.1 Solution Method

We solve for equilibrium using a guess-and-verify method. This includes the following five steps, the first and last of which distinguish our information approach quite sharply from other DGE macro modeling:

1. Information Conjecture: We make a conjecture about information available to agents at each point in time. This involves specifying what information agents receive directly and what they learn by observing trading.

2. Quote Decisions: Based on this information structure, we then guess the form of equilibrium quote functions for spot rates and interest rates (periods 1 and 3).

3. Allocation Decisions: We use log linearized first-order conditions and budget constraint to approximate agents' optimal consumption, investment, and currency choices given the spot and interest rates from step 2.

4. Market Clearing: We check that agent choices for consumption, investment, and currency holdings clear markets.

5. Information Conjecture Verified: We verify that the conjectured information structure (from step 1) can be supported by an inference problem based on exogenous information available to each agent, and their observations of quotes and trading activity.

3.2 Log Approximations

Step 3 of our solution method requires the use of log approximations. In particular, we need log approximations for the following: (1) within-month returns, (2) across-month returns, (3) budget constraints, (4) first-order conditions, and (5) capital-stock dynamics.[22] For within-month returns, we use the definition of the period-3 return $H^3_{t,z}$ from the previous section, which yields the log approximation:

$$h^3_{t,z} \cong \lambda_{t,z}\left(s^3_t - s^1_t\right) + \tfrac{1}{2}\lambda_{t,z}\left(1-\lambda_{t,z}\right)\mathsf{V}^2_{t,z}\left[s^3_t\right] - \mathsf{CV}^2_{t,z}\left[s^3_t, \xi_{t,z}\right], \tag{24}$$

where lowercase letters denote natural logs. (Thus, $h^x_{t,z}$ denotes the log excess return on the wealth of agent z realized in period x of month t.) $\mathsf{V}^j_{t,z}[.]$ and $\mathsf{CV}^j_{t,z}[.]$ denote the variance and covariance conditioned on agent $z's$ information at the start of period j in month t. This approximation is similar to those adopted by Campbell and Viceira (2002) and is based on a second-order approximation that holds exactly in continuous time when the change in spot rates and unexpected order flow follow Wiener processes.

Monthly returns are approximated in a similar fashion. Specifically, for US agents (i.e. $z < 1/2$) we use:

$$\begin{aligned} h^1_{t+1,z} \cong\ & \alpha_{t,z}\left(s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right) + \gamma_{t,z}\left(r^k_{t+1} - r_t\right) + \tfrac{1}{2}\alpha_{t,z}\left(1-\alpha_{t,z}\right)\mathsf{V}^4_{t,z}\left[s^1_{t+1}\right] \\ & + \tfrac{1}{2}\gamma_{t,z}\left(1-\gamma_{t,z}\right)\mathsf{V}^4_{t,z}\left[r^k_{t+1}\right] - \alpha_{t,z}\gamma_{t,z}\mathsf{CV}^4_{t,z}\left[s^1_{t+1}, r^k_{t+1}\right] \\ & - \mathsf{CV}^4_{t,z}\left[s^1_{t+1}, \varsigma_{t,z}\right] - \mathsf{CV}^4_{t,z}\left[r^k_{t+1}, \zeta_{t,z}\right], \end{aligned} \tag{25}$$

and for UK agents ($z \geq 1/2$):

$$\begin{aligned} h^1_{t+1,z} \cong\ & \alpha_{t,z}\left(s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right) + \gamma_{t,z}\left(\hat{r}^k_{t+1} - \hat{r}_t\right) + \tfrac{1}{2}\left(\alpha_{t,z} - \gamma_{t,z}\right)\left(1 - \left(\alpha_{t,z} - \gamma_{t,z}\right)\right)\mathsf{V}^4_{t,z}\left[s^1_{t+1}\right] \\ & + \tfrac{1}{2}\gamma_{t,z}\left(1-\gamma_{t,z}\right)\mathsf{V}^4_{t,z}\left[\hat{r}^k_{t+1} + s^1_{t+1}\right] - \left(\alpha_{t,z} - \gamma_{t,z}\right)\gamma_{t,z}\mathsf{CV}^4_{t,z}\left[s^1_{t+1}, \hat{r}^k_{t+1} + s^1_{t+1}\right] \\ & - \mathsf{CV}^4_{t,z}\left[s^1_{t+1}, \varsigma_{t,z}\right] - \mathsf{CV}^4_{t,z}\left[r^k_{t+1}, \hat{\zeta}_{t,z}\right]. \end{aligned} \tag{26}$$

Notice that unexpected period-4 order flows $\varsigma_{t,z}$ and export demand $\zeta_{t,z}$ affect returns through the last covariance terms shown in each equation. These terms represent the effects of non-diversifiable

[22] Complete derivations are contained in the appendix, section 4.

risk that arise from liquidity provision. Unexpected currency orders and export orders during period 2 and 4 trading represent a source of risk that agents cannot fully hedge.

The monthly budget constraint is approximated by combining the two periodic budget constraints in (10) and (12):

$$\Delta w^4_{t+1,z} \cong r_t + h^3_{t+1,z} + \ln(1-\mu) + \left(\frac{1}{1-\mu}\right) h^1_{t+1,z} - \left(\frac{\mu}{1-\mu}\right)\delta_{t,z}, \tag{27}$$

where:

$$\delta_{t,z} \equiv c_{t,z} - w^4_{t,z} - \ln(\mu/2)$$

is the log consumption wealth ratio. μ is a positive constant equal to the steady-state value of $2C_{t,z}/W^4_{t,z}$.

A fascinating feature of the model is that transmission of price-relevant information via trading can push a wedge between the marginal utilities of wealth and consumption. To see this, we combine the log linearized versions of equations (13) - (19) and our assumption of log utility to obtain the log marginal utility of wealth $v_{t,z}$ as:

$$v_{t,z} = -c_{t,z} - \phi_{t,z}, \tag{28}$$

where the wedge, $\phi_{t,z}$, is defined as follows: $\phi_{t,z} \equiv \mathsf{CV}^4_{t,z}\left[s^1_{t+1}, \varsigma_{t,z}\right] + \mathsf{CV}^4_{t,z}\left[r^k_{t+1}, \zeta_{t,z}\right]$ for $z < 1/2$ (US agents) and $\phi_{t,z} \equiv \mathsf{CV}^4_{t,z}\left[s^1_{t+1}, \varsigma_{t,z}\right] + \mathsf{CV}^4_{t,z}\left[\hat{r}^k_{t+1}, \hat{\zeta}_{t,z}\right]$ for $z \geq 1/2$ (UK agents). In the absence of unexpected period-4 currency orders and export demand, the shocks $\varsigma_{t,z}$, $\zeta_{t,z}$ and $\hat{\zeta}_{t,z}$ are zero and the (log) marginal utility of wealth equals the marginal utility of consumption. When these shocks are present and correlated with the future spot rate, and/or returns on capital, the return on wealth is exposed to these sources of systematic risk that may push up or down the log return on wealth according to the sign of the covariance terms. As we shall see, the covariance between currency orders and the future spot rate, $\mathsf{CV}^4_{t,z}\left[s^1_{t+1}, \varsigma_{t,z}\right]$, will differ from zero when period-4 currency trading provides information relevant to the setting of future spot rates.

Approximations to the model's first-order conditions are derived by substituting for $v_{t,z}$ ($=\ln V_{t,z}$) in the log linearized versions of (13) - (18):

$$\lambda_{t,z} \;:\; \mathsf{E}^2_{t,z}s^3_t - s^1_t + \tfrac{1}{2}\mathsf{V}^2_{t,z}\left[s^3_t\right] = \mathsf{CV}^2_{t,z}\left[c_{t,z} + \phi_{t,z}, s^3_t\right], \tag{29}$$

$$\alpha_{t,z} \;:\; \mathsf{E}^4_{t,z}\left[s^1_{t+1} - s^3_t + \hat{r}_t - r_t\right] + \tfrac{1}{2}\mathsf{V}^4_{t,z}\left[s^1_{t+1}\right] = \mathsf{CV}^4_{t,z}\left[c_{t+1,z} + \phi_{t+1,z} - h^3_{t+1,z}, s^1_{t+1}\right], \tag{30}$$

$$c_{t,z} \;:\; \ln\beta + r_t = \mathsf{E}^4_{t,z}\left[\Delta c_{t+1,z} + \phi_{t+1,z} - h^3_{t+1,z}\right] - \tfrac{1}{2}\mathsf{V}^4_{t,z}\left[c_{t+1,z} + \phi_{t+1,z} - h^3_{t+1,z}\right], \tag{31}$$

$$\hat{c}_{t,z} \;:\; c_{t,z} = s^3_t + \hat{c}_{t,z}, \tag{32}$$

for both US and UK agents. The linearized versions of (17) and (18) are:

$$\gamma_{t,z<1/2} \quad : \quad \mathsf{E}^4_{t,z}\left[r^k_{t+1} - r_t\right] + \tfrac{1}{2}\mathsf{V}^4_{t,z}\left[r^k_{t+1}\right] = \mathsf{CV}^4_{t,z}\left[c_{t+1,z} + \phi_{t+1,z} - h^3_{t+1,z}, r^k_{t+1}\right], \tag{33}$$

$$\gamma_{t,z\geq 1/2} \quad : \quad \mathsf{E}^4_{t,z}\left[\hat{r}^k_{t+1} + s^1_{t+1} - s^3_t - r_t\right] + \tfrac{1}{2}\mathsf{V}^4_{t,z}\left[\hat{r}^k_{t+1} + s^1_{t+1}\right] = \mathsf{CV}^4_{t,z}\left[c_{t+1,z} + \phi_{t+1,z} - h^3_{t+1,z}, \hat{r}^k_{t+1} + s^1_{t+1}\right]. \tag{34}$$

Notice that the presence of liquidity provision in the model only affects the first-order conditions for agent behavior through the $\phi_{t,z}$ terms. When combined with the linearized budget constraint, these equations provide analytic approximations for the solution to the optimizations problems facing agents at the beginning of periods 2 and 4 (i.e., expressions for the two portfolio shares $\lambda_{t,z}$ and $\alpha_{t,z}$, real investment $\gamma_{t,z}$, and consumptions $c_{t,z}$ and $\hat{c}_{t,z}$) given the r^k_t and $\hat{r}^k_t$ processes, and the equilibrium dynamics of spot exchange rates and interest rates (determined below).

Capital stock dynamics are approximated from the market-clearing conditions in (22) and (23):

$$k_{t+1} - k_t \cong r^k_{t+1} + \ln(1-\mu) - \left(\frac{\mu}{2(1-\mu)}\right)\left(s^3_t + \hat{k}_t - k_t + \int \delta_{t,z} dz\right), \tag{35}$$

$$\hat{k}_{t+1} - \hat{k}_t \cong \hat{r}^k_{t+1} + \ln(1-\mu) - \left(\frac{\mu}{2(1-\mu)}\right)\left(k_t - s^3_t - \hat{k}_t + \int \delta_{t,z} dz\right). \tag{36}$$

In deriving these equations for capital dynamics, we have assumed that deposit holdings always represent a small fraction of agent wealth. This condition is met trivially in the steady state because both US and UK agents hold all their wealth in the form of domestic capital. The accuracy of these approximations deteriorates when away from the steady state if agents accumulate substantial financial assets/liabilities relative to their capital holdings.

4 Exchange Rate Dynamics

This section examines equilibrium exchange rate dynamics. In particular, we focus on how dispersed information concerning productivity becomes embedded in spot rates. Recall that the processes for (log) capital returns the US and UK, respectively, follow:

$$r^k_t = r + u_t + e_t + \theta(e_{t-1} - \hat{e}_{t-1}),$$

$$\hat{r}^k_t = r + \hat{u}_t + \hat{e}_t + \theta(\hat{e}_{t-1} - e_{t-1}),$$

where we allow the transitory components u_t and $\hat{u}_t$ have correlation ρ, but the persistent components e_t and $\hat{e}_t$ are independent across countries (for tractability). We assume that information about the return on capital arrives as follows:

1. US agents all observe the realization of their home shocks $\{u_t, e_t\}$ in period 1 of month t,
2. UK agents all observe the realizations of their home shocks $(\hat{u}_t, \hat{e}_t)$ in period 1 of month t, and
3. All agents in both countries observe the realized values of log capital returns in month t, r_t^k and $\hat{r}_t^k$, when they are publicly announced in period 1 of the following month t+1.[23]

The equilibrium exchange rate process implied by this information structure is presented in the form of a series of propositions. These propositions clarify the model's essential features and provide insights into the specific role of information revelation. Formal derivations of the propositions are in the appendix. We begin by characterizing the determination of spot prices in periods 1 and 3 of each month:

Proposition 1 *The log nominal exchange rate implied by spot quotes in periods 1 and 3 are given by*

$$s_t^1 = \mathsf{E}_t^1 \nabla k_t, \tag{38}$$

$$s_t^3 = \mathsf{E}_t^3 \nabla k_t, \tag{39}$$

where the operator ∇ denotes the difference between US and UK values (e.g., $\nabla k_t = k_t - \hat{k}_t$).

Proposition 1 ties the period-1 and period-3 spot rates to the expected difference between the log US and UK capital stocks, where expectations are conditioned on common information, $\Omega_t^j = \{1, 3\}$. To develop intuition for this result, we first note that markets are incomplete in our model, so that the spot rate is not determined by the ratio of marginal utilities of consumption of US versus UK goods.[24] Rather, the spot rate is tied down by the international distribution of wealth. This can be seen by combining the definitions of the realized capital shares $\gamma_{t,z} - \zeta_{t,z}$ (see the definition of $H_{t+1,z}^1$ in equation 12) with the dynamics of US and UK capital:

$$\frac{W_{t,\mathsf{US}}^4}{W_{t,\mathsf{UK}}^4} = \left(\frac{\gamma_{t,\mathsf{UK}} - \zeta_{t,\mathsf{UK}}}{\gamma_{t,\mathsf{US}} - \zeta_{t,\mathsf{US}}}\right)\left(\frac{K_t}{S_t^3 \hat{K}_t}\right)^{\frac{1}{1-\mu}}.$$

[23] This third assumption does not imply that the exchange rate is re-established at its full-information value in period 1 of every month, for two reasons: (1) there are new, unobserved shocks to productivity that arrive at the same time and (2) real decisions in period 4 of the previous month are based on expectations at that time, and errors in these expectations have effects on real allocations that persist (with persistent effects on exchange rates, as clarified below).

[24] The nature of market incompleteness is somewhat novel in this model, so we discuss it in some detail in section 5.

The ratio of US to UK wealth is proportional to the ratio of US to UK capital, with the proportionality factor that depends on the ratio of realized capital shares. In equilibrium, changes in the wealth ratio are highly correlated with changes in the capital ratio because allocation choices (i.e., $\gamma_{t,z}$) are determined by expected excess returns that are comparatively stable. This means that any equilibrium restrictions on the distribution of wealth will have their counterpart on the distribution of capital. One such restriction is that the wealth of each consumer remains positive (i.e., $W^i_{t,z} > 0$ for $i = \{2, 4\}$), or equivalently that log wealth remains bounded. In equilibrium, order flows aggregate dispersed information about productivity because consumers have an incentive to trade based on their individual information. This process of social learning is a crucial element in the equilibrium (see Propositions 3 and 4 below), but it breaks down if the wealth of either US or UK consumers falls to zero. (For example, if $W^2_{t,\mathsf{US}} = 0$, then there is no period-2 order flow that can convey dispersed information about US productivity shocks, u_t and e_t.) It is this bound on log wealth that ties down the spot rate. In, particular, the period-3 spot rate must satisfy:

$$s^3_t = \mathsf{E}^3_t \nabla k_t + \mathsf{E}^3_t \sum_{i=1}^{\infty} (1-\mu)^i \left\{ r^k_{t+i} - \left(\hat{r}^k_{t+i} + \Delta s^3_{t+i} \right) \right\}. \tag{40}$$

Equation (40) identifies the unique value for the spot rate that places $K_t / S^3_t \hat{K}_t$ on an expected future path consistent with the equilibrium bound on log wealth. To see why this is so, consider what would happen if the expected $t+1$ return on US capital rose relative to the return on UK capital, with no change in current or future spot rates. This change in returns would raise the expected ratio of US to UK capital in $t+1$. It would also lower $W_{t+1,\mathsf{UK}}/K_{t+1}$ and raise $W_{t+1,\mathsf{US}}/S^3_{t+1}\hat{K}_{t+1}$, thereby reducing US exports and raising UK exports (relative to domestic capital). These wealth effects induce a self-perpetuating cycle of higher growth in US capital and lower growth in UK capital from $t+1$ onwards. And, as a result, $K_t / S^3_t \hat{K}_t$ would rise without bound and $W^4_{t,\mathsf{UK}}$ would be driven to zero. This outcome can be avoided if the current spot rate is raised to offset the effects of higher returns on the distribution of capital in $t+1$. The present value term in equation (40) shows the extent to which the current spot rate must be raised to offset the effects of future return differentials, such that the international distribution of log capital and wealth remain bounded.

The quote equations of Proposition 1 follow in a straightforward manner from (40). The equilibrium dynamics of spot rates insure that expected future returns on US and UK capital are equal (when expressed in terms of a common currency). Under these circumstances, the present value term disappears from (40), leaving $s^3_t = \mathsf{E}^3_t \nabla k_t$ as shown in equation (39). Period-1 spot rate quotes are set so that expected intra-month returns are equal.[25] Since no intra-month interest is paid on

[25] This property of the equilibrium arises from the absence of hedging terms in the period-2 portfolio choices (see appendix for further details).

US or UK deposits, this requirement implies that $s_t^1 = \mathsf{E}_t^1 s_t^3 = \mathsf{E}_t^1 \nabla k_t$ as in equation (38).

Proposition 1 enables us to identify the different factors that contribute to the dynamics of spot rates. In particular, combining (38) and (39) with the dynamics of US and UK capital in (35) and (36) we find:

$$s_t^1 - s_{t-1}^3 = \mathsf{E}_t^1 \nabla r_t^k + \left(\frac{1}{1-\mu}\right) \mathsf{E}_t^1 (\nabla k_{t-1} - \mathsf{E}_{t-1}^3 \nabla k_{t-1}), \tag{41}$$

$$s_t^3 - s_t^1 = \left(\mathsf{E}_t^3 - \mathsf{E}_t^1\right) \nabla r_t^k + \left(\frac{1}{1-\mu}\right) \left(\mathsf{E}_t^3 - \mathsf{E}_t^1\right) (\nabla k_{t-1} - \mathsf{E}_{t-1}^3 \nabla k_{t-1}). \tag{42}$$

These equations show how changing expectations about the distribution of capital and the return on capital contribute to spot rate dynamics. Specifically, equation (41) shows that the revision in spot rate quotes between periods t-1:3 and t:1 has two components. (Hereafter, we use the shorthand $t{:}x$ to denote period x in month t.) The first is the common knowledge expectation of the difference in capital returns, $\mathsf{E}_t^1 \nabla r_t^k \equiv \mathsf{E}_t^1 [r_t^k - \hat{r}_t^k]$. The second component is proportional to the current estimate (conditional on Ω_t^1) of the last month's error in estimating the distribution of capital, $\nabla k_{t-1} - \mathsf{E}_{t-1}^3 \nabla k_{t-1}$. The spot rate change between t:1 and t:3 shown in (42) is also comprised of two components. The first term $\left(\mathsf{E}_t^3 - \mathsf{E}_t^1\right) \nabla r_t^k$ conveys what agents learn about capital returns during the current month. The second term identifies what they learned during the current month about last month's error in estimating the distribution of capital.

Equations (41) and (42) make it clear that exchange rate dynamics are driven by information flows. In particular, the evolution of common-knowledge information through time is key to understanding the contribution of the various components of spot-rate changes. We shall now study this evolution in detail.

Proposition 2 *Immediate revelation of new information about the month-t state of the economy occurs only when* $\rho = -1$.

Recall that US (UK) agents learn the values of e_t and u_t ($\hat{e}_t$ and $\hat{u}_t$) at the start of period 1 in month t. Although all four shocks contribute to the current difference in capital returns, ∇r_t^k, they cannot affect the spot rate until they become common knowledge. In the special case where $\rho = -1$, both u_t and $\hat{u}_t$ are common knowledge, and thereby affect period-1 quotes via $\mathsf{E}_t^1 \nabla r_t^k$ in (41). Thus, some dispersed information about the current state of the economy is immediately reflected in the spot rate. When $\rho > -1$, none of the dispersed information regarding current returns is immediately common knowledge, so the period-1 spot rate does not reflect any new information about the current state of the economy (despite the information existing in dispersed form).

Proposition 3 shows the extent to which dispersed information is learned via period-2 trading.

Proposition 3 *Let* $T_t^2 \equiv \int T_{t,z}^2 dz$ *denote aggregate order flow for pounds in period-2 trading. In equilibrium, aggregate order flow augments the common-knowledge information set between the start of periods 2 and 3:* $\Omega_t^3 = \left\{T_t^2 \cup \Omega_t^2\right\}$. *In the special case where* $\rho = -1, \nabla e_t \in \Omega_t^3$. *For the general case where* $\rho > -1$ $(\neq 0)$, $\{\nabla e_t, \nabla u_t\} \notin \Omega_t^3$, *and*

$$\mathsf{E}\left[\nabla e_t + \nabla u_t | \Omega_t^3\right] = \psi \xi_t, \tag{43}$$

where $\xi_t \equiv S_t^1 \left(T_t^2 - \mathsf{E}_t^2 T_t^2\right) / \beta R W_{t-1}^2$ *is the scaled innovation to period-2 order flow (relative to* Ω_t^2*) that depends on all four returns shocks:*

$$\xi_t \cong \xi_e \nabla e_t + \xi_u \nabla u_t. \tag{44}$$

At the start of period 3, residual uncertainty about the true distribution of capital is:

$$\begin{aligned} \nabla k_t - \mathsf{E}\left[\nabla k_t | \Omega_t^3\right] &= \nabla e_t + \nabla u_t - \mathsf{E}\left[\nabla e_t + \nabla u_t | \Omega_t^3\right] \\ &= \pi_e \nabla e_t + \pi_u \nabla u_t, \end{aligned} \tag{45}$$

where $\pi_e = (1 - \psi \xi_e) \neq 0$ *and* $\pi_u = (1 - \psi \xi_u) \neq 0$.

This proposition shows the pace at which period-2 trading aggregates dispersed information. (Coefficient values are in the appendix.) Period-2 order flow is informative because $s_t^3 - s_t^1$ is forecastable based on agents' individual information, $\Omega_{t,z}^2$. Hence, each agent has an incentive to trade, and in so doing some of their individual information is revealed to others via order flow. When $\rho = -1$, the innovation in order flow is a function of e_t and $\hat{e}_t$. This means that each agent can infer the value of ∇e_t from incoming order flow and their individual information. Under these circumstances, dispersed information concerning e_t and $\hat{e}_t$ becomes common knowledge after a single trading period. The key to this result is that the value of e_t or $\hat{e}_t$ represents the sole source of individual information that motivates trade. In particular, u_t and $\hat{u}_t$ play no role because they are common knowledge and their implications are fully reflected in the period-1 spot rate, s_t^1. When $\rho > -1$, by contrast, the values of e_t and u_t ($\hat{e}_t$ and $\hat{u}_t$) are both sources of superior information to US (UK) agents because the values of u_t and $\hat{u}_t$ are not reflected in s_t^1. This means that order flow innovations contain information on all four shocks, as approximated by (44). As a consequence, it is not generally possible for any agent to precisely infer the values of $\nabla e_t + \nabla u_t$ by combining their individual information with their observation of period-2 order flow.[26] Consequently, aggregation

[26] The exception occurs when $\rho = 0$. In this case, the trades of US (UK) consumers are a function of $e_t + u_t$ ($\hat{e}_t + \hat{u}_t$),

of dispersed information at the end of period-2 trading is incomplete.

We can gain further perspective on the inference problem by considering the composition of period-2 order flow:

$$T_t^2 = \int \lambda_{t,z} \left(W_{t,z}/S_t^1 \right) dz - \hat{K}_t + \int \mathsf{E}_{t,z}^2 T_{t,z^*}^2 dz.$$

This equation shows that order flow aggregates information from: (i) the portfolio allocation decisions of US and UK consumers $\lambda_{z,t}$, (ii) the distribution of wealth $W_{t,z}$, (iii) the outstanding UK capital stock $\hat{K}_t$, and (iv) expectations of incoming order flow $\mathsf{E}_{t,z}^2 T_{t,z^*}^2$. This means that order flow reflects both individual information about the current state and many other variables that affect the distribution of wealth, capital stock and so on. In general, these additional variables are not common-knowledge. Rather, they represent a source of noise that makes precise inferences about the current state from observations of order flow impossible. This source of informational inefficiency is likely to occur in any model that combines dispersed information with constant relative risk aversion (CRRA): because CRRA asset demands depend on wealth, less-than-full information about the distribution of wealth creates noise, more difficult signal extraction, and informational inefficiency.

Next we turn to period-4 trading.

Proposition 4 *After period-4 trading, information aggregation is complete. In particular, the components of returns $u_t, \hat{u}_t, e_t$, and $\hat{e}_t$ are all common knowledge:*

$$\{u_t, \hat{u}_t, e_t, \hat{e}_t\} \in \left\{ T_t^4 \cup \Omega_t^4 \right\}.$$

where $T_t^4 \equiv \int T_{t,z}^4 dz$ denotes aggregate order flow for pounds in period-4 trading.

When $\rho > -1$, period-3 spot rates cannot fully reflect all information relevant to the state of the economy. This means that agents still have individual information that is relevant for forecasting returns between t:4 and t+1:1, and hence have an incentive to trade in period 4. Order flow in period 4 will therefore constitute a second signal on the underlying distribution of individual information. This signal contains incremental information sufficient to reveal fully the values of $\hat{u}_t$, and $\hat{e}_t$ to US consumers, and the values of u_t and e_t to UK consumers. As a result, the values of $u_t, \hat{u}_t, e_t$, and $\hat{e}_t$ become common knowledge by the end of period-4 trading.

Two features of our model lie behind the speed of information aggregation. First, each consumer has only to learn about a limited amount of information, namely, the values of two foreign shocks. Second, our model makes trading very transparent because in equilibrium, incoming orders are equally divided among all consumers. This means that the order flow received by each consumer

so observation of ξ_t combined with private information could fully reveal the value of $\nabla e_t + \nabla u_t$ to all consumers.

is completely representative of the market as a whole. This high level of transparency insures that incoming orders are only a function of $u_t, \hat{u}_t, e_t$, and $\hat{e}_t$ in each period. Consequently, consumers can precisely infer the values of the two foreign shocks from incoming orders in periods 2 and 4. Information would not aggregate so quickly with less transparency. For example, suppose incoming orders were randomly assigned to consumers quoting the same price, so that the incoming order received by each consumer contains an idiosyncratic component. This idiosyncratic component would add noise to the signals provided by incoming orders in periods 2 and 4. As a consequence, incoming orders would no longer be jointly sufficient to reveal the values of the foreign shocks to all consumers. Thus, if trading were less transparent, information aggregation at the end of period-4 would still be incomplete.

We may summarize the implications of Propositions 2 through 4 as follows. For the special case where $\rho = -1$, common information evolves according to:

$$\begin{aligned} \Omega_t^1 &= \left\{u_t, \hat{u}_t \cup \Omega_{t-1}^4\right\}, & \Omega_t^2 &= \Omega_t^1, \\ \Omega_t^3 &= \left\{e_t, e_t \cup \Omega_t^2\right\}, & \Omega_t^4 &= \Omega_t^3. \end{aligned}$$

This information structure implies that $\nabla k_{t-1} = \mathsf{E}_{t-1}^3 \nabla k_{t-1}$, $\mathsf{E}_t^1 \nabla r_t^k = 2\theta \nabla e_{t-1} + \nabla u_t$ and $\left(\mathsf{E}_t^3 - \mathsf{E}_t^1\right) \nabla r_t^k = \nabla e_t$, so equations (41) and (42) become:

$$s_t^1 - s_{t-1}^3 = 2\theta \nabla e_{t-1} + \nabla u_t, \tag{46a}$$

$$s_t^3 - s_t^1 = \nabla e_t. \tag{46b}$$

The exchange rate dynamics described by these equations reflect the rapid pace of information aggregation. With perfectly correlated productivity shocks u_t and $\hat{u}_t$, seeing one means seeing the other, so at the time of their realization both are in the common-knowledge information set (i.e., at t:1). Consequently, u_t and $\hat{u}_t$ have an immediate, one-to-one effect on the period-1 spot rate. Given this, all consumers can make precise inferences about the remaining uncertainty (the values of e_t and $\hat{e}_t$) from their observation of period-2 order flow. The period-3 price is perfectly revealing.

In the general case where $\rho > -1$ $(\neq 0)$, common information evolves according to:

$$\begin{aligned} \Omega_t^1 &= \left\{u_{t-1}, \hat{u}_{t-1}, e_{t-1}, \hat{e}_{t-1} \cup \Omega_{t-1}^4\right\}, & \Omega_t^2 &= \Omega_t^1, \\ \Omega_t^3 &= \left\{\xi_t, \cup \Omega_t^2\right\}, & \Omega_t^4 &= \Omega_t^3. \end{aligned}$$

This information structure implies that $\nabla k_{t-1} = \mathsf{E}_{t-1}^3 \nabla k_{t-1} + \pi_e \nabla e_{t-1} + \pi_u \nabla u_{t-1}$, $\mathsf{E}_t^1 \nabla r_t^k =$

$2\theta\nabla e_{t-1}$ and $\left(\mathsf{E}_t^3 - \mathsf{E}_t^1\right)\nabla r_t^k = \psi\xi_t$, so equations (41) and (42) become:

$$s_t^1 - s_{t-1}^3 = 2\theta\nabla e_{t-1} + \left(\frac{1}{1-\mu}\right)\left(\pi_e \nabla e_{t-1} + \pi_u \nabla u_{t-1}\right), \quad (47)$$

$$s_t^3 - s_t^1 = \psi\xi_t. \quad (48)$$

The exchange rate dynamics described here reflect the slower speed of information aggregation. Equation (47) shows that u_t and $\hat{u}_t$ have no immediate impact on the spot rate because they are not common knowledge at the time of their realization. Instead, dispersed information on ∇u_t and ∇e_t becomes gradually embedded in spot rates via the order flows generated in periods 2 and 4. Embedding via period-2 order flow is shown in (48). The second term in (47) shows the embedding effect of period-4 order flow.

The speed of information aggregation also has implications for real allocation decisions. When $\rho > -1$, consumers make real consumption and investment decisions at the start of period 4 before the complete state of the economy is known. This means that real allocations will be distorted by (rational) expectation errors. In Propositions 5 and 6 below we examine the implications of these distortions for the dynamics of fundamentals and the volatility of exchange rates.

Proposition 5 *Expectational errors are embedded in fundamentals via the relation:*

$$\nabla k_{t+1} - \nabla k_t = \nabla r_{t+1}^k + \left(\frac{\mu}{1-\mu}\right)\left(\nabla k_t - \mathsf{E}\left[\nabla k_t | \Omega_t^3\right]\right).$$

Proposition 5 shows that the monthly change in the realized distribution of capital includes two components: the difference in capital returns ∇r_{t+1}^k, and residual uncertainty after period-2 trading concerning the distribution of capital, $\nabla k_t - \mathsf{E}\left[\nabla k_t | \Omega_t^3\right]$. When $\rho = -1$, there is common knowledge about the full state of the economy by period 3 and $s_t^3 = \nabla k_t$. Accordingly, we refer to ∇k_t as identifying common-knowledge fundamentals. In this special case, $\nabla k_t \in \Omega_t^3$, so changes in fundamentals are driven solely by the difference in capital returns. In the general case with $\rho > -1$, both components contribute to the dynamics of fundamentals. In particular, Proposition 3 shows that $\nabla k_t - \mathsf{E}\left[\nabla k_t | \Omega_t^3\right] = \pi_e \nabla e_t + \pi_u \nabla u_t$, so

$$\nabla k_{t+1} = \nabla k_t + \nabla r_{t+1}^k + \left(\frac{\mu}{1-\mu}\right)\left(\pi_e \nabla e_t + \pi_u \nabla u_t\right).$$

Thus, residual uncertainty about the distribution of capital becomes embedded in the dynamics of fundamentals via the π_e and π_u terms. The economic intuition behind this result is straightforward. Recall that $s_t^3 = \mathsf{E}\left[\nabla k_t | \Omega_t^3\right]$, so residual uncertainty creates a gap between the month t spot rate,

s^3_t and its fundamental level, ∇k_t, that affects the international distribution of wealth. This, in turn, affects exports in both the US and UK, thereby influencing the rate of capital accumulation in both countries between month t and $t+1$. Thus, past exchange rates affect the current level of fundamentals. Notice, too, that the effects of residual uncertainty are not transitory. Even though the value of past fundamentals become common knowledge with just a one-month lag, effects on the level of fundamentals persist indefinitely: Although consumers learn about their "consumption mistakes" once information aggregation is complete, their optimal response does not involve immediate reversal of those mistakes.[27]

Proposition 6 *When $\rho = -1$, the volatility of the monthly depreciation rate is determined by the volatility of common-knowledge fundamentals:*

$$\mathsf{V}\left[\Delta s^3_{t+1}\right] = \mathsf{V}\left[\nabla k_{t+1} - \Delta k_t\right] = \mathsf{V}\left[\nabla r^k_{t+1}\right].$$

When $\rho > -1$ and $\pi_u > \bar{\pi}_u \equiv 2\left(\frac{1-\mu}{2-\mu}\right)^2$, the monthly depreciation rate displays volatility in excess of that implied by fundamentals:

$$\mathsf{V}\left[\Delta s^3_{t+1}\right] > \mathsf{V}\left[\nabla r^k_{t+1}\right].$$

This proposition links the speed of information aggregation to excess volatility. Recall that when $\rho > -1$, consumers make real consumption and investment decisions at the start of period 4 before the complete state of the economy is known. Proposition 5 shows how this affects the dynamics of fundamentals via the presence of expectations errors. These errors can also be the source of excess volatility. Consider the monthly rate of depreciation implied by equations (41) and (42):

$$\Delta s^3_{t+1} = \mathsf{E}^3_{t+1}\nabla r^k_{t+1} + \left(\frac{1}{1-\mu}\right)\mathsf{E}^3_{t+1}\left[\nabla k_t - \mathsf{E}^3_t \nabla k_t\right]$$

Here we see that monthly changes in the exchange rate depend on current shocks, via $\mathsf{E}^3_{t+1}\nabla r^k_{t+1}$, and on corrections for past-month expectational errors, via $\mathsf{E}^3_{t+1}\left[\nabla k_t - \mathsf{E}^3_t \nabla k_t\right]$. Squaring both sides of this equation and taking expectations gives:

$$\begin{aligned}\mathsf{V}\left[\Delta s^3_{t+1}\right] - \mathsf{V}\left[\nabla r^k_{t+1}\right] &= \left(\mathsf{V}\left[\mathsf{E}^3_{t+1}\nabla r^k_{t+1}\right] - \mathsf{V}\left[\nabla r^k_{t+1}\right]\right) + \left(\frac{1}{(1-\mu)^2}\right)\mathsf{V}\left[\mathsf{E}^3_{t+1}[\nabla k_t - E^3_t \nabla k_t]\right] \\ &\quad + \left(\frac{2}{1-\mu}\right)\mathsf{CV}\left[\mathsf{E}^3_{t+1}\nabla r^k_{t+1}, \mathsf{E}^3_{t+1}\left[\nabla k_t - \mathsf{E}^3_t \nabla k_t\right]\right].\end{aligned}$$

[27] This embedding effect on consumption and real capital provides a natural link to the current account dynamics at the center of new macro modeling.

Now, $\mathsf{V}\left[\mathsf{E}^3_{t+1}\nabla r^k_{t+1}\right] - \mathsf{V}\left[\nabla r^k_{t+1}\right] < 0$ (from the definition of a variance), so the first term on the right suggests that the lack of common knowledge should *reduce* volatility. (This corresponds to the mistaken intuition that less information here can only translate into less price adjustment, and therefore less volatility.) But, as the equation shows, this argument overlooks the effects of agents' learning about past states of the economy. In our model, $\mathsf{E}^3_{t+1}\nabla k_t = \nabla k_t$, so the second and third terms become:

$$\left(\frac{1}{(1-\mu)^2}\right)\mathsf{V}\left[\nabla k_t - \mathsf{E}^3_t\nabla k_t\right] + \left(\frac{2}{1-\mu}\right)\mathsf{CV}\left[\mathsf{E}^3_{t+1}\nabla r^k_{t+1}, \nabla k_t - \mathsf{E}^3_t\nabla k_t\right].$$

Clearly the first term is positive because it is proportional to the variance of forecast errors for fundamentals. The second term will also be positive when agents use information learned about past fundamentals to estimate capital's current return. The size of these terms depends on how much is learnt from period-2 trading. When order flow is relatively uninformative, the π_e and π_u coefficients will be larger and the effects of learning will contribute more to the volatility of spot rates. Proposition 6 identifies a sufficient condition for excess volatility (i.e., $\pi_u > \bar{\pi}_u$), where the learning effects dominate.

5 Additional Implications

In this section, we study four additional aspects of our model: (1) exchange rate responses to public announcements, (2) the information content of order flows versus "portfolio" flows, (3) trading volume, and (4) the role of incomplete markets. We include the first of these because it remains a puzzle why the large empirical literature on public macro announcements finds so little exchange rate impact. Our model provides a plausible direction for resolution. We include the second of these because our model clarifies why these two flow concepts—order flows versus portfolio flows—differ significantly in terms of information content. (Both are used in empirical work, with little attention to their theoretical differences.) For the third, trading volume, it remains a puzzle why the volume of foreign exchange transactions relative to international real trade is so large, so it is natural to ask whether the presence of dispersed information casts new light. Lastly, we consider the role in the model of incomplete markets and how the aggregation of information could be affected by introducing additional financial assets.

5.1 Announcements

Our model brings new perspective to the link between exchange rates and public macro announcements. Recall that we specified the model with a public announcement at the start of each month that conveys the realized value of the previous months' fundamental (the previous month's realized capital returns). In equilibrium, this announcement comes late enough that it has no impact on spot rates: it is simply an official aggregation of information that has already been fully aggregated via trading by the market.

This feature of our model highlights the deep link between the speed of information aggregation and the impact of public announcements. For announcements to have exchange rate impact, they must either arrive more promptly (i.e., before aggregation by the market is complete), or one must add sources of inference complexity to the model such that information revealed in period-4 actions is no longer sufficient to reveal past fundamentals fully. We formalize the first of these possibilities with the following proposition:

Proposition 7 *When $\rho > -1$, public announcements concerning the values of r_t^k and $\hat{r}_t^k$ will only affect exchange rates if the announcements are made before period 4 in month* t.

When the transitory capital-return shocks u_t and $\hat{u}_t$ have correlation greater than -1, the state of fundamentals is not fully revealed until agents observe actions from period 4. Any announcement of realized capital returns prior to that time would itself convey new information to the market, and the amount of information it would convey would depend on how early in the month it occurs.

The second possibility for announcements to have exchange rate impact is to break the market's ability to achieve full aggregation from period-4 actions. This can be achieved, for example, by introducing additional sources of noise. For example, one could introduce noise in the marketwide order-flow statistic that agents observe (a quite realistic source of noise, given the relatively low transparency of actual foreign exchange trading). It could also be achieved by introducing country-level noise in the period-1 observation of the capital-return shocks u_t, $\hat{u}_t$, e_t, and $\hat{e}_t$ (also quite realistic). We leave these extensions to future work.

5.2 Portfolio Shifts

In our model, signed transaction flows between marketmakers are the central flow concept in terms of facilitating information aggregation. At the same time, portfolios are shifting over time, so it is worthwhile asking whether these agent-level portfolio flows are also useful for understanding how dispersed information is aggregated.[28] Since the answer to this question is quite subtle, we begin with a simple example. The example will make it clear that changes in portfolio holdings need not be associated with information aggregation, even though signed transaction flows are.

Suppose a researcher has data on the asset positions of all agents. As such, she can track aggregate holdings of dollar and pound deposits period-by-period, $B_t^j \equiv \int B_{t,z}^j dz$, and $\hat{B}_t^j \equiv \int \hat{B}_{t,z}^j dz$ for

[28] Froot and Ramadorai (2002), Fan and Lyons (2002), and Rime (2002) use end-user portfolio flow data of this kind.

periods $j = \{1, ..., 4\}$. Would changes in B_t^j and/or $\hat{B}_t^j$ be correlated with exchange rate innovations arising from the aggregation of dispersed information? The answer is no. Changes in aggregate holdings are determined solely by asset supply via the requirement of market clearing, so they are unrelated to the information transmission driving the exchange rate. This is readily apparent in our model because market clearing requires that $B_t^j = \hat{B}_t^j = 0$ every period.

In practice, a researcher will not have access to data on all asset holdings in the economy, so the issue becomes whether data on a subset of asset holdings can be usefully employed. To examine this we need to study how asset positions change at the micro level. Consider the change in a US agent's holdings of pound deposits between periods 1 and 3:

$$\hat{B}^3_{t,\mathsf{US}} - \hat{B}^1_{t,\mathsf{US}} = \left(\lambda_{t,\mathsf{US}}\frac{W^1_{t:\mathsf{US}}}{S^1_t} - \hat{B}^1_{t,\mathsf{US}}\right) - \left(T^2_{t,z^*} - \mathsf{E}^2_{t:\mathsf{US}}T^2_{t,z^*}\right) \tag{49}$$

The first term on the right identifies the individual's desired increase in the foreign asset position. This term depends, in part, on the individual forecast of returns, $\mathsf{E}^2_{t:\mathsf{US}}\left[s^3_t - s^1_t\right]$, via the optimal choice of $\lambda_{t,\mathsf{US}}$, and so may convey some of this agent's individual information $\Omega^2_{t,\mathsf{US}}$. Nevertheless, it is the second term that plays the central role in our model as the medium for transmitting new marketwide information. The second term is the unexpected currency orders from all other agents. Thus, the total change in the individual's position is a noisy signal of the unexpected order flow that carries marketwide information, where the "noise" here is the first term—the individual's desired position change. The noise arises because, for example, an agent could want to change his foreign asset position even when there is no dispersed information in the economy. With no dispersed information, incoming orders in this model can be perfectly predicted, so the second term in (49) vanishes. And, as a result, there need not be any relation between the change in asset holdings and the exchange rate.

In the presence of dispersed information, the relation between changes in asset holdings and exchange rates is more complex. In this case, the change in asset holdings signal the arrival of new information to the agent, but this need not imply that changes in the exchange rate and asset holdings are contemporaneously correlated. The reason is that information transmitted to each agent via unexpected order flow only becomes embedded in the new exchange rate when it augments the common information set. This always happens in our model because news to US agents in the term $(T^2_{t,z^*} - \mathsf{E}^2_{t:\mathsf{US}}T^2_{t,z^*})$ is already known to UK agents and vice versa. In general, however, there is no guarantee that information received by each agent during trade augments the common information set, so there is no guarantee that this information is immediately embedded in the exchange rate.

To summarize, the revelation of information that drives exchange rates here changes the *distribution* of asset holdings (across US and UK agents). But going in reverse—i.e., inferring information from changes in that distribution—is difficult. At the subset-of-agents level, changes in holdings may be informative, but only to the extent that the subset captures those distribution changes that are relevant. Put differently, changes in holdings at the individual-agent level are a noisy estimate of the information in trades, so even when information aggregation is taking place, these individual changes in holdings will not be strongly correlated with exchange rate changes.

5.3 Trade Composition

Our model provides interesting perspectives on the determinants of currency trading. In particular, the model allows us to decompose order flows into three components: a goods-market component (related to the need to purchase foreign goods with foreign currency), a speculative component (related to information about the return on foreign currency), and a hedging component (related to the expected arrival of currency orders from other agents). These three components are readily identified by rearranging the definition of $\alpha_{t,z}$, the end-of-month-t desired fraction of wealth held in pounds (see the period-4 problem in equation 11):

$$T^4_{t,z} = \hat{C}_{t,z} + \left(\alpha_{t,z} \frac{W^4_{t,z}}{S^3_t} - \hat{K}_{t,z} - \hat{B}^3_{t,z} \right) + \mathsf{E}^4_{t,z} T^4_{t,z} \tag{50}$$

The first term on the right shows the period-4 foreign currency purchases for the goods market. As one would expect, the effect is one-to-one. The second term identifies the desired increase in holdings of pound assets. The speculative demand for foreign assets contributes to this term via the choice of $\alpha_{t,z}$, which depends, in turn, on the expected excess return on deposits and domestic capital.[29] The third term identifies hedging against expected currency orders from other marketmakers.

Equation (50) has two noteworthy implications in terms of currency trading volume. First, transactions in international goods and services ($\hat{C}_{t,z}$ in our model) may account for an empirically insignificant amount of FX trading, even if there are no sizable shifts in desired portfolio holdings. Rather, trades may be driven almost exclusively by hedging against incoming orders, $\mathsf{E}^4_{t,z} T^4_{t,z}$. Such a situation is analogous to "hot potato" trading, a phenomenon where risky inventories are passed between marketmakers in the process of wider risk sharing. In this model, agents rationally anticipate incoming orders generated by unwanted inventories, rather than simply waiting for their arrival.

The second implication of (50) is that dispersed information contributes to the variability of the speculative component, thereby contributing to trading volume. In general, dispersed information still exists in period 3. As a result, spot and interest rates do not impound the union of all

[29]Recall that for US agents, holdings of UK capital $\hat{K}_{t,z}$=0.

information sets. Under these circumstances, $\alpha_{t,z}$ varies through time and across agents as they speculate on the basis of their unimpounded information. Hence, dispersed information contributes to the variability of the speculative component, thereby contributing to trading volume.

5.4 Incomplete Markets

Let us consider the sense in which asset markets in our model are incomplete, and under what conditions. Though there are many definitions of asset market completeness, many are not met by our model. For example, in models with symmetric information in which agents consume each period, markets are complete in the sense of Debreu (1959) if all possible date-states are insurable through once-and-for-all trade at t=0. In our model, there are clearly incentives for subsequent trade, so this definition of completeness does not hold (as one might expect, given the information asymmetries). But once-and-for-all trading is not the only basis for defining completeness. Rather, markets are called dynamically complete if all time-states are insurable through trading strategies implemented over time. In symmetric information settings, necessary conditions for dynamic completeness are well understood (see, e.g., Arrow 1970). In settings of asymmetric information, completeness is generally much harder to achieve, and necessary conditions for doing so are much less well understood.

In our particular setting, we focus on dynamic completeness and the conditions under which idiosyncratic risk in our model is fully insurable. In fact, this occurs only in a very special case, per the following proposition:

Proposition 8 *There is complete risk sharing in equilibrium if and only if* $\rho = -1$.

In this special case where the transitory capital-return shocks u_t and $\hat{u}_t$ perfectly reveal one another, we know that period-3 prices are fully revealing. In this case, the equilibrium exchange rate process is such that it insures that the returns to home and foreign capital are equalized. This occurs despite agents not having access to foreign capital: the dollar return a US consumer would receive if he were able to hold UK capital is exactly the same as the dollar return on US capital. The investment opportunity sets are, in effect, not restricted.

In contrast, in the general case with $\rho > -1$, the exchange rate process is such that the returns on US and UK capital are not equalized. Here, the slow pace of information aggregation makes it impossible for marketmakers to set period-3 spot rates that reflect true capital returns fully. The lack of access to foreign capital now constitutes a restriction on agents' investment opportunity sets.

Interestingly, if one were to allow cross-country holdings of real capital in the general case—i.e., a world equity market—risk sharing would still not be complete. Although agents would have access to the same investments, cross-country correlation in wealth would not be one, a necessary condition for complete risk sharing. This is because consumers use their individual information to choose different portfolios. (Recall that agents still have different information about the month t state of the world at the start of period 4.) In equilibrium, this individual information is useful for forecasting returns because some information about the month t state of the economy becomes embedded in the exchange rate only after the end of period-4 trading.

Less than complete risk sharing in the general case is also manifest in agents' choice of consumption/wealth ratio, $\delta_{t,z}$. Since consumers have log preferences in our model, the optimal choice of $\delta_{t,z}$ doesn't vary with the interest rate, only with expected excess returns on wealth. Gradual information aggregation means that consumers have individual information that is useful for forecasting excess returns. As a result, the consumption/wealth ratio, $\delta_{t,z}$, is also a function of individual information. (As one would expect from optimal consumption choice, $\delta_{t,z}$ remains uncorrelated with public information, i.e., with contents of the information set Ω_t^3.)

If allowing cross-country holdings of real capital is not enough in the general case to complete the market, what would be necessary to do so? In effect, what needs to be spanned by the capital markets in this model to achieve completeness is the differences in individual information sets. In effect, one would need "information contingent" securities, i.e., assets whose payoffs are perfectly correlated with the individual information used in making consumption decisions. This would be a state-contingent security of a new kind.

6 Conclusion

The new micro model we develop here connects the DGE and microstructure approaches to exchange rates. Though both approaches are built on solid micro-foundations, there has been a distressing disconnect between them. DGE models need to find more traction in the data; our results suggest that enriching their information structures (as opposed to their preference or production structures) may provide that traction. The shortcomings of microstructure modeling are more on the theoretical side: these models need a richer placement within the underlying real economy if they are to realize their potential for addressing macro phenomena. It is precisely this joint need that motivates our paper.

Methodologically, the DGE environment we study has a number of novel features. First, financial markets in our model are incomplete, which, among other things, makes room for the exchange rate to be determined from more than just the marginal rate of substitution between home and foreign consumption goods (see Duarte and Stockman 2001). In particular, the exchange rate is pinned down by a present-value relation in a manner familiar to the asset approach. Second, the model embeds social learning: agents learn from the equilibrium actions of others. Third, the presence of social learning means that we need to solve each agent's decision problem and inference problem jointly. More concretely, the solution begins with a conjecture about each agent's information set, and concludes with verification that these conjectured information sets line up with information provided by market outcomes. Fourth, our solution accounts for agent risk aversion. Risks associated with incomplete knowledge about the state of the economy influence consumption and trading decisions (which, in turn, affect the inferences agents draw from market outcomes). To our knowledge, this is the first paper to solve a DGE model with this combination of risk-averse decision-making, heterogeneous information, and social learning.

We use the new framework to address the exchange rate determination puzzle. Though a natural puzzle to start with, the model is certainly rich enough to address a host of other important puzzles, including forward discount bias and real exchange-rate persistence. With respect to determination, we offer four main results: (1) persistent gaps between exchange rates and macro fundamentals, (2) excess volatility relative to macro fundamentals, (3) exchange rate movements without macro news, and (4) little or no exchange rate movement when macro news occurs. Persistent gaps between exchange rates and fundamentals arise in the model because the underlying state of fundamentals—which corresponds to the union of all information sets—is revealed only gradually. So, though exchange rates fully reflect all public information, they never reflect *all* information. Volatility in excess of fundamentals occurs because real allocations are distorted by (rational) expectation errors, which we call an "embedding effect". These distorted real allocations induce additional exchange rate volatility because the exchange rate, as an asset price, needs to compensate for the persistence of these distorted real variables (a source of excess volatility missed by partial-equilibrium microstructure analysis). Exchange rates move without macro news because microeconomic actions—in particular, trades—convey information, even when public macro news is not present. On the flipside, macro news has no impact on exchange rates if the microeconomic aggregation of information renders subsequent public announcements redundant.

Finally, our model provides a structural-economic rationale for why transaction flows account for monthly exchange rate changes quite well empirically, whereas macro variables perform poorly. The basic idea is that when dispersed information is present, aggregate transaction flows provide a tighter signal of changing macro fundamentals. But is dispersed information present? Dispersed information characterizes most variables at the center of exchange rate modeling, including output, money demand, inflation, consumption preferences, and risk preferences. These variables are not realized at the macro level, but rather as aggregations of underlying micro realizations. Some of this information is being aggregated by markets, and might well prove to be an important missing piece in exchange rate economics.

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A Appendix

This appendix includes five sections. The first describes the model's solution and presents proofs of Propositions 1-8. The second provides detail on the agents' optimization problems. The third addresses market clearing conditions. The fourth provides detail on the log approximations used to solve the model. The fifth derives the relationship between the marginal utility of wealth and the marginal utility of consumption.

A.1 Solving the Model

A.1.1 Conjectured Equilibrium

For the case where $\rho > -1 (\neq 0)$, equilibrium interest rates and the exchange rate are conjectured to follow:

$$
\begin{aligned}
s_{t+1}^{1} - s_{t}^{3} &= 2\theta \nabla e_t + \tfrac{1}{1-\mu} \left(\pi_e \nabla e_t + \pi_u \nabla u_t \right), &\text{(A1)}\\
s_{t}^{3} - s_{t}^{1} &= \psi \xi_t, &\text{(A2)}\\
r_t &= r + \eta \xi_t, &\text{(A3)}\\
\hat{r}_t &= r - \eta \xi_t, &\text{(A4)}
\end{aligned}
$$

where $\xi_t \equiv S_t^1 \left(T_t^2 - \mathsf{E}_t^2 T_t^2 \right) / \beta R W_{t-1}^2$ is the scaled innovation to period-2 order flow (relative to Ω_t^2) that depends on all four returns shocks:

$$\xi_t \cong \xi_e \nabla e_t + \xi_u \nabla u_t. \tag{A5}$$

The π_i and ξ_i coefficients are related by $\pi_i = (1 - \psi \xi_i) \neq 0$. Country-level information sets evolve according to:

$$
\begin{array}{ll}
\Omega_{t,\mathsf{US}}^{1} = \left\{ u_t, e_t, \hat{u}_{t-1}, \hat{e}_{t-1} \cup \Omega_{t-1,\mathsf{US}}^{4} \right\}, & \Omega_{t,\mathsf{UK}}^{1} = \left\{ \hat{u}_t, \hat{e}_t, u_{t-1}, e_{t-1} \cup \Omega_{t-1,\mathsf{US}}^{4} \right\}, \\
\Omega_{t,\mathsf{US}}^{2} = \Omega_{t,\mathsf{US}}^{1}, & \Omega_{t,\mathsf{UK}}^{2} = \Omega_{t,\mathsf{UK}}^{1}, \\
\Omega_{t,\mathsf{US}}^{3} = \left\{ \xi_{t,\mathsf{US}} \cup \Omega_{t,\mathsf{US}}^{2} \right\}, & \Omega_{t,\mathsf{UK}}^{3} = \left\{ \xi_{t,\mathsf{UK}}, \cup \Omega_{t,\mathsf{UK}}^{2} \right\}, \\
\Omega_{t,\mathsf{US}}^{4} = \Omega_{t,\mathsf{US}}^{3}, & \Omega_{t,\mathsf{UK}}^{4} = \Omega_{t,\mathsf{UK}}^{3},
\end{array}
\tag{A6}
$$

where $\xi_{t,z} \equiv S_t^1 \left(T_{t,z^*}^2 - \mathsf{E}_{t,z}^2 T_{t,z^*}^2\right)/W_{t,z}^2$ is the order flow innovation received by consumer z in period-2 trading ($z = \{\mathsf{US}, \mathsf{UK}\}$). The evolution of public information is given by:

$$\begin{aligned}
&\Omega_t^1 = \left\{u_{t-1}, \hat{u}_{t-1}, e_{t-1}, \hat{e}_{t-1} \cup \Omega_{t-1}^4\right\}, \\
&\Omega_t^2 = \Omega_t^1, \\
&\Omega_t^3 = \left\{\xi_t \cup \Omega_t^2\right\}, \\
&\Omega_t^4 = \Omega_t^3.
\end{aligned} \tag{A7}$$

Based on this information structure, individual and public expectations regarding productivity shocks can be represented by:

$$\mathsf{E}_{t,z}^i[\varepsilon_t] = b_z^i \varepsilon_t, \tag{A8}$$

$$\mathsf{E}_t^i[\varepsilon_t] = b^i \varepsilon_t, \tag{A9}$$

where b_z^i and b^i are 1×4 vectors and $\varepsilon_t \equiv [\ e_t \quad \hat{e}_t \quad u_t \quad \hat{u}_t\ ]'$. (A8) and (A9) imply that $\mathsf{V}_{t,z}^i[\varepsilon_t] = \left(I - b_z^i\right)\Sigma_\varepsilon \left(I - b_z^i\right)$ and $\mathsf{V}_t^i[\varepsilon_t] = \left(I - b^i\right)\Sigma_\varepsilon \left(I - b^i\right)$ where Σ_ε is the (exogenous) unconditional covariance of ε_t. Consumer $z's$ choice of portfolio shares and log consumption-wealth ratio can be written as:

$$\lambda_{t,z} = \lambda_z + \underline{\lambda}_z' \varepsilon_t \tag{A10}$$

$$\omega_{t,z} = \omega_z + \underline{\omega}_z' \varepsilon_t \tag{A11}$$

$$\delta_{t,z} = \delta_z + \underline{\delta}_z' \varepsilon_t \tag{A12}$$

where $\omega_{t,\mathsf{US}}' \equiv [\ \alpha_{t,\mathsf{US}} \quad \gamma_{t,\mathsf{US}}\ ]$, $\omega_{t,\mathsf{UK}}' \equiv [\ \alpha_{t,\mathsf{UK}} - \gamma_{t,\mathsf{UK}} \quad \gamma_{t,\mathsf{UK}}\ ]$. $\underline{\lambda}_z$, $\underline{\omega}_z$ and $\underline{\delta}_z$ are 4×1 vectors of coefficients, while λ_z, ω_z and δ_z are constants.

A.1.2 Verification

Decision Rules: Consider the period-4 portfolio problem. Combining the first-order conditions in (30), (33) and (34) with (27) gives:

$$\begin{aligned}
\mathsf{E}_{t,z}^4\left[s_{t+1}^1 - s_t^3 + \hat{r}_t - r_t\right] &= \mathsf{CV}_{t,z}^4\left[\delta_{t+1,z} + \phi_{z,t+1} + \tfrac{1}{1-\mu} h_{t+1,z}^1, s_{t+1}^1\right] - \tfrac{1}{2}\mathsf{V}_{t,z}^4\left[s_{t+1}^1\right], \\
\mathsf{E}_{t,\mathsf{US}}^4\left[r_{t+1}^k - r_t\right] &= \mathsf{CV}_{t,\mathsf{US}}^4\left[\delta_{t+1,\mathsf{US}} + \phi_{t+1,\mathsf{US}} + \tfrac{1}{1-\mu} h_{t+1,\mathsf{US}}^1, r_{t+1}^k\right] - \tfrac{1}{2}\mathsf{V}_{t,\mathsf{US}}^4\left[r_{t+1}^k\right], \\
\mathsf{E}_{t,\mathsf{UK}}^4\left[\hat{r}_{t+1}^k + s_{t+1}^1 - s_t^3 - r_t\right] &= \mathsf{CV}_{t,\mathsf{UK}}^4\left[\delta_{t+1,\mathsf{UK}} + \tfrac{1}{1-\mu} h_{t+1,\mathsf{UK}}^1, \hat{r}_{t+1}^k + s_{t+1}^1\right] - \tfrac{1}{2}\mathsf{V}_{t,\mathsf{UK}}^4\left[\hat{r}_{t+1}^k + s_{t+1}^1\right].
\end{aligned}$$

We verify below that $\phi_{z,t} = \phi$. Substituting for $h^1_{t+1,z}$ with (A74) and imposing this restriction, the equations above become:

$$\mathsf{E}^4_{t,z} x_{t+1,z} + \tfrac{1}{2}\Lambda_z = \Psi_z + \left(\frac{1}{1-\mu}\right)\Sigma_z \omega_{t,z}, \tag{A13}$$

where

$$\begin{aligned} x_{t+1,\mathsf{US}} &\equiv \left[\begin{array}{cc} s^1_{t+1} - s^3_t + \hat{r}_t - r_t & r^k_{t+1} - r_t \end{array}\right], \\ x_{t+1,\mathsf{UK}} &\equiv \left[\begin{array}{cc} s^1_{t+1} - s^3_t + \hat{r}_t - r_t & \hat{r}^k_{t+1} + s^1_{t+1} - s^3_t - r_t \end{array}\right], \end{aligned}$$

$\Sigma_z \equiv \mathsf{V}^4_{t,z}\left[x_{t+1,z}\right]$, $\Lambda_z \equiv \operatorname{diag}[\Sigma_z]$ and $\Psi_z \equiv \mathsf{CV}^4_{t,z}\left[x_{t+1,z}, \delta_{t+1,z}\right]$. Equation (A13) implicitly defines the optimal choice of portfolio at the start of period 4. The conjectured equilibrium interest and exchange rate processes imply that $x_{t+1,z} = \mathcal{A}_z \varepsilon_{t+1} + \mathcal{B}_z \varepsilon_t$, with:

$$\mathcal{A}_{\mathsf{US}} = \left[\begin{array}{c} 0 \\ \iota_1 + \iota_3 \end{array}\right], \quad \mathcal{B}_{\mathsf{US}} = \left[\begin{array}{c} 2\theta\left(\iota_1 - \iota_2\right)\left(I - b^i\right) + \left(\frac{1}{1-\mu}\right)\left(\pi_e\left(\iota_1 - \iota_2\right) + \pi_u\left(\iota_3 - \iota_4\right)\right) \\ \theta\left(\iota_1 - \iota_2\right)\left(I - b^i\right) \end{array}\right],$$

$$\mathcal{A}_{\mathsf{UK}} = \left[\begin{array}{c} 0 \\ \iota_2 + \iota_4 \end{array}\right], \quad \mathcal{B}_{\mathsf{UK}} = \left[\begin{array}{c} 2\theta\left(\iota_1 - \iota_2\right)\left(I - b^i\right) + \left(\frac{1}{1-\mu}\right)\left(\pi_e\left(\iota_1 - \iota_2\right) + \pi_u\left(\iota_3 - \iota_4\right)\right) \\ 2\theta\left(\iota_1 - \iota_2\right)\left(I - b^i\right) + \left(\frac{1}{1-\mu}\right)\left(\pi_e\left(\iota_1 - \iota_2\right) + \pi_u\left(\iota_3 - \iota_4\right)\right) \end{array}\right],$$

where ι_i selects the i'th element in ε_t. Combining these equations with (A8) implies that $\mathsf{E}^4_{t,z} x_{t+1,z} = x_z + \mathcal{B}_z b^4_z \varepsilon_t$, $\Psi_z = \mathcal{B}_z\left(I - b^4_z\right)\Sigma_\varepsilon\left(I - b^4_z\right)'\underline{\delta}'_z$, and $\Sigma_z = \mathcal{A}_z \Sigma_\varepsilon \mathcal{A}'_z + \mathcal{B}_z\left(I - b^4_z\right)\Sigma_\varepsilon\left(I - b^4_z\right)'\mathcal{B}'_z$. Hence (A13) becomes

$$\begin{aligned} \omega_{t,z} &= (1-\mu)\Sigma_z^{-1}\left(\tfrac{1}{2}\Lambda_z - \Psi_z + \mathsf{E}^4_{t,z} x_{t+1,z}\right) \\ &= (1-\mu)\Sigma_z^{-1}\left(\tfrac{1}{2}\Lambda_z - \Psi_z\right) + (1-\mu)\Sigma_z^{-1}\mathcal{B}_z b^4_z \varepsilon_t = \omega_z + \underline{\omega}'_z \varepsilon_t, \end{aligned} \tag{A14}$$

as shown in (A11).

To verify the form of the log consumption-wealth ratio, we first combine (31) (with $\phi_{t,z} = \phi_z$) and (27) to give

$$\delta_{t,z} = (1-\mu)\phi_z + \mathsf{E}^4_{t,z} h^1_{t+1,z} - \left(\tfrac{1}{2(1-\mu)}\right)\mathsf{V}^4_{t,z}\left[h^1_{t+1,z} - (1-\mu)\,\delta_{t+1,z}\right].$$

Substituting for $h^1_{t+1,z}$ with (A74) and (A13), we can rewrite this equation as

$$\delta_{t,z} = -\mu\phi_z + \omega'_{t,z}\Psi_z + \left(\tfrac{\mu}{2(1-\mu)}\right)\left(\omega'_{t,z}\Sigma_z\omega_{t,z}\right) - \left(\tfrac{(1-\mu)}{2}\right)\underline{\delta}_z \Sigma_\varepsilon \underline{\delta}'_z + \omega'_{t,z}\mathcal{A}_z \Sigma_\varepsilon \underline{\delta}_z. \tag{A15}$$

Combining this expression with (A11) and taking a second-order approximation around $\varepsilon_t = 0$ gives

$$\begin{aligned}\delta_{t,z} &\cong -\mu\phi_z + \omega_z'\Psi_z + \tfrac{\mu}{2(1-\mu)}\left(\omega_z'\Sigma_z\omega_z\right) - \tfrac{(1-\mu)}{2}\underline{\delta}_z\Sigma_\varepsilon\underline{\delta}_z' + \omega_z'\mathcal{A}_z\Sigma_\varepsilon\underline{\delta}_z \\ &\quad + \tfrac{\mu}{2(1-\mu)}\mathrm{tr}(\Sigma_z\underline{\delta}_z'\Sigma_\varepsilon\underline{\delta}_z) + \left(\Psi_z' + \tfrac{\mu}{(1-\mu)}\Sigma_z\omega_z + \underline{\delta}_z'\Sigma_\varepsilon\mathcal{A}_z'\right)\underline{\omega}_z'\varepsilon_t, \\ &= \delta_z + \underline{\delta}_z'\varepsilon_t,\end{aligned} \tag{A16}$$

as shown in (A12).

To verify the form of the period-2 portfolio choice, we write the linearized first-order condition for $\lambda_{t,z}$ (with $\phi_{t,z} = \phi_z$) as

$$\mathsf{E}^2_{t,z}s^3_t - s^1_t + \tfrac{1}{2}\mathsf{V}^2_{t,z}\left[s^3_t\right] = \mathsf{CV}^2_{t,z}\left[w^4_{t,z}, s^3_t\right] + \mathsf{CV}^2_{t,z}\left[\delta_{t,z}, s^3_t\right]. \tag{A17}$$

Using the results in (A14) and (A16) above we can evaluate the last covariance term as

$$\begin{aligned}\mathsf{CV}^2_{t,z}\left[\delta_{t,z}, s^3_t\right] &= \left(\Psi_z' + \tfrac{\mu}{(1-\mu)}\Sigma_z\omega_z + \underline{\delta}_z'\Sigma_\varepsilon\mathcal{A}_z'\right)\mathsf{CV}^2_{t,z}\left[\underline{\omega}_z'\varepsilon_t, s^3_t\right] \\ &= \left(\Psi_z' + \tfrac{\mu}{(1-\mu)}\Sigma_z\omega_z + \underline{\delta}_z'\Sigma_\varepsilon\mathcal{A}_z'\right)\mathsf{CV}^2_{t,z}\left[\underline{\omega}_{t,z}, s^3_t\right] \\ &= \left(\Psi_z' + \tfrac{\mu}{(1-\mu)}\Sigma_z\omega_z + \underline{\delta}_z'\Sigma_\varepsilon\mathcal{A}_z'\right)(1-\mu)\Sigma_z^{-1}\mathsf{CV}^2_{t,z}\left[\mathsf{E}^4_{t,z}x_{t+1,z}, s^3_t\right].\end{aligned}$$

Under our proposed solution for the interest and exchange rate processes in (A1)- (A4), $x_{t+1,z}$ is uncorrelated with all elements of Ω^3_t, including s^3_t. Since $\mathsf{CV}^2_{t,z}\left[\mathsf{E}^4_{t,z}x_{t+1,z}, s^3_t\right] = \mathsf{CV}^2_{t,z}\left[x_{t+1,z}, s^3_t\right]$ by rational expectations, our solution implies that $\mathsf{CV}^2_{t,z}\left(\delta_{t,z}, s^3_t\right) = 0$. Substituting this restriction into (A17) and combining the result with (27) gives

$$\lambda_{t,z} = \tfrac{1}{2} + \left(\frac{1}{\mathsf{V}^2_{t,z}\left[s^3_t\right]}\right)\mathsf{E}^2_{t,z}\left[s^3_t - s^1_t\right]. \tag{A18}$$

(A2) and (A5) imply that $\mathsf{E}^2_{t,z}\left[s^3_t - s^1_t\right] = \psi\xi_e\mathsf{E}^2_{t,z}\nabla e_t + \psi\xi_u\mathsf{E}^2_{t,z}\nabla u_t$. Now, given the information structure in (A6), $\mathsf{E}^2_{t,\mathsf{US}}\nabla e_t = e_t$, $\mathsf{E}^2_{t,\mathsf{UK}}\nabla e_t = -\hat{e}_t$, $\mathsf{E}^2_{t,\mathsf{US}}\nabla u_t = (1-\rho)\,u_t$ and $\mathsf{E}^2_{t,\mathsf{UK}}\nabla u_t = (\rho - 1)\,\hat{u}_t$. Combining these results with the expression above gives

$$\lambda_{t,\mathsf{US}} = \tfrac{1}{2} + \left(\tfrac{\psi\xi_e}{\sigma_s^2}\right)e_t + \left(\tfrac{\psi\xi_u(1-\rho)}{\sigma_s^2}\right)u_t, \tag{A19}$$

$$\lambda_{t,\mathsf{UK}} = \tfrac{1}{2} - \left(\tfrac{\psi\xi_e}{\sigma_s^2}\right)\hat{e}_t - \left(\tfrac{\psi\xi_u(1-\rho)}{\sigma_s^2}\right)\hat{u}_t, \tag{A20}$$

where $\sigma_s^2 \equiv (\psi\xi_e)^2\,\sigma_e^2 + (\psi\xi_u)^2\,(1-\rho^2)\sigma_u^2$. This is the form of (A10).

Finally, we verify that $\phi_{t,z}$ is a constant where

$$\phi_{t,\mathsf{US}} \equiv \mathsf{CV}^4_{t,\mathsf{US}}\left[s^1_{t+1}, \varsigma_{t,\mathsf{US}}\right] + \mathsf{CV}^4_{t,\mathsf{US}}\left[r^k_{t+1}, \zeta_{t,\mathsf{US}}\right], \tag{A21}$$

$$\phi_{t,\mathsf{UK}} \equiv \mathsf{CV}^4_{t,\mathsf{UK}}\left[s^1_{t+1}, \varsigma_{t,\mathsf{UK}}\right] + \mathsf{CV}^4_{t,\mathsf{UK}}\left[\hat{r}^k_{t+1}, \hat{\zeta}_{t,\mathsf{UK}}\right]. \tag{A22}$$

We establish in (A39) that $\varsigma_{t,z} \cong \frac{1}{2}\Phi\left(I - b^4_z\right)\varepsilon_t$. Combining this approximation with (A1), rewritten as $s^1_{t+1} - s^3_t = \mathcal{B}\varepsilon_t$, gives

$$\begin{aligned} \mathsf{CV}^4_{t,z}\left[s^1_{t+1}, \varsigma_{t,z}\right] &= \tfrac{1}{2}\mathcal{B}\mathsf{V}^4_{t,z}\left[\varepsilon_{t,}\right]\left(I - b^4_z\right)'\Phi' \\ &= \tfrac{1}{2}\mathcal{B}\left(I - b^4_z\right)\Sigma_\varepsilon\left(I - b^4_z\right)'\left(I - b^4_z\right)'\Phi'. \end{aligned} \tag{A23}$$

In (A42) and (A43) we show that $\zeta_{t,\mathsf{US}} \cong \frac{\mu}{2}\left(\delta_{t,\mathsf{UK}} + s^3_t - \nabla k_t\right)$ and $\hat{\zeta}_{t,\mathsf{UK}} \cong \frac{\mu}{2}\left(\delta_{t,\mathsf{US}} - s^3_t + \nabla k_t\right)$. Now (A47) and (A51) imply that $s^3_t - \nabla k_t = \pi_e \nabla e_t + \pi_u \nabla u_t = \pi'\varepsilon_t$. Combining these results with the processes for capital returns gives

$$\begin{aligned} \mathsf{CV}^4_{t,z}\left[r^k_{t+1}, \zeta_{t,\mathsf{US}}\right] &= \tfrac{\mu\theta}{2}\left(\iota_1 - \iota_3\right)\mathsf{V}^4_{t,\mathsf{US}}\left[\varepsilon_t\right]\left(\underline{\delta}_{\mathsf{UK}} + \pi\right) \\ &= \tfrac{\mu\theta}{2}\left(\iota_1 - \iota_3\right)\left(I - b^4_{\mathsf{US}}\right)\Sigma_\varepsilon\left(I - b^4_{\mathsf{US}}\right)'\left(\underline{\delta}_{\mathsf{UK}} + \pi\right), \end{aligned} \tag{A24}$$

$$\begin{aligned} \mathsf{CV}^4_{t,\mathsf{UK}}\left[\hat{r}^k_{t+1}, \hat{\zeta}_{t,\mathsf{UK}}\right] &= -\tfrac{\mu\theta}{2}\left(\iota_1 - \iota_3\right)\mathsf{V}^4_{t,\mathsf{UK}}\left[\varepsilon_t\right]\left(\underline{\delta}_{\mathsf{US}} - \pi\right) \\ &= -\tfrac{\mu\theta}{2}\left(\iota_1 - \iota_3\right)\left(I - b^4_{\mathsf{US}}\right)\Sigma_\varepsilon\left(I - b^4_{\mathsf{US}}\right)'\left(\underline{\delta}_{\mathsf{US}} - \pi\right). \end{aligned} \tag{A25}$$

Substituting the results in (A23), (A24) and (A25) into the definitions shown in (A21) and (A22) establishes that $\phi_{t,z}$ is a constant.

Information Structure: At the start of period 1, consumers observe home productivity shocks so that $\{e_t, u_t\} \in \Omega^2_{t,\mathsf{US}}$ and $\{\hat{e}_t, \hat{u}_t\} \in \Omega^2_{t,\mathsf{UK}}$. Expectations of the productivity shocks can be calculated from the (Kalman Filter) updating equation

$$\mathsf{E}\left[\varepsilon_t|\Omega^1_{t,z}\right] = \mathsf{E}\left[\varepsilon_t|\Omega^4_{t-1,z}\right] + \mathcal{K}^1_z\varepsilon_{t,z},$$

where $\varepsilon_{t,\mathsf{US}} \equiv (\iota_1 + \iota_3)\varepsilon_t$, $\varepsilon_{t,\mathsf{UK}} \equiv (\iota_2 + \iota_4)\varepsilon_t$, and $\mathcal{K}^1_z \equiv \mathsf{V}\left[\varepsilon_{t,z}\right]^{-1}\mathsf{CV}\left[\varepsilon_t, \varepsilon'_{t,z}\right]$. $\varepsilon_{t,z}$ denotes the vector of shocks directly observed by consumer z at the start of period 1. Since productivity shocks are serially uncorrelated, $\mathsf{E}\left[\varepsilon_t|\Omega^4_{t-1,z}\right] = 0$. The updating equation can therefore be rewritten as

$$\mathsf{E}\left[\varepsilon_t|\Omega^1_{t,z}\right] = \left(\iota_z\Sigma_\varepsilon\iota'_z\right)^{-1}\Sigma_\varepsilon\iota'_z\iota_z\varepsilon_t = b^1_z\varepsilon_t$$

where $\iota_{\mathsf{US}} \equiv (\iota_1 + \iota_3)$ and $\iota_{\mathsf{UK}} \equiv (\iota_1 + \iota_3)$ as shown in (A8) for $i = 1$. Since no new information arrives during period 1, $\Omega^2_{t,z} = \Omega^1_{t,z}$ and hence $b^1_z = b^2_z$. (A8) implies that $\mathsf{E}^2_{t,\mathsf{US}}\hat{e}_t = \mathsf{E}^2_{t,\mathsf{UK}}e_t = 0$,

$\mathsf{E}^2_{t,\mathsf{US}}\hat{u}_t = \rho u_t$ and $\mathsf{E}^2_{t,\mathsf{UK}} u_t = \rho\hat{u}_t$. Since the elements of ε_t are not common knowledge by the start of period 2, $\mathsf{E}\left[\varepsilon_t|\Omega^2_t\right] = \mathsf{E}\left[\varepsilon_t|\Omega^1_t\right] = \mathsf{E}\left[\varepsilon_t|\Omega^4_{t-1}\right] = 0$. This is the form of (A9) for $i = \{1, 2\}$ with $b^i = 0$.

Next, we consider the information that accrues between the start of periods 2 and 3. Under the rules of trading, all consumers receive the same incoming orders in equilibrium, so aggregate order flow, $T^2_t \equiv \int T^2_{t,z^*}dz^*$ is observed by all consumers by the end of period-2 trading. Hence $\Omega^3_t = \left\{T^2_t, \Omega^1_t\right\}$. Combining the market clearing condition, $\int T^2_{t,z^*}dz^* = \int T^2_{t,z}dz$, with the definitions of T^2_t and the target fraction of wealth in pounds, $\lambda_{t,z}$, we obtain:

$$\begin{aligned} S^1_t T^2_t &= \int S^1_t T^2_{t,z} dz \\ &= \int \left\{ \lambda_{t,z} W^2_{t,z} - S^1_t \left(\hat{B}_{t,z} + \hat{K}_t \right) + S^1_t \mathsf{E}^2_{t,z} T^2_{t,z^*} \right\} dz \\ &= \int \lambda_{t,z} W^2_{t,z} dz - S^1_t \hat{K}_t + S^1_t \int \mathsf{E}^2_{t,z} T^2_{t,z^*} dz. \end{aligned}$$

The scaled innovation in order flow is defined as $\xi_t \equiv S^1_t\left(T^2_t - \mathsf{E}^2_t T^2_t\right)/\beta R W^2_{t-1}$, where $W^2_{t-1} = \int W^2_{t-1,z}dz$ is world-wide wealth. Bond-market clearing implies that $W^2_{t-1} = S^1_{t-1}\hat{K}_{t-1} + K_{t-1}$, which according to the conjectured information structure in (A7) is common-knowledge at t:1 (i.e., $W^2_{t-1} \in \Omega^1_t$). We may therefore represent common-knowledge information at t:3 as $\Omega^3_t = \left\{\xi_t, \Omega^1_t\right\}$. Substituting for T^2_t in the definition of ξ_t we get:

$$\begin{aligned} \xi_t &= \lambda_{t,\mathsf{US}} \left(\frac{W_{t,\mathsf{US}}}{\beta R W^2_{t-1}} \right) + \lambda_{t,\mathsf{UK}} \left(\frac{W_{t,\mathsf{UK}}}{\beta R W^2_{t-1}} \right) - \left(\frac{S^1_t \hat{K}_t}{\beta R W^2_{t-1}} \right) \\ &\quad + \left(\frac{\left(\mathsf{E}^2_{t,\mathsf{US}} - \mathsf{E}^2_t\right) S^1_t T^2_t}{\beta R W^2_{t-1}} \right) + \frac{1}{2} \left(\frac{\left(\mathsf{E}^2_{t,\mathsf{UK}} - \mathsf{E}^2_t\right) S^1_t T^2_t dz}{\beta R W^2_{t-1}} \right). \end{aligned} \tag{A26}$$

This expression can be written as:

$$\begin{aligned} \xi_t &= \left(\frac{\lambda_{t,\mathsf{US}} \exp\left(w^2_{t,\mathsf{US}} - k_t\right) \exp\left(\Delta k_t - \Delta k\right)}{\left(\exp\left(s^3_{t-1} - \nabla k_{t-1}\right) + 1\right)} \right) \\ &\quad + \left(\frac{\lambda_{t,\mathsf{UK}} \exp(w^2_{t,\mathsf{UK}} - s^1_t - \hat{k}_t) \exp\left(s^1_t - s^3_{t-1} + \Delta\hat{k}_t - \Delta k\right)}{\left(\exp\left(\nabla k_{t-1} - s^3_{t-1}\right) + 1\right)} \right) \\ &\quad - \left(\frac{\exp\left(s^1_t - s^3_{t-1} + \Delta\hat{k}_t - \Delta k\right)}{\left(\exp\left(\nabla k_{t-1} - s^3_{t-1}\right) + 1\right)} \right) + \frac{1}{2}\mathsf{E}^2_{t,\mathsf{US}}\xi_t + \frac{1}{2}\mathsf{E}^2_{t,\mathsf{UK}}\xi_t. \end{aligned} \tag{A27}$$

Recall that bond-market clearing implies that $W^i_{t,\mathsf{US}} + W^i_{t,\mathsf{UK}} = K_t + S^{i-1}_t \hat{K}_t$, or

$$w^i_{t,\mathsf{US}} - k_t = \ln\left((1 + \exp\left(s^{i-1}_t - \nabla k_t\right) - \exp\left(w^i_{t,\mathsf{UK}} - k_t\right)\right)$$

for $i = \{1, 3\}$. Approximating the right hand side around the steady state gives,

$$w^i_{t,\mathsf{US}} - k_t \cong s^{i-1}_t + \hat{k}_t - w^i_{t,\mathsf{UK}}, \tag{A28}$$

for $i = \{1, 3\}$. Linearizing (A27) around the steady state and combining the result with (A28) for $i = 2$, we find:

$$\xi_t \cong \tfrac{1}{2}(\lambda_{t,\mathsf{US}} - \tfrac{1}{2}) + \tfrac{1}{2}(\lambda_{t,\mathsf{UK}} - \tfrac{1}{2}) - \tfrac{1}{4}\left(s^1_t - \nabla k_t\right) + \tfrac{1}{2}\mathsf{E}^2_{t,\mathsf{US}}\xi_t + \tfrac{1}{2}\mathsf{E}^2_{t,\mathsf{UK}}\xi_t. \tag{A29}$$

Substituting for $\lambda_{t,\mathsf{US}}$ and $\lambda_{t,\mathsf{UK}}$ with (A19) and (A20) gives

$$\xi_t \cong \tfrac{1}{2}\lambda_e \nabla e_t + \tfrac{1}{2}\lambda_u \nabla u_t - \tfrac{1}{4}\left(s^1_t - \nabla k_t\right) + \tfrac{1}{2}\mathsf{E}^2_{t,\mathsf{US}}\xi_t + \tfrac{1}{2}\mathsf{E}^2_{t,\mathsf{UK}}\xi_t, \tag{A30}$$

where $\lambda_e = \psi\xi_e/\sigma^2_s$ and $\lambda_u = \psi\xi_u(1-\rho)/\sigma^2_s$. Substituting for $s^1_t - \nabla k_t$ with (A52) (derived below) gives

$$\xi_t \cong \tfrac{1}{2}\left(\lambda_e + \tfrac{1}{2}\right)\nabla e_t + \tfrac{1}{2}\mathsf{E}^2_{t,\mathsf{US}}\xi_t + \tfrac{1}{2}\mathsf{E}^2_{t,\mathsf{UK}}\xi_t + \left(\tfrac{1}{2}\lambda_u + \tfrac{1}{2}\right)\nabla u_t. \tag{A31}$$

To determine the expectations terms, $\mathsf{E}^2_{t,\mathsf{US}}\xi_t$ and $\mathsf{E}^2_{t,\mathsf{UK}}\xi_t$, we guess and verify that $\xi_t = \varpi_e \nabla e_t + \varpi_e \nabla u_t$ for some coefficients ϖ_i. Under our information structure, this guess implies that $\mathsf{E}^2_{t,\mathsf{US}}\xi_t = \varpi_e e_t + \varpi_u (1-\rho) u_t$ and $\mathsf{E}^2_{t,\mathsf{UK}}\xi_t = -\varpi_e \hat{e}_t - \varpi_u (1-\rho)\hat{u}_t$. Substituting these expressions into our guess for ξ_t and equating coefficients gives:

$$\begin{aligned} \xi_t &\cong \left(\lambda_e + \tfrac{1}{2}\right)\nabla e_t + \tfrac{1}{1+\rho}\left(\lambda_u + \tfrac{1}{2}\right)\nabla u_t \\ &\cong \xi_e \nabla e_t + \xi_u \nabla u_t = \underline{\xi}'\varepsilon_t, \end{aligned} \tag{A32}$$

as shown in (A5).

Inferences about the vector of productivity shocks based on Ω^3_t are derived from the Kalman filter updating equation:

$$\mathsf{E}\left[\varepsilon_t|\Omega^3_t\right] = \mathsf{E}\left[\varepsilon_t|\Omega^1_t\right] + \mathcal{K}^3\left(\xi_t - \mathsf{E}\left[\xi_t|\Omega^1_t\right]\right), \tag{A33}$$

where $\mathcal{K}^3 \equiv \mathsf{V}^1_t\left[\xi_t\right]^{-1}\mathsf{CV}^1_t\left[\varepsilon_t, \xi_t\right]$. Now (A5) and (A7) imply that $\mathsf{E}\left[\varepsilon_t|\Omega^1_t\right] = 0$, so

$$\mathsf{E}\left[\varepsilon_t|\Omega^3_t\right] = \left(\underline{\xi}'\Sigma_\varepsilon\underline{\xi}\right)^{-1}\Sigma_\varepsilon\underline{\xi}\underline{\xi}'\xi\varepsilon_t = b^3\varepsilon_t,$$

which is the form of (A9) with $i=3$.

Inferences about the productivity shocks based on $\Omega^4_{t,z}$ are calculated as follows. Let $\tilde{\xi}_{t,z} \equiv \left(W^2_{t,z}/\beta R W^2_{t-1}\right)\xi_{t,z}$ denote the re-scaled unexpected order flow consumer z received during period-2 trading. Since $W^2_{t,z}/\beta R W^2_{t-1} \in \Omega^1_{t,z}$, we can use $\tilde{\xi}_{t,z}$ to represent individual information accruing to consumer z between the start of periods 2 and 4. (Since period-3 spot rates are a function of Ω^3_t, no new individual information accrues between the start of periods 3 and 4.) Combining the definitions of $\tilde{\xi}_{t,z}$ and ξ_t with (A5) gives

$$\tilde{\xi}_{t,z} \equiv \left(\xi_t - \mathsf{E}^2_{t,z}\xi_t\right) \cong \xi_e\left(\nabla e_t - \mathsf{E}^2_{t,z}\nabla e\right) + \xi_u\left(\nabla u_t - \mathsf{E}^2_{t,z}\nabla u_t\right).$$

Using (A7) to evaluate the expectations terms on the right, we find that

$$\tilde{\xi}_{t,\mathsf{US}} \cong -\xi_e\hat{e}_t + \xi_u\left(\rho u_t - \hat{u}_t\right) = \underline{\tilde{\xi}'_{\mathsf{US}}}\varepsilon_t, \tag{A34}$$

$$\tilde{\xi}_{t,\mathsf{UK}} \cong \xi_e e_t + \xi_u\left(u_t - \rho\hat{u}_t\right) = \underline{\tilde{\xi}'_{\mathsf{UK}}}\varepsilon_t. \tag{A35}$$

Inferences about the productive shocks can now be calculated using these expressions and the updating equation

$$\mathsf{E}\left[\varepsilon_t|\Omega^4_{t,z}\right] = \mathsf{E}\left[\varepsilon_t|\Omega^1_{t,z}\right] + \mathcal{K}^4_z\left(\tilde{\xi}_{t,z} - \mathsf{E}\left[\tilde{\xi}_{t,z}|\Omega^1_{t,z}\right]\right),$$

where $\mathcal{K}^4_z \equiv \mathsf{V}^1_{t,z}\left(\tilde{\xi}_{t,z}\right)^{-1}\mathsf{CV}^1_{t,z}\left(\varepsilon_t, \tilde{\xi}_{t,z}\right)$. Now equations (A34) and (A35) imply that $\mathsf{V}^1_{t,z}\left(\tilde{\xi}_{t,z}\right) = \underline{\tilde{\xi}'_z}\mathsf{V}^1_{t,z}\left(\varepsilon_t\right)\underline{\tilde{\xi}_z}$ and $\mathsf{CV}^1_{t,z}\left[\varepsilon_t, \tilde{\xi}_{t,z}\right] = \mathsf{V}^1_{t,z}\left[\varepsilon_t\right]\underline{\tilde{\xi}_z}$. Further, recall that $\mathsf{E}\left[\varepsilon_t|\Omega^1_{t,z}\right] = b^1_z\varepsilon_t$ and $\mathsf{V}^1_{t,z}[\varepsilon_t] = \left(I - b^1_z\right)\Sigma_\varepsilon\left(I - b^1_z\right)$. Substituting these results into the updating equation above gives

$$\begin{aligned}\mathsf{E}\left[\varepsilon_t|\Omega^4_{t,z}\right] &= \left(b^1_z + \left(\underline{\tilde{\xi}'_z}\left(I - b^1_z\right)\Sigma_\varepsilon\left(I - b^1_z\right)\underline{\tilde{\xi}_z}\right)^{-1}\left(I - b^1_z\right)\Sigma_\varepsilon\left(I - b^1_z\right)\underline{\tilde{\xi}_z\tilde{\xi}'_z}\right)\varepsilon_t \\ &= b^4_z\varepsilon_t,\end{aligned}$$

which is the form of (A8) for $i=4$.

Next, we examine the information revealed by period-4 order flow. As in period 2, all consumers receive the same incoming orders in equilibrium, so aggregate order flow, $T^4_t \equiv \int T^{42}_{t,z^*}dz^*$ is observed by all consumers by the end of period-4 trading. Hence $\Omega^1_{t+1} = \left\{T^4_t, \Omega^4_t\right\}$. Combining the market clearing condition, $\int T^4_{t,z^*}dz^* = \int T^4_{t,z}dz$, with the definitions of T^4_t, the target fraction of wealth in pounds $\alpha_{t,z}$, and the log consumption/wealth ratio $\delta_{t,z}$, we obtain:

$$S^3_t T^4_t = \int\left(\alpha_{t,z} + \tfrac{\mu}{2}\exp\left(\delta_{t,z}\right)\right)W^4_{t,z}dz - S^3_t\hat{K}_t + S^3_t\int\mathsf{E}^4_{t,z}T^4_{t,z^*}dz.$$

The scaled innovation in period-4 order flow is defined as $\varsigma_t \equiv S^3_t\left(T^4_t - \mathsf{E}^4_t T^4_t\right)/\beta R W^2_{t-1}$. Substituting for T_t^4 in this definition gives

$$\begin{aligned}\varsigma_t &= \left(\alpha_{t,\mathsf{US}} + \tfrac{\mu}{2}\exp\left(\delta_{t,\mathsf{US}}\right)\right)\left(\frac{\exp\left(w^4_{t,\mathsf{US}} - k_t\right)\exp\left(\Delta k_t - \Delta k\right)}{\left(\exp\left(s^3_{t-1} - \nabla k_{t-1}\right) + 1\right)}\right) \\ &\quad + \left(\alpha_{t,\mathsf{UK}} + \tfrac{\mu}{2}\exp\left(\delta_{t,\mathsf{UK}}\right)\right)\left(\frac{\exp(w^4_{t,\mathsf{UK}} - s^3_t - \hat{k}_t)\exp\left(s^3_t - s^3_{t-1} + \Delta\hat{k}_t - \Delta k\right)}{\left(\exp\left(\nabla k_{t-1} - s^3_{t-1}\right) + 1\right)}\right) \\ &\quad - \left(\frac{\exp\left(s^3_t - s^3_{t-1} + \Delta\hat{k}_t - \Delta k\right)}{\left(\exp\left(\nabla k_{t-1} - s^3_{t-1}\right) + 1\right)}\right) + \frac{1}{2}\mathsf{E}^2_{t,\mathsf{US}}\varsigma_t + \frac{1}{2}\mathsf{E}^2_{t,\mathsf{UK}}\varsigma_t.\end{aligned}$$

Linearizing this expression around the steady state (where $\alpha_{t,z} = \alpha_z = (1-\mu)/2$, $\delta_{t,z} = 0, W^2_{t,\mathsf{US}} = K_t$, $W^2_{t,\mathsf{UK}} = \hat{K}_t$, and $S^3_t = K_t/\hat{K}_t$), gives

$$\begin{aligned}\varsigma_t &\cong \tfrac{1}{2}\left(\alpha_{t,\mathsf{US}} - \alpha_{\mathsf{US}}\right) + \tfrac{\mu}{4}\delta_{t,\mathsf{US}} + \tfrac{1}{4}\left[\left(w^4_{t,\mathsf{US}} - k_t\right) + \left(\Delta k_t - \Delta k\right)\right] \\ &\quad + \tfrac{1}{2}\left(\alpha_{t,\mathsf{UK}} - \alpha_{\mathsf{UK}}\right) + \tfrac{\mu}{4}\delta_{t,\mathsf{UK}} + \tfrac{1}{4}\left[\left(w^4_{t,\mathsf{UK}} - s^3_t - \hat{k}_t\right) + \left(s^3_t - s^3_{t-1} + \Delta\hat{k}_t - \Delta k\right)\right] \\ &\quad - \tfrac{1}{2}\left(s^3_t - s^3_{t-1} + \Delta\hat{k}_t - \Delta k\right) - \tfrac{1}{4}\left(\nabla k_{t-1} - s^3_{t-1}\right) + \tfrac{1}{2}\mathsf{E}^4_{t,\mathsf{US}}\varsigma_t + \tfrac{1}{2}\mathsf{E}^4_{t,\mathsf{UK}}\varsigma_t. \qquad \text{(A36)}\end{aligned}$$

Combining this approximation with (A28) (for $i = 4$),we find that

$$\begin{aligned}\varsigma_t &= \tfrac{1}{2}\left(\alpha_{t,\mathsf{US}} - \alpha_{\mathsf{US}}\right) + \tfrac{1}{2}\left(\alpha_{t,\mathsf{UK}} - \alpha_{\mathsf{UK}}\right) \\ &\quad + \tfrac{\mu}{4}\left(\delta_{t,\mathsf{US}} + \delta_{t,\mathsf{UK}}\right) + \tfrac{1}{4}(\nabla k_t - s^3_t) + \tfrac{1}{2}\mathsf{E}^4_{t,\mathsf{US}}\varsigma_t + \tfrac{1}{2}\mathsf{E}^4_{t,\mathsf{UK}}\varsigma_t. \qquad \text{(A37)}\end{aligned}$$

Substituting for $\nabla k_t - s^3_t$ with (A51) and the decision rules for $\delta_{t,z}$ and $\alpha_{t,z}$ in (A11) and (A12) gives

$$\varsigma_t = \tfrac{1}{2}\varrho_{\mathsf{US}}\underline{\omega}'_{\mathsf{US}}\varepsilon_t + \tfrac{1}{2}\varrho_{\mathsf{UK}}\underline{\omega}'_{\mathsf{UK}}\varepsilon_t + \tfrac{\mu}{4}\left(\underline{\delta}'_{\mathsf{US}} + \underline{\delta}'_{\mathsf{UK}}\right)\varepsilon_t + \tfrac{1}{4}\pi'\varepsilon_t + \tfrac{1}{2}\mathsf{E}^2_{t,\mathsf{US}}\varsigma_t + \tfrac{1}{2}\mathsf{E}^2_{t,\mathsf{UK}}\varsigma_t.$$

where $\alpha_{t,z} \equiv \varrho_z\omega_{t,z}$. As above, we solve this equation with the guess and verify method using (A8) to give

$$\varsigma_t \cong \Phi_e e_t + \Phi_{\hat{e}}\hat{e}_t + \Phi_u u_t + \Phi_{\hat{u}}\hat{u}_t. \qquad \text{(A38)}$$

where the Φ_i coefficients are implicitly defined by

$$\Phi_i = \iota_i\left(\frac{1}{2}\varrho_{\mathsf{US}}\underline{\omega}'_{\mathsf{US}} + \frac{1}{2}\varrho_{\mathsf{UK}}\underline{\omega}'_{\mathsf{UK}} + \frac{\mu}{4}\left(\underline{\delta}'_{\mathsf{US}} + \underline{\delta}'_{\mathsf{UK}}\right) + \frac{1}{4}\pi + \frac{1}{2}\Phi b^4_{\mathsf{US}} + \frac{1}{2}\Phi b^4_{\mathsf{UK}}\right),$$

with $\Phi \equiv [\ \Phi_e \quad \Phi_{\hat{e}} \quad \Phi_u \quad \Phi_{\hat{u}}\ ]$.

To establish that $\phi_{t,z}$ is constant, we need to identify the unexpected order flows received by each consumer in period-4 trading, and unexpected export orders. Innovations in period-4 order flow can be written as $\varsigma_{t,z} \equiv \left(\beta RW_{t-1}^{2}/W_{t,z}^{4}\right)\left(\varsigma_t - \mathsf{E}_{t,z}^{4}\varsigma_t\right)$. Taking a log-linear approximation around the steady state values of $W_{t-1}^{2}, W_{t,z}^{4}$ and ς_t produces $\varsigma_{t,z} \cong \frac{1}{2}\left(\varsigma_t - \mathsf{E}_{t,z}^{4}\varsigma_t\right)$. Combining this approximation with (A38) and (A9) gives

$$\varsigma_{t,z} \cong \tfrac{1}{2}\Phi\left(I - b_z^4\right)\varepsilon_t. \tag{A39}$$

We approximate unexpected export orders $\zeta_{t,\mathsf{US}}$ and $\hat{\zeta}_{t,\mathsf{UK}}$ in a similar manner. To approximate $\zeta_{t,\mathsf{US}}$, we start with the following identities:

$$\begin{aligned} C_{t,\mathsf{UK}}/W_{t,\mathsf{US}}^{4} &\equiv \tfrac{\mu}{2}\exp\left(\delta_{t,\mathsf{UK}} + w_{t,\mathsf{UK}}^{4} - s_t^3 - \hat{k}_t + s_t^3 - \nabla k_t + k_t - w_{t,\mathsf{US}}\right), \\ C_{t,\mathsf{US}}/W_{t,\mathsf{UK}}^{4} &\equiv \tfrac{\mu}{2}\exp\left(\delta_{t,\mathsf{US}} + w_{t,\mathsf{US}}^{4} - k_t + \nabla k_t - s_t^3 + s_t^3 + \hat{k}_t - w_{t,\mathsf{UK}}\right). \end{aligned}$$

Combining the right hand side of each equation with (A28) and log linearizing the around the steady state gives

$$\begin{aligned} C_{t,\mathsf{UK}}/W_{t,\mathsf{US}}^{4} &\cong \tfrac{\mu}{2}\exp\left(\delta_{t,\mathsf{UK}} + s_t^3 - \nabla k_t + 2\left(k_t - w_{t,\mathsf{US}}\right)\right) \\ &\cong \tfrac{\mu}{2}\left(\delta_{t,\mathsf{UK}} + s_t^3 - \nabla k_t + 2\left(k_t - w_{t,\mathsf{US}}\right)\right), \qquad \text{(A40)} \\ C_{t,\mathsf{US}}/W_{t,\mathsf{UK}}^{4} &\cong \tfrac{\mu}{2}\exp\left(\delta_{t,\mathsf{US}} + \nabla k_t - s_t^3 + 2\left(s_t^3 + \hat{k}_t - w_{t,\mathsf{UK}}\right)\right) \\ &\cong \tfrac{\mu}{2}\left(\delta_{t,\mathsf{US}} + \nabla k_t - s_t^3 + 2(s_t^3 + \hat{k}_t - w_{t,\mathsf{UK}})\right). \qquad \text{(A41)} \end{aligned}$$

Using (A40) to substitute for $C_{t,\mathsf{UK}}/W_{t,\mathsf{US}}^{4}$ in the definition

$$\zeta_{t,\mathsf{US}} \equiv \left(C_{t,\mathsf{UK}}/W_{t,\mathsf{US}}^{4}\right) - \mathsf{E}_{t,\mathsf{US}}^{4}\left(C_{t,\mathsf{UK}}/W_{t,\mathsf{US}}^{4}\right)$$

produces

$$\zeta_{t,\mathsf{US}} \cong \tfrac{\mu}{2}\left(\delta_{t,\mathsf{UK}} + s_t^3 - \nabla k_t\right). \tag{A42}$$

Since $S_t^3\hat{C}_{t,\mathsf{US}} = C_{t,\mathsf{US}}$ in equilibrium,

$$\hat{\zeta}_{t,\mathsf{UK}} \equiv S_t^3\left(\hat{C}_{t,\mathsf{US}} - E_{t,,\mathsf{UK}}^{4}\hat{C}_{t,\mathsf{US}}\right)/W_{t,\mathsf{UK}}^{4} = \left(C_{t,\mathsf{US}}/W_{t,\mathsf{UK}}^{4}\right) - \mathsf{E}_{t,\mathsf{UK}}^{4}\left(C_{t,\mathsf{US}}/W_{t,\mathsf{UK}}^{4}\right).$$

So substituting for $C_{t,\mathsf{US}}/W_{t,\mathsf{UK}}^{4}$, with (A41) in the latter expression gives

$$\hat{\zeta}_{t,\mathsf{UK}} \cong \tfrac{\mu}{2}\left(\delta_{t,\mathsf{US}} - s_t^3 + \nabla k_t\right). \tag{A43}$$

The final step is to show how (A32) and (A38) can be combined with elements of $\Omega_{t,z}^{4}$ so that $\{e_t, \hat{e}_t, u_t, \hat{u}_t\} \in \left\{\varsigma_t \cup \Omega_{t,z}^{4}\right\}$ for $z = \{\mathsf{US}, \mathsf{UK}\}$. For the case of US consumers, we rewrite (A32) and

(A38) as:

$$\begin{aligned}
\chi^2_{t,\mathsf{US}} &\equiv \xi_t - \xi_e e_t - \xi_u u_t = -\xi_e \hat{e}_t - \xi_u \hat{u}_t, \\
\chi^4_{t,\mathsf{US}} &\equiv \varsigma_t - \Phi_e e_t - \Phi_u u_t = \Phi_{\hat{e}} \hat{e}_t + \Phi_{\hat{u}} \hat{u}_t.
\end{aligned}$$

$\chi^2_{t,\mathsf{US}}$ and $\chi^4_{t,\mathsf{US}}$ provide two signals of the values of $\hat{e}_t$ and $\hat{u}_t$ that can be constructed from information available to US consumers at the end of period-4 trading (i.e., $\left\{\chi^2_{t,\mathsf{US}}, \chi^4_{t,\mathsf{US}}\right\} \in \left\{\varsigma_t \cup \Omega^4_{t,\mathsf{US}}\right\}$). Combining these equations, we find that:

$$\begin{aligned}
\hat{e}_t &= \left(\frac{1}{\Phi_{\hat{e}}\xi_u - \Phi_{\hat{u}}\xi_e}\right)\left(\Phi_{\hat{u}}\chi^2_{t,\mathsf{US}} + \xi_u \chi^4_{t,\mathsf{US}}\right), \\
\hat{u}_t &= -\left(\frac{1}{\Phi_{\hat{e}}\xi_u - \Phi_{\hat{u}}\xi_e}\right)\left(\Phi_{\hat{e}}\chi^2_{t,\mathsf{US}} + \xi_e \chi^4_{t,\mathsf{US}}\right).
\end{aligned}$$

Similarly, UK consumers can combine their observations of order flow from periods 2 and 4 with their knowledge of $\hat{u}_t$ and $\hat{e}_t$ to infer the values of e_t and u_t precisely. Thus, $\{e_t, \hat{e}_t, u_t, \hat{u}_t\}$ are indeed common knowledge after period-4 trading. This completes the verification of the information structure shown in (A6) - (A9).

Exchange and Interest Rates: We now verify that processes for exchange rates and interest rates implied the equilibrium quotes made in periods 1 and 3 follow (A1)-(A4). To derive the exchange rate process we first combine the capital accumulation equations, (35) and (36), to give:

$$\nabla k_{t+1} = \nabla k_t + \nabla r^k_{t+1} - \tfrac{\mu}{1-\mu}\left(s^3_t - \nabla k_t\right). \tag{A44}$$

Combining this equation with the identity $s^3_{t+1} - \nabla k_{t+1} \equiv \Delta\left(s^3_{t+1} - \nabla k_{t+1}\right) + s^3_t - \nabla k_t$ gives

$$s^3_{t+1} - \nabla k_{t+1} = \left(\tfrac{1}{1-\mu}\right)\left(s^3_t - \nabla k_t\right) + \Delta s^3_{t+1} - \nabla r^k_{t+1}. \tag{A45}$$

Next, we take conditional expectations of both sides of this equation:

$$\mathsf{E}^3_t\left[s^3_{t+1} - \nabla k_{t+1}\right] = \left(\tfrac{1}{1-\mu}\right)\left(s^3_t - \mathsf{E}^3_t \nabla k_t\right) + \mathsf{E}^3_t\left[\Delta s^3_{t+1} - \nabla r^k_{t+1}\right].$$

By iterated expectations, the left-hand side is equal to $\mathsf{E}^3_t\left[s^3_{t+1} - \mathsf{E}^3_{t+1}\nabla k_{t+1}\right]$. Substituting this expression on the left and iterating forward gives

$$s^3_t = \mathsf{E}^3_t \nabla k_t + \mathsf{E}^3_t \sum_{i=1}^{\infty} (1-\mu)^i \left\{r^k_{t+i} - \left(\hat{r}^k_{t+i} + \Delta s^3_{t+i}\right)\right\}. \tag{A46}$$

which is equation (40) in the text.

Next we note from (A1), (A2) and (A5) that

$$\begin{aligned}\mathsf{E}_t^3[\Delta s_{t+1}^3 - \nabla r_{t+1}^k] &= \mathsf{E}_t^3\left[\tfrac{\pi_e}{1-\mu}\nabla e_t + \tfrac{\pi_u}{1-\mu}\nabla u_t - \pi_e \nabla e_{t+1} - \pi_u \nabla u_{t+1}\right] \\ &= \mathsf{E}_t^3\left[\tfrac{\pi_e}{1-\mu}\nabla e_t + \tfrac{\pi_u}{1-\mu}\nabla u_t\right] \\ &= \left(\tfrac{1}{1-\mu}\right)\mathsf{E}_t^3\left[\nabla k_t - \mathsf{E}_t^3 \nabla k_t\right] = 0.\end{aligned}$$

Since $\mathsf{E}_t^3\left[\Delta s_{t+i}^3 - \nabla r_{t+i}^k\right] = \mathsf{E}_t^3\left[\mathsf{E}_{t+i-1}^3\left[\Delta s_{t+i}^3 - \nabla r_{t+i}^k\right]\right]$ for $i > 1$ by iterated expectations, the expression above implies that $\mathsf{E}_t^3\left[\Delta s_{t+i}^3 - \nabla r_{t+i}^k\right] = 0$ for $i > 0$, so (A46) simplifies to

$$s_t^3 = \mathsf{E}_t^3 \nabla k_t. \tag{A47}$$

In equilibrium, the expected return on holding pounds conditioned on public information Ω_t^1 (i.e., $\mathsf{E}_t^1\left[s_t^3 - s_t^1\right]$) must equal zero. To establish why this must be the case, recall from (A26) that under market clearing

$$\begin{aligned}\xi_t &= \lambda_{t,\mathsf{US}}\left(\frac{W_{t,\mathsf{US}}}{\beta R W_{t-1}^2}\right) + \lambda_{t,\mathsf{UK}}\left(\frac{W_{t,\mathsf{UK}}}{\beta R W_{t-1}^2}\right) - \left(\frac{S_t^1 \hat{K}_t}{\beta R W_{t-1}^2}\right) \\ &\quad + \left(\frac{\left(\mathsf{E}_{t,\mathsf{US}}^2 - \mathsf{E}_t^2\right) S_t^1 T_t^2}{\beta R W_{t-1}^2}\right) + \frac{1}{2}\left(\frac{\left(\mathsf{E}_{t,\mathsf{UK}}^2 - \mathsf{E}_t^2\right) S_t^1 T_t^2 dz}{\beta R W_{t-1}^2}\right).\end{aligned}$$

Applying the conditional expectations operator E_t^1 to both sides of this equation gives

$$0 = \mathsf{E}\left[\lambda_{t,\mathsf{US}}\left(\frac{W_{t,\mathsf{US}}}{\beta R W_{t-1}^2}\right) + \lambda_{t,\mathsf{UK}}\left(\frac{W_{t,\mathsf{UK}}}{\beta R W_{t-1}^2}\right) - \left(\frac{S_t^1 \hat{K}_t}{\beta R W_{t-1}^2}\right)\middle|\,\Omega_t^1\right],$$

which implies that

$$0 = \mathsf{E}\left[\lambda_{t,\mathsf{US}} W_{t,\mathsf{US}} + \lambda_{t,\mathsf{UK}} W_{t,\mathsf{UK}} - S_t^1 \hat{K}_t | \Omega_t^1\right], \tag{A48}$$

because $W_{t-1}^2 \in \Omega_t^1$. Notice that this restriction follows as an implication of market clearing and rational expectations (it does not rely on any approximations). As such, it must hold true for any equilibrium distribution of wealth, including the case where $W_{t,\mathsf{US}} = W_{t,\mathsf{UK}} = S_t^1 \hat{K}_t \in \Omega_t^1$. Under these circumstances, (A48) simplifies further to

$$\mathsf{E}\left[\lambda_{t,\mathsf{US}} - \tfrac{1}{2}|\Omega_t^1\right] = -\mathsf{E}\left[\lambda_{t,\mathsf{UK}} - \tfrac{1}{2}|\Omega_t^1\right].$$

Substituting for $\lambda_{t,\mathsf{US}}$ and $\lambda_{t,\mathsf{UK}}$ with (A18) gives

$$\mathsf{E}\left[\left(\frac{1}{\mathsf{V}^2_{t,\mathsf{US}}\left[s^3_t\right]}\right)\mathsf{E}^2_{t,\mathsf{US}}\left[s^3_t - s^1_t\right]\middle|\,\Omega^1_t\right] = -\mathsf{E}\left[\left(\frac{1}{\mathsf{V}^2_{t,\mathsf{UK}}\left[s^3_t\right]}\right)\mathsf{E}^2_{t,\mathsf{UK}}\left[s^3_t - s^1_t\right]\middle|\,\Omega^1_t\right].$$

When period-3 spot rates are set according to (A47) and the distribution of capital follows (A44), $\mathsf{V}^2_{t,\mathsf{US}}\left[s^3_t\right] = \mathsf{V}^2_{t,\mathsf{UK}}\left[s^3_t\right] = \sigma^2_s$, a constant. This means that the equation above further simplifies to

$$\mathsf{E}\left[s^3_t - s^1_t\middle|\,\Omega^1_t\right] = -\mathsf{E}\left[s^3_t - s^1_t\middle|\,\Omega^1_t\right],$$

a condition that can only be met when $\mathsf{E}\left[s^3_t - s^1_t\middle|\,\Omega^1_t\right] = 0$. Notice that we would not be able to derive this simple implication of rational expectations and market clearing if hedging terms were present in the period-2 portfolio decisions. Combining $\mathsf{E}\left[s^3_t - s^1_t\middle|\,\Omega^1_t\right] = 0$ with (A47) gives us the equilibrium exchange rate quoted by all consumers in period 1:

$$s^1_t = \mathsf{E}^1_t \nabla k_t. \tag{A49}$$

Equilibrium exchange rate dynamics are derived by combining (A47) and (A49) with (A44). For this purpose, we take expectations conditioned on Ω^1_{t+1} on both sides of (A44) to give

$$s^1_{t+1} = \mathsf{E}^1_{t+1}\nabla k_t + \mathsf{E}^1_{t+1}\nabla r^k_{t+1} + \tfrac{\mu}{1-\mu}\mathsf{E}^1_{t+1}\left[\nabla k_t - s^3_t\right].$$

Subtracting s^3_t from both sides yields

$$\begin{aligned} s^1_{t+1} - s^3_t &= \mathsf{E}^1_{t+1}\nabla r^k_{t+1} + \tfrac{1}{1-\mu}\mathsf{E}^1_{t+1}\left[\nabla k_t - s^3_t\right] \\ &= \mathsf{E}^1_{t+1}\nabla r^k_{t+1} + \tfrac{1}{1-\mu}\mathsf{E}^1_{t+1}\left[\nabla k_t - \mathsf{E}^3_t\nabla k_t\right], \end{aligned} \tag{A50}$$

as shown in (41) in the text. Now (A44) implies that

$$\nabla k_t - \mathsf{E}^3_t\nabla k_t = \nabla r^k_t - \mathsf{E}^3_t\nabla r^k_t - \tfrac{\mu}{1-\mu}\left((s_{t-1} - \nabla k_{t-1}) - \mathsf{E}^3_t\left[s_{t-1} - \nabla k_{t-1}\right]\right).$$

Under our information structure, $\{s_{t-1}, \nabla k_{t-1}, \nabla e_{t-1}\} \in \Omega^3_t$, so this expression simplifies to

$$\begin{aligned} \nabla k_t - \mathsf{E}^3_t\nabla k_t &= \nabla e_t + \nabla u_t - \mathsf{E}\left[\nabla e_t + \nabla u_t|\Omega^3_t\right], \\ &= \pi_e\nabla e_t + \pi_u\nabla u_t = \pi'\varepsilon_t, \end{aligned} \tag{A51}$$

where $\pi_e = \left(1 - \imath\mathcal{K}^3\xi_e\right)$ and $\pi_u = \left(1 - \imath\mathcal{K}^3\xi_u\right)$ with $\imath \equiv [\begin{array}{cccc} 1 & -1 & 1 & -1 \end{array}]$. The form of the π_i coefficients follow from (A33) and (A5). Combining (A51), (A50) and the fact that $\mathsf{E}^1_{t+1}\nabla r^k_{t+1} =$

$2\theta\nabla e_t$ under our information structure, gives

$$s^1_{t+1} - s^3_t = 2\theta\nabla e_t + \tfrac{1}{1-\mu}\left(\pi_e\nabla e_t + \pi_u\nabla u_t\right),$$

as shown in (A1).

We can also use (A50) and (A44) to calculate the value of $s^1_t - \nabla k_t$ used in the derivation of period-2 order flow above. Specifically, by combining (A50) and (A44) we can write

$$s^1_t - \nabla k_t = \left(\mathsf{E}^1_t - 1\right)\nabla r^k_t + \tfrac{1}{1-\mu}\left(\mathsf{E}^1_t - 1\right)(\nabla k_{t-1} - \mathsf{E}^3_{t-1}\nabla k_{t-1}).$$

According to the conjectured information structure in (A7), ∇k_{t-1} and ∇e_{t-1} are common knowledge by t:1, i.e., $\{\nabla k_{t-1}, \nabla e_{t-1}\} \in \Omega^1_t$. This means that the second term in the expression above equals zero. (A7) also implies that $\left(\mathsf{E}^1_t - 1\right)\nabla r^k_t = -\nabla e_t - \nabla u_t$. Substituting these results into the equation above gives

$$s^1_t - \nabla k_t = -\nabla e_t - \nabla u_t. \tag{A52}$$

To derive (A2), we take expectations conditioned on Ω^3_t on both sides of (A44) (lagged one month), to give

$$s^3_t = \mathsf{E}^3_t\nabla k_{t-1} + \mathsf{E}^3_t\nabla r^k_t + \tfrac{\mu}{1-\mu}\mathsf{E}^3_t\left[\nabla k_{t-1} - s^3_{t-1}\right].$$

Subtracting s^1_t from both sides and combing the result with (A50) gives

$$s^3_t - s^1_t = \left(\mathsf{E}^3_t - \mathsf{E}^1_t\right)\nabla r^k_t + \tfrac{1}{1-\mu}\left(\mathsf{E}^3_t - \mathsf{E}^1_t\right)\left[\nabla k_{t-1} - s^3_{t-1}\right]. \tag{A53}$$

Under the information structure, $\{\nabla k_{t-1}, \nabla e_{t-1}\} \in \Omega^1_t$, so $\left(\mathsf{E}^3_t - \mathsf{E}^1_t\right)\left[\nabla k_{t-1} - s^3_{t-1}\right] = 0$ and $\left(\mathsf{E}^3_t - \mathsf{E}^1_t\right)\nabla r^k_t = \left(\mathsf{E}^3_t - \mathsf{E}^1_t\right)[\nabla e_t + \nabla u_t]$. Since $\mathsf{E}^1_t\varepsilon_t = 0$, the latter term simplifies to $\mathsf{E}^3_t\left[\nabla e_t + \nabla u_t\right]$. Now (A33) and (A5) imply that $\mathsf{E}^3_t\left[\nabla e_t + \nabla u_t\right] = \imath\mathcal{K}^3\xi_t$. Combining these results with the equation above gives

$$s^3_t - s^1_t = \imath\mathcal{K}^3\xi_t = \psi\xi_t,$$

as shown in (A2).

Finally, we turn to the interest rate quotes made in period 3. From (A39), (A42) and (A43) we see that innovations to period-4 order flow, $\varsigma_{t,z}$, and exports, $\zeta_{t,\mathsf{US}}$ and $\hat{\zeta}_{t,\mathsf{UK}}$, depend on the choices for $\omega_{t,z}$ and $\delta_{t,z}$ made at the start of the period. This means that $\omega_{t,z}$ and $\delta_{t,z}$ cannot be functions of Ω^3_t, otherwise $\varsigma_{t,z}$, $\zeta_{t,\mathsf{US}}$ and $\hat{\zeta}_{t,\mathsf{UK}}$ would not be orthogonal to Ω^3_t as rational expectations requires. For this to be the case, expected excess returns on capital cannot be correlated with elements of

Ω_t^3. Thus, market clearing requires that the interest rates quoted in period 3 satisfy:

$$r_t = \mathsf{E}_t^3 r_{t+1}^k, \quad (A54)$$
$$\hat{r}_t = \mathsf{E}_t^3 \hat{r}_{t+1}^k. \quad (A55)$$

Given the process for capital returns, and the conjecture information structure, these equations become

$$\begin{aligned} r &= r + \theta \mathsf{E}_t^3 \nabla e_t \\ &= r + \theta (\iota_1 - \iota_2) \mathcal{K}^3 \xi_t, \end{aligned} \quad (A56)$$
$$\begin{aligned} \hat{r} &= r - \theta \mathsf{E}_t^3 \nabla e_t \\ &= r - \theta (\iota_1 - \iota_2) \mathcal{K}^3 \xi_t, \end{aligned} \quad (A57)$$

where $\mathcal{K}^3$ is defined in (A33). Equations (A56) and (A57) take the same form as (A3) and (A4) with $\eta = \theta (\iota_1 - \iota_2) \mathcal{K}^3$.

A.1.3 Equilibrium when $\rho = -1$

When $\rho = -1$, equilibrium interest rates and the exchange rate follow

$$s_{t+1}^1 - s_t^3 = \nabla u_{t+1} + 2\theta \nabla e_t, \quad (A58)$$
$$s_t^3 - s_t^1 = \nabla e_t, \quad (A59)$$
$$r_t = r + \theta \nabla e_t, \quad (A60)$$
$$\hat{r}_t = r - \theta \nabla e_t, \quad (A61)$$

Individual information sets evolve according to

$$\begin{aligned} \Omega_{t,\mathsf{US}}^2 = \Omega_{t,\mathsf{US}}^1 &= \left\{ u_t, \hat{u}_t, e_t \cup \Omega_{t-1,\mathsf{US}}^{43} \right\}, & \Omega_{t,\mathsf{UK}}^2 = \Omega_{t,\mathsf{UK}}^1 &= \left\{ u_t, \hat{u}_t, \hat{e}_t \cup \Omega_{t-1,\mathsf{US}}^4 \right\}, \\ \Omega_{t,\mathsf{US}}^4 = \Omega_{t,\mathsf{US}}^3 &= \left\{ \hat{e}_t \cup \Omega_{t,\mathsf{US}}^2 \right\}, & \Omega_{t,\mathsf{UK}}^4 = \Omega_{t,\mathsf{UK}}^3 &= \left\{ e_t \cup \Omega_{t,\mathsf{UK}}^2 \right\}, \end{aligned} \quad (A62)$$

and the evolution of public information is given by

$$\begin{aligned} \Omega_t^1 &= \left\{ u_t, \hat{u}_t \cup \Omega_{t-1}^4 \right\}, & \Omega_t^2 &= \Omega_t^3, \\ \Omega_t^3 &= \left\{ e_t, \hat{e}_t \cup \Omega_t^2 \right\}, & \Omega_t^4 &= \Omega_t^3. \end{aligned} \quad (A63)$$

Unexpected order flow in period-2 is perfectly correlated with ∇e_t, while order flows in period 4 are perfectly predictable. Period-2 portfolio choices are given by

$$\lambda_{t,\mathsf{US}} = \frac{1}{2} + \frac{1}{\sigma_e^2} e_t, \qquad \text{and} \qquad \lambda_{t,\mathsf{UK}} = \frac{1}{2} - \frac{1}{\sigma_e^2} \hat{e}_t. \tag{A64}$$

The consumption-wealth ratio and period-4 portfolio shares are constant.

We can verify that these equations describe the equilibrium following the procedure described above. In this special case things are much simpler, so we will just outline the argument. We start with the observation that $\{u_t, \hat{u}_t\} \in \Omega_t^1$ because the "u" shocks are perfectly (negatively) correlated. Thus, the "e" shocks are the only source of individual information at the start of period-2 trading. In equilibrium, consumers use this information in choosing their desired portfolio, as (A64) shows, with the result that the innovation in order flow, ξ_t, is a function of ∇e_t. Thus, $\{e_t, \hat{e}_t\} \in \left\{\xi_t \cup \Omega_{t,z}^1\right\}$ for all z, so the "e" shocks become common knowledge by the start of period 3. This means that $\mathsf{E}_t^3 \nabla k_t = \nabla k_t$, $\mathsf{E}_t^1 \nabla R_t^k = 2\theta \nabla e_{t-1} + \nabla u_t$ and $\left(\mathsf{E}_t^3 - \mathsf{E}_t^1\right) \nabla r_t^k = \nabla e_t$. Substituting these results into (A50) and (A53) gives (A58) and (A59). The information structure also implies that $\mathsf{E}_t^3 r_{t+1}^k = \theta \nabla e_t$ and $\mathsf{E}_t^3 \hat{r}_{t+1}^k = -\theta \nabla e_t$, so (A60) and (A61) follow from (A54) and (A55). All that now remains is to verify the form of the decision rules. (A58) - (A62) imply that the vector of expected excess returns $\mathsf{E}_{t,z}^4 x_{t+1,z}$ are zero. Under these circumstances, (A14) and (A15) imply that $\omega_{t,z}$ and $\delta_{t,z}$ are constant. Equation (A64) follows from (A17), (A59) and (27).

A.1.4 Proofs of Propositions

Proposition 1: The results in this proposition follow directly from (A47) and (A49) above.

Proposition 2: The equilibria above show that $\{e_{t-1}, \hat{e}_{t-1}\} \in \Omega_t^1$, so the shocks u_t, e_t $\hat{u}_t$ and $\hat{e}_t$ represent all the new information about the month t state of the economy. Under our assumptions, $\{u_t,\, e_t\} \in \Omega_{\mathsf{US},t}^1$ and $\{\hat{u}_t,\, \hat{e}_t\} \in \Omega_{\mathsf{UK},t}^1$, so $\{u_t,\, e_t\ \hat{u}_t\ \hat{e}_t\} \notin \Omega_t^1 \equiv \cap_z \Omega_{z,t}^1$ when $\rho > -1$. When $\rho = -1$, $\{u_t,\, \hat{u}_t\} \in \Omega_t^1 \equiv \cap_z \Omega_{z,t}^1$, so some new information about the month t state becomes common knowledge in period t:1.

Proposition 3: The only part of the proposition not covered in section A.1.2 concerns the values of π_e and π_u. We argue by contradiction to show that $\pi_e \neq 0$ and $\pi_u \neq 0$. If $\pi_e = (1 - \psi\xi_e) = 0$ and $\pi_u = (1 - \psi\xi_u) = 0$, then $\xi_e = \xi_u$ so (A32) implies that

$$(1+\rho)\left(\lambda_e + \frac{1}{2}\right) = \lambda_u + \frac{1}{2}.$$

The assumption that $\pi_e = \pi_u = 0$, also implies that

$$\begin{aligned}\lambda_e &= \left(\sigma_e^2 + (1-\rho^2)\sigma_u^2\right)^{-1}, \\ \lambda_u &= (1-\rho)\left(\sigma_e^2 + (1-\rho^2)\sigma_u^2\right)^{-1},\end{aligned}$$

from (A18). Combining these expressions with the equation above gives $\left(\sigma_e^2 + (1-\rho^2)\sigma_u^2\right) = -4$; a contradiction.

Proposition 4: This proposition is proved in the subsection verifying the form of the information structure.

Proposition 5: To derive the equation in this proposition we simply combine the results in (A44) and (A47).

Proposition 6: The first variance expression follows directly from the capital returns processes (8a) and (8b), and the exchange rate equations (A58) and (A59). To derive the second expression we combine (A1) and (A2) to give

$$\Delta s_{t+1}^3 = \psi\xi_{t+1} + 2\theta\nabla e_t + \left(\tfrac{1}{1-\mu}\right)(\pi_e\nabla e_t + \pi_u\nabla u_t).$$

Substituting for ξ_{t+1} with (A5), we obtain

$$\begin{aligned}\Delta s_{t+1}^3 &= \psi\xi_e\nabla e_{t+1} + \psi\xi_u\nabla u_{t+1} + 2\theta\nabla e_t + \left(\tfrac{1}{1-\mu}\right)(\pi_e\nabla e_t + \pi_u\nabla u_t) \\ &= \psi\xi_e\nabla e_{t+1} + \psi\xi_u\nabla u_{t+1} + \nabla r_{t+1}^k - \nabla e_{t+1} - \nabla u_{t+1} + \left(\tfrac{1}{1-\mu}\right)(\pi_e\nabla e_t + \pi_u\nabla u_t) \\ &= \nabla r_{t+1}^k - (\pi_e\nabla e_{t+1} + \pi_u\nabla u_{t+1}) + \left(\frac{1}{1-\mu}\right)(\pi_e\nabla e_t + \pi_u\nabla u_t).\end{aligned}$$

Using the last line in this expression we compute

$$\begin{aligned}\mathsf{V}\left[\Delta s_{t+1}^3\right] - \mathsf{V}\left[\nabla r_{t+1}^k\right] &= \left(\tfrac{\pi_e}{1-\mu} + \pi_e\right)^2 2\sigma_e^2 + \left(\tfrac{\pi_u}{1-\mu} + \pi_u\right)^2 2(1-\rho)\sigma_u^2 \\ &\quad + 4\pi_e\left(\tfrac{1+\mu}{1-\mu}\right)\sigma_e^2 - 4\pi_u(1-\rho)\sigma_u^2 \\ &= \left(2\pi_e^2\left(\tfrac{2-\mu}{1-\mu}\right)^2 + 4\pi_e\left(\tfrac{1+\mu}{1-\mu}\right)\right)\sigma_e^2 + \left(2\pi_u^2\left(\tfrac{2-\mu}{1-\mu}\right)^2 - 4\pi_u\right)(1-\rho)\sigma_u^2.\end{aligned}$$

The first term in unambiguously positive because $\pi_e > 0$. The second term is positive if $\pi_u > \bar{\pi}_u \equiv 2\left(\tfrac{1-\mu}{2-\mu}\right)^2$. Note that $\bar{\pi}_u < 1$ because $1 > \mu > 0$, so $\bar{\pi}_u$ is the lower bound on π_u sufficient to generate excess volatility.

Proposition 7: We established in (A47) and (A49) that the equilibrium log exchange rate can be written as $s_t^i = \mathsf{E}\left[\nabla k_t|\Omega_t^i\right]$, for $i = \{1,3\}$, where Ω_t^i denotes the public information set at $t{:}i$ identified in (A7) without announcements. Thus, a public announcement concerning the values of r_t^k and $\hat{r}_t^k$ in $t{:}i$ will have no impact on the exchange rate if $\mathsf{E}\left[\nabla k_t|\Omega_t^i\right] = \mathsf{E}\left[\nabla k_t|\Omega_t^i, r_t^k, \hat{r}_t^k\right]$. Since $\nabla k_t \in \Omega_{t+1}^1$, announcements made after $t{:}4$ have no exchange rate effects because all the information they contain has been aggregated by consumers via trading. Suppose the announcement is made in $t{:}3$. Equation (A51) implies that $\nabla k_t = \mathsf{E}_t^3 \nabla k_t + \pi_e \nabla e_t + \pi_u \nabla u_t$, and (A44) with (A7) imply that $\mathsf{E}\left[\nabla k_t|\Omega_t^3, r_t^k, \hat{r}_t^k\right] = \nabla k_t$, so

$$\mathsf{E}\left[\nabla k_t|\Omega_t^1, r_t^k, \hat{r}_t^k\right] - \mathsf{E}\left[\nabla k_t|\Omega_t^1\right] = \pi_e \nabla e_t + \pi_u \nabla u_t.$$

Under these circumstances, the effect of the announcement on the exchange rate is identified by the second term in

$$s_t^3 - s_t^1 = \psi \xi_t + \left(\pi_e \nabla e_t + \pi_u \nabla u_t\right).$$

Period-1 announcements will also affect the exchange rate because

$$\begin{aligned}\mathsf{E}\left[\nabla k_t|\Omega_t^1, r_t^k, \hat{r}_t^k\right] - \mathsf{E}\left[\nabla k_t|\Omega_t^1\right] &= \nabla k_t - \mathsf{E}\left[\nabla r_t^k + \nabla k_{t-1} - \tfrac{\mu}{1-\mu}\left(s_{t-1}^3 - \nabla k_{t-1}\right)|\Omega_t^1\right] \\ &= \nabla e_t + \nabla u_t.\end{aligned}$$

Proposition 8: When $\rho = -1$, we established above that the consumption-wealth ratios $\delta_{t,z}$ are constant. We therefore need to show that $corr(\Delta w_{t,\mathsf{US}}, \Delta w_{t,\mathsf{UK}}) = 1$ to establish complete risk sharing. For this purpose, we use the definitions of $\zeta_{t,\mathsf{US}}$ and $\hat{\zeta}_{t,\mathsf{UK}}$ to write

$$\begin{aligned}K_{t+1} &= R_{t+1}^k(\gamma_{t,\mathsf{US}} - \zeta_{t,\mathsf{US}})W_{t,\mathsf{US}}^4, \\ S_{t+1}^3 \hat{K}_{t+1} &= \left(\frac{S_{t+1}^3 \hat{R}_{t+1}^k}{S_t^3}\right)(\gamma_{t,\mathsf{UK}} - \hat{\zeta}_{t,\mathsf{UK}})W_{t,\mathsf{UK}}^4.\end{aligned}$$

Combining these equations, we find that:

$$w_{t,\mathsf{US}}^4 = w_{t,\mathsf{UK}}^4 + \left(\nabla k_{t+1} - s_{t+1}^3 + \Delta s_{t+1}^3 - \nabla r_{t+1}^k\right) + \ln\left(\frac{(\gamma_{t,\mathsf{UK}} - \hat{\zeta}_{t,\mathsf{UK}})}{(\gamma_{t,\mathsf{US}} - \zeta_{t,\mathsf{US}})}\right). \tag{A65}$$

When $\rho = -1$, $s_t^3 = \nabla k_t$, and $\Delta s_{t+1}^3 - \nabla r_{t+1}^k$, so the first term in parentheses on the right equals zero. To evaluate the second term, recall that when $\rho = -1$ the month t state of the economy is common knowledge by that the start of period 3. Hence, $\hat{\zeta}_{t,\mathsf{UK}} = \zeta_{t,\mathsf{US}} = 0$. Furthermore, $\gamma_{t,\mathsf{UK}}$ and $\gamma_{t,\mathsf{US}}$ are constant. Thus, the (A65) simplifies to $w_{t,\mathsf{US}}^4 = w_{t,\mathsf{UK}}^4 +$ constant, so that $corr\left(w_{t,\mathsf{US}}^4, w_{t,\mathsf{UK}}^4\right) = 1$.

To establish the absence of complete risk sharing in the $\rho > -1$ case, we argue by contradiction.

Suppose there is complete risk sharing so that the marginal utilities of US and UK consumers are always equal. Given log utility, this implies that $c_{t,\mathsf{US}} = c_{t,\mathsf{UK}}$ in all states of the world, including the state where $w_{t,\mathsf{US}} = w_{t,\mathsf{UK}}$. In this state, complete risk sharing implies that $\delta_{t,\mathsf{US}} = \delta_{t,\mathsf{UK}}$. It also implies that the demand for exports is perfectly predictable because home and foreign consumption are perfectly correlated. This means that $\delta_{t,\mathsf{UK}} = \nabla k_t - s_t^3$ and $\delta_{t,\mathsf{US}} = s_t^3 - \nabla k_t$ from (A42) and (A43). Combining these equations with the other implication, $\delta_{t,\mathsf{US}} = \delta_{t,\mathsf{UK}}$, implies that $s_t^3 = \nabla k_t$, a restriction that is violated by the equilibrium spot rate when $\rho > -1$.

A.2 Optimization Problems (Equations 9 – 19)

To derive the budget constraint in (10), we use the definitions of the intra-month desired portfolio share in pounds, $\lambda_{t,z}$, and the period-2 order flow, ξ_t, together with the intraday dynamics of US and UK deposits to obtain:

$$\begin{aligned} S_t^3\left(\hat{B}_t^3 + \hat{K}_{t,z}\right) &= \tfrac{S_t^3}{S_t^1}(\lambda_{t,z} - \xi_t) W_{t,z}^2, \\ B_t^3 &= \left[1 - (\lambda_{t,z} - \xi_{t,z})\right] W_t^2 - K_{t,z}. \end{aligned}$$

(Note that agents only hold domestic capital, so that $K_{t,z} = 0$ for $z \geq 1/2$ and $\hat{K}_{t,z} = 0$ for $z < 1/2$.) Substituting these expressions into the definition of $W_{t,z}^4$, gives (10):

$$W_{t,z}^4 = \left(1 + \left(\frac{S_{t+1}^3}{S_t^1} - 1\right)(\lambda_{t,z} - \xi_{t,z})\right) W_{t,z}^2.$$

Now we turn to deriving equations (13)–(19), the first-order conditions describing consumption and portfolio choices. Let $\varsigma_{t,z} \equiv S_t^3(T_{t,z*}^4 - \mathsf{E}_{t,z}^4 T_{t,z*}^4)/W_{t,z}^4$, $\zeta_{t,z} \equiv \left(C_{t,z*} - \mathsf{E}_{t,z}^4 C_{t,z*}\right)/W_{t,z}^4$ and $\hat{\zeta}_{t,z} \equiv \left(\hat{C}_{t,z*} - \mathsf{E}_{t,z}^4 \hat{C}_{t,z*}\right)/W_{t,z}^4$ respectively denote unexpected order flow, unexpected US export demand, and unexpected UK export demand (all measured relative to period-4 wealth). Then using the definitions of $\alpha_{t,z}$, and $\gamma_{t,z}$, together with the overnight dynamics of deposits and capital for US agents, we obtain:

$$\begin{aligned} S_{t+1}\hat{B}_{t+1}^1 &= \left(\frac{S_{t+1}^1 \hat{R}_t}{S_t^3}\right)(\alpha_{t,z} - \varsigma_t)\, W_t^4, \\ B_{t+1}^1 &= R_t\left[1 - (\alpha_{t,z} - \varsigma_{t,z})\right] W_{t,z}^4 - R_t\left(C_{t,z} + S_t^3 \hat{C}_{t,z}\right) \\ &\quad - R_t(\gamma_{t,z} - \zeta_{t,z}) W_{t,z}^4, \\ K_{t+1,z} &= R_{t+1}^k(\gamma_{t,z} - \zeta_{t,z}) W_{t,z}^4. \end{aligned}$$

Substituting these expressions into the definition of $W^2_{t+1,z}$ gives the US version of (12):

$$\begin{aligned} W^2_{t+1,z} &= R_t\left(1+\left(\frac{S^1_{t+1}\hat{R}_t}{S^3_t R_t}-1\right)(\alpha_{t,z}-\varsigma_{t,z})+\left(\frac{R^k_{t+1}}{R_t}-1\right)\left(\gamma_{t,z}-\zeta_{t,z}\right)\right)W^4_{t,z} \\ &\quad -R_t\left(C_{t,z}+S^3_t\hat{C}_{t,z}\right). \end{aligned}$$

In the case of UK agents, we have

$$\begin{aligned} S_{t+1}\hat{B}^1_{t+1} &= \left(\frac{S^1_{t+1}\hat{R}_t}{S^3_t}\right)\left[(\alpha_{t,z}-\varsigma_{t,z})-(\gamma_{t,z}-\hat{\zeta}_{t,z})\right]W^4_t, \\ B^1_{t+1} &= R_t\left[1-(\alpha_{t,z}-\varsigma_{t,z})\right]W^4_{t,z}-R_t\left(C_{t,z}+S^3_t\hat{C}_{t,z}\right), \\ S^3_t\hat{K}_{t+1,z} &= \hat{R}^k_{t+1}(\gamma_{t,z}-\hat{\zeta}_{t,z})W^4_{t,z}. \end{aligned}$$

Substituting these expressions into the definition of $W^2_{t+1,z}$ gives:

$$\begin{aligned} W^2_{t+1,z} &= R_t\left(1+\left(\frac{S^1_{t+1}\hat{R}_t}{S^3_t R_t}-1\right)(\alpha_{t,z}-\varsigma_{t,z})+\left(\frac{S^1_{t+1}\hat{R}^k_{t+1}}{S^3_t R_t}-\frac{S^1_{t+1}\hat{R}_t}{S^3_t R_t}\right)\left(\gamma_{t,z}-\hat{\zeta}_{t,z}\right)\right)W^4_{t,z} \\ &\quad -R_t\left(C_{t,z}+S^3_t\hat{C}_{t,z}\right), \end{aligned}$$

which is the UK version of (12).

The first-order and envelope conditions from the period-2 optimization problem are

$$0 = \mathsf{E}^2_{t,z}\left[\mathcal{D}J^4_z\left(W^4_{t,z}\right)\left(\frac{S^3_t}{S^1_t}-1\right)\right], \tag{A66}$$

$$\mathcal{D}J^2_z(W^2_{t,z}) = \mathsf{E}^2_{t,z}\left[\mathcal{D}J^4_z(W^4_{t,z})H^3_t\right], \tag{A67}$$

where $\mathcal{D}J_z(.)$ denotes the derivative of $J_z(.)$. The first-order conditions for $C_{t,z}$, $\hat{C}_{t,z}$, and $\lambda_{t,z}$ in the period-4 problem take the same form for US and UK agents:

$$\lambda_{t,z} \;:\; 0=\mathsf{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})\left(\frac{S^1_{t+1}\hat{R}_t}{S^3_t R_t}-1\right)\right], \tag{A68}$$

$$C_{t,z} \;:\; U_c(\hat{C}_t,C_t)=R_t\beta\mathsf{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})\right], \tag{A69}$$

$$\hat{C}_{t,z} \;:\; U_{\hat{c}}(\hat{C}_t,C_t)=R_t\beta S^3_t\mathsf{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})\right]. \tag{A70}$$

The first-order conditions for $\gamma_{t,z}$ differ:

$$\gamma_{t,z<1/2} \quad : \quad 0 = \mathsf{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})\left(\frac{R^k_{t+1}}{R_t} - 1\right)\right], \tag{A71}$$

$$\gamma_{t,z\geq 1/2} \quad : \quad 0 = \mathsf{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})R_t\left(\frac{S^1_{t+1}\hat{R}^k_{t+1}}{S^3_t R_t} - \frac{S^1_{t+1}\hat{R}_t}{S^3_t R_t}\right)\right]. \tag{A72}$$

The envelope condition for US and UK agents is

$$\mathcal{D}J^4_z(W^4_{t,z}) = \beta R_t \mathsf{E}^4_{t,z}\left[\mathcal{D}J^2_z(W^2_{t+1,z})H^1_{t+1,z}\right]. \tag{A73}$$

Equations (13) - (19) are obtained by combining (A66) - (A73) with $V_{t,z} \equiv \mathcal{D}J^4_z(W^4_{t,z})$.

A.3 Market Clearing Conditions

For any variable X, let $X_{t,\mathsf{US}}$ denote $X_{t,z}$ for $z < 1/2$, and $X_{t,\mathsf{UK}} = X_{t,z}$ for $z \geq 1/2$. Market clearing in US deposits in period 1 of day $t+1$ implies that (see equations 2, 4, and 21):

$$(B^3_{t,\mathsf{US}} + S^3_t T^4_{t,z*} - S^3_t T^4_{t,\mathsf{US}} + C_{t,\mathsf{UK}} - I_{t,\mathsf{US}}) + (B^3_{t,\mathsf{UK}} + S^3_t T^4_{t,z*} - S^3_t T^4_{t,\mathsf{UK}} - C_{t,\mathsf{UK}}) = 0.$$

With deposit-market clearing in period 3, this condition further simplifies to:

$$S^3_t T^4_{t,z*} - S^3_t T^4_{t,\mathsf{US}} + S^3_t T^4_{t,z*} - S^3_t T^4_{t,\mathsf{UK}} - I_{t,\mathsf{US}} = 0.$$

Since market clearing in currency markets implies that $\int T^j_{t,z}dz = \int T^j_{t,z^*}dz^*$, this condition implies that $I_{t,\mathsf{US}} = 0$. Imposing this restriction on the overnight dynamics of US capital gives (22). Similarly, market clearing in the UK deposit markets implies that:

$$\begin{aligned} 0 &= (\hat{B}^1_{t,\mathsf{US}} + T^4_{t,\mathsf{US}} - T^4_{t,z*} - \hat{C}_{t,\mathsf{US}}) + (\hat{B}^1_{t,\mathsf{UK}} + T^4_{t,\mathsf{UK}} - T^4_{t,z*} + \hat{C}_{t,\mathsf{US}} - \hat{I}_{t,\mathsf{US}}) \\ &= T^4_{t,\mathsf{US}} - T^4_{t,z*} + T^4_{t,\mathsf{UK}} - T^4_{t,z*} - \hat{I}_{t,\mathsf{US}} \\ &= -\hat{I}_{t,\mathsf{US}}. \end{aligned}$$

Imposing $\hat{I}_{t,\mathsf{UK}} = 0$ on the overnight dynamics of UK capital gives (23).

A.4 Log Approximations

To approximate log portfolio returns we make use of a second-order approximation similar to the one employed by Campbell and Viceira (2002). Both $h^1_{t,z} \equiv \ln H^1_{t,z}$ and $h^3_{t,z} \equiv \ln H^3_{t,z}$ can be expressed as:

$$h^j_{t,z} = \ln\left[1 + (e^x - 1)(a - u) + (e^y - 1)(b - w)\right]$$

where x, y, u, and w are random variables that are zero in the steady state. Taking a second-order Taylor series approximation to $h^j_{t,z}$ around this point gives:

$$h^j_{t,z} \cong ax + by + \frac{1}{2}\left(a - a^2\right)x^2 + \frac{1}{2}(b - b^2)y^2 - abxy - xu - yw.$$

The final step is to replace x^2, y^2, xy, xu, and yw by their respective moments:

$$h^j_{t,z} \cong ax + by + \frac{1}{2}\left(a - a^2\right)\mathsf{V}[x] + \frac{1}{2}(b - b^2)\mathsf{V}[y] - ab\mathsf{CV}[x, y] - \mathsf{CV}[x, u] - \mathsf{CV}[y, w]$$

Campbell and Viceira (2002) show that the approximation error associated with this expression disappears in the limit when x, y, u and w represent realizations of continuous-time diffusion processes.

Applying this approximation to the definitions of $\ln H^1_{t+1,z}$ and $\ln H^3_{t,z}$ yields equations (24), (25) and (26). In deriving the solution of the model, it is useful to write the latter two equations as:

$$h^1_{t+1,z} = \omega'_{t,z}x_{t+1,z} + \tfrac{1}{2}\omega'_{t,z}\Lambda_z - \tfrac{1}{2}\omega'_{t,z}\Sigma_z\omega_{t,z} - \phi_{t,z}, \tag{A74}$$

where $\Sigma_z \equiv \mathsf{V}^4_{t,z}(x_{t+1,z})$ and $\Lambda_z \equiv \mathrm{diag}(\Sigma_z)$, with

$$\begin{aligned}
\omega'_{t,z} &\equiv \begin{bmatrix} \alpha_{t,z} & \gamma_{t,z} \end{bmatrix}, \\
x_{t+1,z} &\equiv \begin{bmatrix} s^1_{t+1} - s^3_t + \hat{r}_t - r_t & r^k_{t+1} - r_t \end{bmatrix}, \\
\phi_{t,z} &\equiv \mathsf{CV}^4_{t,\mathsf{US}}\left[s^1_{t+1}, \varsigma_{t,z}\right] + \mathsf{CV}^4_{t,\mathsf{US}}\left[r^k_{t+1}, \zeta_t\right],
\end{aligned}$$

for $z < 1/2$ (i.e. US consumers), and

$$\begin{aligned}
\omega'_{t,z} &\equiv \begin{bmatrix} \alpha_{t,z} - \gamma_{t,z} & \gamma_{t,z} \end{bmatrix}, \\
x_{t+1,z} &\equiv \begin{bmatrix} s^1_{t+1} - s^3_t + \hat{r}_t - r_t & \hat{r}^k_{t+1} + s^1_{t+1} - s^3_t - r_t \end{bmatrix}, \\
\phi_{t,z} &\equiv \mathsf{CV}^4_{t,z}\left[s^1_{t+1}, \varsigma_{t,z}\right] + \mathsf{CV}^4_{t,z}\left[\hat{r}^k_{t+1}, \hat{\zeta}_t\right],
\end{aligned}$$

for $z \geq 1/2$.

Our solution also makes use of approximations to the capital dynamics in (35) and (36). To derive these approximations we start by writing the dynamics of US capital as:

$$\frac{K_{t+1}}{K_t} = R^k_{t+1}\left(1 - \frac{C_{t,\mathsf{US}}W^4_{t,\mathsf{US}}}{W^4_{t,\mathsf{US}}K_t} - \frac{C_{t,\mathsf{UK}}W^4_{t,\mathsf{UK}}}{W^4_{t,\mathsf{UK}}K_t}\right).$$

Log linearizing this equation gives:

$$k_{t+1} - k_t \cong r^k_{t+1} + \ln(1-\mu) - \left(\frac{\mu}{2(1-\mu)}\right)\left(w^4_{t,\mathsf{US}} - k_t + \delta_{t,\mathsf{US}}\right) - \left(\frac{\mu}{2(1-\mu)}\right)\left(w^4_{t,\mathsf{UK}} - k_t + \delta_{t,\mathsf{UK}}\right).$$

Now, deposit-market clearing implies that $K_t + S^3_t \hat{K}_t = W^4_{t,\mathsf{US}} + W^4_{t,\mathsf{UK}}$, so:

$$w^4_{t,\mathsf{US}} - k_t = \ln\left(1 + \frac{S^3_t \hat{K}_t}{K_t} - \frac{W^4_{t,\mathsf{UK}}}{K_t}\right) \cong s^3_t + \hat{k}_t - k_t - (w^4_{t,\mathsf{UK}} - k_t).$$

Combining these equations gives (35). We approximate dynamics of UK capital in a similar manner. Deposit-market clearing implies that:

$$\begin{aligned}
\frac{\hat{K}_{t+1}}{\hat{K}_t} &= \hat{R}^k_{t+1}\left(1 - \frac{\hat{C}_{t,\mathsf{US}} W^4_{t,\mathsf{US}}}{W^4_{t,\mathsf{US}} \hat{K}_t} - \frac{\hat{C}_{t,\mathsf{UK}} W^4_{t,\mathsf{UK}}}{W^4_{t,\mathsf{UK}} \hat{K}_t}\right) \\
&= \hat{R}^k_{t+1}\left(1 - \frac{C_{t,\mathsf{US}} W^4_{t,us}}{W^4_{t,\mathsf{US}} S^3_t \hat{K}_t} - \frac{C_{t,\mathsf{UK}} W^4_{t,UK}}{W^4_{t,\mathsf{UK}} S^3_t \hat{K}^3_t}\right),
\end{aligned}$$

where the second line follows from the fact that the first-order conditions for consumption imply that $C_{t,z} = S^3_t \hat{C}_{t,z}$ for all z. Log linearizing this equation gives (36).

A.5 Marginal Utility of Wealth

The section addresses the marginal utility of wealth and why it can depart in this model from the marginal utility of consumption. To derive the relationship between the marginal utility of wealth and the marginal utility of consumption for US agents, we first combine (A67)-(A69) and (A73):

$$\begin{aligned}
0 &= \mathsf{E}^4_{t,z}\left[V_{t+1,z} H^3_{t+1,z}\left(\frac{S^1_{t+1}\hat{R}_t}{S^3_t R_t} - 1\right)\right], \\
0 &= \mathsf{E}^4_{t,z}\left[V_{t+1,z} H^3_{t+1,z}\left(\frac{R^k_{t+1}}{R_t} - 1\right)\right], \\
U_c(\hat{C}_{t,z}, C_{t,z}) &= \beta R_t \mathsf{E}^4_{t,z}\left[V_{t+1,z} H^3_{t+1,z}\right], \\
V_{t,z} &= \beta R_t \mathsf{E}^4_{t,z}\left[V_{t+1,z} H^3_{t+1,z} H^1_{t+1,z}\right].
\end{aligned}$$

Log linearizing these equations, with $U_c(\hat{C}_{t,z}, C_{t,z}) = \frac{1}{2}C_{t,z}^{-1}$, we find:

$$\mathsf{E}_{t,z}^4\left[s_{t+1}^1 - s_t^3 + \hat{r}_t - r_t\right] = -\mathsf{CV}_{t,z}^4\left[v_{t+1} + h_{t+1,z}^3, s_{t+1}^1\right] - \tfrac{1}{2}\mathsf{V}_{t,z}^4\left[s_{t+1}^1\right], \tag{A75}$$

$$\mathsf{E}_{t,z}^4\left[r_{t+1}^k - r_t\right] = -\mathsf{CV}_{t,z}^4\left[v_{t+1} + h_{t+1,z}^3, r_{t+1}^k\right] - \tfrac{1}{2}\mathsf{V}_{t,z}^4\left[r_{t+1}^k\right], \tag{A76}$$

$$c_t + \ln\beta + r_t = -\mathsf{E}_{t,z}^4\left[v_{t+1,z} + h_{t+1,z}^3\right] - \tfrac{1}{2}\mathsf{V}_{t,z}^4\left[v_{t+1,z} + h_{t+1,z}^3\right], \tag{A77}$$

$$v_{t,z} - \ln\beta - r_t = \mathsf{E}_{t,z}^4\left[v_{t+1,z} + h_{t+1,z}^3 + h_{t+1}^1\right] + \tfrac{1}{2}\mathsf{V}_{t,z}^4\left[v_{t+1,z} + h_{t+1,z}^3 + h_{t+1}^1\right]. \tag{A78}$$

Stacking (A75) and (A76), combining (A77) and (A78), and substituting for h_{t+1}^1 gives

$$\mathsf{E}_{t,z}^4\left[x_{t+1,z}\right] + \tfrac{1}{2}\Lambda_z = -\mathsf{CV}_{t,z}^4\left[x_{t+1,z}, v_{t+1,z} + h_{t+1,z}^3\right], \tag{A79}$$

$$v_{t,z} + c_t + \phi_{t,z} = \omega_{t,z}'\mathsf{E}_{t,z}^4\left[x_{t+1,z}\right] + \tfrac{1}{2}\omega_{t,z}'\Lambda_z + \omega_{t,z}'\mathsf{CV}_{t,z}^4\left[x_{t+1,z}, v_{t+1,z} + h_{t+1,z}^3\right]. \tag{A80}$$

Combining these expressions we obtain equation (28). In the case of UK agents, we work with log linearized versions of (A67), (A68), (A70) and (A73):

$$\mathsf{E}_{t,z}^4\left[s_{t+1}^1 - s_t^3 + \hat{r}_t - r_t\right] = -\mathsf{CV}_{t,z}^4\left[v_{t+1} + h_{t+1,z}^3, s_{t+1}^1\right] - \tfrac{1}{2}\mathsf{V}_{t,z}^4\left[s_{t+1}^1\right],$$

$$\mathsf{E}_{t,z}^4\left[\hat{r}_{t+1}^k + s_{t+1}^1 - s_t^3 - r_t\right] = -\mathsf{CV}_{t,z}^4\left[v_{t+1} + h_{t+1,z}^3, r_{t+1}^k\right] - \tfrac{1}{2}\mathsf{V}_{t,z}^4\left[\hat{r}_{t+1}^k + s_{t+1}^1\right],$$

$$c_t + \ln\beta + r_t = -\mathsf{E}_{t,z}^4\left[v_{t+1,z} + h_{t+1,z}^3\right] - \tfrac{1}{2}\mathsf{V}_{t,z}^4\left[v_{t+1,z} + h_{t+1,z}^3\right],$$

$$v_{t,z} - \ln\beta - r_t = \mathsf{E}_{t,z}^4\left[v_{t+1,z} + h_{t+1,z}^3 + h_{t+1}^1\right] + \tfrac{1}{2}\mathsf{V}_{t,z}^4\left[v_{t+1,z} + h_{t+1,z}^3 + h_{t+1}^1\right].$$

Proceeding as before with our approximation for $h_{t+1,z}$ for $z \geq 1/2$ gives (A79) and (A80). Hence, equation (28) holds for UK agents.