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Jeffrey R. Campbell  
Zvi Hercowitz

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**ABSTRACT**

This paper characterizes the labor supply and borrowing of a household facing collateral requirements that limit its debt and compel it to accumulate equity in its durable goods stock. The household's discount rate exceeds the market rate of interest, so it would otherwise finance increased current consumption by borrowing against future wages. Collateral constraints generate a positive comovement between the household's debt, the stock of durable goods and labor supply following wage or interest rate shocks---as the household's labor supply adjusts to finance downpayments on new durable good purchases and the subsequent debt repayment. Increasing the speed of debt repayment amplifies these movements.

Jeffrey R. Campbell  
Federal Reserve Bank of Chicago  
230 South LaSalle Street  
Chicago, IL 60604  
and NBER  
jcampbell@frbchi.org

Zvi Hercowitz  
Eitan Berglas Scholl of Economics  
Tel Aviv University  
P.O.B. 39040  
Ramat Aviv, Tel Aviv 69978  
ISRAEL  
zvih@post.tau.ac.il

# 1 Introduction

This paper characterizes the consumption and labor supply of a household facing collateral requirements that limit its debt. The household's rate of time preference is higher than the average interest rate, so it wishes to trade future earnings for current consumption. However, the need for collateral implies that the household can borrow only for purchasing durable goods, including housing. If the collateral requirements impose a minimum down-payment and accelerated amortization of the debt, then the choices of hours worked and durable purchases become directly linked to each other through the household's flow budget constraint.

In the face of stochastic wages and interest rates, the choices of this household differ substantially from those of a standard unconstrained household:

First, a permanent wage increase induces an unconstrained household to finance a one-time increase in its durable goods stock by borrowing against future earnings (or by reducing financial assets). Assuming that household's preferences are consistent with balanced growth as in King Plosser, and Rebelo (1988), the wage's income and substitution effects on leisure offset each other and leave labor supply unchanged. Under collateral constraints building up the durable stock takes time, during which debt accumulates and labor supply is higher.

Second, a temporary decline in the interest rate reduces hours worked of an unconstrained household, as consumption and leisure become cheaper relative to their future counterparts (Barro and King (1984)). The collateral constraint on debt makes these choices infeasible, as the household cannot borrow to finance leisure and nondurable consumption. Instead, the constrained household *increases* its labor supply to acquire the down payment required for the desired additional purchases of durable goods.

Clearly, the household under consideration here cannot be a representative consumer in a closed economy, because the market for loaned funds must clear, along with the labor and the commodity markets. The funds should come from a more patient household, as in Becker (1980) and Krusell and Smith (1998), which is likely to be wealthier than the household considered here—as the patient household accumulates assets. If the wealth differential is large, the choices of the representative debtor will disproportionately affect aggregate hours worked.

The present model may be used to interpret the marked increase in the stock of consumer debt since the middle 1980's. The model predicts that as

the repayment of principal slows down—which in practice can follow from less costly access to refinancing—hours worked become both less responsive to wage or interest rate shocks and less persistent. Thus, our model is consistent with some aspects of the decline in business cycle volatility following 1984 documented by McConnell and Quiros (2000) and Stock and Watson (2003), given financial innovation that reduces consumers' equity in their durable goods.

Our analysis builds on the literature on the implications of capital market imperfections for household behavior. Regarding consumption, the empirical failure of Euler equations based on unconstrained intertemporal substitution was documented by Zeldes (1989) and others. More directly related to this paper are the results in Fortín (1995) and Del Boca and Lusardi (2003), who found that married women's labor supply increases with their household's mortgage debt, using Canadian and Italian data. Here, we model this connection as arising from an empirically relevant collateral constraint, and consider its implications for the household's responses to wage and interest rate shocks.

Rupert, Rogerson, and Wright (2000) stress that the accumulation of durable goods plays in general no role for the dynamic labor supply decisions of a household facing complete markets—or even incomplete markets but with access to a one period bond.<sup>1</sup> In contrast, with collateral constraints the accumulation of durable goods is central for labor supply, given the interaction generated through the budget and borrowing constraints.

The remainder of this paper proceeds as follows. In the next section, we present evidence on the comovement of hours worked, consumer debt, and the stock of consumer durables at business cycle frequencies and over the long run. In Section 3, we specify the household's preferences, endowments and constraints, and characterize the solution to its utility maximization problem. To build intuition for the quantitative results, we consider in Section 4 a simpler version of the model without accelerated debt repayment. Section 5 presents the household's dynamic responses to wage and interest rate

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<sup>1</sup>In their setup, no interaction in utility between home capital and leisure is introduced. Greenwood and Hercowitz (1991) consider a setup with such an interaction, stressing complementarity of home capital and time spent in non-work activities. These differences, however, are related to the specification of home production, which is another channel of generating a link between durable purchases and labor supply.

shocks in a calibrated version of the model, and it assesses how reducing collateral requirements affects the volatility and persistence of the household's choices. The disaggregation of household durables is also addressed. Section 6 contains concluding remarks.

## 2 Some Evidence on Households' Debt, Work, and Durables

To illustrate the potential importance of collateral constraints on households, we present here aggregate evidence on the comovement of household debt, hours of work, and the stock of household durables in the U.S. Intuition suggests that collateral constraints should induce a positive comovement of debt, hours worked and durable goods of debtor households, and our analysis below verifies this. Observing the aggregate comovements can indicate whether this mechanism is strong enough to be potentially relevant for macroeconomic fluctuations. We consider three aspects of the data: Cyclical comovements, long-run trends, and changes of cyclical comovements over time. Household debt is measured by mortgage debt plus consumer credit, and 'household durables' are measured as real estate plus the stock of durable consumer goods. Both are nominal values deflated by the consumption price index, and the three variables are expressed in per-capita terms.<sup>2</sup> Figure 1 plots the three HP-filtered variables, showing strong positive comovement of both the stocks of household debt and durables with hours worked. The correlation coefficients are 0.79 and 0.60, respectively. As Figure 1 suggests, the correlation between the stocks of debt and durables is also large and positive, 0.79.

The strong positive comovement of household debt with hours worked at the aggregate level suggests that studying the behavior of debtor households can add to the understanding of labor fluctuations. In particular, the positive correlations of the durables' stock with both hours worked and household debt suggests that collateral constraints, which link debt with durable purchases and labor supply—to earn the downpayment and the repayment of the debt—may be an important factor in generating the aggregate fluctuations

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<sup>2</sup>The debt and durables are end of quarter values, and the deflator is the total consumption chain price index from NIPA, base 1996. The per-capita values are computed using the civilian noninstitutional population.

of these variables.

The unfiltered observations of these variables are shown in Figure 2. Household debt and the stock of durable goods are scaled by the wage—the nominal per-capita stocks divided by an index of the nominal hourly wage—and logged. A particular interesting year for this figure is 1983, when the debt starts to increase at a rapid rate. As it can be seen from the logarithmic scale on the left, the debt, relative to the wage, increased by 60% from 1983:1 to the end of the sample (19 years). In comparison, from 1954:1 to 1982:4 (29 years) the rate of change is only 33%. The expansion of the debt from the early 1980s onwards is consistent with a relaxation of borrowing constraints around this time, which induced borrowing against a large fraction of the durable stock.

An additional feature of Figure 2 is that the trend of per-capita hours changes from negative to positive around the same time the debt starts to expand faster. The present model suggests a link between relaxing the borrowing constraint and higher levels of debt and hours worked. Using steady-state comparative statics, it follows from the model that improving the terms of collateralized borrowing increases the attractiveness of durable goods relative to leisure and nondurables. This leads to a lower level of leisure and higher levels of the stocks of durables and debt relative to the wage.

A relaxation of collateral requirements raises the possibility that the cyclical comovement of hours worked with debt and durables as well as the volatility of hours decline—as the mechanism stressed in this paper is weakened. The statistics for the periods prior and after 1983:1 are consistent with this consideration. In the first part of the sample, the correlations of debt and durables with hours are 0.86 and 0.71, respectively, and in the second part of the sample they decline to 0.53 and 0.40, respectively. Additionally, as it can be seen from Figure 1, the volatility of hours is lower in the second part of the sample: The standard deviation of hours before 1983:1 is 1.9 percent, and from 1983:2 onwards it is 1.4 percent.

### 3 The Model

The model characterizes the choices of a single infinitely lived household that faces a collateral constraint on its borrowing. This constraint has three features that are typical of consumer loan contracts in the United States. First, debt collateralized by homes and vehicles is almost 90% of total house-

hold debt.<sup>3</sup> Here, we make the assumption that all debt is collateralized by durable goods. Second, the majority of new home mortgages in the United States have loan to value ratios at or below 80%, and a typical loan to value ratio for a new car purchase is 90%.<sup>4</sup> Accordingly, we assume that borrowing requires a minimum downpayment that exceeds the user cost. Third, typical loan contracts are for a fixed term that is much less than the useful life of the durable good, so we assume that loan contracts require accelerated repayment.<sup>5</sup>

Given that our goal is to analyze the implications of collateral constraints for a representative debtor's choices, the collateral constraint is the model's only nonstandard feature. A possible extension of our model would include nonconvex costs of durable goods adjustment—which would be relevant for housing and automobiles—and aggregation over heterogeneous debtor households. King and Thomas (2003) examine models of lumpy labor demand by heterogeneous employers and find that they are observationally similar to models of smooth adjustment by a representative employer. This suggests to us that the interaction of debt and labor supply through the budget constraint that we consider here would manifest itself in such an extension of the present paper.<sup>6</sup>

### 3.1 Preferences, Prices and Trade

The model's household values three goods: leisure, nondurable consumption, and the service-flow from durable goods. Each period the household has one

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<sup>3</sup>Using data from the 2002 Survey of Consumer Finances, Aizcorbe, Kennickell, and Moore (2003) report that borrowing collateralized by residential property account for 81.5% of households' debt in 2001 (Table 10), and installment loans, which include both collateralized vehicle loans and unbacked education and other loans, amounts to an additional 12.3%. Credit card balances and other forms of debt account for the remainder. The reported uses of borrowed funds (Table 12) indicate that vehicle debt represents 7.8% of total household debt, and, hence, collateralized debt (by homes and vehicles) is almost 90% of total household debt.

<sup>4</sup>Evidence on the terms of mortgages comes from Federal Housing Finance Board's Monthly Interest Rate Survey. Federal Reserve Statistical Release G.19 reports the terms of new automobile loans.

<sup>5</sup>We refer below to the possibility of refinancing, which practically amounts to extending the horizon of repayment.

<sup>6</sup>Single automobiles and homes are not divisible, but divisibility of the durable goods stock will arise if the consumer can make supplemental expenditures to improve the current durable good's quality (home improvement).

unit of time which it splits between leisure and work. Let  $N_t$ ,  $C_t$ , and  $S_t$  denote the household's hours worked, nondurable consumption, and durable goods stock in period  $t$ .<sup>7</sup> A time-separable expected utility function with a constant elasticity of intertemporal substitution expresses the household's preferences over stochastic sequences of these goods.

$$\begin{aligned} \mathbf{E} \left[ \sum_{t=0}^{\infty} e^{-\rho t} (S_t^\theta C_t^{1-\theta})^{1-\sigma} (1 - N_t)^{1-\lambda} / (1 - \sigma) \right] & \text{ if } \sigma \neq 1, \\ \mathbf{E} \left[ \sum_{t=0}^{\infty} e^{-\rho t} (\theta \ln S_t + (1 - \theta) \ln C_t + \ln(1 - N_t)) \right] & \text{ if } \sigma = 1. \end{aligned} \quad (1)$$

In (1),  $\rho$  is the household's rate of time preference. We restrict  $(1 - \sigma)(1 - \lambda)$  to be positive if  $\sigma \neq 1$  to ensure that the utility function is concave. The assumption that preferences are additively separable in consumption and leisure when the intertemporal elasticity of substitution equals one guarantees that they satisfy the balanced growth restrictions of King, Plosser, and Rebelo (1988).

The household funds its expenditures from the labor and the credit markets. The corresponding stochastic prices are  $W_t$ —the real wage—and  $R_t$ —the (gross) real interest rate on borrowing at time  $t$ . Both prices are taken as given by the household. The mean growth rate of the real wage is  $\mu$ , and interest rate is stationary with mean  $R$ , where  $R > \mu$ .

The impatient nature of the household is characterized by the assumption that  $e^\rho \mu^\sigma > R$ . As in Becker (1980) and Krusell and Smith (1998), this inequality can arise if the source of borrowed funds is a more patient household, whose rate of time preference determines the market rate of interest.

Denoting the household's debt issued at  $t$  and repayable in  $t + 1$  with  $B_t$  and the depreciation rate of durables with  $\delta$ , the budget constraint is

$$R_{t-1}B_{t-1} + C_t + (S_t - (1 - \delta)S_{t-1}) \leq W_t N_t + B_t. \quad (2)$$

We now turn to the specification of the constraint on the household's borrowing. All debt must be backed by collateral, and the durable good's collateral value is generally less than its replacement cost. The collateralizable value of the household's current durable goods stock is

$$V_t = (1 - \pi) \sum_{j=0}^{\infty} (1 - \phi)^j (S_{t-j} - (1 - \delta)S_{t-j-1}). \quad (3)$$

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<sup>7</sup>In this model, the flow of services from the durable goods stock is proportional to that stock. We normalize this constant of proportionality to equal one and use the same notation for the durable goods stock and its service flow.



Here,  $\pi$  represents the fraction of a newly purchased good's value that cannot serve as collateral. Hence, this is the downpayment required for the purchase of a new durable good. The rate at which the durable good's collateral value depreciates is  $\phi \geq \delta$ . If this rate of collateral depreciation exceeds the rate of physical depreciation, then the household must accumulate equity in its previous durable goods purchases. With this specification for the evolution of collateral value, the household's borrowing constraint is simply<sup>8</sup>

$$B_t \leq V_t. \tag{4}$$

We have excluded holdings of financial assets or productive capital from the right-hand side of (4). This is without loss of generality, because no household will choose to hold such assets if (4) constrains its intertemporal substitution.

### 3.2 Household Choices

We now develop the solution to the household's problem given the assumption that (4) always binds. Replacing  $V_t$  with  $B_t$  in (3), the constraint can be rewritten as

$$B_t = (1 - \phi) B_{t-1} + (1 - \pi) (S_t - (1 - \delta) S_{t-1}). \tag{5}$$

Given its initial stocks of debt and durable goods,  $B_{-1}$ , and  $S_{-1}$ , the borrower chooses state-contingent sequences of  $C_t$ ,  $S_t$ ,  $N_t$ , and  $B_t$  to maximize the utility function in (1) subject to the sequences of budget and borrowing constraints in (2) and (5).

Denote the appropriately discounted Lagrange multiplier on (2) with  $\Psi_t$ , which will always be positive. If we express the discounted Lagrange multiplier on (5) as  $\Xi_t \Psi_t$ , then  $\Xi_t$  measures the value (in units of either consumption good) of relaxing the borrowing constraint. In addition to the two binding constraints, the first order conditions for this utility maximization

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<sup>8</sup>Note that accumulated equity is by assumption not collateralizable. Hence, the implied assumption is that transaction costs in the appropriation and liquidation of a used durable good generate a wedge between the market value,  $S_t$ , and the net value for the lending institution,  $V_t$ .

problem are

$$\Psi_t = (1 - \theta) (S_t^\theta C_t^{1-\theta})^{-\sigma} \left( \frac{S_t}{C_t} \right)^\theta (1 - N_t)^{1-\lambda}, \quad (6)$$

$$\frac{\theta}{1 - \theta} \left( \frac{C_t}{S_t} \right) = (1 - \Xi_t (1 - \pi)) - e^{-\rho} (1 - \delta) E \left[ \frac{\Psi_{t+1}}{\Psi_t} (1 - \Xi_{t+1} (1 - \pi)) \right], \quad (7)$$

$$W_t = \frac{1 - \lambda}{(1 - \sigma)(1 - \theta)} \frac{C_t}{1 - N_t}, \quad (8)$$

$$\Xi_t = 1 - e^{-\rho} E \left[ \frac{\Psi_{t+1}}{\Psi_t} \right] R_t + e^{-\rho} (1 - \phi) E \left[ \frac{\Psi_{t+1}}{\Psi_t} \Xi_{t+1} \right]. \quad (9)$$

A state-contingent sequence of  $(C_t, S_t, N_t, B_t, \Psi_t, \Xi_t)$  that satisfies these first order conditions, the two constraints, and the transversality conditions

$$\lim_{t \rightarrow \infty} E[\Psi_t] = \lim_{t \rightarrow \infty} E[\Psi_t \Xi_t] = 0 \quad (10)$$

is a solution to the household's utility maximization problem.

The interpretation of (6) and (8) is familiar. The first equates  $\Psi_t$  to the marginal utility of current nondurable consumption, and the second is a labor supply condition. Equations (7) and (9) arise from differentiating the Lagrangian with respect to  $S_t$  and  $B_t$ . In the absence of a binding constraint on  $B_t$ , the household would equate the marginal utility of durable consumption with the good's purchase price minus its discounted and depreciated expected resale value. This is the condition that arises if we artificially set  $\Xi_t$  and  $\Xi_{t+1}$  to zero in (7). If we define  $1 - \Xi_t (1 - \pi)$  to be the net purchase price of a durable good—the actual price less the benefit from relaxing the borrowing constraint by purchasing one more unit—then (7) has a similar interpretation when  $\Xi_t$  and  $\Xi_{t+1}$  are positive.

Similarly, setting  $\Xi_t$  and  $\Xi_{t+1}$  to zero reduces equation (9) to the familiar condition that the consumer equates the marginal rate of intertemporal substitution with the real interest rate. In general, (9) determines the marginal value of debt,  $\Xi_t$ , to equal the expected wedge between the real interest rate and the marginal rate of intertemporal substitution plus its discounted and depreciated expected future value.

### 3.3 The Balanced Growth Path

Here we consider the deterministic balance growth path with  $R_t = R$  and  $\log(W_{t+1}/W_t) = \mu$ , and derive the long-run comparative statics to changes in the parameters of the collateral constraint,  $\pi$  and  $\phi$ . On the balanced growth path,  $N_t$  and  $\Xi_t$  equal constants,  $N$  and  $\Xi$ , all remaining quantities grow at the rate  $\mu$  (e.g.  $B_t = B\mu^t$ ,  $C_t = C\mu^t$ , ...), and the marginal utility of nondurable consumption shrinks at the rate  $\mu^\sigma$ .

We begin with (9), which immediately implies that

$$\Xi = \frac{1 - e^{-\rho}\mu^{-\sigma}R}{1 - e^{-\rho}\mu^{-\sigma}(1 - \phi)}. \quad (11)$$

Given the assumption that  $e^{-\rho}\mu^{-\sigma}R < 1$ ,  $\Xi$  is positive and the associated constraint always binds on the balanced growth path. From (11), we can interpret  $\Xi$  as the present discounted value of the violation of the standard Euler equation.

From (7), the ratio of durable to nondurable consumption is

$$\frac{S}{C} = \frac{\theta}{1 - \theta} \frac{1}{(1 - \Xi(1 - \pi)) [1 - e^{-\rho}\mu^{-\sigma}(1 - \delta)]}. \quad (12)$$

This is a usual expression for the ratio of durable goods to nondurable consumption, involving the ratio of the two expenditure shares and their relative price. In this case, the relative price is the net cost of durables, discussed above.<sup>9</sup> Using (11) to replace  $\Xi$ ,  $S/C$  can be expressed as function of the model's primitive parameters.

The steady state borrowing constraint immediately implies that

$$\frac{B}{S} = \frac{(1 - \pi)(1 - (1 - \delta)/\mu)}{1 - (1 - \phi)/\mu}. \quad (13)$$

This is the household's leverage ratio. If  $\phi$  exceeds  $\delta$ , then  $B/S$  is less than  $(1 - \pi)$ , the fraction of new durable goods purchases financed by debt. Thus, the household's equity share in the most recent purchases is lower than the corresponding share in the entire durable goods stock.

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<sup>9</sup>Note that (12) implies that a household facing a binding constraint on borrowing will direct its consumption more heavily towards durable goods than a household without such constraint, given that the net purchase price of durables is lower than 1 when  $\Xi > 0$ .

The derivation of the steady-state growth path is complete after expressing  $C$  in terms of  $W$ ,  $R$  and the exogenous parameters. To do so, we substitute (8) into (2) and use (12) and (13) to get

$$C = \frac{W}{1 + \frac{S}{C}(1 - (1 - \delta)/\mu) + \frac{1-\lambda}{(1-\sigma)(1-\theta)} + (R/\mu - 1)\frac{B}{S}\frac{S}{C}}. \quad (14)$$

The balanced growth solution can be used to derive the long-run implications of changes in the collateral requirements, represented by the values of  $\pi$  and  $\phi$ . Increasing the downpayment rate,  $\pi$ , directly reduces  $S/C$  and  $B/S$  in (12) and (13). Hence,  $C/W$  increases and  $N$  declines according to (8). From the budget constraint and (13), both  $B/W$  and  $S/W$  decline as well. Intuitively, increasing the downpayment rate increases the net cost of durable goods by decreasing the debt associated with their purchase (equation (12)), and this induces the household to shift expenditures away from durable goods towards both leisure and nondurable consumption. Increasing the rate of debt repayment,  $\phi$ , has the same qualitative implications on  $S/C$ . In this case, however, the effect on the net cost of durables works through  $\Xi$ .

These comparative statics can be used to interpret the observations shown in Figure 2. In particular, the figure shows an acceleration in the debt/wage ratio and an increase in hours worked starting around 1983. This is a useful reference point because it can be associated with the acceleration of financial innovation in the United States in the early 1980s, which facilitated refinancing of home loans. Increasing the frequency of loan refinancing makes possible a reduction of the borrower's rate of equity accumulation, and hence it can be captured in our model by reducing  $\phi$ . The model predicts that a reduction in  $\phi$  permanently increases  $B/W$ ,  $S/W$ , and  $N$ .

## 4 Results from a Simple Case

To develop intuition for the household's choices in a stochastic environment under the present borrowing constraint, we consider in this section a version of the model with two simplifying assumptions. The first is that  $\phi = \delta$ , so that there is no accelerated repayment requirement, but there is still a downpayment requirement. The second assumption is that  $\sigma = \lambda = 1$ , so that the household's preferences are additively separable in the two consumption goods and leisure.

With  $\phi = \delta$ , it follows from the borrowing constraint in (5) that if the household starts off with no assets and no durables, that is  $B_{-1} = S_{-1} = 0$ , then  $B_t = (1 - \pi) S_t$  for all  $t \geq 0$ . Replacing  $B_t$  and  $B_{t-1}$  with  $(1 - \pi) S_t$  and  $(1 - \pi) S_{t-1}$ , the budget constraint (2) can be expressed as

$$C_t + \pi S_t \leq W_t N_t + R_{t-1} \left( \pi - \frac{R_{t-1} - 1 + \delta}{R_{t-1}} \right) S_{t-1}. \quad (15)$$

The household's sources of funds in period  $t$  are its labor income,  $W_t N_t$ , and the value of the depreciated durable goods net of debt repayment,  $(1 - \delta) S_{t-1} - R_{t-1} (1 - \pi) B_{t-1}$ —which, when  $\phi = \delta$ , can be written as  $R_{t-1} (\pi - (R_{t-1} - 1 + \delta) / R_{t-1}) S_{t-1}$ . Nondurable consumption and downpayments on its current durable goods stock are the uses of funds. Also, the first-order conditions can be combined to yield

$$\frac{\theta}{S_t} = \frac{\pi}{W_t (1 - N_t)} - e^{-\rho} E \left[ \frac{R_t \left( \pi - \frac{R_t - 1 + \delta}{R_t} \right)}{W_{t+1} (1 - N_{t+1})} \right]. \quad (16)$$

Here, the marginal utility of durable goods consumption is equated with the utility cost of working to acquire the downpayment less the expected utility in the following period from the leisure equivalent of accumulated equity.

We begin by using the budget constraint to illustrate a key implication of the present framework for household's choices. In (15), the current decision variables  $N_t$ ,  $C_t$ , and  $S_t$  appear along with predetermined variables and  $W_t$ . From the first order condition for labor in (8),  $C_t$  and  $W_t (1 - N_t)$  are proportional. Hence, if the household decides to expand its durable goods' stock because of an interest rate cut or expected future changes in wages or interest rates, (15) dictates that hours of work must go up. This positive comovement between hours worked and durable consumption contrasts sharply with the results from a model of a financially unconstrained household, as in Barro and King (1984). In such a model, when the current wage is held constant, a change in hours worked should be accompanied by changes in all forms of consumption in the *opposite* direction.<sup>10</sup>

A key term in both equations is  $\pi - (R_{t-1} - 1 + \delta) / R_{t-1}$ —the difference between the downpayment rate and the conventionally defined user cost of durable goods in period  $t - 1$ . When the downpayment is higher than the

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<sup>10</sup>The model in Barro and King has no durable consumption goods, but their result carries over to a model with durables if the household faces frictionless rental markets.

user cost, the borrowing constraint forces the household to acquire some ownership of its durable goods stock. We focus next on two cases regarding this term.

## 4.1 Full Collateral

A benchmark case consists of setting  $\pi = (R - 1 + \delta)/R$ , where  $R$  is the mean of  $R_t$ , and  $\phi = \delta$ , as assumed earlier in this section: The downpayment covers only the average user cost and there is no accelerated repayment requirement. We call this the case of *full collateral*, because the values of the outstanding debt and the depreciated durable goods stock are equal at the average interest rate.

Consider the effects of changes in  $W_t$ , holding constant  $R_t$  at  $R$ . Because the last terms in both (15) and (16) are now equal to zero, these equations and the first order condition for  $N_t$  are satisfied only by an immediate and full adjustment of  $C_t$  and  $S_t$  to the wage change, while  $N_t$  is unchanged. If the wage change is permanent, then these choices correspond exactly to those of a household facing no borrowing constraints. Here, however, this result holds regardless as to whether the change is permanent or transitory. An unconstrained household borrows to finance leisure when the wage falls temporarily, but this option is unavailable to the the present household because borrowing must be backed by purchases of durables goods. Therefore, full collateral eliminates completely the variation of hours worked following wage changes.

In contrast to the lack of response to wage changes, hours worked do respond to interest rates changes. The interest rate affects durable purchases through the user cost, and then, as discussed above, hours worked comove with durables so as to satisfy the budget constraint.

## 4.2 Partial Collateral

When  $\pi > (R - 1 + \delta)/R$ , the borrowing constraint forces the household to accumulate equity on its durable goods stock. Correspondingly, only a fraction of the durable stock can serve as collateral, and thus we label this case as one of *partial collateral*.

Here, when  $W_t$  changes permanently, the choice of immediate proportional adjustment in  $C_t$  and  $S_t$  leaving  $N_t$  unchanged violates the budget constraint. Hence, the adjustment of  $C_t$  and  $S_t$  to their new long-run levels is

gradual. The optimal labor supply condition (8) and the gradual adjustment of  $C_t/W_t$  imply that  $N_t$  exceeds its long-run level during the adjustment. In the present context, a temporary wage change is less interesting because the household's response is qualitatively similar to the case of unconstrained borrowing.<sup>11</sup>

The value of  $\pi$  has opposite implications for the responses of hours worked to wage and to interest rate changes. A higher  $\pi$  increases the sensitivity of  $N$  to the real wage, but lowers its sensitivity to the interest rate. The first implication was already stressed as the result of moving from full to partial collateral. The second implication follows from the fact that a higher downpayment reduces the amount of allowed borrowing, and hence it makes the household less sensitive to changes in  $R$ . The extreme case is of course  $\pi = 1$ , when the interest rate becomes irrelevant.

The main results from considering this simple version of the model can be summarized as follows. First, the lack of access to uncollateralized credit overturns the standard model's prediction that labor comoves negatively with all consumption goods when the interest rate or expectations about the future change. Second, with full collateral, the borrowing constrained household does not change its hours worked in response to wage variation, permanent or temporary, while under the realistic assumption of partial collateral, labor does respond positively to wages even when the changes are permanent. Third, with partial collateral the household's adjustment of the durable goods stock is gradual.<sup>12</sup>

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<sup>11</sup>Both with unconstrained borrowing and here,  $C_t$  and  $N_t$  comove positively from (8).

<sup>12</sup>An alternative causality story which also leads to positive comovement of hours, debt and durable goods is a Keynesian-type one, where  $N$  changes exogenously, given a shifting demand for labor under excess supply. Because of the collateral constraint, the household durable purchases—and thus downpayments and the debt—comove positively with  $N$  and thus with labor income. In contrast, if this household had perfect access to the capital market, the exogenous changes in  $N$  would lead to *opposite* changes in the net debt. The difference between this type of causality story and the one addressed in this paper has to do with nondurable consumption. In this alternative setup, nondurable consumption moves positively with labor, given that the labor supply condition ceases to be relevant, while in the present model this condition dictates opposite movements of nondurable consumption and hours worked.

## 5 Results from a Calibrated Model

For the general case, we follow the procedure of calibrating the model's parameters and then calculating the household's optimal responses. We first describe the parameter choices and then the household's optimal responses to wage and interest rate shocks. We also present the results of experiments designed to illustrate the role of the borrowing constraint in generating the results.

### 5.1 Calibration

The assignment of values to the model's parameters is straightforward. We set  $\mu = 1.0047$ , the average quarterly growth rate of the real hourly compensation in the business sector from 1954 through 2001. The depreciation rate  $\delta$  is equated to its empirical analogue constructed from the Bureau of Economic Analysis' *Fixed Tangible Reproducible Wealth*. The estimate of  $\delta$  is 0.0115, the average of 0.0018 for residential structures and 0.034 for other durable goods, with weights 0.7 and 0.3, respectively. We set  $R = 1.01$ .

The impatience parameter is harder to calibrate. We set  $\rho = 0.015$ , i.e., half of a percentage point higher than the interest rate. This degree of impatience is similar in magnitude to that used by Krusell and Smith (1998). Using a model with 3 levels of time preference, they calibrate the differences between each type as 0.36%; or 0.72% between the two extremes. We have experimented with various values for this parameter with almost identical results to those reported below.

For  $\pi$ , we use 0.15, which is between 0.20, a typical downpayment fraction for home loans, and 0.1, a typical value for car loans. The rate of repayment  $\phi$  is computed using the average term of home loans of 104 quarters and the average term of car loans of 12 quarters during the 1952-2002 sample. The corresponding linear repayment rates are 0.0096 and 0.083. During that sample, the average shares of the two types of loans are 0.78 and 0.28, respectively, and thus  $\phi$  is set equal to the weighted average repayment rate of 0.03.<sup>13</sup> The parameter  $\theta$  is set so as to match the ratio of quarterly durable consumption expenditures to nondurable consumption expenditures,  $((1 - (1 - \delta)/\mu) S/C)$ , which is 0.25 during the 1952-2002 sample. The resulting value is 0.26. We adopt  $\sigma = 2$  as our baseline case, and experimented

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<sup>13</sup>See footnote 4 for the sources of our observations of loan terms for automobile installment loans and residential mortgages.



also with  $\sigma = 1$ . Finally, the value of  $\lambda$  is computed using the balanced growth version of the condition for  $N$  in (8) and the other parameters which determine the  $C/W$  ratio, so that  $N = 0.3$ .

## 5.2 Baseline Results

The solution procedure is standard, we log-linearize the first order conditions (6), (7), (8) and (9) as well the constraints (2) and (5) around the balanced-growth path and solve the resulting log-linear system for particular stochastic processes of the wage and the interest rate. The assumption that the borrowing constraint always binds seems reasonable because only small deviations from the balanced growth path are considered. We assume that  $\log W_t$  follows a random walk with drift  $\mu$  and that  $\log R_t$  follows a first-order autoregression with autoregressive coefficient 0.95.

Figure 3 plots the household's responses—expressed as percentage deviations from the initial balanced growth path—to an unexpected 1% permanent increase in  $W_t$ . Immediately after the wage shock, the stock of durables increases 0.25%, nondurable consumption 0.91% and hours worked 0.22%. As described in Section 4, this partial adjustment of the two consumption goods and the increase in hours worked stand in contrast with the behavior of an unconstrained household—which would adjust the two consumption goods immediately by 1% and leave hours worked unchanged.

After three years, the accumulation of the stock of durables to its new level is close to be complete, while the debt behaves quite differently. The debt *overshoots* its long-run increase of 1%, and after two years it is more than 1.5% higher than its initial level. Then, the debt falls very slowly towards its long-run level. The source of the marked difference between the behavior of the debt and the behavior of the stock of durable goods is the accelerated repayment of the debt. Because  $\phi$  exceeds  $\delta$ , the collateral value of a durable good as a fraction of its current value declines with its age—newer goods can support more borrowing.<sup>14</sup> For the parameter values used,  $B/S = 0.4$ , so that a new purchase can collateralize more than twice as much debt as the average durable good in the household's stock. Hence, the surge in the durable goods stock allows the consumer to borrow more. Even after the durable goods stock has approached its long-run level, its average age remains below the long-run average age. The long decline of the debt

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<sup>14</sup>For goods of age  $t$ , this fraction equals  $(\frac{1-\phi}{1-\delta})^t$ .

after its initial surge reflects the gradual aging of the household’s durable goods stock. The slow convergence of the household’s debt accounts for the sluggish behavior of its nondurable consumption and hours worked. While debt remains high, the consumer reduces consumption and leisure to finance its repayment. Thus, accelerated repayment causes both higher volatility of the debt—because of the overshooting behavior—and persistent variation in nondurable consumption and hours worked.

We now turn to consider the household’s response to a transitory but persistent interest rate shock—as the autoregression coefficient is 0.95. Figure 4 plots the responses following a 0.25% decrease in the interest rate. The household’s stocks of the durable good and debt are extremely interest sensitive. In the period of the interest rate reduction, the stock of the durable good rises about 1% and the debt rises by more than 2%. The strong positive response of hours worked, 1.4%, reflects the point stressed in Section 4 about the positive comovement of hours worked and durables when the interest rate or expected future variables change. It is this link that leads to the nonstandard negative effect of the interest rate on labor supply. This argument also implies a negative response of nondurable consumption, whose initial change is  $-0.6\%$ . The movements in hours worked and in nondurable consumption persist for several quarters. The stock of durables and the debt peak after six quarters. The durable goods stock peaks at 3.7% above its value before the shock, while the stock of debt rises over 7% before beginning to fall.<sup>15</sup>

The negative comovement of the durable goods stock and nondurable consumption expenditure following an interest rate change raises the question of how does total consumption of this household comove with hours worked. We calculated the responses of a total consumption measure that adds the user cost of the durable goods stock to nondurable consumption.<sup>16</sup> Initially, this consumption measure falls slightly,  $-0.2\%$ , following the interest rate shock. Thereafter, it rises above its long-run level and achieves a peak of 0.7% after eight quarters, before slowly reverting to its steady state value. Thus, the household’s hours worked covary positively with this particular measure of its total consumption.

Whether the household faces variation in the wage, the interest rate, or both; the calibrated model’s simulations indicate that its debt and labor

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<sup>15</sup>When  $\sigma = 1$  is used, the responses are larger in magnitude and with a similar degree of persistence.

<sup>16</sup>We calculated the service flow from the durable goods stock using the user cost of durable goods,  $(R - 1 + \delta)/R$ , with a constant interest rate.

supply comove positively. This behavior is qualitatively consistent with the observations presented in Section 2.

### 5.3 Relaxing the Borrowing Constraint

We now turn to consider two versions of the model in which the borrowing constraint requires either a slower repayment or a lower downpayment. Figures 5 and 6 plot the household's responses to the wage and interest rate shocks when  $\phi$  is lowered to the level of  $\delta$ , so that there is no accelerated repayment, while holding all the other parameters constant.

The contrast between the responses to the 1% permanent wage increase in Figure 5 and those in Figure 3 illustrates the implications of accelerated repayment. Hours worked become less volatile and much less persistent. The response of the durable goods stock is very similar in the two cases, but here it converges faster. By construction, debt perfectly tracks the durable goods stock when  $\phi = \delta$ , so that also the debt converges very quickly to its long-run level. Given that debt does not overshoot, it becomes less volatile following wage changes.

Figure 6 plots the household's responses to the interest rate decline examined earlier. The most important difference between these responses and those from the baseline version is that they are now weaker and less persistent. This is particularly the case for hours worked.

We also lowered  $\pi$  from 0.15 to 0.1. These responses, not shown, are quantitatively very similar to those from the calibrated model. As discussed in Section 4, the effects of changing the downpayment rate on the responses of hours worked are mixed. Hours worked become less sensitive to the wage, but more sensitive to the interest rate. Also, the easier access to credit amplifies the responses of debt and durables.

Typical downpayment rates on automobile loans and home mortgages changed little over the sample used in Figures 1 and 2, but the frequency of refinancing fixed term mortgages, or borrowing against accumulated home equity, went up dramatically during this period.<sup>17</sup> This is reflected in the sharp increase in household debt since around 1983. In the model,  $\phi - \delta$  represents the rate at which a debtor builds equity in durable goods. Lowering this rate increases the levels of a representative debtor's debt and hours

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<sup>17</sup>Brady, Canner, and Maki (2000) report that 8% of homeowners in 1977 had refinanced the first mortgage on their current home. The analogous statistics in 1989, 1994, and 1999 were 20%, 45%, and 47%.

worked, and at the same time it decreases their volatility because debt co-moves more closely with the durable goods stock. Hence, the present model of a representative debtor suggests a consistent interpretation of the salient features of both levels and variances of household debt and hours worked shown in Figures 1 and 2. Around the same time, about 1983, when the trends of debt and hours increase, the volatilities of both variables decline.

## 5.4 Disaggregation: Housing and Automobiles

In reality, two distinct goods account for most of a typical household's durable goods: houses and automobiles. Houses depreciate much slower than automobiles, both physically and for collateral purposes, and downpayments are typically higher than for automobiles. Additionally, the two goods may have different utility parameters.

To examine the robustness of our results to disaggregating the durable goods stock, we extended the model to include two durable goods, each with a separate collateral constraint. The downpayment and repayment rates were calibrated for houses and automobiles separately using the values referred to in Section 5. The utility parameters were inferred from the expenditure shares of residential investment (owner occupied) and automobiles: 0.05 and 0.14, respectively.

The main conclusion from this extension is that the results—not reported in the paper—are very similar to those reported above. Here, we also obtain the behavior of the two stocks and two types of debt. For example, when the wage increases permanently, the immediate percentage increase in car debt is much higher than for home debt—given the lower downpayment rate—and converges faster to the long-run one-percent increase—given the higher repayment rate.

This extension of our model could be used to address relative price changes of the two durable goods. We do not pursue this avenue because the mechanism triggered by relative price changes in the present model is not essentially different than in the standard household framework. In both, a relative price decline will shift demand toward that good, and away from the other goods, including leisure.

## 6 Concluding Remarks

Limiting a household's borrowing by imposing collateral constraints fundamentally alters its intertemporal choices. In models of unconstrained intertemporal substitution with preferences consistent with balanced growth, a permanent wage gain increases immediately both consumption and net indebtedness while leaving hours worked unchanged. In the same models, a reduction in interest rates induces the household to work less and consume more in the present at the expense of the future. Collateral constraints render these choices infeasible. A household facing collateral constraints responds to both of these shocks by working more to accumulate downpayments for durable goods purchases and later to repay the principal. The result is a gradual accumulation of durable goods. Surprisingly, forcing the household to repay its debts at a rate faster than durable goods' depreciation rate *amplifies* fluctuations in the debt. This arises from new durable goods having higher value as collateral.

Because our household is a net debtor, it obviously cannot be considered a representative household for the analysis of aggregate data. Nevertheless, we find the positive comovement of household debt with hours worked and the stock of durable goods in the aggregate U.S. data to be striking enough to merit the investigation of a representative debtor's choices. The natural generalization of the present framework adds a more patient household to serve as a source of funds. Because the patient representative creditor holds all of the economy's tangible wealth minus the debtor's equity in his durable goods stock, its supply of labor is likely to be smaller than that of the borrower. On these grounds, if the wealth differential is large enough, total hours worked in such an economy will reflect primarily the debtor's choices.

One of the issues suggested by the present analysis for a general equilibrium analysis of this type is the implications of financial innovation for aggregate fluctuations. The current analysis suggests that financial innovation may reduce the volatility of business cycles, given that the positive comovement between household durable purchases, debt and hours worked is weakened. The analysis of a general equilibrium setup of this type is the subject of our next research project.

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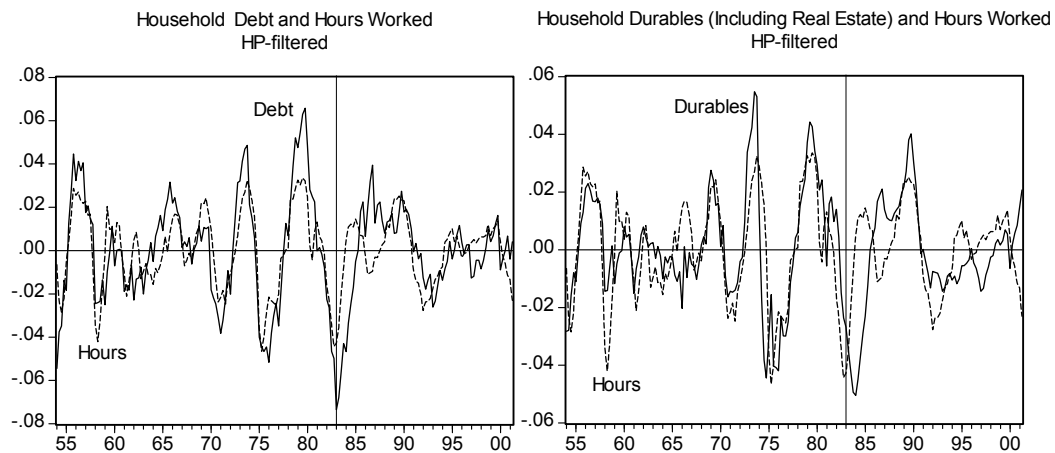


Figure 1: Cyclical Fluctuations of Household Debt, the Durable Goods Stock, and Hours Worked

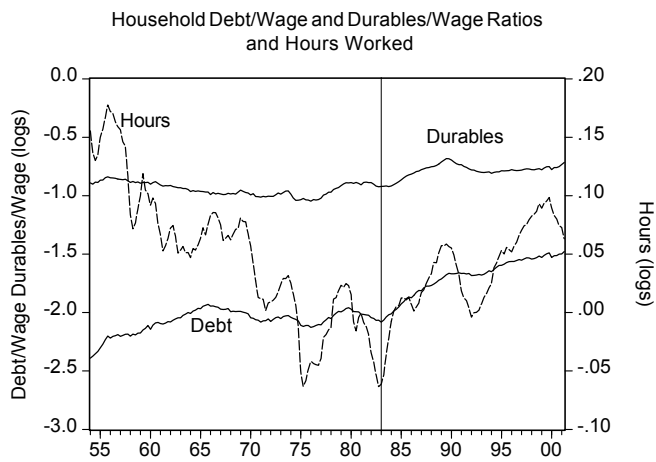


Figure 2: Levels of Household Debt, the Durable Goods Stock, and Hours Worked



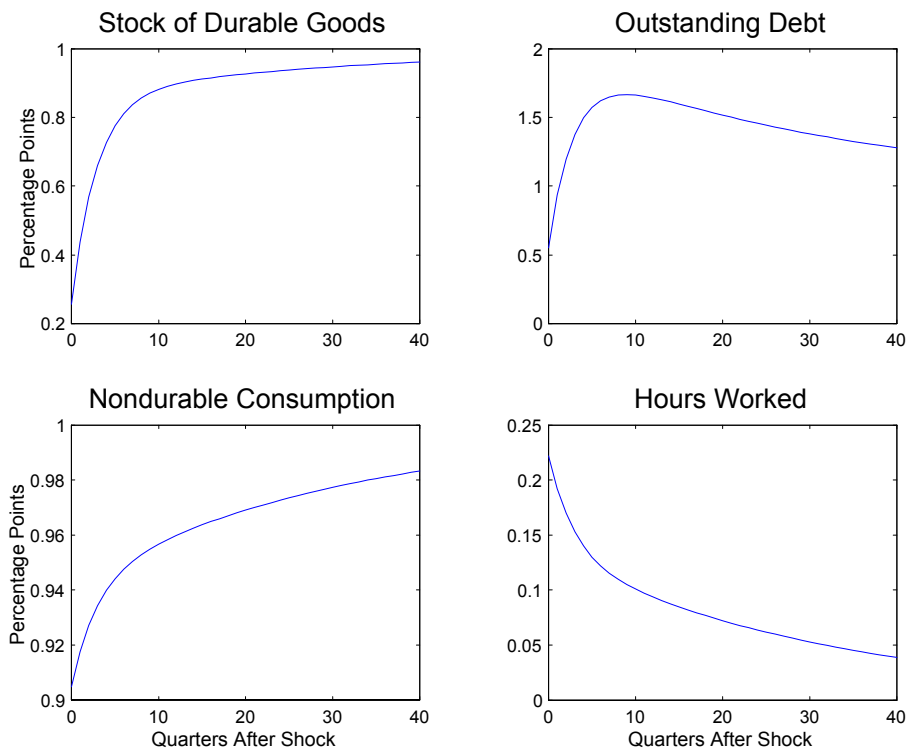


Figure 3: Baseline Responses to a 1% Permanent Wage Increase

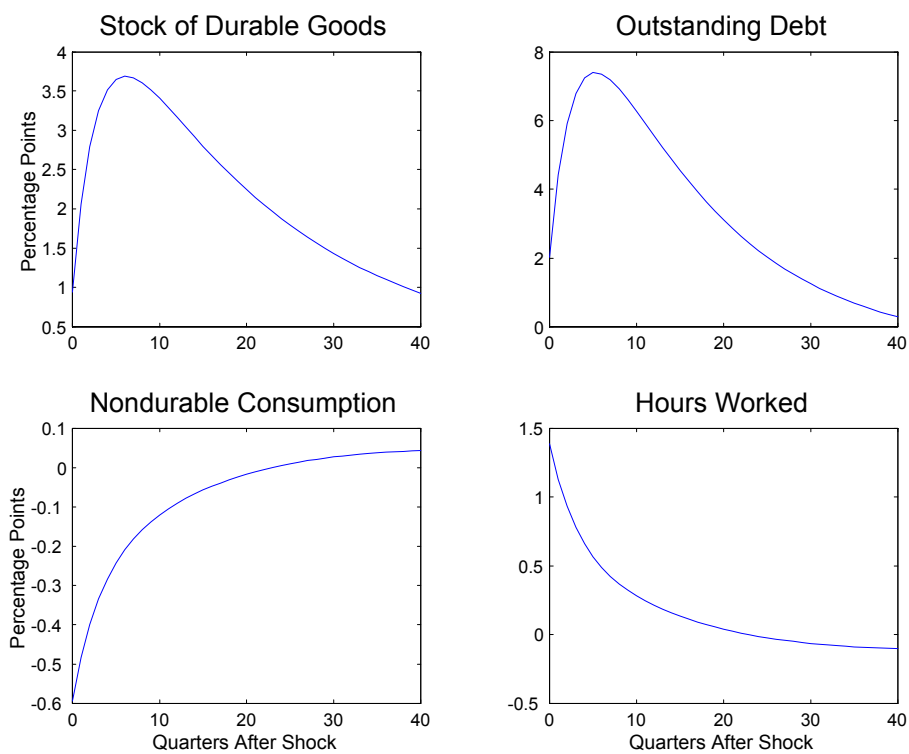


Figure 4: Baseline Responses to a Persistent 1/4% Interest Rate Decrease



Figure 5: Responses to a Permanent 1% Wage Increase when  $\phi = \delta$

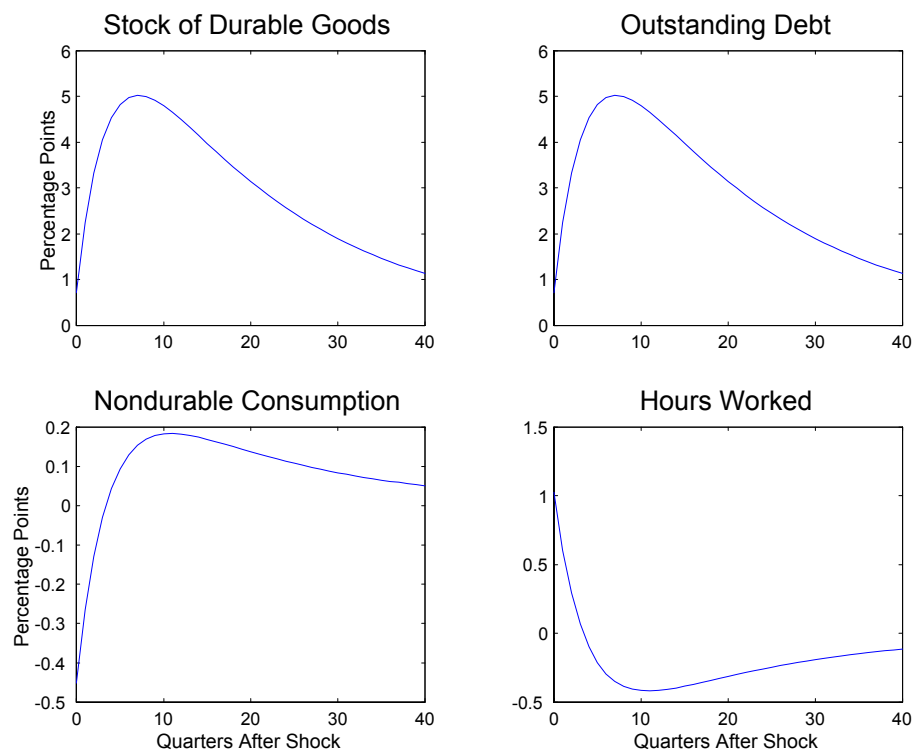


Figure 6: Responses to a Persistent 1/4% Interest Rate Decrease when  $\phi = \delta$