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OPTIMAL FINANCIAL AID POLICIES FOR A SELECTIVE UNIVERSITY

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ABSTRACT

Recent federal cut-backs of financial support for undergraduates have worsened the financial position of colleges and universities and required them to debate how they will allocate their scarce financial aid resources. Our paper contributes to the debate by providing a model of optimal financial aid policies for a selective university--one that has a sufficient number of qualified applicants that it can select which ones to accept and the type of financial aid package to offer each admitted applicant.

The university is assumed to derive utility from "quality-units" of different categories (race, sex, ethnic status, income class, alumni relatives, etc.) of enrolled students. Average quality in a category declines with the number of applicants admitted and the fraction of admitted applicants who enroll increases with the financial aid package offered the category. The university maximizes utility subject to the constraint that its total subsidy of students (net tuition revenue less costs including financial aid) is just offset by a predetermined income flow from nonstudent sources (e.g., endowment). The model implies that the financial aid package to be offered to each category of admitted applicants depends on the elasticity of the fraction who accept offers of admission with respect to the financial aid package offered them, the propensity of the category to enroll, the elasticity of the category's average quality with respect to the number admitted, and the relative weight the university assigns in the utility function to applicants in the category.

While the latter must be subjectively determined by university administrators, the former parameters are subject to empirical estimation. The paper concludes with a case study of one selective institution's data and illustrates how they may be estimated. Based upon data from the university's admissions and financial aid files, as well as questionnaire data which ascertained what alternative college most admitted freshman applicants were considering and the financial aid packages at the alternative, probit probability of enrollment equations are estimated as are equations that determine how average quality varies with the number admitted for each category. These estimates are then applied to illustrate what the "optimal" financial aid policy would be for the university.

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I. INTRODUCTION

Recent and proposed future cut-backs of federal financial support for undergraduate students from lower- and middle-income families have worsened the financial positions of colleges and universities and required them to rethink how scarce financial aid resources should be allocated. For example, the Ivy League universities and a number of other highly selective universities have been following a policy of meeting the "full financial need" of all accepted applicants who enroll. That is, after evaluating how much an applicant and his or her family should be able to contribute to the applicant's education, the difference between the total cost of attending the university (tuition and fees, room and board, books, and other expenses) and this sum was provided to the applicant in the form of a package consisting of scholarships, loans, and in-school employment opportunities. This policy was designed to assure equal access for all qualified students to these universities; family income ceased to be a key determinant of whether a student could enroll at one of them. The worsening financial situation that these universities face has, in many cases, called the continuation of these policies into question.

What options do these universities, and other universities that have never had the resources to meet the need of all accepted applicants, face? What financial aid policies will maximize a university's welfare? Out paper contributes to the debate by providing in Section II a model of optimal financial aid policies for a selective university -- one that has a sufficient number of qualified applicants that it can select both which ones to accept and the type of financial aid package to offer each admitted applicant.

The university is assumed to derive utility from "quality-units" of different categories (race, sex, ethnic status, income class, alumni relatives, etc.) of enrolled students. The average quality of a category declines with the number admitted and the fraction of admitted students who enroll increases with the financial aid package offered the category. The university maximizes utility subject to the constraint that its total subsidy of students (its tuition revenue less costs including financial aid) is just offset by a predetermined income flow from nonstudent sources (e.g., endowment). The model implies that the financial aid package to be offered to each category of admitted applicants should depend on (1) the elasticity of the fraction who accept offers of admission with respect to the financial aid package offered them and, in some cases, the propensity of applicants to enroll (2) the elasticity of the category's average quality with respect to the number admitted, and (3) the relative weight the university assigns in its utility function to applicants in the category.

While the latter must be subjectively determined by university administrators, the former parameters are subject to empirical estimation. Section III presents a case study using data from one selective university, Cornell University, that illustrates how the parameters may be estimated. Based upon data from the university's admissions and financial aid files, as well as questionnaire data that ascertained the alternative college that most admitted freshman applicants were considering or enrolled at, and the financial aid packages at the alternative, probit probability of enrollment equations are estimated as are equations that determine how average quality varies with the number of applicants admitted from each category. The implications of these estimates for the optimal structure of the university's financial aid policy are then discussed in Section IV, as are some qualifications and implications for future research.

II. A MODEL OF OPTIMAL FINANCIAL AID AND ADMISSIONS POLICIES FOR A SELECTIVE UNIVERSITY

By a selective university, we mean one in which the number of applicants far exceeds the number of available positions, and thus the university exercises considerable discretion in the admissions process. For example, Cornell University admits only about one-quarter of its freshman applicants, and undoubtedly a large number of potential applicants fail to apply because they believe that their chances of admission are so low. Selective universities can increase their applicant flows by publicly announcing slightly lower admission standards.[2]

Suppose that a selective university faces N different categories of applicants. The categories may depend upon family income, minority status, academic quality, region of the country, parent alumni status, student athletic ability, or any other attributes that the university is concerned about. To take an example, if the university was concerned only about whether an applicant had an alumni parent and whether the applicant was a minority, there would be four categories (yes/yes, yes/no, no/yes, no/no). Let X_i denote the number of applicants from category i that are admitted. Since the university's applicant pool is assumed to far exceed the number of available freshman enrollment opportunities, we treat the X_i as choice variables.[3]

To simplify, we assume that over the range of potential total enrollment levels, the university faces a fixed marginal cost per student of C. Although it may establish a tuition level, which does not vary across students, it awards different financial aid packages (treated here entirely as scholarships) to different categories of students. Let P_i denote the total net cost of attending the university for applicants from category i. Then the share of the costs that the university bears for students from this category, S_i, is given by

$$(1) \qquad S_i = (C-P_i)/C = 1 - (P_i/C) \qquad\qquad i = 1,2,\ldots,N$$

We assume that students who have applied to the university are aware of its cost level C, which we treat as predetermined. The fraction of accepted applicants in category i that actually enroll, F_i, is assumed to depend on the share of the costs that the university bears.[4] Other things equal, the proportion that will enroll (which is sometimes called the yield) will increase as the university's share of the cost increases. That is,

$$(2) \qquad F_i = F_i(S_i) \qquad F_i' > 0 \qquad 0 \leq F_i(S_i) \leq 1 \qquad i = 1,2,\ldots N .$$

Note that (2) asserts that the lower the net price charged applicants, the more likely that they will enroll. Also, there is nothing in this formulation that prevents F_i from also depending upon the level of the university's costs.

The product of the number of admitted applicants and the fraction who accept admission is the number of students who actually enroll in each category. The university is not indifferent, however, to the composition of the enrolled students. Specifically, suppose that we can rank admitted applicants by some <u>objective</u> measure of their academic quality, such as

SAT scores or class rank, and that within each category the average quality of admitted applicants (q_i) declines with the number admitted.[5]

$$(3) \qquad q_i = q_i(X_i) \qquad q_i'(X_i) < 0 \qquad i = 1,2,\ldots N$$

We assume that the university values the total "quality units" of students who actually enroll in each category and that the weights it places on attracting different categories of students can be summarized by the quasi-concave utility function

$$(4) \qquad U = U\,[F_1(S_1)X_1q_1(X_1),\ F_2(S_2)X_2q_2(X_2),\ \ldots,\ F_N(S_N)X_Nq_N(X_N)]$$

where $F_iX_iq_i$ represents the total quality units of enrolled students from category i.[6]

Note that this formulation distinguishes between the academic quality of applicants and their relative attractiveness to the university. Quality is an objective measure, and depends upon measurable academic attributes as noted above. Relative attractiveness, however, depends upon the subjective valuations of university decision-makers of applicant characteristics such as minority status, athletic ability, family income class, and whether the applicant has relatives who are alumni, that are summarized by the form of the utility function in (4).

We can define category i applicants as being relatively more attractive to the university than category j applicants if for any equal number of quality units of enrolled students of each type, the university obtains greater marginal utility from enrolling an additional quality unit of type i applicants. That is, if for equal numbers of enrolled quality units

(5) $U_i/U_j > 1,$

where the subscripts after the U indicate marginal utilities.

The university seeks to maximize its utility from enrolling different categories of students subject to a constraint that the net revenue it receives from students plus its revenue from nonstudent sources that can be applied to subsidizing students' education can not be less than the total cost of the students.[7] Let R denote the total applicable revenue from nonstudent sources; one can think of this as the revenue that is available to the university's operating budget from endowment and current year gifts. Recalling, that S is the share of per student costs borne by the university (average cost per student less the net price actually charged students divided by average costs), the constraint can be written

(6) $$\sum_{i=1}^{N} S_i C F_i(S_i) X_i - R \leq 0$$

Alternatively, if we let r be the total number of students the university can enroll if it bears the entire cost of education $(r = R/C)$, this can be expressed as

(7) $$\sum_{i=1}^{N} S_i F_i(S_i) X_i - r \leq 0.$$[8]

Assuming that an interior solution exists, with the university enrolling all categories of students and just exhausting the revenue from nonstudent sources, maximizing the utility function (4), subject to the constraint (7) requires the university to pursue admissions (X_i) and financial aid (S_i) policies that satisfy the following constraints

(8) $U_i\ [F_iq_i + F_iX_iq_i'] + \lambda S_iF_i = 0 \qquad i = 1,2,\ldots N$

(9) $U_i\ [F_i'X_iq_i] + \lambda[S_iF_i'X_i + F_iX_i] = 0 \qquad i = 1,2,\ldots N$

(10) $\sum_{i=1}^{N} S_iF_iX_i - r = 0.$

λ is a Lagrangian multiplier that has the interpretation of minus the marginal utility of an additional quality unit of enrolled student divided by the marginal cost. Conditions (8) and (9) require that the university admit applicants from different categories and vary the share of the per student cost that it bears across categories up until the point that the ratio of the marginal utility to the marginal cost of obtaining an additional quality unit is equal for each action.

Some algebraic manipulations of (8) and (9) indicates that they can be rewritten as

(11) $(S_i/S_j) = [U_iq_i(1+n_i)]/[U_jq_j(1+n_j)] \quad \forall\ i,j$

and

(12) $(1+n_i)(1+(1/\varepsilon_i)) = (1+n_j)(1+(1/\varepsilon_j)) \quad \forall\ i,j$

where n_i (< 0) is the elasticity of the average quality of category i with respect to the number of category i applicants admitted $(q_i'X_i/q_i)$ and ε_i (> 0) is the elasticity of the fraction of category i applicants who will enroll (the yield of category i applicants) with respect to the share of the costs borne by the university $(F_i'S_i/F_i)$. One can show that given the utility function in (4) sufficient (but <u>not</u> necessary) conditions for a maximum to exist are either

(13) ε_i constant, $n'(X_i) < 0$ for each i

or

(14) n_i constant, $\varepsilon'(S_i) < 0$ for each i

The former requires that the percentage decline in average quality associated with a given percentage increase in admissions gets larger (in absolute value) as the number of admitted applicants increases. The latter requires that the percentage increase in the yield associated with a given percentage increase in the share of costs borne by the university becomes smaller as the share of cost borne by the university becomes larger.[9]

While in principle one could totally differentiate the system of 2N + 1 equations in (8)-(10) to obtain comparative static results for the model, it is simpler to work with a specific utility function. One tractable form is the Cobb-Douglas function

$$(15) \quad U = \prod_{i=1}^{N} (F_i X_i q_i)^{\alpha_i} \qquad \alpha_i > 0$$

Other things equal, the greater the value of α_i the greater the relative attractiveness of category i applicants. Moreover, with this formulation, the university must enroll some applicants from all N categories to obtain any utility.[10] Given this utility function, equation (11) becomes

$$(16) \quad \frac{S_i F_i(S_i)}{S_j F_j(S_j)} = \frac{\alpha_i}{\alpha_j} \frac{(1+n_i)\, X_j}{(1+n_j)\, X_i} \qquad \forall\ i,j\ .$$

One can make use of equations (12) and (16) to derive a number of propositions about how financial aid resources should be allocated across different categories of applicants. These propositions, which are formally proven in the appendix, can be summarized as follows.

Suppose we first consider the case when the elasticity of the fraction of admitted applicants who enroll with respect to the university's share of costs is constant within each category (but may differ across categories). In this case, for each category $F_i = K_i S_i^{\varepsilon_i}$ and larger values of K_i indicate greater propensities to enroll, other things equal.

Proposition 1: Suppose the enrollment elasticity ε_i is equal across two categories, both face the same elasticity of average quality function, and the elasticity of average quality varies with the number admitted. Then the relative price the university should charge to each category of applicants depends only on their relative weights in the university's utility function and on their relative propensity to enroll if admitted. Other things equal, the group which is relatively more attractive to the university should be charged a lower price. Similarly, other things equal, the group with a greater propensity to enroll should be charged a higher price.

Proposition 2: Suppose the elasticity of the fraction of admitted applicants who enroll differs across groups but that the two groups have the same elasticity of quality function, the same values of the propensity to enroll parameter (K), and are equally relatively attractive to the university. In this case the group whose enrollment elasticity is greater should be charged a lower price.

Proposition 3: Suppose at any given number of admitted applicants, the elasticity of quality with respect to applicants differs across groups, but that the two groups have the same (constant) elasticity of yield with respect to the university's share of cost, the same values of the propensity to enroll parameter, and are equally relatively attractive to the university. In this case, the group whose elasticity of quality with respect to admissions is larger (in absolute value) should be charged the lower price.

Intuitively, these propositions all make sense. The first asserts the university should charge lower prices to categories of applicants who it derives more utility from and/or who are less likely to enroll at any given price. The second asserts that applicants whose enrollment decisions are more sensitive to price should be charged a lower price. Finally, the third asserts that applicants whose average quality falls off more rapidly with the number admitted should be charged a lower price to increase the yield and thus reduce the number who need be admitted to reach any desired enrollment figure.

In an anlogous manner, one can consider the case when the elasticity of average quality with respect to the number admitted is a constant for each category. Here, for each category $q_i = q_{0i}X_i^{n_i}$ and larger values of q_{0i} indicate higher average quality, other things equal.

Proposition 4: Suppose n_i is equal across two categories and both face the same probability of enrollment function. In this case, the relative number of applicants the university should admit depends only on the relative weights it places on the different categories, with relatively more attractive students being admitted with greater frequency. Moreover, both categories should be charged the same price.

Proposition 5: Suppose the elasticity of average quality with respect to the number admitted differs across two groups but that both face the same elasticity of yield function. In this case, the university should charge a lower price to the category whose elasticity of quality with respect to the number admitted is more elastic (larger in absolute value).

Proposition 6: Suppose n_i is equal across two categories and that, at any given share of cost that the university bears one category's elasticity of yield is larger than the other's. In this case, the university should charge a lower price to the group whose elasticity of yield with respect to the subsidy is higher.

Given the explanation of the earlier propositions these latter three are almost self-evident. Moreover, as Table A1 in the appendix summarizes, most of these six propositions continue to hold when several alternative forms of utility functions are used.

Our model implies then, that the financial aid package to be offered to each category of admitted applicants should depend on (1) the elasticity of the fraction who accept offers of admission with respect to the financial aid package offered them and the propensity of the applicants to enroll, (2) the elasticity of the category's average quality with respect to the number admitted, and (3) the relative weight the university assigns in the utility function to applicants in the category. While the latter must be subjectively determined by university administrators, the other parameters are subject to empirical estimation, and the next section illustrates how this may be done.

III. ESTIMATING THE RELEVANT PARAMETERS: A CASE STUDY

A number of prior studies have presented estimates of the determinants of the college enrollment decisions of high school seniors, using data on national or regional samples of individuals.[12] While such studies are of use in simulating the effects of various federal financial aid policies on the level and composition (say by race or income class) of aggregate college enrollments, they are of little use to an individual college or university that is trying to optimally set its own financial aid policies. What is required in the latter case are institutionally based studies of the determinants of the enrollment decision and in the first part of this section, we indicate how such a study can be conducted, using data from one selective university, Cornell University.[13] Similarly, to optimally frame financial aid policies, university decision-makers need institutionally based information on the elasticity of average quality with respect to the number admitted for the various categories of applicants they face. The second part of the section shows how these parameters can be estimated, again using data for Cornell University freshmen applicants.

A. The Probability of Enrollment Decision

Consider applicant k who has been admitted at one selective university, s, and also at an alternative university, o. The decision as to which university to enroll at presumably is based upon a comparison of the net utility, V, that the applicant would receive from each option, with the net utility in each option depending upon observable characteristics of the individual (X_k), the net costs to the applicant of the two options (N_s, N_o), other characteristics of the two options (Z_s, Z_o), and random variables

(e_{sk}, e_{ok}), that represent unobservable differences in tastes that the individual has for the two options. Hence

$$(17)\quad V_{sk} = V(X_k, N_s, Z_s) + e_{sk}$$

$$V_{ok} = V(X_k, N_o, Z_o) + e_{ok}$$

and

$$(18)\quad H_k^* = V_{sk} - V_{ok} = V(X_k, N_s, Z_s) - V(X_k, N_o, Z_o) + (e_{sk} - e_{ok})$$

$$H_k = 1 \quad \text{if} \quad H_k^* > 0$$

$$= 0 \quad \text{otherwise.}$$

Here H_k^* is an unobservable continuous variable that represents the net utility individual k would obtain from choosing to enroll at university s rather than at the alternative. Although it is unobservable, we can arbitrarily scale its cutoff value so that when it is greater than zero we observe individual k enrolling at university s $(H_k = 1)$. Conversely, if it is less than zero, the applicant will turn down the offer of admission from s and enroll elsewhere $(H_k = 0)$. If we further assume that the e_{sk} and e_{ok} can be treated as normally distributed random variables that are uncorrelated with the explanatory variables in the model, equation (18) represents a probit model of the determinants of the probability of enrollment at the selective university.[14] Put another way, estimates of it can be used to generate the yield curve faced by the university (F(S) in (2)).

Variants of equation (18) are used below to obtain estimates of the determinants of the probability of enrollment for freshmen applicants who were admitted to Cornell University for the year that began in September of 1981 (the class of 1985). Cornell is a selective Ivy League university that has approximately 6,000 graduate and 11,000 undergraduate students. The latter attend its seven undergraduate divisions -- the Colleges and Schools of Arts and Sciences, Architecture, Art and Planning, Engineering, Hotel Administration, Agriculture and Life Sciences, Human Ecology, and Industrial and Labor Relations.

A unique institutional feature of Cornell is that it is partially privately supported and partially State supported. The first four divisions mentioned above are in the private part of the university and are referred to as the endowed sector. The latter three are operated by Cornell under contract with the State of New York and have some relationship with the State University of New York; these are referred to as the statutory sector. Because tuition charges differ substantially between the sectors, as does the subject matters taught, and because the statutory sector draws students primarily from New York State, all of the estimates below are done separately for the endowed and statutory sectors.[15]

The data required to conduct the study came from a variety of sources. The university's admissions file had data for all freshmen applicants on the division the student applied to, objective measures of student ability (SAT scores, rank in class), characteristics of the student (e.g., race, sex, alumni parents, region of the country), the admission decision, and, if admitted, the applicant's enrollment decision. The university's financial aid management information file had information for all admitted financial aid applicants on family income, and the financial aid package offered the applicants from Cornell and other sources (grants or scholarships, loans, academic year earnings opportunities).[16] A university admissions research questionnaire, mailed to all admitted freshmen applicants and returned by approximately 60 percent of them, yielded data on the name of the other college the applicant was considering (or enrolled at), the distance of both that college and Cornell from the applicant's home, and the financial aid package offered to the applicant at the other college.[17] Finally, once the name of the other college was known, published sources provided data on the total costs of attending the other college and the average SAT scores of the entering freshman class there.

Tables 1 presents probit enrollment equations for both the endowed and statutory college financial aid applicants. The explanatory variables correspond to applicant characteristics, the applicant's net costs of attendance at Cornell and the other option, and other characteristics of Cornell and the option. Definitions of all of the variables appear in the table, while some descriptive statistics for various subgroups of applicants appear in Appendix Table A2.

These estimates conform quite well to what one would expect from a model of individual enrollment decisions. The enrollment decision does depend upon the characteristics of the option; the better the other college, as indicated by its average freshman SAT score (endowed) or by its being an Ivy League college (both), the less likely that financial aid applicant will enroll at Cornell. Minority students, who other selective universities compete for, are less likely and applicants with alumni relatives (in the endowed sector) are more likely to enroll at Cornell. Cornell is also less likely to attract its highest "quality" admitted applicants, as measured by SAT scores or class standing; these students tend to find their way into even more prestigious universities.

Financial aid and cost variables at both Cornell and the other option are also seen to play an important role. Other things equal, a $500 increase in the scholarship offer at Cornell will raise the yield on accepted applicants by roughly 6½ to 8 percent in the endowed sector and 7 to 10 percent in the statutory sector.[18] The loan the student can get if he or she attends Cornell also marginally (in a statistical sense) appears to matter in the endowed sector, although not unexpectedly its marginal effect is less than that of the Cornell grant.[19] Similarly, other things equal, increasing the size of the scholarship elsewhere decreases the probability of attending Cornell while increasing the total cost of the other college increases the probability of attending Cornell. Finally, while the provision of employment opportunities at Cornell does not seem to influence the enrollment decision, the provision of a loan or employment opportunities elsewhere does reduce the probability of attending Cornell.

How sensitive are these results to the specification chosen? Additional estimates, not reported here for brevity, suggest that allowing family income to nonlinearly influence the enrollment probabilities or including the region of the country that the applicant is from in the model does not alter the other results. The results are also quite similar between the two specifications ((1) and (2)) reported in Table 1; in the latter the coefficients of the Cornell total cost and scholarship offer are constrained to be equal and opposite in sign, as are the analogous coefficients for the other college.[20]

Since a key parameter is the sensitivity of different applicant groups to the Cornell net cost of attendance, attempts were made to ascertain if the probit coefficient of the net cost variable varied with four sets of characteristics that the university might use to define applicant groups.

These are the SAT range that the applicant falls in, the family income class he or she is from, whether the applicant has an alumni parent, and whether he or she is a minority applicant in Cornell's minority admissions program (COSEP).[21] The results are reported in Appendix Table A3; the interactions with only one set of characteristics at a time (columns (1) to (4)) would be relevant if the university's financial aid policy was to vary along only one dimension, while the interactions with all sets simultaneously (column (5)) would be relevant if the university's financial aid policy was to consider all of the factors.

Most strikingly, the probit coefficient of the Cornell net cost variable does not appear to vary with the applicant's SAT score, nor does it vary significantly with the applicant's parent alumni status. In the endowed sector, however, the response of yield to net cost does appear to be lower for minority applicants and there is some evidence that it is higher for applicants in the \$18,000-\$36,000 family income range than it is for applicants from lower-income families.

Table 2 summarizes the implications of these results for the elasticities of the probabilities of enrollment with respect to Cornell's net cost; the variable that is analogous to ε in our model. Elasticities are calculated for the probit net cost model without interactions (Col. 1), the model with one type of interaction at a time (Col. 2) and the model with all four sets of interactions occurring simultaneously (Col. 3). Because most of the interactions in the latter specifications proved to be statistically insignificant, most attention should be directed to the results in the first column.

In both the statutory and endowed sectors, it appears that the elasticity is lowest for the lowest income class of financial aid recipients.[22] There also is evidence, especially in the endowed sector, that the elasticity is higher for applicants with alumni parents than for other applicants, and that it is highest for the highest quality applicants (as measured by SAT scores). Finally, again especially in the endowed sector, there is evidence that the elasticity is lower for minority applicants than it is for non-minority applicants.

This latter result may not square with some readers' a prior expectations given the intense competition at selective unversities for minority students and probably warrants some explanation. Applicants to Cornell's minority admission program (COSEP) who enroll at Cornell receive academic support through a program that seeks to provide them with study skills, remedial teaching, tutorial services for large freshman required courses and a pre-freshmen summer program. The COSEP program is an extremely well-regarded one. In addition, given their "computed need", applicants often receive a greater share of their financial aid in the form of grants than do non-COSEP students. Because of these two factors, it is quite plausible to find COSEP applicants' enrollment decisions less responsive at the margin to scholarship offers than the decisions of nonminorities.

B. The Average Quality of Applicants and the Number Admitted

The estimation of the elasticity of average quality of applicants with respect to the number admitted is more straightforward. An objective measure of average quality can be constructed from admission committee rankings, scores on standardized tests, or data on rank in class. Applicants can then be ordered by the measure of quality and the average quality for the top X applicants (q_x) computed. One can then estimate for each category of applicants equations of the form,

$$(19) \quad q_x = q(X) \qquad X = 1,2...,A,$$

where A represents the total number of applicants in a category, to obtain estimates of the elasticities of average quality.

For our Cornell case study we have used the sum of an applicant's verbal and quantitative SAT scores as the measure of quality. For simplicity, we have considered only two functional forms for q, the constant elasticity form $q(X) = q_0X^n$ which yields the estimating equation

$$(20) \qquad \log q(X) = \log q_0 + n\log X,$$

and the form $q(X) = e^{n_o}e^{n_1X}$ which yields the estimating equation

$$(21) \qquad \log q(X) = n_o + n_1X .$$

In the former case the elasticity is given by n and in the latter case by n_1X.

Estimates of (20) and (21) are found in Table 3 for all applicants and for the subsample of admitted financial aid applicants. SAT scores are expressed in hundreds of points and separate estimates are presented for the three ability groups of applicants, for five family income classes of applicants, for COSEP and non-COSEP applicants, and for alumni and non-alumni applicants. The pattern of elasticities one obtains from the two specifications ((20) and (21)) is virtually identical (one should recall in the latter case we are comparing elasticities at a given number of applicants).

Among the different ability classes, the elasticity of average quality with respect to the number of applicants admitted is lowest in absolute value for applicants in the middle (for Cornell) ability range (1150 < SATSUM $\leq$ 1300) and, in most cases, highest for applicants in the high ability range. Among the different income classes the elasticity tends to decline in absolute value as family income increases, although for statutory applicants it levels out rapidly after an initial decline.[23] Finally, the elasticity of average quality with respect to the number of applicants admitted is much larger for minority (COSEP) applicants than it is for non-minority applicants and, in some specifications, it is larger for applicants from alumni families than it is for other applicants.[24]

IV. CONCLUSIONS AND IMPLICATIONS FOR FUTURE RESEARCH

What are the implications of the results presented so far for the optimal financial aid policy of one selective institution, Cornell University. Subject to the proviso that all of our results should be considered preliminary and that one should not seriously recommend any policies until it is shown that the pattern of relevant parameters remains roughly constant across categories over several years, the following tentative conclusions are in order.

The model of optimal financial aid policies that we presented in Section II suggested that, other things equal, financial aid packages should be more generous for groups that the university considers relatively more attractive (i.e., that have greater weight in the utility function), for groups that have lower propensities to enroll, for groups that have higher elasticities of the probability of enrollment (i.e., the yield) with respect to the university's share of costs, and for groups with higher (in absolute value) elasticities of average quality with respect to the number of applicants admitted. Table 4 summarizes the implied effect of each of these parameters for four classes of applicants, those with alumni relatives, minorities, those from low-income families (defined here to be less than $18,000) and those who are from the highest ability class (SAT scores that exceed 1300). We arbitrarily assume, for expository purposes that the university finds applicants from each of these groups relatively more attractive than the average Cornell freshman applicant.[25]

Our results unambiguously suggest that the highest ability students should receive above-average financial aid packages, since they tend to have a lower propensity to enroll, a higher than average elasticity of yield (in the endowed sector) and higher than average elasticities of average quality with respect to the number admitted. A financial aid policy based on scholarly merit, within each need class, appears to make sense for the university.

They also support the notion that minorities should receive larger aid packages, other things equal, because, in addition to being relatively attractive to the university, minorities have a lower propensity to enroll and a higher elasticity of average quality. The optimality of such a policy of more generous financial aid for minorities is not unambiguous, however, because minorities have a relatively low elasticity of yield, especially in the endowed sector.

Results for applicants who are relatives of alumni and for applicants from low-income families are more ambiguous. While alumni applicants are assumed to be relatively attractive to the university and there is some evidence that they have higher elasticities of average quality and of yield, their high propensity to enroll in the endowed sector may offset these effects. Similarly, while applicants from low-income families have high elasticities of average quality, which call for larger aid packages, they also have low estimated elasticities of yield, which call for less-generous packages.[26] On balance, then, the model probably yields no unambiguous implications for these groups.[27]

Numerous extensions and implications for future research are suggested by our paper. In addition to the ones we have discussed so far, one might note that our optimal financial aid model takes the financial aid packages at other universities as given. At the theoretical level one might attempt to model optimal financial aid policies for interdependent selective universities, using either a cartel framework or a framework that incorporated competitors' reaction functions and seeks to ascertain the "market equilibrium". Such a framework would make the yield functions in equation (2) explicitly dependent upon the costs and financial aid packages at other universities; a generalization already incorporated into our empirical work. At the empirical level one could use the empirical results to simulate the effects of simultaneous changes in both the institution's and its competitors' financial aid policies.

Finally, we must stress that one's ability to apply the theoretical model to particular institutional settings hinges on the stability of the relative ranking of key parameters across categories of applicants over time at an institution. For example, are the quality elasticity for minorities and the sensitivity of yield to net price for high quality applicants both always relatively high vis-a-vis the comparable parameters for other applicants. While we have pointed out the tentative implications of our current results for the financial aid policies at one selective institution, obviously repeated replication of the empirical estimates at individual institutions are required before the institutions should seriously consider implementing policies based on such studies. This is especially true in a period like the early 1980's when the external environment facing universities is changing so rapidly.

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Footnotes

1. The model presented in this section in some respects is a generalization and extension of the model presented in Stephen Hoenack (1971). Hoenack did not distinguish, however, between the applicant acceptance and enrollment decisions, nor did he allow applicant quality to decline with the number admitted. As will become apparent, these are key features of our work.

2. There is, of course, the danger that announcing lower standards might cause some of the "better" potential applicants to self-select themselves out of the applicant pool.

3. Actually, we are assuming that the applicant pool is sufficiently large in each category, to allow it to choose the number to admit. In some cases, for example for minorities or low-income applicants, this may not always be true unless the university first expends resources to attract applicants. One obvious extension of our model is to build in such recruitment costs directly, possibly by allowing the marginal cost per student which is defined below to vary across categories. The marginal cost might also vary across categories, if the propensity to choose different majors varies across categories, since certain types of education (e.g., chemistry) may be more expensive than others (e.g., economics).

4. Stated university tuitions rarely cover student costs, so $C - P_i$ will typically exceed the reported scholarship that an applicant receives. Indeed, for some public universities, P_i may equal the actual tuition and there may be no reported scholarship but $C - P_i$ will be large because of public subsidies. What is relevant to applicants is the share or absolute level of the total costs they will have to pay, not the stated tuition and scholarship per se.

5. Allowing the average quality function to also depend on S_i is another generalization that we do not pursue here.

6. If increases in X_i led to proportionately greater reductions in q_i, it would always be in the university's best interests to reduce X_i to zero. To avoid this, we make the innocuous assumption that total quality units of type i applicants never decrease with the number admitted; this requires that the elasticity of average quality with respect to the number admitted, n_i $(=q_i'X_i/q_1)$ always be greater than minus one $(-1 < n_i \leq 0)$.

7. Alternatively, one could model the university as maximizing the indicated utility function subject to constraints that (a) its total enrollment not exceed a specified capacity and (b) that the difference between its total student costs and the net revenue the students actually provide not exceed the available revenue from nonstudent sources. With a constant marginal cost per student, unless the enrollment constraint is binding, the two models would yield the same form of solution.

8. There is no general agreement on how one measures the adequacy of endowments, either over time for a given university, or across universities at a point in time (references will be added here). Equation (7) suggests that, at least for the former, a reasonable measure is r, the total number of students the university can enroll if it bears the entire cost of education, perhaps computed as a fraction of its actual enrollment.

9. For later reference, note that a constant elasticity of average quality function has the form $q(X) = q_o X^n$ where $q_o > 0$ and $-1 < n < 0$. Higher values of q_o imply higher average quality at each applicant level. A function that satisfies $n' < 0$ is $q(X) = q_o e^{q_1 X}$ where $q_o > 0, q_1 < 0$. For this function $n = q_1 X$.

Similarly, a constant elasticity of yield function has the form $F = kS^{\varepsilon}$ where $0 < k \leq 1$ and $\varepsilon > 0$. Given this function, no applicants would enroll if the university bore none of the costs $(S = 0)$ and the fraction k would enroll if the university bore the full cost $(S = 1)$. Finally, a function that satisfies $\varepsilon' < 0$ is $F = e^{K_o-(K_1/S)}$ with $0 < K_o \leq K_1$. For this function, $\varepsilon(S)$ equals K_1/S and the yield is again zero if the university bears none of the costs and is $e^{K_o-K_1}$ if the university bears all of the cost. Moreover, provided that K_1 is less than 2, the yield curve is "S" shaped, increasing first at an increasing and then at a decreasing rate as the share of the costs borne by the university increases.

10. One could, of course, consider alternative forms for the utility function such as the additive utility function $U = \sum_{i=1}^{N} \Theta_i(F_iX_iq_i)$, where Θ_i represents the constant relative attractiveness to the university of category i applicants, or the constant elasticity of substitution utility function $U = \left[\sum_{i=1}^{n} \delta_i(F_iX_iq_i)^{-B}\right]^{-(1/B)}$, where $-1 < B$ and $\sigma = 1/(1+B)$ is the elasticity of substitution between different applicant categories. Table A1 in the appendix summarizes the differences in the results that occur if these alternative utility functions are used.

11. Although we do not pursue it here, the model can also be used to generate results concerning the effects of increases in nonstudent forms of revenue, such as income from endowment (R). For example, in the Cobb-Douglas utility function case, with $\varepsilon_i = \varepsilon_j$ = constant and n_i the same function as n_j, one can show that the level of nonstudent income effects only the share of costs that the university bears and not the number of students to be admitted. Other things equal, increases in R leave the ratios of the share of costs the university should bear across categories constant, lowering the net price faced by all categories in the following

manner -- $(\partial P_i/\partial R)/(\partial P_j/\partial R)) = (C-P_i)/(C-P_j)$. Of course, since decreasing net prices will increase yields, the number of enrolled students will increase in each category.

12. Examples include Winship Fuller, Charles Manski, and David Wise (forthcoming), John Abowd (1977), John Bishop (1977), Meir Kohn, Charles Manski, and David Mundel (1976), Roy Radner and Lawrence Miller (1980), and Gregory Jackson (1978).

13. There have been several earlier institutionally based studies. See, for example, Donald Elliot and Jerry Hollenhorst (1981), Randall Chapman (1977), James Scannell (1980) and David Hopkins and William Massy (1981), Ch. 8. None of their models are as general as ours, for example none contain data on the characteristics of the other institution the applicant is considering including the explicit financial aid package offered to him or her there.

14. One complication is that an individual's being observed in the sample is conditional on his or her having applied to Cornell and been admitted. Because of this, the error terms in (17) and (18) may not be uncorrelated with the explanatory variables and we may confound the effects of an explanatory variable on the yield, with its effect on the probability of being observed in the sample. This is nothing more than the now standard "selection bias problem" (see James Heckman (1979)), complicated by our not observing the characteristics of individuals who fail to apply to Cornell. We leave it to future research in the area to address this problem.

We should also stress that different assumptions about the distribution of error terms (e.g., log-normal) would lead to different statistical models (e.g., logit). While in most cases, conclusions derived from the two methods are quite similar, individuals applying the model in future applications might want to test for the sensitivity of their results to the functional form estimated.

15. We also allow the intercept term of the yield curve to vary with the Cornell college or school to which the student applied. While it would be desirable to estimate separate yield curves for each of the seven Cornell colleges and schools, the sample sizes are not large enough to permit this.

16. Financial "need", as mechanically computed by the College Scholarship Service, was also available but its inclusion never significantly altered any of the results that followed. It would have been desirable to have data on the underlying factors that, in addition to income, influence a family's ability to pay for education, such as wealth, family size, and number of other college-age students. Unfortunately, such data was unavailable for the class of 1985.

17. The return rate was greater for students that actually enrolled at Cornell and this introduces an obvious choice-based sampling problem (see Charles Manski and Steven Lerman (1977)) which we ignore here.

18. To obtain estimates of the marginal effect of a unit ($1,000) change in the Cornell scholarship offer, the probit coefficient for the Cornell scholarship variable (CGRANT) in column (1) or for the Cornell net cost variable (CNCOST = total cost of attending Cornell less the scholarship offer at Cornell) are multiplied by the adjustment factors described in footnote a of Table 4. The products are then multiplied by .5 to reflect a $500 change.

19. Previous studies, including Fuller, Manski, and Wise, have not treated loans or employment opportunities as forms of financial aid. Our findings in Table 1 do not support these omissions. We must caution, however, that loans are relevant only to the extent that applicants can receive below market rates of interest. Changes in federal loan programs, faced first by the class of 1966, reduced the size of the implicit subsidy and tightened

eligibility restrictions. As such, one might expect that the marginal effect of the loan package on enrollments would be smaller for classes subsequent to the class of 1985.

20. While Table 1 suggests that the latter constraints may not be formally valid, imposing them facilitates our testing for interactions below.

21. We include interactions with SAT scores because Cornell University adopted a financial aid policy for the class of 1986 in which the share of the aid package in the form of grants was higher for more "attractive" applicants. Attractiveness was to be determined by the individual college admissions committees and the criteria to be applied were intentionally left to the colleges' discretion. However, it was understood by everyone that academic quality would play a major role. Since the ranking scheme was not in effect for the class of 1985, the data which we have analyzed, we use SAT score as a proxy for the ranking scheme.

22. Previous researchers have often found that students from low-income families have larger elasticities of enrollment with respect to net cost than do students from higher-income families (see Gregory Jackson and George Weathersby (1975) for a review of the evidence). These studies, however, are typically not institutionally based and refer to the decision to attend any college.

Numerous possible explanations exist as to why the elasticity might be lowest for lower-income applicants. One is that they may apply to fewer colleges and have fewer and/or less attractive alternatives. The data in Table A2, however, suggest that this is not the case. Lower-income students do not apply to fewer colleges (Col. 4), do not have fewer alternatives at which they are accepted (Col. 5), and are equally as likely to have the option of going to another Ivy League college (Col. 6).

On the other hand, the data also suggest that the scholarship packages received by lower-income applicants at Cornell are relatively more attractive vis-a-vis the other college's offer than they are for other income classes of applicants. Contrasting the Cornell net cost (Col. 2) with the net cost of the alternative (Col. 3), one observes that Cornell is substantially "cheaper" for the lowest income applicants ($5,000 in the endowed and $4,200 in the statutory) than the alternative college. The differential for other income classes of aid applicants is much smaller; in some cases the Cornell net cost proves to be higher.

Given that Cornell has chosen to be more generous to its lower-income applicants than other colleges, it is not surprising that their applicants' net cost elasticity is lower. Two factors enter in here. First, a 10 percent increase in their Cornell net cost represents a much smaller absolute increase in costs than it does a comparable percentage increase for higher-income applicants (Col. 2). Second, such a percentage increase in the Cornell net cost would hardly influence the gap between the net cost at Cornell and the alternative, for lower-income applicants, while it would virtually eliminate the gap for other income classes of applicants.

23. Family income data is available only for accepted financial aid applicants and hence the elasticities by income class cannot be computed for the larger sample.

24. One statistical issue should be noted here. Since the SAT score of the n^{th} applicant is used in the construction of the average quality score for the first m applicants, for all m greater than $n - 1$, it is likely that a complicated form of serial correlation is introduced into the residuals implicit in (20) and (21). Because of this, the reported standard errors in Table 3 probably overstate the precision of the estimates in the table.

One way around this problem, suggested to us by our colleague Bob Hutchens, is to try to estimate the parameters of the average quality function from data on the marginal quality of each applicant. For example, suppose the quality of the j^{th} ranked applicant is given by

(i) $q_j = \alpha_0 j^n$ or $\log q_j = \log \alpha_0 + n \log j$

In this case, the average quality of the first j applicants is

(ii) $q(j) = \frac{1}{j}\int_0^j q_X dX = \frac{1}{j}\int_0^j \alpha_0 X^n dX = \frac{\alpha_0}{n+1} j^n$ or

(iii) $\log q(j) = \log (\alpha_0/n+1) + n \log j$

But (iii) is nothing more than (20) in the text and hence estimation of (i) will allow one to recover estimates of n, without worrying about the serial correlation problem.

When this was done, the pattern of results proved to be virtually identical to those reported in the text. Since not all average quality equations correspond to easily estimable marginal quality equations, we have chosen to emphasize the former in the text.

25. Children of alumni might be considered relatively more attractive because of the belief that failure to admit them will result in reduced alumni giving. To our knowledge, there is no evidence on this point, however.

26. Note that if the elasticity of the yield curve had been greater for low-income students, as previous studies suggest (see footnote 22), unambiguously it would be optimal to give low-income students more generous financial aid packages. Moreover, one should recall that we have argued (see footnote 22) that it is the university's relatively generous financial

aid policies for these students that in fact leads to their less elastic yield curve.

27. One could, of course, obtain unambiguous implications for each of the categories if one were willing to adopt explicit functional forms for all of the equations in the model, including the weight each category should receive in the utility function. Hoenack (1971) adopted such a strategy.

Table 1

Probit Enrollment Equations for the
Class of 1985 Financial Aid Applicants
(absolute value t statistic)

	Statutory[a]		Endowed[a]	
	(1S)	(2S)	(1E)	(2E)
ILR	.342 (1.2)	.425 (1.4)		
HUMEC	-.025 (0.1)	.001 (0.0)		
HOTEL			.413 (1.1)	.504 (1.3)
ARCH			.136 (0.3)	.175 (0.4)
ENGR			-.027 (0.3)	-.037 (0.4)
COSEP	-.773 (2.6)	-.903 (3.1)	-.258 (1.6)	-.337 (2.1)
LEGACY	-.082 (0.4)	-.035 (0.2)	.256 (1.8)	.286 (2.0)
SEX	-.361 (2.0)	-.356 (2.0)	-.002 (0.0)	-.027 (0.3)
SATSUM	-.046 (0.6)	-.073 (1.0)	-.204 (4.7)	-.202 (4.7)
STAND	5.121 (2.9)	5.391 (2.9)	1.369 (1.9)	1.369 (1.9)
DISTCU	-.023 (1.1)	-.023 (1.2)	-.006 (1.0)	-.003 (0.4)
DISTOTH	.026 (1.4)	.024 (1.3)	.002 (0.2)	-.002 (0.3)
IVY	-.981 (3.7)	-1.169 (4.5)	-.701 (6.4)	-.781 (7.2)
OTHSAT	-.014 (0.1)	-.100 (1.0)	-.248 (4.0)	-.359 (6.3)
CGRANT	.222 (2.2)		.256 (9.4)	
CLOAN	.031 (0.2)	.036 (0.3)	.063 (0.9)	.100 (1.4)
CWORK	-.163 (0.9)	-.239 (1.4)	-.029 (0.3)	-.107 (0.2)
OGRANT	-.219 (4.3)		-.196 (8.5)	
OLOAN	.105 (1.0)	-.178 (2.1)	.091 (1.7)	-.161 (3.9)
OWORK	.096 (0.5)	-.437 (2.8)	.061 (0.7)	-.262 (3.6)
CCOST	-.213 (1.8)			
CNCOST		-.162 (2.5)		-.210 (8.5)
ONCOST		.103 (2.6)		.117 (6.7)
PARINC	.031 (0.4)	.023 (0.3)	.012 (0.3)	.028 (0.8)
LOG L	-161.15	-167.02	-553.58	-564.77
χ^2 (DOF)	136.40 (24)	124.66 (22)	376.98 (24)	354.60 (23)
n	491	491	1094	1094

All variables are defined below. An intercept term and dummy variables for non-reporting of the applicant's SAT scores, the applicant's class standing, and the average SAT score of freshmen in the applicant's alternative college are also included in the equations.

Log L - log of the likelihood function

χ^2 - chi-square statistic to test the hypothesis that the entire vector of probit coefficients, except for the intercept, is zero -- with DOF degrees of freedom

[a]To obtain slopes of the conditional mean function at the regression means (i.e., the marginal effect of a unit change in an explanatory variable on the probability of enrollment) multiply the probit coefficients by the following:

(1S) .896 (1E) .634
(2S) .891 (2E) .626

Table 1 (continued)

where:

ILR	- 1=applicant to School of Industrial & Labor Relations, 0=other	Ag & Life Sciences is the reference group for the statutory sector
HUMEC	- 1=applicant to College of Human Ecology, 0=other	
HOTEL	- 1=applicant to Hotel College, 0=other	Arts & Science is the reference group for the endowed sector
ARCH	- 1=applicant to College of Architecture, Art and Planning, 0=other	
ENGR	- 1=applicant to College of Engineering, 0=other	
COSEP	- 1=minority admissions program applicant, 0=other	
LEGACY	- 1=parent, grandparent, or sibling is a Cornell alumnus, 0=other	
SEX	- 1=male, 0=female	
SATSUM	- sum of applicant's score on math and verbal SAT tests (in hundreds)	
STAND	- applicant's percentile class ranking in high school (0=highest, 1=lowest)	
DISTCU	- distance from applicant's home to Cornell in miles (hundreds)	
DISTOTH	- distance from applicant's home to best alternative or college he enrolled in miles (hundreds)	
IVY	- 1=best alternative or college applicant actually enrolled in is an Ivy League school, 0=no	
OTHSAT	- sum of the average math and verbal SAT scores of freshmen at the best alternative or college that the applicant actually enrolled in (in hundreds)	
CGRANT	- total scholarship at Cornell, from Cornell and all other sources (in thousands)	
CLOAN	- total loan package at Cornell (in thousands)	
CWORK	- total work-study and other employment opportunities at Cornell (in thousands)	
OGRANT	- total scholarship at the alternative (in thousands)	
OLOAN	- total loan package at the alternative (in thousands)	
OWORK	- total employment opportunities at the alternative (in thousands)	
CCOST	- total cost of attending Cornell (tuition and fees, books and supplies, room and board, and personal expenses) (in thousands)	
OCOST	- total cost of attending the alternative (including an estimate of average travel costs) (in thousands)	
CNCOST	- total cost of attending Cornell less scholarship offer at Cornell (in thousands)	
ONCOST	- total cost of attending the alternative less the scholarship offer there (in thousands)	
PARINC	- parents' income (in ten thousands)	

Sources of Data:

(1) Cornell University Admissions Office "Admissions File" - HOTEL, ARCH, ENGR, COSEP, LEGACY, SEX, SATSUM, STAND.

(2) Cornell University Financial Aid Office "Financial Aid Management Information System File" - CGRANT, CLOAN, CWORK, PARINC.

(3) Cornell University Admissions Office "Admissions Research Questionnaire File" - DISTCU, DISTOTH, IVY, OGRANT, OLOAN, OWORK.

(4) College Entrance Examination Board, The College Cost Book, 1981-82 edition (plus telephone interviews).

(5) Case and Birnbaum's, Guide to Colleges (1981 edition) - OTHSAT.

Table 2

Implied Elasticities of the Probability of Enrollment with Respect to Cornell's Net Cost[a]

	Statutory			Endowed		
	(1)	(2)	(3)	(1)	(2)	(3)
0 < SATSUM ≤ 1150	-1.07	-.92	-1.18	-1.65	-1.46	-1.64
1150 < SATSUM ≤ 1300	-1.03	-1.01	-1.22	-1.78	-1.73	0.50
1300 < SATSUM	-1.06	-1.03	-1.42	-1.84	-1.87	-2.03
0 < PARINC ≤ 18,000	-.54	-.60	-.58	-.92	-.75	-.78
18,000 < PARINC ≤ 28,000	-.92	-.51	-.48	-1.20	-1.62	-1.18
28,000 < PARINC ≤ 36,000	-.99	-.53	-.51	-1.69	-2.00	-1.39
36,000 < PARINC ≤ 44,000	-1.25	-.63	-.90	-1.71	-1.95	-1.42
44,000 < PARINC	-1.11	-.05	+.06	-2.08	-2.26	-2.25
COSEP = 0	-1.09	-1.04	-1.20	-1.87	-1.90	-2.04
COSEP = 1	-.95	-.95	-.88	-1.59	-.95	-1.52
LEGACY = 0	-1.06	-.94	-1.22	-1.69	-1.79	-1.64
LEGACY = 1	-1.05	-1.60	-1.61	-1.95	-2.87	-2.33
Overall	-1.06	--	--	-1.83	--	--

[a]Computed using the mean values of the relevant variables for each subsample and the estimated coefficients from

(1) the probit net cost model without interactions (Table 1)
(2) the probit net cost models with one type of interaction at a time (Appendix Table A3)
(3) the probit net cost model with all four types of interactions occurring together (Appendix Table A3)

For (2) and (3) the only interactions that prove to be even marginally statistically significant are those for the endowed colleges for COSEP and family incomes in the \$18,000-\$28,000 and \$28,000 - \$36,000 ranges.

Table 3

Average Quality as a Function of Number of Accepted Applicants
(standard errors)

Group Accepted Applicant Total Sample	Elasticity of Average Quality WRT the Number of Enrollees: Constant Elasticity Specification (1)	(2)	Coefficient of Applicant Rank Variable Elasticity Specification[a] (1)	(2)
ENDOWED				
0 < SATSUM ≤ 1150 [181/4377]	-.021 (.0012)	-.025 (.0003)	-.046 (.0007)	-.002 (.0000)
1150 < SATSUM ≤ 1300 [378/4733]	-.012 (.0004)	-.014 (.0001)	-.012 (.0001)	-.001 (.0000)
1300 < SATSUM [555/3243]	-.027 (.0003)	-.028 (.0001)	-.016 (.0002)	-.003 (.0000)
0 < PARINC ≤ 18,000 [110]	-.056 (.0019)		-.171 (.0024)	
18,000 < PARINC ≤ 28,000 [161]	-.044 (.0012)		-.095 (.0014)	
28,000 < PARINC ≤ 36,000 [174]	-.041 (.0011)		-.080 (.0010)	
36,000 < PARINC ≤ 44,000 [176]	-.038 (.0010)		-.074 (.0008)	
44,000 < PARINC [228]	-.036 (.0009)		-.055 (.0006)	
COSEP = 0 [929/14,388]	-.036 (.0004)	-.053 (.0002)	-.013 (.0001)	-.001 (.0000)
COSEP = 1 [185/1529]	-.064 (.0014)	-.076 (.0007)	-.116 (.0016)	-.017 (.0000)
LEGACY = 0 [998/11029]	-.040 (.0005)	-.053 (.0002)	-.015 (.0001)	-.002 (.0000)
LEGACY = 1 [116/1330]	-.044 (.0014)	-.052 (.0002)	-.127 (.0021)	-.014 (.0000)
STATUTORY				
0 < SATSUM ≤ 1150 [171/2130]	-.018 (.0010)	-.027 (.0004)	-.042 (.0007)	-.005 (.0000)
1150 < SATSUM ≤ 1300 [220/1381]	-.015 (.0006)	-.017 (.0002)	-.026 (.0001)	-.005 (.0000)
1300 < SATSUM [103/449]	-.026 (.0006)	-.028 (.0002)	-.081 (.0020)	-.020 (.0003)
0 < PARINC ≤ 18,000 [44]	-.058 (.0045)		-.045 (.0058)	
18,000 < PARINC ≤ 28,000 [76]	-.050 (.0018)		-.022 (.0040)	
28,000 < PARINC ≤ 36,000 [90]	-.049 (.0016)		-.018 (.0035)	
36,000 < PARINC ≤ 44,000 [76]	-.043 (.0017)		-.019 (.0033)	
44,000 < PARINC [89]	-.049 (.0017)		-.018 (.0028)	
COSEP = 0 [425/3560]	-.046 (.0007)	-.058 (.0003)	-.038 (.0003)	-.006 (.0000)
COSEP = 1 [69/402]	-.074 (.0024)	-.075 (.0012)	-.340 (.0082)	-.065 (.0006)
LEGACY = 0 [408/3359]	-.048 (.0083)	-.059 (.0003)	-.041 (.0003)	-.006 (.0000)
LEGACY = 1 [82/605]	-.057 (.0017)	-.065 (.0001)	-.219 (.0048)	-.037 (.0003)

(1) Accepted applicant sample.

(2) Total applicant sample.

[a] Coefficient and standard error have been multiplied by 100.

Table 4

Implied Effects of Characteristics on the Size of the "Optimal" Financial Aid Package

Characteristic	Effect Operating Through: Assumed Relative Attractiveness	Propensity to Enroll	Elasticity of Yield	Elasticity of Average Quality
ENDOWED				
Alumni Relatives	+	-	+ or 0	0 or +
Minority	+	+	-	+
Low Family Income	+	0	-	+
High Ability	+	+	+	+
STATUTORY				
Alumni Relatives	+	0	+ or 0	+
Minority	+	+	- or 0	+
Low Family Income	+	0	- or 0	+
High Ability	+	+	0	+

Appendix

PROOF OF PROPOSITIONS

This appendix provides a proof of all of the propositions presented in the text, as well as a table that indicates how the results differ when alternative utility functions are used.

Proposition 1: Suppose $\varepsilon_i = \varepsilon_j = \varepsilon$ and n_i is the same function as n_j. From (12), we require $n_i = n_j$ and hence $X_i = X_j$. Thus (16) becomes $(S_i F_i(S_i)/S_j F_j(S_j)) = (\alpha_i/\alpha_j)$. But since ε is a constant $F_i = K_i S_i^{\varepsilon}$ and $F_j = K_j S_j^{\varepsilon}$. Hence $(S_i/S_j)^{\varepsilon+1} = (\alpha_i/\alpha_j)(K_j/K_i)$. Recall that $(S_i/S_j) > 1 \Rightarrow (P_i/P_j) < 1$.

Q.E.D.

Proposition 2: Suppose $\varepsilon_i > \varepsilon_j$, both are constants, and that n_i is the same function as n_j. From (12), we require in equilibrium $[(1+n_j)/(1+n_i)] < 1$. Since $n_i(X) = n_j(X)$ and $n' < 0$, this requires that $X_j > X_i$. From (16) then $(S_i F_i(S_i)/S_j F_j(S_j)) > (\alpha_i/\alpha_j)$. Substituting for F_i and F_j this becomes $(S_i/S_j)^{\varepsilon j+1} S^{\varepsilon i-\varepsilon j} > (\alpha_i/\alpha_j)(K_j/K_i)$. Now suppose, as is done in the text, that $\alpha_i = \alpha_j$ and $K_i = K_j$, then equilibrium requires that $(S_i/S_j)^{\varepsilon j+1} S_i^{\varepsilon i-\varepsilon j} > 1$. If $\varepsilon_i > \varepsilon_j$, $S_i^{\varepsilon i-\varepsilon j} \leq 1, \forall\ S_i \leq 1$. Hence $S_i > S_j$ and $P_i < P_j$.

Q.E.D.

Proposition 3: Suppose $\varepsilon_i = \varepsilon_j = \varepsilon$, and n_i is greater than n_j at the same X (j is more elastic). Suppose also, as is assumed in the text, that $\alpha_i = \alpha_j$ and $K_i = K_j$. From (12), we require $n_i = n_j$ at equilibrium and, since $n' < 0$, we need $X_j < X_i$. From (16) then $(S_i/S_j)^{\varepsilon+1} = (X_j/X_i)$. Hence $S_i < S_j$ or $P_i > P_j$.

Q.E.D.

Proposition 4: Suppose $n_i = n_j = n$ and $F(S_i)$ is the same function as $F(S_j)$. From (12) we require $\varepsilon_i = \varepsilon_j$ and hence $S_i = S_j$. From (16) it immediately follows that $(X_i/X_j) = (\alpha_i/\alpha_j)$.

Q.E.D.

Proposition 5: Suppose $n_i > n_j$ so that j is more elastic and both are constants and that ε_i is the same function as ε_j. From (12) we require $(1/\varepsilon_j) > (1/\varepsilon_i)$ and hence $\varepsilon_j < \varepsilon_i$. Since $\varepsilon_i = \varepsilon_j$ at the same S and $\varepsilon' < 0$, it immediately follows that $S_i < S_j$ or $P_i > P_j$.

Q.E.D.

Proposition 6: Suppose $n_i = n_j = n$ and ε_i is greater than ε_j at the same S. If $S_i = S_j$ than $\varepsilon_i > \varepsilon_j$. But from (12), we require $\varepsilon_i = \varepsilon_j$ and, since $\varepsilon' < 0$, this implies that $S_i > S_j$ and hence $P_i < P_j$.

Q.E.D.

Table A-1

Summary of Results:
Alternative Utility Functions

Case/Utility Function	Cobb-Douglas	Additive	CES
1. $\varepsilon_i = \varepsilon_j = \varepsilon$ n_i same function as n_j	Other things equal--lower price to group with a) greater weight in utility b) lower propensity of enrolling	same*	same
2. $\varepsilon_i > \varepsilon_j$, both constant n_i same function as n_j same weights in utility same propensity of enrolling	Lower price to group whose elasticity of yield w.r.t. university's share of cost is higher (i)	same*	$\sigma \geq 1$ same $\sigma < 1$ ambiguous
3. $\varepsilon_i = \varepsilon_j = \varepsilon$ $n_i > n_j$, at same X (j is more elastic) same weights in utility same propensity of enrolling	Lower price to group whose elasticity of quality w.r.t. admissions is higher (j)	**	**
4. $n_i = n_j = n$ $F(S_i)$ is the same function as $F(S_j)$	Relative number admitted equals relative weights in utility. Both charged the same price	same	same***
5. $n_i > n_j$, both constant (j is more elastic) ε_i same function as ε_j	Lower price to group whose elasticity of quality w.r.t. admissions is higher (j)	same	same
6. $n_i = n_j = n$ $\varepsilon_i > \varepsilon_j$ at same S	Lower price to group whose elasticity of yield w.r.t. university's share of costs higher	same	same

*Propensity of enrollment not matter in (1) nor are equal ones required in (2).

**Ambiguous without further assumptions about the form of the q(X) functions.

***Also $\sigma > 1$ ($\sigma < 1$) increase in (q_{oi}/q_{oj}) increases (decreases) X_i/X_j.

Table A2

Descriptive Statistics

Group	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Endowed								
Overall	.590	8.03	8.15	4.55	3.45	.247	.181	.237
0 < PARINC ≤ 18,000	.615	4.01	9.09	4.74	3.50	.284	.136	.419
18,000 < PARINC ≤ 28,000	.605	5.70	6.26	4.14	3.24	.266	.150	.244
28,000 < PARINC ≤ 36,000	.584	7.13	6.95	4.43	3.37	.212	.179	.219
36,000 < PARINC ≤ 44,000	.577	8.09	7.70	4.74	3.72	.206	.200	.204
44,000 < PARINC	.540	9.69	8.47	4.76	3.64	.247	.215	.177
0 < SATSUM ≤ 1150	.765	6.99	6.92	4.45	3.36	.169	.273	.227
1150 < SATSUM ≤ 1300	.650	7.84	7.87	4.40	3.35	.217	.194	.237
1300 < SATSUM	.494	8.58	8.76	4.73	3.56	.293	.136	.241
COSEP = 1	.603	6.57	6.78	4.75	3.86	.318	.168	.218
COSEP = 0	.588	8.37	8.31	4.52	3.39	.236	.183	.240
LEGACY = 1	.702	8.83	8.79	3.93	2.87	.242	.196	.250
LEGACY = 0	.567	7.92	8.02	4.68	3.57	.248	.178	.235
Statutory								
Overall	.834	5.94	6.70	3.65	3.03	.097	.419	.189
0 < PARINC ≤ 18,000	.867	2.80	7.07	3.64	3.00	.099	.422	.333
18,000 < PARINC ≤ 28,000	.838	5.11	6.22	3.02	2.59	.095	.351	.217
28,000 < PARINC ≤ 36,000	.850	5.86	6.11	3.36	2.95	.112	.486	.182
36,000 < PARINC ≤ 44,000	.773	6.54	6.79	3.80	3.12	.067	.371	.163
44,000 < PARINC	.870	6.65	7.08	4.10	3.30	.190	.420	.120
0 < SATSUM ≤ 1150	.862	5.55	6.26	3.56	2.92	.067	.437	.182
1150 < SATSUM ≤ 1300	.859	6.14	6.66	3.59	2.99	.072	.445	.196
1300 < SATSUM	.733	6.12	7.80	4.14	3.44	.211	.273	.186
COSEP = 1	.675	4.89	6.53	4.15	3.36	.218	.309	.184
COSEP = 0	.856	6.14	6.73	3.58	2.99	.079	.435	.189
LEGACY = 1	.867	6.05	6.75	3.07	2.59	.076	.458	.215
LEGACY = 0	.826	5.91	6.69	3.78	3.13	.101	.410	.184

where:
(1) proportion of accepted applicants that enrolls (yield)
(2) mean Cornell net cost
(3) mean other net cost
(4) mean number of other colleges applied to
(5) mean number of other colleges accepted at
(6) fraction with another Ivy school as best alternative
(7) fraction with a public university as best alternative
(8) mean ratio of Cornell net cost/family income

Table A3

Testing for the Effects of Applicant Characteristics on the Sensitivity of Enrollment Decisions to Financial Variables (absolute value t statistics)

	Statutory					Endowed				
	(1)	(2)	(3)	(4)	(5)	(1)	(2)	(3)	(4)	(5)
CNCOST	-.144 (1.7)	-.194 (1.7)	-.184 (2.4)	-.149 (2.2)	-.191 (1.4)	-.218 (7.8)	-.164 (3.8)	-.232 (8.1)	-.206 (7.9)	-.186 (4.1)
CNSAT1	-.011 (0.2)				-.060 (0.1)	.024 (0.8)				.017 (0.4)
CNSAT2	-.017 (0.4)				-.012 (0.3)	.009 (0.6)				.002 (0.1)
CNINC1		.106 (1.2)			.110 (1.3)		-.058 (1.6)			-.056 (1.5)
CNINC2		.107 (1.2)			.110 (1.2)		-.057 (1.5)			-.054 (1.4)
CNINC3		.054 (0.5)			.062 (0.6)		-.035 (0.9)			-.031 (0.8)
CNINC4		.185 (1.4)			.196 (1.5)		-.026 (0.6)			-.019 (0.4)
CNCOSP			.060 (0.5)		.039 (0.3)			.074 (1.8)		.072 (1.7)
CNLEGY				-.120 (0.7)	-.071 (0.4)				-.072 (1.2)	-.065 (1.1)

where: CNCOST - applicant's estimated cost of attending Cornell less scholarship offer at Cornell (in thousands)
ONCOST - applicant's estimated cost of attending the other college less scholarship offer there (in thousands)
CNSAT1 - CNCOST if applicant's SAT score is less than 1150, 0 = otherwise
CNSAT2 - CNCOST if applicant's SAT score is greater than 1300, 0 = otherwise
} $1150 \leq SAT \leq 1300$ is the omitted class
CNINC1 - CNCOST if applicant's family income lies in the \$18,000-\$28,000 range, 0 = otherwise
CNINC2 - CNCOST if applicant's family income lies in the \$28,000-\$36,000 range, 0 = otherwise
CNINC3 - CNCOST if applicant's family income lies in the \$36,000-\$44,000 range, 0 = otherwise
CNINC4 - CNCOST if applicant's family income exceeds \$44,000, 0 = otherwise
} family income < \$18,000 is the omitted class
CNCOSP - CNCOST if applicant is a COSEP student, 0 = otherwise
CNLEGY - CNCOST if applicant is a legacy student, 0 = otherwise