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THE ROLE OF OVERLAPPING-GENERATIONS MODELS IN MONETARY ECONOMICS

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Abstract

The main arguments of this paper can be summarized as follows. (1) The overlapping-generations (OG) structure provides a useful framework for the analysis of macroeconomic issues involving intertemporal allocation. (2) As a "model of money," the basic OG setup--which excludes cash-in-advance or money-in-the-utility-function (MIUF) features--is inadequate and misleading because it neglects the medium-of-exchange property that is the distinguishing characteristic of money. (3) That this neglect obtains is verified by noting that, in contrast with an axiomatic "traditional presumption," the same aggregate leisure/consumption bundles are available in equilibria in which "money" is valued and valueless. (4) That the model may be misleading is demonstrated by examples in which three of its most striking properties--tenuousness of monetary equilibrium, optimality of zero money growth, and price level invariance to open-market exchanges--disappear in the presence of modifications designed to reflect the medium-of-exchange property. (5) There is no compelling reason why cash-in-advance, MIUF, or other appendages should not be used in conjunction with the OG framework.

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I. Introduction

A substantial and sophisticated body of research has grown up, over the past few years, in which the overlapping-generations framework of Samuelson (1958) has served as the analytical basis for models designed to explain monetary phenomena. A far-from complete list of examples might include papers by Bryant (1981), Bryant and Wallace (1979) (1980a), Kareken and Wallace (1981), Martins (1980), Peled (1982), Sargent and Wallace (1981), Wallace (1980) (1981), and Walsh (1982). In fact, the profoundly influential analysis of Lucas (1972) falls into this category.

Proponents of the overlapping generations or "OG" approach have argued vigorously not only that it is useful for monetary analysis, but also that other existing approaches are highly unsatisfactory--see, for example, Kareken and Wallace (1980), Bryant (1980), and Cass and Shell (1980). In Wallace's words, the OG theory "includes what's essential for a good theory of money. Economists should recognize it as the best available, and the Fed should not ignore its policy implications" (1977, p. 2).

Other leading theorists have disputed these suggestions. Most prominently, Tobin (1980) has expressed disbelief that "the overlapping generations model is the key to the theory of money." Indeed, he suggests, its analytical parable "should not be taken seriously as an explanation of the existence of money in human society" (1980, p.83). More recently, Hahn (1981, p. 11) has remarked "how very bad these models are." And, of course, crucial implications of prominent OG models conflict sharply with those of other types. For example, Wallace (1980) (1981) finds that the method of financing government deficits has no impact on subsequent inflation rates, while Friedman (1974) and Barro (1981) suggest that the method of finance is of central importance.

The purpose of the present paper is to examine the issues underlying this controversy and develop tentative conclusions regarding the appropriate role of the OG framework in macro and monetary economics. We begin in Section II with a discussion of some general features of OG models and an exposition of two examples, both non-monetary in nature. Then in Section III the basic monetary OG model, as described by Wallace (1980), is discussed together with some important implications. The extent to which this model reflects the medium-of-exchange role of money is considered next, in Section IV, while related methodological controversies are taken up in Section V. A brief discussion of some more elaborate models is included in Section VI, and a conclusion appears in Section VII.

II. The Overlapping Generations Framework

The distinguishing characteristic of an OG model is that agents in an everlasting economy have finite life spans of two or more periods. These agents change as they age in the sense that behavior during youth becomes sunk and choices are made with regard to consumption, leisure, etc., in fewer remaining periods of life. Also, agents' endowments and wealth may change as they grow older. At any point in time, then, the economy includes agents of different ages who have different perspectives on the accumulation of wealth, even if all agents are alike "before birth"--if the lifetime utility functions and endowments are the same for all generations. This feature provides a cogent rationale for some kinds of economic activity that are hard to justify in other types of models, without, that is, introducing the complication of agents that are heterogeneous before birth. Thus OG models help to avoid analytical difficulties that may be of no importance for various important topics in monetary or macro economics. They do so, moreover, in a context in which some agents--as yet unborn--cannot possibly meet in a marketplace with those currently alive. This rules out certain types of exchanges and thereby introduces an explicit and understandable "friction," some type of which is widely considered to be necessary for providing a sound foundation for the analysis of monetary phenomena.

In addition, OG models are typically characterized by the assumption of perfect foresight or rational expectations, depending on whether the analysis is deterministic or stochastic. While this characteristic is not inevitable--one can imagine an OG model in which agents have static or adaptive expectations--there seems to be something of a natural relationship: the motivation for models with the virtues described above arises more clearly in the presence of the severe discipline imposed by the assumption of rationality. In any event, perfect foresight or rational expectations will be presumed throughout the following discussion.

A large fraction of the OG literature employs the assumption that agents live for but two periods, with a new generation born each period. Clearly, it is not possible to explain high-frequency empirical phenomena--with a periodicity of (say) 10 years or less--with such models. They can be used, nevertheless, to acquire understanding of various types of dynamic behavior of macroeconomic variables. Furthermore, some qualitative properties of two-period OG models will carry over to versions in which a larger number of phases of life are recognized. For most of the issues discussed below, it does not matter how many periods an agent lives, so for simplicity a two-period lifetime will be used in all that follows.

At this point it should be useful to describe a sample OG model, so as to fix some notation and provide a warmup for subsequent discussion. Since the more controversial aspects of OG analysis concern monetary applications, it will be best to begin with a model involving only real magnitudes. A notable example is provided by the model of Diamond (1965), a version of which we now describe.

Let c_t be consumption during period t (t = 1,2,...) by a representative young person and let x_t be consumption during t by an old person.[1/] When young, a person born in t seeks to maximize $u(c_t, x_{t+1})$, where $u(.,.)$ is a utility function that yields unique differentiable demand functions for c_t and x_{t+1}. It is assumed that c_t and x_{t+1} are normal goods and that the marginal rate of substitution u_1/u_2 approaches zero (infinity) as c_1/c_2 approaches infinity (zero). This assures that positive quantities of c_t and x_{t+1} will be chosen whenever possible. Each young person is endowed with one unit of labor, each old person with none. When old, a person's only object is to consume as much as possible.

Production is carried out by old persons according to

$$(1) \qquad y_t = f(n_t,k_t),$$

where n_t and k_t are the labor employed and capital held in t by a representative old person, with y_t the resulting output. The production function f(.,.) has positive but diminishing marginal products, constant returns to scale, and satisfies the Inada conditions (which assure that positive quantities of n_t and k_{t+1} will be chosen). Capital, which is stored output, depreciates at the rate δ.

In this setting, the consumption of an old person in period t is

$$(2) \qquad x_t = f(n_t,k_t) - w_t n_t + (1-\delta)k_t,$$

where w_t is the real wage. Since the only choice variable is n_t, labor is employed so that

$$(3) \qquad f_1(n_t,k_t) - w_t = 0.$$

Given the degree-one homogeneity of f(.,.), then, $f(n_t,k_t) = n_t f_1(n_t,k_t) + k_t f_2(n_t,k_t)$ so that

$$(4) \qquad x_t = k_t f_2(n_t,k_t) + (1-\delta)k_t.$$

At the same time, each young person seeks to maximize $u(c_t, x_{t+1})$ subject to

(5) $$w_t = c_t + x_{t+1}/(1 + \rho_t),$$

where ρ_t is the competitive interest rate on loans[2/] from t to t+1 and where the left-hand side reflects the inelastic supply of one unit of labor. To determine the optimum choices, we form the Lagrangian function

$$L = u(c_t, x_{t+1}) + \lambda_t[w_t - c_t - x_{t+1}/(1 + \rho_t)],$$

calculate first-order conditions,[3/] eliminate λ_t, and obtain

(6) $$u_1(c_t, x_{t+1}) = u_2(c_t, x_{t+1}) [1 + \rho_t].$$

Demand functions for c_t and x_{t+1} are implied by (5) and (6); the former we write as

(7) $$c_t = c(w_t, \rho_t).$$

Market equilibrium requires $n_t = 1$ (and also $n_{t+1} = 1$) so w_t and n_t are determined as functions of the predetermined variable k_t by relationships (3) and (4). Equilibrium also requires that the rate of interest satisfy

(8) $$\rho_t = f_2(1, k_{t+1}) - \delta$$

and that consumption plus investment equals output, viz.,

(9) $$c_t + x_t + k_{t+1} - (1 - \delta)k_t = f(1, k_t).$$

Thus equilibrium values of c_t, ρ_t, and k_{t+1} are determined by (7), (8), and (9), again as functions of the state variable k_t. Given an initial per-capita[4/] stock of capital, k_1, the model in principle defines time paths for t = 1,2,... for all of the system's variables.

While the foregoing describes an extremely simple model, it has some substantial analytical merits. In particular, it is an internally consistent model in which optimizing agents make non-trivial intertemporal allocation (saving) decisions, in which expectations are correct, and in which markets clear. Since utility and production functions are clearly specified, it is in principle possible to use the model to obtain answers to questions in welfare economics--answers expressed, it should be emphasized, in terms of the utility of individual agents.[5/]

Before turning to monetary applications, let us briefly consider another simple model involving only real magnitudes.[6/] In particular, consider the model of Calvo (1978), in which the second factor of production is not capital but land: i.e., a commodity that is non-augmentable, non-depreciable, and non-consumable, but which is useful in production. Output, moreover, is nonstorable.

In this model preferences are as above and (2) again describes production possibilities but with k_t now denoting the number of land units held during t by a representative old person. The per-capita stock of land is fixed at k (say), but k_{t+1} is nevertheless a choice variable to each individual young person in period t. Claims to land parcels, which are transferred after production takes place, are bought from old persons (who supply them inelastically) at the market-determined (real) price q_t. As before, old persons hire labor from the young at the real wage rate w_t.

In this economy, consumption of the old during t is

$$(10) \qquad x_t = f(n_t, k_t) - w_t n_t + q_t k_t.$$

Again n_t is an old agent's only choice variable so (3) holds as before and with constant returns to scale we have

$$x_t = k_t[f_2(n_t,k_t) + q_t]. \tag{11}$$

With $\rho_t = f_2(n_{t+1},k_{t+1})$ taken parametrically as before, the relevant Lagrangian expression for a young person is now

$$L = u(c_t,x_{t+1}) + \lambda_t[w_t - c_t - x_{t+1}q_t/(\rho_t + q_{t+1})]. \tag{12}$$

After elimination of λ_t, the first-order conditions imply

$$\frac{u_2(c_t,x_{t+1})}{u_1(c_t,x_{t+1})} = \frac{q_t}{q_{t+1} + \rho_t} \tag{13}$$

and the budget constraint is

$$w_t - c_t - x_{t+1}\, q_t/(q_{t+1} + \rho_t) = 0. \tag{14}$$

These imply demand functions for c_t and x_{t+1}, with those choices dependent upon w_t, q_t, ρ_t, and the expectation of q_{t+1}.

Market equilibrium in this model requires $k_t = k_{t+1} = k$ as well as $n_t = n_{t+1} = 1$. Thus (3) and the definition of ρ_t imply that w_t and ρ_t are constants, say w and ρ. The other equilibrium condition is that the per-capita supply of land, k, equals the demand, which from (11) is

$$k_{t+1} = x_{t+1}/(q_{t+1} + \rho). \tag{15}$$

Equations (13), (14), and (15) then determine q_t, c_t and x_{t+1} as functions of q_{t+1}. Thus we write

$$(16) \qquad q_t = \phi(q_{t+1}),$$

an equation that is analogous to the "pseudo reduced form" expressions that appear in linear stochastic rational expectations models.[7] Such an equation need not be treated as an ordinary difference equation in q_t, for q_{t+1} is in fact the expectation formed at t of the price of land that will obtain in t+1, an expectation that happens to equal the realization only because of the absence of stochastic elements in the present model. In other words, equation (16) does not describe the market determination of q_{t+1} given q_t, but instead the market determination of q_t given the period-t expectation of q_{t+1}.

In light of this discussion, it is natural to look for a solution expression that relates q_t to the relevant state variables. The per-capita stock of land in existence, k_t, is a state variable just as in the previous model but now its value is constant through time. Consequently, the solution for q_t is also a constant, say $q_t = q$. Accordingly, the expectation of q_{t+1} equals q also, and we evaluate q in terms of the model's basic parameters by solving $q = \phi(q)$. Under our assumptions there will be a single solution to this equation.

Nevertheless, the solution path for q that we have just described may not be unique: for some admissible u(.,.) and f(.,.) specifications there will be time paths other than $q_t = q$ (t = 1,2,...) that satisfy (13)-(15).[8] Indeed, Calvo's development of this model was designed to show that it is possible to have a multiplicity of rational expectations solutions in a non-monetary model with optimizing agents. This multiplicity arises, however, because Calvo treats (16) as an ordinary difference equation.[9/] In McCallum (1981) it is argued at length that such a treatment amounts to assuming that agents use extraneous state variables in forming expectations of future values, thereby inducing effects that occur only because they are arbitrarily expected to occur. In a wide class of models, however, there is only one solution that excludes such "bootstrap" or "bubble" effects. To avoid the analytical paralysis that would result from the admission of an infinity of solutions, therefore, it is often desirable to focus attention on the single "minimal state variable" solution that is free of bootstrap effects.[10/] In the present case that happens to be the stationary solution $q_t = q$ but, in general, minimal-state-variable solutions will not be constants. Focussing attention on minimal-state-variable solutions conforms to the practice and recommendation of Wallace (1980, p.55), so it seems doubly appropriate for the discussion of monetary OG models of the type championed by Wallace.[11/]

III. The Basic Overlapping-Generations Model of Money

We now turn to the more controversial class of OG models, that in which there is a non-productive and "intrinsically useless" entity identified as fiat money. Since Neil Wallace has been an influential leader in the development and application of such models, we shall begin with an exposition of the basic setup in his 1980 paper. Substantively, this model is modified only by taking population growth to be zero, but our notation will differ considerably from Wallace's.

In this model, as in those of Section II, young persons endowed with one unit of labor seek to maximize $u(c_t, x_{t+1})$. Now, however, labor is the only input to the productive process, which is characterized by

$$(17) \qquad y_t = f(n_t).$$

Consequently, production will be effected by the young and there will be no loss in proceeding as if their endowments were directly in the form of output, in the amount $y = f(1)$ units per young person. This output is in general storable, but depreciates at the rate δ. More specifically, $\gamma = 1/(1 + \delta)$ units are available in period t+1 for each unit stored in t. By letting $\delta \rightarrow \infty$, we can then represent a completely perishable commodity. We permit negative values for δ, reflecting autonomous reproduction of output, but require that $\delta > -1$.

Let k_t denote the amount of output stored in t by a representative young person. Output neither consumed nor stored can be traded for money; the money price of a unit of output is P_t. The real quantity of money held by a young person at the end of t is denoted m_t.

Old persons have zero endowments but each receives, in period t, a lump-sum monetary transfer payment from the government of real magnitude v_t. Consequently, each old person is able to purchase output in the amount $v_t + m_{t-1}P_{t-1}/P_t$ in period t; per capita consumption by the old is

(18) $$x_t = \gamma k_{t-1} + v_t + m_{t-1}P_{t-1}/P_t.$$

Given the present setup, the choice problem of a young person is to select c_t, x_{t+1}, k_t and m_t to maximize $u(c_t,x_{t+1})$ subject to

(19) $$c_t + k_t + m_t = y$$

and to a version of (18) applicable to t+1. Our assumptions on $u(.,.)$ guarantee that c_t will be strictly positive, but non-negativity constraints must be imposed for x_{t+1}, k_t, and m_t. Consequently, the first-order conditions obtained from the Lagrangian expression

(20) $$L = u(c_t,x_{t+1}) + \lambda_{1t}[y-c_t-k_t-m_t] + \lambda_{2t}[\gamma k_t + v_{t+1} + m_tP_t/P_{t+1} - x_{t+1}]$$

are as follows:

(21a) $$u_1(c_t,x_{t+1}) - \lambda_{1t} = 0$$

(21b) $$u_2(c_t,x_{t+1}) - \lambda_{2t} \leq 0$$

(21c) $$x_{t+1}[u_2(c_t,x_{t+1}) - \lambda_{2t}] = 0$$

(21d) $$\gamma\lambda_{2t} - \lambda_{1t} \leq 0$$

(21e) $$k_t[\gamma\lambda_{2t} - \lambda_{1t}] = 0$$

(21f) $$\lambda_{2t}P_t/P_{t+1} - \lambda_{1t} \leq 0$$

(21g) $$m_t[\lambda_{2t}P_t/P_{t+1} - \lambda_{1t}] = 0$$

(21h) $y - c_t - k_t - m_t = 0$

(21i) $\gamma k_t + v_{t+1} + m_t P_t / P_{t+1} - x_{t+1} = 0$

These conditions imply demand functions for c_t, x_{t+1}, m_t, and k_t in which the arguments are v_{t+1} and P_t/P_{t+1}. In particular, conditions (21) imply a per-capita demand function for money balances,

(22) $m_t = m(v_{t+1}, P_t/P_{t+1})$.

Market equilibrium in the model at hand requires that supplies and demands are equated for money and goods, but by using Walras's Law we can express equilibrium in terms of the single condition

(23) $m(v_{t+1}, P_t/P_{t+1}) = M_t/P_t$,

where M_t is the per-capita supply of money. Following Wallace, we limit our attention to monetary policies of the form

(24) $M_t = (1+\mu)\, M_{t-1} \qquad \mu \geq -1$

with μ constant. Then we can write

(25) $$v_{t+1} = \frac{M_{t+1} - M_t}{P_{t+1}} = \frac{\mu M_t}{P_{t+1}} = \mu \frac{M_t}{P_t} \frac{P_t}{P_{t+1}}$$

and express the equilibrium condition as

(26) $$m\left(\mu \frac{M_t}{P_t} \frac{P_t}{P_{t+1}} ; \frac{P_t}{P_{t+1}}\right) = \frac{M_t}{P_t}$$

The task of this equilibrium condition is to determine P_t; the variable P_{t+1} is (as described above) an expectation. The sole state variable is M_t, so if P_t is finite the minimal-state-variable solution should be of the form

$$(27) \qquad P_t = \Pi(M_t).$$

If we guess that this function Π is of the form

$$(28) \qquad P_t = \pi M_t,$$

with π a finite constant, then we find that the expectation is

$$(29) \qquad P_{t+1} = \pi M_{t+1} = \pi(1+\mu) M_t$$

and the equilibrium condition becomes

$$(30) \qquad m\left(\frac{\mu M_t}{\pi(1+\mu)M_t}, \frac{\pi(1+\mu)M_t}{\pi M_t}\right) = \frac{M_t}{\pi M_t} .$$

Cancelling M_t from the latter results in a single equation that determines the value of π as a function of μ. Consequently, we verify the guess reflected in (28) and conclude that (28) in fact governs the behavior of P_t, when P_t is finite, in the model at hand.[12/]

The possibility must be emphasized, however, that money will be valueless in the model's equilibrium, i.e., that $1/P_t = 0$ for all t. In this case we can nevertheless utilize the equations written above if we define P_{t+1}/P_t as equal to $1+\mu$, an equality which also holds when $1/P_t > 0$. Let us consider, then, what conditions lead to monetary and non-monetary equilibria--i.e., to equilibria in which money is valuable and valueless, respectively, for all $t = 1,2,\ldots.$

First, consider the case in which money-stock growth is slow relative to physical depreciation; that is, in which $\mu < \delta$. Then $\lambda_2/(1+\delta) < \lambda_2/(1+\mu)$ so (21f) implies that (21d) holds as a strict inequality. Equation (21e) then requires $k_t = 0$; i.e., that no storage take place. But in this case x_{t+1} can be positive only if money has value, and our conditions on $u(c_t, x_{t+1})$ imply the existence of a solution with $x_{t+1} > 0$--see Wallace (1980, pp.54-55). Consequently, we see that a monetary equilibrium will prevail if $\mu < \delta$.[13/]

By contrast, when $\mu > \delta$, we have $\lambda_2/(1+\delta) > \lambda_2/(1+\mu)$ so that (21f) is required by (21d) to hold as a strict inequality. Then (21g) implies that $m_t = 0$ so that the equilibrium is one in which money is not valued. More generally, if population grows steadily at the rate ν, money will be valueless whenever $\mu > \nu + \delta + \nu\delta$.[14/]

Clearly, this "tenuousness" of monetary equilibria is an unusual and striking feature of the OG model. In the case in which there is no population growth or depreciation, the foregoing result implies that money will be valueless if policy makes the money stock growth rate any number greater than zero! And even with growth and depreciation taken into account, a money stock growth rate in excess of (say) 10% per year would be predicted to result in a non-monetary equilibrium.[15/]

For many persons, a first reaction to this result might be, I would guess, to conclude that the OG model is "obscenely at variance" with actual experience.[16/] Kareken and Wallace (1981) have emphasized, however, that such results presume that all agents confidently believe that the constant money growth policy will be maintained permanently. Furthermore, the result would be weakened by the introduction of stochastic elements--see, e.g., Peled (1982). So it is actually not a straightforward matter to bring evidence to bear on the empirical validity of the basic OG model. Let us then continue to explore its properties, reserving judgment for the moment on its merits as a model of money.

Doing so, we find that other striking implications are readily obtainable. Prominent in Wallace's (1980) discussion, in particular, are several strong propositions concerning the efficiency--that is, Pareto-optimality--of monetary and non-monetary equilibria. Consider, for example, Wallace's Propositions 5 and 6. These pertain to the case in which $\mu < \delta$, so that the stationary equilibrium has valued money, and assert that this monetary equilibrium is efficient if $\mu \leq 0$ and inefficient if $\mu > 0$.

To demonstrate the latter result, let us designate the equilibrium values of c_t, x_{t+1}, and m_t as c^*, x^*, m^* and note that, since $k^* = 0$, $c^* + x^* = y$. These c^*, x^* values refer to young persons alive during each period of the economy's evolution, $t = 1,2,\ldots$. Also relevant to the Pareto-optimality criterion is the consumption of the old during period 1, the initial period under consideration.[17/] Now, for each period $t = 1,2,\ldots$ the feasible values of c_t and x_{t+1} are those on or below the dotted line extending from $c_t = y$ to $x_{t+1} = y$ in Figure 1. The budget line as seen by a young person is, however, $c_t = y + (P_{t+1}/P_t)(v_{t+1} - x_{t+1})$. Thus choices are made relative to a budget line with slope $-(1+\mu)$, such as the one shown in Figure 1. Thus with $\mu > 0$ there will exist consumption

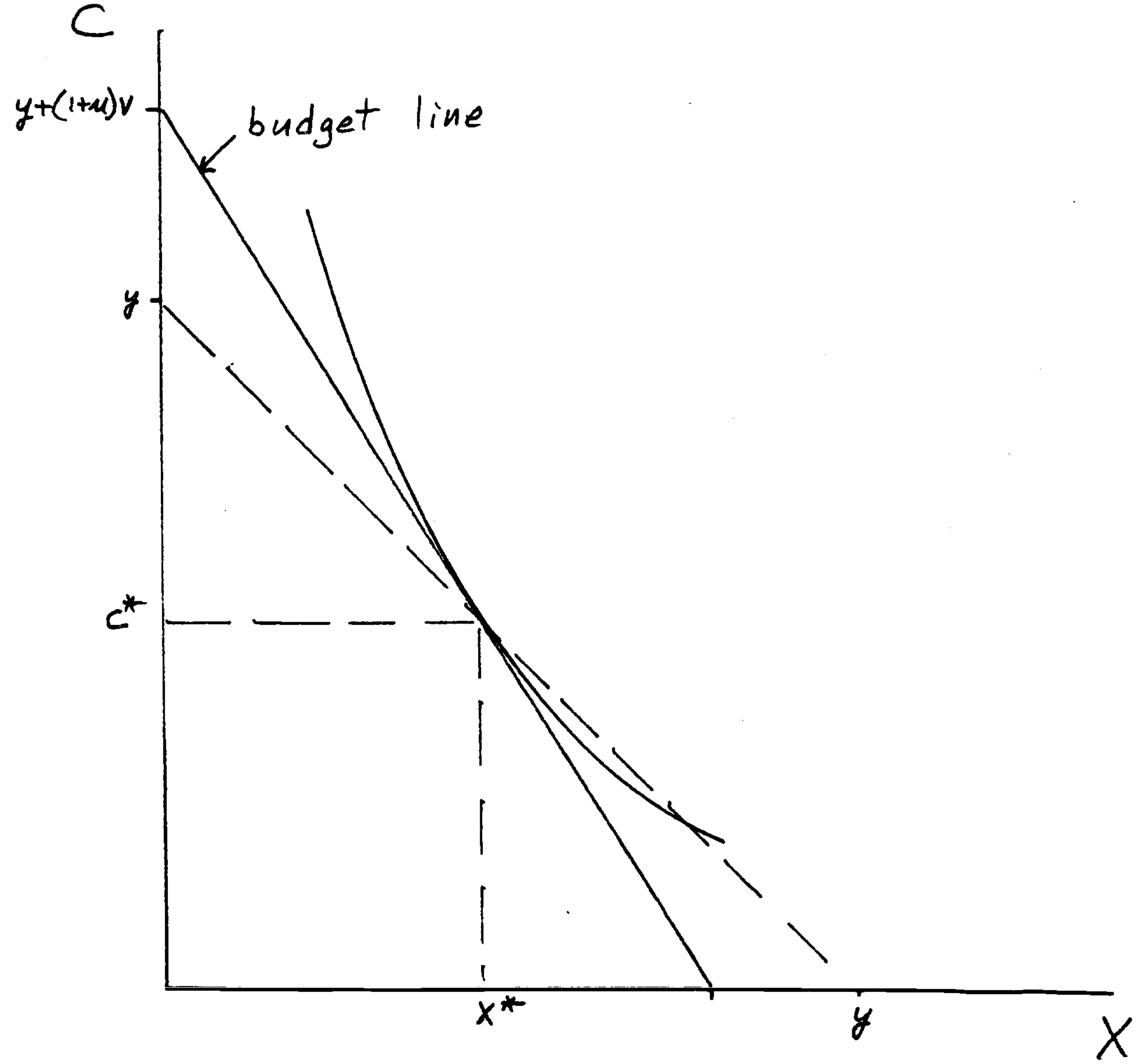

Figure 1

bundles c_t, x_{t+1} with $c_t < c^*$ and $x_{t+1} > x^*$ that are feasible <u>and</u> preferred by the young of each period to their chosen values, c^* and x^*. Furthermore, the old of period 1 would prefer a larger value for x_1, which is implied by $c_1 < c^*$, so the equilibrium c^*, x^* is not efficient.

If, by contrast, we had $\mu = 0$, the budget line faced by the young would coincide with the socially-feasible tradeoff and the equilibrium position would be optimal. And if $\mu < 0$ the budget line would be less steep than the feasibility frontier so that feasible reallocations away from c^*, x^* that are desired by the young would involve $c_t > c^*$ and $x_{t+1} < x^*$, which would diminish the utility of the old in period 1.

The results of these last two paragraphs are not affected, it should be added, by population growth: it is the aggregate, not the per-capita, growth rate of money that is relevant. The slope of the socially-feasible budget line differs from that perceived by a young person when μ differs from zero, not when the inflation rate differs from zero.[18/]

Thus monetary equilibria with positive money-stock growth rates are socially inferior, in the OG model, to monetary equilibria with constant or decreasing quantities of (nominal) money. But how do the latter compare with non-monetary equilibria, in which money is valueless and intertemporal consumption patterns differ from endowments because of storage ($k^* > 0$) of the consumable commodity? The basic answer is extremely simple: unless the commodity grows while in storage ($\delta < 0$), storage is inefficient. That is, barring

$\delta < 0$, intertemporal reallocations away from endowment patterns can be brought about by the use of money as a store of value. None of the good needs to be kept in storage, so depreciation (if any) can be avoided and a one-time increase in consumption effected. Thus any non-monetary equilibrium with $\delta \geq 0$ is inefficient.

Taken together, these results suggest, as Wallace points out (1980, p. 58), "that the quantity of fiat money should not be increased. For [they] imply that if [$\mu \leq 0$], then an optimal monetary equilibrium exists whenever the nonmonetary [equilibrium] is nonoptimal" (p. 58). This is, as Wallace emphasizes, a rather drastic conclusion--one that contrasts sharply with the conduct of monetary policy in most actual economics. The obvious question--indeed, the main issue of this paper--is, then, whether it is reasonable to use the OG model as a basis for reaching conclusions regarding monetary issues.

In this regard, some critics might be inclined to reject the OG model simply because it does not incorporate labor or product market disequilibria; i.e., because it is an equilibrium model. But that reason for rejection seems unwarranted. Whether pure, flexible-price equilibrium models--as opposed to sticky-price equilibrium models of the type mentioned by Lucas (1980a, 712, n. 14)--will ultimately prove fruitful for the analysis of cyclical fluctuations and stabilization policy is at present unclear. But even if flexible-price equilibrium models do not prove adequate for cyclical issues, they may nevertheless provide a useful basis for issues involving inflation, monetary growth rates, etc. There are various issues in monetary economics that have traditionally been examined in full-employment models.

More serious, perhaps, is the question--mentioned by Kareken and Wallace (1980, p. 8), Tobin (1980, p. 87), and many others--of whether the analytical entity called "fiat money" functions in the OG model as a medium of exchange. That this entity serves only as a store of value is suggested both by the OG model's structure and by the tenuousness property described above. To investigate this issue is the next task on our agenda.

IV. The Functions of Money

Let us begin the discussion by recalling that it has long been agreed, by economists and laymen alike, that for something to be called "money" it should function both as a store of value and as a medium of exchange.[19] Indeed, as Wicksell (1935) observed, it is only the latter function that is distinctive;[20] many assets serve as stores of value. But the important issue is not, of course, terminological. It is, rather, the entirely substantive matter of whether the OG model can provide useful answers to questions regarding the behavior of actual economies in which there is a medium of exchange.

To approach this issue let us first try to establish whether the entity called fiat money does or does not serve as a medium of exchange in the OG model. Now it might be thought that the answer to this question is obvious; that one can simply look at the specification of the model and immediately see that there is no medium of exchange.[21] Lucas (1972, p. 107) has suggested, however, that the matter is not so simple--indeed, that the model is not rich enough to justify an answer one way or the other. Presumably the idea is that the absence of an explicit description of some activity is not enough to imply that this activity is nonexistent in the modelled economy. A one-good model, for example, can be given a many-good interpretation, as is done in Lucas's (1980b) colored-marbles economy. That example, furthermore, shows that a one-good economy can reasonably be thought of as possessing a medium of exchange, one that serves to facilitate transactions of types--say, among the members of a single generation--that are invisible in the OG framework.

This notion--that activities may exist even if not explicitly described--may be accepted, however, without agreement that it is impossible to make progress on the issue at hand. What the notion implies is that the approach must be indirect: instead of merely looking at the model, we must look at its implications. If some can be found that are inconsistent with the absence (or presence) of a medium of exchange, then we can conclude that the model's money does (or does not) function in that capacity.

Taking that approach, it appears that a reasonably satisfactory argument can in fact be developed. This argument builds upon the traditional presumption that an economy with a medium of exchange will be more productive than it would be if no such medium existed. [22/] More specifically, the presumption is that leisure-consumption possibilities that would be unavailable under barter conditions become feasible in the presence of a medium of exchange. The relevant comparison, it should be noted, is one that holds constant (or corrects for) the stock of capital: extra consumption or leisure resulting from a larger capital stock does not reflect any medium-of-exchange effects. For a given value of k, then, agents in the aggregate are able to enjoy more leisure and/or consumption (without loss of the other) than would be possible under barter. Given this condition, it becomes clear that money in the basic OG model does not serve as a medium of exchange, for precisely the same leisure-consumption totals are available in the monetary and non-monetary equilibria when k is the same in each. This is clearest in the no-storage case in which $\gamma = 0$. More generally, consumption $c+x$ equals $y + (\gamma-1)k$ regardless of the value of γ.

Correcting for the magnitude of k, then, c+x is the same in monetary and non-monetary equilibria. And to complete the argument, we note that the entity called money certainly does not function as a medium of exchange in the latter type of equilibrium, for its exchange value is zero.

To the foregoing it might be objected by OG advocates that the traditional presumption--that economies with a medium of exchange are more productive than without--has not been firmly established by evidence or analysis. In response, it must be admitted that conclusive direct evidence is not available: the experiment of switching abruptly from barter to monetary exchange has probably not been performed on an economy-wide basis. And it might even be granted that the analyses of the presumption carried out by Brunner and Meltzer (1971), Clower (1970), and Neihans (1978, pp. 99-117) are not marked by formal rigor and explicitness to the extent that is typical in the OG literature. But it is difficult to believe that any reader of these three items--or, for that matter, of Jevons (1875) or Pigou (1949)--would doubt the validity of the presumption. The objection seems to me unsustainable; consequently, I contend that it is necessary to conclude that the OG model's money does not serve as a medium of exchange.

Parenthetically, it might be noted that, while Wallace (1980) does not mention the presumption,[23/] it is implicitly denied by his "confession" that, to him, non-monetary equilibria and commodity-money equilibria "are simply different names for the same thing." More specifically, in models with several equilibria in which $1/P_t = 0$, "there will in general be no basis for classifying these into ... commodity money equilibria and ... barter equilibria" (1980, p. 52). Wallace attempts to justify this position by suggesting that the main difference between commodity-money and barter equilibria is in the differing patterns of transaction velocities and conjecturing that "most models with frictions are likely to display some intermediate pattern" (p. 52). But while the pattern of transaction velocities is directly and definitionally related to the distinction under consideration, that does not rule out the possibility of identification by means of implications for leisure-consumption bundles (a possibility that may remain in settings in which transaction velocities cannot be observed). Thus Wallace's denial of the presumption seems unwarranted.

Next we turn to the question of whether it matters if the monetary entity in OG models is not a medium of exchange. But evidently it does matter; some of the most striking properties of these models are crucially sensitive in this regard. It is the absence of the medium-of-exchange function, for example, that accounts for the tenuousness of monetary equilibria in the model of Section III. For if the model's money served as a medium of exchange, it would provide its holders with enhanced leisure-consumption possibilities by reducing transaction costs. And this would induce agents to demand the asset even when its rate of return is exceeded by that of other equally-riskless assets; thus $\mu > \delta$ does not require $m_t = 0$.

In order to demonstrate this particular claim, let us provisionally adopt the artifice of making real balances $\xi_t = m_t P_t / P_{t+1}$ an argument of each young person's utility function, which then becomes $u(c_t, x_{t+1}, \xi_t)$ with $u_3 \geq 0$.[24/] Given this modification to the model of Section III, the first-order conditions (21) are replaced with the following:

(31a) $\quad u_1(c_t, x_{t+1}, \xi_t) - \lambda_{1t} = 0$

(31b) $\quad u_2(c_t, x_{t+1}, \xi_t) - \lambda_{2t} \leq 0$

(31c) $\quad x_{t+1}[u_2(c_t, x_{t+1}, \xi_t) - \lambda_{2t}] = 0$

(31d) $\quad \gamma \lambda_{2t} - \lambda_{1t} \leq 0$

(31e) $\quad k_t [\gamma \lambda_{2t} - \lambda_{1t}] = 0$

(31f) $\quad u_3(c_t, x_{t+1}, \xi_t) P_t/P_{t+1} - \lambda_{1t} + \lambda_{2t} P_t/P_{t+1} \leq 0$

(31g) $\quad m_t[u_3(c_t, x_{t+1}, \xi_t) P_t/P_{t+1} - \lambda_{1t} + \lambda_{2t} P_t/P_{t+1}] = 0$

(31h) $\quad y - c_t - k_t - m_t = 0$

(31i) $\quad \gamma k_t + v_{t+1} + m_t P_t/P_{t+1} - x_{t+1} = 0$

Now in this case, $\mu > \delta$ implies $\lambda_2/(1+\delta) > \lambda_2/(1+\mu)$ as before, but the presence of $u_3(c_t, x_{t+1}, \xi_t) > 0$ in (31f) makes it possible for (31f) to hold as an equality nevertheless. Thus $m_t = 0$ is not required by $\mu > \delta$; money may be valued when the money stock growth rate exceeds the depreciation rate. In particular, money may be valued even though $\mu > 0$ and $\delta < 0$.

The claim of the last two paragraphs--that tenuousness obtains in the OG model because of the absence of the medium-of-exchange function--has been disputed by Scheinkman (1980). In particular, Scheinkman examines OG, cash-in-advance, and money-in-the-utility-function models, and finds that "the mathematical conditions that insure the absence of tenuousness are very similar in all three classes of models" (1980, p. 91).[25/] The basis for this statement is Scheinkman's demonstration that all three types of models may have solution paths along which $m_t \rightarrow 0$ as $t \rightarrow \infty$ unless utility and production functions are such that money is in some sense "essential" or "necessary to the system" (1980, p. 95). It is important to recognize, therefore, that these equilibria in which $m_t \rightarrow 0$ are quite different from the non-monetary equilibria emphasized by Wallace and described above. Specifically, $m_t = 0$ for all $t = 1,2,...$ in the latter, while $m_t > 0$ for all finite t in Scheinkman's examples. In fact, examination indicates that the solution paths discussed by Scheinkman are bootstrap or bubble paths in the sense defined in Section II--paths along which effects arise only because they are arbitrarily expected to arise. Those discussed by Wallace, by contrast, are bubble-free, stationary paths along which money is valueless in each period. Thus Wallace's tenuousness is much more severe than the "asymptotic" variety considered by Scheinkman. Consequently, the latter's examples shed no light on conditions tending to bring about tenuousness of the relevant type.

It is of course a matter of considerable interest whether Wallace's optimality conclusions are also invalidated by changing the young agents' utility function to $u(c_t, x_t, m_t P_t / P_{t+1})$. In considering this issue it seems appropriate, given our argument concerning tenuousness, to interpret Wallace's Propositions 5 and 6 as being applicable whenever money is valuable, a less restrictive condition than $\mu < \delta$. Doing so, we find that the former proposition--that any monetary equilibrium is efficient if $\mu \leq 0$--is not valid in the modified model. This conclusion follows from consideration of a monetary equilibrium in which storage is productive ($\delta < 0$) and μ satisfies $\delta < \mu \leq 0$. Starting from such an equilibrium, it is possible to increase the stationary value of m without reducing c or x, since the relevant physical constraint,

$$(32) \qquad y + (\gamma - 1)\, k = c + x,$$

does not involve m. Thus young agents' utility can be increased for each t = 1,2,... without any reduction in x_1. Indeed, it is shown in the Appendix that optimality of a monetary equilibrium with $k > 0$ requires $\mu = \delta$, just as in Friedman's (1969) essay on the "optimum quantity of money."

It would obviously be possible to object to the reinterpretation utilized in the last paragraph, but to do so would make the propositions too model-specific to be of much interest. It seems more likely that objections by OG theorists would focus upon the use of a specification in which real money balances are treated as an argument of agents' utility functions. We need, then, to address the implied issue in some detail.

V. Methodological Issues

It is no doubt widely known that most OG theorists object strenuously to the use of money-in-the-utility-function or "MIUF" models. In fact, the objection is shared by some monetary theorists who are not proponents of the OG framework (e.g., Tobin (1980, p.86)); yet MIUF models continue to be widely used. Let us consider why.

The rationale that users of MIUF models have in mind is presumably something like the following.[26] Individuals do not derive utility directly from money balances; the only arguments of the direct utility function pertain to consumption of commodities and leisure. Thus an individual who lives for two periods (beginning with t) has a utility function such as $u(c_t, \ell_t, c_{t+1}, \ell_{t+1})$. (Note that the notation is different from that of previous sections.) But in order to obtain consumption goods, this individual must acquire them in the marketplace--here a multigood interpretation like that in Lucas (1980b) is helpful--and shopping takes time. Thus leisure in t is constrained by

$$\ell_t = 1 - n_t - s_t, \tag{33}$$

where n_t and s_t are amounts of time spent working and shopping, measured in units that make the total time endowment per period equal to 1. Finally, shopping time per unit of consumption is constrained by a technological relationship that reflects the transaction-facilitating properties of the medium of exchange. Specifically, we assume

$$s_t/c_t = \psi(m_t/c_t) \tag{34}$$

with $-\infty < \psi' < 0$ and $0 < \psi(\infty) < \psi(0) < \infty$, where m_t is the quantity of real money balances held at some point of time in period t.[27/] Then substitution readily yields

$$(35)\qquad u\left[c_t,\ 1-n_t-c_t\psi(m_t/c_t),\ c_{t+1},\ 1-n_{t+1}-c_{t+1}\psi(m_{t+1}/c_{t+1})\right]$$

$$= \tilde{u}(c_t, n_t, m_t, c_{t+1}, n_{t+1}, m_{t+1}),$$

and it is a function analogous to $\tilde{u}$ that appears in the MIUF models.

There are, of course, a number of issues concerning the specification of ψ. In addition to the timing ambiguity mentioned in footnote 26, there is the question of why this transaction specification is not as in the cash-in-advance models.[28/] More fundamentally, Kareken and Wallace (1980) have argued that such specifications involve implicit theorizing, which makes it difficult or impossible to examine all aspects of the model for logical consistency. Furthermore, the approach implies a failure of the analysis to explain which physical objects are used as the medium of exchange--something, they suggest, that should be determined endogenously.

It is, I think, clear that the Kareken-Wallace criticisms of this sort of approach have considerable merit. But there are several responses that can be made. First, analogous criticisms are applicable to virtually all of the "fundamental" relationships employed in neoclassical (and other) economic theory. Standard production-function specifications, for example, are not actually dependent only upon the laws of physics. There is implicit theorizing involved with the treatment of "capital services" and "technical progress," even at the purely theoretical level.[29/] And at the level of empirical application, the task of selecting a recorded time series to represent "labor services" is not greatly different than when the task is required for "money." Thus it appears that whole-hearted adoption of the Kareken-Wallace principles would eliminate most existing neoclassical analysis.[30/] Second, whatever their merits, the criticisms do not provide much support for the basic OG specification. Specifically, it is difficult to believe that the description of the medium-of-exchange role of money provided by equation (34) could be more inaccurate than one that requires this role to be nonexistent. To adopt the latter condition would seem to be rather like erecting a theory of the firm upon an assumption that makes output insensitive to the quantity of labor services employed. In addition, one cannot explain what physical objects serve as media of exchange by requiring that none do.

The upshot of this discussion seems to be that the use of MIUF models is not as unreasonable as some have suggested.[31/] But it should be kept in mind that the underlying rationale is provided by some sort of specifications like (33)-(35), and the implied properties of the indirect utility function should be observed. Thus, for example, equations (33)-(35) imply that $\partial\tilde{u}/\partial m_t = -(\partial u/\partial \ell_t)\psi'/c_t$, which is inconsistent with an assumption such as $\partial\tilde{u}/\partial m_t \rightarrow \infty$ as $m_t \rightarrow 0$. With $\psi(0) < \infty$, the shopping-time approach therefore implies that the medium of exchange may be abandoned if its holding costs become too great. Also, a specification such as that used to obtain equation (31) can be justified only with some very special assumptions; more generally, one would expect labor time to enter as an additional variable. More positively, the approach suggests that agents can become satiated with finite magnitudes of m.

Now it should be clear that the foregoing argument has nothing to do with the essential generational structure of OG models; it concerns instead the methodological predilections of some economists who happen to be leading proponents of the OG framework. Thus there is no reason, according to this argument, why OG models should not incorporate shopping-time constraints such as (34)-(35) or even--if care is exercised--MIUF specifications. Indeed, given the substantial merits of the OG structure as a vehicle for the analysis of certain intertemporal issues, such a combination seems attractive.

In fact, Helpman and Sadka (1979) have used a model of precisely that type: an OG model in which young agents' utility depends upon consumption while young and old, labor while young, and real money balances carried into old age.[32/] A question that immediately arises, given the foregoing discussion, is whether that particular specification is consistent with the shopping-time approach. As it happens, an affirmative answer can be obtained if it is specified that persons cannot supply labor services (or that they are unproductive) when old. Then if each person begins life without money, equation (35) will specialize to

$$(36)\qquad u\left[c_t,\ 1-n_t-c_t\psi(0),\ c_{t+1},\ 1-c_{t+1}\psi(m_{t+1}/c_{t+1})\right] = \tilde{u}(c_t,n_t,c_{t+1},m_{t+1}).$$

And since Helpman and Sadka make no explicit assumptions regarding derivatives with respect to m_{t+1}, they do not violate any properties implied by u and ψ.

This may be an appropriate point at which to consider, as suggested by Wallace (1980, p. 77), whether giving money a medium-of-exchange role requires abandonment of the idea that money is "intrinsically useless." The answer to this question must depend, obviously, on precisely what is meant by intrinsic uselessness. Presumably, it would require that money not directly appear in any person's utility function. But what about specifications like (33)-(35), which make money balances an argument of indirect utility functions? It is my impression that most economists would not consider such a specification as inconsistent with intrinsic uselessness; even the store-of-value role can, after all, be utility-enhancing. But if it were agreed that a proper definition makes intrinsic uselessness inconsistent with a medium-of-exchange role for money, then it has to be the former that is abandoned if one wishes to understand the workings of a monetary economy.

It might also be said--perhaps unnecessarily--that this paper's disagreements with certain specific positions taken by Wallace should not be interpreted as a denial of the fruitful products of his work with monetary OG models. In addition to its analytical results, this work has usefully emphasized the need for internal consistency and reliance upon policy-invariant relationships in macroeconomic modelling, as well as the desirability of individual-utility-based policy analysis. Furthermore, questions raised by Wallace and his collaborators--e.g., why do government bonds not serve as a medium of exchange?--have drawn attention to important issues.

VI. Other Monetary OG Models

Thus far, our discussion of monetary issues has concentrated on the basic model of Section III. It has, consequently, neglected some more elaborate OG setups that have been developed and utilized in a number of notable papers. A brief discussion of a few of these richer monetary models will accordingly be presented in this section.

One of the more striking results in the OG literature is provided by Wallace's (1980, pp. 71-73) (1981) "Modigliani-Miller theorem for open-market operations," which suggests that changes in the money stock brought about by open-market operations will have no effect on the price level. Analytically, an open-market operation is defined in these papers as a government purchase [33] with money of some other asset, together with the associated adjustments in (lump-sum) transfer payments that are needed to keep the path of government consumption unchanged.[34] For such an exchange to be of interest, it is of course necessary that both money and the second asset be valued in equilibrium. Of necessity, then, the model must be richer than that of Section III. In fact, the main discussion in Wallace's cited papers utilizes a model that is quite similar to that of Section III but in which the outcome of storage activity is stochastic. Specifically, for each unit of output stored in period t, γ_{t+1} units are available in period t+1, where γ_{t+1} is random and drawn from a stationary distribution with mean greater than 1.0. Then with the aggregate money stock held constant over time, risk-averse young agents will--for some γ distributions--wish to both hold money and store positive quantities of output. The stochastic aspect of the model therefore makes it applicable to issues that cannot be addressed with the basic version.

In this model the government can also store output, subject to the same stochastic technology, which it obtains in exchange for money. Let k_t and k_t^g denote the per-capita quantities stored by young persons and by the government at the end of period t. The formal counterpart of an open-market operation is then a comparison of alternative stationary equilibria in which the government stores different amounts of output, with the quantities related by the condition $dM = P\ dk^g$. Here dM and dk^g refer to differences across equilibria, while P is the price of output in the "initial" equilibrium.

The result that the same value of P is consistent with both equilibria may be explained as follows. Since interest earned on government storage is returned to the private sector via transfer payments (positive or negative), the ultimate constraints on c and x are the same whether the government chooses a large or small value for k^g--provided that this value does not exceed the quantity that young agents would choose in the absence of government storage.[35/] Thus young agents' choices for c and x will be the same for different settings of k^g, which requires that total storage $k + k^g$ be invariant to k^g. But the young agents' first-period budget constraint is $y = c+m+k$. Thus with c invariant to k^g, so must be m+k. Consequently, real balances m must be directly related to k^g unit-for-unit across steady states: $dm = dk^g$. But this is consistent with the open-market purchase condition $dM = P\ dk^g$ together with $dP = 0$. So the same price level satisfies equilibrium requirements in both steady states.

The foregoing argument can be modified to show that the price-level invariance result will not obtain if real money balances provide transaction services. For in that sort of an economy, young agents care about the magnitude of m, as well as c and x. So these agents will not be indifferent to changes in m brought about by changes in k^g, and will accordingly adjust their choices of c,x, and k. And that adjustment process will prevent m from moving unit-for-unit with M and k^g; changes in P will be required. In particular, m will tend to move less than M: an increase in M will induce an increase in P.

Reflection upon the relative usefulness of assets in Wallace's model makes his price-level invariance result less surprising than it seems at first glance. Thus, the result can be restated more generally as follows: an open-market exchange of a productive asset for an unproductive asset will leave the relevant real quantity of the former unchanged, with the real quantity of the latter adjusting as required. When stated in this manner, the result becomes consistent with the predictions of "Ricardian" models in which open-market operations in government bonds induce price level changes proportional to changes in the nominal money stock.[36] To see this note that in Ricardian models money provides transaction services and so is useful, while government bonds are--because agents take account of associated future tax payments--of no consequence; useless. Thus the Ricardian application is simple: the real quantity of the useful asset, money, is unaffected by an open-market exchange for a useless asset. Finally, note that our restatement of the result is also consistent with the predictions of non-Ricardian models in which both money and bonds are useful: the real quantities of both must then adjust in response to an open-market exchange.

A different way of providing room for two assets has been suggested by Bryant and Wallace (1980b) and utilized (with modifications) by Sargent and Wallace (1981). In the models of these papers, the second asset is interest-bearing government or private debt, i.e., bonds that are issued only in large denominations. Young agents who are potential lenders are of two types: ones with relatively large endowments (termed "rich") and ones with small endowments ("poor"). Rich lenders are obviously motivated to hold bonds rather than money in order to obtain interest payments--there is no other difference (except denomination of issue) in the two paper assets. Poor lenders, however, can not hold bonds because each one's endowment is too small to permit the purchase of a single bond and it is by assumption illegal for individuals to share ownership of a bond. Thus there may exist (possibly deterministic) equilibria in which both bonds and money are valued.

Without going into the details of the models in question, one can, I think, conclude from the foregoing description and the discussion of Section IV that "money" in these models does not serve as a medium of exchange. Each of the paper assets functions as does "money" in the basic OG model; both are valued in equilibrium only because of the rather arbitrary assumptions that keep poor lenders from holding bonds. This conclusion is supported, moreover, by application of the criterion proposed in Section IV--aggregate consumption/leisure possibilities are not enhanced by the existence of valued money.

Next, let us consider an ambitious attempt by Peled (1982) to generate, without resorting to a MIUF or cash-in-advance specification, a more comprehensive role for money than exists in the basic OG structure. In Peled's model, in which each young person has the same endowment of a completely perishable consumption good and the same risk-averse utility function, there are two spatially-separated islands to which are assigned randomly-determined fractions of each new generation. In addition, there is a second source of randomness: at the end of each period half of the occupants of each island are selected at random and relocated (with their money holdings) to the other island for the next period. The agents thus selected will then spend their old age in a location with conditions that, because of the random assignment of newly-born persons, are different from those faced by their contemporaries who stay on their island of birth. Since this selection process is random, there is a certain type of risk that young agents on an island can (if they wish) agree to share. They can do so by negotiating bilateral contracts specifying money transfers that are contingent upon the second-period population of the islands on which each party will spend his or her old age.[37/]

Consequently, Peled's model is one in which money serves to facilitate <u>intragenerational</u> communication, something that appears to be non-existent in the basic OG model and its variants with random endowments or storage processes. Under certain conditions the existence of money therefore shifts outward the "utility possibility frontier;" if risk-avoidance were a commodity it could be said that money makes feasible aggregate consumption bundles not available in its absence.[38/] Nevertheless, it would appear that money still does not serve as a medium of exchange. It is a transportable store of value which makes possible a type of insurance contract, but there is no evidence that it serves to facilitate exchange of commodities.[39/]

Finally, let us consider Townsend's (1980) models in which money serves to facilitate exchange among spatially-separated agents. In particular, consider Townsend's version of the Cass-Yaari (1967) "circle" model. This is not itself an overlapping-generations model, but is related in the sense that it involves a highly-stylized setting in which the role of money in facilitating certain desirable transactions is described with great explicitness.[40/] In Townsend's version, the model features a countable infinity of infinite-lived agents, indexed by the integers, and a countable infinity of different perishable commodities.[41/] In each period household i, which is endowed with a positive quantity of good i only, is physically able to make contact only with households i-1 and i+1. The tastes of household i are such that it obtains satisfaction from consumption of goods i and i+1. Similarly, household i+1 is endowed only with good i+1 and desires to consume goods i+1 and i+2, etc. Thus there is no possibility of barter exchange, since there is no pair of agents each of which has a commodity that the other values. In the absence of money, each household simply consumes its own endowment. But if agents hold positive quantities of fiat money--some physical entity that can be stored and transferred costlessly--they can use these holdings to make purchases of the second desired commodity. Townsend shows that a monetary equilibrium exists in this setting and is Pareto superior to the nonmonetary, autarkic equilibrium.

Providing a full discussion of the strengths and weaknesses of this model as a vehicle for monetary analysis is beyond the scope of the present paper. A few comments are nevertheless in order. First, as Sargent (1982) has noted, Townsend's models are somewhat less tractable than OG models. Second, the type of exchange facilitation provided by money in the Cass-Yaari-Townsend model is more extreme than in the traditional literature, where the absence of a medium of exchange makes trades more costly rather than impossible. As a result, the criterion of my Section IV would suggest that money does not serve as a medium of exchange in this model. But that conclusion would not hold under a less extreme specification of transaction costs. Indeed, it does not hold when the production technology described by Cass and Yaari (1967, pp.364-365) is used instead of fixed endowments. Third, the continued development of models emphasizing spatial separation seems desirable.

VII. Conclusion

The arguments of the present paper can be briefly summarized as follows. The overlapping-generations structure provides an attractive framework for the analysis of macroeconomic issues involving the intertemporal allocation of resources. The attractiveness of the OG framework is a consequence of the essential generational feature that permits an everlasting economy to be populated with agents who live for only a limited number of periods. By assuming that agents are alike before birth, one obtains a model in which it is possible to have a competitive equilibrium that is time-stationary although the economy is populated at each point of time with agents that are unalike with respect to wealth and incentives to save. The stationarity facilitates analysis, while the point-in-time heterogeneity of agents increases the types of economic activity that can be considered.

As a "model of money," the basic OG structure--which excludes cash-in-advance or money-in-the-utility-function (MIUF) appendages--seems inadequate and potentially misleading, the reason being that it neglects the medium-of-exchange property of money. That this type of model does, in fact, neglect the medium-of-exchange property is argued in the following way: an economy that possesses a medium of exchange can attain aggregate consumption/leisure bundles that would be unattainable in its absence;[42/] in the basic OG model the same bundles are attainable (with a given capital stock) in equilibria in which "money" is valuable and valueless; hence this money does not serve as a medium of exchange. That the models may be misleading is demonstrated by examples in which three of their most striking properties--tenuousness of monetary equilibria, optimality of a constant money supply, and invariance of the price level to open-market asset purchases--disappear in the presence of modifications designed to reflect the medium-of-exchange property.

The paper also contends that, as the previous sentence implicitly suggests, there is no particular reason why cash-in-advance, MIUF, or other appendages designed to reflect the medium-of-exchange property should not be used in conjunction with the OG framework. The usefulness of each potential appendage or elaboration is a distinct issue that may involve difficult and important methodological considerations, but in most cases these considerations have nothing to do with the essential generational structure of OG models.

APPENDIX

In the MIUF model of Section IV, conditions for Pareto optimality of a stationary equilibrium can be obtained by maximizing $u(c,x,\xi) + \theta\hat{u}(x_1)$, where θ is any positive constant and $\hat{u}(x_1)$ is the utility of the initial old, subject to the constraints $y + (\gamma-1)k - c - x = 0$, $y + k_0 - k - c - x_1 = 0$, $k \geq 0$, $x_1 \geq 0$, and $\xi \geq 0$.[43/] If λ_1 and λ_2 are the Lagrangian multipliers, the optimality conditions include the following:[44/]

$$\text{(A-1)} \qquad u_1(c,x,\xi) - \lambda_1 - \lambda_2 = 0$$

$$\text{(A-2)} \qquad u_2(c,x,\xi) - \lambda_1 = 0$$

$$\text{(A-3)} \qquad u_3(c,x,\xi) \leq 0, \qquad \xi u_3(c,x,\xi) = 0$$

$$\text{(A-4)} \qquad \theta\hat{u}'(x_1) - \lambda_2 \leq 0, \qquad x_1[\theta\hat{u}'(x_1) - \lambda_2] = 0$$

$$\text{(A-5)} \qquad \lambda_1(\gamma-1) - \lambda_2 \leq 0, \qquad k[\lambda_1(\gamma-1) - \lambda_2] = 0.$$

Now consider a monetary equilibrium in which storage is positive, i.e., a case with $\xi > 0$ and $k > 0$. From (A-3) we see that a satiation level of money balances must be held, i.e., that $u_3(c,x,\xi) = 0$. From (A-5) we further see that $\lambda_2 = \lambda_1(\gamma-1)$, which with (A-1) and (A-2) implies

$$\text{(A-6)} \qquad u_1(c,x,\xi) = u_2(c,x,\xi) + (\gamma-1)u_2(c,x,\xi).$$

But from equations (31g), (31a), and (31b) in Section IV we see that agents' behavior implies

$$\text{(A-7)} \qquad (1+\mu)\, u_1(c,x,\xi) = u_2(c,x,\xi) + u_3(c,x,\xi).$$

Thus with $u_3(c,x,\xi) = 0$ it is clear that (A-6) and (A-7) together require $\gamma = 1/(1+\mu)$, which immediately reduces to $\mu = \delta$. This is the Friedman-type result mentioned toward the end of Section IV.

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FOOTNOTES

1. Note that this notational convention differs from that typically found in the OG literature, where t indexes generations rather than time.

2. With homogeneous young agents the loan market will be inactive but, as in Diamond (1965), we assume that ρ_t is taken parametrically nevertheless. In (5), and throughout the paper, inequality constraints are written as equalities when they are certain to be binding in equilibrium.

3. Under the stated conditions, these will be necessary and sufficient for optimality.

4. Here and in what follows, "per capita" refers to quantities per member of the relevant generation, not per person alive.

5. Barro (1974) has shown the existence of an operative bequest motive will under some conditions make an OG model analytically equivalent to one with agents with infinite planning horizons. Drazen (1978) has described one set of conditions under which this equivalence does not obtain. In my opinion, there is ample reason to believe that the continued development of both types of models--infinite-horizon and no-bequest OG models--is appropriate. Barro's result is important but does not warrant a dismissal of no-bequest OG models.

6. The reason for including this second example is to demonstrate that multiple solutions may obtain in non-monetary OG models, thereby indicating that solution multiplicities are not necessarily related to monetary issues of interest.

7. See, for example, Sargent and Wallace (1975, p.247).

8. See Calvo (1978, pp.322-324). These other paths are such that $q_t \rightarrow q$ as $t \rightarrow \infty$; they thus arise when $\emptyset'$ is large, making its reciprocal less than 1.0.

9. When (16) is viewed as a difference equation, q_t appears to be a state variable relevant to the determination of q_{t+1} or, equivalently, q_{t-1} as a state variable relevant to the determination of q_t. That viewpoint suggests a solution of the form $q_t = Q(q_{t-1})$ and substitution into (16) gives $Q(q_{t-1}) = \emptyset[Q(q_t)]$ which "verifies" the guess.

10. That this strategy is feasible is demonstrated in McCallum (1981). It will be desirable if, as in the present case, the issues at hand are not directly concerned with the possible existence of bubbles.

11. Wallace's argument is compatible with, but less complete than, the one in McCallum (1981). In particular, Wallace does not consider the possibility of extraneous state variables other than time and does not offer any procedure for selection of the appropriate solution in cases in which the solution to undetermined-coefficient identities is not unique.

12. There are, as in the Calvo model, other solution paths. We shall ignore them, however, for the reasons given in Section II.

13. To be precise, the theory implies only that a monetary equilibrium will exist--there is also an autarkic equilibrium in which $1/\rho_t = 0$ for all t.

14. In the case in which $\mu = \delta$, there is a continuum of equilibria in which money is valued and goods are stored. Since μ and δ are parameters of independent processes, I will devote no attention to this borderline case.

15. Here 10% is simply an order-of-magnitude figure reflecting a depreciation rate of about 0.06 and an output growth rate--resulting from population growth and technical progress--of about 0.04.

16. The phrase is taken from Sargent (1976).

17. Here period 1 is simply the point of reference for the optimality calculation--a period within which a reallocation is contemplated. If it is also the first period of the economy's existence, and the latter begins without old persons, then $\mu < 0$ is not optimal--as consideration of the next paragraph in the text will reveal.

18. In the case in which population grows at the rate ν, the per-capita notation employed above becomes inadequate for representing the transfer-payment injections of money. The slope of an individual's budget line remains $-P_{t+1}/P_t$, which now equals $-(1+\mu)/(1+\nu)$, while the feasible values satisfy $c_t = y - x_{t+1}/(1+\nu)$.

19. To quote Wicksell (1935, p. 6), "The conception of money is involved in its functions ...: as a _measure of value_, as a _store of value_, and as a _medium of exchange_ ...." Also see Schumpeter (1954, pp. 62-63) and Jevons (1875).

20. "Of the three main functions, only the last is in a true sense characteristic of money" (Wicksell, 1935, pp. 6-7). According to Schumpeter (1954, pp. 62-63), a similar view was held by Aristotle. Also see Clower (1970, p. 14).

21. This seems to be the view of Hahn (1981) and Helpman and Sadka (1979, p. 156).

22. See, for example, Clower (1970, pp. 8-14), Neihans (1978, p. 1-3), and Pigou (1949, pp. 25-26).

23. Wallace's discussion on p. 52 is concerned with the relative efficiency of fiat vs. commodity money, a concern that is significant but entirely different. On p. 78, he denies or overlooks the presumption in his assertion that "not a single proposition in monetary theory makes use of" the distinction between the medium-of-exchange and store-of-value roles of money.

24. Whether there exists a reasonable justification for this procedure is discussed below. Note that equations (31) also apply if the real-balance term ξ_t is defined as $v_{t+1} + m_t P_t / P_{t+1}$.

25. Scheinkman explicitly interprets his cash-in-advance model as one in which money serves as a medium of exchange. To represent that function of money is the usual reason for the MIUF assumption.

26. The development in this paragraph reflects influences from a number of sources. These include Brock (1974), Dornbusch and Frenkel (1973), Dutton and Gramm (1973), Fischer (1974), and Saving (1970). There are doubtless other relevant references; I have listed only those that have had an influence on my own views.

27. Most writers would probably specify real balances at the start of period t. Some formulations in the literature imply, however, that it is the end-of-period balances that matter (Brock, 1975). Some average over the period is another possibility.

28. One answer is that the cash-in-advance specification is a special case, one with $\psi = 0$ for $m_t/c_t \leq 1$ and $\psi = \infty$ for $m_t/c_t > 1$.

29. Implicit theorizing must be involved, for example, in Wallace's (1980) storage technology--especially in the case $\delta < 0$. Note that this argument is more general than that of Jensen and Meckling (1979), which hinges on a distinction between firms and other agents.

30. That does not imply that the principles are necessarily misguided, but it does suggest that one proceed cautiously in embracing them.

31. Use for some problems, it should be added. Obviously such models are not going to explain which objects serve as the medium of exchange.

32. Another example is provided by Weiss (1980).

33. Purchase quantities may of course be positive or negative for individual operations.

34. These transfer-payment adjustments are, I believe, implicit in traditional analyses of open-market operations. Their explicit recognition is not the reason for Wallace's result.

35. Here and in what follows we drop the time subscripts since steady-state equilibria are being discussed.

36. For an example and references, see Barro (1981), Chapter 6.

37. Old persons' money will be more valuable on islands in which there is a high ratio of young to old persons.

38. Compare the argument of Section IV.

39. This view is supported by application of the criterion of Section IV, conditional upon realizations of the random shocks. Details of the timing or transactional assumptions of the model are also open to objection. In particular, newly-born young agents arrive before the selected old agents are relocated--otherwise, how could their number be known?--but trades with them are not permitted.

40. Sargent (1982) states that Townsend's models and OG models "embody the same sort of impediment to private borrowing and lending which provided a role for currency." (Townsend's Section 3 describes a generalized OG setup; that will not be discussed here.)

41. Townsend's device of arranging agents in an infinite sequence rather than a closed finite "circle," seems to me unconstructive. The object of this device is to provide an "endogenous" exclusion of private loans. But this exclusion results because of an analytical condition--an infinity of agents at each point of time--that must be literally untrue. If it matters whether one takes the number of agents to be infinite, as opposed to large and finite, then to do so is inappropriate.

42. This "classical presumption" is taken to be axiomatic.

43. It would not affect the following argument if we used $\hat{u}(x_1, \xi_1)$ as the utility of the initial old.

44. To avoid clutter, the discussion proceeds under the condition $x > 0$.