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A SIMULATION-BASED WELFARE LOSS CALCULATION  
FOR LABOR TAXES WITH PIECEWISE-LINEAR BUDGETS

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**ABSTRACT**

Graduated income tax rates and transfer programs create piecewise-linear budget constraints that consist of budget segments and kink points. With any change in these tax rules, each individual may switch between a kink point and a budget segment, between two budget segments, or between two kink points. With errors in the estimated labor supply equation, the new choice is uncertain, and so the welfare effects of a tax change are uncertain. We propose a simulation-based method to compute expected welfare effects that is easy to implement and that fully accounts for uncertainties about choices around kink points. Our method also provides information about expected changes in working hours.

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# 1 Introduction

Graduated income tax rates and income transfer programs create piecewise-linear budget constraints that are composed of a collection of budget segments and kink points. A considerable body of work estimates labor supply under such budget sets.<sup>1</sup> Key insights in this literature are that the consumer may choose a budget segment or a kink, whichever provides maximum utility, and that this behavior is estimated with error.

Economists also calculate welfare loss due to taxation of labor supply. Many use labor supply estimates to calculate average and marginal welfare loss, and many evaluate the economic effects of proposed and real tax reforms. As reviewed below, however, existing welfare cost calculations often do not fully account for the errors of estimation and their interaction with the nonlinear budget constraint for each individual. In particular, with a change of tax schedule, the stochastic specification means that each individual has a distribution of possible outcomes: she may switch to another budget segment, switch to a kink point, or even switch to or from participating in the labor force. In general, each different budget segment produces a different net wage and a different virtual income.

In this paper, we develop a method to calculate welfare cost that employs the full stochastic specification of any estimated labor supply model. In particular, we account for uncertainties that arise from estimating errors by using Monte Carlo simulation across heterogeneous individuals. For each individual, this method uses the estimated probabilities of switching from each segment or kink point to another to calculate “expected” welfare loss for each individual. This method also identifies the expected change of working hours. Moreover, it provides a natural way to aggregate welfare loss and the change in working hours for various types of heterogeneous individuals. We then illustrate this method using three existing samples of individuals and estimates of labor supply behavior.

The problem of welfare loss from labor taxes under piecewise-linear budget constraints is essentially the same problem as calculating consumer surplus or willingness-to-pay in discrete choice models where choices are mutually exclusive. Similarly, in the labor supply

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<sup>1</sup>For surveys, see Hausman (1985), Moffit (1990, 2002), and Blundell and MaCurdy (1999).

model, a worker may choose only one budget segment or kink point. Small and Rosen (1981) were among the first to study systematically the effect of a price change on welfare for discrete choice models. However, their study did not account for the possibility of changing income. McFadden (1999) thoroughly discusses a willingness-to-pay problem in discrete choice models by explicitly comparing the choices that yield maximum utilities before and after changes in some specific attributes of arguments in the utility function. Possible changes in income, prices or attributes may change the choice that maximizes utility and hence affect the values of the compensating variation (CV) and equivalent variation (EV). While his study concerns fishing,<sup>2</sup> other examples concern housing<sup>3</sup> or wealth accumulation.<sup>4</sup>

Previous literature on calculating welfare loss of labor taxation with piecewise budget constraints is based on analytical solutions. Examples include Hausman (1983) and Blomquist (1983). In order to allow for this analytical solution in his study of the change from one piecewise-linear budget constraint to another, Hausman assumes that each person's new optimal choice is on a segment of the new budget constraint. Blomquist allows for kinks in the existing tax system, but calculates the welfare gains of moving to a proportional tax system (with no kinks). By using a simulation approach, we can allow for changes to or from a kink.

The framework we adopt here is pioneered by various studies of Hausman in the 1980's

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<sup>2</sup>In McFadden's example, evaluating environmental damages at various fishing sites, the attributes include the quality and quantity of fish at each site. The CV or EV are those that equalize the maximum utilities before and after some change in fishing quality.

<sup>3</sup> In a study of housing and taxes, Berkovec and Fullerton (1992) use a simulation approach to calculate welfare loss. They employ eight mutually exclusive regimes, with discrete choices about whether to hold owner housing, rental housing, and corporate equity. For each household, they compare the utility levels in each regime before the tax change, and again after the tax change. Within each regime, they consider what tax bracket the person would face. Since they study housing choice, however, they ignore the choice of working hours. The implicit assumption is that hours do not change in response to a change in tax rate.

<sup>4</sup>Hubbard, Skinner and Zeldes (1995) show that the often-assumed monotonic relationship between wealth and consumption may not be valid anymore due to piecewise-linear budget constraints. Also, the breakdown of this monotonic relationship may have important effects on wealth accumulation and life-cycle behavior.

(Hausman, 1981b, 1983, 1985). Blundell and MaCurdy (1999) discuss several attractive features of this framework: it explicitly recognizes the institutional features of the tax system, and it readily incorporates the fixed cost of holding a job. However, some concerns on how to estimate labor supply in this framework have also emerged. The most notable concern is of Heckman (1983), that the budget set for each individual often cannot be accurately determined and that a special type of errors-in-variable bias results.<sup>5</sup> Yet a recent paper by Gan and Stahl (2002) shows that the Heckman concern can indeed be addressed in the Hausman framework by introducing measurement error in non-labor income, because it creates a random budget set. Such a labor supply equation can be estimated in a framework of piecewise-linear budget constraints without suffering from the Heckman concern.

This paper does not provide any assistance in estimating labor supply functions. Rather, the point is to employ the stochastic specifications of such models along with their parameter estimates when calculating welfare effects of tax changes. It is to be consistent with those labor supply models that we suggest a Monte Carlo method. These models often have multiple random errors, and they have no closed-form solution for welfare cost. Our method yields strikingly different results compared to use of point estimates in a simple welfare cost formula. Then, once the Monte Carlo method is employed, several other complications can easily be incorporated as well.

In particular, this paper makes several contributions relative to existing welfare cost calculations. First, we calculate welfare cost using labor supply estimates that account for the Heckman concern. Second, earlier analytical approaches had to assume that each person's new indifference curve is tangent to a line segment on the new budget, while our approach allows movement to or from a kink point. Third, we account for the fact that the EV or CV itself is a transfer that may also affect the person's choice. Fourth, our simulation

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<sup>5</sup>Also, MaCurdy, Green and Paarsch (1990) argue that the likelihood setup in Hausman's framework may create artificial constraints on the parameter values. Blundell and MaCurdy (1999) suggest that the Triest (1990) dual random error model is not subject to this problem. In fact, however, Hausman's random coefficient model is not subject to this problem since the Triest model is a special case of Hausman's model. See Gan and Stahl (2002) for a detailed discussion on this point.

method is easy to implement and to calculate, with no additional difficulty for a non-convex budget set. Finally, earlier analytical approaches could not employ the entire estimated distributions of multiple error terms. For example, Hausman (1983) allows for measurement error and for heterogeneity in one of the preference parameters. To get a probability-weighted choice of hours, one needs to integrate over both distributions. To simplify, one might use just the mean of each distribution. Later we call this the simple “Harberger” method (Harberger, 1964), because the person’s choice is only one point. Instead, Hausman uses an approximation, evaluating the distribution at the means of intervals. Here, we employ the entire estimated distributions of both error terms. For *each* individual in the data set, our Monte Carlo simulation takes a large number of random drawings from the two estimated distributions. For each drawing, it calculates the chosen segment or kink, and the resulting welfare cost. We then have a probability distribution of the welfare cost. Because welfare cost increases with the square of the tax rate, the expected welfare cost exceeds the welfare cost at the expected point. Compared to the simple Harberger method, this procedure might be important, especially if the errors are large and the tax system is steeply graduated.<sup>6</sup>

Indeed, we find larger welfare effects in each of our three illustrations. In one calculation, Harberger’s welfare cost is 26% of tax revenue, Hausman finds 58%, and we find 75%. For the rate reduction of the Tax Reform Act of 1986, Harberger’s gain is 6% of tax revenue, and ours is 35%. In a final example where the point estimate of the compensated labor supply elasticity is near zero, the Harberger-type welfare cost is near zero but ours is not: the elasticity is estimated with error, and the possibility of a positive elasticity implies positive expected welfare cost.

In Section 2, we define and provide a framework to estimate the CV and EV under budget constraints that are piecewise linear. These budget constraints are discussed in

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<sup>6</sup>Suppose, for example, that the mean of the distribution places the person in the 20% tax bracket but that the person actually has a 40% probability of being in the 30% bracket. The simple welfare cost is some constant times .2 squared (which is .04), while the true welfare cost involves the same constant times  $[(.6)(.2)(.2) + (.4)(.3)(.3)]$ , which is .06. In this simple example, the welfare cost measure is raised by 50%.

Section 2.1, while the issues related to CV and EV under piecewise budget constraints are in Section 2.2. Then Section 3 provides a framework to calculate welfare loss using the simulation method. Section 4 offers three empirical examples to compare the values of welfare loss derived from alternative methods. Section 5 concludes the paper.

## 2 A Basic Framework

In this study, we consider a static partial equilibrium labor supply model. The before-tax wage is constant, with no inter-temporal optimization of labor supply. All of the following variables are individual-specific, but we suppress the index for notational convenience.

We begin with a typical labor supply model of utility maximization with respect to choices about leisure and other consumption goods  $x$ . The hours of work are defined to be  $h$ , so  $-h$  is leisure. With no taxes, the person's non-labor income is  $y$ , and the real wage is  $w$ . The indirect utility  $v(w, y)$  is the maximum value of the direct utility  $u(x, h)$  that can be obtained when facing the budget constraint:

$$\begin{aligned} v(w, y) &= \max_{x, h} u(x, h) \\ \text{s.t. } & x - wh = y \end{aligned} \tag{1}$$

where the price of  $x$  is normalized to 1, and the cost of leisure is the wage rate  $w$ .

### 2.1 Budget segments and tax revenues

Graduated tax rates and income transfers imply different combinations of real wage rates and incomes in Equation (1). Let a tax bracket be represented by  $\{t_j; Y_{j-1}, Y_j\}$ , where  $t_j$  is the marginal tax rate for a person whose before-tax income lies within the interval  $[Y_{j-1}, Y_j]$ . Information about  $\{t_j; Y_{j-1}, Y_j\}$  can often be found from tax tables. Note that the relevant budget set is based on after-tax income. Let the end points of the segment in a budget set that corresponds to bracket  $\{Y_{j-1}, Y_j\}$  be  $\{y_{j-1}^a, y_j^a\}$ , where  $y^a$  refers to after-tax income. A complete characterization of budget segments requires information on working hours that correspond to the set  $[y_{j-1}^a, y_j^a]$ , and we denote these hours as  $[H_{j-1}, H_j]$ . To

calculate the location of each budget segment, we start with the first budget segment and proceed through all budget segments. Besides the before-tax wage rate  $w$ , another critical piece of information necessary is  $Y^n$ , the non-labor income this person may have. Let  $y^n$  be after-tax non-labor income, where the tax is calculated as if the person had no labor income. Then labor income pushes the person into successively higher tax brackets. We summarize information on budget segments in Table 1.

One interesting observation from Table 1 is that non-labor income affects the location of the budget segments for each individual, since the end points of a budget segment are functions of  $Y^n$  or  $y^n$ :

$$\begin{aligned} H_j &= (Y_j - Y^n)/w \\ y_j^a &= y^n + \sum_{k=2}^j (1 - t_k)(Y_k - Y_{k-1}) \end{aligned} \tag{2}$$

A change in non-labor income  $Y^n$  will lead to a change of the whole budget set. If  $Y^n$  is measured with error, the whole budget set will be measured with error. This point is used by Gan and Stahl (2002) as a way to resolve the critique that Heckman (1983) raises with respect to the Hausman labor supply estimates.

It is well known in the literature that a person's optimal hours may be at a kink point instead of being on the interior of a segment, in the framework of piecewise-linear budget constraints. Define

$$\begin{aligned} S_j &\equiv \begin{cases} 1 & \text{if on the interior of segment } j, \\ 0 & \text{otherwise;} \end{cases} \\ K_j &\equiv \begin{cases} 1 & \text{if at kink } j, \\ 0 & \text{otherwise.} \end{cases} \end{aligned} \tag{3}$$

The conditions determining the values of  $S_j$  and  $K_j$  require knowledge of the labor supply function. For example, consider a commonly estimated linear labor supply function

$$h = \begin{cases} \alpha w_j + \beta y_j^v + s, & \text{if positive} \\ 0 & \text{otherwise} \end{cases} \tag{4}$$



where  $w_j \equiv w(1-t_j)$  and where  $s$  includes  $z\gamma$  (the effect of other socio-demographic variables  $z$ ) and the statistical error. In this equation,  $y_j^v$  is virtual income, defined as the intercept of the line that extends budget segment  $j$  to the zero-hours axis. Given that labor supply function, the conditions for  $S_j = 1$  or  $K_j = 1$  are:

$$\begin{aligned} S_j = 1 & \quad \text{if} \quad H_{j-1} < \alpha w_j + \beta y_j^v + s < H_j \\ K_j = 1 & \quad \text{if} \quad \alpha w_{j+1} + \beta y_{j+1}^v + s \leq H_j \leq \alpha w_j + \beta y_j^v + s \end{aligned} \quad (5)$$

If a budget set is globally convex, the highest indifference curve must either touch a single kink point or be tangent to a single segment. Only one of  $S_j$  or  $K_j$  will be 1. However, often a budget set is not convex due to the fixed cost of working or some income transfer program (such as AFDC or TANF).<sup>7</sup> A possibility then arises that more than one of the  $S_j$  and/or  $K_j$  is 1. In this case, we must compare the utility levels for for all  $S_j = 1$  and  $K_j = 1$  and pick the segment or kink point that yields the highest utility level.

Another key variable in the calculation of welfare cost is the tax revenue from this person, which can be obtained based on the information in Table 1. Let working hours be  $h \in [H_{j-1}, H_j)$  as in the table. Then the tax revenue  $R$  for this individual is:

$$\begin{aligned} R &= R^n + \sum_{k=1}^{j-1} (H_k - H_{k-1})wt_k + (h - H_{j-1})wt_j \\ &= R_{j-1} + (h - H_{j-1})wt_j \end{aligned} \quad (6)$$

where  $R^n$  is the tax revenue from non-labor income, and  $R_{j-1}$  is defined as the tax revenue if the working hours were  $h = H_{j-1}$  (which may be obtained from the tax table and Table 1 when the wage rate  $w$  is given).

## 2.2 CV and EV under piecewise budget constraints

The welfare cost of the tax may be based on either the compensating variation (CV) or the equivalent variation (EV). In a simple proportional tax system, consider the case

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<sup>7</sup>Aid to Families with Dependent Children (AFDC) was replaced in 1996 by Temporary Assistance for Needy Families (TANF).

where a change in tax moves the pair of after-tax wage and virtual income from  $(w^0, y^0)$  to  $(w', y')$ . The CV and EV may be formally defined as:

$$\begin{aligned} u^0 = v(w^0, y^0) &= v(w', y' + CV) \\ v(w^0, y^0 - EV) &= v(w', y') = u' \end{aligned} \tag{7}$$

Calculating welfare cost in the framework of piecewise-linear budget constraints is similar to the problem of calculating willingness-to-pay in a discrete choice model. After a tax change, when a utility-maximizing individual chooses new working hours on a budget segment that provides the highest utility, the chosen segment or kink point has likely changed. We then compare the difference between the old and new utility levels and find a CV or EV value to equalize them. This basic idea is in McFadden (1999), but in our case the CV or EV is a transfer that may itself affect the person's choice of kink point or segment.

At any kink point where  $K_j = 1$ , we use the direct utility function  $u(x, h)$ , where  $x = y_j^a$ , and  $h = H_j$ . A person whose optimal hours are zero or negative does not participate in the labor force. The utility level of this person is  $u(y_0^a, 0)$ , where  $y_0^a = y_0^v = y^n$ .

Suppose  $k^0$  and  $k'$  are the total numbers of segments before and after the tax change. For a convex budget set, since only one of the  $S_j$ s and  $K_j$ s is 1, we can find the utility levels before and after a tax change as:

$$\begin{aligned} u^0 &= \sum_{j=1}^{k^0} S_j^0 v(w_j^0, y_j^{v0}) + \sum_{j=0}^{k^0} K_j^0 u(y_j^{a0}, H_j^0) \\ u' &= \sum_{j=1}^{k'} S_j' v(w_j', y_j^{v'}) + \sum_{j=0}^{k'} K_j' u(y_j^{a'}, H_j') \end{aligned} \tag{8}$$

Note, in general, that  $S_j^0 \neq S_j'$  and  $K_j^0 \neq K_j'$ . Under the new tax regime, a person may switch to a different kink point or segment.

When the budget set is not convex, we must consider the possibility that more than one of the  $S_j$ s and/or  $K_j$ s is one (while other segments and kinks are not relevant). Define

$$\begin{aligned} v_j &\equiv v(w_j, y_j^v)S_j + (1 - S_j)m \\ u_j &\equiv u(y_j^a, H_j)K_j + (1 - K_j)m \end{aligned} \tag{9}$$

where  $m$  is a large negative number used to represent a floor under all possible utility evaluations:  $m < \min_j \{v(w_j, y_j^v), u(y_j^a, H_j)\}$ . The utility levels before and after a change in

tax can be written as

$$\begin{aligned} u^0 &= \max_j \{v_j^0, u_j^0; j = 1, \dots, k^0\} \\ u' &= \max_j \{v_j', u_j'; j = 1, \dots, k'\} \end{aligned} \tag{10}$$

where  $v_j$  and  $u_j$  are defined in (9).<sup>8</sup>

Additional complications arise because a lump-sum transfer of CV or EV may change a person's entire budget set. The new budget set is still piecewise linear, in a way that corresponds to tax rules, but the extra transfer means that the person can buy more leisure (as well as other goods). For the end point of budget segment  $j$ ,  $H_j$  does not change, but  $y_j^a$  and virtual income  $y_j^v$  do change – by the amount of lump sum transfer. As a consequence, the optimal working hours change. Therefore, it is entirely possible that a person moves to a different segment or kink point. Let  $''$  represent variables after the person is given the EV:

$$\begin{aligned} v_j'' &= v(w_j^0, y_j^{v0} - EV)S_j'' + (1 - S_j'')m \\ u_j'' &= u(y_j^{a0} - EV, H_j^0)K_j'' + (1 - K_j'')m \end{aligned} \tag{11}$$

In (11), the values of  $S_j''$  and  $K_j''$  are functions of the unknown EV, and  $m$  is the same as in (9). A correct measure of EV must take this complication into account, as the solution to:

$$EV : u' = \max_j \{v_j'', u_j''; j = 0, \dots, k^0\} \tag{12}$$

where  $u'$  is defined in Equation (10). Because  $S_j$  and  $K_j$  depend on the unknown  $EV$ , a solution to (12) must be obtained iteratively. A similar calculation can be undertaken for CV.

In order to compare these procedures to those suggested in Hausman (1983), we first rewrite Hausman's methods in our notation. In particular, consider the expenditure function, Equation (2.4) in Hausman (1983). The calculation of EV based on such an expenditure function depends on the condition that a person must fall on a particular segment. In

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<sup>8</sup>The purpose of introducing  $m$  is to compare utility levels  $v_j$  and  $u_j$  only at the relevant segments and kinks. Equations (9) assign this large negative number  $m$  to the segments and kinks that are not relevant.

our notation, suppose  $j^0$  is the segment chosen under old tax rules, such that:

$$\begin{aligned} \text{let } u_{j^0}^0 &\equiv \max_j \{v(w_j^0, y_j^{v^0}); j = 1, \dots, k^0\}, \\ \text{then } v(w_{j^0}, y_{j^0}^v - EV) &= u' \equiv \max_j \{v(w_j', y_j^{v'}); j = 1, \dots, k'\} \end{aligned} \tag{13}$$

Equation (13) can be compared to (11)-(12), revealing two differences: first, (13) does not consider a kink point, and second, it does not consider the case that a transfer of EV may further change the chosen segment. Also, Hausman (1983) mentions that calculation of (13) by integration over the error terms' distributions is numerically difficult when the budget set is non-convex. Therefore, he uses various simplifications to calculate a good approximate solution. Because we use Monte Carlo simulations, however, these simplifications are no longer necessary. Finally, note that the simulation method based on (11)-(12) is not affected by whether the budget set is convex or non-convex.

### 3 Welfare Loss Based on Stochastic Simulations

In this section, we introduce a stochastic specification into the model of the previous section, and we provide a simulation-based method to calculate expected welfare loss for each individual.

#### 3.1 Specifying the utility function

Calculations based on Equation (10) require complete knowledge of a person's direct and indirect utility functions. Two approaches have been proposed in the literature. In the first approach, one may start with an assumed utility specification and then solve for demand functions including leisure demand (labor supply). For example, Dickens and Lundberg (1993) use a CES-type of utility function. After estimating the corresponding demand function, they can use the parameters to calculate welfare loss. In the second approach, introduced in Hausman (1981a), one starts with and estimates a specification of the demand function, such as a linear specification, and "recovers" the utility function for that demand

function by using Roy's identity. That is, using

$$\frac{\partial v(w, y)/\partial w}{\partial v(w, y)/\partial y} = h, \quad (14)$$

one can solve a differential equation to get  $v(w, y)$ . Although Slesnick (1998) points out that closed-form solutions to Equation (14) can only be obtained for a limited class of demand functions, Hausman and Newey (1995) show that a relatively simple algorithm can numerically solve the differential equation. Thus, more general functional forms could be used for labor supply.

Nevertheless, we adopt the second approach and use labor supply functions that yield closed-form solutions. In particular, we consider a linear labor supply function as in (4). Following Hausman (1981a), when  $h > 0$ , the corresponding indirect utility function is:

$$v(y_j^v, w_j) = e^{\beta w_j} \left( y_j^v + \frac{\alpha}{\beta} w_j - \frac{\alpha}{\beta^2} + \frac{s}{\beta} \right) \quad (15)$$

When a person is at a kink point, the indifference curve is not tangent to the budget set, so the utility level can only be obtained from the direct utility function. At kink point  $j$ , the direct utility function corresponding to the labor supply function in (4) is:

$$u(y_j^a, H_j) = \exp \left( \frac{\beta y_j^a + s - H_j}{H_j - \alpha/\beta} \right) \left( \frac{H_j - \alpha/\beta}{\beta} \right) \quad (16)$$

### 3.2 A stochastic specification and simulation procedures

So far, we have discussed how to obtain utility functions from empirically estimated labor supply functions, but these functions are estimated with stochastic error. Part of this error may represent the deviation between actual hours and desired hours (which econometricians do not observe). Another part may be a deliberate effort by the econometrician to represent the heterogeneity of preferences or to represent specification errors. A typical example is in a random coefficient model where a parameter of the model is assumed to be randomly distributed, and where the task of the estimation is to obtain the parameters of that random distribution.

When the stochastic errors enter into an objective function linearly, they tend to cancel out. In that case, a non-stochastic calculation might be sufficient. In our case, however,

the welfare loss is a non-linear function of the stochastic errors. Comparing a stochastically specified model and a non-stochastic one, the welfare loss calculation may be significantly different. We show this difference below.

Researchers may obtain information from a stochastic model that would be difficult or impossible to obtain from a non-stochastic model. For example, if one is interested in the probability of switching segments, or of switching from participating in the labor force to non-participation, one can acquire this information rather easily in a stochastically-specified model. That information may be very hard to obtain from a non-stochastic model.

In this section, we consider a stochastic specification based on empirically estimated labor supply equations. The stochastic errors in different specifications of labor supply have different forms. In Hausman (1981b), for example, the labor supply equation is:

$$h = \alpha w_j + (\bar{\beta} + \eta)y_j^v + z\gamma + \zeta \quad (17)$$

where  $\bar{\beta}$  is the mean value of  $\beta$ , the coefficient on virtual income  $y_j^v$ . Equation (17) has two errors:  $\eta$  represents heterogeneity of preferences, and  $\zeta$  is the error in measuring working hours. Another example is in Triest (1990), where the labor supply equation is:

$$h = \alpha w_j + \beta y_j^v + z\gamma + \eta + \zeta \quad (18)$$

In this equation,  $\eta$  is an optimization error. It is not observed by the econometrician but only observed by the individual to determine her segment or kink point. Again,  $\zeta$  serves as measurement error for working hours. At a kink point in this model, we only have error  $\eta$ , but both  $\eta$  and  $\zeta$  are present when a person is on a line segment.

For both (17) and (18), the indirect and direct utility functions are given in (15) and (16), respectively. Often, when labor supply equations are estimated, the density forms of  $\eta$  and  $\zeta$  are assumed, and the parameters of the density functions are estimated. Our simulation procedure is based on random draws of  $\eta$  and  $\zeta$  from the estimated densities. We now describe the basic procedure of this simulation method.

We start with the choice of estimated labor supply equation (17) or (18), and then for each worker we take  $I = 1,000$  draws of the error term  $\epsilon \equiv (\eta, \zeta)$ . The draws may

come from a “known” parametric distribution specified and estimated for the labor supply function. Alternatively, it may come from the empirical distribution of the residuals of the labor supply function.<sup>9</sup>

For the  $i$ th random draw,  $\epsilon_i = (\eta_i, \zeta_i)$ , we find the values of  $S_{ij}^0, S'_{ij}, K_{ij}^0$  and  $K'_{ij}$  from (5). Then from (9) and (10), we find the optimal segment or kink point in each of the two tax regimes, given  $\epsilon_i$ . This procedure applies whether the budget set is convex or non-convex.

Let  $j_i^0$  and  $j'_i$  be the optimal choice of segment or kink in the two tax regimes, given the  $i$ th draw of  $\epsilon$ , and let  $u_i^0$  be the optimal utility in the old tax regime given  $\epsilon_i$ . We can obtain the  $EV_i$ , given  $\epsilon_i$ , and  $u'_i$ , using (11) and (12). Note that the chosen segment or kink point reflects the transfer of  $EV_i$ . Solving (12) requires numerical iteration.

For any individual worker, we know the  $j_i^0$  and  $j'_i$  for the  $i$ th draw, so it is easy to obtain the tax revenues in the two tax regimes  $R_i^0$  and  $R'_i$  (and  $\Delta R_i \equiv R'_i - R_i^0$ ). One definition of deadweight loss (DWL) for this person, just for the  $i$ th drawing from the whole distribution of  $\epsilon$  is:<sup>10</sup>

$$DWL_i \equiv - (EV_i - \Delta R_i). \quad (19)$$

Naturally, the mean of all these  $DWL_i$  can be made arbitrarily close to the expectation of  $DWL$  by increasing the number of draws  $I$  (and similarly for  $\Delta R_i$ ):

$$\begin{aligned} E(DWL) &= \int DWL_i dF(\epsilon_i) \approx \frac{1}{I} \sum DWL_i, \\ E(\Delta R) &= \int \Delta R_i dF(\epsilon_i) \approx \frac{1}{I} \sum \Delta R_i. \end{aligned} \quad (20)$$

The mean square error of the simulation is proportional to  $1/I$  (see Geweke and Keane, 2001). One may also calculate the probability of moving from segment  $j^0$  to segment  $j'$ .

$$\text{Prob}(\text{segment } j^0 \Rightarrow \text{segment } j') = \frac{1}{I} \sum_{i=1}^I S_{j_i^0} \times S_{j'_i},$$

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<sup>9</sup>In principle, using this kind of simulation method, one could also introduce errors on parameters to account for standard errors of parameter estimates.

<sup>10</sup>See Moring (1971) and Auerbach (1985). Since  $EV < 0$  for a gain, we take the negative of  $(EV - \Delta R)$  in order to show a positive number for a welfare gain from removing the tax (loss from having the tax).

or the probability of moving from segment  $j^0$  to kink  $j'$ :

$$\text{Prob}(\text{segment } j^0 \Rightarrow \text{kink } j') = \frac{1}{I} \sum_{i=1}^I S_{j_i^0} \times K_{j_i'}$$

In addition, we can calculate the change of working hours. If labor supply is estimated using (18), for example, and if  $j_i^0$  and  $j_i'$  are chosen segments, for each random draw  $\epsilon_i$ , then working hours can be calculated as:

$$\begin{aligned} h_i' &= \hat{\alpha}w_{j_i'} + \hat{\beta}y_{j_i'}^v + z\hat{\gamma} + \eta_i + \zeta_i \\ h_i^0 &= \hat{\alpha}w_{j_i^0} + \hat{\beta}y_{j_i^0}^v + z\hat{\gamma} + \eta_i + \zeta_i \end{aligned}$$

The difference between  $h_i^0$  and  $h_i'$  is the change in labor supply,  $\Delta h_i$ . The average from all random draws provides a number that converges to the expected value of the change in working hours:

$$E(\Delta h) = \int \Delta h_i dF(\epsilon_i) \approx \frac{1}{I} \sum_{i=1}^I (h_i' - h_i^0)$$

All the estimated factors are calculated conditional on the wage rate  $w$ , virtual income  $y^v$ , and other socio-demographic variables  $z$ . We can then integrate over these factors to get the population average. In practice, we just repeat the previous process for each successive individual in the sample and take the average of all individuals (applying sample weights, if available).

## 4 Examples

This section provides three illustrations of the procedures just described.

### **Example 1: The welfare loss of taxation for a married woman.**

As in Hausman (1981b), we consider a married woman whose wage rate is \$4.15 an hour, and whose husband is earning a fixed \$10,000 (both in 1975 dollars). She works full time (1,925 hours per year) and files a joint return. The tax regime she faces is shown in Table 2, the federal tax brackets of 1975 (the sample year for Hausman, 1981b). The “new tax” regime is no tax at all. We choose this example for several reasons: First, this



example is considered in Hausman (1981b), where he estimates labor supply using data from the Panel Study of Income Dynamics (PSID) and applies the estimates to calculate welfare loss. Second, the standard deduction for a married couple filing a joint return in 1975 creates a non-convex budget set.<sup>11</sup> Third, this example has only one person, so it can be used to illustrate how the stochastic specification yields various possibilities for the chosen segment or kink point. It is also easy to compare the results with a traditional welfare cost calculation such as the Harberger triangle.

The estimated hours equation is given by

$$h = \alpha w_j + \beta(y_j^v - FC) + z\gamma + \zeta \quad (21)$$

where  $h$  is in thousands of hours,  $y_j^v$  is in thousands of dollars,  $w_j$  is in dollars per hour, and  $FC$  is the fixed cost of working (1.26 thousand dollars per year). Hausman estimates that  $\hat{\alpha} = 0.4608$ , with a standard error .106, and  $\beta$  is a random coefficient representing variations in taste, with a truncated normal distribution (i.e.,  $\beta = \beta_k$  where  $\beta_k \sim N(2.0216, 0.5262^2)$  and  $\beta_k < 0$ ). The mean of this truncated normal is  $E(\beta) = -.123$ . Also,  $\zeta \sim N(0, 0.2801^2)$ . Then we obtain  $z\gamma = 0.2595$ , from the equation

$$z\gamma = h - \hat{\alpha}w_j + E(\beta)(y_j^v - \hat{FC}) \quad (22)$$

where  $h = 1.925$  thousand hours, and  $j$  is the chosen segment. At the means of the parameters and of the error distribution, the marginal tax rate for this woman is 28%.

The random draws represent both the preference heterogeneity and measurement errors of working hours among all those married women who have exactly the same observed set of characteristics as this woman (working full time at 1,925 hours per year, filing joint tax returns, having non-labor income of \$10,000, and earning \$4.15 per hour). Therefore, our simulation results can be said to estimate welfare effects for a subset of the population that has the observed characteristics of the woman in this example.

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<sup>11</sup>In 1975, for income below \$11,800, the standard deduction for a married couple filing a joint return was \$1,900. Then, when total income is between \$11,800 and \$16,250, the true marginal tax rate falls because the standard deduction is \$1,900 plus 16% of the income that exceeds \$11,800.

Table 3 first shows that the DWL estimate is \$471 per year for this person using a simple Harberger triangle approximation.<sup>12</sup> Our stochastic specification not only yields an expected DWL that is substantially larger (\$1,401), but it also provides an estimate of the standard error for DWL (\$716). The welfare loss based on the Harberger triangle is about 26.0% of tax revenue for this woman, but the expected DWL over expected revenue is 75.5%. The estimate by Hausman is in between, at 58.1% of tax revenue. All of these numbers are large because of the large compensated elasticity from the Hausman estimates.

In the stochastic specification, this person has probabilities of being on different segments or kink points, as shown in Table 4. The probability that this working woman chooses the segment with the 28% tax rate (segment 5) is 47.2%. The sum of the probabilities of choosing kink points is 13.1%. Generally speaking, segments or kink points closer to segment 5 have higher probabilities, with two exceptions. First, kink points 1 and 2 have zero probabilities, since the budget set is non-convex around these two kink points. Second, the probability of being at kink point 0 (not working) is positive (.032) because of the fixed cost of working in this model. Not working yields highest utility for some random draws where the optimal working hours are relatively small. In the new tax regime with no tax at all, the person has 96% probability of working, and 4% chance of not working.

The simple Harberger calculation is possible when the individual switches from an observed segment of a non-linear budget constraint to a known segment after the tax change (such as the zero tax rate in the example above). With a switch from one non-linear tax system to another, however, the simulation of the new tax regime may place the individual on a number of possible segments or kinks. Thus, no direct Harberger calculation is feasible for our next two examples, the Tax Reform Act of 1986 and the Bush tax cut of 2001. Instead, for comparison, we use the results of our simulations to calculate the income-weighted average of marginal tax rates before and after reform, and use those to calculate a Harberger-type DWL before and after reform – a calculation that would not be possible

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<sup>12</sup>For this purpose, we use Equation (4) in Browning (1987) for DWL as a function of the compensated labor supply elasticity, the fixed gross wage rate, labor hours, and the marginal tax rate.

without our model.

**Example 2: The Tax Reform Act of 1986 for married women.**

The parameter estimates used in this example are from Triest (1990), and they are applied to a cross-section of married women extracted from the 1983 PSID.<sup>13</sup> Table 5 lists the tax rates and income brackets for both tax regimes. We assume all individuals take the standard deduction and file jointly.<sup>14</sup>

The basic labor supply function in Triest (1990) appears above as Equation (18). In our data set, we observe each woman’s working hours and wage rate in 1983. The non-labor income is calculated from the husband’s income and other family income. We can therefore derive the budget constraint for each woman, and determine her chosen segment or kink point. Triest assumes no fixed cost of working. If the chosen segment is  $j$ , with observed net wage  $w_j$  and virtual income  $y_j^v$ , then we can use Triest’s parameter estimates  $\hat{\alpha} = .235$  and  $\hat{\beta} = -.022$  to calculate for each observation:

$$z\gamma = h - \hat{\alpha}w_j - \hat{\beta}y_j^v$$

The random errors  $\eta_i$  and  $\zeta_i$  are distributed as  $\eta \sim N(0, 0.67^2)$  and  $\zeta \sim N(0, 0.77^2)$ . Since the mean of the observed workings hours is 1.074 thousand hours in a year, the standard deviations of the random errors  $\eta$  (0.67 thousand hours) and  $\zeta$  (0.77 thousand hours) represent substantial variations in working hours.

For each individual, we take 1,000 random draws from the joint distribution of  $(\eta, \zeta)$ .<sup>15</sup>

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<sup>13</sup>We extract data from the 1983 PSID following the procedures described in Triest (1990), but some differences appear between our data and the Triest data. Our data set has 1,136 observations, while Triest has only 978 observations, but the summary statistics for our data and the Triest data are very close. One possible explanation is that the new version of the PSID has fewer missing values.

<sup>14</sup>We model only the reduced rates of the 1986 Act, not the redefinition of taxable income to broaden the tax base, so we probably overestimate tax rate reduction for women whose loss of tax deductions push them back into higher brackets.

<sup>15</sup>The errors are large enough, however, that a few extreme drawings yield implausible results. To avoid unreasonably large EV, we constrain the absolute value of EV to be smaller than before-tax total family income. The EV hits this constraint for .43% of all individuals at all random draws.

We first calculate EV, working hours, and taxes for each random draw, and then we average over 1,000 random draws to get this individual's EV, working hours and taxes. Each random draw can be considered to represent a different person with the same observed variables as the current individual. Different random draws yield different initial working hours, although averaging over 1,000 random draws yields working hours very close to the observed working hours of the individual. Together, the 1,000 random draws represent a subset of population that shares the same observed variables as this individual. By averaging over all random draws, we get the average welfare effect for that subset of the population. Since we have a representative sample of married women, averaging over all 1,136 individuals yields estimates for the population of married women.<sup>16</sup>

Table 6 shows the change in tax revenue, the change in working hours, the equivalent variation, and the net welfare gain from this tax reform for the population represented by our sample. Interestingly, even though the tax reform generally reduces tax rates, it slightly reduces average working hours. This “backward bending” labor supply behavior indicates that the income effect dominates. The income-weighted average of marginal tax rates is reduced from 33.9% to 28.7%.<sup>17</sup> Tax revenue in the new regime falls by 37.7%. Since marginal tax rates are reduced more for initial brackets, the percentage fall is larger for tax revenue than for the average of all marginal tax rates.

Since  $EV$  on average is negative, the utility level in the new tax regime is higher. Because of the large standard deviations of the random errors  $(\eta, \zeta)$ , the EV in Table 6 also has a large standard deviation. The expected net welfare effect is \$1,790 per family, or 34.9% of old tax revenue.

If Triest's estimates are evaluated at the mean wage and mean marginal tax rate of his sample, the compensated labor supply elasticity is .686 for a full time worker.<sup>18</sup> This

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<sup>16</sup>We did not consider the fact that the PSID oversamples minorities.

<sup>17</sup>This summary statistic is used only in the simple Harberger formula, and it reflects the fact that a higher income person contributes more to tax revenue (and aggregate DWL) than a lower income person.

<sup>18</sup>To compare the Triest and Hausman estimates, we can apply the Triest estimates to the woman in the first example above. Her compensated elasticity would then be only .430 instead of 1.084.

elasticity could be used in a simple Harberger formula to calculate DWL of the old tax, compared to no tax system. When the new marginal tax rate is unknown, however, the new DWL is not so simple. Our method is useful to predict the new marginal tax rate of each person. When we employ the predicted rates in the Harberger formula, before and after reform, Table 6 shows that the change in DWL is only 5.7% of old tax revenue.

**Example 3: The tax change of 2001 for married women.**

In this example, we apply the parameter estimates of Gan and Stahl (2002) to the Economic Growth and Tax Reconciliation Act of 2001 (the Bush tax cut). Their model assumes measurement error in non-labor income  $Y^n$ ,

$$Y^n = Y^{n*} - \eta \tag{23}$$

where  $Y^n$  is observed non-labor income, and  $Y^{n*}$  is the true non-labor income (known to the individual herself but not to the econometrician). The measurement error,  $\eta \sim N(0, \sigma_\eta^2)$ , in non-labor income produces a random budget set: the end points of each segment are random variables. Such a model is not subject to the Heckman critique. In fact, it conforms to the insights in Heckman (1983). Gan and Stahl show that such a model yields very different parameter estimates and performs better statistically. Given the error in  $Y^n$ , which affects  $y_j^v$ , the estimated labor supply equation is:

$$h = \alpha w_j + \beta y_j^v + z\gamma + \zeta \tag{24}$$

Here, we use the same data set as in Gan and Stahl (2002): married women in the Current Population Survey (CPS) of March 2001 between the ages of 25 and 55. This data set has 16,829 observations. The parameter estimates and summary statistics are listed in Table 7. One interesting aspect of the parameter estimates is the large standard deviation of the measurement error in nonlabor income ( $\sigma_\eta = 1.33$ , where income is in thousands of dollars). Another interesting aspect is that the income elasticity is very small (the compensated wage elasticity and the uncompensated wage elasticity are both .012).<sup>19</sup>

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<sup>19</sup>We recognize that these low elasticity estimates may be controversial. The purpose here is not to endorse

Their estimates are used here to evaluate the effect of the Economic Growth and Tax Relief Reconciliation Act of 2001, or simply the “Bush tax cut.” We consider only the changes in marginal tax rates, and since the changes are phased in, we use only the rates after 2006 when all changes are fully implemented. Table 8 compares the regimes before and after the Bush tax cut. Again, we assume that all of these married women take the standard deduction and file jointly.

We take 50 random draws of the error in equation (24),  $\zeta \sim N(0, 0.65^2)$ , where hours are in thousands, and for each  $\zeta$ , we take 50 random draws of the error in Equation (23),  $\eta \sim N(0, 1.33^2)$ . Note that each different  $\eta$  yields a different budget constraint.

Averaging the 16,829 individuals, Table 9 shows the change in tax revenue, the change in working hours, the equivalent variation, and the welfare gain from this tax reform. The net result is almost no change in expected working hours. The tax revenue is lowered by an average of \$843 per person, and the expected welfare gain is \$357 per person. This number has a large error, but the point estimate is 3.5% of revenue in the old tax regime. In contrast, using the point estimate of the compensated labor supply elasticity (.012) yields a change in Harberger DWL that is \$.40, or zero percent of old tax revenue.

## 5 Conclusion

The calculation of welfare loss suggested in this paper depends on estimates of labor supply. An ongoing debate concerns how to estimate the labor supply function under piecewise-linear budget constraints, but recent estimates are able to address the Heckman concern within Hausman’s framework. The first contribution of this paper, relative to existing literature, is to calculate the welfare cost of labor taxes using labor supply estimates that address this concern. Second, we allow each individual to move from any kink or linear segment of the original budget constraint to any kink or linear segment of the new budget

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their method, or even to repeat discussion of it, but just to show that the simulation method described here is applicable to any estimated model. This example also is useful to show that the expected welfare effect may still be positive even when the point estimate of the labor supply elasticity is near zero.

constraint. Third, we account for the fact that the equivalent variation is a transfer that itself would change the choice of each individual. Fourth, the method we propose is relatively easy to implement and to calculate. Finally, our method uses Monte Carlo simulation in order to employ the entire estimated distribution of each error term. Thus, we need not assume that the person chooses one particular point, which would ignore the fact that labor supply is estimated with error.

Using this new method, we calculate the welfare effect of three illustrative labor tax changes. First, we employ the example of Hausman (1981b) with one married woman who works full time. We show that the welfare effect of eliminating the tax system in this example using the stochastic evaluation is significantly larger than when using a simple Harberger triangle approximation. Second, we employ Triest's (1990) estimates to consider 1,136 married women in the 1983 PSID data. In this case, we show that the mean welfare gain from the tax rate reduction of Tax Reform Act of 1986 is 34.9% of the original tax revenue. In the third case, we apply the estimates from Gan and Stahl (2002) to a recent data set from the CPS (March, 2001) to investigate the welfare gains of the Bush tax cut of 2001. We find almost no change in working hours for these married women. Even though the point estimate of the labor supply elasticity is near zero, the use of all error distributions yields an expected welfare gain that is 3.5% of the old tax revenue.

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Table 1: Summary of budget segments

	Budget segment 1	Budget segment $j > 1$
Function for after-tax income $y^a$	$y^a = y^n + w(1 - t_1)h$	$y^a = y_{j-1}^a + w(1 - t_j)(h - H_{j-1})$
Kink points for income $y^a$	$y_0^a = y^n$	$y_j^a = y_{j-1}^a + w(1 - t_j)(H_j - H_{j-1})$ $= y^n + \sum_{k=2}^j (1 - t_k)(Y_k - Y_{k-1})$
Kink points for working hours $h$	$H_0 = 0$ $H_1 = (Y_1 - Y^n)/w$	$H_j = (Y_j - Y^n)/w$
Virtual income $y^v$	$y_1^v = y^n$	$y_j^v = y_{j-1}^a - w(1 - t_j)H_{j-1}$ $= Y^n(2 - t_1 - t_j) - Y_j(1 - t_j)$ $+ \sum_{k=1}^j (1 - t_k)(Y_k - Y_{k-1})$

We define  $t_1$  as the first tax rate applied to labor income of this person (after taxation of non-labor income). Using the person's non-labor income,  $t_j$  and  $Y_j$  are also individual-specific, but can be found from the tax table.

Table 2: Tax schedule for example 1  
(married woman filing a joint 1975 tax return)

Income	Rates
0 – \$1,900	.0
\$1,900 – \$2,900	.14
\$2,900 – \$5,900	.16
\$5,900 – \$9,900	.19
\$9,900 – \$11,800	.22
\$11,800 – \$13,900	.185
\$13,900 – \$16,250	.21
\$16,250 – \$17,900	.25
\$17,900 – \$21,900	.28
\$21,900 – \$25,900	.32
\$25,900 – \$29,900	.36
\$29,900 – \$33,900	.39
\$33,900 – \$37,900	.42
\$37,900 – \$41,900	.45
\$41,900 – \$45,900	.48
\$45,900 +	.50

Table 3: Welfare effect in example 1 with the 1975 tax system

before-tax wage = 4.15; before-tax non-labor income = \$10,000

<u>Deterministic evaluation</u>	
Working hours	1,925
Marginal tax rate	.28
Compensated elasticity	1.084
Tax revenue	\$1,815
Harberger DWL	\$471
DWL as % of tax revenue	26.0%
<u>Means using stochastic evaluation<sup>a</sup></u>	
Old working hours (with 1975 taxes)	2,019 (753)
New working hours (with no taxes)	2,386 (943)
Change in tax revenue	\$1856 (\$187)
EV <sup>b</sup>	-\$3,257 (\$895)
DWL <sup>c</sup>	\$1,401 (\$716)
E(DWL) as % of tax revenue, $E(R^0)$	75.5%

a. Standard errors are in parentheses.

b.  $EV < 0$  means a gain from removal of the tax.

c. The DWL is  $-(EV - \Delta R)$ .

Table 4: The probability that the working woman in example 1 is on each initial budget segment or kink

Number	Segments		Kink points
	Marginal tax rate	Probabilities	Probabilities
0			.032
1	.22 <sup>a</sup>	.020	.0
2	.185	.042	.0
3	.21	.111	.021
4	.25	.128	.039
5	.28 <sup>b</sup>	.472	.039
6	.32	.086	.0

- a. This woman has \$10,000 of non-labor income, so the first tax rate applied to any of her labor income is 22% (even though that is the fifth bracket of the 1975 tax system shown in Table 2).
- b. Using only the mean of the distribution, this woman would be on segment 5 in the old tax regime.

Table 5: Tax schedules for example 2, the Tax Reform Act of 1986  
(married women filing joint 1983 tax returns)

Old tax regime		New tax regime	
Income	Rates	Income	Rates
0 – \$3,400	0	0 – \$3,000	.11
\$3,400 – \$5,500	.11	\$3,000 – \$28,000	.15
\$5,500 – \$7,600	.13	\$28,000 – \$45,000	.28
\$7,600 – \$11,900	.15	\$45,000 – \$90,000	.35
\$11,900 – \$16,000	.17	\$90,000 –	.385
\$16,000 – \$20,200	.19		
\$20,200 – \$24,600	.23		
\$24,600 – \$29,900	.26		
\$29,900 – \$35,200	.30		
\$35,200 – \$45,800	.35		
\$45,800 – \$60,000	.40		
\$60,000 – \$85,600	.44		
\$85,600 – \$109,000	.48		
\$109,000 +	.50		

Table 6: Welfare effect in example 2, the Tax Reform Act of 1986

	Old tax regime	New tax regime	Difference
Tax revenue	\$5,132 (\$5,566) <sup>a</sup>	\$3,196 (\$3,693)	-\$1,936 (\$1,915)
Working hours	1,230 (711)	1,170 (715)	-60 (410)
Marginal Tax Rates	33.9% (26.1%)	28.7% (25.2%)	5.2% (4.1%)
EV <sup>b</sup>			-\$3,725 (\$3,706)
Welfare effect <sup>c</sup>			\$1,790 (\$3,057)
Welfare effect as a % of old tax revenue			34.9%
% with negative welfare effect			18.6%
Harberger DWL <sup>d</sup>	\$874	\$581	\$293
as % of tax revenue	17.0%	11.3%	5.7%

a. Standard errors are in parentheses.

b.  $EV < 0$  means a gain.

c. The welfare gain is  $-(EV - \Delta R)$ .

d. Evaluated at the mean wage and mean marginal tax rate for a full time worker.

Table 7: Estimation results and summary statistics from the CPS (March, 2001)<sup>a</sup>

	Coefficient estimates	Summary statistics
Working hours (in 1,000 hours per year)		1.353 (.954) <sup>b</sup>
Constant	-.218 (.027)	
Wage (in \$ per hour)	.00012 (0.000034)	16.16 (25.5)
Non-labor income (in \$1000 per year)	-0.00176 (.00013)	58.59 (59.3)
# kids ages 0-5	-.214 (.008)	.369 (.675)
# kids ages 6-18	-.088 (.0046)	.880 (1.06)
Age minus 40	-.012 (.0036)	.36 (2.65)
Unemployment rate (%)	-.011 (.0037)	4.01 (1.61)
Education (in years)	.034 (.0020)	10.14 (3.14)
# of observations		16,829
% labor participation		75.2%
Std dev of measurement error ( $\sigma_\eta$ )	1.33 (.12)	
Std dev of optimization error ( $\sigma_\zeta$ )	.65 (.0034)	
Elasticities (evaluated at means)		
Uncompensated	.012	
Compensated	.012	

Source: Gan and Stahl (2002).

a. Married women between ages 25 and 55.

b. Standard errors are in parentheses.



Table 8: Tax schedules for example 3, the Bush tax cuts of 2001  
(married women filing joint 2001 tax returns)

Old tax regime		New tax regime <sup>a</sup>	
Income	Rates	Income	Rates
0 – \$7,600	0	0 – \$7,600	0
\$7,601 – \$51,450	.15	\$7,601 – \$13,600	.10
\$51,451 – \$113,550	.28	\$13,601 – \$51,450	.15
\$113,551 – \$169,050	.31	\$51,451 – \$113,550	.25
\$169,051 – \$295,950	.36	\$113,551 – \$169,050	.28
\$295,951 +	.396	\$169,051 – \$295,950	.33
		\$295,951 +	.35

a. The new tax regime is the Bush tax cut, the Economic Growth and Tax Reconciliation Act of 2001.

Table 9: Welfare effect in example 3, the 2001 Bush tax cuts

	Old tax regime	New tax regime	Difference
Tax revenue	\$10,061 (\$16,610) <sup>a</sup>	\$9,218 (\$15,050)	-\$843 (\$1,568)
Working hours	1,390 (340)	1,381 (344)	-9 (113)
Marginal Tax Rates	27.5% (33.2%)	27.3% (33.3%)	-.2% (1.1%)
EV <sup>b</sup>			-\$1,200 (\$3,570)
Welfare effect <sup>c</sup>			\$357 (\$3,054)
Welfare effect as a % of old tax revenue			3.5%
% with negative welfare effect			31.6%
Harberger DWL <sup>d</sup>	\$19.50	\$19.10	\$.40
as % of old tax revenue	.19%	.19%	0.0%

a. Standard errors are in parentheses.

b.  $EV < 0$  means a gain.

c. The welfare gain is  $-(EV - \Delta R)$ .

d. Evaluated at mean wage and mean marginal tax rate for a full time worker.