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TIME-SEPARABLE PREFERENCES AND INTERTEMPORAL-SUBSTITUTION MODELS OF BUSINESS CYCLES

Robert J. Barro

Robert G. King

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Abstract

Time-separability of utility means that past work and consumption do not influence current and future tastes. This form of preferences does not restrict the size of intertemporal-substitution effects--notably, we can still have a strong response of labor supply to temporary changes in wages. However, there are important constraints on the relative responses of leisure and consumption to changes in relative-price and in permanent income.

When the usual aggregation is permissible, time-separability has some important implications for equilibrium theories of the business cycle. Neglecting investment, we find that changes in perceptions about the future--which might appear currently as income effects--have no influence on current equilibrium output. With investment included, no combination of income effects and shifts to the perceived profitability of investment will yield positive co-movements of output, employment, investment and consumption. Therefore, misperceived monetary disturbances or other sources of changed beliefs about the future cannot be used to generate empirically recognizable business cycles.

Some richer specifications of intertemporal production opportunities may eventually yield more satisfactory answers. Because of the positive correlation between cyclical movements of consumption and work, equilibrium theories with time-separable preferences inevitably predict a procyclical behavior for the real wage rate, arising from shifts to labor's marginal product. Empirically, we regard the cyclical behavior of real wages as an open question. Aside from analyzing autonomous real shocks to productivity, we suggest that such shifts may occur as firms vary their capital utilization in response to intertemporal relative prices. However, we still lack some parts of a complete theory.

Robert J. Barro
Robert G. King
Dept. of Economics
University of Rochester
Rochester, NY 14627
(716) 275-3895

Intertemporal substitution of goods and leisure is a central component in modern equilibrium theories of the business cycle. These models seek to explain macroeconomic phenomena under the twin disciplines of rational expectations and cleared markets. In particular, when intertemporal margins of substitution are relevant to economic agents' current market decisions, equilibrium theories are capable of rationalizing a broad class of phenomena, as Lucas (1977) stresses. In such models macroeconomic quantities at a point in time reflect (i) economic agents' intertemporal preferences for goods and leisure, (ii) intertemporal production opportunities, and (iii) expectations held by economic agents.

As yet, there has been little systematic research aimed at identifying those aspects of preferences and production opportunities that are consistent with the observed cyclical behavior of quantities and relative prices. This paper discusses a benchmark case. Economic agents are immortal families with time-separable preferences for goods and leisure. Production opportunities are neoclassical, with current output depending on current labor services, previously accumulated capital, and exogenous technological conditions. Surprisingly, these two assumptions substantially restrict the types of disturbances that can generate positive co-movements of aggregate production, employment, investment and consumption, which we take to be a minimal empirical description of the dominant component of business fluctuations.

In Section I we describe the implications of time-separable utility for individual demands for goods and leisure. One set of implications relates individual response to changes in intertemporal relative prices to individual response to changes in lifetime income. These restrictions on demands for goods and leisure play a central role in our subsequent macroeconomic analysis. Another set of results indicates that time-separable preferences are consistent with several common assertions about intertemporal substitution of goods and leisure. First, current labor supply will respond more elastically to a temporary increase in the wage rate than to a permanent increase, even if we consider income-compensated changes. Second, compensated increases in the real interest rate will increase current labor supply and decrease current consumption demand. In this section we also discuss some departures from time-separable utility. These alternatives imply that the usual list of "state" variables for an individual--asset stocks, etc.--must be augmented to include past leisure and consumption. The history of these variables affects current "tastes" for goods and leisure.

The discussion then turns to macroeconomic analysis under two alternative specifications of technology. In Section II we assume that production opportunities are static in the sense that goods are not storable and production activity is accomplished within a single period. In this setup time-separable preferences imply that current quantities of consumption and leisure are isolated from future events, such as prospective changes in technology. These changes show up as wealth effects on current supply and demand. We demonstrate that the market-equilibrating movements in rates of return fully offset these wealth effects. This exercise is important for two reasons. First, it emphasizes that the relevance of intertemporal substitution to the equilibrium behavior of quantities depends on substitution in both preferences and production opportunities. In the case of static production opportunities, substantial intertemporal substitutability of goods and leisure is compatible with negligible equilibrium variation in quantities. Second, more specifically, the results of this section have strong implications for theories of the business cycle constructed along the lines of Barro (1976). These models stress that monetary disturbances have real effects because a representative agent over- or under-estimates the future value of money. However, if agents have time-separable preferences, then movements in rates of return neutralize the impact of variations in perceived wealth.

If capital--that is, previously accumulated stocks of goods--is a factor of production, then current quantities are no longer isolated from future events. However, under time-separable preferences, we cannot generate positive co-movements in consumption, investment, and work effort as responses to future shocks--either to income, the perceived marginal product of capital or both. This conclusion is important for those monetary theories of the business cycle--such as Lucas (1975) and King (1982)--that stress cyclical variations in investment and asset values arising as a consequence of misperceived monetary disturbances. The finding also restricts the impact of future shocks in real-business-cycle theories, such as King and Plosser (1981), which specify time-separable preferences.

In order to generate positive co-movements of consumption, investment and work effort, we must have a procyclical pattern in the terms of trade between goods and leisure--that is, the real wage rate. With the neoclassical production structure employed in this paper, procyclical variations in the real wage can arise only if there are appropriate shifts to the schedule for the marginal product of labor. The possibilities for technological disturbances are stressed in the real-business-cycle analysis of Long and Plosser (1980). However, our hunch is that richer structures of intertemporal production opportunities will ultimately permit the current marginal product of labor to respond to intertemporal prices. These possibilities may provide links to future conditions and, perhaps, to monetary shocks. These linkages do not exist in our present setup.

I. Allocation of Goods and Leisure over Time

Our analysis of economic aggregates begins with an individual price-taking consumer, who has preferences defined over a time stream of consumption, $(C_0, C_1, \ldots)$, and work effort, $(L_0, L_1, \ldots)$. Throughout, we neglect consumer durables, so that consumption and consumer expenditure coincide. At the present time, date 0, the household evaluates its total utility as

$$(1) \qquad U = U(C_0, C_1, C_2, \ldots; L_0, L_1, L_2, \ldots; q_0),$$

where q_0 is a vector of variables that are important for "tastes." Utility derives from consumption and "leisure" (time spent away from work) at these various dates, so that we assume $\partial U/\partial C_t > 0$ and $\partial U/\partial L_t < 0$.

The individual participates in markets for consumption goods, labor services and credit. The real wage rate in period t is w_t and the real rate of return on one-period loans between periods t and t+1 is r_t. The present-value price of a unit of consumption in period $t > 0$ is equal to $1/[(1+r_0)(1+r_1) \ldots (1+r_{t-1})]$. Analogously, the present-value price of a unit of leisure is $w_t/[(1+r_0)(1+r_1) \ldots (1+r_{t-1})]$ for $t > 0$. Defining the discount-factor, ρ_t, to be one for $t = 0$ and to equal $1/[(1+r_0)(1+r_1) \ldots (1+r_{t-1})]$

for t > 0, the household's intertemporal budget constraint (over an infinite horizon) is

$$(2) \qquad \sum_{t=1}^{\infty} [\rho_t C_t + \rho_t w_t(-L_t)] = \sum_{t=0}^{\infty} \rho_t \pi_t + I_0,$$

where I_0 is initial assets and π_t is non-labor income for period t.

Individual demand behavior for current goods and leisure, as well as the remainder of the household's plan, derives from maximization of equation (1), subject to the budget constraint in equation (2). The general forms of the current demand and supply functions are as follows,

$$(3a) \qquad C_0^d(1, \rho_1, \rho_2, \ldots, w_0, w_1\rho_1, w_2\rho_2, \ldots, \Omega_0, q_0),$$

$$(3b) \qquad L_0^s(1, \rho_1, \rho_2, \ldots, w_0, w_1\rho_1, w_2\rho_2, \ldots, \Omega_0, q_0).$$

In these demand functions we are assuming that changes in relative prices are income-compensated. Ω_0 is a measure of lifetime income/wealth, measured in current commodity units, so that $\partial\Omega_0/\partial I_0 = 1$, $\partial\Omega_0/\partial\rho_t = \pi_t + w_t L_t - C_t$, $\partial\Omega_0/\partial\pi_t = \rho_t$ and $\partial\Omega_0/\partial w_t = \rho_t L_t$.

What information do we have about the forms of these demand and supply functions? Lucas (1977, pp. 16-17) discusses some observations that are important in restricting these functions. First, consumption and leisure appear to be superior goods, so that an increase in income/wealth motivates an increase in consumption demand and a decrease in labor supply. Second, individuals exhibit a willingness to alter their allocations of work over time in response to wages that are temporarily high or low. Thus, current and future leisure are substitutes. Third, permanent changes in the real wage do not seem to have a major effect on labor supply. This result reflects the absence of possibilities for intertemporal substitution, as well as the income effect from a permanent change in the real wage.

Time-Separable Utility

The expression for total utility from equation (1) is often specialized to a time-separable form. This specification appears in studies that range from the demand for goods and leisure over the life cycle--as in MaCurdy (1981)--to capital theory, as surveyed by McKenzie (1980). We consider the time-additive form,[1]

$$(4) \qquad U = u^0(C_0, L_0) + \frac{1}{1+\gamma} u^1(C_1, L_1) + (\frac{1}{1+\gamma})^2 u^2(C_2, L_2) + \ldots$$

We assume a simple form of time preference, where utility for period t is discounted to a starting date, t_0, by the factor, $1/(1+\gamma)^{t-t_0}$. (γ is a positive constant and we have set t_0 equal to zero in equation (4).) This formulation ensures time-consistency in the sense of Strotz (1956). That is, in a situation of perfect foresight, optimizing households will follow through on plans formulated at date zero as the starting date, t_0, advances.

Equation (4) embodies the idea that actions taken in period t (or utility achieved in period t) do not affect utility in other periods. However, we do not require the utility functions to be identical in each period. Thus, variations in the form of u^t can account for life-cycle elements or time-varying features of the aggregate economy (such as shifts in government spending that substitute for private spending).

The form of equation (4) suggests that goods and leisure may interact differently within a period than across periods. Thus, for example, current consumption, C_0, may substitute for the contemporaneous amount of leisure, determined by L_0, in a manner different from future consumption or leisure. Looking forward in time, there might seem to be no special rationale for this asymmetry. However, we can equivalently think of time separability as restricting the manner in which past consumption and leisure influence current preferences. Equation (4) says that a person's tastes for current and future quantities of consumption and leisure do not depend on that person's history of consumption and leisure. Accordingly, the consumption-work plan that someone formulates at the present time, say date 0, can depend on previous settings for consumption and work only to the extent that these earlier choices show up in current state variables, such as wealth, knowledge, productivity, and so on. The past can matter through budget constraints, but not through shifts in "tastes." In other words the current taste parameter, q_0 in equation (3), does not depend on lagged values of consumption and work.

We can indicate some possibilities for intertemporal interactions that are ruled out by time-separability. If a person works hard in some period--that is, L_t is high--then fatigue may be a significant consideration in future allocations of work and consumption. In this case work is like a durable good, in the sense that the level of fatigue is a current state variable. In particular, the satisfaction attached to leisure may be especially high when the accumulated amount of fatigue is great. Looking ahead, the prospect of a high level of work during some future period, L_t, would raise the value of relaxation beforehand--that is, individuals would choose low values for L_{t-1}, L_{t-2}, etc. Analogously, the memory of past consumption experiences (or prospects of future ones) may bear on subsequent (previous) tastes for consumption and leisure.

If we do not impose time-separability on utility, then past flows of consumption and work can appear as current state variables in analyzing the plan for consumption and work--that is, as elements of the taste parameter, q_0, in equation (3). We could not restrict the pertinent state variables to measures of initial wealth, technology, etc. This viewpoint conflicts with the bulk of existing macroeconomic analyses,[2] which do not incorporate the history of consumption and work as state variables. In other words most existing macroeconomic theories assume implicitly that utility functions are separable over time. We do not intend to quarrel with this viewpoint in the present paper--rather, our main argument is that insufficient mileage has been derived from this powerful assumption.

The treatment of utility as time-separable would be satisfactory for most purposes if the memory of past consumption and work had important effects on subsequent tastes for consumption and leisure only over a brief interval. For purposes of business cycle analyses, we are thinking that departures from separability would matter significantly only for days or weeks, rather than for months or years. So, a period of unusually hard work might have little direct effect on the taste for leisure after one or two months. (Recall that we are not ruling out influences that work through the budget constraint--the discussion here concerns the effects of past choices on the form of the utility function, as specified to apply from the present date onward.) In order to focus the issue with an extreme example, consider the intense work effort of U.S. residents during World War II. Abstracting from effects on the country's capital stock, people's information, etc., did this history of hard work have a major prolonged impact on the tastes for consumption and leisure from 1946 onward? More specifically, did lingering fatigue from World War II imply a persisting downward shift in the willingness to work? It is precisely this type of effect that we neglect by postulating time-separable utility.

While introspection cannot be definitive, our inclination is to proceed under the assumption that time-separable utility is a satisfactory approximation, at least for the purposes of most macroanalyses. Thus, we rule out a role for the history of consumption and work as current state variables. Although this assumption is implicit in most macroeconomics, the full implications have not been clarified and exploited. These implications provide potentially refutable empirical propositions. Therefore, we generate bases for assessing time-separable utility that should ultimately prove more convincing than introspection about the nature of people's preferences.

Implications of Time-Separability for Demand Functions

Houthakker (1960, pp. 247, ff.) and Goldman-Uzawa (1964, pp. 392, ff.) describe the implications of block-addivite utility for the forms of consumer demand functions. (In our case each block corresponds to the consumption and leisure from a single time period.) Block additivity implies that there is a specific relationship between income effects and compensated price effects, when the prices pertain to goods from different blocks (that is, different dates). Our appendix provides a detailed description of the demand-behavior of a price-taking consumer who is selecting quantities of M goods each period, (x_{jt}, $j = 1, 2, \ldots, M$). Each good has a present-value price, $\rho_t p_{jt}$, where the ρ_t are discount factors, as discussed previously. Consequently, the agent's intertemporal budget constraint over an infinite horizon has the form, $\sum_{t=0}^{\infty}(\rho_t \sum_{j=1}^{M} p_{jt}x_{jt}) \leq I_0 + \sum_{t=0}^{\infty}(\rho_t \sum_{j=1}^{M} p_{jt}z_{jt})$, where z_{jt} is an endowment of good j at date t.[3] The set of necessary and sufficient conditions that follow from time-separability are

$$(5) \qquad \frac{\partial x_{kt}^{d}}{\partial(\rho_\tau p_{j\tau})} = \left(\frac{\lambda}{\mu}\right) \frac{\partial x_{kt}^{d}}{\partial I_0} \frac{\partial x_{j\tau}^{d}}{\partial I_0}, \quad \text{for } t \neq \tau, \text{ and for all } (j, k) = 1, \ldots, M.$$

In this expression the left side refers to an income-compensated relative price effect. On the right side, the positive number, λ/μ, does not depend on the choices of goods or periods.

For our simplified case of leisure and a single consumption good, let $x_{1t} = -L_t$ and $x_{2t} = C_t$. In our case we are dealing with relative prices with consumption as the numeraire, so that the present-value price of date t leisure, $x_{1t} = -L_t$, is $\rho_t w_t$, and the price of date t consumption, $x_{2t} = C_t$, is ρ_t. If we think of t as representing the present, while τ represents any future period, then the left side of equation (5) is the income-compensated effect of any future present-value price on today's leisure or consumption. The equation relates this substitution effect to two income effects, one contemporaneous and the other pertaining to the future period in which the price change takes place.

For the bulk of our analysis, we focus on the implications of time-separability for the quantities of consumption and leisure that are chosen at the same date. Using equation (5), we find that time-separability implies the following relation between relative-price and income effects,

$$(6)\qquad \frac{\partial x_{2t}/\partial \rho_\tau p_{j\tau}}{\partial x_{1t}/\partial \rho_\tau p_{j\tau}} = \frac{\partial x_{2t}/\partial I_0}{\partial x_{1t}/\partial I_0}, \qquad \text{for } t \neq \tau \text{ and } j = 1, 2.$$

The left side of equation (6) is the ratio of the responses of today's consumption and leisure to an income-compensated change in the future price, $\rho_\tau p_{j\tau}$. This expression equals the ratio of the responses of today's consumption and leisure to a change in lifetime income. We refer to conditions such as equation (6) as cross-preference relations for consumption versus leisure. Such cross-preference relations play a central role in the equilibrium analysis below.

The intuition behind equation (6) is straightforward. Consider the choices of x_{1t} and x_{2t} for any period t. Time-separability implies that quantities of leisure and consumption at other dates are important for decisions at date t only through the budget constraint. Thus, we can break the agent's decisions into two stages: (i) for a given amount of date t expenditure, e_t, divide this amount optimally between leisure and goods; and (ii) select a pattern of expenditure over time that maximizes utility, subject to the lifetime budget constraint, $\sum_{t=0}^{\infty} \rho_t e_t = I_0 + \sum_{t=0}^{\infty} (\rho_t \sum_{j=1}^{M} p_{jt} z_{jt})$. We can think of "static" demand functions for leisure and goods, which are $x_1^{dt}(e_t, p_{1t}, p_{2t})$ and $x_2^{dt}(e_t, p_{1t}, p_{2t})$, where the time t dependence derives from time-varying features built into u^t above. These functions treat expenditure for the period, e_t, as given. The individual responds to a change in lifetime wealth, I_0, by altering the pattern of expenditure. Let the change for period t be $\partial e_t/\partial I_0$. Further, an individual responds to a compensated change in the present-value price of a good in some other period, $\rho_\tau p_{j\tau}$, by altering expenditure in period t. Let that change be $\partial e_t/\partial\rho_\tau p_{j\tau}$. Then, the income and compensated price effects satisfy the following conditions,

$$\frac{\partial x_{1t}^d}{\partial\rho_\tau p_{j\tau}} = \left(\frac{\partial x_1^{dt}}{\partial e_t}\right)\left(\frac{\partial e_t}{\partial\rho_\tau p_{j\tau}}\right), \qquad \frac{\partial x_{2t}^d}{\partial\rho_\tau p_{j\tau}} = \left(\frac{\partial x_{2t}^d}{\partial e_t}\right)\left(\frac{\partial e_t}{\partial\rho_\tau p_{j\tau}}\right),$$

$$\frac{\partial x_{1t}^d}{\partial I_0} = \left(\frac{\partial x_1^{dt}}{\partial e_t}\right)\left(\frac{\partial e_t}{\partial I_0}\right), \qquad \frac{\partial x_2^{dt}}{\partial I_0} = \left(\frac{\partial x_2^{dt}}{\partial e_t}\right)\left(\frac{\partial e_t}{\partial I_0}\right).$$

The cross-preference relation from equation (6) follows by inspection. The key point is that changes in prices for date τ induce reallocations of expenditure toward or away from period t. The individual treats these changes in expenditure as identical to those arising for any other reason, such as a change in income, I_0.

Note that we assumed that changes in the present-value price for period τ, $\rho_\tau p_{j\tau}$, were income-compensated. However, all that counts in the previous derivation is the impact of this price change on e_t, the total expenditure for period t. Consequently, uncompensated changes in prices also satisfy the cross-preference relation, as set out in equation (6).

We have focused on the present-value price, $\rho_\tau p_{j\tau}$. Our derivation of the cross-preference relation in equation (6) remains valid if we consider instead any one-period rate of return, r_τ, or real wage rate, w_τ (where $\tau \neq t$). In particular, we can replace $\rho_\tau p_{j\tau}$ in equation (6) by either r_τ (for all values of τ) or w_τ (for $\tau \neq t$). The cross-preference relation holds for either income-compensated or non-compensated changes in r_τ (for all τ) or w_τ (for $\tau \neq t$).

Current Consumer Demand and Labor Supply

We focus now on representations for current consumption demand and labor supply, C_0^d and L_0^s, under the assumption that preferences are time-separable. We are interested in two kinds of properties. First, we want to develop several implications of separability that will be important for the equilibrium analysis below. Second, we want to discuss whether time-separable preferences are compatible with certain individual observations on consumption demand and labor supply.

Time-separable preferences imply the following cross-preference condition, which relates real-interest-rate effects to income effects:

$$\text{(7)} \qquad \frac{\partial C_0^d}{\partial r_0} \Big/ \frac{\partial L_0^s}{\partial r_0} = \frac{\partial C_0^d}{\partial I_0} \Big/ \frac{\partial L_0^s}{\partial I_0} .$$

In equation (7) we refer to an income-compensated change in r_0, while holding fixed all other rates of return, r_1, r_2, ..., and the array of real wage rates, w_0, w_1, ... Thus, we are examining income-compensated changes in ρ_1, ρ_2, ..., and $w_1\rho_1$, $w_2\rho_2$, ..., all of which derive from a change in r_0. Equation (7) remains valid if the change in r_0 is not income-compensated.

The form of equation (7) holds if we substitute any prospective one-period rate of return, r_t, for r_0. In the aggregate comparative-statics analysis below, the following form of this condition will be useful:

$$\text{(8)} \qquad \left[\frac{\partial C_0^d}{\partial r_0} \Big/ \frac{\partial L_0^s}{\partial r_0}\right] = \left[\frac{\partial C_0^d}{\partial r_t} \Big/ \frac{\partial L_0^s}{\partial r_t}\right], \qquad \text{for all } t = 1, 2, \ldots$$

Another property that we will employ concerns the relationship between responses to changes in interest rates and responses to changes in future real wage rates. This condition is

$$(9) \qquad \left[\frac{\partial C_0^d}{\partial r_0} \Big/ \frac{\partial L_0^s}{\partial r_0}\right] = \left[\frac{\partial C_0^d}{\partial w_t} \Big/ \frac{\partial L_0^s}{\partial w_t}\right] \qquad \text{for all } t = 1, 2, \ldots$$

As with the interest rate, we can interpret changes in future real wage rates (appearing on the right side of equation (9)) as income-compensated or not. Note, however, that equation (9) will not hold for the current real wage rate, w_0.

Income and Substitution Effects

Time-separability of utility says nothing about whether goods (consumption and leisure in various periods) are superior or inferior. However, any two superior goods (consumption or leisure) from different periods must be substitutes (see equation (5)). For example, if $\partial C_t^d/\partial I_0 > 0$ and $\partial C_\tau^d/\partial I_0 > 0$, where $t \neq \tau$, then the compensated price effect, $\partial C_t^d/\partial \rho_\tau$, must be positive.

In the bulk of our subsequent analysis, we assume that consumption and leisure in all periods are superior goods. Equation (5) then implies that all goods from different periods are substitutes. This result means that the current choices for consumption and leisure both decline with a compensated change in any prospective rate of return, r_t, where $t \geq 1$. Current consumption and leisure both rise with a compensated change in any prospective real wage rate, w_t, where $t \geq 1$.

Under time-separable preferences (with restrictions on the form of u^t to ensure that consumption and leisure are superior), the behavior of individuals accords with Lucas's observations. Specifically, there are two reasons why individual labor suppliers will respond more elastically to a temporary change in their wage than to a permanent change of an identical amount. First, since current and future leisure are substitutes, higher future wage rates will tend to depress current labor supply at any given current wage rate. Put alternatively, permanent wage changes do not exert intertemporal-substitution effects on labor supply. Second, a temporary increase in the wage has a smaller income effect than does a permanent change. These two features imply that time-separable preferences are compatible with the secular evidence on labor supply--permanent changes in real wages have negligible or negative effects--as well as the short-run evidence--temporary changes in wages have substantial positive effects.

Aggregation

We assume that the properties of individuals' demand functions--notably, the cross-preference relations in equations (7)-(9)--carry over to the aggregate level. This assumption permits us to do macroeconomics in the usual manner, where we focus on the behavior of a representative agent. However, we have not as yet considered any detailed justification for this assumption, nor the sensitivity of our conclusions to departures from its validity.[4]

If such aggregation is permissible, then we need not worry about the distribution of claims on the credit market, but rather can focus on a representative agent. The internal character of the loan market implies that such a representative individual will have a zero net position. Uncompensated changes in any rate of return will then have a zero income effect. Therefore, properties of compensated changes in rates of return carry over to uncompensated changes at the aggregate level. In the bulk of our analysis, we assume that consumption and leisure are superior goods in all periods. It follows that an increase in any prospective rate of return lowers current aggregate consumption demand and raises current aggregate labor supply.

II. Equilibrium Analysis with Static Production Opportunities

We consider here the implications of time-separable utility for a basic equilibrium business-cycle model, where goods are not storable and production activity occurs within a single period. (Barro (1976, 1980) employs this model to analyze the real effects of imperfectly perceived nominal disturbances.)

In this model we think of households and firms as integrated units, rather than explicitly analyzing a market for labor services.[5] In particular, households produce non-durable goods in the quantity Y by means of their own labor effort, L. The production function for each household is

$$(10) \qquad Y_t = F(L_t;\ \alpha_t),$$

where α_t is a shift parameter.

The marginal product of labor is positive, but diminishing in L. The schedule for the marginal product determines the real wage rate--that is, the shadow price of time relative to goods in each period--w_t, which affects households' choices as discussed before. To start with, we fix the set of parameters, α_t, which fixes the schedule for the marginal product of labor in each period. We introduce the services from capital stock as a productive input in a later section. At this stage each household's stock of capital can be regarded as fixed--in particular, there is no investment.

Households face current and prospective real rates of return, r_0, r_1, ... We can think of these returns as applying to one-period, real-valued loans on a consumer-credit market. Using equation (2), along with $Y_t = w_t L_t + \pi_t$, we can write each household's budget constraint as

$$\sum_{t=0}^{\infty} \rho_t (C_t - Y_t) = I_0,$$

where the ρ_t are the present-value prices for commodities in period t. The level of output varies with the amount of labor input in accordance with the production function in equation (10). The income term, I_0, which may be positive or negative for a particular individual, depends on the individual's starting position for net claims on the credit market.

Given the parameters of the production function, α_0, α_1, ..., the rates of return, r_0, r_1, ..., and its income position, I_0, each household chooses labor effort, L_t, and consumption demand, C_t^d, in each period. The choice of work, L_t, determines the supply of goods, Y_t^s, from equation (10). We can write each household's choices for demand and supply of commodities in the current period, date 0, in the functional forms,

(11) $$C_0^d = C_0^d(\alpha_0, I_0, r_0, \ldots),$$

(12) $$Y_0^s = Y_0^s(\alpha_0, I_0, r_0, \ldots).$$

The omitted arguments of the functions in equations (11) and (12) are future values for the production parameters, α_1, α_2, ..., and rates of return, r_1, r_2, ...

The cross-preference relations from equations (7) and (8) apply to each household's choices of current consumption and work effort. Output supply depends on work effort through the production function in equation (10). Therefore, the cross-preference relations hold also in terms of current output supply--namely,

$$(13)\qquad \frac{\partial C_0^d/\partial r_0}{\partial Y_0^s/\partial r_0} = \frac{\partial C_0^d/\partial I_0}{\partial Y_0^s/\partial I_0} = \frac{\partial C_0^d/\partial r_t}{\partial Y_0^s/\partial r_t}, \qquad \text{for all } t = 1, 2, \ldots$$

In equation (13) we are considering uncompensated changes in rates of return, r_0 and r_t. That is, these changes apply when income, I_0, and the prospective rates of return for other periods are held constant.

Changes in future parameters of the production function, α_t, entail income effects and shifts in the schedules for the marginal product of labor. The latter effects amount to changes in prospective real wage rates. Changes in lifetime income and in prospective real wage rates both satisfy the cross-preference condition for current consumption demand and labor supply (equations (7) and (9)). Hence, the effects of changes in α_t on current consumption demand and output supply satisfy the cross condition,

$$(14)\qquad \frac{\partial C_0^d/\partial r_0}{\partial Y_0^s/\partial r_0} = \frac{\partial C_0^d/\partial \alpha_t}{\partial Y_0^s/\partial \alpha_t}, \qquad \text{for all } t = 1, 2, \ldots$$

As discussed in the previous section, we assume that aggregate representations for commodity demand and supply can be written in forms that parallel those of equations (11) and (12). In particular, we assume the validity of the cross-preference relations from equations (13) and (14) when expressed in terms of aggregate variables.

Since consumption is the only use of output in the present context, we have the market-clearing condition,

$$(15) \qquad \overline{Y}_0 = \overline{C_0^d}(\alpha_0, \overline{I}_0, r_0, \ldots) = \overline{Y_0^s}(\alpha_0, \overline{I}_0, r_0, \ldots),$$

where $\overline{Y}_0$ is current aggregate output, $\overline{C_0^d}$ and $\overline{Y_0^s}$ are aggregate functions, and $\overline{I}_0$ is an aggregate income term.

The Basic Result

The initial disturbance that we consider is a change in $\overline{I}_0$. An increase in $\overline{I}_0$ means an improvement in the representative person's lifetime income prospects. We can generate this type of change in the present model by postulating an upward shift in the anticipated position of future production functions. If we think of purely parallel upward shifts in these functions--where the schedules for the marginal product of labor do not change--then a pure income effect is involved.

Equation (15) permits us to calculate the effects of this type of disturbance on current aggregate output and the real rate of return. We take as givens the omitted arguments of the functions in equation (15), which include the future (anticipated) real interest rates, r_1, r_2, ...[6] We also neglect prospective shifts to future schedules for the marginal product of labor. We consider here only changes to perceived future production opportunities that can be categorized as pure income effects. These shifts are equivalent to changes in the income term, $\overline{I}_0$.

The results are

$$(16)\qquad \frac{dr_0}{d\bar{I}_0} = \left[\frac{\partial \bar{C}_0^d}{\partial \bar{I}_0} - \frac{\partial \bar{Y}_0^s}{\partial \bar{I}_0}\right] \Big/ \left[\frac{\partial \bar{Y}_0^s}{\partial r_0} - \frac{\partial \bar{C}_0^d}{\partial r_0}\right] > 0 \text{ if all goods are superior,}$$

$$(17)\qquad \frac{d\bar{Y}_0}{d\bar{I}_0} = \left[\frac{\partial \bar{C}_0^d}{\partial \bar{I}_0}\frac{\partial \bar{Y}_0^s}{\partial r_0} - \frac{\partial \bar{Y}_0^s}{\partial \bar{I}_0}\frac{\partial \bar{C}_0^d}{\partial r_0}\right] \Big/ \left[\frac{\partial \bar{Y}_0^s}{\partial r_0} - \frac{\partial \bar{C}_0^d}{\partial r_0}\right] = 0.$$

We can think of these results as follows. Assuming that current consumption and leisure are superior goods, the rise in perceived aggregate income raises $\bar{C}_0^d$ and lowers $\bar{Y}_0^s$ (reduces work effort). Because currently desired saving declines, the current real rate of return, r_0, rises. This conclusion follows from equation (16) if $\partial \bar{Y}_0^s/\partial r_0 > 0$ and $\partial \bar{C}_0^d/\partial r_0 < 0$, which hold unambiguously if consumption and leisure are superior goods in all periods. The rise in the real interest rate, r_0, achieves commodity-market clearing, partly by stimulating current output supply (by inducing more work today) and partly by lowering current consumption demand.

The effects on current output are offsetting. The positive forces are the income effect on demand, $\partial \bar{C}_0^d/\partial \bar{I}_0$, and the intertemporal-substitution effect on supply, $\partial \bar{Y}_0^s/\partial r_0$. Offsetting these elements are the negative income effect on supply, $\partial \bar{Y}_0^s/\partial \bar{I}_0$, and the negative intertemporal-substitution effect on demand, $\partial \bar{C}_0^d/\partial r_0$. The net impact on output depends on the composite term, $(\partial \bar{C}_0^d/\partial \bar{I}_0)(\partial \bar{Y}_0^s/\partial r_0) - (\partial \bar{C}_0^d/\partial r_0)(\partial \bar{Y}_0^s/\partial \bar{I}_0)$. (In Barro (1976, 1980), the analogues to these forces are combined into a net term called H, which involves elasticities rather than sensitivities of commodity demand and supply.)

As indicated in equation (17), the conditions needed for a net positive response of current output cannot obtain if utility is time-separable (and the implied cross-preference relation holds in aggregate form). Rather, the net response of current output to an aggregate income effect is zero. This result does not require any limitations on the sizes of any individual substitution or income effects--for example, the substitution effect on supply, $\partial\overline{Y}_0^s/\partial r_0$, can be arbitrarily large. Rather, the outcome reflects the cross-preference relation, which limits the various sensitivities in relation to each other. Suppose, for illustrative purposes, that we had already prescribed the magnitudes of the income and rate-of-return effects on demand, $\partial\overline{C}_0^d/\partial\overline{I}_0$ and $\partial\overline{C}_0^d/\partial r_0$. It remains possible to observe any value for the substitution response of supply, $\partial\overline{Y}_0^s/\partial r_0$. However, under time-separability, higher values for this sensitivity must be accompanied by equiproportionately higher magnitudes for the income effect on supply, $\partial\overline{Y}_0^s/\partial\overline{I}_0$. Given time-separable utility, it is infeasible to vary the intertemporal-substitution effect, $\partial\overline{Y}_0^s/\partial r_0$, arbitrarily while maintaining that the income effect, $\partial\overline{Y}_0^s/\partial\overline{I}_0$, is--for example--of negligible significance. (This conclusion assumes that the sizes of the demand sensitivities are being held fixed.)

Our basic result is the invariance of current output from pure income effects. We derive this result from an alternative perspective in the following section.

Defoe's "Island" Model

Consider the situation of an isolated individual--Robinson Crusoe.[7] A positive income effect can arise here if Crusoe anticipates the arrival of some free goods in a future period. (Crusoe expects for some future period a higher position of the production function but, for the sake of argument, no change in the schedule for the marginal product of labor.) With no investment opportunities and time-separable utility, Crusoe's behavior today is divorced entirely from that at other times. In particular, changes in this period's work and production have no implications for any state variables that will apply for later periods. Hence, today's optimal choices of consumption and leisure are invariant with any shifts in future prospects. The invariance of current output from income effects holds immediately for Robinson Crusoe. (However, the shadow discount rate that connects tomorrow's consumption or leisure with today's, r_0, does tend to rise when future prospects improve.[8])

The argument for Robinson Crusoe carries over to the aggregate of individuals in a market economy when there are no investment opportunities and utility functions are time-separable. Decisions on today's aggregates of work and production do not matter for the levels of aggregate state variables in future periods. (The distribution of claims on the credit market could be affected.) If only the aggregate levels of these state variables matter for aggregate choices--as we have been assuming--then current behavior is separated from perceptions about future conditions. Hence, current output is invariant to aggregate income effects that result from changes in future prospects.

General Implications

The invariance of current output applies in our model to any change in prospective conditions. These include any source for a shift in the perceived aggregate income term, $\overline{I}_0$. Changes in prospective technological parameters, α_t, combine income effects, which amount to shifts in $\overline{I}_0$, with alterations in future schedules for the marginal product of labor. Because of the separation between time periods--when we exclude investment and assume time-separable utility--there is no spillover from these changes in future prospects to the current choices of work and production. Formally, we can derive this result for a change in $\alpha_t (t = 1, 2, ...)$ by using the cross-preference relation from equation (14).[9]

Non-Separable Utility and Fatigue

We can clarify briefly how a specific form of non-separable perferences would alter the previous results. Suppose that today's utility from leisure depends on "fatigue"--namely, the greater is the amount of past work (possibly expressed as a distributed lag of prior work levels), the higher is the marginal utility of today's leisure relative to today's consumption. In this context past work becomes a pertinent state variable for the current period.

Consider again the consequences of an aggregate income effect--that is, a rise in $\overline{I}_0$. We hold fixed the prospective schedules for the marginal product of labor in future periods. From the perspective of Robinson Crusoe, we can think of the prospect of more goods--but not a higher marginal product of labor--in some future period. In this situation people expect to take more leisure in the future period where goods have become more abundant. Accordingly, there is less cost attached to being fatigued at this future date. Therefore, people become more willing to work harder during periods--including the present--that precede this time of abundance. In our previous discussion we found that the equilibrium quantities of today's work and production were invariant with a pure income effect. Because the new element that considers fatigue is favorable to current work, we will now conclude that current work and production are higher on net in response to a positive income effect. (In equilibrium, the positive effect of the higher rate of return, r_0, on $\overline{Y}_0^s$ in equation (15) will end up dominating over the negative income effect.)

Improvements in future labor productivity, which mean rises in some future real wage rates, are likely to imply increases in future work levels. In this case it becomes more important to rest today in order to prepare for the subsequent strenuous activity. Therefore, we will find that current work and production are now reduced by any disturbance--such as a rise in a future schedule for the marginal product of labor--that leads to an increase in future work effort.

We assume in our main analysis that cumulated fatigue is not an important consideration over time intervals that are interesting for macroeconomics. Therefore, we abstract in our principal discussion from the types of effects that were discussed in this section.

Monetary Theories of Business Fluctuations

If nominal disturbances are mistakenly perceived as representing shifts in intertemporal relative prices, then such shocks can be non-neutral toward aggregate output. Suppose that each economic agent produces and consumes in only one of many decentralized markets, while no one observes directly nominal aggregates or general price indices. Then, Lucas (1973) and Barro (1976, 1980) demonstrate that the basic equilibrium model can rationalize a positive correlation of money, aggregate output and the price level. In these models a positive monetary disturbance (arising as a surprise transfer from the government) causes the average household to overestimate its income position, which can be represented by an increase in $\overline{I}_0$ in the present setup. The "normal" positive response of output to a monetary disturbance obtains in the models of Barro (1976, 1980) only if the parameter H, which is analogous to $(\partial\overline{C}_0^d/\partial\overline{I}_0)(\partial\overline{Y}_0^s/\partial r_0) - (\partial\overline{Y}_0^s/\partial\overline{I}_0)(\partial\overline{C}_0^d/\partial r_0)$, is positive. However, under time-separable preferences, monetary disturbances--even if imperfectly perceived--will have no impact on current output.[10] This conclusion follows from the cross-preference relation, which ensures that $H = 0$.

Government Purchases

Some recent equilibrium analyses (Hall, 1980; Barro 1981) consider the effects of government purchases on aggregate output and rates of return. These studies stress the distinction between permanent and temporary changes in government purchases. Time-separable preferences also have strong implications for this type of macro disturbance.

Suppose that the government demands commodities in the amount G_t in period t. Assume for the moment that these expenditures are financed by lump-sum taxes, which are equal for each household. The government uses its purchases of goods to provide some services to the private sector. These services may appear in households' functions for utility or production.

With respect to utility, we assume that the effects of G_t appear only in period t's flow--that is, as $u^t(C_t, L_t, G_t)$. Hence, there is still time-separability in the utilities derived from goods, which now include public services as well as private consumption and leisure. Similarly, the productive-input effect of G_t is purely contemporaneous. That is, we write the production function for period t as

$$(18) \qquad Y_t = F(L_t, G_t; \alpha_t).$$

The present value of (lump-sum) taxes appears as a negative item in each household's budget constraint. Neglecting transfers and public debt, the present value of each household's taxes equals the per capita value of government purchases, $\sum_{t=0}^{\infty} \rho_t G_t$. The household's intertemporal budget constraint still has the form,

$$\sum_{t=0}^{\infty} \rho_t (C_t - Y_t) = I_0,$$

if we include the per capita value of $-\sum_{t=0}^{\infty} \rho_t G_t$ as part of the household's income term, I_0. The aggregate income term, $\overline{I}_0$, now includes the negative of the present value of government purchases.

The introduction of the government implies that the form of the aggregate commodity-market-clearing condition, equation (15), must be modified to accommodate public purchases of final product. The revised condition is

$$(19) \qquad \overline{Y}_0 = \overline{C}_0^d(\alpha_0, \overline{I}_0, r_0, G_0, \ldots) + G_0 = \overline{Y}_0^s(\alpha_0, \overline{I}_0, r_0, G_0, \ldots).$$

The omitted terms in equation (19) may include effects of the prospective time path of government purchases, G_1, G_2, ... These variables can matter for future choices of consumption and work, which then can alter the net amount of funds available to an individual household for expenditure during the current period.

Consider a rise in current government purchases, G_0, when all future levels of purchases are held fixed. There is a one-to-one expansion of current aggregate demand, which is offset by any influences of G_0 on $\overline{C}_0^d$ and $\overline{Y}_0^s$. Since the change in purchases is temporary, there will not be a large effect on the income term, $\overline{I}_0$. However, the change in G_0 may interact directly with the household's choices of work effort, production and consumption. As one possibiliby (see Bailey (1971, Ch. 9) and Barro (1981, pp. 1090-93)), the public services substitute for some contemporaneous private consumption--so that $\overline{C}_0^d$ declines--and enhance private production--so that $\overline{Y}_0^s$ expands. (There might also be direct effects of public services on desired work effort.) If the sum of the magnitudes of the responses of $\overline{C}_0^d$ and $\overline{Y}_0^s$ is less than one-to-one with the change in G_0, then an increase in current government purchases causes current aggregate demand to rise by more than supply. Therefore, a rise in r_0 is typically required in order to clear the commodity market. The equilibrium is likely to entail higher current work effort and output, but lower consumption. (These results depend on the precise manner in which public services enter the utility and production functions.)

We are not concerned here so much with the responses to a change in government purchases. Rather, we want to study the differences in results when the change in purchases applies for only one period--as assumed above--instead of for many periods. We can convert our case of a temporary change to a more or less permanent one by altering the levels of G_1, G_2, ..., and seeing how these shifts influence the market-clearing condition in equation (19). Increases in these future purchases--which entail future taxes--imply reductions in the aggregate income term, $\bar{I}_0$. The future levels of purchases may also appear directly in the functions for $\bar{C}_0^d$ and $\bar{Y}_0^s$. As noted before, these effects involve alterations in planned future levels of work and consumption. However, with time-separable utility, neither changes in $\bar{I}_0$ nor other shifts to future work and consumption can influence the equilibrium levels of current work and consumption. Therefore, the changes entailed by converting from a temporary change in purchases to a more or less permanent change have no influences on current quantities. With time-separable utility, temporary and permanent changes in government purchases have identical effects on current quantities of work, production, and consumption.

This result can again be motivated by reference to Robinson Crusoe. Suppose that the "government" acquires 100 units of Crusoe's goods (on a lump-sum basis) for some worthwhile purpose. With time-separable utility and no investment, the response of Crusoe's work effort, etc. does not depend on his expectations of future governmental activities. His optimization problem for today is isolated from subsequent events.

The distinction between temporary and permanent government purchases does matter for the response of the real rate of return, r_0. Under the conditions assumed before, increases in future levels of purchases, G_1, G_2, ..., imply declines in future levels of consumption and leisure. Typically, these anticipated changes lead to a lower value for the current real rate of return, r_0, (The full results depend on the manner in which government purchases enter the utility and production functions.) The effect on r_0 of an increase in anticipated future government purchases is analogous to the effect from a decline in the aggregate income term, $\bar{I}_0$. That is, the present case is the reverse of the one that was explored in previous sections, where $\bar{I}_0$ increased. We conclude that the positive pressure from a rise in current government purchases on the rate of return, r_0, is reduced to the extent that future purchases are also expected to increase.[11] (The rate of return may not increase at all if the rise in purchases is permanent.)

Distorting Taxes and Public Debt

Suppose that taxes are levied on effort, rather than being lump-sum. Then, an increase in government purchases implies more taxes, which motivate substitutions away from market activity and toward "leisure." However, the output effect of permanent and temporary shifts in government purchases would still be identical if the government were not permitted to issue debt. That is, if date t expenditures were financed solely by date t taxes, then an analogue to our previous argument would demonstrate that permanent and temporary shifts in government purchases have equal effects on current work effort, production and consumption.

If the government is permitted to borrow or lend on the economy's credit market, then a potential difference arises between permanent and temporary shifts in purchases. Suppose, as in Barro (1979), that the government uses debt issue to smooth the behavior of (income) tax rates over time. Then, a permanent change in government purchases necessitates a larger adjustment of current tax rates than does a temporary one. Hence, the effect on current output tends to be greater if the change in government purchases is temporary. Because of the smaller increase in tax rates, the induced substitutions away from market activities will be smaller in the case of a temporary change.

The introduction of public debt permits time patterns of tax rates that are infeasible under the balanced-budget option. When taxes are distorting, we must include the outstanding stock of public debt in a description of the "state of the economy." This stock functions as a current state variable in a manner analogous to "cumulated fatigue," which we discussed before, or the capital stock, which we discuss later.

III. Intertemporal Production Opportunities

Previously, production opportunities in each period were separated from economic actions taken in other periods. The most natural way to connect different time periods on the production side is to permit economic agents to accumulate physical stocks of goods (or of human capital or knowledge). The key point is that the potential for accumulation/decumulation at the aggregate level permits a representative agent to behave, in equilibrium, more like an individual who faces a given interest rate on the credit market. That is, the economy as a whole may respond to a temporary change in income or the marginal product of labor by altering the stocks of goods carried over to the future.

Here we sketch out the simplest possible model of capital accumulation and draw out some of its implications. To a certain extent, the discussion is meant to be suggestive rather than definitive. The linkages across time that are introduced by capital accumulation substantially complicate the equilibrium analysis. In this paper we do not analyze fully "Robinson Crusoe's" dynamic choice problem for production and consumption, which would yield a detailed description of equilibrium quantities.

Suppose now that the economy faces the production function for period t,

$$(20) \qquad Y_t = F(L_t, K_{t-1}; \alpha_t),$$

where K_{t-1} is the stock of capital available to a household at the start of period t. The stock, K_{t-1}, includes the investment from period t-1, but excludes any investment during period t. We can think of K as encompassing either producers' or consumers' stocks of durables. We assume a one-sector technology with no adjustment costs--that is, K is just the accumulation of past Y's that have been designated as capital goods. Further, we assume that capital can be reconverted on a one-for-one basis to consumables, which can then be "eaten up." These assumptions imply that the marginal product of capital is always equated to the one-period real rate of return--that is,

$$(21) \qquad \partial F/\partial K_t = r_t \text{ for all t.}$$

Investment demand, $i_t^d \equiv K_t^d - K_{t-1}$, depends inversely on r_t, given the position of the schedule (versus K_t) for the marginal product of capital. The technological shift parameter, α_{t+1}, and any elements that affect the level of future work, L_{t+1}, can influence the position of this marginal-product schedule. Therefore, these variables can affect investment demand. For our purposes we write the aggregate investment-demand function as

$$(22) \qquad \bar{i}_t^d = \bar{i}_t^d(r_t, \bar{K}_{t-1}, \ldots),$$

where $\partial\bar{i}_t^d/\partial r_t < 0$ and $\partial\bar{i}_t^d/\partial\bar{K}_{t-1} < 0$. Omitted components of equation (22) include technological parameters, α_{t+1}, and forces that affect L_{t+1}.

Next, we augment the goods-market equilibrium condition, equation (15), to incorporate aggregate investment demand. The revised condition is

$$(23) \qquad \bar{Y}_0 = \bar{C}_0^d(\alpha_0, \bar{I}_0, r_0, \ldots) + \bar{i}_0^d(r_0, \bar{K}_{-1}, \ldots)$$

$$= \bar{Y}_0^s(\alpha_0, \bar{I}_0, r_0, \bar{K}_{-1}, \ldots).$$

We abstract here from government purchases.

As mentioned before, there are various omitted arguments in the functions shown in equation (23). We omit changes in these variables when considering some economic disturbances. Then, it is straightforward to calculate effects on the current rate of return, r_0, and on current quantities. This procedure is suggestive but cannot give the exact change in current output, because we are dealing with a non-trivial model of general equilibrium over time. That is, changes in current capital accumulation (arising from a particular disturbance) will cause variations in the future rates of return, r_1, r_2, ..., and future marginal-product schedules, which are suppressed in equation (23). In turn, these variables will feed back into the commodity-market equilibrium condition, and thereby alter current levels of capital accumulation, etc.

Consider again the effect of a change in aggregate income, $\overline{I}_0$, when the starting capital stock, $\overline{K}_{-1}$, is held fixed. The effects are now given by

$$(24) \qquad \frac{dr_0}{d\overline{I}_0} = \left[\frac{\partial \overline{C}_0^d}{\partial \overline{I}_0} - \frac{\partial \overline{Y}_0^s}{\partial r_0}\right] \Big/ \left[\frac{\partial \overline{Y}_0^s}{\partial r_0} - \frac{\partial \overline{C}_0^d}{\partial r_0} - \frac{\partial \overline{i}_0^d}{\partial r_0}\right] > 0 \text{ (if all goods are superior)},$$

$$(25) \qquad \frac{d\overline{Y}_0}{d\overline{I}_0} = \left[\frac{\partial \overline{C}_0^d}{\partial \overline{I}_0}\frac{\partial \overline{Y}_0^s}{\partial r_0} - \frac{\partial \overline{Y}_0^s}{\partial \overline{I}_0}\frac{\partial \overline{C}_0^d}{\partial r_0} - \frac{\partial \overline{Y}_0^s}{\partial \overline{I}_0}\frac{\partial \overline{i}_0^d}{\partial r_0}\right] \Big/ \left[\frac{\partial \overline{Y}_0^s}{\partial r_0} - \frac{\partial \overline{C}_0^d}{\partial r_0} - \frac{\partial \overline{i}_0^d}{\partial r_0}\right]$$

$$= -\left[\frac{\partial \overline{Y}_0^s}{\partial \overline{I}_0}\frac{\partial \overline{i}_0^d}{\partial r_0}\right] \Big/ \left[\frac{\partial \overline{Y}_0^s}{\partial r_0} - \frac{\partial \overline{C}_0^d}{\partial r_0} - \frac{\partial \overline{i}_0^d}{\partial r_0}\right] < 0 \text{ (if all goods are superior)}.$$

As before, the rise in $\overline{I}_0$ increases $\overline{C}_0^d$ and lowers $\overline{Y}_0^s$ (assuming superior goods), which tends to drive up the current rate of return, r_0. Because the increase in this rate of return reduces investment demand, the necessary increase in r_0 (shown in equation (24)) is less than that calculated before, assuming that the income and substitution effects on $\overline{Y}_0^s$ and $\overline{C}_0^d$ are the same as previously. In our earlier discussion current work and consumption ended up unchanged on net. Therefore, with a smaller rise in the interest rate, we find that current work now declines on net, while current consumption rises. The fall in work means a reduction in current production (since $\overline{K}_{-1}$ and the form of the aggregate production function in period 0 are fixed). The rise in current consumption is made possible by a decline in current investment. The results for current output appear in equation (25), which uses the cross-preference relation from equation (13).

We can again explain the findings from the perspective of Robinson Crusoe. A positive income effect signals the prospect of better times ahead (although not necessarily higher marginal-product schedules for labor and capital). Given this expectation, Crusoe has less incentive to work hard and consume little today in order to accumulate capital. In fact, Crusoe wants to use up previous stores of goods (capital) in order to raise present consumption and leisure. Cutbacks in investment effectively enable Crusoe to use future abundance in order to provide for current consumption and leisure. Therefore, when we include a variable amount of investment--which ties different time periods together--we find that a positive income effect tends to lower current work, production and investment, while raising current consumption and leisure.

Notice that our analysis does include an intertemporal-substitution variable, r_0, as a positive--possibly strong--determinant of today's work effort. Further, the positive aggregate income effect does tend to raise the equilibrium value of this substitution variable. However, any enhancement in the reward to working today rather than later--as reflected by an increase in r_0--implies a parallel expansion in the cost of consuming goods today rather than later. The disturbance being considered--a pure positive income effect--does not alter the terms on which people can exchange today's leisure for today's consumption. These terms are dictated by today's production function. Therefore--given time-separable utility and the assumption that today's consumption and leisure are superior goods--it is impossible for current consumption and leisure to move in opposite directions in response to a pure income effect. Consumption and leisure also cannot react in opposite directions to movements in the intertemporal-substitution variable, r_0. Changes in any prospective real wage rate or rate of return also move current consumption and leisure in the same direction. In the case described above, where investment can be lowered, we find that a positive aggregate income effect raises current consumption and leisure. This response means a decline in today's output and employment, as well as a reduction of investment. (A negative income effect would increase today's output, employment, and investment, but would lower current consumption.)

We take as a minimal empirical description of business fluctuations the positive co-movements of aggregate production, employment, investment and consumption.[12] We cannot generate this typical pattern of business cycles from pure income effects in our present model. This result follows from the following properties: 1) time-separable utility, 2) consumption and leisure in all periods are superior goods, and 3) aggregation is permissible in the usual manner. This observation is significant since some equilibrium models of business cycles (Lucas (1973), Barro (1976), et. al.) treat monetary shocks as operating initially through an income effect on consumer demand.[13]

Shocks to Investment Demand

We want to consider shocks to investment demand as a possible source of business fluctuations. King (1982) shows that monetary surprises can alter perceptions about the prospective marginal revenue product of capital, which then lead to shifts in desired investment. We want to see whether this type of disturbance--possibly combined with the sorts of income effects that we examined before--can yield the typical pattern of business cycle response. In particular, we are looking for co-movements of output, employment, investment and consumption.

The current equilibrium condition for the commodity market is given in equation (23). It is straightforward to introduce a positive, autonomous shift to investment demand, $\bar{i}_0^d$. The appropriate change in the technological parameter, α_1, would generate this response. We assume that there is no shift in the current production function--that is, α_0 is unchanged. The main effects of this disturbance are an increase in the current rate of return, r_0, increases in current output, work effort and investment, and a decline in current consumption. In particular, consumption and leisure move in the same direction--downward in this case--in response to the rise in r_0. With time-separable utility and all goods superior, it is again impossible for consumption and leisure to respond in opposite directions to changes in the relative prices of future goods--that is, to a change in r_0 (or, more generally, to other prospective rates of return or real wage rates). Therefore, although this disturbance can generate positive responses of current production, work effort and investment, it cannot simultaneously generate a positive reaction of current consumption.

One might conjecture that the addition of an aggregate income effect could provide the requisite boost to current consumption demand. (In King (1982) monetary surprises create positive income effects as well as boosts to the perceived marginal value product of capital.) But, under the maintained hypothesis that preferences are time-separable, this route turns out to be unsatisfactory. If the positive aggregate income effect is sufficiently strong to generate a net increase in current consumption, then it must do so by more than offsetting the negative influence from the rise in the current real rate of return, r_0. But--because of the cross-preference relation from equation (13)--the same balance of forces then implies that current leisure must also increase on net. In this case current work effort and production decline. Assuming that all goods are superior and that utility is additive over time, no combination of aggregate income effects and shifts in the relative prices of future goods (that is, changes in r_0, r_1, ..., or shifts in prospective real wage rates) can move current consumption and leisure in opposite directions. There is no package of shocks to investment demand and perceived aggregate income that can lead simultaneously to increases in current employment and consumption.[14]

The Contemporaneous Real Wage Rate

If utility is time-separable and all goods are superior, then we can generate an increase in today's consumption and work effort--hence, a decline in today's leisure--only if we generate an upward shift in today's schedule for the marginal product of labor.[15] In particular, we require an increase in the current real wage rate, w_0, which equals labor's current marginal product.

Suppose that we introduce separate markets for commodities and labor services, but that we stick to the plausible story that individuals are buying today's leisure and today's consumption at prices that are observed simultaneously. (People may not observe the prices of all goods at once, but they at least know the prices of those goods that they actually buy or sell.) Then, it will not be possible for monetary surprises or aggregate disturbances to generate misperceptions about the ratio of current nominal wages to the prices of consumer goods that are bought currently. In our world with time-separability, we will find an increase in today's consumption and a decrease in today's leisure only if the (observed) price of today's consumption falls relative to the (observed) price of today's leisure. A procyclical pattern of the actual real wage rate is central to our analysis.

We should stress that the real wage rate in our theory refers to the typical person's shadow price for current leisure relative to current consumption. There are at least two difficulties in using reported series on average wages to measure this concept. First, (efficient) long-term contracts are consistent with a discrepancy between reported wage rates and the true shadow value of current time. (See, for example, Barro, 1977.) Second, since wage rates vary cross-sectionally, average wage data (for employed persons) may be misleading when the composition of the employed labor force varies. Notably, if workers with relatively low productivity tend to be laid off first, then a spurious element of countercyclical wage movement will be present in data on average wages (of employed persons). There are also complications in using over-time versus straight-time wage rates. Since the shadow value of the marginal unit of time is pertinent to our analysis, it would be inappropriate simply to use a measure of straight-time wage rates, which attempt to exclude over-time payments. We look forward to a careful study of the cyclical behavior of real wage rates, which takes account of the factors cited above.

Shifts to Productivity

In our setup a natural way to generate an increase in today's real wage rate, w_0, is to postulate a general upward shift to the current production function. In particular, we could have a shift to the current technological parameter, α_0, which raises the current schedule for the marginal product of labor.

If this type of general upward shift to technology applies to future periods, we would also have a positive aggregate income effect and a boost to the marginal product of capital--effects that were discussed before. When we add in the upward shift to the current schedule for the marginal product of labor, we can resolve our earlier dilemma. It now becomes possible to observe increases in current output, employment, investment and consumption. In particular--because of the upward shift in the current real wage rate--current consumption can rise while current leisure falls.

Sources of Shifts

Exogenous changes in productivity are central driving variables in the real-business-cycle theories of Kydland and Prescott (1980) and Long and Plosser (1980). These analyses use versions of the neoclassical production function in which there are more than one capital-stock variable. For the present purpose, the key feature of this structure is that there are no current-period actions that can alter the position of the marginal-product schedule for labor. This conclusion follows because capital stocks are not adjustable within the period.

More general descriptions of intertemporal production opportunities do not share this characteristic. For example, King (1980) and Merrick (1981) indicate how variations in the utilization of existing capital goods--in response to intertemporal relative prices--can alter the position of the current schedule for labor's marginal product. That is, a higher current flow of "capital services" raises labor's marginal product if the factors are complementary. Yet, variable utilization alone is not sufficient to generate the cyclical co-movements that we are looking for. In the simplest formulation of the utilization decision, it is impossible to get current investment and current flows of capital services to move in the same direction. That is, this model fails to generate a procyclical pattern for both investment and capacity utilization. Our hunch is that intertemporal structures that mix variable utilization and "time-to-build" requirements for capital will ultimately deliver the co-movements that we seek. Conceivably, in this framework, misperceived monetary shocks may generate a procyclical pattern of investment, capital utilization, employment, output and consumption. These possibilities will be explored in future research.

Conclusions

Time-separability of utility means that past work and consumption do not influence current and future tastes. This type of separation may be a reasonable approximation over time periods--such as quarters or years--that are of primary interest for macroeconomic analysis.

The assumption that preferences are time-separable is implicit in much macroeconomic analysis. For example, Friedman's (1956) linkage of consumption to permanent income derives much of its attractive empirical content from the fact that past consumptions are bygones, which are unimportant for current decisions. This preference condition--made explicit by Hall (1978)--generates strong testable restrictions that are not implied by other theories, such as the habit-persistance model, which implicitly incorporates non-time-separable preferences.

In our analysis of dynamic labor supply and consumption decisions, time-separable preferences do not restrict the size of intertemporal-substitution effects--notably, we can still have a strong response of labor supply to temporary changes in real wages. The important restrictions arise as cross-preference conditions--constraints on the relative responses of leisure and consumption to relative-price and income effects. There are also restrictions on the relative responses of today's work or consumption to prospective wage rates or interest rates from different future (or past) periods. While these types of cross conditions are testable, we do not know of empirical evidence that contradicts them.

When the usual aggregation is permissable, time-separability has some important implications for equilibrium theories of the business cycle. On the one hand, we find it difficult to use some existing versions of these models to generate the typical cyclical pattern of quantities. Specifically, combinations of income effects and shifts to the perceived profitability of investment do not yield positive co-movements of output, employment, investment and consumption. Therefore, we are unable to use misperceived monetary disturbances or other sources of changed beliefs about the future in order to generate empirically recognizable business cycles.

On the other hand, our analysis points to modifications in existing theories that may yield more satisfactory answers. First, we stress that variable investment is essential in order to link current choices to perceptions about the future. Because of time-separable utility, such a linkage does not arise--in equilibrium--from the side of preferences. Second, we use the observed positive correlation between cyclical movements of consumption and work--that is, inverse movements in consumption and leisure--to argue that the real wage rate must move procyclically. Empirically, we regard the cyclical pattern of real wages as an open question. At the theoretical level, we are led to stress disturbances that alter the current schedule for the marginal product of labor. Aside from autonomous real shocks to productivity, we mention the role of capacity utilization. Changed prospects about future conditions may motivate firms to work their capital harder. Complementarity between capital services and labor services then generates an upward shift to the current schedule for the marginal product of labor. Hence, the current real wage rises. We suggest that misperceived monetary disturbances might function in this manner. While we regard this route as promising, we have so far been unsuccessful in combining this story with a procyclical pattern of investment.

Footnotes

[1]This utility function is strongly separable with respect to a partition by time periods.

[2]Kydland and Prescott (1981) deal with a particular form of non-time-separable preferences. In their analysis a distributed lag of past work appears as a current state variable.

[3]The length of the horizon is unimportant for present purposes.

[4]Lucas (1972) constructs a model where simple aggregation does not work. The distribution of income between young and old is important in Lucas's setup, as it is generally in overlapping-generations models that neglect private intergenerational transfers.

[5]We can equivalently deal with a separate labor market. We would then use the condition that the marginal product of labor equals the real wage rate, w_t, in each period.

[6]More generally, we would substitute the future market-clearing values of these interest rates, as perceived by the representative individual. The prospective values of r_1, r_2, ..., will not change if the average person does not expect the disturbance to have aggregate consequences in future periods. This expectation may be reasonable for the context of monetary surprises, which are not perceived as aggregate shocks. Generally, the presence of financial futures markets or markets for long-term loans will affect the information that people have about the future one-period interest rates, r_1, r_2, ... In the present context variations in these future interest rates are, in any case, inconsequential for current output. This result follows from equation (13), which implies that changes in any future interest

(continued)

rate, r_t, will alter the current rate, r_0, so as to leave current output unchanged. The calculated effects on the current interest rate in equation (16) will be inexact for this reason.

[7]Long and Plosser (1980) similarly describe the position of a Robinson Crusoe with time-separable preferences, within a "real business-cycle" model that incorporates many consumption and capital goods.

[8]Shadow prices may be read off the derivatives of Crusoe's maximized utility function. Note that the announcement of a receipt of goods k > 1 periods in the future would not alter the one-period real rate of return, $(1+r_0) = (1+\gamma)(\partial u^0/\partial C_0)(\partial u^1/\partial C_1)$, in such a setup. In this sense the calculated effect on the current real interest rate in equation (16) is inexact in ways that are economically important (see fn. 6, above).

[9]The income effect from a change in α_t is covered by equations (16) and (17). If the change in α_t raises the prospective real wage rate, w_t (by raising the schedule for the marginal product of labor), we will find an additional positive effect on r_0 (because $\overline{C}_0^d$ rises and $\overline{Y}_0^s$ falls). However, $\overline{Y}_0$ is again unchanged.

[10]As is traditional in business-cycle analysis, we abstract from the effects of imperfect information on aggregate real balances and possible implications for agents' willingness to engage in market activities. In particular, a positive monetary disturbance that was under-estimated would lead economic agents to over-estimate the future value of money. The nominal interest rate would be lower than under full information, and real balances

(continued)

would be correspondingly higher. We rule out any temporary real affects of such variations in real balances. Essentially, these potential effects correspond to those that would arise in a fully-perceived, permanent inflation. These influences are commonly regarded as a minor element in the business cycle.

[11]The current one-period interest rate, r_0, depends only on quantities for periods 0 and 1. These quantities are insensitive to changes in government purchases after period 1--that is, to G_2, G_3, ... Therefore, prospective purchases after date 1 cannot affect r_0 in the present model. The current, one-period rate of return, r_0, depends on G_0 and G_1. Basically, r_0 rises when G_0 increases relative to G_1. Changes in G_2, G_3, ..., affect r_1, r_2, ... (Note that these rates of return enter among the omitted arguments of the functions in the market-clearing condition of equation (19).) The effects on future short rates show up in current long-term interest rates (or in interest-rate futures), although not in r_0. Hence, prospective variations in government purchases affect the term structure of real interest rates. (See Benjamin and Kochin, 1982, in this context.) When investment is added to the model (below), the prospective path of purchases, G_2, G_3, ..., will also influence the current short rate, r_0.

[12]We do not mean that all aggregate business fluctuations exhibit these characteristics. For example, expansions associated with major wars tend to show declines in private investment and in at least the durable-goods component of consumer spending. This pattern is especially evident during World War II.

[13]The inclusion of investment means that temporary and permanent changes in government purchases no longer have identical effects on current output (even when we ignore the effects implied by distorting taxes). Current investment tends to decline more when the change in purchases is temporary. (This finding is consistent with the tendency of the current interest rate, r_0, to rise in response to temporary changes in purchases.) In effect, society (or Robinson Crusoe) can meet emergencies partly by working off the existing stock of capital--or, at least, by investing less than otherwise. This channel reduces people's incentives to work hard and consume little during periods where government purchases are temporarily high. Because of the reduced motivation to work hard, the overall effect on current output now tends to be greater when the change in purchases is permanent, rather than temporary. The effects of distorting taxation, which were described earlier, have the opposite implications. Therefore, we cannot say whether temporary or permanent changes in government purchases have a greater overall effect on current output.

[14]Grossman (1973, p. 1367) pointed out that market-clearing macroeconomic models predict a negative association between consumption and employment, if the primary disturbances are variations in "autonomous" expenditures, such as shifts to investment demand. Without restrictions implied by time-separability, however, King (1982, pp. 12-15) demonstrates that positive co-movements of consumption, investment and employment may arise if factors that raise investment demand also increase a representative economic agent's perceived wealth. The conclusion that no package of shocks can lead to the desired positive co-movements is a consequence of time-separable preferences.

[15] We neglect shifts in the forms of the representative household's preferences for consumption and leisure. That is, we rule out shifts in tastes as significant sources of aggregate business fluctuations.

Appendix

Implications of Time-Separability for Demand Functions

This appendix discusses the properties of consumer demand functions that are implied by time-separability. More specifically, an individual's preferences for goods over time have the form,

$$\text{(A1)} \qquad \sum_{t=0}^{T-1} \Gamma^t u^t(x_t),$$

where $x_t = (x_{1t}, x_{2t}, \ldots X_{Mt})'$ is a vector of M activities undertaken at date t. Individuals have a pure rate of time preference, γ, with $\Gamma = (1+\gamma)^{-1}$, which is independent of the level of x_t. The "momentary utility function," u^t, is increasing in each of its arguments, twice continuously differentiable and strictly concave.

An intertemporal budget constraint with a T-period horizon (assuming no bequests) is

$$\text{(A2)} \qquad \sum_{t=0}^{T-1} \rho_t(p_t x_t) = I_0 + \sum_{t=0}^{T-1} \rho_t(p_t z_t),$$

where $p_t = (p_{1t}, p_{2t}, \ldots p_{Mt})$ is a vector of prices, $z = (z_{1t}, z_{2t}, \ldots z_{Mt})'$ is a vector of endowments, and ρ_t is a present-value factor, ($\rho_0 = 1$, $\rho_1 = 1/(1+r_0)$, $\rho_2 = 1/[(1+r_0)(1+r_1)]$, etc.).

A straightforward means of describing the consumer's demand behavior is to use conventional comparative-statics results, as discussed by

Intrilligator (1971, Ch. VII). Treat the consumer's problem as (A3), where we stack the vectors of date t activities sequentially into larger vectors, $X' = (x_0', x_1', \ldots x_{T-1}')$, $P = (p_0, \rho_1 p_1, \ldots \rho_{T-1} p_{T-1})$ and $Z' = (z_0', z_1', \ldots z_{T-1}')$:

$$\text{(A3)} \qquad \max_X U(X), \text{ subject to } PX \leq I + PZ.$$

There are a total of N = MT activities. Let H be the NxN Hessian matrix of second-partial derivatives of U. Then, as shown by Intrilligator, the comparative-statics results for a change in income and utility-compensated changes in prices are

$$\text{(A4)} \qquad \frac{\partial X^d}{\partial I} = -\mu H^{-1} P',$$

$$\text{(A5)} \qquad \frac{\partial X^d}{\partial P}\Big|_U = \mu H^{-1} P' P H^{-1} \lambda + H^{-1} \lambda,$$

where λ is the Lagrange multiplier attached to the constraint, $I + PZ - PX \geq 0$, and $\mu = -[PH^{-1}P']^{-1}$. Economically, λ corresponds to the life-time marginal utility of wealth, and $\mu = -\frac{\partial \lambda}{\partial I}$. Finally, the total effect of a price change is given by

$$\text{(A6)} \qquad \frac{\partial X^d}{\partial P} = \frac{\partial X^d}{\partial P}\Big|_U - \left(\frac{\partial X^d}{\partial I}\right)(X-Z)'.$$

Now, write the compensated price sensitivities of demand as

$$\text{(A7)} \qquad \frac{\partial X^d}{\partial P}\Big|_U = \left(\frac{\lambda}{\mu}\right)\left(\frac{\partial X^d}{\partial I}\right)\left(\frac{\partial X^d}{\partial I}\right)' + H^{-1}\lambda.$$

Thus, zeroes in H^{-1} link price sensitivities of demand to income sensitivities.

Under time-separability, the Hessian matrix H is block-diagonal, having the following form

$$H = \begin{bmatrix} A_0 & 0 & 0 & \cdots & 0 \\ 0 & \Gamma A_1 & 0 & \cdots & 0 \\ 0 & 0 & \Gamma^2 A_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & \cdots & \Gamma^{T-1} A_{T-1} \end{bmatrix},$$

where A_t is the (negative-definite) MxM matrix of second derivatives of u^t above, and 0 is an MxM matrix of zeroes. Correspondingly, the matrix inverse is also block diagonal,

$$H^{-1} = \begin{bmatrix} A_0^{-1} & 0 & 0 & \cdots & 0 \\ 0 & \frac{1}{\Gamma} A_1^{-1} & 0 & \cdots & 0 \\ 0 & 0 & (\frac{1}{\Gamma})^2 A_2^{-1} & \cdots & 0 \\ \vdots & \vdots & \vdots & & \\ 0 & 0 & 0 & & (\frac{1}{\Gamma})^{T-1} A_{T-1}^{-1} \end{bmatrix}.$$

Thus, with H^{-1} block diagonal, (A7) implies the following condition, which links demand behavior for good m at date t to changes in the price of good k (which may or may not be m) at some other date τ,

$$\text{(A8)} \qquad \frac{\partial x^d_{mt}}{\partial(\rho_\tau p_{k\tau})}\Big|_U = (\frac{\lambda}{\mu}) \frac{\partial x^d_{mt}}{\partial I} \frac{\partial x^d_{k\tau}}{\partial I} \qquad \text{for all } t \neq \tau,$$

which is the cross-condition described by Houthakker, Goldman-Uzawa, and in the main text.

Some further results can be obtained by utilizing the special form of H^{-1} implied by time-separable preferences,

$$\text{(A9)} \qquad \frac{\partial x^d_t}{\partial I} = \mu \frac{\rho_t}{\Gamma^t} A_t^{-1} p_t' \qquad \text{for } t = 0, 1, \ldots T-1,$$

$$\text{(A10a)} \qquad \frac{\partial x^d_t}{\partial(\rho_t p_t)}\Big|_U = \mu\lambda(\frac{\rho_t}{\Gamma^t})^2 A_t^{-1} p_t' p_t A_t^{-1} + \lambda A_t^{-1},$$

$$\text{(A10b)} \qquad \frac{\partial x^d_t}{\partial(\rho_s p_s)}\Big|_U = \mu\lambda(\frac{\rho_t}{\Gamma^t})(\frac{\rho_s}{\Gamma^s}) A_t^{-1} p_t' p_s A_s^{-1} \qquad \text{for } t \neq s,$$

where $\mu = -[p_0 A_0^{-1} p_0' + \frac{\rho_1^2}{\Gamma} p_1 A_1^{-1} p_1' + \ldots + (\frac{\rho_{T-1}^2}{\Gamma^{T-1}}) p_{T-1} A_{T-1}^{-1} p_{T-1}']^{-1}$.

Defining expenditure in period t as $e_t = p_t x_t^d$, it follows from these conditions that

$$\frac{\partial e_0}{\partial r_0}\Big|_U = \sum_{t=1}^{T-1} \frac{\partial e_0}{\partial(\rho_t p_t)} (- \frac{1}{1+r_0} \rho_t p_t')$$

$$= - \frac{1}{1+r_0} \mu\lambda[\sum_{t=1}^{T-1} (\frac{\rho_t^2}{\Gamma^t}) p_0 A_0^{-1} p_0' \, p_t A_t^{-1} p_t']$$

(continued)

$$= (-\frac{1}{1+r_0})\ \mu\lambda[-\mu^{-1} - p_0A_0^{-1}p_0'][p_0A_0^{-1}p_0']$$

$$= (-\frac{1}{1+r_0})(\frac{\lambda}{\mu})[-1 + \frac{\partial e_0}{\partial I}][-\frac{\partial e_0}{\partial I}].$$

Consequently, it follows that the proportional factor in (A8) may be interpreted as follows,

$$\text{(A11)} \qquad (\frac{\lambda}{\mu}) = \frac{-(\frac{\partial e_0}{\partial r_0}|_U)(1+r_0)}{(1-\frac{\partial e_0}{\partial I})(\frac{\partial \bar{e}_0}{\partial I})}\ .$$

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