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THE BEHAVIOR OF MONEY, CREDIT AND PRICES IN A REAL BUSINESS CYCLE

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ABSTRACT

This paper analyzes the interaction of money and the price level with a business cycle that is fully real in origin, adopting a view which differs sharply from traditional theories that assign a significant causal influence to monetary movements. The theoretical analysis focuses on a banking system that produces transaction services on demand and thus reflects market activity. Under one regime of bank regulation and fiat money supply by the monetary authority, the real business cycle theory predicts that (i) movements in external monetary measures should be uncorrelated with real activity and (ii) movements in internal monetary measures should be positively correlated with real activity. Preliminary empirical analysis provides general support for this focus on the banking sector since much of the correlation between monetary measures and real activity is apparently with inside money.

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I. INTRODUCTION

The correlation between variations in the quantity of money and fluctuations in real activity is perhaps the most discussed and least easily explained element of macroeconomics. Traditional monetary theories of business fluctuations stress market failure as the key to understanding this relationship, interpreting monetary movements as a primary source of impulses to real activity.

This paper describes an initial attempt to account for the relationship between money and business cycles in terms that most economists would label reverse causation. The basic idea is that the real sector drives the monetary sector, in contrast to the traditional view of monetary movements as business cycle impulses. In our model, variations in the real opportunities of the private economy--which include shifts in government policies such as government purchases and tax rates, as well as technical and environmental conditions--are assumed to cause variations in both measures of real activity and monetary quantities. The formal theoretical framework involves two key elements: (i) market clearing determination of prices and quantities and (ii) rational expectations based on accurate knowledge of the current values of all pertinent state variables. Real business cycles emerge in this setup as a result of the optimal plans of private agents who are confronted by variations in real opportunities.

Since the idea that certain monetary quantities are endogenous is an old one, it is useful to outline two recurrent themes. First, viewing monetary services as a privately produced good, changes in internal money can be systematically related to economic activity through the operation of an unregulated banking sector.[1] Second, in a setting that contains a mix of external (fiat) money and internal money, changes in external money can be related to either economic activity or to the forces that drive economic activity through central bank policy response. For example, Tobin (1970) provides an earlier analysis of a model with endogenous money that emphasizes central bank policy response. Tobin's deterministic treatment involves the Keynesian idea that money and real activity respond to the same causal influence--aggregate demand.[2]

Our analysis, on the other hand, employs a stochastic neo-classical growth model in which movements in money and real activity respond to variations in real opportunities. The analysis focuses on the banking system and draws heavily on another analysis of Tobin (1963) and the important recent contribution by Fama (1980a). In the absence of central bank policy response, the model predicts that movements in external money measures should be uncorrelated with real activity. Some preliminary empirical analysis (using annual data for the 1953-1978 period) provides general support for our emphasis on private banks since the correlation between monetary measures and real activity is primarily with inside money.

Our motivation for pursuing this line of research is to produce an equilibrium model that is capable of explaining the joint time series behavior of real quantity variables, relative prices, the price level, and alternative monetary measures. It is a common observation, however, that such models must include a causal role for money [e.g., Lucas (1977)]. Yet, economic theory has yet to provide a convincing rationale for such a "nonneutrality" of money. Traditional explanations focus on central, but implausible, market failures: Keynesian models most obviously so and recent equilibrium theories in terms of the use of information, particularly contemporaneous monetary information [(see King 1981)].[3] Consequently, it seems worthwhile to consider alternative hypotheses concerning money and business cycles.

The organization of the paper is as follows. In Section II we describe a simple model that is capable of generating real business cycles. This model is used to discuss correlations between an "internal" monetary quantity and real activity. In Section III, with fiat money grafted onto the model, we analyze the relationship between monetary quantities, output, and the price level in an unregulated banking environment. The analysis also deals with a regulated banking system where it becomes crucial to spell out the form of central bank policy. In Section IV we discuss some of the empirical implications of the theory and provide a preliminary analysis of the post-war U.S. experience.

II. THE REAL ECONOMY

In this section a simple model economy is described that embodies the potential for real business cycles. At this stage of research, government is taken to exert no influence on the economy through tax policy or purchases of final product. Rather, the model economy is driven solely by shocks to current production opportunities that are assumed to have a technological interpretation. Basically, the model is a stochastic growth model with a single final product. Stochastic growth models of a more general variety have recently been employed to study business cycles by Long and Plosser (1980). Since such models incorporate many final products and a more general pattern of production inter-relationships, richer patterns of variation in economic activity can be generated. However, the main aspects of the interactions of money, credit, and prices with a real business cycle can be outlined in the present, simpler framework.

A. The Final Goods Industry

There is a single final product that can serve as either a consumption good (x) or a capital good (k). This final product is produced by a constant returns to scale production process that uses labor (n), capital (k), and transaction services (d) as inputs. The output generated from given inputs is assumed to be uncertain due to stochastic technical factors.

The production technology is given by

(1) $$y_{t+1} = f(k_{yt}, n_{yt}, d_{yt})\phi_t\xi_{t+1} ,$$

where k_{yt} is the amount of capital services, n_{yt} is the amount of labor services, and d_{yt} is the amount of transaction services used in the final goods industry.[4] The production technology in (1) implies that a unit of time corresponds to the production period for the single final product.

Transaction services in (1) are viewed as an intermediate good purchased by the final good producers from the financial industry (to be described below). Although not involved directly in the production of output in the same sense as labor and capital, transaction services are taken to be a factor necessary to the firm's operation similar to other organizational and control inputs. The final good industry and financial industry could be vertically integrated so that transaction services are provided by the firm internally. We believe, however, that it is more fruitful to view firms specializing either in the production of the final good or in the production of the transaction services.[5]

The production process described by (1) is subject to two random shocks, ϕ_t and ξ_{t+1}, dated by the time of their realization. For simplicity, $\{\phi_t\}$ and $\{\xi_t\}$ are assumed to be strictly positive, mutually uncorrelated $[E(\phi_t \xi_{t+s}) = 0$, for all t and s], stationary stochastic processes. Further, it is assumed that each process is serially independent and that for all $t, E(\phi_t) = E(\xi_t) = 1$. The roles played by these two shocks are quite different as will become clear in the discussions that follow. At this point it is sufficient to recognize that ϕ_t alters expected time t+1 output and affects time t input decisions by altering intertemporal opportunities. On the other hand, ξ_{t+1} represents the basic uncertainty of the production process by altering output in an unexpected manner.[6]

Production is assumed to be under the supervision of N identical competitive firms. Firms operate by selling claims against the future product obtained by combining units of capital, labor, and transaction services that are rented at their respective rental prices, q_t, w_t, and ρ_t. Each firm is assumed to sell one unit of claim for each unit of output as determined by $f(k_{yt}, n_{yt}, d_{yt})$. This assumption amounts to a definition of a "share" in the firm. The going market price of claims is v_t, so that the firm faces a static maximization problem involving the choice of inputs (k_{yt}, n_{yt}, d_{yt}) that maximizes profits, $v_t f(k_{yt}, n_{yt}, d_{yt}) - w_t n_{yt} - q_t k_{yt} - \rho_t d_{yt}$. The assumption of constant returns to scale implies that the firm has a supply of claims that is horizontal at the price v_t^*.

How does this artifical economy correspond to the real world? So far we have described firms that combine factors of production into final goods. The firms rely on external finance to accomplish production activity, issuing claims in the amount $v_t f(n_{yt}, k_{yt}, d_{yt})$, and renting the services of factors of production from their owners. Thus, the economy has the counterparts of trade credit, or circulating capital (shares), and physical capital that play key roles in many macroeconomic theories [as described, for example, by Haberler (1963)].

B. Asset Returns

Within the model economy there are two basic ways for agents to hold wealth--as claims to the ownership of the goods in process at the firm (circulating capital) and as physical capital.

The commodity value at time t+1 of each claim or share issued at time t is simply $\phi_t \xi_{t+1}$, so that the realized rate of return can be expressed as

$$\frac{\phi_t \xi_{t+1} - v_t}{v_t} = r_{yt} \quad .$$

Given that ϕ_t is known at t and independent of ξ_{t+1}, the expected return conditional on all information available at time t, I_t, is just

$$E(r_{yt} | I_t) = \frac{\phi_t}{v_t} - 1 \ ,$$

where it is assumed that ξ_{t+1} is independent of I_t [i.e., $E(\xi_{t+1} | I_t) = E(\xi_{t+1}) = 1$].

An alternative store of value is capital. The owner of a unit of final product used as physical capital in period t earns a return that involves two components. The first component involves the rental rate q_t, paid in period t, and the second incorporates the capital depreciation rate δ. The net return to the capital owner is

$$\left(\frac{1-\delta}{1-q_t}\right) - 1 = r_{kt} \quad .$$

This return is certain in commodity terms because depreciation is not random and rental prices are paid in period t.

It is useful in the analysis that follows to view agents as having the opportunity to borrow or lend at a (real) riskless market interest rate, r_t. Since these bonds are internal in character, in an economy of identical individuals, the representative agent will not execute any such transaction. Nevertheless, the rate at which he/she would do so is well defined and the rental rate on capital is fixed by the requirement that the returns to these two assets be equalized. This follows from the fact noted above that capital owners bear no risk. Thus, in order for no arbitrage opportunities to exist, it must be the case that $q_t = 1 - (1-\delta)/(1+r_t)$.

The claims to production, on the other hand, are uncertain due to ξ_{t+1}. Thus, the yields on these claims, r_{yt}, and the return to the real bond need not be equalized. In the absence of production uncertainty the "share price" v_t is simply $\phi_t/(1+r_t)$. More generally, however, uncertainty leads agents to discount future income at a higher rate resulting in a lower "share price." [7]

C. The Financial Industry

The financial industry provides accounting services that facilitate the exchange of goods. The accounting system of exchange [described by Fama (1980a, pp. 42-43)] operates through bookkeeping entries that permit indirect market transactions without necessarily requiring any physical medium of exchange. The produced output of the financial industry is transactions services and is an intermediate good. The production of transaction services is assumed to be instantaneous with a nonstochastic constant returns to scale production function,

$$(2) \qquad d_t = h(n_{dt},k_{dt}) \ ,$$

where, n_{dt} and k_{dt} are the amounts of labor and capital services allocated to the financial industry.[8] A key element of this production structure is that the production of the intermediate good requires a neglibible amount of time relative to the production of the consumption/capital good. In addition, the constant returns to scale structure implies that at given factor prices, w_t and q_t, the financial industry has a supply curve that is horizontal at ρ_t^*.

So far we have described the real productive activities of the financial industry--flows of services generated from factors of production--in a manner that could describe any intermediate product. The transaction (banking) industry, however, usually provides these services in conjunction with portfolio management or intermediary services. That is, the industry maintains claims on the probability distribution of output ("loans") and issues other claims ("deposits").

At this stage of the analysis, however, households have no demand for transactions services except indirectly through their demand for the final product. Consequently, the scope for the portfolio management role of banks is limited. More specifically, since households do not hold "deposits," transfers of "funds" from firms to individuals in payment for labor and capital services cannot be conducted through book-keeping entries alone. Thus, some form of wealth must "change hands" and we suppose that it is desirable that some "assets" actually be held in the financial industry to facilitate these transfers. For example, one can view firms as selling shares to individuals and depositing the "proceeds" with the financial industry which then provides the accounting and transaction services necessary in disbursing the factor payments to households during the period. We further assume that the size of the "deposit" can be represented as γd_t, indicating that it is proportional to the flow of transaction services.[9]

The activities of the financial industry described above are quite limited. In Section III, the role of the financial industry is expanded by assuming households have an independent desire for transactions services. In that environment, the financial industry becomes more easily identified with a banking system. Nevertheless, the present setup is sufficiently developed to highlight the relationship between real output fluctuations and deposits in an economy that possesses no fiat money.

Consider, for example, the case of an autonomous movement in ϕ_t that raises demand for all factors of production including transactions services and thus deposits. (We assume that the increase is not reversed in equilibrium by increases in the wage and rental rates.) A general strategy for simplifying the structure of the model that is consistent with the above story is to define the effective production function for the final goods producer as $f^*(k_t,n_t)$, where k_t and n_t are the total quantities of capital and labor used to produce final output (both directly and indirectly through transactions services). The comparable "reduced form" production expression for deposits is $h^*(n_t,k_t)$. In the analysis below, the co-movement of deposits with output emerges as a result of underlying movements in total amounts of capital and labor inputs.

D. Equilibrium Prices and Quantities

To complete the specification of the model it is necessary to specify the preferences of a representative agent and to discuss the nature of the constraints that the economy places on his/her behavior.

The representative individual is assumed to be infinite lived and possess the lifetime utility function,

$$U_t \equiv \sum_{j=0}^{\infty} \beta^j \, u(x_{t+j}, \bar{n} - n_{t+j}) \, , \tag{3}$$

which involves a fixed utility discount factor (β) and a single period utility function that depends only on consumption (x_{t+j}) and leisure ($\bar{n} - n_{t+j}$), where $\bar{n}$ is the total hours available in each period. The utility maximand is the expected utility measure $E_tU_t = u(x_t, \bar{n} - n_t) + \beta E_tU_{t+1}$, where E_t denotes the conditional expectation based on full information about economic conditions at time t.

Newly produced units of product, $y_t = f^*(n_{t-1}, k_{t-1})\phi_{t-1}\xi_t$, and capital in place, $\hat{k}_t = (1-\delta)k_{t-1}$, are assumed to be perfect substitutes in production and consumption so that the economy as a whole faces the constraint,

(4) $$x_t + k_t \leq y_t + \hat{k}_t .$$

In this expression, k_t corresponds to the (per capita) amount of resources allocated at date t to the final product and deposit industries ($k_t = k_{yt} + k_{dt}$).[10]

One analytical mechanism for generating equilibrium quantities and prices [see, for example, Lucas (1978) and Long and Plosser (1980)] is the study of the planning problem for the representative agent. This strategy exploits the optimality of a competitive equilibrium in environments such as the present setup to calculate equilibrium quantities. Furthermore, at optimal planned quantities, subjective and technical rates of substitution correspond to competitive exchange rates.

From the point of view of the representative agent, the state of the economy at date t is summarized by the values of two random variables, $s_t = y_t + \hat{k}_t$ and ϕ_t. The first represents a measure of "national wealth" and the second represents a technical factor affecting the current opportunity to transfer resources intertemporally.

The "economic planner" faces the constraint that $x_t + k_t \leq s_t$, in terms of sources and uses of current product. The recursive equation of the planner's problem has the form

(5) $$J(s_t,\phi_t) = \max_{\{k_t, n_t\}} [u(s_t - k_t, \bar{n} - n_t) + \beta E_t\{J(s_{t+1},\phi_{t+1})\}],$$

where $s_t = \phi_{t-1}\xi_t f^*(n_{t-1},k_{t-1}) + (1 - \delta)k_{t-1}$. Equation (5) also assumes that the resource constraint holds as an equality (i.e., consumption is a good). The first-order conditions, derived in Appendix B, can be written as,

$$u_1(s_t - k_t, \bar{n} - n_t) = \beta E_t\{J_1(s_{t+1}, \phi_{t+1})\}[\phi_t f_k^* + (1-\delta)]$$

$$+ \beta E_t\{J_1(s_{t+1}, \phi_{t+1})(\xi_{t+1} - 1)\} f_k^* \phi_t ,$$

and

$$u_2(s_t - k_t, \bar{n} - n_t) = \beta E_t\{J_1(s_{t+1}, \phi_{t+1})\}[\phi_t f_n^*]$$

$$+ \beta E_t\{J_1(s_{t+1}, \phi_{t+1})(\xi_{t+1} - 1)\} f_n^* \phi_t ,$$

which emphasize two distinct features of individual behavior. The first term on the right-hand side of these expressions measures variations in the expected rewards to capital formation and work, respectively. The second term involves the effects of production uncertainty on the household's assessment of the marginal value product of capital formation and work.[11] In the discussion that follows the latter pattern of effects is de-emphasized by simply dropping the final term. This simplification implies that the share price, v_t, equals $\phi_t/(1+r_t)$ and amounts to omitting possibly interesting cyclical variation in "risk premia." In focusing on expected returns, however, the discussion is within the spirit of much macroeconomic analysis.

The decision rules for this problem, $k(s_t, \phi_t)$ and $n(s_t, \phi_t)$, are stationary functions of the state variables, s_t and ϕ_t. In the discussion below, it is assumed that the derivatives of the optimal decision rules have the following signs and magnitudes:

$$0 < \frac{dk_t}{ds_t} < 1 \quad , \quad \frac{dk_t}{d\phi_t} \simeq 0$$

$$\frac{dn_t}{ds_t} > 0 \quad , \quad \frac{dn_t}{d\phi_t} > 0 \; ,$$

(Explicit expressions for these derivatives are given in Appendix B). The economic reasoning underlying these conditions is as follows. First, an increase in the amount of the initial stock, s_t, involves additional wealth so that the consumption of final product and leisure are expected to rise. Agents, however, choose to spread some portion of this wealth increment over time and do so by increasing the amount of commodity allocated to capital services so that $0 < \frac{dk_t}{ds_t} < 1$. This raises the marginal product of labor since capital and labor are complements in production ($f_{kn} > 0$). If the wealth effect on labor supplied, which arises from the increased output of final goods next period, is outweighed by the increase in the real wage (marginal product of labor), then hours worked rises, $\frac{dn_t}{ds_t} > 0$.

Second, an increase in ϕ_t involves both wealth and substitution effects. Given current inputs, future production is higher and the current returns to additional units of factors of production are higher.

These offsetting effects are analogous to the income and substitution effects of a real interest rate change. Essentially, the small impact on capital of a shift in ϕ_t, on the amount of output allocated to capital services $\frac{dk_t}{d\phi_t} \simeq 0$, reflects the idea that the income and substitution effects are roughly offsetting. On the other hand, the substitution effect of such shifts on labor supply is presumed to dominate so that $\frac{dn_t}{d\phi_t} > 0$.

E. Inside Money, Credit, and the Real Business Cycle

In the real business cycle described above, a positive correlation (co-movement) of real production, credit, and "inside money" (created by the financial services industry) arises from the general equilibrium of production and consumption decisions by firms and households. The timing patterns among these variables, however, depends on the source of the variation in real output.

1. Unexpected Output Events (ξ_t)

Real events of this form operate by altering the initial conditions, s_t, pertinent for economic agents' plans for consumption, investment, and hours of work. As discussed above, the implications of an unexpected wealth increment ($\xi_t > 1$) for capital is straightforward: the economy responds with higher net investment than would otherwise have been the case. The implications for units of labor services are less clear, reflecting the offsetting impact of higher national wealth and a higher marginal product. Under the assumption that the wage effect dominates, real output rises and exhibits positive serial correlation. During the course of such an economic expansion the volume of credit is also high as firms finance relatively large amounts of goods in process. Real rates of return move in a countercyclical direction, as agents' opportunities to spread wealth over time are subject to diminishing returns (since total time is in fixed supply).

The movement in final goods production induces a higher volume of financial services and an accompanying increase in real bank deposits due to the role of financial services as an intermediate factor in production. Hence, the volume of "inside money" quantities are positively correlated with output with a rough coincidence in timing.

At least some business cycle episodes, however, are commonly viewed by outside observers as involving a substantially different timing pattern. In particular, traditional business cycle analysts [Burns and Mitchell (1946)]; modern time series macroeconometricians [Sims (1972, 1980)];[12] and monetary historians [Friedman and Schwartz (1963)] view monetary variables as "leading" measures of real activity, with various forms of timing concepts employed.[13]

2. Expected Output Events (ϕ_t)

One way of generating a substantially different timing pattern is through shifts in the intertemporal opportunities of the economy as a whole. Real events of this form, represented by ϕ_t, alter agents' allocations of leisure and consumption between the present and the future for a given level of national wealth. A positive shock ($\phi_t > 1$), under the assumptions outlined above, expands hours worked with little accompanying change in consumption or capital. The fact that financial services are an intermediate product--which can be produced more rapidly than the final product--leads to an expansion of the quantity of such services and of bank deposits. Consequently, movements in hours worked, interest rates and security prices, deposits/financial services, and trade credit all occur prior to the expansion of output. The subsequent increment to time t+1 wealth (stemming from the joint impact of the exogenous shift, ϕ_t , and agents' responses to that shift) works much like the above discussion of unexpected output events.

III. CURRENCY, DEPOSITS, AND PRICES

In order to investigate the relationship between nominal aggregates and the real business cycle, it is necessary to augment the hypothetical economy developed above. In this section, a non-interest bearing government-supplied fiat currency (dollars, in a chauvinistic gesture) is introduced and the factors affecting its value are analyzed.

First, in order for currency to be a well-defined economic good, and thus, to have a determinate price in terms of a unit of output (1/P), there must be a demand function for currency that reflects the economic value assigned to the services of currency by economic agents. It would be desirable to build in basic aspects of the economic environment that give rise to a commodity money and then to examine the potential for a non-backed governmental issue to have positive value. This route, however, is not attempted in the present paper. Instead, we assume that households have a demand for transaction services. These services are produced by combining units of currency purchasing power with the deposit services of the banking industry. The production technology is

$$\tau_t = \tau(c_t, d_{ht}) , \tag{6}$$

where τ_t is the flow of transactions services produced at time t, c_t is the stock of currency purchasing power, and d_{ht} is the quantity of services purchased from the financial industry.

The demand for real transactions services is assumed to be a positive function of the total market transactions that the household engages in, y_t, as well as depending negatively on the cost of producing these services. This demand for real transactions services and the production technology (6) imply a pair of derived demands for real currency and services of the accounting system of exchange. These derived demands are functions of the rental price of real currency, $R_t/(1+R_t)$, and the effective cost of financial services, $\bar{\rho}_t$. In forming these rental prices, two important assumptions are made. First, there is a market for one period nominal bonds that bear interest rate R_t. This nominal rate is the sum of the real component, r_t, and an expected rate of inflation, $E(\pi_t)$.[14] Second, if banks are required to hold noninterest bearing reserves, the returns earned by depositors may not match market rates. Given the proportionate link between financial services (d_t) and deposits (γd_t), the effective cost of a unit of deposit services, $\bar{\rho}_t$, is influenced by this reserve regulation. In the absence of reserve requirements $\bar{\rho}_t = \rho_t$, where ρ_t is the rental price of deposit services in the competitive environment of Section II.

It is assumed that the real demands for currency and financial services take the following forms,

(7a) $$c_t = \ell(\underset{-}{\frac{R_t}{1+R_t}}, \underset{+}{\bar{\rho}_t}, \underset{+}{y_t}) ,$$

(7b) $$d_{ht} = \delta(\underset{+}{\frac{R_t}{1+R_t}}, \underset{-}{\bar{\rho}_t}, \underset{+}{y_t}) .$$

The + or - below the arguments denote the signs of the partial derivative, e.g., $\partial c_t/\partial y_t > 0$. These assumptions about partial derivatives involve some conventional assessments about the impacts of own and cross rental price elasticities.

This formulation involves several ideas that are worth emphasizing. First, the structure of the markets for currency and financial services is analogous to that provided by Fama (1980a). In contrast to the discussion in Section II, the financial industry can now be viewed as providing services that facilitate the exchange of goods through an accounting system. As discussed previously, the accounting system operates through bookkeeping entries without necessarily requiring any physical medium of exchange. Following Fama (1980a), we assume that it is desirable that some "assets" be held in the financial industry. Thus, the financial industry holds claims (shares) on the probability distribution of output and issues deposits.[15] In the process of market exchange, the claims that individuals and firms hold on the banks' portfolio (deposits) are altered, with the banking system recording these transfers. Banks pass on to depositors the return to the portfolio of assets less a fee for services.

Second, the structure of the financial industry implies that the direct cost of the bookkeeping services, $\bar{\rho}_t$, does not hinge on the character of the assets in the bank's portfolio. As discussed by Fama (1980a), it follows that there is no particular reason that deposits would be a homogenous good in an unregulated financial industry. Finally, the idea that financial services and currency are substitutes, but not perfect substitutes, is implicit in (7). In particular, currency yields a real service flow in that there are some transactions (either of magnitude or character) that are more efficiently carried out using currency rather than the accounting system of exchange.[16]

Given the setup of the markets for currency and deposit services described above, determinacy of the price level is insured if the government fixes the nominal quantity of currency--direct or indirect regulation of financial sector quantities and/or characteristics are not necessary. Nevertheless, regulations can be important for two reasons. First, regulations produce a differentiated class of suppliers of financial services (banks) whose deposits are sometimes described as inside money. Second, regulations can have influences on the price level by altering the effective rental price of financial services.

The analysis below focuses on the implications of alternative banking structures for the behavior of currency, deposits, and prices. For clarity, the bulk of the discussion is conducted under the assumption that the treasury/central bank maintains a policy of controlling the issue of nominal currency so that the stock of currency ($C_t = P_t c_t$) is an exogenous random variable. Other models of central bank behavior are discussed in Part B below.

A. Money and Prices--Unregulated Banking

In an unregulated banking environment we assume that the deposit industry would hold virtually no currency. Consequently, the determination of the price level involves the requirement that the real supply of currency (C/P) be equal to the real demand for currency given by (7a) above. The equilibrium price level is then

$$(8) \qquad P_t = C_t/\ell(\cdot) \ ,$$

where $\ell(\cdot)$ is the demand function for real currency. Using the arguments of $\ell(\cdot)$ we can rewrite this condition as

$$(9) \qquad P_t = P(\underset{+}{C_t}, \underset{-}{y_t}, \underset{+}{R_t}, \underset{-}{\bar{\rho}_t}) \ .$$

The signs of the respective derivatives in (9) are straightforward and warrant little explanation.[17]

An important feature of (9) is that nominal demand deposits do not appear. Thus, as stressed by Fama (1980a), there is no need for government control of banking or the supply of deposits to insure a determinate price level. Banks, in a competitive, unregulated environment, simply pass portfolio returns on to their depositors less a fee charged for the provision of transactions services, so that $\bar{\rho}_t = \rho_t$.

Although the signs of the partial derivations in (9) are straightforward, it is important to stress that we have ruled out real effects of sustained increases in the growth of currency.[18] Such behavior leads to sustained inflation and a rise in the nominal interest rate, R, which implies a fall in the demand for real currency and a rise in real deposits.

Since an increase in real deposit services involves the use of real resources, the economy is made worse off by sustained inflation. We assume, however, that this increase in the size of the financial sector has no important implications for the real general equilibrium. It is not obvious that this is a good assumption from an empirical point of view. Nevertheless, it does serve to bring into sharp focus the distinction between "inside" and "outside" money, particularly with respect to the neutrality and superneutrality of government currency issue.

B. Money and Prices--Regulated Banking

In the unregulated environment the price level is determined in the "currency market" and deposits play no essential role. There are a number of bank regulations, however, that serve to distinguish banks from other financial intermediaries. This section discusses the extent to which these regulations alter the nature of price level determination. As it turns out, the impact of regulations depends on (i) the interaction of banking regulation with the external money supply policy of the central bank/ treasury and (ii) the extent to which government mandates can be offset by countervailing private substitutions.

1. Portfolio Regulations and Reserve Requirements

It is useful to start by discussing a set of regulations that do not have any important consequences for the price level. Suppose that the government specifies the "risk composition" of the underlying assets against which deposits are claims. As long as agents can offset this restriction by rebalancing the contents of their portfolios (i.e., the distribution of total wealth between the banking sector and other portfolio managers), then this regulation will have no impact on any real variables or the price level.[19]

On the other hand, restrictions specifying that banks must hold some fraction (θ) of their nominal asset portfolio in the form of noninterest bearing reserves issued by the central bank may have important effects. For example, the central bank could specify that reserve accounts are deposits of securities, with nominal interest accruing to the central bank. This mechanism is one way of imposing a deposit tax with the consequence that the cost of deposit services would be $\bar{\rho}_t > \rho_t$. Such a deposit tax results in a reduction in the size of the banking sector and an increase in the real demand for currency.[20] The impact of this reserve requirement on price level determination depends on the central bank policy. For example, if the treasury/central bank makes currency in the hands of the public an exogenous quantity, unresponsive to developments in the banking sector,[21] then the price level continues to be determined by the requirement that the real stock of currency outstanding (C_t/P_t) be equal to the real demand.

2. Alternative Central Bank Policies

The currency market determines the price level if the central bank is assumed to make currency an exogenously controlled quantity. There are, however, other control methods available to the central bank. For example, if the central bank combines a reserve requirement with a policy of controlling the sum of currency and nominal bank reserves (high powered money), then the price level can be viewed as being determined in the market for high powered money.

Let $B_t = \theta(P_t\gamma d_t)$ be the nominal stock of bank reserves and $H_t = B_t + C_t$ be the exogenous total of bank reserves and currency. Under this regime, the price level may be viewed as arising from the requirement that the total private demand for fiat money equals the supply. That is, $H_t = P_t\{C_t + \theta\gamma d_t\} = P_t\{C_t + \frac{B_t}{P_t}\}$. The equilibrium price level can be expressed as

(10) $$P_t = \frac{H_t}{\ell(\cdot) + (B_t/P_t)} \quad ,$$

or using the arguments of $\ell(\cdot)$,

(11) $$P_t = P(\underset{+}{H_t}, \underset{-}{y_t}, \underset{+}{R_t}, \underset{-}{\bar{\rho}_t}, \underset{-}{(B_t/P_t)}).$$

Once again the signs of the partial derivatives are straightforward. Note, in particular, that an increase in the demand for real reserves (B_t/P_t) holding high powered money fixed necessitates a fall in the price level.

There are a variety of mechanisms for accomplishing control of high powered money by the central bank. One possibility is that the government could require that banks hold dollar denominated, non-interesting bearing reserve accounts and for the central bank to increase or decrease currency in response to shifts in bank reserves.[22] Under this scheme the central bank could allow banks to freely exchange reserves for currency, but would not allow banks to freely convert securities into reserves.

As discussed in Fama (1980a, pp. 52-53) there are other central bank policies that could be used to make the price level determinate. In particular, the central bank could choose to make nominal bank reserves an exogenous quantity and supply currency on demand. In this case (which some argue are the current policies of the Federal Reserve), the price level can be viewed as being determined in the market for reserves. The equilibrium price level would be determined by the exogenous supply of nominal reserves (B_t) and the total real demand for deposit services.[23]

C. The Price Level and the Real Business Cycle

Price level movements in response to the two shocks (ϕ_t and ξ_t) involve two important factors. First, there is the impact of movements in real output on the demand for outside money. Second, there is the impact of nominal interest rates on the demand for outside money. Since variation in the price level also depends on central bank policy, we focus on the case of a regulated banking system with the central bank assumed to make the quantity of high powered money exogenous.

It is convenient to summarize household and bank behavior in the following demand function for outside money[24]

$$h_t^d = p_t + \lambda y_t - \psi R_t \quad , \qquad \lambda > 0,\ \psi > 0 \ ,$$

where h_t is the logarithm of high powered money, y_t is the logarithm of real output, p_t is the logarithm of the price level, and R_t is the nominal interest rate. Using the fact that $R_t = r_t + (E_t p_{t+1} - p_t)$ and the monetary equilibrium condition that $h_t = h_t^d$, it follows that a rational expectations solution for the price level along the lines of Sargent-Wallace (1975) can be written as

$$p_t = \frac{1}{1+\psi} \{ \sum_{j=0}^{\infty} (\frac{\psi}{1+\psi})^j E_t[h_{t+j} + \psi r_{t+j} - \lambda y_{t+j}]\} .$$

Unexpected wealth increments ($\xi_t > 1$) lead to a business cycle where output is high and the real rate of return is low. Consequently, a wealth increment leads to lower prices due to both lower real returns ($r_t, E_t r_{t+1}, \ldots$) and higher income (y_t, $E_t y_{t+1}, \ldots$).

In Section II we describe how a better than average opportunity to transfer resources intertemporally ($\phi_t > 1$) leads to an increase in r_t. In addition, the increase in wealth that is brought about by such a shift leads to lower future returns ($E_t r_{t+1}$, $E_t r_{t+2}, \ldots$) and higher future outputs ($E_t y_{t+1}$, $E_t y_{t+2}, \ldots$). Thus, the overall impact on the price level is ambiguous, involving the positive influence of the higher current real return, and the negative influence of the lower expected future returns and higher expected future outputs.

The above two examples indicate that the model produces a price level that is likely to be countercyclical. For some macroeconomists, the procyclical character of the general price level is such a well established empirical regularity that this feature alone is sufficient to reject real business cycle theory [e.g., Lucas (1977, p. 20)]. If it is indeed necessary to generate procyclical price movements, then there appear to be two principle channels. First, an alternative structure that involves a more permanent, capital-augmenting form of technological change could heighten the real return effects discussed above. Combining this structure with a sufficiently interest sensitive demand for money could lead to procyclical prices. Second, policy response to real activity also could generate procyclical price movements.[25] For example, a positive response of outside money creation to output could lead to a positive correlation between prices and output.

IV. EMPIRICAL ANALYSIS

The preceding sections describe a simple model economy with business cycles that are completely real in origin. Nevertheless, correlations between real activity and monetary measures arise from the operation of the banking system and central bank policy responses. In this section, we discuss some of the predictions that our model makes concerning the joint time series behavior of output, monetary aggregates, rates of return, and the price level. In addition, we discuss U. S. business cycle experience during the post World War II period, providing some gross correlations that bear on the potential relevance of our theoretical stories.

Before proceeding, it is useful to briefly consider general strategies for investigating the empirical importance of 'real business cycle' theories and to discuss how the present analysis of money and the price level could be related to such investigations.

One empirical strategy is to attempt to isolate a group of observable real disturbances that provide an explanation of much of a particular nation's business cycle experience, in the sense of delivering a "good fit." Candidates for such real shocks include government purchase, tax, and regulatory actions; changes in technological and environment conditions; and movements in relative prices that are determined in a world market. In pursuing this line of research, the goal is to provide a direct substitute for the high explanatory power of monetary variables in other business cycles studies [e.g., Friedman and Schwartz (1963) and Barro (1981a)]. The natural extension would be to study the explanatory power of such real factors for monetary variables and the price level. In our framework, many of these real variables would be restricted to influence monetary quantities (particularly, inside money) through their influence on output and a small set of relative prices. In this sense, aspects of the present type of monetary theory do provide meaningful restrictions on the data.

Another approach is to treat the fundamental real shocks as unobservable and to focus on the interactions between sectors that arise during business cycles; a strategy that is the empirical analogue of the theoretical analysis of Long and Plosser (1980). Since a particular real business cycle theory restricts own and cross serial correlation properties of industry output and relative prices, this route can provide valuable information about the regular aspects of business cycles even though the sources of shocks are not identified. Again, the principal testable restrictions of theorizing along the above lines would arise from the restricted fashion that variations in production in other sectors were allowed to influence developments in the monetary sector.

Unfortunately, analysis of monetary phenomena using either of these strategies is not feasible given the state of real business cycle models. Consequently, the present empirical investigation is limited to providing some admittedly crude correlations among the variables suggested by the theory.

A. Summary Statistics

Summary measures of the series to be discussed below are presented in Table I. The data are annual--generally yearly averages--for

TABLE I

SUMMARY STATISTICS

Annual Data: 1953-1978

Series	Mean	Std. Deviation	ρ_1	ρ_2	ρ_3	ρ_4
		A. Real Variables				
Growth Rate of:						
Real GNP (y_t)	.0327	.0249	-.01	-.24	-.12	.29
Government Purchases						
Nondefense (g_t^n)	.0362	.1153	.02	.00	-.07	-.19
Defense (g_t^d)	-.0147	.0706	.40	-.12	-.33	-.14
Real Energy Price (P_t^e/P_t)	.0120	.0761	.36	.13	.22	.05
Real Deposits (γd_t)	-.0002	.0226	.36	-.22	-.19	.20
Real Currency (C_t/P_t)	.0101	.0209	.65	.39	.26	.25
Real High Powered Money (H_t/P_t)	.0027	.0253	.40	.33	.08	.19
Real Reserves (B_t/P_t)	-.0110	.0449	.32	-.01	-.05	-.02
Real Service Charges (ρ_t)	-.0252	.0601	.81	.69	.66	.56
		B. Nominal Variables				
Growth Rate of:						
Price Level (P_t)	.0371	.0233	.84	.64	.66	.84
Deposits ($P_t\gamma d_t$)	.0373	.0211	.58	.35	.43	.58
Currency (C_t)	.0481	.0329	.93	.88	.85	.82
High Powered Money (H_t)	.0398	.0338	.71	.76	.59	.68
Reserves (B_t)	.0260	.0455	.37	.03	.09	.32
Change in the Interest Rate (R_t)	.2177	1.471	.03	-.71	-.29	.68

Note--ρ_i is the sample autocorrelation coefficient at lag i, for i = 1,...,4. The large sample standard error is .20.

the period 1953-1978.[26] We focus on the 1953-1978 interval primarily to avoid the period when the Federal Reserve maintained a policy of pegging the yields on U. S. government securities. The implications of such a policy may be very different from those described in the previous section where the central bank controls some measure of money.

The most noticeable feature in Table I is the different behavior of nominal and real variables. Typically, the growth rates of real variables display much less serial correlation than the growth rates of nominal variables. For example, the growth of real demand deposits is much less autocorrelated than the growth rate of nominal demand deposits. Indeed, as previously noted by Nelson and Plosser (1982) and other authors, many real variables display random walk like behavior in logarithmic form. The most notable exceptions to this random walk behavior are real currency (C_t/P_t) and real service charges (ρ_t), both of which display significant positive serial dependences.

B. Real Factors and Aggregate Output

As an example of how real factors could serve as important impulses to business fluctuations, Table II reports regressions of aggregate output on two components of government purchases of goods and services (real federal defense and nondefense components) and the relative price of energy. Table II indicates that defense purchases exert a positive influence on output (though statistically a weak one) and that nondefense purchases are unimportant. These results are consistent with Barro's (1981b) longer period evidence that temporary increases in government purchases have an expansionary

TABLE II

REAL FACTORS AND OUTPUT GROWTH

Annual Data: 1953-1978

$$\Delta \ell n y_t = \gamma_0 + \gamma_1 \Delta \ell n g_t^n + \gamma_2 \Delta \ell n g_t^d + \gamma_3 \Delta \ell n (P_t^e/P_t) + \varepsilon_t$$

$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	R^2	s.e.$(\hat{\varepsilon})$	ρ_1
.035** (.005)	-.003 (.045)	.097 (.073)	-.107* (.063)	.191	.0239	-.04

Note--See Table I for the definition of the variables. $\Delta \ell n(\cdot)$ indicates the change in the logs of the variable. R^2 is the coefficient of determination, s.e.$(\hat{\varepsilon})$ is the standard error of the regression, ρ_1 is the estimated first-order autocorrelation coefficient of the residuals, which has a large sample standard error of .20, and standard errors of the coefficients are in parentheses.

*Indicates significance at the 10% level.

**Indicates significance at the 5% level.

impact. Since our sample period involves only the Vietnam war, it is perhaps not surprising that the t-statistic on defense purchases is low. Table II also indicates a weak negative impact of the relative price of energy. At present, we are seeking to augment these real factors with other tax and expenditure measures. The main difficulty lies in constructing the average marginal tax rates that theory predicts would be important in output determination.

Additional evidence on the importance of real disturbances in output fluctuations is offered in Nelson and Plosser (1982). Using an unobserved components model of output and the observed autocovariance structure of real GNP, Nelson and Plosser conclude that real (non-monetary) disturbances are the primary source of variance in real activity. This result is based on the commonly held view that monetary disturbances should have no permanent effects on real output and thus disturbances that are of a permanent nature must be associated with real rather than monetary sources.

C. Money-Output Correlations

The theoretical model stresses that internal real monetary balances should be positively correlated with real activity, since money is a produced input. Further, the model predicts that autonomous external nominal money creation/destruction is neutral with respect to output growth. These two ideas suggest the value of analyzing money-output correlations in two forms: real versus nominal balances and internal versus external monetary measures.

Table III presents some information on the contemporaneous relationships between output growth and growth rates of alternative monetary measures.

TABLE III

CONTEMPORANEOUS MONEY-OUTPUT REGRESSIONS

Annual Data: 1953-1978

$$\Delta \ell n y_t = \alpha_0 + \alpha_1 \Delta \ell n M_t + \varepsilon_t$$

		Independent Variables (M_t)								
		Real Monetary Measures			Nominal Monetary Measures					
Equation	$\hat{\alpha}_0$	γd_t	H_t/P_t	C_t/P_t	$P_t \gamma d_t$	H_t	C_t	R^2	s.e.$(\hat{\varepsilon})$	ρ_1
(1)	.033** (.004)	.740** (.167)						.450	.0188	-.08
(2)	.031** (.004)		.510** (.103)					.337	.0206	-.18
(3)	.025** (.005)			.664** (.202)				.311	.0211	-.01
(4)	.015 (.009)				.465** (.222)			.155	.0233	.10
(5)	.026** (.007)					.171 (.146)		.054	.0247	.00
(6)	.027** (.009)						.111 (.153)	.022	.0251	.01
(7)	.025** (.006)	.742** (.161)				.176* (.108)		.507	.0182	-.11
(8)	.023** (.006)	.784** (.162)					.194* (.111)	.514	.0181	-.08
(9)	.017* (.010)				.558* (.326)	-.080 (.203)		.161	.0238	.10
(10)	.015 (.010)				.661** (.307)		.181 (.197)	.185	.0234	.07

Note--See Table II for definitions and explanations of entries.

Equation (1) shows the strong, positive contemporaneous correlation that exists between real demand deposits and economic activity. This strong contemporaneous correlation is shared by real external balances measured as currency or as high powered money (equations (2) and (3)). In nominal balance form, equations (4), (5), and (6) show demand deposits are more strongly correlated with real activity than either of the nominal external money measures.

Further, (7) and (8) indicate that nominal high powered money and currency growth have a positive partial correlation with output given real demand deposits although the correlation is not as strong as that with real deposits (either in terms of the magnitude of the coefficient or the pertinent t-statistic).

From the standpoint of our theoretical discussion, the key aspects of these contemporaneous correlations are as follows. First, the fact that much of the correlation is with internal monetary measures is consistent with our general view of the relationship between money and real activity. Second, the fact that nominal money measures may be positively correlated with real activity is at odds with our theory if the monetary authority makes nominal monetary measures such as currency or high powered money evolve in an autonomous manner.

To provide some further information on the relationship between output growth and growth rates of alternative monies, Table IV reports some analogous regression results that incorporate lags of the alternative monetary measures.

TABLE IV

MONEY GROWTH AND OUTPUT GROWTH REGRESSIONS

Annual Data: 1953-1978

$$\Delta \ell n y_t = \alpha_0 + \sum_{i=0}^{2} \beta_i \Delta \ell n \gamma d_{t-i} + \sum_{i=0}^{2} \gamma_i \Delta \ell n H_{t-i} + \sum_{i=0}^{2} \delta_i \Delta \ell n C_{t-i} + \varepsilon_t$$

		Independent Variables											
		Real Deposits (γd_t)			High Powered Money (H_t)			Currency (C_t)					
Equation	$\hat{\alpha}_0$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\delta}_0$	$\hat{\delta}_1$	$\hat{\delta}_2$	R^2	s.e.$(\hat{\varepsilon})$	ρ_1
(1)	.034** (.003)	.644** (.159)	.135 (.166)	-.352** (.159)							.651	.0152	.01
(2)	.035** (.008)				.489** (.239)	-.399* (.210)	-.175 (.243)				.236	.0225	.05
(3)	.032** (.010)							.342 (.487)	-.577 (.493)	.262 (.360)	.068	.0249	.18
(4)	.034** (.006)	.607** (.160)	.066 (.189)	-.296* (.162)	.263 (.182)	-.229 (.145)	-.059 (.185)				.713	.0150	.00
(5)	.031** (.006)	.644** (.173)	.119 (.178)	-.333* (.171)				.302 (.317)	-.313 (.321)	.017 (.238)	.674	.0160	.00
(6)	.033 (.003)	.605** (.150)	.091 (.157)	-.305* (.151)	.233* (.118)	-.233* (.118)					.710	.0142	.02
(7)	.033 (.003)	.642** (.158)	.114 (.167)	-.343** (.159)				.297 (.280)	-.297 (.280)		.670	.0152	.00

Note--See Table II for definitions and explanations of entries. Equations (6) and (7) are the results of the regressions that constrains $\gamma_0 = -\gamma_1$ and $\delta_0 = -\delta_1$.

Equation (1) in Table IV shows the results of adding two years of lagged real deposits to the output regression. The F statistic pertinent for evaluating the marginal contribution of these lags is 2.48, which is well below the 95% critical value of 3.49, so that there is no strong evidence that these lags are important. Equations (2) and (3) show analogous results for nominal money growth measures.

In order to investigate the extent to which nominal money growth is correlated with real activity, after accounting for real deposit growth, equations (4) and (5) must be examined. The contemporaneous and two lags of high powered money and currency in the hands of the public are not important explanatory variables (the 95% critical value for $F(3, 17)$ is 3.20 and the F statistics for the lags of high powered money and currency terms are 1.22 and .40, respectively). However, the estimated coefficient on current and lagged high powered money are opposite in sign and nearly identical in magnitude, so that the change in high powered money growth appears to be positively correlated with real activity (see equation (6)).

Overall, our interpretation is that the correlations reported in Tables III and IV indicate that much of the relationship between money and fluctuations in real activity is apparently with inside money, which is comforting given the key role that the banking system plays in our theoretical story. Nevertheless, somewhat weaker correlations between real activity and nominal outside money may exist, suggesting it probably is necessary to analyze policy response in greater detail for 1953-1978 period.

D. Money-Inflation Correlations

The theoretical model predicts that variations in external money, real activity, the nominal interest rate and a measure of the cost of banking services should be important in explaining movements in the price level. Table V provides estimates of the price level equations (9) and (11) of Section III under the assumption that a log-linear functional form is appropriate. Although the nominal interest rate is endogenous and the above discussion indicates that high powered money and/or currency may also be endogenous due to policy response, ordinary least squares methods are employed. Since there is a substantial empirical literature on price level/money demand equations, our discussion focuses principally on new aspects that are raised by the theoretical discussion above.

First, the theory suggests that a measure of external money, such as currency a high powered money, is the relevant nominal aggregrate for price level determination [Fama (1980b) stresses this point]. Table V reports results for both currency and high powered money.

Second, in the regulated banking environment described in Section III, the relevant cost of deposit services (denoted $\bar{\rho}_t$) involves both the direct cost of providing an accounting system of exchange (denoted ρ_t) and the interest that the bank/depositor must forego due to reserve requirements. The empirical counterpart to the nominal unit cost of deposit services that we have constructed is the ratio of total services charges on demand deposits at Federal Reserve member banks to total check clearings by the Federal Reserve. Deflating this measure by the price level leads to a measure of real costs of deposit services, entered in Table V as ρ_t. However, during some portions of the period under study, banks faced apparently binding constraints on the level of interest payments that could be paid on demand

TABLE V

INFLATION REGRESSIONS

Annual Data: 1953-1978

$$\Delta \ell n P_t = \alpha_0 + \alpha_1 \Delta \ell M_t + \alpha_2 \Delta \ell n y_t + \alpha_3 \Delta R_t + \alpha_4 \Delta \ell n(B_t/P_t) + \alpha_5 \Delta \ell n \rho_t + \varepsilon_t$$

Equation	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$	$\hat{\alpha}_4$	$\hat{\alpha}_5$	R^2	s.e.($\hat{\varepsilon}$)	ρ_1
			A. Currency as External Money						
(A1)	.023** (.005)	.590** (.070)	-.445** (.100)	.0025 (.0017)			.790	.0114	.33
(A2)	.020** (.005)	.583** (.067)	-.396** (.100)	.0037** (.0017)	-.099 (.059)		.815	.0109	.29
(A3)	.025** (.005)	.513** (.105)	-.441** (.100)	.0023 (.0017)		-.056 (.057)	.800	.0114	.37
(A4)	.023** (.005)	.489** (.101)	-.387** (.100)	.0035** (.0017)	-.108* (.059)	-.069 (.055)	.828	.0108	.32
(A5)	.005 (.006)	1.00	-.520** (.155)	.0022 (.0026)			.463	.0178	.64
(A6)	.003 (.007)	1.00	-.483** (.163)	.0032 (.0029)	-.077 (.097)		.478	.0180	.61
(A7)	.008 (.005)	1.00	-.494** (.138)	.0029 (.0023)		.143** (.053)	.596	.0158	.39
(A8)	.006 (.006)	1.00	-.461** (.145)	.0038 (.0025)	-.070 (.086)	.142** (.054)	.608	.0158	.36
			B. High Powered Money as External Money						
(B1)	.030** (.006)	.536** (.092)	-.436** (.128)	.0003 (.0022)			.651	.0147	.28
(B2)	.019** (.005)	.629** (.067)	-.317** (.093)	.0027 (.0017)	-.248** (.058)		.837	.0103	-.17
(B3)	.033** (.006)	.371** (.127)	-.428** (.123)	.0001 (.0021)		-.122* (.068)	.697	.0139	.37
(B4)	.021** (.005)	.537** (.100)	-.321** (.092)	.0027 (.0165)	-.265** (.059)	-.063 (.051)	.848	.0101	-.05
(B5)	.015* (.007)	1.00	-.522** (.183)	.0030 (.0031)			.248	.0211	.28
(B6)	.004 (.006)	1.00	-.342** (.142)	.0016 (.0025)	-.374** (.085)		.599	.0151	.12
(B7)	.017** (.007)	1.00	-.500** (.175)	-.0025 (.0030)		.122* (.068)	.342	.0201	-.01
(B8)	.006 (.005)	1.00	-.324** (.130)	.0021 (.0023)	-.368** (.077)	.114** (.048)	.684	.0143	-.32

Note--See Table II for definitions and explanations of entries.

deposits. It is frequently argued that explicit service charges would be reduced as a means of avoiding the interest rate constraint. As a result, we are not completely comfortable with our interpretation of this variable.

Third, when reserve requirements are present and the central bank is controlling the quantity of high powered money, the theory predicts that the volume of real reserves should negatively influence the price level given the stock of high powered money. On the other hand, when currency is the controlled external quantity, real reserves should not be relevant.

In Panel A of Table V, equations (A4) and (A8) report the results of estimating the price level (inverse money demand) equation over the sample period 1953-1978, with currency as the measure of external money and employing real reserves and the service charge measure as explanatory variables. The main features of these equations are broadly consistent with other studies: a negative impact of real activity, positive impact of nominal money growth, and minor impact of the short-term interest rate (4 to 6 month commercial paper rate). If currency is the appropriate measure of external money the theory predicts a zero coefficient on real reserves. In equation (A4) this coefficient is negative and marginally significant. Imposition of the unit coefficient on currency reduces the significance of this coefficient. The tendency of our service charge measure to switch sign with the imposition of the unit constraint is troubling, although the presence of that additional explanatory variable does not appear to substantially influence other coefficient estimates.

In Panel B of Table V, equations (B4) and (B8) report analogous results for high powered money as the measure of external money with general features that are again broadly consistent with other studies. Under our theory, real bank reserves should enter negatively in such price level equations if high powered money is the controlled measure of external money. This is borne out by significant negative coefficients in both the unconstrained equation (B4) and constrained equation (B8). As previously, the service charge variable has a tendency to change sign when the unit constraint is imposed.

Overall, the results of Table V are broadly consistent with the theoretical stories told in the sections above. The negative influence of real reserves on the price level potentially is important, both in terms of explaining post-war price level behavior and, potentially, in explaining the apparently anomolous behavior of the price level during the interwar period. Finally, additional work needs to be done in producing measures of the market prices of bank services.

CONCLUSIONS

This paper describes an initial attempt to account for the relationship between money, inflation, and economic activity within the framework of real business cycle theory. Although the empirical work presented above is simplistic, we draw two main lessons from it. First, much of the contemporaneous correlation of economic activity and money is apparently with inside money, with inflation principally resulting from changes in the stock of fiat money and variations in real activity. Second, future work along these lines may have to consider policy responses that are broad enough to produce variations in outside money that are contemporaneously correlated with real activity.

A main component of our future work in this area will be to develop the implications of the analysis for security returns, so that the general equilibrium predictions for these variables can be exploited in tests of the model. This topic is especially important because Sims (1980) and Fama (1981) have provided some hints about the interactions of money, asset returns, and real activity. In addition, we are in the process of casting the predictions of theory within the class of linear multivariate time series models so that broader patterns of policy response can be examined and the results compared to other studies. Thus, we feel there is a substantial amount of work to be done.

In conclusion, it seems worthwhile to discuss two recurrent comments on this line of research that we have received. First, there has been a surprising willingness on the part of the many individuals to simultaneously argue that our model (a real business cycle model with an explicit banking sector and central bank) is probably observationally equivalent to existing theories and that a "common sense" view leads one to prefer alternative models as descriptions of reality.[27] This line of argument puzzles us, since it was presumably on empirical grounds that the profession rejected pre-Keynesian "equilibrium theories" of the business cycle that stressed real causes of economic fluctuations.

Second, some individuals have argued that market failure is central to both the understanding of cyclical fluctuations and the primary reason for economists to study these phenomena. Our view is that widespread market failure need not be a necessary component of a theory of business fluctuations and we (at least) remain interested in real business cycle theory as potentially an important contribution to positive economics. This perspective, however, is not inconsistent with the view that the accumulation of scientific knowledge may lead to the design of more desirable governmental policies toward business fluctuations (such as tax and expenditure policies) or toward the regulation of the financial sector.

Footnotes

1. A related story involves the operation of a commodity money economy where changes in the production of the commodity money are related to economic activity through variations in the relative price of the commodity money.

2. Black (1972) also describes a model where both internal and external money are endogenously determined.

3. Not surprisingly, this single sentence dismissal of received doctrine on the relationship between money and business cycles has provoked a sharp reaction from a number of readers. Although a footnote is not an appropriate vehicle for a survey of contemporary macro theory, some additional comments are perhaps in order. Keynesian models typically rely on implausible wage rigidities, from the textbook reliance on exogenous values to recent, more sophisticated efforts of Fischer (1977) and Phelps and Taylor (1977) that rely on existing nominal contracts. As Barro (1977) points out, a key feature of the Fischer-Phelps-Taylor model is that agents select contracts that do not fully exploit potential gains from trade. In addition, Azariadis's (1978) micro-based model of wage-employment contracts implies that perceived monetary disturbances do not alter output.

   Recent analyses of monetary nonneutrality that stress expectation errors based on "imperfect information" [Lucas (1977) provides a summary of this viewpoint] similarly rely on an apparent failure in the market for information. For example, information on monetary statistics is cheap and readily available. King (1981) demonstrates that in Lucas' (1973) model, real output should be uncorrelated with contemporaneously available monetary information. Boschen and Grossman (1982) empirically investigate this proposition and find that it is rejected by the data.

4. Capital services are measured in commodity units allocated to production at time t, labor services are hours worked, and transaction services can be viewed as the number of bookkeeping entries used (described more fully below).

5. An interesting and useful discussion of the role of money as a factor of production can be found in Fischer (1974).

6. The multiplicative nature of the randomness in total production implies a technological neutrality of the shocks with respect to individual factors of production. A more general specification might allow different stochastic elements to be associated with particular factors of production. Such a model could provide a richer set of co-movements among various measures of economic activity.

7. In a certainty steady state--where real rates of return must be equal to the subjective rate of time preference (α) and there are no shocks ($\phi_t = \xi_t = 1$)--the above definitions imply that

$$v^* = \frac{1}{1+\alpha} \;,\; q^* = \frac{\alpha+\delta}{1+\alpha} = \frac{1}{1+\alpha} \{f_k(k^*, n^*, d^*)\} \;.$$

That is, the requirement that production takes time means that the rental price of capital involves the marginal product of capital, which influences output next period, discounted at the rate α.

8. We assume that transaction services are produced deterministically for several reasons. The first reason is to keep the model as simple as possible by limiting the number of independent sources of random fluctuation. The second reason involves the assumption that stochastic variations in transaction services are "small" in that they have little impact on the real equilibrium in the final good industry.

It may, however, prove interesting to allow stochastic variation in the provision of financial services as an independent source of shocks to the real system. One might then proceed to decompose real fluctuations into those attributable to shocks to the final goods industry and the financial industry.

9. Assuming that transaction services and the stock of deposits are related by a positive constant (any positive function is sufficient for our purposes) implicity rules out firms (and in Section III, individuals) using a given stock of deposits more or less intensively (i.e., getting varying amounts of bookkeeping entries from a given stock of deposits). We recognize that this is a strong assumption (although it is commonly used in specifying production functions) but it (over?) simplifies the analytical tractability of the model.

10. The details of household and firm optimization and equilibrium conditions are discussed in Appendix A.

11. The sign of these terms is a function of the contemporaneous covariance of J_1 and ξ. Assuming ξ_{t+1} and s_{t+1} are positively correlated and J_1 is concave ($J_{11} < 0$), this covariance is negative.

12. Sims (1980) discusses reverse causation of money and output working through central bank operating policies. The present setup is a first step toward the type of small scale general equilibrium model that is necessary to evaluate the reverse causation argument.

13. We deliberately employ the idea of a "leading variable" in a loose manner so as to capture the common elements of these alternative discussions. Sargent (1979, pp. 247-248) discusses how the timing concepts employed by business cycle forecasters and time series econometricians differ, relating a particular formal definition of a "leading indicator" to "causality" in the sense of Granger, Sims, and Weiner.

14. At this point we note that we are ignoring any "risk premium" due to inflation risk in our definition of the nominal rate.

15. Deposits in an unregulated environment are obviously not riskless assets and banks would presumably offer deposits (claims on an underlying portfolio of assets) with varying levels of risk depending on demand.

16. It is also worth noting that consumers and producers are treated asymmetrically in that the demand for real currency is only a household demand and is not partly a derived demand by firms. This reflects our view of the type of transactions that are accomplished with cash and also a desire to keep the model as simple as possible.

17. Although our model is similar in some respects to Black (1972), our view of price level determination is quite different. In Black's model, the price level is exogenous to the money supply process. Equilibrium requires that the central bank supply fiat currency on demand (passively) or else it loses economic value as a medium of exchange.

18. In other words, our model is "super neutral," in the language of monetary growth theory. It is worthwhile pointing out that this literature does not provide a clearcut guide to the nature of departures from super neutrality. For example, Tobin (1965), has argued that an increase in inflation will lower real rates of return and raise capital formation, by lowering the real value of money and, consequently, raising saving. By contrast, Stockman (1982) argues that inflation acts as a tax on the saving process (in which money is an input) and, hence, depresses capital formation.

19. This application of the Modigliani-Miller theorem requires us to ignore transactions costs in a specific manner that is stronger than usual. In particular, the cost of running the accounting system of exchange must be the same across regulated and unregulated systems.

20. We are assuming that the reduction in the size of the deposit industry has a negligible impact on the real general equilibrium of the model (in particular, on the final goods production, which enters as a scale variable in the demand for currency).

21. The central bank, therefore, "passively supplies" any quantity of reserves required by the banking system.

22. Another possibility is to require banks to hold cash directly. For the current discussion, however, it is not necessary to distinguish between those operating schemes.

23. We have not analyzed the implications for price level determination when the central bank attempts to control the interest rate. However, this may be important for some periods in order to conduct an appropriate empirical investigation.

24. For simplicity, we ignore movements in the cost of deposit services ($\bar{\rho}_t$) as an important factor affecting the price level.

25. Recent work using post-World War II data [e.g., Hodrick and Prescott, (1980)] seems to suggest that the positive correlation between output and price level movements may be not as robust as sometimes thought.

26. Data sources are as follows: Real GNP, government purchase variables, and the GNP deflator are taken from <u>The National Income and Product Accounts of The United States 1929-1974</u> and various issues of the

Survey of Current Business. Currency in the hands of the public, demand deposits, and bank reserves are from Business Statistics 1979. High powered money is the sum of currency in the hands of the public and bank reserves. The interest rate is the 4 to 6 month prime commercial paper rate taken from Banking and Monetary Statistics 1941-1970 and various issues of the Annual Statistical Digest. The energy price variable is the fuels, power, and related products component of the Producer Price Index and is taken from Business Statistics 1979. Finally, the service charge variable is the ratio of total service charges on demand deposits accrued by Federal Reserve member banks to total check clearings by the Federal Reserve. Both series are taken from Banking and Monetary Statistics 1941-1970 and various issues of the Annual Statistical Digest.

27. Grossman (1982) makes an explicit statement of this view.

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APPENDIX A

Market Behavior

The discussion in the text focuses almost entirely on the planning problem and it may be worthwhile to discuss some aspects of economic agents' market behavior in this hypothetical economy. The present appendix dicussses the choice problems faced by households and firms as well as the conditions of market equilibrium.

Household Optimization

The agent under study chooses current flows of consumption (x_t) and leisure ($\bar{n}-n_t$), as well as purchases/sales of real bonds (b_t), claims to circulating capital (a_t), and physical capital (k_t) so as to maximize the expected value of lifetime utility. The agent is assumed to view the prices $\frac{1}{1+r_t}$, v_t, q_t and w_t as functions of aggregate state variables but not of his/her own actions and, further, to employ the objective probability distributions of these aggregate random variables in forming expectations.

Wealth at date t is $z_t \equiv b_{t-1} + \xi_t \phi_{t-1} a_{t-1} + (1-\delta)k_{t-1}$.

The recursive equation of the pertinent discrete time dynamic programming problem is

$$H(z_t, \frac{1}{1+r_t}, v_t, q_t, w_t)$$

$$= \max_{\{x_t, n_t, b_t, a_t, k_t\}} [u(x_t, \bar{n}-n_t) + \beta E_t H \{z_{t+1}, \frac{1}{1+r_{t+1}}, v_{t+1}, q_{t+1}, w_{t+1}\}]$$

subject to $x_t = w_t n_t + (q_t - 1)k_t + z_t - b_t \frac{1}{1+r_t} - v_t a_t$.

The first order conditions representing the agent's decisions in the four markets (real bonds, labor, circulating capital, and physical capital) are as follows.

$$\frac{1}{1+r_t} u_1 = \beta E_t\{H_1\} \qquad \text{bond}$$

$$w_t u_1 = u_2 \qquad \text{labor (supply)}$$

$$u_1 v_t = \beta E_t\{H_1 \phi_t \xi_{t+1}\} \qquad \text{circulating capital (demand)}$$

$$u_1(q_t - 1) = -\beta E_t\{H_1(1-\delta)\} \qquad \text{physical capital (supply)}$$

where $H_1 = \frac{\partial H}{\partial z_{t+1}} (z_{t+1}, 1+r_{t+1}, r_{t+1}, q_{t+1}, w_{t+1})$, $u_1 = \frac{\partial u}{\partial x_t}$ and $u_2 = \frac{\partial u}{\partial(\bar{n}-n_t)}$.

Viewing the first and fourth of these conditions, it follows that the absence of arbitrage requires that $q_t = 1 - \frac{1-\delta}{1+r_t}$, as real bonds and physical capital are perfect substitutes as a store of value from the household's viewpoint. This will be presumed to hold here and below.

Rearranging the first and third expressions leads to the following condition, which can be interpreted as equality between the cost of a unit of "circulating capital" and the household's risk adjusted demand price for these claims.

$$v_t = \frac{\phi_t}{1+r_t}\left(1 + \frac{E_t\{H_1(\xi_{t+1} - 1)\}}{E_t\{H_1\}}\right)$$

From an application of the envelope theorem, it follows that $\frac{\partial H}{\partial z_t} = u_1$ so that $H_1 = u_1(x_{t+1}, \bar{n} - n_{t+1})$. Hence, it follows that $H_{11} < 0$. Using this fact, it is straightforward that $E\{H_1(\xi_{t+1} - 1)\} < E\{H_1\}E\{\xi_{t+1} - 1)\} = 0$, so that a household requires a risk premium -- in the sense of a lower share price and a higher expected return -- to hold any specified positive quantity of circulating capital. In the absence of uncertainty about future returns (e.g., ξ_{t+1} has a degenerate distribution at its mean of unity), however, it follows that $v_t = \frac{\phi_t}{1+r_t}$, i.e., the security price is simply the present value of the certain future earnings.

The first two conditions simply involve efficient selection of an intertemporal consumption-saving plan at interest rate r_t and a contemporaneous allocation between labor and leisure, i.e., a labor supply decision.

Firm Optimization

Following the production example above, it simplifies matters to "vertically integrate" the final product and financial industries so that there is a single production decision being made, based on a production function of the firm, $y_{t+1} = \phi_t\xi_{t+1}f^*(n_t, k_t)$.

The purely atemporal decisions of the firms in this model economy then involve the following comparable efficiency conditions in the four markets.

none	bond
$v_t f_n^* = w_t$	labor (demand)
$v_t f^* = w_t n_t + q_t k_t$	circulating capital (supply)
$v_t f_k^* = q_t$	physical capital (demand)

Equilibrium Conditions

Given the decisions of the representative household (b_t, a_t, k_t^s, and n_t^s) and the decisions of the representative firm (f_t^*, k_t^d, n_t^d) implicit in the above marginal conditions, it is straightforward to discuss the equilibrium of the four markets under study. Equilibrium requires $b_t = 0$ in the market for real bonds and $a_t = f_t^*$ in the market for circulating capital. In the factor markets, equilibrium requires that $n_t^d = n_t^s$ and $k_t^d = k_t^s$.

Imposing these market clearing conditions implies that $z_{t+1} = \xi_{t+1}\phi_t f_t^* + (1-\delta)k_t$, etc., so that there are four equations that may be solved for the equilibrium values of r_t, v_t, n_t and k_t.

$$(1+r_t) = \beta \frac{E_t\{H_1\}}{u_1} \qquad \text{bonds}$$

$$v_t f_n^*(n_t, k_t) u_1 = u_2 \qquad \text{labor}$$

$$v_t = \frac{\phi_2}{1+r_t}\left(1 + \frac{E_t\{H_1(\xi_{t+1} - 1)\}}{E_t\{H_1\}}\right) \qquad \text{circulating capital}$$

$$v_t f_k^*(n_t, k_t) = 1 - \frac{1-\delta}{1+r_t} \qquad \text{physical capital}$$

where H_1 is evaluated at equilibrium values, i.e.,

$$H_1(\xi_{t+1}\phi_t f_t^* + (1-\delta)k_t, \frac{1}{1+r_{t+1}^e}, v_{t+1}^e, q_{t+1}^e, w_{t+1}^e) \qquad \text{where}$$

r_{t+1}^e, v_{t+1}^e, q_{t+1}^e and w_{t+1}^e satisfy updated versions of the above four conditions.

APPENDIX B

The Planning Problem

This appendix provides some additional details on the planner's problems discussed in Section II of the text.

The "economic planner" faces the constraint that $x_t + k_t \le s_t \equiv y_t + (1-\delta)k_{t-1}$, in terms of sources and uses of current product. The recursive equation of the planner's problem has the form

$$J(s_t, \phi_t) = \max_{\{k_t, n_t\}} [u(s_t - k_t, \bar{n} - n_t) + \beta E_t J(s_{t+1}, \phi_{t+1})]$$

given that the constraint holds as an equality. The first order conditions to this problem are

$$u_1(s_t - k_t, \bar{n} - n_t) = \beta E_t\{J_1(s_{t+1}, \phi_{t+1}) \cdot \xi_{t+1}\} f_k^* \phi_t + \beta E_t\{J_1(s_{t+1}, \phi_{t+1})\}(1-\delta)$$

$$u_2(s_t - k_t, \bar{n} - n_t) = \beta E_t\{J_1(s_{t+1}, \phi_{t+1}) \cdot \xi_{t+1}\} f_n^* \phi_t \quad .$$

These may be rewritten in a more useful form that emphasizes the two distinct features of individual behavior.

$$u_1(s_t - k_t, \bar{n} - n_t) = \beta E_t\{J_1(s_{t+1}, \phi_{t+1})\}[\phi_t f_k^* + (1-\delta)]$$

$$+ \beta E_t\{J_1(s_{t+1}, \phi_{t+1})(\xi_{t+1} - 1)\} f_k^*$$

$$u_2(s_t - k_t,\ \bar{n} - n_t) = \beta E_t\{J_1(s_{t+1}, \phi_{t+1})\}[\phi_t f_n^*]$$

$$+ \beta E_t\{J_1(s_{t+1}, \phi_{t+1})(\xi_{t+1} - 1)\} f_n^* .$$

The first component involves agent's optimal response to variations in the expected response to capital formation and work. The second involves the effects of production "uncertainty" on the household's assessment of the marginal value product of capital formation and work. The present analysis deemphasizes the latter pattern of effects by simply dropping the final term (which corresponds to a degenerate distribution of ξ_{t+1} at its mean of unity). Totally differentiating the above expressions leads to the following:

$$- \begin{bmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{bmatrix} \begin{bmatrix} dk_t \\ dn_t \end{bmatrix} = \begin{bmatrix} -u_{11} \\ -u_{12} \end{bmatrix} ds_t + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} dk_t \\ dn_t \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} d\phi_t$$

where

$$a_{11} = [\beta E_t\{J_{11}\}[f_k^* \phi_t + 1 - \delta]^2 + \beta E_t\{J_1\} f_{kk}^* \phi_t]$$

$$a_{12} = [\beta E_t\{J_{11}\}[f_k^* \phi_t + 1 - \delta]\phi_t f_n^* - \beta E_t\{J_1\}\phi_t\ f_{kn}^*] = a_{21}$$

$$a_{22} = [\beta E_t\{J_{11}\}[\phi_t \; f^*_n]^2 + \beta E_t\{J_1\} \; f^*_{nn}\phi_t]$$

$$b_1 = [\beta E_t\{J_{11}\} \; f^*[f^*_k\phi_t + 1-\delta]\phi_t + \beta E_t\{J_1\}f^*_k]$$

$$b_2 = [\beta E_t\{J_{11}\}f^*f^*_n\phi_t + \beta E_t\{J_1\} \; f^*_n]$$

Solving this system leads to the following set of expressions for the derivatives of the "policy functions" $k(s_t, \phi_t)$ and $n(s_t, \phi_t)$, where $\Delta = (u_{11} + a_{11})(u_{22} + a_{22}) - (u_{12} + a_{12})(u_2 + a_{21})$ is positive.

$$\frac{dk_t}{ds_t} = \Delta^{-1}\{u_{11}u_{22} - u_{12})^2 - (a_{22}u_{11} + u_{12}a_{12})\}$$

$$\frac{dk_t}{d\phi_t} = \Delta^{-1}\{b_2(u_{21} + a_{21}) - b_1(u_{22} - a_{22})\}$$

$$\frac{dn_t}{ds_t} = \Delta^{-1}\{a_{11}u_{12} - a_{12}u_{11}\}$$

$$\frac{dn_t}{d\phi_t} = \Delta^{-1}\{b_1(u_{12} + a_{12}) - b_2(u_{11} + a_{11})\} \quad .$$

The discussion in the main text, assumes that these derivatives have the following signs and magnitudes:

$$0 < \frac{dk_t}{ds_t} < 1 \quad , \qquad \frac{dk_t}{d\phi_t} \gtreqless 0 \quad ,$$

$$\frac{dn_t}{ds_t} > 0 \quad , \qquad \frac{dn_t}{d\phi_t} > 0 \quad .$$

The economic basis for these conditions is discussed in the text.