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PRICE DESYNCHRONIZATION AND PRICE LEVEL INERTIA

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Abstract

If price decisions are taken neither continuously nor in perfect synchronization, the process of adjustment of all prices to a new nominal level will imply temporary movements in relative prices. It might then well be that, to avoid these movements in relative prices, each price setter will want to move his own price slowly compared to others. The result will be a slow movement of all prices to their new nominal level, and substantial inertia of the price level. This paper formalizes this intuitive argument and reaches four main conclusions:

(1) Even small departures from perfect synchronization can generate substantial price level inertia.

(2) If price decisions are desynchronized, even anticipated movements in money will usually have an effect on economic activity. It is however possible to find paths of money deceleration which reduce inflation at no cost in output.

(3) Price desynchronization has implications for relative price movements as well as for the price level. Goods early in the chain of production have more price and profit variability than goods further down the chain.

(4) Price inertia, if it is due to price desynchronization, may be difficult to remove. It may well be that, given the timing decisions of others, no agent has an incentive to change his own timing decision: the time structure of price desynchronization may be stable.

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I. Introduction

It is often informally argued that given the complexity of the price system and the inherent problems of coordination, the apparent inertia of the price level should come as no surprise.[1] A rather appealing argument along these lines is the following:

When a nominal disturbance requires a change in the price level, what is required is not a change of a single price, but a change of a complex structure of final good, intermediate good and input prices. Price decisions for each of these goods are not taken continuously. Furthermore, price decisions across goods are unlikely to be perfectly synchronized. The process of adjustment of all prices to a new nominal level will therefore imply movements of relative prices along the way. If price setters do not want large changes in relative prices, the path of adjustment of all prices may be slow, the price level may adjust slowly.

The purpose of this paper is to formalize this argument and to see whether and how it survives formalization. The paper focuses on three sets of questions:

The first is whether desynchronization of individual price decisions can generate "substantial" price inertia. It is obvious that, with so many price decisions, the price level will not adjust overnight to changes in aggregate demand; the question is whether, if each price is set for a relatively short period of time, say a month or two at most, desynchronization can generate the degree of price inertia we appear to have in the United States. The answer of the paper is that this is indeed possible.

The second set of questions addresses whether the price level inertia so generated coincides with the usual notion of inertia or "stickiness." Does it for example imply that decreases in money or decelerations in money growth necessarily lead to recessions? The answer is mixed: In general, movements in money will lead to movements in real money balances and economic activity. There are however paths of monetary deceleration which lead to disinflation with no output loss. These paths are reasonable and, apart from issues of credibility, easy to implement.

The third set of questions considers the implications of desynchronization for the relation between disturbances, the price level and the structure of relative prices. This is of interest both in itself and because it provides a way of differentiating this theory of price inertia from other theories and potentially testing it.[2] Desynchronization implies snake effects, i.e. movements in factor prices slowly transmitted to intermediate and final good prices. It also implies more variability of profits and prices for primary inputs than for intermediate goods, for intermediate goods than for final goods; these implications seem to be in accordance with facts.

This paper therefore suggests that desynchronization of price decisions is capable of generating price level inertia. If price level inertia is indeed partly due to desynchronization, the prospects for reducing it are not good. Given the time structure of price decisions, each price setter chooses its price optimally and frequently. Reducing inertia requires better overall synchronization of price decisions; this may be difficult to achieve, either by agents or by policy.

The paper is organized as follows: Section II presents the model. Sections III and IV look at the implications for price level inertia and the effects of monetary policy. Section V looks at the implications for relative prices. Section VI concludes.

II. The Model

In order to focus later on the effects of desynchronization, I start with an economy in which all price decisions are perfectly synchronized.

Equilibrium with Synchronized Prices

The economy is characterized by its technology, a specification of input supply and of output demand.

Final output is produced in n stages, each of them carried under constant returns to scale by competitive firms. Technology is given by n relations:

$$(1) \qquad y_i = y_{i-1} + \theta_i \qquad\qquad i = 1, \cdots, n.$$

y_i denote good i so that y_o denotes the primary input and y_n the final output. All variables, here and in the rest of the paper, are in logarithms. The θ_i are constants which are unimportant for our purposes and will be deleted in what follows. Production is instantaneous[3] and, to avoid issues of inventories, all goods are perishable.

Competitive zero profit equilibrium implies that, if p_i is (the log of) the price of good i, the following relations hold (forgetting the θ_i):

$$(2) \qquad p_i = p_{i-1} \qquad\qquad i = 1, \cdots, n \qquad\qquad \Rightarrow p_n = p_o$$

Increasing the number of production stages, n, keeping the sum of θ_i's constant, allows us to increase the number of price decisions, while leaving the technology unchanged.[4,5] In this economy with synchronized prices, the number of price decisions is clearly irrelevant: y_n is always equal to y_o and p_n to p_o.

The model is closed by a specification of input supply and output demand:

(3) $$y_o = \beta(p_o - p_n) + \xi \qquad \beta \geq 0$$

(4) $$y_n = m - p_n$$

Input supply is an increasing function of its real price and of a disturbance ξ.[6] Output demand depends positively on real money balances.[7]

Equilibrium is characterized by the price relations given by (2) and equilibrium in the primary input market; the derived demand for the primary input must equal the supply:

(5) $$\beta(p_o - p_n) + \xi = m - p_n$$

Combining (2) and (5) gives:

$$y_n = y_o = \xi \quad ; \quad p_n = \cdots = p_i = \cdots = p_o = m - \xi$$

Money is neutral and affects only the level of all prices. Supply disturbances increase output, decrease all nominal prices and leave relative prices unchanged.

Price Desynchronization

I now relax the assumption that price decisions are taken every period and are perfectly synchronized:

All price decisions are now taken every two periods. The basic period is presumably short and can be thought of as a month at most.[8]

Price decisions are not all taken at the same time. Half of them are taken every period, in the following way: Firms at the same stage of production take decisions at the same time for two periods. At even stages (i even) firms take decisions at t, t+2 and so on; at odd stages (i odd) firms take decisions at t-1, t+1 and so on. n is assumed for convenience to be even, so that firms producing y_n take decisions at t, t+2, $\cdots$, firms producing y_{n-1} take decisions at t-1, t+1, $\cdots$, suppliers of the primary input take decisions at t, t+2, $\cdots$.

Firms choosing p_i at time t for periods t and t+1 face two possibly different input prices for t and t+1. The competitive zero profit condition used above is now replaced by an expected zero profit condition over the two periods. This is formalized by:[9]

$$p_{it} = \frac{1}{2}\,(p_{i-1\ t-1} + E(p_{i-1\ t+1}|t)) \qquad i = 2, 4, \cdots, n. \tag{6}$$

$E(\cdot|t)$ denotes the expectation conditional on information available at time t. $p_{i-1\ t-1}$ is the current input price in period t which was set in period t-1 and $E(p_{i-1\ t+1}|t)$ the expected input price for period t+1. A corresponding formula holds for i odd, at time t-1 or t+1.

As nominal prices are fixed at each stage for two periods, they may not clear the market in both periods and an assumption must be made about quantity determination. I assume the outcome to be demand determined: when a firm fixes its price for two periods, it stands ready to supply on demand. This is feasible as production is instantaneous and all input suppliers also supply on demand. Demand for the final good therefore determines the demand for intermediate inputs and for the primary input.[10]

Prices in the primary input market are set in period t at the average expected market clearing levels over periods t and t+1. For convenience, we assume m and ξ to move only every two periods, so that $m_t = m_{t+1}$ and $\xi_t = \xi_{t+1}$. As $p_{nt} = p_{nt+1}$, the derived demand and the supply of the primary input are the same in periods t and t+1. The primary input price for t and t+1 is therefore given by:[11]

$$(7) \qquad \beta(p_{ot} - p_{nt}) + \xi_t = m_t - p_{nt}$$

To summarize, all firms choose their relative price every two periods. Their price decision depends on current and expected input prices for the next two periods. Half of the prices change every period.

The only deviation from the flexible price world is the presence of desynchronization: other sources of price inertia are excluded in order to isolate the effects of desynchronization. This excludes in particular such elements as labor contracts with nominal wages predetermined for long periods of time.[12] As a result, it is not clear whether the primary input should be thought of as labor or as a raw material. If thought of as labor, its price has probably more inertia than formalized in equation (7).

Equilibrium with Desynchronized Prices

Equilibrium is now characterized by equations (6) and (7). Input market equilibrium, equation (7), gives us a first relation between p_{ot} and p_{nt}, given m_t and ξ_t. The other relation between p_{nt} and p_{ot} follows from the set of pricing relations given by (6). We now derive it by recursive substitution.

Starting from i = n, equation (6) gives:

$$p_{nt} = \frac{1}{2}\left(p_{n-1\ t-1} + E(p_{n-1\ t+1}|t)\right) \tag{8}$$

For i = n-1, it gives for t-1 and t+1:

$$p_{n-1\ t-1} = \frac{1}{2}\left(p_{n-2\ t-2} + E(p_{n-2t}|t-1)\right)$$

$$p_{n-1\ t+1} = \frac{1}{2}\left(p_{n-2t} + E(p_{n-2\ t+2}|t+1)\right)$$

Assuming rational expectations, taking expectations of $p_{n-1\ t+1}$ at time t, using iterated expectations and replacing in equation (8) gives:

$$p_{nt} = \left(\frac{1}{2}\right)^2 \left[p_{n-2\ t-2} + E(p_{n-2\ t}|t-1) + p_{n-2\ t} + E(p_{n-2\ t+2}|t)\right] \tag{9}$$

By induction, we can express p_n as a function of p_o:

$$\begin{aligned} p_{nt} = 2^{-n} \Bigg[& \sum_{i=1}^{n/2} \sum_{j=0}^{(n/2)-i} b_{nj}\, E(p_{ot-2i}|t - \tfrac{n}{2} - i + j) \\ & + \sum_{j=0}^{n/2} b_{nj}\, E(p_{ot}|t - \tfrac{n}{2} + j) \\ & + \sum_{i=1}^{n/2} \sum_{j=0}^{(n/2)-i} b_{nj}\, E(p_{ot+2i}|t - \tfrac{n}{2} + i + j) \Bigg] \end{aligned} \tag{10}$$

with $\quad b_{nj} \equiv \binom{n}{j} - \binom{n}{j-1} \quad ; \quad b_{no} \equiv 1$

This formula is quite formidable but has a simple structure. Consider first the case of perfect foresight, so that expectations are equal to actual values. This gives:

$$(10') \qquad p_{nt} = 2^{-n}\left[\sum_{i=1}^{n/2}\binom{n}{(n/2)-i}p_{ot-2i} + \binom{n}{n/2}p_{ot} + \sum_{i=1}^{n/2}\binom{n}{(n/2)-i}p_{ot+2i}\right]$$

This shows the first effect of desynchronization: the price level depends on input prices up to n periods in the past and n periods in the future. The weights are simply the coefficients of a binomial expansion, normalized by their sum, 2^n.

When we relax the assumption of perfect foresight and allow for uncertainty, actual values of input prices in (10') are replaced by expectations. The price level depends then on three sets of terms. The first double sum involves past input prices, both actual and expected; the term in p_{ot-2i} for example includes both the actual value of p_{ot-2i} and the values of p_{ot-2i} expected prior to t-2i, from $t - \frac{n}{2} - i$ to t-2i-1. The second sum involves both the actual value and past expectations of the current input price. The third involves both past and current expectations of future input prices. Note, and we shall return to this below, that many terms in this last double sum are past expectations of future prices and thus are predetermined at time t. Thus, the symmetry between the effects of the future and the past which obtains under perfect foresight (equation (10')) does not obtain under uncertainty and rational expectations.

A visually more explicit representation of (10) is given in Figure 1 for n = 10. Each line represents a set of terms in equation (10). The right end

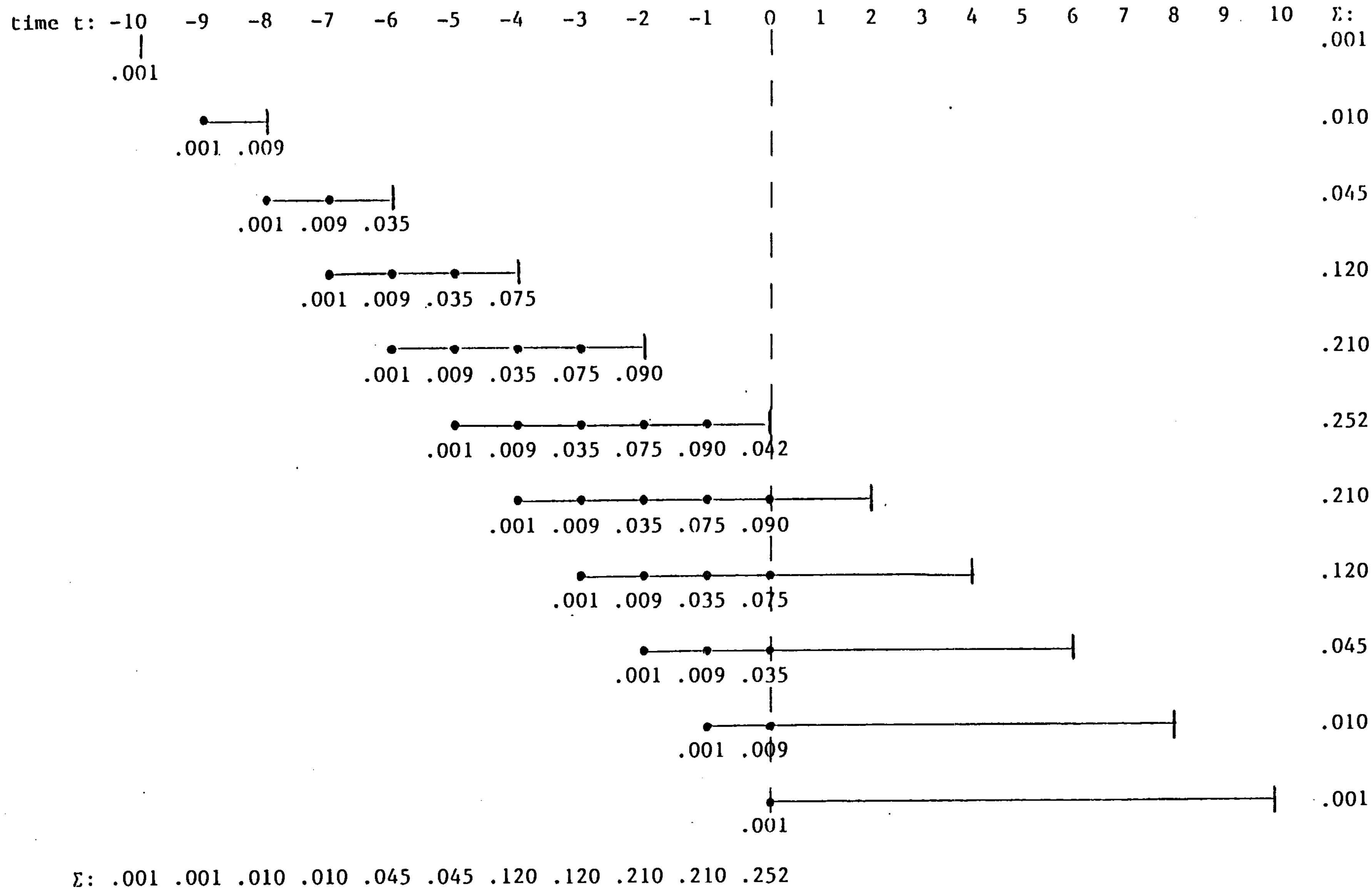

Figure 1. Weights on the primary input price for n = 10.

of a line indicates for what period the expectation of p_o is held. The dots on each line indicate when these expectations were formed. The numbers under the dots are the relative weights, $2^{-n} b_{nj}$. All elements strictly to the left of the vertical line t = 0 are predetermined at time t.

III. Price Level Inertia and the Number of Price Decisions

A Simple Measure of Inertia

Producers of the final good freely choose their own nominal price, the price level, every two periods and would not characterize it as sluggish. Their price decision however depends directly and indirectly on past input prices and, in a well-defined sense, the price level is sluggish: looking at equation (10), we can usefully think of the price level as the sum of 2^n components, some of them determined in the past and thus predetermined at time t, some of them free to move at time t.

This suggests a simple measure of price level inertia, namely the ratio of the number of predetermined components to the number of nonpredetermined components in (10). From equation (10), this ratio, R, is given by:[13]

$$R = 1 - 2^{-n} \binom{n}{n/2}$$

$$= .5 \text{ for } n = 2, \ .75 \text{ for } n = 10, \ .92 \text{ for } n = 100.$$

Thus this ratio is higher than the proportion of prices which are not free to adjust at any given time--one half--and is increasing with the number of price decisions. If n is large, most of the elements which compose the price level are predetermined.

That, as n increases, desynchronization implies a dependence of the price level on input prices further in the past and expected further in the future is quite intuitive. That, as n increases, the degree of predetermination increases is less intuitive. Figure 2 helps understand why by showing how the price level depends on input prices, as we go down the chain of production. Any element below the line is predetermined and thus can only depend in turn on predetermined elements. Any element above the line is not predetermined and may in turn depend on both predetermined and nonpredetermined elements. p_{nt} depends on predetermined $p_{n-1\ t-1}$ and nonpredetermined $E(p_{n-1\ t+1}|t)$. $E(p_{n-1\ t+1}|t)$ however depends itself on partly predetermined elements such as $p_{n-3\ t-1}$. As we extend the graph to the right, more and more elements go below the line: the ratio, R, of predetermined to nonpredetermined elements increases and tends to 1 as n gets large.

This measure of price level inertia is a bit crude: it tells us how much of the price level is predetermined and cannot change in response to disturbances in the current period, but tells us nothing about the path of price level adjustment thereafter. We now look at the complete path; this requires solving the model.

The Effects of an Increase in Money

As characterized by (10), the effect of the input price, actual or expected, on the price level is unambiguously positive. The effect of the price level on the input price is however ambiguous, as shown in (7). An increase in the price level decreases real balances, aggregate demand, the derived input demand and thus the equilibrium real input price; the net effect of a higher price level and a lower real price is ambiguous. If $\beta = 1$,

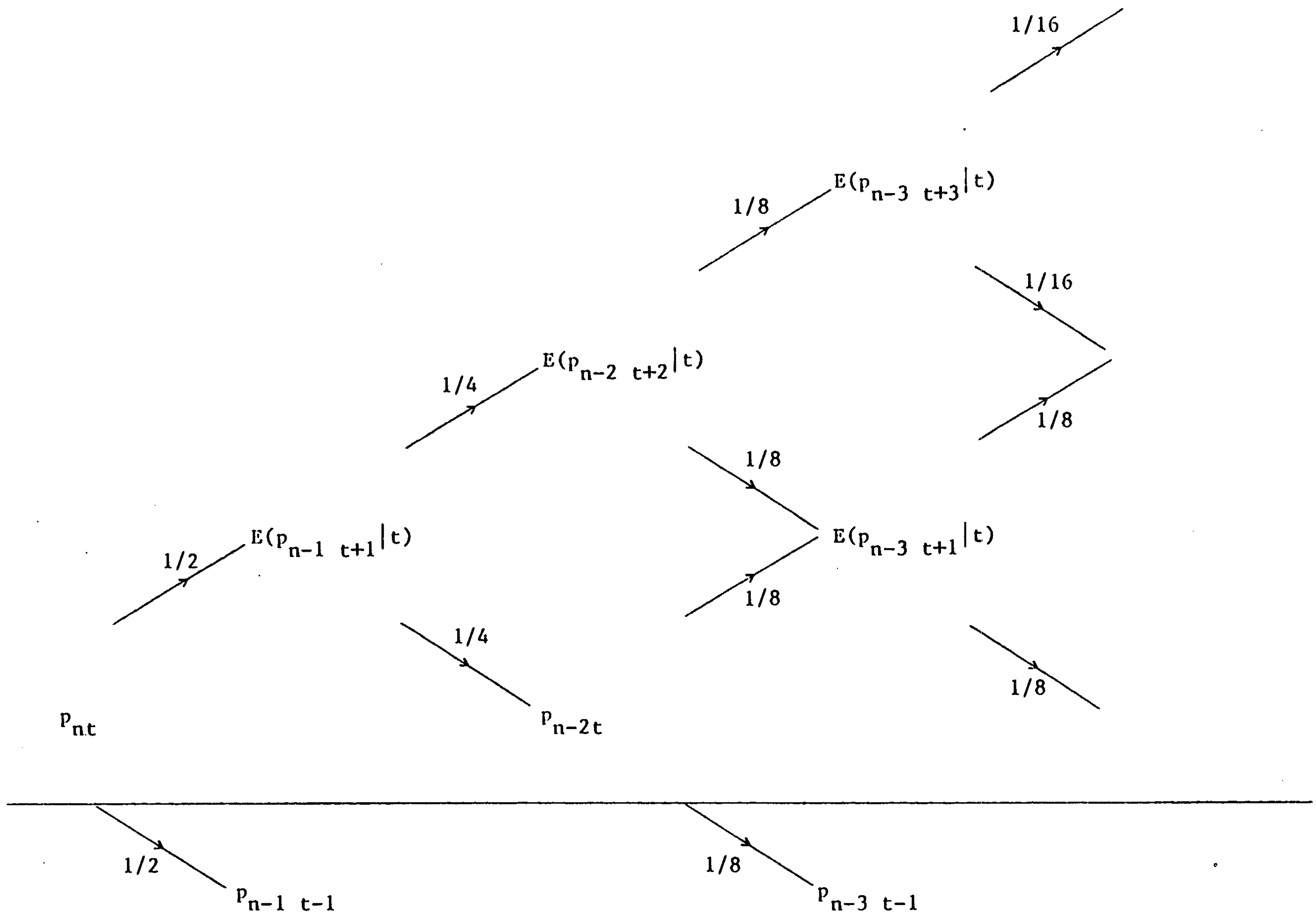

Figure 2. The degree of predetermination of the price level p_n.

the net effect is zero and the input price does not depend on the price level. The system is then recursive, the price level depending on the input price, and the input price depending on money and the supply disturbance. We start with this case; the general case will be analyzed in the next section. If $\beta = 1$, replacing p_{ot} from (7) in (10) gives

$$p_{nt} = 2^{-n}\left[\sum_{i=1}^{n/2}\sum_{j=0}^{(n/2)-i} b_{nj}\, E(\psi_{t-2i}|t-\tfrac{n}{2}-i+j) + \sum_{j=0}^{n/2} E(\psi_t|t-\tfrac{n}{2}+j) + \sum_{i=1}^{n/2}\sum_{j=0}^{(n/2)-i} b_{nj}\, E(\psi_{t+2i}|t-\tfrac{n}{2}+i+j)\right] \tag{11}$$

with $\quad \psi_t \equiv m_t - \xi_t$

Consider a permanent unanticipated increase in money at time t_o. Because of the long-run neutrality of money in this model, the long-run elasticity of the price level with respect to money is unity. We can derive from (11) incremental and cumulative price level elasticities over time. Denoting the proportional increase in money by dm we get:

$$
\begin{aligned}
p_{nt_o+2i} - p_{nt_o+2i-2} &= 0 && \text{if} \quad i < 0 \\
&= \binom{n}{n/2} dm && \text{if} \quad i = 0 \\
&= 2\binom{n}{(n/2)-i} dm && \text{if} \quad i = 1, \cdots, n/2 \\
&= 0 && \text{if} \quad i > n/2
\end{aligned}
$$

Tables 1 and 2 give incremental and cumulative elasticities of p_n over time for different values of n. They show a monotonic adjustment with the rate of adjustment increasing initially before decreasing later.

The adjustment of the price to its higher level takes exactly n periods. The adjustment is however substantially complete before that: assuming the period to be a month, the adjustment after a year is 99% complete if n = 20, 90% complete if n = 50, 75% complete if n = 100. Values of n of 100 may therefore generate the amount of price inertia we observe in the United States. Given the highly idealized nature of the model, it is difficult to decide whether such values for n are or are not reasonable.

There is an interesting distinction between demand disturbances, m, and supply disturbances, ξ. Note from equation (11) that they have an identical dynamic effect on the price level. Demand disturbances however affect demand and production all along the chain of production and thus are immediately perceived by all producers. The assumption made above that the change in money immediately known by all is therefore reasonable. Supply disturbances on the other hand have no direct effect on demand (this results from the assumption of demand determination). Thus, producers of y_i $i = 2, \cdots, n$ will perceive no change in their demand or imput price at time t_o. If their information included only the demand they face and the input price they pay, they would not revise expectations. In this case, the increase in the primary input price would slowly be transmitted to the structure of prices. p_n would not be given by equation (11) but by $p_{nt} = p_{ot-n}$, which implies substantially more inertia.

Table 1. Effects of an unanticipated increase in money at time t_o
Incremental effects on p_n

n =		t_o	t_o+2	t_o+4	t_o+6	...	t_o+10	...	t_o+20	...	t_o+50
2	p_2	.5	.5	0							
4	p_4	.375	.5	.125	0						
6	p_6	.312	.468	.187	.031	...	0				
10	p_{10}	.247	.410	.234	.086	...	0				
20	p_{20}	.176	.322	.240	.148	...	.028	...	0		
50	p_{50}	.113	.214	.192	.158	...	.084	...	.004	...	0
100	p_{100}	.079	.156	.148	.132	...	.098	...	.022	...	0
500	p_{500}	.035	.072	.070	.068	...	.064	...	.048	...	.006

Table 2. Effects of an unanticipated increase in money at time t_o
Cumulative effects on p_n

n =		t_o	t_o+2	t_o+4	t_o+6	...	t_o+10	...	t_o+20	...	t_o+50
2	p_2	.5	1.0								
4	p_4	.375	.875	1.0							
6	p_6	.312	.770	.957	.988	...	1.0				
10	p_{10}	.247	.547	.891	.977	...	1.0				
20	p_{20}	.176	.498	.738	.886	...	.988				
50	p_{50}	.113	.327	.519	.677	...	.881	...	.997	...	
100	p_{100}	.079	.235	.383	.515	...	.730	...	.966	...	.999
500	p_{500}	.035	.107	.177	.245	...	.378	...	.654	...	.998

The Effects of Money Deceleration

Characterizing the effects of a change in the level of money is a useful first step but the experiment lacks empirical relevance. Of more direct relevance are the effects of money deceleration. Suppose that money and prices are both growing at rate g per period and that this rate of inflation is considered too high by policy makers. What are the effects on real output of a sudden deceleration, say sudden zero growth of money?[14] The effects differ, depending on whether this change is anticipated or not. Let's first assume that the policy is announced at time t_o, to take place at time t_o+n: the rate of money growth remains equal to g until t_o+n and is equal to zero thereafter. From (11), real money balances from t_o on are given by:

$$(m-p)_{t_o+2i} = (2^{n-1}g) \sum_{j=0}^{i-1} (i-j) \binom{n}{j} \quad \text{for} \quad i = 1, \cdots, n/2$$

$$= (2^{n-1}g) \sum_{j=0}^{i-1} \left((n/2)-j\right) \binom{n}{j} \quad \text{for} \quad i = n/2, \cdots, n$$

The paths of money and prices are plotted in Figure 3. Real money balances, and therefore output, increase slowly after the announcement. They reach their maximum value at t_o+n when money growth stops. If for example n = 50 and g = 1% which corresponds, if the basic period is a month, to 12% annually, real money balances are higher by 1.3% at time t_o+50. They decrease thereafter and return to their normal level at t_o+2n. Thus deflation is achieved not with a recession but with a (mild) expansion!...

What is this due to? The announcement of a lower money growth leads price setters to slow down their rate of increase of prices before money deceleration takes place. When zero money growth actually takes place, real money balances are higher but progressively return to their normal level as prices keep increasing until t_o+2n. This is a very general feature of the "new" models of price inertia and holds for example also in the Taylor-Phelps (Taylor [1980], Phelps [1979]) model of overlapping labor contracts.[15] What is required however is a decrease in inflation before the decrease in money growth: for this to happen, the announcement of the future change in policy must be credible. In practice, the lack of credibility is probably what makes this result unlikely to occur. If for example agents do not believe zero money growth before it is actually implemented, this deceleration leads to a temporary loss in output. The path of prices in this case is also plotted in Figure 3.

IV. Price Level Inertia and the Elasticity of Input Supply

In traditional empirical macroeconometric models, prices are approximately markups over wages. Wages in turn depend on labor market conditions; of central importance for price inertia and the effects of money on real activity is the elasticity of nominal wages to the unemployment rate, the slope of the "short-run Phillips curve." These models have however been criticized for their formalization of expectations and the critique is as follows: expectations of inflation should be included in the Phillips curve and, with rational expectations, anticipated movements in money will have no effect on output, independently of the slope of the short-run Phillips curve.

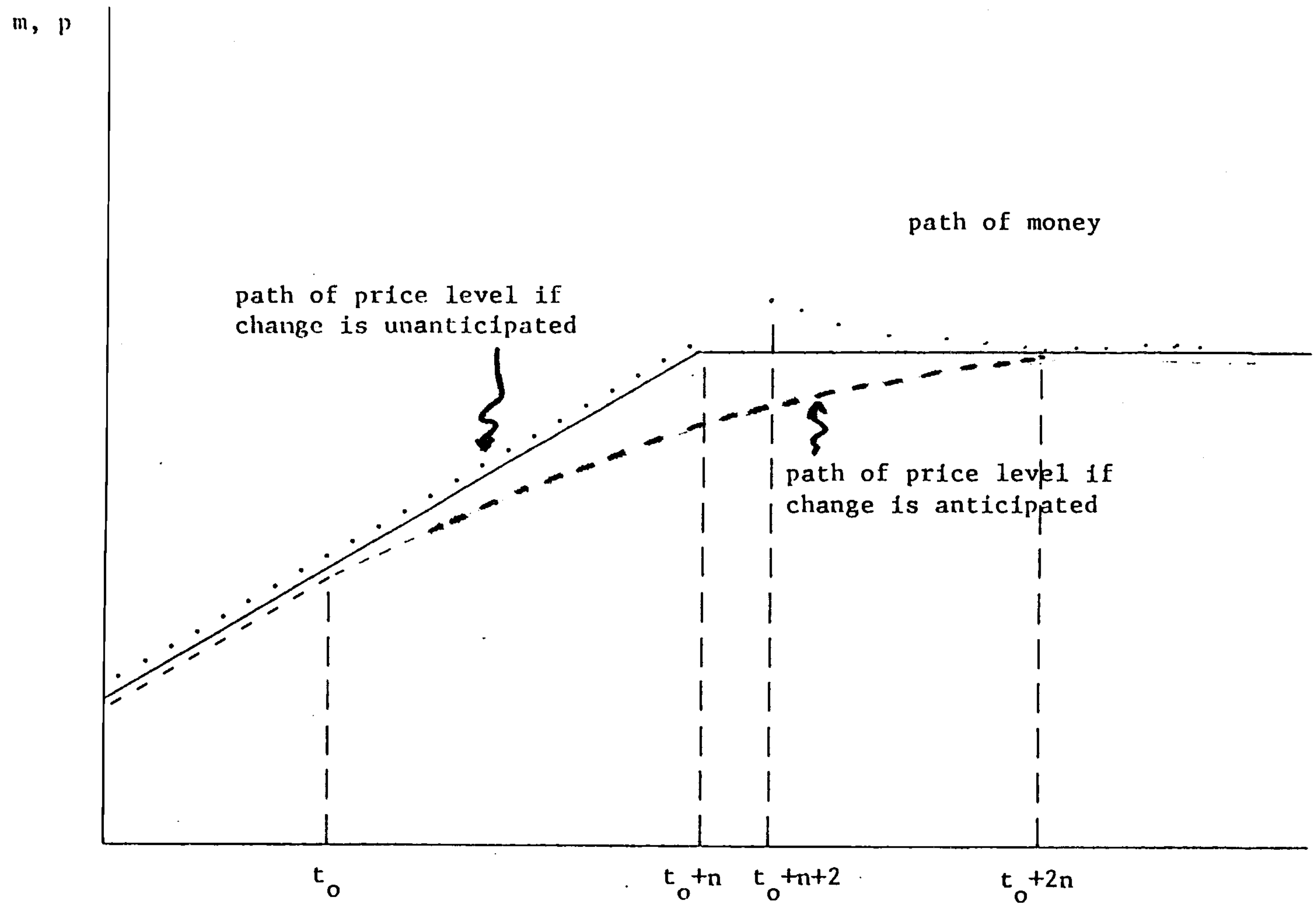

Figure 3. The effects of money deceleration.

This section shows that, if prices are desynchronized, the slope of the "short-run Phillips curve" is, even with rational expectations, an important determinant of the degree of price inertia. More precisely, it shows that the flatter the input supply is, the slower the price level will adjust, the larger the effect of money on real output will be.

The case n = 2 can be solved analytically. As β is not necessarily equal to unity, the model is no longer recursive and is a little more difficult to solve. To focus on the effects of demand disturbances, ξ is put equal to zero. Replacing (7) in (9) gives us an equation in p_{nt}:

$$(12) \qquad p_{nt} = ((1-\beta^{-1})/4)(p_{nt-2} + E(p_{nt}|t-2) + p_{nt} + E(p_{nt+2}|t))$$
$$+ (\beta^{-1}/4)(m_{t-2} + E(m_t|t-2) + m_t + E(m_{t+2}|t))$$

Taking expectations at time t-2, denoting $E(\cdot|t-2)$ by a hat and defining $\hat{\phi}_t \equiv \beta^{-1}(\hat{m}_{t-2} + 2\hat{m}_t + \hat{m}_{t+2})$:

$$(13) \qquad (1-\beta^{-1})\hat{p}_{nt+2} - 2(1+\beta^{-1})\hat{p}_{nt} + (1-\beta^{-1})\hat{p}_{nt-2} = -\hat{\phi}_t$$

Equation (13) can be solved by factorization to give:

$$(14) \qquad \hat{p}_{nt} = \lambda\hat{p}_{nt-2} + \lambda(1-\beta^{-1})^{-1} \sum_{i=0}^{\infty} \lambda^i \hat{\phi}_{t+2i}$$

with $\lambda \equiv (1-\beta^{-\frac{1}{2}})^2(1-\beta^{-1})^{-1}$.

λ gives the direct dependence of $\hat{p}_{nt}$ on $\hat{p}_{nt-2}$; it is an increasing function of β. For $\beta = 1$ (the value assumed in the previous section), taking limits appropriately, $\lambda = 0$ and equation (14) reduces to the equation of the previous section. If input supply is relatively inelastic, i.e. for β between 0 and 1, λ is negative and tends to -1 as β tends to zero. If input supply is relatively elastic, i.e. for β greater than 1, λ is positive and tends to 1 as β tends to infinity. Thus, the flatter input supply, the larger the direct dependence of the price level on the past.

What we want however is not $\hat{p}_{nt}$ but the actual value of p_{nt}. Consider as in the previous section an unanticipated permanent increase in money at time t_o and assume for notational convenience that the increase is from zero from unity. As there are no unanticipated movements in money or prices after t_o, equation (14) together with the assumed path of money implies in this case:

$$(15) \qquad p_{nt+2} = \lambda p_{nt} + (1-\lambda) \quad ; \quad t \geq t_o$$

Thus, given p_{nt_o}, we can solve for the sequence of prices after t_o. Equation (12) and the assumptions about the path of money give us another relation between p_{nt_o+2} and p_{nt_o} and thus the initial condition we need:

$$(16) \qquad p_{nt_o} = ((1-\beta^{-1})/4)(p_{nt_o} + p_{nt_o+2}) + \beta^{-1}/2$$

Equations (15) and (16) allow us to solve for the path of prices at and after t_o.

Table 3 gives the path of prices for different values of β. It shows in particular that if real input prices are insensitive to market conditions, i.e. if β is large, the price level reacts less and adjusts more slowly to changes in money: money has larger and more lasting effects on output. If we think of the input as labor, this shows the importance of the "short-run Phillips curve" slope, even in an economy with rational expectations.

Table 3. Cumulative effects of a permanent unanticipated increase in money at time t

		β=10.0	β=2.0	β=1.25	β=1.0	β=0.80	β=0.66	β=0.50	β=0.10
n=2	p_t	.24	.41	.47	.50	.52	.55	.58	.76
	p_{t+2}	.73	.90	.97	1.00	1.02	1.05	1.07	1.12
	p_{t+4}	.86	.98	.99	1.00	.99	.99	.99	.93
	p_{t+6}	.92	1.00	1.00	1.00	1.00	1.00	1.00	1.03
n=10	p_t		.19	.23	.25	.26	.27	.30	
	p_{t+2}		.55	.62	.66	.68	.72	.77	
	p_{t+4}		.77	.85	.89	.92	.95	.99	
	p_{t+6}		.90	.96	.98	.99	1.01	1.03	
	p_{t+8}		.95	.99	1.00	1.00	1.01	.99	
	p_{t+10}		.98	1.00	1.00	1.00	1.00	1.00	

Extending the analysis to values of n larger than 2 presents no particular difficulty and the method is sketched in the appendix. Results for $n = 10$ and different values of β are presented in Table 3. The conclusions are the same as above. As the analysis is substantially simpler when $\beta = 1$, the last section makes this assumption; this section has shown how the results would be modified if the assumption were relaxed.

V. Variability of Relative Prices and Profits

Desynchronization of price decisions has implications not only for the dynamics of the price level but for the dynamics of the structure of relative prices. The equation giving the behavior of any nominal price p_k is, if k is even, the same equation as for p_n, i.e. equation (10) with n replaced by k. The formula for k odd is slightly different but, as there are no particular insights to be obtained from it, we shall limit our attention to prices for which k is even.

Snake Effects

To see the effects of a permanent increase in money on the structure of prices, we can return to Tables 1 and 2 in Section III: they can also be interpreted as giving the cross section time series of prices. The first column gives the values of p_{kt} for values of k ranging from 2 to 500, the second column the values of p_{kt+2} for the same values of k and so on.

Table 2 shows how the increase in money twists the structure of prices. Prices early in the chain of production move more and adjust faster, prices further in the chain move less and adjust more slowly. If we measure profit rates by $(p_k - p_{k-2})$ for sector k,[16] it also appears that profit rates move more for low values of k. These results would be unchanged if we were looking at a supply disturbance, ξ, instead of a demand disturbance, m.

Variance of Prices and Profits

Instead of looking at effects of once-and-for-all changes in m or ξ, we may look at the stochastic behavior of prices for a given process for m or ξ. Assume for example that ξ and m are white--keeping for convenience the assumption that for t even, realizations of ξ and m are the same for t and $t+1$. If, as before, we define ψ_t as $m_t - \xi_t$, the behavior of p_{kt} is, from (10):

$$p_{kt} = 2^{-k}\left[\sum_{i=0}^{k/2} b_{k,(k/2)-i}\,\psi_{t-2i}\right] \tag{17}$$

Thus, the standard deviations of nominal prices, real prices and profit rates are given by:

$$\sigma(p_k) = 2^{-k}\left[\sum_{i=0}^{k/2} \left(b_{k,(k/2)-i}\right)^2\right]^{\frac{1}{2}} \sigma_\psi$$

$$\sigma(p_k - p_n) = \left[\sum_{i=0}^{n/2} \left(2^{-k}\, b_{k,(k/2)-i} - 2^{-n}\, b_{n,(n/2)-i}\right)^2\right]^{\frac{1}{2}} \sigma_\psi \quad , \quad b_{kj} = 0 \text{ if } j < 0$$

$$\sigma(p_k - p_{k-2}) = 2^{-k}\left[\sum_{i=0}^{k/2}\left(b_{k,(k/2)-i} - 4b_{k-2,(k/2)-i}\right)^2\right]^{\frac{1}{2}} \sigma_\psi \quad , \quad b_{kj} = 0 \text{ if } j < 0$$

Using identities associated with the hypergeometric distribution (Feller [1950], p. 62), the first expression can be rewritten as:

$$\sigma(p_k) = 2^{-k}\left(\binom{2k}{k} - \binom{2k}{k-1}\right)^{\frac{1}{2}} \sigma_\psi$$

Table 4. Standard deviations of prices and profits

Sector	Standard deviation of		
	Nominal prices	Real prices	Profits
$n = k = 10$	.126	.0	.025
$k = 8$	.146	.025	.044
$k = 6$	.178	.067	.070
$k = 4$	.253	.135	.153
$k = 2$	.353	.276	.790
$k = 0$	1.00	.966	—

$\sigma^2_{\psi} = \sigma^2_{\xi} + \sigma^2_{m}$ is normalized to unity

The values of these standard deviations, for $n = 10$ and $k = 0, \cdots, 10$, are reported in Table 4. The standard deviations of nominal prices, real prices and profit rates are all decreasing in k. This ordering is again independent of whether the economy is affected by supply or demand disturbances.

This result is fairly robust, being due to desynchronization rather than to the other assumptions of the model. There are two ways to potentially reverse it. The first is to relax the assumptions of constant returns and no inventory. In this case faced for example with a temporary increase in demand, a firm may decrease its price, decumulate inventory and not change its derived demand; it would therefore not transmit the disturbance further down the chain of production. Its price may then vary more than prices further down the chain. The second is to allow for disturbances to the technology itself, for example to allow the θ_i in equation (1) to be stochastic. In this case sectors affected by large technological disturbances may experience more price variability than the others.

If we think of the primary input as raw materials--there are clearly other factors at work in the labor market--the result is in accordance with facts. In the United States, the variance of raw materials is larger than the variance of intermediate products, which is itself larger than the variance of the WPI, both for periods dominated by demand disturbances and periods dominated by supply disturbances.[17]

VI. Conclusions and Extensions

This paper has shown that desynchronization of individual price decisions generates both inertia of the price level and movements in relative prices which appear in accordance with the facts.

It is time to return to the assumptions and face the question addressed to other models of price inertia. Are there obvious opportunities for profit left unused? Is every agent acting optimally? There are two crucial assumptions in the model:

The first is that price setters choose the same nominal price for two periods, rather than choosing different nominal prices for both periods, or allowing the second-period price to be contingent. We have purposefully chosen a basic period short enough that such schemes are likely to have costs which outweigh their benefits. Indexation of the second-period price on the price level is clearly unfeasible if the basic period is short: there may well be no reliable price level index.

The second is the structure of timing decisions. Given the timing decisions of others, does an agent have an incentive to maintain his own timing decision? In our model, the answer that he has an incentive to change it: each producer has an incentive to synchronize its price decisions with those of his supplier. This feature is however a characteristic of the simple structure of the model and is easily removed: if for example each producer uses two inputs, the prices of which change at different times, he cannot achieve synchronization with both. It is easy to construct structures of timing decisions such that no price setter has an incentive to change his own timing given the timing of others. With such structures, desynchronization and the implied inertia of the price level will remain: no agent has an incentive to change his timing or behavior.

This model can be seen as an alternative to the model of overlapping labor contracts developed by Akerlof [1969], Phelps [1979] and Taylor [1980]. Both explanations of price inertia are however probably empirically relevant. The comparative advantage of this model is twofold. The first is that it is more explicitly grounded in maximizing behavior; this allows for an easier treatment of normative aspects of policies. The second and more important is that it derives the complete structure of prices together with the price level. Thus it is well adapted to analyze questions involving both nominal and relative prices. It can for example easily be used to look at the desirability of exchange rate indexation under various sources of disturbances, a question analyzed by Dornbusch [1982] using the Taylor model.

Appendix. Price Solution for Arbitrary n and β

Replacing equation (7), with $\xi_t \equiv 0$, in equation (10) gives:

$$\text{(A1)}\quad p_{nt} = 2^{-n}\Bigg[\sum_{i=1}^{n/2}\sum_{j=0}^{(n/2)-i} b_{nj}E\big((1-\beta^{-1})p_{nt-2i} - \beta^{-1}\, m_{t-2i}\,|\,t-(n/2)-i+j\big)$$

$$+ \sum_{j=0}^{(n/2)-i} b_{nj}E\big((1-\beta^{-1})p_{nt} - \beta^{-1}\, m_t\,|\,t-(n/2)+j\big)$$

$$+ \sum_{i=1}^{n/2}\sum_{j=0}^{(n/2)-i} b_{nj}E\big((1-\beta)^{-1}p_{nt+2i} - \beta^{-1}\, m_{t+2i}\,|\,t-(n/2)+i+j\big)\Bigg]$$

We proceed in two steps. The first is to derive the behavior of $E(p_{nt}|t-n)$. Taking expectations in (A1) at time t-n and denoting them with a hat:

$$\text{(A2)}\qquad \hat{p}_{nt} = 2^{-n}(1-\beta^{-1})A(L)\hat{p}_{nt} + 2^{-n}\,\beta^{-1}\,A(L)\hat{m}_t$$

with $\quad L : Lx_t = x_{t-2}$

and $\quad A(L) \equiv \sum_{i=-n/2}^{n/2} \binom{n}{(n/2)+i} L^i$

Consider the polynomial $1 - 2^{-n}(1-\beta^{-1})A(L)$ associated with the homogenous part of this difference equation. It is symmetric so that if λ is a root, λ^{-1} is also a root. Thus it can be factorized as:

$$1 - 2^{-n}(1-\beta^{-1})A(L) = \sigma B(L)B(L^{-1})$$

where σ is a scalar and $B(L) = 1 + b_1 L + \cdots + b_{n/2} L^{n/2}$ has all roots inside the unit circle.

This implies that $E(p_{nt}|t-n)$ follows:

$$\text{(A3)} \qquad B(L)\hat{p}_{nt} = 2^{-n} \beta^{-1} \sigma^{-1} (B(L^{-1}))^{-1} A(L)\hat{m}_t$$

The second step is to solve for the actual value of p_{nt}. This is easily done for any specific path of--or process for--money. In the case of a permanent unanticipated increase in money at time t_o from zero to unit, it is derived as follows:

As there are no unanticipated movements of money or prices after t_o, (A3) implies for $t \geq t_o + n$:

$$B(L)p_{nt} = 2^{-n} \beta^{-1} \sigma^{-1} (B(L^{-1}))^{-1} A(L)m_t$$

The path of money considered here is such that all values of m_t on the right hand side are equal to unity. Thus:

$$B(L)p_{nt} = 2^{-n} \beta^{-1} \sigma^{-1} (B(1))^{-1} A(1) \Rightarrow$$

$$\text{(A4)} \qquad B(L)p_{nt}^{\cdot} = (\beta\sigma B(1))^{-1} \quad \text{as} \quad 2^{-n} A(1) = 1$$

For $t = t_o+n, \cdots, t_o+2n-2$ this gives a system of n/2 equations in n unknowns, $p_{ot_o+2n-2}, \cdots, p_{ot_o}$. In turn, equation (A1) gives for

$t = t_o, \cdots, t_o+n-2$ and given the path of money, n/2 equations in the same unknowns. This gives a system of n equations in n unknowns. Once this system is solved, values of p_{ot} for $t \geq t_o+2n$ can be derived using (A4). This is the method used to construct the second part of Table 3.

Footnotes

1. Many arguments along this line are presented in Gordon [1981].

2. The implications of various theories for the relation between disturbances, the price level and relative prices are presented in Fischer [1981].

3. It is sometimes argued that a source of price inertia is the length of the production process (for example, Coutts et al. [1978]). The argument is that if price is based on historical cost, a longer production process will lead to longer lags in price adjustment. Although this argument seems to have some empirical success, it appears difficult to reconcile with rational behavior on the part of firms.

4. An alternative formalization, which would extend work by Akerlof [1969], would postulate a large number of imperfectly substitutable final outputs produced under monopolistic competition. An increase in the number of price decisions would be obtained by increasing the number of products. The problem for our purposes is that the "technology" would not remain invariant as the number of price decisions increased. Otherwise, results are very similar.

5. An alternative is to formalize production as iterations of an input-output matrix. This turns out to be difficult to analyze.

6. The supply disturbance ξ does not affect the technology. It would be easy to allow for technological disturbances as well, by letting the θ_i be stochastic in equation (1).

7. It is well known that this relation can either be seen as a velocity equation or as a reduced form ISLM. Allowing for an interest rate explicitly would complicate the analysis but bring few insights. The unitary elasticity of output with respect to money balances assumption can be easily relaxed.

8. Although we do not derive the decision about period length from an optimization problem, this can be done by equalizing the marginal cost of more frequent changes to the marginal benefit of more accurate relative prices. This analysis has been pursued by Sheshinski and Weiss (for example [1981]).

9. This voluntarily abstracts from issues of monopoly power which may arise with desynchronized price setting. Condition (6) differs in two minor ways from the correct expected zero-profit condition: it neglects the fact that the second-period expected profit should be discounted by the interest rate; equivalently it assumes the real interest rate to be equal to zero. It implicitly assumes that the firm sells the same quantities in both periods, so that the weights on profit rates in period t $(p_{it} - p_{i-1\ t-1})$ and period t+1 $(p_{it} - E(p_{i-1\ t+1}|t))$ are equal. Both shortcuts simplify the analysis considerably and are not the source of the main results of the paper.

10. In a more realistic model, firms would have the choice of supplying demand out of either production or inventories. The initial effect of an increase in aggregate demand on the derived demand for the primary input would in general be smaller.

11. Using the fixed-price equilibrium terminology, our model allows for overemployment or underemployment of the primary input but not for unemployment as the input market is always in equilibrium. If we allowed for changes in m and ξ every period, the price would not necessarily clear the market in both periods and there could be unemployment.

12. The nominal rigidity of labor contracts is of a different nature from the rigidities considered in this paper. It is usually of much longer duration and the assumption of demand determination is certainly more questionable.

13. All the expressions in this paper are computed using binomial distribution tables (Aiken [1955]). These give $F(n,r,p) = \text{Prob}(x > r)$ if x follows an n-binomial with probability p. Then:

$$2^{-n} \binom{n}{r} = F(n,r,\tfrac{1}{2}) - F(n, r+1, \tfrac{1}{2})$$

14. The usual caveat about policy invariance of the structure applies. Such a drastic change may lead price setters to change price decisions more often or to try to achieve better synchronization.

15. In his paper [1979], Phelps considers a slightly different question. The question is whether, starting from steady inflation, there is a path of money such that inflation disappears over time and there is no change, positive or negative, in output. Phelps shows that there is such a path in his model but that the path is unappealing, involving oscillations in the rate of inflation along the way. Our model also has such a path, with the same unappealing features for $n > 2$.

16. This is more precisely the profit rate of the consolidated sector (k, k-1). We use this definition to avoid having to introduce p_k for odd k. The change in definition does not affect any of the conclusions.

17. This statement is based on comparisons of standard deviations of residuals from regressions on a quadratic trend, for subsamples of 47-1 to 80-1, for the following three series, "finished goods" producer price index (WPISOP3000NS in the DRI U.S. price bank), "intermediate materials, supplies and components" index (WPISOP2000NS), and "crude materials for further processing" index (WPISOP1000NS).

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