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VALUING PENSIONS (ANNUITIES) WITH DIFFERENT TYPES OF INFLATION PROTECTION IN TOTAL COMPENSATION COMPARISONS

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ABSTRACT

Pensions provided in the public sector are often indexed, while pensions in the private sector typically are not. To conduct the total compensation comparisons that ostensibly guide government pay policy, one must value annuities which differ in their degree of inflation protection. This paper conducts this exercise from the viewpoint of modern finance theory, and contrasts the results with those of a representative government, the Government of Canada. The results suggest that governments may typically understate the value of indexed pensions and overstate the value of pensions which receive incomplete inflation protection. A contributing factor is the apparent belief that standardizing actuarial assumptions is sufficient to ensure comparability, in spite of the fact that risk is ignored and that interest rate and inflation assumptions are typically not those of the market.

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1. INTRODUCTION

In both Canada and the United States, governments are ostensibly guided in their pay policies by the principle that total compensation in the public sector should be comparable to that in the private sector. Typically, pensions provided in the public sector receive better inflation protection. The pensions provided to federal government employees, for example, are fully indexed to the price level, while full indexing is quite rare in the private sector. If valid comparisons of total compensation are to be made, then fully indexed annuities as well as annuities which receive incomplete inflation protection must be properly valued. This problem is made difficult by the fact that fully indexed annuities as well as those which receive incomplete inflation protection are not sold in - and are thus not explicitly priced by - the capital market.

This paper uses the principles of modern finance theory to value five annuities which differ in their degree of inflation protection. Each corresponds in a stylized way to current practice in Canada, and most are relevant to the United States as well (Table 1). They are: (1) a fully indexed annuity; (2) an indexed annuity subject to a cap, either fixed (the maximum adjustment is limited to $\underline{x}$ percent per year) or floating (the adjustment equals the inflation rate less $\underline{y}$ percent); (3) an annuity which receives cost-of-living adjustments based on pension fund earnings in excess of an assumed interest rate; (4) an annuity which is partially indexed and receives adjustments equal to $\underline{z}$ percent of the inflation rate; and (5) a nominal annuity. The paper contrasts the results suggested by modern finance theory with those of a representative government, the Government of Canada. The analysis suggests that governments may typically

understate the value of fully indexed annuities and overstate the value of annuities which receive incomplete inflation protection. In part, this is due to the mistaken belief that standardizing the assumptions used in actuarial valuations is sufficient to ensure comparability. The fact that actuarial valuations ignore risk and typically embody interest rate and inflation assumptions which are not those of the market receives inadequate attention.

The paper is organized as follows. In the first section, the choice of a discount rate for fully indexed annuities is discussed. The analysis focuses on the problem of estimating the expected real rate of return on the minimum variance portfolio. The second section reviews the problem of valuing capped annuities. The third analyzes the problem of valuing annuities receiving cost-of-living protection equal to pension fund earnings in excess of an assumed interest rate, or by "excess" earnings. The analysis focuses on the conditions under which these can be reduced to standard variable annuities (and thus readily valued), as well as the complications posed by the possible exclusion of unrealized gains and losses from the measure of fund earnings. The fourth section analyzes the problem of valuing partially-indexed annuities which receive adjustments equal to a designated fraction of the inflation rate. The fifth section briefly reviews the problem of valuing nominal annuities, and draws attention to the accounting veil implicit in most actuarial valuations. The total compensation comparisons conducted by the Federal government of Canada are then reviewed, in order to contrast the procedures used by a representative government with those implied by modern finance theory. A summary section concludes the paper.

2. FULLY INDEXED ANNUITIES

The pensions provided to Federal government employees in both Canada and the United States, and to employees of many other levels of government as well, are fully indexed to the consumer price index. Since fully indexed pensions are rare in the private sector,[1] the proper valuation of these annuities is essential if the total compensation of public sector employees is to be properly compared to that of employees in the private sector.

In a world of no inflation, a life insurance company could perfectly hedge an n-period, nominal (and hence real) annuity by holding a portfolio of insured, n-period mortgages. The expected return on such a portfolio would be the competitive rate at which such annuities would be sold. If indexed mortgages were available in a world of uncertain inflation, an analogous result would obtain since a life company could perfectly hedge an n-period, fully indexed annuity by holding a porfolio of n-period, indexed mortgages.

In North America, (uncertain) inflation does exist and indexed debt instruments are not available in the capital market. Prior studies (Bodie (1976), Pesando and Rea (1977)) have shown that the real rate of return on a diversified portfolio of common stocks is negatively correlated with unanticipated inflation, as is the real rate of return on fixed-income securities. To the extent that a life company is constrained by regulation or by habit to hold a portfolio that consists of stocks and fixed-income securities, the life company cannot create a portfolio which is devoid of inflation risk. Thus the life company cannot perfectly hedge a fully indexed

annuity. To minimize its underwriting risk, the best the life company can do is to hold that portfolio which has minimum variance and thus the most stable real return. The expected real return on this portfolio would be the upper bound on the competitive interest rate at which a fully indexed annuity would be sold, since shareholders of the life company would have to be compensated for bearing the residual investment risk.

Data on the (pretax) real returns to 91-day Treasury bills, long-term Government of Canada bonds, and common stocks for the period 1953-1980 are presented in Table 2. The annual real return on Treasury bills averaged 0.82 percent, with a standard deviation of 2.02 percent. Common stocks are much riskier, with a mean real return of 7.97 percent and a standard deviation of 17.02 percent. The mean real return on long-term Canada bonds is actually negative, reflecting the impact of unanticipated (permanent) inflation, and has a standard deviation of 7.60 percent. The real returns on bills, bonds and stocks are positively correlated, and all are negatively correlated with the rate of inflation. In the absence of short selling[2], the minimum variance portfolio is a portfolio comprised exclusively of Treasury bills. This is the portfolio that a life company would hold if it wished to minimize its underwriting risk in issuing an n-period, fully indexed annuity.

The expected real return on bills, or about one percent, is the upper bound on the competitive interest rate at which an n-period, indexed annuity would be sold in Canada. One percent is an upper bound because shareholders of the life company would have to be compensated for bearing

the residual investment risk, since the real return on bills is not constant. Note that this is the upper bound regardless of the portfolio that the life company actually chooses to hold. If the life company holds a portfolio of common stocks to hedge the indexed annuity, the higher expected real return is simply the compensation due shareholders for assuming the additional investment risk. The fact that one percent is an upper bound is also apparent if one considers the situation of an individual who wished to provide his own retirement annuity. If he wished to construct a variable annuity[3] which provided the most stable stream of real annuity payments, he would construct a variable annuity backed by Treasury bills. The expected real return on this portfolio would be about one percent, and the real stream of payments - unlike those of a fully indexed annuity - would still be uncertain. If the individual were sufficiently risk averse that he wished to purchase a fully indexed annuity, he would have to compensate shareholders of the life company for eliminating the residual investment risk. He would do so by accepting a lower interest rate on the indexed annuity than the expected real return on Treasury bills.

Bodie (1980) reviews similar empirical evidence for the United States and reaches a similar conclusion. The minimum variance portfolio is a portfolio consisting (almost)[4] exclusively of Treasury bills, and the expected real return on this portfolio is close to zero. He concludes that the competitive interest rate at which a life company would sell a fully indexed annuity in the United States would not exceed zero percent.

The interest rates at which n-period, indexed annuities would be priced competitively in North American capital markets may appear rather

low to many observers. In part, this result may be viewed as a direct cost of increased price level uncertainty. A life company must hedge an n-period, indexed annuity by holding a one-period rather than an n-period debt instrument. Notwithstanding this portfolio response, the residual investment risk borne by the life company is positive and thus exceeds that borne by the life company in a world where inflation uncertainty is absent. Consider, for example, what would happen if indexed debt instruments were bought and sold in North American capital markets. Modern finance theory indicates that an n-period, index bond would be priced to yield a higher expected real return than a one-period, index bond (or an indexed Treasury bill) if real interest rate risk is non-diversifiable. If so, a life company could hedge an n-period, indexed annuity by holding n-period, index bonds (or, more precisely, n-period index mortgages) and thus earn a higher expected real return than if it were constrained to hold only indexed Treasury bills. In the absence of a market for indexed debt instruments, the life company is effectively forced to hold only bills if it wishes to issue an n-period, indexed annuity and simultaneously minimize its risk exposure. The opportunity to bear real interest rate risk by perfectly hedging an n-period indexed annuity with an n-period, index mortgage is at present denied life companies in North America, although this opportunity would clearly exist in a world in which price level uncertainty was absent or in which indexed debt instruments did exist. Index bonds have recently been issued by the Government in the United Kingdom. Significantly, their market - determined yields do rise with their terms to maturity,[5] thus providing evidence that real interest rate risk is indeed non-diversifiable.

3. ANNUITIES WHOSE INFLATION PROTECTION IS "CAPPED"

There are two ways that inflation protection may be capped. The first, termed a fixed cap, limits the inflation adjustment to x percent even if the change in the consumer price index exceeds that amount. The second, termed a floating cap, provides the annuitant with inflation protection equal to the change in the consumer price index less y percent. The Province of Ontario, for example, limits the cost-of-living adjustments to 8 percent even if the inflation rate exceeds that amount. Fixed caps also exist in some private sector plans in Canada (Tomenson-Alexander (1978)), and in both private and state retirement systems in the United States (Myers (1978)). Anecdotal evidence suggests that at least some firms in Canada follow a policy of allowing their retired workers to absorb the impact of (say) the first 3 percent of any increase in the consumer price index, and then make ad hoc adjustments to offset the impact of inflation above this amount. In effect, they also provide annuities subject to a floating cap.

If one assumes that firms whose ad hoc adjustments are based on the floating cap make such adjustments with certainty, it is easy to value the annuities so protected. If the floating cap is the inflation rate less y percent, then the real value of the annuity will decline with certainty at y percent per year.[6] These payments must be discounted at the risk-free real rate of interest (i_f), to yield a net discount rate of i_f plus y. At present, there is no asset in the North American capital markets which provides a risk-free real rate of return. The expected real

return on the minimum variance or bills portfolio does, however, place an upper bound on i_f. Thus i_f is not likely to exceed one in Canada and zero in the United States, so that an upper bound can readily be placed on the net discount rate.

If the inflation rate (π) were known with certainty, then the valuation of an annuity subject to a fixed cap would be straightforward. If the certain inflation rate is less than the cap (i.e. $\pi \leq x$), then the real value of the annuity is constant and the relevant discount rate is simply i_f. If $\pi > x$, then the real value of the annuity declines with certainty at a rate equal to $\pi - x$. When this stream of payments is discounted at the risk-free real rate of interest, the end result is a net discount rate equal to $(\pi - x)$ plus i_f.

If the inflation rate is uncertain, the valuation of an annuity subject to a fixed cap is far more complicated. Let b_0 be the real value of the annuity payment at the beginning of the current period, let b_1 be the real value of the annuity payment at the beginning of the next period, and let $\tilde{\pi}_1$ be the realization of the inflation process during the period. If $\tilde{\pi}_1 \leq x$, then $b_1 = b_0$. If $\tilde{\pi}_1 > x$, then $b_1 = b_0 * \frac{(1+x)}{(1+\tilde{\pi}_1)} < b_0$. In effect, the annuitant has claim to the certain real benefit b_0, and has sold a call option on the rate of inflation with a striking price equal to x. If $\sigma(\tilde{\pi})$ is the relevant measure of risk for option pricing, then:

$$b_1 = b_0 - \text{CALL}(b_0, x, \sigma(\tilde{\pi})) \qquad (1)$$

The value of the call option is an increasing function of $\sigma(\tilde{\pi})$, and a decreasing function of x. The important point of (1) is that the valuation

of the capped annuity cannot be reduced to the choice of a single and (approximately) unchanging discount rate. To value (1), one must know the risk-free real rate of interest (in order to take the present value of the certain real component of b_1) and one must explicitly value the indicated call option at the beginning of each period. The value of this option is likely to exhibit considerable variation over time in view of the substantial serial correlation in the inflation rate. Less formally, if today's inflation rate is high, then tomorrow's inflation rate is also likely to be high. This result is an important input into any attempt to value the capped annuity, and provides an intuitive explanation as to why it cannot be assigned a unique and thus unchanging value.

4. ANNUITIES WHOSE INFLATION PROTECTION IS FROM "EXCESS" FUND EARNINGS

The lack of formal provisions notwithstanding, most large firms in Canada grant cost-of-living adjustments to retired plan members. These adjustments are apparently financed - at least in part - by pension fund earnings in excess of the interest rate assumed in the plan's valuation.[7] Indeed, future government initiatives are likely to formalize this use for "excess" earnings (Ontario (1982)). Large firms in the United States typically grant cost-of-living adjustments as well, although these are less generous and there is less evidence that they are linked to pension fund performance (Bankers Trust Company (1980)). Many observers, including Munnell (1982), have recommended that excess earnings be used by private pension plans in the United States to provide inflation protection.

The valuation of an annuity whose inflation protection is delivered through excess earnings is straightforward if the annuity can be shown to be equivalent to a standard variable annuity. Assume that the plan sponsor will lower the nominal value of the annuity payment if pension fund earnings

fall short of the assumed interest rate, so that there is no nominal floor on the annuity payment. Assume also that the plan sponsor will apply all fund earnings in excess of the assumed interest rate to escalate the annuity payment, even if this means that the real value of the payment increases. If so, one can proceed as if two separate transactions occur. First, the plan member receives a lump sum payment at the date of his retirement equal to the promised pension benefit capitalized at the plan's assumed interest rate. This first step establishes the assumed interest rate as the correct discount rate to value the promised stream of pension payments. Secondly, the plan member uses the lump sum payment to purchase a standard variable annuity, and the assumed interest rate is used to set the initial annuity payment. This initial annuity payment is then equal to the promised pension benefit, and rises or falls as the nominal return on the supporting assets exceeds or falls short of the assumed interest rate.[8] Note that the assumed interest rate is the correct discount rate independent of the assets actually held in the pension fund.

There are, however, two potential complications. The first occurs if there is a floor and/or ceiling on the use of excess earnings. The second occurs if a non-market measure of pension fund performance is used to define excess earnings.

Consider, for simplicity, only the case in which there exists a nominal floor on the annuity payments. This possibility is motivated by the stylized fact that the ad hoc adjustments provided by most firms tend to be permanent. Once a nominal pension has been increased, it is never reduced. If these enrichments are financed by excess earnings, the implication is that the plan sponsor absorbs the shortfall when pension fund

earnings are less than the assumed interest rate. In effect, the plan sponsor guarantees that the pension fund will never earn less than the assumed interest rate in the excess earnings scheme. This is explicitly the case in the widely-cited Rockefeller Foundation Plan in the United States, which grants permanent cost-of-living increases equal to the average prime interest rate for the year less 3 percent (Heaton (1980)).

If the annuitant is provided with a variable annuity subject to a nominal floor, the use of the assumed interest rate in the excess earnings scheme to value the annuity will in general understate its value. In effect, the pensioner receives a standard variable annuity plus a put option on the nominal investment earnings of the pension fund with a striking price equal to the assumed interest rate. If A_0 is the lump sum necessary to buy a standard variable annuity, RV is the plan's assumed interest rate and $\sigma(\tilde{R})$ the measure of the risk of the nominal return that is relevant to option pricing, then the lump sum necessary to buy this same annuity subject to a nominal floor is A_0' such that:

$$A_0' = A_0 + \text{PUT}(A_0, \text{RV}, \sigma(\tilde{R})) \tag{2}$$

If the assumed interest rate increases, then the put option becomes more valuable since the probability increases that pension fund earnings will fall short of this guaranteed rate. A similar result obtains if the risk of the assets in the pension fund increases. The latter implies, unlike the case of a standard variable annuity, that the value of the annuity depends upon the assets in the pension fund. Unless the value of the put option is zero, the use of the assumed interest rate in the excess earnings scheme to value the annuities so protected will understate

their value. If the annuity provides for a single payment at the end of the first period, then the value of this option will be zero if the pension fund holds only the risk-free nominal asset (bills) and if the bill rate exceeds the plan's assumed interest rate. In the more relevant multiperiod setting, the put option will have zero value only if there is no probability in some future period that the bill rate will fall beneath the assumed rate.

If unrealized capital gains or losses are not included in the measure of fund earnings, as is the explicit practice of the Teachers Insurance and Annuity Association (TIAA) in the United States, then intergenerational risk sharing is imposed on plan members. Suppose the pension fund holds only long-term bonds. If long-term rates rise in anticipation of future inflation, the resulting capital losses go unrecorded. Excess earnings remain relatively stable, implying that the real value of the annuity payments will decline in subsequent periods when the inflation rate is higher. If it is known that unrecognized capital losses exist on the date that an individual retires, then the value of the annuity to him will be overstated, and conversely if unrecognized capital gains are inherited. On average, however, the assumed interest rate in the excess earnings scheme remains the correct discount rate to be used in total compensation comparisons. So long as participation is mandatory, so that retired plan members cannot game againt the non-market measure of fund performance, the exclusion of unrealized capital gains and losses does not complicate the valuation problem.[9]

5. VALUING ANNUITIES WHICH ARE PARTIALLY INDEXED TO THE INFLATION RATE

Private surveys often focus on the average degree of cost-of-living protection provided by ad hoc adjustments as a summary measure to be used in total compensation comparisons (Tomenson-Alexander (1978)). For simplicity, it is useful to consider the problem of valuing an annuity which receives adjustments equal to z percent of the inflation rate.[10] The special cases where z equals zero (i.e. the annuity is purely nominal) and where z equals 100 (i.e. the annuity is real) are ruled out by assumption.

If these adjustments are made with certainty, then the valuation procedure is conceptually straightforward. First, one forecasts the inflation rate (π^*) expected to prevail over the life of the annuity, as well as specifying its risk characteristics. The real stream of annuity payments is expected to decline at a rate equal to $(1-z)\pi^*$ percent per year. The appropriate interest rate to calculate the present value of this stream of payments is the expected real return on that efficient portfolio which generates a real income stream with the same risk characteristics. This expected real return, plus $(1-z)\pi^*$, is then the net discount rate to be used to value the indicated annuity.

In practice, it is likely to prove difficult to obtain a precise estimate of this net discount rate. In large part, this is due to the necessity of specifying in advance the distribution of future rates of inflation. In addition, the procedure as outlined invokes the strong assumption that adjustments equal to z percent of the inflation rate are made with certainty. In view of the fact that firms have elected not to incorporate this provision into their formal pension contracts, but to proceed on an ad hoc basis, this assumption is clearly suspect.

6. VALUING NOMINAL ANNUITIES WHICH RECEIVE NO INFLATION PROTECTION

At present, life companies in Canada sell life annuities to individuals aged 65 at an interest rate of 14 percent. This is obviously the correct discount rate to value a purely nominal stream of promised pension benefits. In spite of the fact that pension benefits may be purely nominal (i.e. the sponsoring firm may have no intention of providing cost-of-living adjustments), annuities due under the terms of pension plans in Canada are typically valued at interest rates which do not exceed 7 percent (Pesando (1981)). If workers are rational, they presumably see through this accounting veil and value their accruing pension benefits at prevailing annuity rates in order to make the appropriate wage concessions. As discussed in the next section of this paper, however, it appears likely that government officials use interest rates closer to those assumed by actuaries than those prevailing in the capital market to value nominal pension benefits.

7. INFLATION PROTECTION ACCORDED PENSIONS AS AN INPUT INTO TOTAL COMPENSATION COMPARISONS: THEORY VERSUS PRACTICE

It is instructive to contrast the valuations conducted by the Government of Canada, which are probably typical, with those implied by modern finance theory. To conduct total compensation comparisons, the Government must value its fully indexed annuities relative to a composite of those provided in the private sector.

By combining a nominal interest rate assumption of 6½ percent with an inflation rate assumption of 3 percent, the Government effectively uses a discount rate of (approximately) 3½ percent to value the fully indexed

annuities due under the terms of its pension plan. As noted, the risk-free real rate of interest in Canada is likely to be bounded above by one percent. Abstracting from the possible political risk that indexing provisions might be cut back or eliminated at some future date,[11] the Government is clearly understating the value of its fully indexed annuities.

To value annuities which receive no inflation protection, the Government uses its assumed nominal interest rate of 6½ percent. This is far beneath the 14 percent at which life companies in Canada now sell life annuities to those aged 65.

To value annuities which receive inflation protection through excess earnings, the Government uses the interest rate used to define excess earnings. In the absence of floors or ceilings in the excess earnings scheme, this is the correct procedure.

To value annuities which are partially indexed, the Government first postulates that adjustments in the private sector typically equal 50 percent of the inflation rate, and proceeds as if each firm made such adjustments with certainty. The Government then uses its assumed inflation rate of 3 percent to imply that inflation adjustments will equal 1½ percent per year. This annual rate of escalation of the annuity payments is then subtracted from the assumed nominal interest rate of 6½ percent to yield a net discount rate of 5 percent. At present, the nominal yield on long-term Canada bonds exceeds 15 percent, suggesting an implicit inflation forecast of at least 10 percent. This implies that the expected rate of decline in the real value of the partially indexed annuity is at least 5 percent per year. Because the real value of these payments is uncertain, they must be discounted at a rate in excess of the risk-free rate. If

a real interest rate of (say) 3 percent is appropriate, this would imply a net discount rate of 8 percent, which considerably exceeds the 5 percent used by the Government.

Because the most visible plans in Canada which contain capped indexing provisions are in the public sector, the Federal government does not appear to explicitly value them. The Province of Ontario, in what may be a representative response, uses an assumed inflation rate which is less than its 8 percent fixed cap to project its nominal annuity payments, and then discounts them by an assumed nominal interest rate. In the 1976 valuation (Ontario (1978)), the Province uses a nominal interest rate of 7.25 percent in conjunction with an inflation assumption of 5½ percent. By implicitly treating these assumptions as certain, the Province implies that its real annuity payments will be constant. If true, this would require that they be discounted at the risk-free real rate of interest, which is less than the real rate of (approximately) 1¾ percent implied by the inflation and nominal interest rate assumptions. Because the inflation rate is uncertain, and thus the call option in (1) has a positive value, it is not possible without more formal enquiry to establish whether or not the capped annuities are indeed being undervalued.[12]

To sum up, the Government of Canada employs procedures which understate the value of indexed annuities and typically overstate the value of annuities which receive incomplete inflation protection. In part, this is due to excessive reliance on standard actuarial valuations, which ignore risk and typically use non-market interest rate and inflation assumptions. The fact that the same actuarial assumptions are used to value public and private sector plans, although touted by practitioners (Martel et. al.

(1980), Carow (1981)), is largely irrelevant. Whatever their merits, standard actuarial valuations do not provide a satisfactory basis for calculating the value to employees of annuities which differ sharply in their degree of inflation protection.

Uncritical dependence on standard actuarial valuations is certainly not unique to the Government of Canada. The Civil Service Retirement System in the United States was valued in 1979 with an interest rate assumption of 7 percent and an inflation rate assumption of 6 percent. These assumptions, which the Office of Personnel Management (1981) then used in its total compensation comparisons, suggest that fully indexed annuities are valued at a real interest rate of one percent.[13] Although Bodie (1980) argues that a real interest rate of zero may be more appropriate, this discrepancy is clearly less important than its Canadian counterpart. Of perhaps more consequence is the apparent use by the Office of Personnel Management of the nominal interest rate assumption of 7 percent - based on the assumed inflation rate of 6 percent - to value the nominal annuities provided by most private sector plans. This 7 percent is considerably less than the then prevailing 9 to 10 percent yield on long-term U.S. Government bonds, and suggests that the value of these nominal annuities was overstated. As in Canada, there is no evidence that the issue of inflation uncertainty and its ramifications was addressed.

Finally, because pension costs as a percent of payroll are significant,[14] the potential scope for errors in these total compensation comparisons is large. The Government of Canada uses a real rate of 3½ percent to value its fully indexed annuities, so that the present value of each dollar of a 15-year annuity is calculated to be $11.52. At an interest rate of

one percent, an upper bound on the risk-free real rate, this annuity would cost $13.86 or 20.3 percent more. The present value of this annuity, when treated as nominal and valued at the assumed interest rate of 6½ percent, is $9.40. At a market interest rate of 14 percent, this annuity would sell for $6.14 or 53.1 percent less. Thus the Government understates the value of the indexed annuity that it provides by 20 percent, while overstating the value of the nominal annuity provided in the private sector by 53 percent.

8. CONCLUSIONS

The procedures suggested by modern finance theory for valuing annuities which differ in their degree of inflation protection are summarized in Table 1. These are not the procedures used by the Federal government in Canada, which are probably representative of those used by governments in general. In Canada as well as in the United States, inflation protection is typically far better in public than in private sector pension plans. The analysis in this paper suggests that governments may typically undervalue pensions provided in the public sector, while overvaluing those provided in the private sector. The net result is that total compensation comparisons are seriously flawed. To the extent that these comparisons guide pay policy in the public sector, and to the extent that total compensation paid in the public sector is not disciplined by competitive forces,[15] total compensation in the public sector may be too high on this account.

FOOTNOTES

1. In Canada in 1980, 98.3 percent of pension plans in the private sector (covering 95.1 percent of plan members) had no formal provisions to provide cost-of-living adjustments (Statistics Canada (1982)). Myers (1978) reports that automatic-adjustment provisions are also rare among private sector plans in the United States. The discussion in the text presumes, as appears to be the case, that nominal annuity payments will be reduced if the inflation rate is negative. If this is not the case, then the procedures discussed in the text will understate the value of "indexed" annuities so long as there is a non-zero probability of deflation in some subsequent period.

2. If short selling is permitted, the minimum variance portfolio is long on bills (108 percent), and short on both bonds (minus 7 percent) and stocks (minus one percent). The mean bond return reported in Table 2 cannot be viewed as an equilibrium return, and a corresponding caveat is in order. In a portfolio consisting only of bills and stocks, minimum variance is obtained with a long position in bills (101 percent) and a short position in stocks (minus one percent).

3. Let A_0 be his initial capital. Let RV be the assumed interest rate. Then the base payment B_0 of the N-period variable annuity is:

$$B_0 = RV[1 - (1+RV)^{-N}]^{-1} * A_0 \qquad (F1)$$

If $\tilde{R}_t$ is the realized nominal return in period t on the underlying portfolio, then the nominal benefit B_t paid in period t

$(t = 1 \text{ to } N)$ is:

$$B_t = \frac{(1+\tilde{R}_t)}{(1+RV)} * B_{t-1} \qquad (F2)$$

4. Bodie (1980) argues that the minimum variance portfolio in the United States, although comprised mostly of Treasury bills, would include a small (long) position in a diversified set of commodity futures.

5. As of 31 March 1982, the 2's of '96 were priced at par to yield 2 percent; the 2's of '06 were priced at 96 to yield 2.32 percent; and the 2½'s of '11 were priced at 98 to yield 2.58 percent.

6. For convenience, continuous time results are cited here and elsewhere in the text, unless explicitly noted to the contrary.

7. The pension plans provided by most large firms in Canada and the United States are defined benefit plans. These are plans in which the employee receives a benefit equal to a given fraction of his average or of his final earnings for each year of service, or a fixed dollar amount for each year of service. If the pension fund earns more than the interest rate used to value the plan, which range from 4 to 7 percent in Canada, the plan experiences an actuarial surplus.

8. Let B_0 now represent the base pension payment defined by the terms of the plan. Using the notation of footnote 3, it is as if the retiring plan member receives the lump sum payment A_0:

$$A_0 = B_0 \div RV[1 - (1+RV)^{-N}]^{-1} \qquad (F3)$$

He then uses A_0 to purchase a standard variable annuity, in which

RV is the assumed interest rate. His base pension payment is thus B_0, as required by the plan, and the actual payments are those indicated by (F2). Note that under excess earnings, the defined benefit plan is effectively transformed into a defined contribution plan at the date of the plan member's retirement. This appears to be an additional innovation in response to the apparent increase in price level uncertainty, as discussed by Pesando (1982).

9. Since investment opportunities have not changed, this intergenerational risk sharing does not improve the overall efficiency of risk bearing in the economy. A potential disadvantage is that retired workers may fail to realize the implication of (say) a rise in long-term interest rates which produces capital losses on the bond portfolio. These losses, although unrealized, require an immediate reduction in real consumption if expected real consumption is to be stabilized over the remaining lifetimes of the annuitants (Rea (1981)).

10. In their study of inflation protection in the U.K., Brealey and Hodges (1980) address the problem of valuing fully indexed annuities provided to government employees against private sector pensions which are presumed to receive cost-of-living adjustments equal to 62 percent of the inflation rate.

11. After the first draft of this paper was complete, the Government of Canada announced as part of its austerity measures in its (June) 1982 budget that the cost-of-living adjustments to the indexed pensions of civil servants would be limited to 6 percent and 5 percent, respectively, in 1982 and 1983. Subsequent discussion in the text

continues to abstract from political risk, although this caveat is clearly important. There is no apparent way to quantify political risk, other than to note that it would cause the risk-free real rate to overstate the value of the ostensibly indexed annuities. It should be noted that the possibility of capping, either temporarily or permanently, the inflation adjustments due retired Federal government employees has also been raised in the United States.

12. The procedure used by either level of government to value annuities subject to a floating cap is not known. Parallel treatment, however, would be to escalate the projected annuity payments by the assumed inflation rate less y percent, and then discount this stream of payments by the assumed nominal interest rate.

13. Munnell (1982) discusses a proposal which would permit individuals, at the time of their retirement, to purchase a limited quantity of index bonds from the U.S. Government. These bonds would provide a guaranteed real return of one percent. This guaranteed rate, which implies that the bonds would be the equivalent of index Treasury bills, is clearly an above market rate based on the analysis of Bodie (1980). Although real Treasury bill yields in both Canada and the United States are currently well above their historical means of one and zero percent, there is as yet no persuasive case for arguing that the expected real return on bills has permanently increased.

14. The Government of Canada estimates that its share of pension costs for current service is 9.5 percent of salary, compared to 4.5 to 5.5 percent of salary for private sector firms (Martel et. al. (1980)).

As reported by Gajda (1981), employer contribution rates (which appear to blend current and past service costs) for municipal employees in the United States may exceed 70 percent of payroll! Note also that these contribution rates are as conventionally measured and do not take into account the limitations analyzed in the text.

15. Carow (1981) acknowledges the possibility that there may be longer queues of workers for federal government jobs in the United States than for nonfederal jobs, and lower quit rates for civil service employees. The fact that governments in both Canada and the United States appear to underprice their fully indexed annuities suggests that it is not possible to infer that these workers are sufficiently risk averse to demand fully indexed pensions. This issue is discussed by Feldstein (1981) and Pesando (1981).

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Table 1

PENSIONS (ANNUITIES) WITH DIFFERENT TYPES OF INFLATION PROTECTION

Type of Annuity	Inflation Protection	Where Found	Method of Valuation (Modern Finance Theory)
I Indexed Annuity	$A_t = \tilde{\Pi}_t$	Federal Governments, Canada and the U.S.; some other governments	i_f
II "Capped" Annuity			
Fixed Cap	$A_t = \begin{cases} \tilde{\Pi}_t \text{ if } \tilde{\Pi}_t \leq x \\ x \text{ if } \tilde{\Pi}_t > x \end{cases}$	Private sector, some governments, both Canada and the U.S.	i_f - Call(b_o, x, $\sigma(\tilde{\Pi})$)
Floating Cap	$A_t = \tilde{\Pi}_t - y$	"Stylized" practice, private sector in Canada	$i_f + y$
III Annuity Subject to "Excess" Earnings Escalation			
No Floor (Variable Annuity)	$A_t = (1+\tilde{R}_t)/(1+RV) - 1$	"Stylized" practice, private sector in Canada; Rockefeller Foundation in U.S.	RV
Nominal Floor	$A_t = \begin{cases} (1+\tilde{R}_t)/(1+RV) - 1 \text{ if } \tilde{R}_t \geq RV \\ 0 \text{ if } \tilde{R}_t < RV \end{cases}$	"Stylized" practice, private sector in Canada and the U.S.	RV + Put (A_o, RV, $\sigma(\tilde{R})$)
IV Partially indexed Annuity	$A_t = z\,\tilde{\Pi}_t$ $\quad 0 < z < 1$	"Stylized" practice, private sector in Canada and the U.S.	$i_z + (1-z)\Pi^*$
V Nominal Annuity	$A_t \equiv 0$	Private sector, Canada and the U.S.	r_f

where A_t = cost-of living adjustment in period t

$\tilde{\Pi}_t$ = inflation rate in period t

x = amount (expressed as a percentage) of the fixed cap

y = amount (expressed as a percentage) of the floating cap

$\tilde{R}$ = nominal return on the pension fund in period t

RV = assumed interest rate used to value the pension plan

z = fraction of the inflation rate that is offset by the partially indexed annuity

i_f = risk-free real rate of interest

b_o = real value of base annuity payment

A_o = capital sum necessary to purchase variable annuity with same base payment and assumed interest rate

i_z = expected real return on efficient portfolio with same risk characteristics as partially indexed annuity

Π^* = expected rate of inflation

r_f = risk-free nominal rate of interest

Table 2

Real Rates of Return in Canada 1953-1980, Bills, Bonds and Stocks

Year	91-day Treasury Bills	Long-term Canada Bonds	Common Stocks	Inflation (C.P.I.)
1953	1.71	3.61	2.72	0.0
1954	0.83	9.20	38.46	0.60
1955	1.32	-0.61	27.32	0.30
1956	0.17	-6.45	8.90	3.10
1957	1.58	4.08	-22.37	2.15
1958	-0.26	-8.17	28.16	2.52
1959	3.39	-5.84	3.04	1.37
1960	1.83	5.56	0.33	1.35
1961	2.68	9.48	32.41	0.13
1962	2.42	1.42	-8.70	1.59
1963	1.70	2.66	13.50	1.83
1964	1.79	4.48	22.96	1.93
1965	1.05	-1.87	3.57	2.90
1966	1.39	-1.92	-10.28	3.55
1967	0.48	-5.99	13.32	4.14
1968	2.09	-4.37	17.53	4.09
1969	2.50	-6.47	-5.29	4.58
1970	4.46	19.67	-5.01	1.46
1971	-1.41	5.98	2.84	5.04
1972	-1.46	-3.76	21.14	5.09
1973	-3.34	-6.78	-8.70	9.12
1974	-4.13	-12.48	-34.76	12.46
1975	-1.91	-6.09	9.31	9.48
1976	2.88	11.99	4.82	5.82
1977	-1.98	-3.32	0.43	9.50
1978	0.23	-6.56	19.25	8.43
1979	1.76	-11.14	31.64	9.76
1980	1.42	-8.22	16.83	11.21
Mean	0.82	-0.78	7.97	4.41
Standard Deviation	2.02	7.60	17.02	3.67

Correlation Coefficients	(1)	(2)	(3)	(4)
(1) Bills	-	.526	.216	-.623
(2) Canada's		-	.048	-.599
(3) Stocks			-	-.226
(4) Inflation				-

Notes: Data are drawn from C.G. Carlton, D.D. Ezra and K.P. Sharp, "Canadian Investment Returns and Other Economic Statistics, 1926-1980", where details regarding the calculation of the individual series may be found. The annual return on Treasury Bills is obtained by the successive purchase of 91-day Treasury Bills at the end of each quarter.