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TARIFFS AS INSURANCE: OPTIMAL COMMERCIAL POLICY WHEN DOMESTIC MARKETS ARE INCOMPLETE

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ABSTRACT

Free trade is not optimal for a small country that faces uncertain terms of trade if some factors are immobile *ex post*, and markets for contingent claims are incomplete. The government can improve social welfare by using commercial policy that serves as a partial substitute for missing insurance markets. Using a combination of analytical and simulation techniques we demonstrate that optimal policy for this purpose will often have an anti-trade bias. We also show that the usual preference by economists for factor or product taxes and subsidies over tariffs and export subsidies may not be justified in this context.

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1. Introduction

An important accomplishment of international economics in the past decade has been an extension of the theory of international trade to situations of uncertainty. A major result has been that, if appropriate risk sharing arrangements exist among domestic consumers and producers, the traditional arguments in favor of free trade remain intact. For a number of reasons, however, falling for the most part into the categories of "moral hazard" and "adverse selection", domestic risk sharing arrangements are likely to be incomplete. In such situations free trade may no longer constitute optimal commercial policy.

The implications of incomplete insurance markets for the optimality of competitive equilibrium have been explored by Hart (1975), Newbery and Stiglitz (1981), and Cornes and Milne (1981). Newbery and Stiglitz (1981) have shown explicitly that trade intervention may be optimal when domestic markets fail to allocate risks optimally. Commercial policy can act as a partial substitute for insurance markets.

A number of trade theorists have recognized, at a somewhat informal level, the role of commercial policy as insurance. See, in particular, Corden (1974, pp. 320-321), Cassing (1980, pp. 396-397) and Baldwin (1981, pp. 20-21). Quoting the last author:

> Workers and capital owners who are risk-averse wish to avoid human and physical capital losses due to sudden and significant increases in imports that compete with the domestic products they produce. However, private markets to insure against this risk fail to exist, apparently for reasons of inadequate data or "moral hazard". The import relief legislation involving recommendations from the International Trade Commission can, for example, be viewed as a means of providing the desired insurance (p. 21).

In fact, some analysts have explained the existence of tariffs as the manifestation of a social desire to provide such insurance. See for example, Corden (1974, p. 321).

While the role of commercial policy as a substitute for insurance has been recognized on both a theoretical and practical level, little has been done to determine (1) conditions under which intervention is desirable, (2) the form that intervention should take, and (3) the optimal level of intervention. The purpose of this paper is to address these issues in a framework that is familiar to trade theorists.[1]

In an economy in which all individuals have identical tastes, own identical amounts of each factor of production, and can diversify their factor endowments among activities, the issue of insurance does not arise. Risk is spread equally among all individuals, who are equally willing to bear it. Any market for insurance will be inactive. If there are asymmetries either in tastes or in initial factor endowments, or else if individuals must completely specialize their use of a factor in some activity, a reallocation of risk is likely to be optimal. In this paper we assume that <u>ex ante</u>, i.e., before uncertainty is resolved, all individuals are identical in their tastes and factor endowments. Individuals must, however, allocate their endowment of one factor to a particular activity before the actual terms of trade of the country are known. <u>Ex post</u>, the terms of trade that materialize will benefit some individuals relative to others. We thus focus on commercial policy as a method to insure individuals against unfavorable outcomes in the terms of trade. This insurance motive for trade intervention is distinct from a redistributive motive that would arise if workers differed in their initial endowments or in their attitudes toward work.

We consider a small, open economy characterized by the standard Heckscher-Ohlin assumptions: two factors of production, capital and labor, produce two traded commodities, both of which are consumed domestically. Production of both commodities takes place under perfectly competitive conditions. Following, for example, Rothenberg and Smith (1971) and Eaton (1979), we assume that, at the time capital must be allocated between productive activities, the terms of trade are unknown. Labor, unlike capital, can move between the production of exportables and import substitutes after the uncertainty is resolved. In the short-run, then, the model is equivalent to the two-commodity, three factor Ricardo-Viner model analyzed extensively by Jones (1971).[2] Individuals have identical tastes and initial endowments of labor and capital, but an individual must engage his capital entirely in one activity or the other. This assumption may be justified by the presence of indivisibilities and set-up costs that make specialization highly efficient. For example, human capital is frequently most productive when it takes the form of specialized training; an individual farm is usually more productive when it produces a small number rather than a wide variety of products.[3]

An individual's pre-tax income, then, is determined by the wage earned by labor, which is the same for everyone, and the rate of return on capital, which depends upon the activity in which it is engaged. Ex post, individuals with capital in different sectors earn different incomes. We assume that there is no market, such as an insurance or a stock market, in which agents can trade claims to capital income across states of nature.[4] We also assume that redistribution via an income tax is infeasible, perhaps because of transactions costs and evasion.

We do not attempt to explain the non-existence of optimal redistribution methods, but assume it, appealing to the well-known reasons of moral hazard and adverse-selection. See Arrow (1970) and Shavell (1979) for a discussion.

We show, in this context, that an interventionist commercial policy in general raises social welfare. Furthermore, when individuals anticipate the policy when they make their capital allocation decisions, intervention is Pareto-improving in the sense that it makes everyone's expected utility higher, ex ante.

The main form of policy that we consider is import tariffs. We consider a number of variations in institutional format of the tariff policy. As is generally the case in models of optimal taxation, interesting comparative static results are hard to come by for very general cases. For this reason much of our analysis takes the form of simulations.

Section 2 sets forth the basic assumption of our model and simulation analysis. In section 3 we show that intervention, in a number of variants, is likely to be welfare improving. Section 4 examines the optimal level of intervention and how it is affected by changes in (1) the amount of uncertainty; (2) tastes; and (3) technology. Section 5 summarizes our major conclusions.

While we examine a number of factors influencing optimal commercial policy, three general results seem to emerge. One is that, as long as the government budget must always be in balance, the net effect of policy is likely to favor import-competing industries relative to export industries; i.e., there is an anti-trade bias to optimal policy. A second result is that an increase in substitutability between commodities in consumption or between factors in production is likely to diminish the optimal level of intervention. Finally, tariffs may dominate production subsidies or taxes as a means of insuring against shifts in the terms of trade.

2. The Model

We consider an economy that can produce two commodities, commodity 1 and commodity 2, with capital and labor. Outputs of the two commodities in state of nature i are at levels X^{1i} and X^{2i}, given by

$$X^{1i} = F^1(K^1, L^{1i}) \qquad i = A, B \tag{2.1a}$$

$$X^{2i} = F^2(K^2, L^{2i}) \qquad i = A, B \tag{2.1b}$$

where K^j and L^{ji} denote the amount of capital and labor, respectively, engaged in producing commodity j in state i. The functions F^j are homogeneous of degree one in K^j and L^{ji}, quasi-concave and twice-differentiable. Factor allocation is subject to the constraints

$$K^1 + K^2 \leq K \tag{2.2a}$$

$$L^{1i} + L^{2i} \leq L, \qquad i = A, B \tag{2.2b}$$

where K and L represent the aggregate endowments of capital and labor. We assume that each individual is endowed with one unit of labor and k units of capital. We normalize the number of individuals to equal one. Thus $L = 1$ and $K = k$.

We assign the international price of commodity 2 the role of numeraire and denote the price of commodity 1 in state of nature i as P^i. We assume, for simplicity, that there are two states of nature, A and B, characterized by $P^A > P^B$. Without loss of generality (via the Lerner symmetry theorem) we assume that tariff intervention takes the form of taxes or subsidies on imports of commodity 2. Thus the domestic price of commodity 2 in state of nature i is $1 + t^i$, where t^i is the ad valorem tariff rate in state of nature i.

Since L^1 and L^2 are determined after P^i and t^i are known, ignoring corner solutions competition implies that

$$(2.3) \qquad w^i = P^i F_L^1(K^1, L^{1i}) = (1 + t^i)F_L^2(K^2, 1 - L^{1i}), \ 0 < L^{1i} < 1; \ i = A, B$$

where w^i denotes the wage in state of nature i. Here $F_L^j \equiv \partial F^j/\partial L^j$ and $F_K^j = \partial F^k/\partial K^j$. Implicitly, the second part of (2.3) defines a labor allocation function $L^{1i}(P^i, t^i, K^1, K^2)$. The rates of return on capital in sectors 1 and 2 are, respectively, given by

$$(2.4a) \qquad R^{1i} = P^i F_K^1(K^1, L^{1i}) \qquad i = A, B$$

$$(2.4b) \qquad R^{2i} = (1 + t^i)F_K^2(K^2, L^{2i}) \qquad i = A, B.$$

Individuals will have allocated capital either to one sector or the other. The income of individual j, having allocated his capital to sector j, is thus, in state i,

$$(2.5) \ Y^{ji} = R^{ji}k + w^i + T^{ji} \qquad j = 1, 2; \ i = A, B$$

were T^{ji} denotes tariff revenue distributed to individual j in state i. Total tariff revenue is given by $t^i M^i$, where M^i denotes imports of commodity 2 in state i. We assume that this is distributed equally among all individuals in a lump-sum fashion.[5] Thus $T^{ji} = t^i M^i$, j = 1, 2.

In state i the utility of individual j is given by the indirect utility function $V(Y^{ji}, P^i, 1 + t^i)$. Consumption of commodity 2 by individual j in state i, C^{ji}, is given from Roy's identity, by

$$(2.6) \qquad C^{ji} = - V_t^{ji}/V_Y^{ji} \qquad j = 1, 2; \; i = A, B$$

where $V^{ji} \equiv V(Y^{ji}, P^i, 1 + t^i)$, $V_Y^{ji} \equiv \partial V^{ji}/\partial Y$ and $V_t^{ji} = \partial V^{ji}/\partial t^i$. Imports of commodity 2 in state i are thus

$$(2.7) \qquad M^i = \lambda C^{1i} + (1 - \lambda)C^{2i} - F[(1 - \lambda)K, 1 - L^{1i}]; \quad i = A, B$$

where $\lambda \equiv K^1/K$, the share of capital in sector 1 (i.e. exportables).

If capital is allocated to both sectors in positive amounts then the expected utility of placing capital in one sector or the other must be the same. Thus the condition

$$(2.8) \qquad \Sigma\pi^i(V^{1i} - V^{2i}) = 0,$$

where π^i denotes the probability that $P = P^i$, i = A, B, must obtain in equilibrium.

Together, conditions (2.1) - (2.8) constitute a system of 26 equations which determine equilibrium levels of X^{1i}, X^{2i}, L^{1i}, L^{2i}, K^1, K^2, w^i, R^{1i}, R^{2i}, Y^{1i}, Y^{2i}, C^{1i}, C^{2i}, and M^i as functions of the capital endowment K, the terms of trade P^i, the probabilities π^i, and the tariff rates t^i.

In our simulations we assume that the utility function and both production functions are of the constant elasticity of substitution (CES) form. We also assume constant relative risk aversion. The indirect utility function can be shown to be of the form

$$(2.9) \qquad V(Y, P^i, 1 + t^i) = \frac{1}{\gamma} Y^\gamma \left[\frac{(\alpha_c^{-\rho_c} + 1 - \alpha_c)^{-1/\rho_c}}{1 + t^i + \phi P^i}\right]^\gamma$$

where $\phi \equiv [(1 - \alpha_c)P^i/\alpha_c(1 + t^i)]^{-1/(1 + \rho_c)}$.

The production functions are given by

$$(2.10) \qquad X^1 = [\alpha_1(K^1)^{-\rho_1} + (1 - \alpha_1)(L^1)^{-\rho_1}]^{-1/\rho_1}$$

$$(2.11) \qquad X^2 = [\alpha_2(K^2)^{-\rho_2} + (1 - \alpha_2)(L^2)^{-\rho_2}]^{-1/\rho_2} .$$

The elasticities of substitution are

$$\sigma_k = \frac{1}{1 + \rho_k}, \qquad k = c, 1, 2 .$$

In our simulations we set $\pi^A = \pi^B = 1/2$ and, $P^A = 1 + u$ and $P^B + 1/(1 + u)$. We thus consider uncertainty in terms of a geometric mean preserving spread around a price of one. Such a characterization has the virtue of being insensitive to the choice of numeraire. See Flemming, Turnovsky and Kemp (1977).

The following are the parameter values that are used throughout most of our simulation analysis, and which constitute our "base" case:

$u = .15, .25, .35$

$\gamma = -1$

$\rho_c = .01$

$\alpha_c = .25$

$\rho_1 = .01$

$\rho_2 = .01$

$\alpha_1 = .5$

$\alpha_2 = .25$

$L = 1$

$K = 1$

Note that our parameter values imply a degree of relative risk aversion of two, and elasticities of substitution in both consumption and production very near unity. We also assume in our simulations that the export sector is more capital intensive.

3. Optimal Tariff Intervention

We assume that the goal of policy is to maximize expected social welfare, where welfare in any state i is the sum of all individuals' levels of utility in that state. The social welfare function W may therefore be defined by

$$W \equiv \sum_i \pi^i[\lambda V^{1i} + (1 - \lambda)V^{2i}] \quad (3.1)$$

In steady state it is reasonable to assume that intervention policy is anticipated at the time capital is allocated. If the political environment has changed, however, capital may have been allocated under the assumption that free trade would always prevail. In this case unanticipated policies may be implemented.

Anticipated policies are, of course, of greater interest than unanticipated ones. We find it useful, however, to consider first the case in which capital was allocated between sectors under the assumption that there would be no intervention. For this case we can ignore the effects of the policy on the allocation of capital. We introduce a capital-allocation effect subsequently.

3.1 Optimal Unanticipated Tariff Policy

Even when intervention has no effect on the allocation of capital, a policy of free trade is not optimal. Appropriate intervention transfers income toward the group with the higher marginal utility of income in each state of nature. To demonstrate the desirability of intervention we first show the following results:

Result 1: An increase in the tariff on commodity 2 shifts labor from sector 1 to sector 2, given the allocation of capital.

This result follows from differentiating the second part of the condition (2.3) to obtain

$$\left.\frac{dL^{1i}}{dt^{i}}\right|_{\bar{K}^1} = \frac{F_L^2}{P^i F_{LL}^1 + (1 + t^i)F_{LL}^2} < 0 \tag{3.2}$$

Result 2: An increase in the tariff on commodity 2 distributes income away from individuals with capital in sector 1 toward those with capital in sector 2, given the allocation of capital.

This follows from differentiating $\Delta Y^i \equiv Y^{2i} - Y^{1i}$ where, from (2.5),

$$\left.\frac{d(\Delta Y^i)}{dt^i}\right|_{\bar{K}^1} = \{ F_K^2(K^2, 1 - L^{1i}) - [(1 + t^i)F_{KL}^2 + P^i F_{KL}^1]\left.\frac{dL^{1i}}{dt^i}\right|_{\bar{K}^1} \}K \tag{3.3}$$

which, since $F_{KL}^j > 0$, is positive.

Analogously, an increase in P, the price of commodity 1, shifts labor from sector 2 to sector 1, and distributes income from individuals with capital in sector 2 (type 2 individuals) to those with capital in sector 1 (type 1 individuals). It is therefore the case that, in the absence of tariffs, type 1 individuals have relatively higher incomes when $P = P^A$;

i.e., that $Y^{1A} - Y^{2A} \geq Y^{1B} - Y^{2B}$. Since the expected utility from engaging capital in the two sectors must be equal, we must have either $V^{1A} > V^{2A}$ and $V^{1B} < V^{2B}$, or the converse. As all individuals face the same commodity prices, differences in utility between individuals in a given state derive solely from differences in income. Since type 1 individuals have relatively higher income in state A it must be that $V^{1A} > V^{2A}$ while $V^{1B} < V^{2B}$ and that $Y^{1A} > Y^{2A}$ while $Y^{1B} < Y^{2B}$: type 1 individuals earn absolutely more income when $P = P^A$ and conversely when $P = P^B$.

To show that a nonzero tariff is optimal we differentiate W with respect to t^i, holding K^1 and K^2 constant, and evaluate the resulting expression at $t^i = 0$. Using the derivative of the equilibrium condition (2.3) with respect to t^i, and exploiting Euler's theorem and Roy's identity, we obtain the expression[6]

$$(3.4) \qquad \left.\frac{dW}{dt^i}\right|_{\bar{K}^1} = \lambda(1-\lambda)\pi^i(V_Y^{2i} - V_Y^{1i})\left[\left.\frac{d(\Delta Y^i)}{dt^i}\right|_{\bar{K}^1} + (C^{1i} - C^{2i})\right] .$$

Consider first the case i = A. Since $Y^{1A} > Y^{2A}$, diminishing marginal utility of income implies that $V_Y^{2A} > V_Y^{1A}$. From (3.3) $\frac{d(\Delta Y^i)}{dt^i}$ is positive. Finally, if commodity 2 is non-inferior, $C^{1A} > C^{2A}$. Thus all the terms in expression (3.4) are positive. From a position of free trade imposing a small <u>positive</u> tariff raises social welfare.

The tariff distributes incomes from individuals with a low marginal utility of income (type 1's) to individuals for whom the marginal utility of income is high (type 2's). It does so in two ways. First, because the tariff raises the relative <u>producer</u> price of commodity 2, it transfers income from type 1 to type 2 individuals, via result 2. Second, because the tariff raises the <u>consumer</u> price of commodity 2 above the world price, it taxes consumption of commodity 2. The higher income group (type 1) pays more of the tax since they consume more of commodity 2. Since tax revenue is distributed as a poll tax, the result is a transfer to the group with lower income.

Consider next the case i = B. Since $Y^{1B} \leq Y^{2B}$, $V_Y^{2B} \leq V_Y^{1B}$ and, because of noninferiority, $C^{1B} < C^{2B}$. Thus (3.4) is ambiguous in sign. The effect of the tariff on producer prices tends to make an import subsidy optimal, since a subsidy transfers income toward type 1 individuals. But an import subsidy also lowers the consumer price of commodity 2, which tends to benefit type 2 individuals, who consume relatively more commodity 2. For this reason the sign of expression (3.4) is ambiguous.

Welfare improving commercial policy thus requires a tariff on imports of commodity 2 when the world price of commodity 2 is below mean, but may require either an import subsidy or a tariff when the price is above mean.[7] The effect of a positive tariff on consumer prices is always to transfer income toward the lower income group. There is thus some reason to think that optimal intervention, on average, will tend to reduce the average amount of trade. This presumption was supported by our simulation analysis, in which we found that the optimal tariff, when the price of commodity 1 was high, was in every case as large or larger in absolute value than the optimal import subsidy when the terms of trade were unfavorable. The optimal tariffs as well as the share of the capital stock invested in sector 1 are reported in Table 3.1. We note once again that because the policies we are considering here were unanticipated when capital was allocated between sectors, the optimal tariffs in Table 3.1 need not Pareto-dominate free trade; policy may lower the expected utility of individuals in one sector or the other. It is true, however, that some set of state-contingent tariff rates can be found such that each type of individual benefits in an *ex ante* (i.e. expected utility) sense.

3.2 Anticipated, Time-Consistent Tariff Policy

We now consider how policy is modified when its effects on capital allocation are taken into account. A problem that arises in this context is that, at the time tariffs are actually imposed, the capital stock is fixed in place. The commercial policy that maximizes social welfare from the perspective of the period in which the policy is implemented can ignore its effects on the allocation of capital, which at that point is a bygone. Policy makers may wish to affect the allocation of capital in the previous period by announcing policies that affect capital allocation in the direction desired. But once the capital has been allocated, policy makers will typically have an incentive to deviate from the announced policy.

Unless policy makers have a means of constraining themselves to policies that were announced previously, they will pursue policies that are optimal from the perspective of the period in which they are implemented. These are referred to as time-consistent policies. If individuals are rational in forming their expectations, these are the policies that they anticipate when deciding where to invest their endowment of capital.

In this section we consider time-consistent policies. In the sections that follow we assume, instead, that policy makers are able to precommit themselves credibly to policies at the time that capital is allocated between sectors, and actually pursue the policies announced. We refer to these as optimal, anticipated policies.[8]

Since time-consistent policy takes the allocation of capital as given it is formulated according to the same principle as unanticipated policy. Thus expression (3.4) also indicates the direction that time-consistent intervention will take. Individuals now, however, anticipate the policies that are in fact pursued, rather than free trade, when making their investment decisions. Therefore the equilibrium condition (2.8) now applies to expected utilities under time-consistent policy rather than under free trade. The difference between optimal unanticipated policy and time-consistent, anticipated policy is that the second affects the allocation of capital while the first does not.

Anticipated, time consistent tariffs for our base case are reported in Table 3.2. Note that, as with optimal unanticipated policy, there is an anti-trade bias: there is always a higher tariff rate in state A than subsidy in state B. Comparing this table with Table 3.1 note that the effect of intervention is always to shift capital into sector 2, the import-competing sector. The two tables also show that, for our base case, whether policy is unanticipated or anticipated and time-consistent, trade intervention offsets roughly one-third of the variability in international prices.

3.3 Optimal Anticipated Tariff Policy

We now consider optimal commercial policy when the government can commit itself credibly to a particular policy before capital is allocated between sectors. The government may bind itself legally to a particular response, or else it may act out of concern for the effect of its current policies on its future reputation.[9]

As in the previous cases we considered, a small positive tariff when the terms of trade are favorable is welfare improving, while a welfare improving small deviation from free trade in the state with unfavorable terms of trade may involve either a tariff or an import subsidy.

To see this, consider first commercial policy in state of nature A. The first derivative of the welfare function with respect to t^A, evaluated at $t^A = t^B = 0$, is given by

$$\text{(3.5)} \qquad \frac{dW}{dt^A} = \lambda(1-\lambda)\{\pi^A(V_Y^{2A} - V_Y^{1A})[\frac{\partial \Delta Y^A}{\partial t^A} + \frac{\partial \Delta Y^A}{\partial K^1}\frac{dK^1}{dt^A} + (C^{1A} - C^{2A})]$$

$$+ \pi^B(V_Y^{2B} - V_Y^{1B})[\frac{\partial \Delta Y^B}{\partial K^1}\frac{dK^1}{dt^A}]\} .$$

Since $V_Y^{2A} \geq V_Y^{1A}$ and $V_Y^{1B} \geq V_Y^{2B}$, a tariff (subsidy) can only be welfare improving if <u>either</u> the expression in the first square bracket on the r.h.s. is positive (negative)or the expression in the second square bracket is negative (positive). It is straightforward to show that the first bracketed expression is positive if and only if $d(V^{1A} - V^{2A})/dt^A < 0$, i.e. if a tariff in state of nature A increases the relative utility of type 2 individuals in that state. Similarly, the second bracketed expression is negative if and only if $d(V^{1B} - V^{2B})/dt^A > 0$. But note that a fully anticipated commercial policy cannot, in equilibrium, raise the relative utility level of the same group of capital owners in both states of nature, since expected utilities must remain equal. Therefore, $d(V^{1A} - V^{2A})/dt^A < 0 <=> d(V^{1B} - V^{2B})/dt^A > 0$, and thus a tariff (subsidy) is welfare improving only if <u>both</u> the first bracketed expression is positive (negative) and the second bracketed expression is negative (positive). We will show that the conditions for an import subsidy to be welfare improving lead to a contradiction.

Suppose that an import subsidy in state of nature A were welfare improving. We know from (3.3) that $\partial \Delta Y^A/\partial t^A > 0$, and if importables are non-inferior $C^{1A} - C^{2A} > 0$. Evaluating the effect of a change in K^1 on the income dif-erential we obtain

$$\text{(3.6)} \quad \frac{\partial \Delta Y^i}{\partial K^1} = -K\{P^i F_{KK}^1 + (1 + t^i)F_{KK}^2 + [P^i F_{KL}^1 + (1 + t^i)F_{KL}^2]\frac{dL^{1i}}{dK^1}\}, \ i = A, B.$$

From (2.3)

$$(3.7) \qquad \frac{dL^{1i}}{dK^{1}} = - \frac{P^{i}F^{1}_{LK} + (1 + t^{i})F^{2}_{LK}}{P^{i}F^{1}_{LL} + (1 + t^{i})F^{2}_{LL}} > 0.$$

Substituting (3.7) into (3.6) yields an expression that is positive (since the determinant of the Jacobian of a two-factor constant returns production function is zero).

Thus, if the first bracketed expression in (3.5) is to be negative, dK^{1}/dt^{A} must be negative. However, this would imply that the second bracketed expression in (3.5) is also negative, which violates the equilibrium condition requiring that the total effect of a commercial policy be to transfer utility in opposite directions in the two states of nature. Hence, we conclude that $dW/dt^{A} > 0$, i.e. that a fully anticipated small tariff when terms of trade are favorable increases social welfare.[10] It does so directly by transfering income to type 1 individuals in state A, and indirectly, via the capital reallocation, by transfering income to type 2 individuals in state B.

Turning now to optimal policy in state B, we have an expression analogous to (3.5) for dW/dt^{B} and we can apply exactly the same reasoning to conclude that a small welfare-improving policy in state B must cause capital to reallocate to sector 1, so that type 2 individuals benefit in state A. And similarly, in equilibrium intervention must benefit type 1 individuals in state B. As in the unanticipated and time-consistent cases, this may involve either a tax or a subsidy. Again there is a presumption that optimal policy has an anti-trade bias, since the effect of an import <u>tariff</u>, as opposed to an import subsidy, on consumer prices is to transfer income from the high income to the low income group.

Our simulation results for this case are presented in Table 3.3.

Comparing this table with Table 3.2 note that time-consistent and optimal anticipated policies are virtually identical. At the three-digit level of accuracy of our calculations we can discern differences only when the degree of price variation reaches 35 per cent. For this case optimal policy requires a slightly higher subsidy to the export sector when the terms of trade are unfavorable. The share of capital allocated to the export sector is consequently larger: when policy makers take into account the effects of their policies on capital allocation, they reduce the anti-trade bias of intervention, but only very slightly. The share of capital allocated to the export sector is still much lower than what it is if individuals anticipate free trade. We conclude the major channel through which optimal anticipated intervention raises social welfare is not through its effect on capital allocation.

3.4 Optimal One-State Tariffs

The tariff authority may find itself constrained to set tariffs only at non-negative rates: import subsidies, which require that revenue be raised via a poll tax, may be politically infeasible. If, in fact, a non-negative tariff is optimal in both states, this constraint is not binding. Otherwise, the optimal tariff in state B will equal zero while the tariff in state A will be modified. Optimal state A tariffs, when $t^B = 0$, are presented in Table 3.4. The optimal tariff in state A is always lower than in the unconstrained case, but the net effect on capital allocation is much larger: when commercial policy is constrained to non-negative tariff rates it results in shifting more capital to the import-competing sector than otherwise; the anti-trade bias is stronger.

3.5 Non-State Contingent Tariffs

So far we have assumed that tariff rates may depend upon the terms of trade that materialize. This assumption is appropriate to situations in which (i) policy makers are very flexible or (ii) variation in the terms of trade is of rather low frequency. If neither condition is met a state-contingent policy may in fact be infeasible. A policy of imposing a tariff at a fixed rate still dominates free trade, however. To illustrate this result we differentiate the social welfare function W with respect to $t = t^A = t^B$, and evaluate the resulting expression at t=0, to obtain

$$\text{(3.8)} \qquad \frac{dW}{dt} = \lambda(1 - \lambda) \sum_{i=A}^{B} (V_Y^{2i} - V_Y^{1i})\left[\frac{d\Delta Y^i}{dt} + (C^{1i} - C^{2i})\right]$$

Since $(V_Y^{2i} - V_Y^{1i})$ reverses sign while $\frac{d\Delta Y^i}{dt} > 0$, i = A, B the sign of the expression is ambiguous. However, since the sign of $(V_Y^2 - V_Y^{1i})$ is, if commodity 2 is non-inferior, always equal to the sign of $(C^{1i} - C^{2i})$, there are three positive and one negative terms. A tariff at a positive level will, via its effect on consumer prices, always transfer income from the rich to the poor. The effect on producer prices is, of course, always to transfer income from type 1 to type 2 individuals. Our simulations for this case, presented in table 3.5, do, in fact, always indicate that a small, positive tariff is optimal. Once again, optimal policy has an anti-trade bias.

3.6 Production Taxes/Subsidies vs. Commercial Policy

A policy of taxing or subsidizing production can affect producer prices while allowing consumers to buy commodities at world prices. If taxes/subsidies can be state contingent a welfare-improving ad valorem subsidy s^i to sector 1 in state i has the sign of

$$(3.9) \qquad \left.\frac{dW}{ds^i}\right|_{s^i=o} = \lambda(1 - \lambda) \sum_{j=A,B} \pi^j (V_Y^{2j} - V_Y^{1j}) \frac{d\Delta Y^j}{ds^i}$$

where $\frac{d\Delta Y^j}{ds^i} = \frac{d\Delta Y^j}{dt^i}$. Optimal policy will necessarily require a subsidy on sector 2 output in state A and a tax in state B. Table 3.6 presents optimal subsidies and taxes (defined as negative subsidies) to sector 2, the import-competing sector.

If a _state-contingent_ tax/subsidy scheme is infeasible, the optimal fixed _ad valorem_ subsidy has the sign

$$(3.10) \qquad \left.\frac{dW}{ds}\right|_{s=o} \lambda(1 - \lambda) \sum_{j=A,B} \pi^j (V_Y^{2j} - V_Y^{1j}) \frac{d\Delta Y^j}{ds} .$$

While there is less presumption that this magnitude is positive, a non-zero value is still likely. One reason is that $\frac{dY^A}{ds} \neq \frac{dY^B}{ds}$ in general. A given subsidy rate may transfer more income in one state than in the other. Another reason is that the condition $\sum_j \pi^j (V^{2j} - V^{1j}) = 0$ does not necessarily imply that $\sum_j \pi^j (V_Y^{2j} - V_Y^{1j}) = 0$.[11]

Economists often recommend taxes and subsidies over tariffs as a means of correcting domestic factor market imperfections because they effect only producer prices, leaving consumer decisions undistored. This effect operates in favor of a production tax/subsidy policy in our context as well. A factor operating in the other direction, however, is the effect of a tariff, via its impact on consumer prices, to redistribute income from the rich to the poor in either state of nature. Our simulations indicate that, in fact, a policy of imposing taxes and subsidies on _trade_ may Pareto dominate a policy that imposes taxes and subsidies on production, especially when the elasticity of substitution in consumption is low.

3.7 Stochastically Balanced Budget

So far in our discussion we have assumed that, in each state of nature, tariff or tax revenue is redistributed equally to all individuals in a lump-sum fashion or, alternatively, that a poll tax is imposed to finance an import or production subsidy. Such poll taxes and subsidies might not be feasible policies. The government may, however, be able to borrow and lend in international capital markets so that the budget need only be in balance on average. We consider optimal tariff policy when poll taxes are infeasible, but when the government is constrained to balance the budget only in an expected sense, facing the constraint,

$$(3.11) \qquad \sum[\pi^i t^i (C^i - F^i].$$

Our simulations, presented in Table 3.7, still indicate that a tax on imports is optimal when i = A, while a subsidy is optimal when i = B. The optimal import subsidy in state B is now, however, much larger than the optimal tariff in state A: it is optimal for the government to run a budget deficit when the terms of trade are unfavorable, giving out larger subsidies, and to run a surplus under favorable terms of trade. In this way the government smooths out the effect of terms of trade fluctuations on income. The consequence of such a policy is to attract capital to the export sector. The rest of the world, through government borrowing and lending, is acting to insure the entire economy against fluctuations in its terms of trade. It is optimal for individuals to behave in a more risk neutral fashion, and to specialize more in producing the export good.

4. Sensitivity Analysis

In section 3 we established that, when domestic risk sharing arrangements are incomplete, an interventionist commercial policy is generally welfare improving. We characterized optimal intervention under various institutional arrangements for a given set of parameter values. In this section we restrict ourselves to commercial policy that involves taxes or subsidies on (i) imports, where tariff rates (ii) are anticipated, (iii) are state-contingent, (iv) may be positive or negative and (v) are subject to a balanced-budget constraint in each state. We assume that policy makers can credibly commit themselves to their actual policies before investment decisions are made. We analyze how optimal intervention changes with changes in tastes and technology.

4.1 Risk Aversion

We calculated optimal tariffs for the values of γ indicated in Table 4.1, holding other parameters at the levels given in section 2, and setting $u = .25$. Note that if individuals are risk neutral ($\gamma = 1$), free trade is optimal. Optimal intervention rises with the degree of relative risk aversion R, (where $R = 1 - \gamma$), as does the share of capital allocated to the import-competing sector.

4.2 Elasticity of Substitution in Consumption

We calculated optimal tariffs for the values of ρ_c given in Table 4.2. As the elasticity of substitution rises (ρ_c falls), the optimal tariff rates fall in absolute value, as does the share of capital allocated to the export sector. As substitutability between the two goods rises, so does the distortion implied by a given level of tariff protection. Thus the optimal tariff falls as σ_c rises. In contrast to the effect of varying the risk aversion parameter γ, large changes in σ_c have rather insignificant effects on the optimal tariff rates.

4.3 The Share of Exports in Consumption

As Table 4.3 indicates, as the imported good comes to occupy a larger share of expenditures (α_c falls) the optimal tariff becomes smaller in absolute value. As the share of imports in consumption becomes larger, a given reallocation of income can be obtained with a smaller tariff rate.

4.4 The Elasticity of Substitution in Production

As ρ_1 and ρ_2 fall, (implying higher values of σ_1 and σ_2) the optimal tariff falls, while the share of capital allocated to the export sector grows. These results are reported in Table 4.4. As production becomes more elastic, more labor is transferred between sectors in response to the state of nature. A given tariff therefore has a greater distorting effect on production. Thus the optimal tariff rate falls as σ_1 and σ_2 rise. Greater substitutability between factors acts, also, to reduce the effects of terms of trade variations on the returns to capital, thereby providing an insurance effect. The optimal amount of capital allocated to the export sector therefore rises as technology becomes more elastic.[12] Note that changing the elasticity of substitution in production has a much more pronounced effect on the optimal tariff rates than does changing the elasticity of substitution in consumption.

4.5 Capital Shares

Reducing the capital shares α_1 and α_2 reduces the optimal amount of tariff intervention, as shown in Table 4.5. The reason is that as the share of the mobile factor (labor) rises, so does the *ex post* flexibility of the economy. The distortion implied by a given tariff rises. At the same time the wage is higher relative to capital income. The overall differences in income between the two types of individuals becomes lower. For this reasons we found that reducing α_1 and α_2, but holding α_1/α_2 constant, increases the share of capital allocated to the export sector.

4.6 The Capital -Labor Ratio

As we raise the capital-labor ratio, the optimal level of protection also rises, as reported in Table 4.6. The reason is that as this magnitude rises, so does the share of capital income in total income. Differences in income between the two types of individuals across states of nature are consequently larger. The *share* of capital allocated to the export sector also rises with the total capital-labor ratio, as would be expected from Rybczynski Theorem considerations.

5. Conclusion

We have shown how in an economy in which agents must specialize in their use of a factor endowments, and in which domestic risk sharing arrangements are incomplete, departures from free trade are likely to be welfare improving even for a small, open economy. We emphasize, however, that an interventionist commercial policy constitutes a *second best* solution. A first best policy would redistribute income directly without distorting consumer or producer prices. In our simple model a tax on income would serve this purpose. Problems of evasion may make an income tax difficult to administer, however, especially in a less developed country where much domestic economic activity takes place outside the market. For this reason commercial policy may be the only available method of pooling risk.

Our results may be compared with Johnson's (1965) and Bhagwati's (1971) analysis of commercial policy in the presence of other domestic market imperfections such as factor market distortions. Here as well an interventionist commercial policy is not the first best means of correcting the distortion, but if other instruments are not available then free trade is not optimal.

Finally, we note that in this paper we have restricted ourselves to consider the terms of trade as the source of uncertainty. Uncertainty may also arise in domestic preferences and technologies. The essential arguments that we have made here are not affected if the source of uncertainty changes. We performed a number of simulations in which the source of uncertainty was a multiplicative disturbance term in the production function of the exportable good. No results emerged that were qualitatively different from what we report above.

FOOTNOTES

1. Eaton and Rosen (1980a, 1980b) examine the implications of incomplete wage insurance markets for optimal income tax policy. In their model, however, all private risk is eliminated at the aggregate level by the law of large numbers. When uncertainty arises from the terms of trade it is aggregate risk which is not diversifiable.

2. Grossman (1981) develops a model which allows for gradations of inter-sectoral factor mobility, in constrast to the sharp distinction between perfect mobility and perfect immobility we draw here. As long as some degree of immobility is present, the basic implications of our analysis remain intact.

3. See Grossman and Shapiro (1981) for a model that endogenizes the efficiency gains from specialization in sector-specific training.

4. If perfect insurance markets existed, the outcome would be identical to the one that would obtain if individuals could divide their capital between activities. It is this essential indivisibility of capital, as well as its immobility, that leads to a suboptimal allocation of risk. It is in assuming that capital is indivisible that our analysis here differs from that in Eaton (1979). This second model assumes that the representative individual can divide his capital between activities. The optimality of free trade follows.

5. Note that "tariff revenue" may be negative. In such cases the import subsidy is financed by a poll tax.

6. The derivation of (3.4) is provided in the appendix.

7. Strictly speaking, this statement is necessarily true only for small deviations from free trade. Globally optimal policies could possibly be opposite in sign to small, welfare-improving policies if there are strong interactions between the welfare effect of a trade policy in a given state and the distortion caused by the non-infinitesimal deviation from free trade in the opposite state. We found no examples of such reversals in our simulations, however.

8. Kydland (1977)defines a policy that (i) is time-consistent and (ii) responds to current information about the state of the economy as the feedback solution to the government's control problem. A policy response that (i) is optimal from the perspective of an initial period prior to the period of implementation but (ii) responds to current information is the closed loop solution. The open loop solution (i) is optimal from the perspective of an initial period prior to the period of implementation and (ii) depends only on information available at that initial period. According to this nomenclature this section treats feedback commercial policy. In section 3.3 we consider closed loop policy while open loop policy is taken up in section 3.5

9. Dybvig and Spatt (1980) model the reputation phenomenon formally for a firm concerned with its reputation for product quality. They find that time-consistent policy in general lies between the feedback policy when there is no reputation effect and the closed-loop policy.

10. Since expected utilities are equal an increase in social welfare for an anticipated policy is equivalent to an increase in expected utility of the representative individual.

11. This relationship is, however, implied by constant absolute risk aversion.

12. The last result also appeared in Eaton's (1979) one-individual model.

REFERENCES

Arrow, K.J. (1970) Essays in the Theory of Risk Bearing (Amsterdam: North Holland).

Baldwin, R.E. (1981) "The Political Economy of Protectionism", in Bhagwati, J.N. (ed.), Import Competition and Responde (Chicago: University of Chicago Press).

Bhagwati, J.N. (1971) "The Generalized Theory of Distortions and Welfare", in Bhagwati, J.N., Jones, R.W., Mundell, R.A. and J. Vanek (eds.), Trade, Balance of Payments and Growth (Amsterdam: North Holland).

Cassing, J.H. (1980) "Alternative of Protectionism", in Levenson, J. and J.W. Wheeler (eds.), Western Economics in Transition (Boulder: Westview Press).

Corden, W.M. (1974) Trade Policy and Economic Welfare (Oxford: Oxford University Press).

Cornes, R. and F. Milne (1981) "An Analysis of Mutually Disadvantageous Trades", manuscript.

Dybvig, P.H. and C.S. Spatt (1980) "Does it Pay to Maintain a Reputation?", Financial Research Memorandom No. 32, Princeton University, August.

Eaton, J. (1979) "The Allocation of Resources in an Open Economy with Uncertain Terms of Trade", International Economic Review 20 (2), pp. 391-403.

Eaton, J. and H.S. Rosen (1980a) "Optimal Redistributive Taxation and Uncertainty", Quarterly Journal of Economics 95 (3), pp. 357-364.

Eaton, J. and H.S. Rosen (1980b) "Labor Supply, Uncertainty and Efficient Taxation", Journal of Public Economics 14 (3), pp. 365-374

Fleming, J.S., S.J. Turnovsky and M.C. Kemp (1977) "On the Choice of Numeraire and Certainty Price in General Equilibrium Model of Price Uncertainty", Review of Economic Studies 64(3) pp. 573-584.

Grossman, G.M. (1981) "Partially Mobile Capital: A General Approach to Two-Sector Trade Theory", International Finance Section Discussion Paper G-81-01, Princeton University.

Grossman, G.M. and C. Shapiro (1981) "A Theory of Factor Mobility", Woodrow Wilson School Discussion Papers in Economic No. 1, Princeton University.

Hart, O.D. (1975) "On the Optimality of Equilibrium When the Market Structure is Incomplete", Journal of Economic Theory 11 (3), pp. 418-443.

References, cont'd.

Johnson, H.G. (1965) "Optimal Trade Intervention in the Presence of Domestic Distortions" in Caves, R., Johnson, H.G. and P.B. Kenen, (eds.), Trade, Growth and the Balance of Payments (Chicago: Rand McNally).

Jones, R.W. (1971) "A Three-Facotr Model in Theory, Trade and History", in Bhagwati, J.N., Jones, R.W., Mundell, R.A. and J. Vanek (eds.), Trade, Balance of Payments and Growth (Amsterdam: North Holland).

Kydland, F. (1977) "Equilibrium Solutions in Dynamic Dominant-Player Models," Journal of Economic Theory, 15 (2), pp. 307-324.

Newbery, D.M.G. and J.E. Stiglitz (1981) The Theory of Commodity Price Stabilization (Oxford: Oxford University Press).

Rothenberg, T.J. and K.R. Smith (1971) "The Effect of Uncertainty on Resource Allocation in a General Equilibrium Model", Quarterly Journal of Economics 85 (3), pp. 440-453.

Shavell, S. (1979) "On Moral Hazard and Insurance", Quarterly Journal of Economics 93 (4), pp. 541-562.

Tables

Table 3.1

Optimal Unanticipated Tariff Rates

u	t^A	t^B	K^1
.15	.06	-.06	.671
.25	.09	-.09	.690
.35	.13	-.12	.709

Table 3.2

Anticipated Time Consistent Tariff Rates

u	t^A	t^B	K^1
.15	.06	-.05	.642
.25	.10	-.08	.633
.35	.14	-.10	.613

Table 3.3

Optimal Anticipated Tariff Rates

u	t^A	t^B	K^1
.15	.06	-.05	.642
.25	.10	-.08	.633
.35	.14	-.11	.627

Table 3.4

Optimal One-State Tariff Rates

u	t^A	t^B	K^1
.15	.03	0	.616
.25	.06	0	.585
.35	.08	0	.579

Table 3.5

Optimal Non-State Contingent Tariff Rates

u	$t = t^A = t^B$	K^1
.15	.01	.638
.25	.02	.631
.35	.04	.579

Table 3.6

Optimal Subsidies

u	s^A	s^B	K^1
.15	.08	-.07	.656
.25	.13	-.10	.649
.35	.19	-.13	.638

Table 3.7

Optimal Tariffs with a Stochastically Balanced Budget

u	t^A	t^B	K^1
.15	.12	-.149	.733
.25	.30	-.332	.862
.30*	.36	-.469	.99

*At u = .35 complete specialization occurred.

Table 4.1

Variations in Risk Aversion

$\gamma (R = 1 - \gamma)$	t^A	t^B	K^1
1.0	0.0	0.0	.714
.01	.06	-.05	.667
-1.0	.10	-.08	.633
-2.0	.13	-.10	.608
-9.0	.21	-.13	.522

Table 4.2

Variations in the Elasticity of Substitution in Consumption

$\rho_c (\sigma_c = \frac{1}{1 + \rho_c})$	t^A	t^B	K^1
100.00	.12	-.10	.663
0.50	.11	-.09	.642
0.01	.10	-.08	.633
-0.33	.10	-.08	.621
-0.50	.10	-.08	.613

Table 4.3

Variations in the Share of Exports in Consumption

α_c	t^A	t^B	K^1
0.5	.11	-.09	.614
0.25	.10	-.08	.633
0.1	.10	-.08	.665

Table 4.4

Variations in the Elasticity of Substitution in Production

$\rho = \rho_1 = \rho_2 (\sigma_i = \frac{1}{1+\rho_i})$	t^A	t^B	K^1
4.00	.15	-.13	.447
0.50	.12	-.10	.566
0.01	.10	-.08	.633
-0.33	.07	-.06	.735
-0.50	.05	-.04	.806
-0.67	.02	-.02	.903

Table 4.5

Variations in Capital Shares

α_1	α_2	t^A	t^B	K^1
.75	.5	.14	-.12	.583
.75	.25	.10	-.08	.743
.5	.25	.10	-.08	.633
.1	.05	.03	-.01	.676

Table 4.6

Variations in the Capital-Labor Ratio

K	t^A	t^B	K^1/K
1.15	.10	-.09	.748
1.0	.10	-.08	.633
0.85	.09	-.07	.500

Appendix: Derivation of Equation (3.4)

To derive expression (3.4) in the text we first differentiate the social welfare function W with respect to t^i to obtain:

$$\text{(A.1)} \qquad \left.\frac{dW}{dt^i}\right|_{\bar{K}^i} = \pi^i\{\lambda[V_Y^{1i} \left.\frac{dY^{1i}}{dt^i}\right|_{\bar{K}^i} + V_t^{1i}] + (1 - \lambda)[V_Y^{2i} \left.\frac{dY^{2i}}{dt^i}\right|_{\bar{K}^i} + V_t^{2i}]\} .$$

Differentiating (2.5) with respect to t^i yields

$$\text{(A.2)} \qquad \frac{dY^{1i}}{dt^i} = P^i k\, F_{KL}^{1i}\, L_t^{1i} + \frac{dw^i}{dt^i} + M^i + t^i \frac{dM^i}{dt^i}$$

and

$$\text{(A.3)} \qquad \frac{dY^{2i}}{dt^i} = -k\, F_K^{2i} - (1 + t^i)k\, F_{KL}^{2i}\, L_t^{1i} + \frac{dw^i}{dt^i} + M^i + t^i \frac{dM^i}{dt^i}$$

where

$$\text{(A.3)} \qquad L_t^{1i} \equiv -dL^{1i}/dt^i$$

Since we consider deviations from an initial situation of the free trade we set

$$\text{(A.4)} \qquad t^i = 0$$

From (2.3)

$$\text{(A.5)} \qquad \left.\frac{dw^i}{dt^i}\right|_{\bar{K}^i} = P^i F^{1i}_{LL} L^{1i}_t$$

and also

$$\text{(A.5')} \qquad \left.\frac{dw^i}{dt^i}\right|_{\bar{K}^i} = F^{2i}_L - F^{2i}_{LL} L^{1i}_t$$

Euler's theorem implies that

$$\text{(A.6)} \qquad F^{ji}_{KL} = - F^{ji}_{LL} L^{ji}/K^j \qquad j = 1, 2$$

and that

$$\text{(A.7)} \qquad F^{2i}_K = (F^{2i} - F^{2i}_L L^{2i})/K^2$$

Finally

$$\text{(A.8)} \qquad M^i = \lambda C^{1i} + (1-\lambda)C^{2i} - F^{2i} .$$

Substituting (A.3) through (A.8) into (A.2) and (A.2') gives

$$\text{(A.9)} \qquad \frac{dY^{1i}}{dt^i} = (\frac{\lambda - L^{1i}}{\lambda})\frac{dw^i}{dt^i} + \lambda C^{1i} + (1 - \lambda)C^{2i} - F^{2i}$$

and

(A.9')

$$\text{(A.9')} \qquad \frac{dY^{2i}}{dt^i} = \frac{F^{2i}}{1-\lambda} - \left(\frac{\lambda - L^{1i}}{1-\lambda}\right) + \lambda C^{1i} + (1-\lambda)C^{2i} - F^{2i}$$

Substituting (A.9), (A.9') and Roy's identity (2.6) into (A.1) we obtain

$$\text{(A.10)} \qquad \left.\frac{dW}{dt^i}\right|_{\bar{K}^i;\, t^i=0} = \pi^i\{(V_Y^{1i} - V_Y^{2i})[(\lambda - L^{1i})\left.\frac{dw^i}{dt^i}\right|_{\bar{K}^i} - \lambda G^{2i}]$$

$$+ V_Y^{1i}\,\lambda[\lambda C^{1i} + (1-\lambda)C^{2i} - C^{1i}]$$

$$+ V_Y^{2i}(1-\lambda)[\lambda C^{1i} + (1-\lambda)C^{2i} - C^{2i}]\} \;.$$

Rearranging we get

$$\text{(A.11)} \qquad \left.\frac{dW}{dt^i}\right|_{\bar{K}^i;\, t^i=0} = \pi^i\{\lambda V_Y^{1i}[\left(\frac{\lambda - L^i}{\lambda}\right)\left.\frac{dw^i}{dt^i}\right|_{\bar{K}^i}$$

$$+ \lambda C^{1i} + (1-\lambda)C^{2i} - F^{2i} - C^{1i}]$$

$$+ (1-\lambda)V_Y^{2i}[F^{2i}/(1-\lambda) + \left(\frac{\lambda - L^i}{1-\lambda}\right)\left.\frac{dw^i}{dt^i}\right|_{\bar{K}^i}$$

$$+ \lambda C^{1i} + (1-\lambda)C^{2i} - F^{2i} - C^{2i}]\}$$

$$\text{(A.11')} \qquad = \pi^i[(V_Y^{1i} - V_Y^{2i})(\lambda - L^{1i})\left.\frac{dw^i}{dt^i}\right|_{\bar{K}^i}$$

$$+ \lambda(1-\lambda)(V_Y^{1i} - V_Y^{2i})(C^{2i} - C^{1i}) - \lambda(V_Y^{1i} - V_Y^{2i})F^{2i}]$$

(A.11")

$$= \pi^i (V_Y^{2i} - V_y^{1i}) \lambda(1 - \lambda)\left[\left(\frac{L^{1i} - \lambda}{\lambda(1 - \lambda)}\right)\frac{dw^i}{dt^i}\Bigg|_{\bar{K}^i} + F^{2i}/(1 - \lambda) + (C^{1i} - C^{2i})\right] .$$

From (A.9) and (A.9') we have

(A.12)

$$\frac{d\Delta Y^i}{dt^i}\Bigg|_{\bar{K}^i} = \frac{F^{2i}}{(1 - \lambda)} - \left[\frac{\lambda - L^{1i}}{\lambda(1 - \lambda)}\right]\frac{dw^i}{dt^i}\Bigg|_{\bar{K}^i} .$$

Substituting (A.12) into (A.11") yields expression (3.4). Expression (3.5), (3.8), (3.9) and (3.10) may be obtained via a similar set of substitutions.