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PATENTS, R&D, AND THE STOCK MARKET RATE OF RETURN

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ABSTRACT

The purpose of this paper is to present and estimate a model which allows one to use the recently computerized U.S. Patent Office's data base to identify when and where changes in inventive output have occurred. The model assumes a firm which chooses a research strategy to maximize the expected discounted value of the net cash flows from its activities, and a stock market that evaluates this expectation at different dates (it is a version of the Lucas-Prescott, 1971, investment model). Patents are taken as an indicator of the output of the firm's research laboratories. These assumptions place a set of testable restrictions on the stochastic process generating patents, R&D, and the stock market rate of return on the firm's equity (the econometric framework used is that of a restricted index, or dynamic factor-analysis model (Sargent and Sims, 1977; Geweke, 1977b)). The data contain observations on these three variables for 120 firms over an eight year period. The model fits these data quite well and the final section reports on the implications of the parameter estimates.

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For some time there has been a feeling in the profession that the process of invention and innovation is a major source of growth and structural change in the economy. Unfortunately, however, our analysis of the role of inventive activity is hampered by a lack of empirical evidence on its casues and its effects. A major part of the reason for the lack of empirical results in this area is the difficulty in finding (or constructing) meaningful measures of inventive output. Early studies often used successful patent applications as their output measure (Schmookler and Brownlee, 1962; Griliches and Schmookler, 1963; Scherer, 1965a, 1965b; Schmookler, 1966). The patent variable had the advantage of being a more direct consequence of inventive activity than the other indicators of performance available (examples used include profits, productivity, and sales of new products) and that patent applications were, at least in principle, available in an extremely detailed breakdown (by both grantee and product class, see USDC, 1973-79). In fact, the only other variable available which was directly related to inventive activity was R & D expenditures. R&D, however, is really an input measure (and to analyse many issues one requires measures of both inputs and outputs);[1] and publicly available data on R & D are not nearly as rich as those on patents (particularly when one considers breakdowns by product class). There were, however, two serious problems with the patent variable.

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[1]In Section I and in the appendix I discuss briefly the advantages of having both input and output measures for analysing the determinants of R&D demand.

First, though patent counts were available in principle, they were inaccessible in practice; second, variation in the number of patents granted had no clear interpretation. The recent computerization of the U.S Patent Office's data base has changed this situation. One can now obtain annual patent applications in a variety of different breakdowns at reasonable cost (for an example, see Pakes and Griliches, 1980b). Thus the interpretative problem now takes on renewed importance. That is, to use the Patent Office's data base effectively we require some indication of the relationship between successful patent applications and meaningful measures of the economic value of the output of inventive activity.

The question of the relationship between successful patent applications and different economic magnitudes is not new, but the evidence available on it is still inconclusive (see, in particular, the contributions of Kuznets, Sanders, and Schmookler in Nelson, 1962; Comanor and Scherer, 1969; and Taylor and Silberston, 1973). It is clear that patent applications are only granted when a useful and technologically feasible advance has been made (U.S. Department of Commerce, 1978), and that the patentee expects some positive benefit from the patent (since the process of application is costly in itself). But it is also clear that a variety of circumstances (technological, institutional, and market) can cause patents to vary greatly in their economic value, and that not all useful innovations are patented.

The purpose of this paper is to present and estimate a model which allows us to interpret variations in patent applications in terms of variations in the stock market value of the output of the firm's research activities. The paper, therefore, investigates the relationships between patent applications and a measure of the inputs into the inventive process (R&D expenditures), and between these applications and a well-defined (though indirect) measure of inventive output (stock market values). In this context the use of stock-market values has one major advantage. As noted by Arrow (1962) the public-good characteristics of inventive output make it extremely difficult to market. Returns to innovations are mostly earned by embodying it in a tangible good or service which is then sold or traded for other information which can be so embodied (Wilson, 1975; Von Hippel, forthcoming). There are therefore no direct measures of the value of inventions, while indirect measures of current benefits (such as profits or productivity) are likely to react to the output of the firm's research laboratories only slowly and erratically (see the review by Griliches, 1979). On the other hand, under simplifying assumptions, changes in the stock-market value of the firm should reflect (possibly with error) changes in the expected discounted present value of the firm's entire uncertain net cash-flow stream. Thus, if an event does occur that causes the market to re-evaluate the accumulated output of the firm's research laboratories, its full effect on stock-market values ought to be recorded immediately.[2] This full effect is, of course, the expected effect of the event on future net cash flows and need not be equal to the effect which actually materializes. The fact that we are measuring expectations rather than realizations, however, does have its advantages. In particular it is expectations which ought to determine research demand, so that the use of stock-market values will allow us to check whether the interpretation we give to our parameter estimates is consistent with the observed behavior of the research expenditure series.

Section I describes the model which underlies the interpretation of the empirical results to be presented. It is based on an optimizing firm which chooses a research strategy to maximize the expected discounted value of the net cash flows from its activities and a stock market which evaluates this expectation on the basis of current information (a similar model can be found in Lucas and Prescott, 1971). The model places a set of testable restrictions on the trivariate process generating patents, R&D, and the stock market rate of return on the firm's equity, and therefore, permits one to check whether the interpretation given to the parameter estimates is consistent with the observed behaviour of the data.

[2] A similar point was actually made as far back as 1973 by Griliches; see Griliches, 1973, pp. 68-69.

Econometrically it leads us to a version of index models (see Sargent and Sims, 1977) or dynamic factor-analysis models (see Geweke, 1977b) which have recently been used to analyze macro-economic data.[3] Section II begins by explaining these points and then presents the estimates and associated test statistics. In Section III, the implications of the empirical results, particularly those that concern the interpretation of movements in the patent variable, are considered in some detail.[4] Brief concluding remarks follow.

I. *The Model*

The model used to interpret the empirical results is based on a firm which chooses its research programme to maximize the expected discounted value of the net cash flows from its activities and a stock market which evaluates this expectation at different points in time. It thus invokes the same assumptions as those used in the Lucas and Prescott (1971) investment model -- a model which has led to several recent empirical investigations of the demand functions for traditional factors of production (see, for example, Sargent,

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[3] There are two differences. First, the model used here is estimated on a cross-section of time series, rather than a single one. This allows one to weaken some of the stochastic assumptions that underlie macro-economic dynamic factor-analysis models. Second, the micro-economic foundations of our model suggest more restrictions than are usually available in macro-economic work and lead to simpler testing and estimation procedures.

[4] The appendix goes over some of the more detailed implications of the empirical findings and provides confirmation of them under a more general set of assumptions than those used in the text.

1978, Geweke, 1977a, and Meese, 1980). There is, however, one major distinction between the models considered in those articles and the one used here.[5] The models referred to above specify a quadratic net cash-flow function and a stochastic process which affects it; and then proceed to derive and estimate the relevant factor demand equation(s). From our point of view the disadvantage of this approach is that a measure of changes in the expected discounted value of the net cash flows of the firm never explicitly appears in the equations derived from it; and it is the relationship between this value and patent applications that we are primarily concerned with. An alternative, noted by Lucas and Prescott (1971), is to approximate directly the function determining the stock-market value of the firm (rather than the net cash-flow function which generates it). This allows one to use the observed stock-market rate of return on the firm's equity as an indicator of the change in the expected value of the firm resulting from the events which have occurred in a given period and to relate it to both the patent and the R&D expenditure series.

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[5] There is also one minor distinction of some interest. Investment models assume the existence of a market for ready-made capital goods. Thus, the gradual response of investment to changes in market conditions in these models is generally assumed to be a result of convex adjustment costs, which increase unit costs of installing capital goods in any given period. Since the existence of markets in which a firm can buy and sell the information context of innovations would not, in general, be assumed in the R&D demand literature, there is no need for the convex adjustment cost assumption in this context. Instead one assumes that a firm must produce (or search for) an innovation in order to use it. Moreover, the production (search) process is assumed to take time, or rather, to be more costly the faster it it carried out, and this will induce the firm to respond only gradually to changes in market conditions.

The firm is assumed to engage in two types of activity, research and commercial (production and marketing). Successful research activity either lowers the costs of production or improves the demand conditions that will face the firm in the future. It does not produce an output which can be sold directly on the goods market. Commercial activity transforms non-research inputs into outputs which *are* sold on the market. The demand and supply conditions which will face the firm in the future are in the present random variables whose distribution depends on the firm's R & D efforts until they are realized. It will be assumed that inputs into commercial activities can be adjusted costlessly at the beginning of each period so as to maximize the profits attainable in that period.

In this environment the firm realizes that its demand for commercial inputs in period $t + \tau$ will be a function of its research expenditures until $t + \tau$ and of certain exogenous variables, $Z_{t+\tau}$ which will only be known with certainty at the beginning of period $t + \tau$. The latter include the determinants of demand in the firm's output markets, input prices, and the technological success and failure of the firm's R & D laboratories. The firm can solve for the distribution of future commercial input demands conditional on the realizations of[6] $Z_{t+\tau}$. Substituting this solution into the operating-profit function we have $\pi_{t+\tau} = \pi(R_{t+\tau-1}, R_{t+\tau-2}, \ldots, Z_{t+\tau})$ for all t, and for $\tau \geq 0$. Thus from the vantage point of period t the discounted value of the net cash flows accruing to the firm is the random variable, $\tilde{V}_t$, where

$$(1) \qquad \tilde{V}_t = \sum_{\tau=0}^{\infty} D^{\tau}[\pi(R_{t+\tau-1}, R_{t+\tau-1}, \ldots, Z_{t+\tau}) - R_{t+\tau}] ,$$

and D is the (time-invariant) discount factor.

[6] It is assumed that there exist unique, finite solutions to all maximization problems and that the expectations we shall be dealing with are finite.

It will be assumed that the stochastic process generating the sequence $\{\pi_{t+\tau}\}_{\tau=0}^{\infty}$ is known to all economic agents. We can now formalize our two behavioural assumptions. First, the research-decision problem confronting the manager of the firm is to choose a research programme (a probability distribution for the sequence $\{R_{t+\tau}\}_{\tau=0}^{\infty}$) to maximize $E_t\tilde{V}_t$, where E_t is the expectations operator conditional on the information set available at the beginning of period t, say Ω_t.

The programme is formulated by using Ω_t to choose R_t and to formulate alternative strategies for R_{t+1}, R_{t+2}, ... It is known that the strategy which will actually be implemented will depend on the information available when the research resources are committed. Thus the optimum research programme consists of a number, R_t, and a sequence of random variables[7] $\{R_{t+\tau}(\Omega_{t+\tau})\}_{\tau=1}^{\infty}$.

The second behavioural assumption is that the agents operating in the stock market (who will in general include the manager of the firm) evaluate the expected discounted value of the net cash flows likely to result from the firms's decision. The stock market value of the firm is, therefore, obtained by substituting the optimum research programme into (1), conditioning on the information set currently available, and passing through an expectations operator. This information set will contain current and past research expenditures as well as any other variables which help to predict the distribution of future net cash flows. If one held to the model exactly, then, an additional sequence of random variables, $\{A_t\}$, could be introduced to represent the effect on the value of the firm of all those variables in the

[7] A more detailed discussion of the nature of the solution of this decision problem can be found in Sargent (1979) and the literature cited there.

information sets (other than current and past research expenditures) which are relevant to the prediction of future net cast flows, and the equation for the stock market value could be written as,

$$H(R_t, R_{t-1}, \ldots, A_t) - R_t$$

where

$$H(R_t, R_{t-1}, \ldots, A_t) = E_t\{\sum_{\tau=0}^{\infty} D^{\tau}\pi_{t+\tau} - \sum_{\tau=1}^{\infty} D^{\tau}R_{t+\tau}\} \quad .$$

Since we will, however, be estimating this model, we ought to take account of disturbances in the relationships defined by it. Introducing these disturbances will correspond to allowing for factors which affect market value without affecting research activity.[8] I, therefore, introduce another sequence of random variables, $\{B_t\}$ (from which the disturbances will be derived), and assume that the stock market value of the firm can be written as,

$$(2) \qquad V_t(\Omega_t) = E_t\tilde{V}_t = B_t\ \{H(R_t, R_{t-1}, \ldots, A_t) - R_t\} \quad .$$

(2) is the value equation. Stochastic assumptions on the evolution of B_t and A_t over time will be made below.

Two implications of the behavioral assumptions will be used in the empirical analysis. First, given that (2) provides an expression for the expected discounted value of future net cash flows, it must be the case that the optimal choice of R_t will,

$$(3) \qquad \max_{R_t}\ H(R_t, R_{t-1}, \ldots, A_t) - R_t \quad .$$

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[8] It should be noted that many of the firms in our data set are involved in several lines of business, some of which have little or nothing to do with their R&D activities.

(3) follows from the Bellman condition for this problem. Note that R_t will depend on $\{A_t\}$ (but not on $\{E_t\}$) and on R_{t-1}, $R_{t-2}\ldots$, where the lagged R values will, in turn, depend on past values of A. It follows that the stochastic process generating $\{R_t\}$ can be derived from the process generating $\{A_t\}$ and the specific form of $H(\cdot)$.

The second implication is that, provided dividends are paid out at the beginning of the period, the one-period excess rate of return on the firm's equities (capital gains plus dividends on $1 invested in the firm *minus* the interest rate), q_t, is equal to the percentage increase in the expected discounted value of the firm's net cash flows caused by information which accumulates over the given period:[9]

(4) $$q_t = (E_t - E_{t-1})\tilde{V}_t/E_t\tilde{V}_t \ .$$

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[9] This is a discrete-time approximation to a continuous-time result. Using the notation introduced above and a continuous-time model, we have

$$V_t = E_t \int_{\tau=t}^{\infty} (\pi_\tau - R_\tau) e^{-\rho(\tau-t)} d\tau,$$

$$V_{t+\delta} = E_{t+\delta} \int_{\tau=t+\delta}^{\infty} (\pi_\tau - R_\tau) e^{-\rho(\tau-t-\delta)} d\tau + E_{t+\delta} \int_{\tau=t}^{t+\delta} (\pi_\tau - R_\tau - div_\tau) e^{+\rho(\tau-t)} d\tau \quad ,$$

while $q_t = [\dot{V}_t + div_t]/V_t - \rho$, where div_τ represents dividends paid out at τ [so that $(\pi_\tau - R_\tau - div_\tau)$ equals retained earnings at τ], ρ is the instantaneous discount rate, and $\dot{V}_t = \lim_{\delta\to 0} 1/\delta\ (V_{t+\delta} - V_t)$. Using the first two expressions to solve for $\dot{V}_t$ and substituting the result into the third we obtain

$q_t = \dot{E}_t \tilde{V}_t/E_t\tilde{V}_t$ where $\dot{E}_t = \lim_{\delta\to 0} 1/\delta\ (E_{t+\delta} - E_t)$. Equation 4 is a discrete time approximation to this result. It ignores terms equal to the within-period interest earned on dividends per share and the within-period interest on capital gains per share. A correction for this omission did not change the empirical results.

We still require a specification for the patent equation. A simple model of the patenting process would be based on the production of new bits of information by a firm's research laboratories, and a patenting decision determining how many patents ought to be applied for given the number of bits produced in any given year (here one would normalize the bits of information in terms of their contribution to the value of the firm). The amount of information would be determined by current and past research expenditures as well as by a stochastic process indicating the firm's degree of success in transforming the research expenditures into valuable output. Given the number of bits of information produced, the number of patents applied for would depend on an assortment of factors including the rules governing the behaviour of the patent office, the type of the information produced, the costs of applying for a patent and the advantages to be gained by obtaining them. The total effect of these factors on patenting has been termed by Scherer (1965a, 1965b) the propensity to patent and in what follows we represent it by a stochastic process. Adding a patent equation based on these considerations to (3) and (4) produces a fairly rich model which is briefly discussed in the appendix. That model distinguishes between the effect of events which lead to patentable results only by first increasing R&D expenditures (say demand shocks) and those events which have a direct effect on patents as well as an indirect effect via the research demand they induce (say technological or supply shocks).[10] As is discussed in the appendix, however, to distinguish between

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[10] For an interesting discussion of the importance of distinguishing between the effects of demand and supply factors on inventive activity see Rosenberg (1974) and Schmookler (1960). Clearly, many policy issues hinge on this distinction.

these different events one requires more (and perhaps different) data than is used in this paper. I, therefore, present the results from a simpler model where patents are taken to be a function of current and past R&D expenditures, current and past values of the effect of the other factors which influence R&D demand ($\{A_t\}$) and an additional sequence of random variables $\{G_t\}$ which determine the propensity to patent and, therefore, are assumed to have no effect on either R&D demand or on the market value of the firm. That is,

$$(5) \qquad P_t = P(A_t, A_{t-1}, \ldots, R_t, R_{t-1}, \ldots, G_t).$$

Note that one could use (3) to solve for A_t in terms of current and past R and substitute that solution into (5) to derive an observable relationship between P_t, current and past R, and a term (G_t) which is not (by assumption) related to (current or past) R&D expenditures. In the light of our previous discussion, however, one should not interpret this relationship as a production function for patentable output as it does not distinguish the direct effect of R&D on patents from the effect of supply shocks on R&D <u>and</u> patents.[11]

Taking (3), (4) and (5) together one finds three sequences ($\{A_t\}$, $\{B_t\}$ and $\{G_t\}$) which determine the evolution of q, R, and P. Let the logarithms of the random variables in these sequences be $\{a_t\}$, $\{b_t\}$ and $\{g_t\}$.

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[11]This is the time-series analogue of the classical simultaneous equations problem involved in estimating static production-type relationships; see Marschak and Andrews (1944).

In what follows we shall assume that these latter sequences evolve as three mutually uncorrelated covariance stationary stochastic processes and use their moving average representations (see Wold, 1948), given by

$$(6) \qquad a_t = \sum_{\tau=0}^{\infty} a_\tau \varepsilon_{t-\tau}, \quad b_t = \sum_{\tau=0}^{\infty} b_{1,\tau} \eta_{1,t-\tau}, \quad g_t = \sum_{\tau=0}^{\infty} b_{3,\tau} \eta_{3,t-\tau}$$

where $a_0 = b_{1,0} = b_{3,0} = 1$ and $\{\varepsilon_t\}$, $\{\eta_{1,t}\}$, $\{\eta_{3,t}\}$ are the three mutually uncorrelated white-noise processes (i.e., processes which are serially uncorrelated with constant variance) from which $\{a_t\}$, $\{b_t\}$, and $\{g_t\}$ can be derived.[12] Finally, using a logarithmic approximation to $H(\cdot)$ in (3) and to $P(\cdot)$ in (5), substituting (6) into these equations and into (4), and eliminating all unessential constant terms, one derives the moving average representation of the stochastic process generating $\{q, r, p\}$

($r = \log R$ and $p = \log P$) as:

$$(7) \qquad \begin{aligned} q_t &= \varepsilon_t + \eta_{1,t} \\ r_t &= \sum_{\tau=0}^{\infty} c_{2,\tau} \varepsilon_{t-\tau} \\ p_t &= \sum_{\tau=0}^{\infty} c_{3,\tau} \varepsilon_{t-\tau} + \sum_{\tau=0}^{\infty} b_{3,\tau} \eta_{3,t-\tau} \end{aligned}$$

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[12] Two technical remarks should be made here. First, time dummy variables are added to all equations in the empirical work. These ought to pick up any linearly deterministic component in the processes generating $\{a_t\}$, $\{b_t\}$, and $\{g_t\}$ and we therefore ignore such components in what follows. Second, the appendix considers tests of the basic assumptions of the model which do not require covariance stationarity; these assumptions made no difference to the major results.

It is the system in (7), and its autoregressive transformation, which will be investigated in the next sections.[13]

Suppose now that an unexpected, research-related event occurred during the previous time period which increased the market value of the firm by X percent, i.e., $\varepsilon = X$. The returns on holding the firm's equity over that period, will, as a result be X percent above the market rate of return. This same event will also cause changes in the firm's R&D programme and in its patent applications. Current R&D expenditures will go up by $c_{2,0}X$ percent above what would have been expected for them at t-1 (past ε's can be estimated from past R's; see the autoregressive representation of this system in the next section), while expected R&D expenditures τ periods ahead will go up by $c_{2,\tau}$ percent. Similarly patent applications τ periods ahead will go up by $c_{3,\tau}$ percent. A realization of η_1 equal to, say, X is noise in the sense that it never (either currently or in the future) affects p or r; while a realization of $\eta_3 = X$ will never affect either research expenditures or the value of the firm and in this sense can be interpreted as a change in the propensity to patent given the history of the output of the firm's R&D laboratories.

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[13] Several points should be noted here. First, for purposes of interpretation one should keep in mind that r_t and p_t refer to R&D expenditures and patent applications in the coming year while q_t refers to the stock market rate of return over the previous period. This is a result of the assumption that decisions on r and p are made at the beginning of the year and that assumption was supported well by the data (see the next section). Second, there are at least two approximations of the $H(\cdot)$ equation which lead to the same moving average representation of the model:

$H(\cdot) = A_t \sum_{\tau=0}^{\infty} w_\tau R_{t-\tau}^{\beta}$ and $H(\cdot) = A_t \prod_{\tau=0}^{\infty} R_{t-\tau}^{\mu_\tau}$. The reason one cannot distinguish between them is that equation (7) cannot tell us whether any persistence in the effect of past ε's on current r is a result of the complementarity of past r's or of a persistence of the effect of past ε's on A. Finally, the equation for q_t uses the approximation $e^{\varepsilon_t+\eta_{1,t}}-1 \approx \varepsilon_t+\eta_{1,t}$.

II. *Test Statistics and Parameter Estimates*

The model of the last section is refutable in the sense that it places testable restrictions on the stochastic process generating $y_t' = (q_t, r_t, p_t)$. Before going on to the parameter estimates, it will be useful to consider those restrictions and their interpretation in terms of the assumptions of the model.

To begin, note that if the model of the last section is appropriate, then the moving average representation of the process generating y(t) can be written as,

$$y_t = c(L)\varepsilon_t + B(L)\eta_t, \tag{8}$$

where now $(\varepsilon_t, \eta_{1,t}\ \eta_{2,t}\ \eta_{3,t}) = (\varepsilon_t, \eta_t')$ are four mutually uncorrelated white-noise deviates, c(L) is a column vector, and B(L) is a diagonal matrix of polynomials in L. Models of this form have been called dynamic factor analysis models (Geweke, 1977b) or unobservable index models (Sargent and Sims, 1977). The name is a result of the fact that in (8) there is a single stochastic process built up from the ε, that accounts for all the observed intercorrelations between the components of y_t. That is, each of the components of η affect one, and only one, of the y elements (B(L) is diagonal). In particular, η_3, which is interpreted as the process generating differences in the protensity to patent, is noise in the economic sense that it never affects either the firm's value or its R&D programme. With three observable deviates the single factor model is overidentified and thus can be tested. The model [equation (7)] also implies that the uppermost polynomials in both c(L) and B(L) equal unity. This is Fama's (1970) semi-strong test of market efficiency. Since the history of ε and η can be estimated from the history of y, the implication we are testing here is that movements in the rate of return on firms' stocks cannot be predicted from available information: or that realizations of q_t represent the effect of events which were not known at the beginning of the period. Finally, note that if the model is a good approximation to the data then the middle polynomial in B(L) is irrelevant ($\eta_{2,t} \equiv 0$). Thus all the variance in r_t can be accounted for by the factors affecting p and q; that is, there is no measurement error in r. A model allowing for measurement error in r is discussed briefly in the appendix. There it is shown that one of the more striking implications of the empirical findings is that there is no need to allow for such a measurement error [$var(\eta_{2,t})/var(\varepsilon_t) \simeq 0$].[14]

[14]This contrasts sharply with other studies which relate current and past research expenditures to *indirect* measures of *current* benefits (such as productivity). See the review by Griliches, 1979, and, in particular, the error-variance ratios estimated by Pakes and Schankerman, 1980.

Inverting the matrix polynomial which defines the moving-average representation of the y_t process, one derives the autoregressive representation of this process as,

(9) $$y_t = D(L)y_{t-1} + D_0\zeta_t \quad .$$

The restrictions of the model in terms of this autoregressive representation are

$$D(L) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & d_{22}(L) & 0 \\ 0 & d_{32}(L) & d_{33}(L) \end{bmatrix}$$

and

$$D_0 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & c_{2,0} & 0 \\ 0 & c_{3,0} & 1 \end{bmatrix}$$

where $\zeta_t' = (\eta_{1,t}, \varepsilon_t, \eta_{3,t})$, $[1-d_{22}(L)]^{-1} = c_2(L) \cdot 1/c_{2,0}$,

$d_{32}(L) = \{[1-d_{33}(L)]c_3(L) - c_{3,0}\}c_2(L)^{-1}$, and $[1-d_{33}(L)]^{-1} = b_3(L)$.

Note that the restrictions the model places on $D(L)$ are all exclusion restrictions and thus particularly simple to test. On the other hand, there is a nonlinear restriction on the covariance matrix of disturbances from the projection of y_t on its past values; that is $D_0 \Lambda D_0'$ $[\Lambda = E(\zeta_t \zeta_t')]$ contains only five free parameters, while its unrestricted form has six of them. There is, however, a recursive translation of (9) in which this nonlinear constraint becomes another exclusion restriction, and which (by its recursive nature) permits equation-by-equation estimation techniques. Due to the simplicity of this recursive form (which has q_t as a function of the history of y_t, r_t as a function of q_t and the history of y_t, and p_t as a function of q_t, r_t and the history of y_t) we concentrate on estimating it in what follows.

The data used here are the successful patent applications, the R & D expenditures, and the annual rate of return on the stocks of 120 firms over eight years. The sample of firms and the method of constructing the patent variable are discussed in Pakes and Griliches (1980a). The observations on the one-period rate of return are the same as those on the Crisp Master File (1975).

The test of market efficiency in the recursive form is the test of whether q can be predicted by past values of itself, r, or p. That is, if the market is efficient there should be no simple (in our case linear) trading rule based on the history of y_t which allows one to make excess returns on the stock market.

Table 1 presents test statistics for this hypothesis. Column (1) shows that it is reasonable to assume that q_t is uncorrelated with past values of itself; column (2), that it is uncorrelated with past values of

Table 1: Tests of the Unpredictability of q_t*: Test Statistics for Joint Significance*[a]

Four lagged values of		Included in the equation		
		q (1)	r,p (2)	q,r,p (3)
q	F^4	0.11[b]	n.i.	0.44[b]
r	F^4	n.i.	1.82[b]	2.00[b]
p	F^4	n.i.	0.40[b]	0.32[b]
r,p	F^8	n.r.	1.49[c]	1.56[c]
r,p,q	F^{12}	n.r.	n.r.	1.09[d]

a/ There are 480 observations (120 firms over four years). Time dummies are included in all equations. 'Not included' and 'not relevant' are denoted by n.i. and n.r.

b/ Critical values are 2.39 and 3.36 at 5 and 1 percent respectively.

c/ Critical values are 1.96 and 2.55 at 5 and 1 percent respectively.

d/ Critical values are 1.78 and 2.23 at 5 and 1 percent respectively.

r or p; and column (3) that it is uncorrelated with past value of itself, r, or p. Thus rates of return do seem to represent unpredictable movements in the value of the firm, or at least movements that cannot be predicted with the variables in our data set.

To obtain the recursive form of the r_t equation, note that ε_t can written as

$$(10) \qquad \varepsilon_t = \theta q_t + v_t \quad ,$$

where $\theta = \sigma_\varepsilon^2 / \sigma_q^2$, that is, θ is the signal-to-total-variance ratio in q, and $v_t = (1 - \theta)\varepsilon_t - \theta\eta_{1,t}$, from which it follows that v_t is uncorrelated with q_t and with past values of all variables. Substituting (10) into the equation for r_t in (9) one obtains

$$(11) \qquad r_t = c_{2,0}\theta q_t + d_{22}(L)r_{t-1} + c_{2,0}v_t \quad .$$

Note that the variance of the disturbances in this equation is $\sigma_q^2 c_{2,0}^2 (1-\theta)\theta$ so that (together with the first coefficient and σ_q^2) it can be used to identify θ , and therefore $c_{2,0}$. The model predicts then that in a regression of r_t on lagged r, current and lagged q, and lagged p, all the coefficients but those on current q and lagged r should be close to zero.

The recursive form of the p_t equation can be obtained either by manipulating (9) or directly from (7). Multiplying the latter equation through by $b_3(L)^{-1} = 1 - d_{33}(L)$ and making the substitution, $\varepsilon_t = c_2(L)^{-1}r_t$,

(12) $$p_t = c_2(L)^{-1} [1 - d_{33}(L)]r_t + d_{33}(L)p_{t-1} + \eta_{3,t} \quad .$$

Here the model implies that in a regression of p on current q and r and on lagged values of all variables, all the q coefficients should be close to zero.

Table 2 presents the results. The unrestricted autoregressive forms of these equations have been presented for comparison, while the relevant test statistics are presented at the bottom of the table. Beginning with the R & D equation [column (1)] one finds two rather striking implications of the estimates. First, the events leading the market to re-evaluate the firm are indeed highly and positively correlated with the events leading the firm to change its R & D policy from what would have been predicted given the firm's observable history (i.e., the history of y_t). There is really no doubt on this point as the coefficient of q_t is large and estimated with great precision. Equally striking is the fact that we can be quite sure that each of the coefficients of the lagged p variables in this equation are very close to zero (once again all of the estimates are near zero and their standard errors are small, see also test T_2 of this column). Thus once we account for the influence of past r and past q (or just past r, see appendix) the additional information in movements in past p is information which never affects R & D expenditures. This is confirmation of our interpretation of the $\eta_{3,t}$ process as differences in the propensity to patent for a given history of the firm's R&D programme, since changes in it do not affect r.

The only implication of the model, then, which is not strongly supported by the estimates of column (1), is the zero restriction on the

Table 2: Test Statistics and Parameter Estimates[a]

	R & D equation (r_t)			Patent equation (p_t)		
	Recur-sive (1)	Auto-regres-sive (2)	Con-strained (3)	Recur-sive (4)	Auto-regres-sive (5)	Con-strained (6)
Coefficient of						
r_t	n.i.	n.i.	n.i.	0.60 0.11	n.i.	0.60 0.11
r_{t-1}	0.89 0.05	0.90 0.05	0.92 0.05	-0.21 0.15	0.34 0.12	-0.21 0.15
r_{t-2}	-0.06 0.07	-0.10 0.07	-0.04 0.07	-0.13 0.17	-0.20 0.17	-0.15 0.16
r_{t-3}	0.21 0.07	0.24 0.08	0.14 0.08	0.00 0.18	0.16 0.18	0.04 0.17
r_{t-4}	-0.03 0.05	-0.02 0.06	-0.03 0.05	-0.13 0.13	-0.14 0.14	-0.15 0.12
p_{t-1}	0.00 0.02	0.00 0.02	n.i.	0.45 0.05	0.45 0.05	0.45 0.05
p_{t-2}	0.03 0.02	0.03 0.02	n.i.	0.30 0.05	0.32 0.05	0.30 0.05
p_{t-3}	-0.05 0.03	-0.04 0.03	n.i.	0.00 0.06	-0.02 0.06	0.00 0.06
p_{t-4}	0.00 0.02	0.00 0.02	n.i.	0.14 0.05	0.14 0.05	0.14 0.05
q_t	0.13 0.02	n.i.	0.13 0.02	0.00 0.06	n.i.	n.i.
q_{t-1}	0.05 0.03	0.05 0.03	n.i.	-0.02 0.07	0.01 0.07	n.i.
q_{t-2}	0.08 0.03	0.08 0.03	n.i.	-0.04 0.07	0.01 0.07	n.i.
q_{t-3}	0.04 0.03	0.05 0.03	n.i.	0.05 0.07	0.08 0.07	n.i.
q_{t-4}	-0.02 0.02	-0.02 0.02	n.i.	-0.01 0.05	-0.02 0.04	n.i.
σ^2	0.035	0.036	0.035	0.203	0.215	0.201
Test statistics[b]						
T_1	2196.52	2205.88		14.09[c]	9.92	
T_2	1.91	1.52		358.75	335.62	
T_3	7.54[c]	3.29		0.21[c]	0.40	

Notes on next page

a/ Small numerals are standard errors. See also note a to Table 1.

b/ T_1, T_2, and T_3 are the observed values of the F-test statistic for the joint significance of, respectively, the R & D variables, the patent variables, and the one-period rates of return. The critical values are 2.39 and 3.36 at 5 and 1 percent respectively, except as specified in note d.

c/ Critical values are 2.23 and 3.06 at 5 and 1 percent respectively.

lagged q coefficients. The relevant test statistic here is T_3 of column (2) which is significant at the 5 but not at the 1 percent level. Additional results discussed below indicate that we observe marginally significant lagged q coefficients because the assumption that the process generating r_t has a low-order autoregressive representation is questionable. Since this is a technical problem, and since correcting for it does not change any of the basic implications of the parameter estimates, we shall ignore it below, and accept the column (3) estimates for the r_t equation.[15,16]

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[15]The series available for each of our firms were longer for r_t than for p_t or q_t (see Pakes and Griliches, 1980b). When nine lagged values of r were entered into the r_t equation the eighth and ninth lags were still marginally significant (which indicates that the r_t process is close to not having a low-order autoregressive representation). If one generalizes and assumes that the r_t process does have a low-order autoregressive moving-average representation, one would expect lagged q to enter the r_t equation. A direct set of estimates for the autoregressive moving-average model of the r_t equation can be provided by using current and lagged q as an error-ridden indicator of the moving-average component of that process and then using the excluded earlier values of r as instruments on the included values of that variable. When this was done we found that the implications of the parameter estimates were essentially the same as those of the estimates in column (3). Note that the appendix provides a set of tests of the basic assumptions of the model which does not require the process generating y_t to have an auto-regressive representation, or for that matter to be stationary. These results also support our assumptions.

[16]Another interesting detail can be gleaned from the estimates. They imply that the process generating r_t is quite close to a random walk, though the random walk and the weaker martingale hypothesis can both be rejected at conventional significance levels. Thus firms which experience events which cause them to increase (decrease) their R&D expenditures are not likely to revert to their former level of expenditure for some time (see also the discussion in the next section) and a reasonable predictor for r_{t+1} is simply r_t.

Moving to the patent equation is clear that current and past changes in R&D [past changes only in column (5)] have a significant effect on changes in current patent applications (test T_1). Though this was, perhaps to be expected (see Pakes and Griliches, 1980b) what is more surprising is that once the effect of R&D expenditures on patent applications is taken care of, other factors which lead to a change in the market's evaluation of the firm are not correlated with patent applications (test T_3). In particular, all the q coefficients in the p equation are near zero and this leads one to accept the interpretation of the error in the regression of p_t on the $r_{t-\tau}$ as differences in the propensity to patent, given the market value of the output of the firm's current and past research expenditures.

Since the results support our interpretation, we now go on to explore the implications of the parameter estimates in greater detail.

III. SOME IMPLICATIONS OF THE PARAMETER ESTIMATES

We begin with the implications of our estimates for the interpretation of movements in q and r. Noting that $\sigma_q^2 = 0.10$ and using the parameters of the R&D equation one finds a $\theta(\sigma_\varepsilon^2/\sigma_q^2)$ of 0.05. That is, about 5 percent of the within-period variance in the rate of return is caused by events which also cause changes in both R&D expenditures and patent applications.[17] A θ of 0.05 implies that $c_{2\rho}(=\partial r_t/\partial \varepsilon_t) = 2.60$. This implies that a 1 percent increase in R&D expenditures above what would have been predicted given past information is associated with events that have caused an increase in the value of the firm of 0.39 percent. Evaluating derivatives at the means of all variables one finds that a $100 unexpected increase in R&D is associated with research and patent-related

[17] The firms in our sample are all rather large (the average value of their common shares is $1,514 million) and diversified and do a fair amount of research.

events that have increased the value of the firm by \$1,870.[18] Recall that our results implied that there was no need to allow for measurement error in R & D (see the discussion in Section II and the appendix) so that all unpredictable changes in R & D have this interpretation. The unexpected increase in patents is $c_{3,0}\varepsilon_t + \eta_{3,t}$, where, from our estimates $c_{3,0} = 1.56$. Thus events which lead to a unit increase in ε result in a 1.56 percent increase in successful patent applications. Much of the variance in the unexpected change in the patent variable (about 94 percent of it) is noise, so that one finds that a 1 percent increase in patents will, again on average, reflect only a 0.044 percent increase in the market value of the firm; alternatively, one additional patent indicates that events have occurred which increase the firm's market value by \$810,000.

Figure 1 presents the estimates of the distributed lags from ε to r [labelled $c_2(\tau)$] and from ε to p [$c_3(\tau)$], while Figure 2 presents the distributed lags from r to p [$\gamma(L)$] and from η_3 to p [$b_3(L)$]. Figure 1 makes it clear that the events which change the market value of a firm's research programme have a persistent effect on both patents and R & D expenditures. As a result interfirm differences in R & D expenditures are quite stable over time, and if we are seeking their causes we should look for factors in the firm's environment whose effects are likely to persist. On the other hand, the small changes that do occur in the firm's R & D expenditures are almost entirely determined by recent events.

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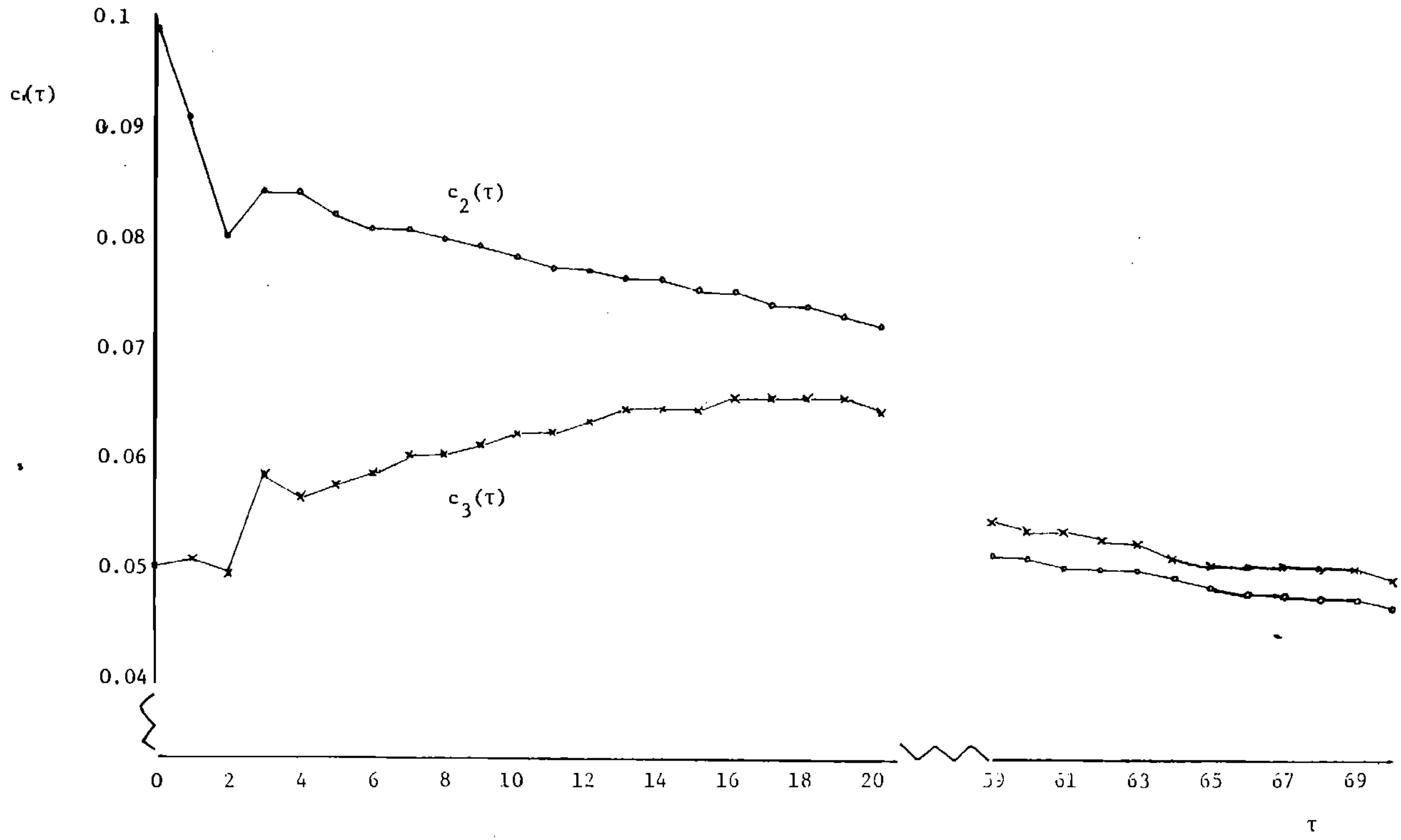
[18] The means reported here are sample means, i.e., they are calculated over all observations (N firms and T years), and thus require the use of price deflators. The consumer price index was used to deflate stock-market values while the R&D deflator discussed in Pakes and Griliches (1980a) was used for R&D expenditures. The base year for these deflators is 1972, so all dollar figures in the text are in 1972 dollars.

Figure 1: The Distributed Lag From ε to r [$c_2(\tau)$] and From ε to p [$c_3(\tau)$].

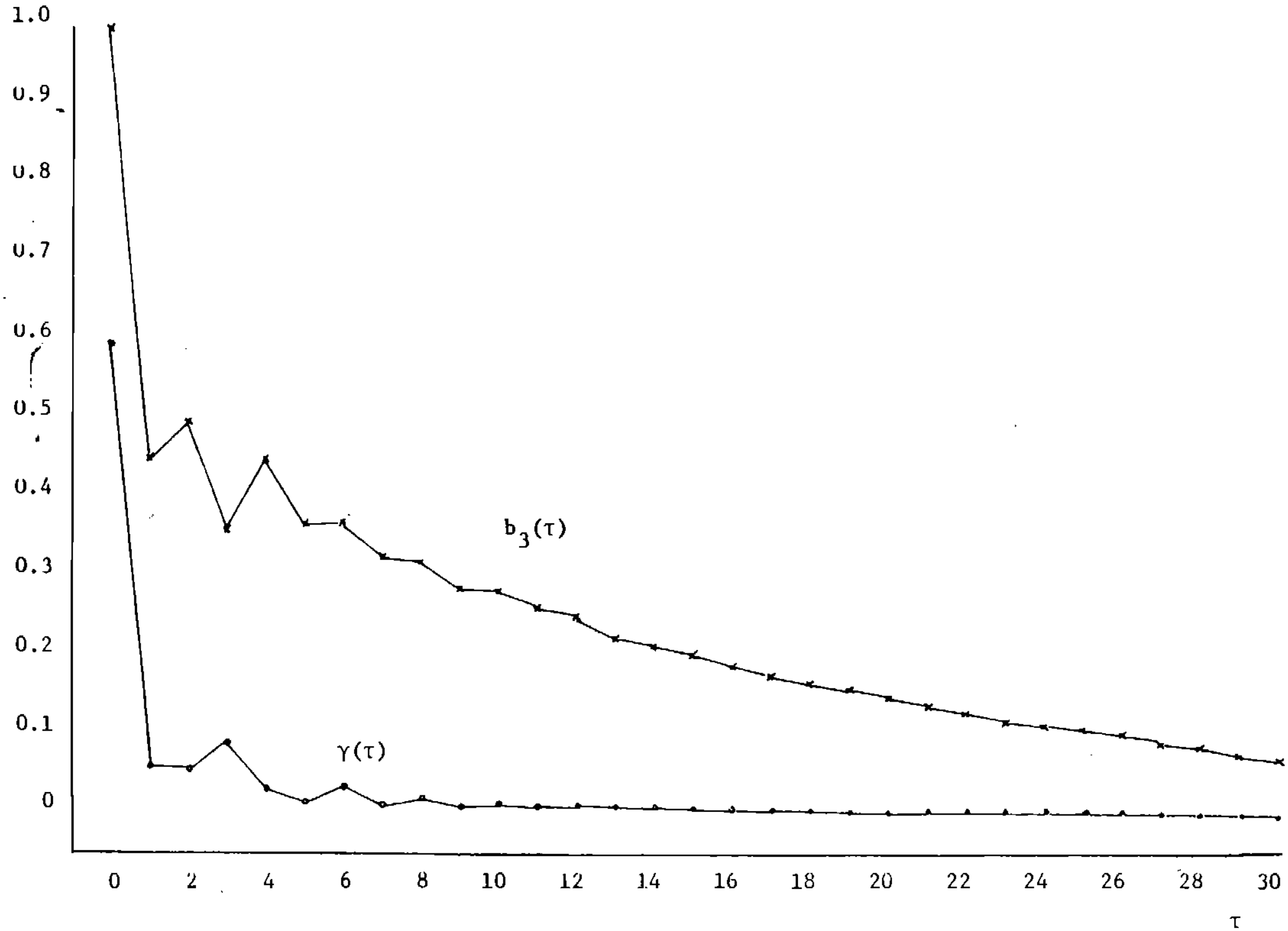


Thus events that occurred over three years earlier will have essentially the same effect on r_t as on r_{t-1} and cannot cause differences between them. The estimates of $c_3(\tau)$ is similar to that of $c_2(\tau)$ except for the fact that the effect of the ε on p tends to increase before declining, giving the impression that p reacts to the ε a little more slowly than r does. Thus moving to Figure 2, one sees that patent applications follow the factors determining the productivity of current R & D expenditures (and hence R & D demand) quite closely.

The sum of the coefficients in the distributed lag from r to p is 1.18, implying that the events leading to a 1 percent increase in R & D expenditures will, eventually, lead to a 1.18 percent increase in patented innovations. About 50 percent of these patents will be applied for in the same year as the R & D expenditures are incurred, while 70 percent will be applied for within three years. In fact, if from $c_2(\tau)$ one gets the impression that events which cause unexpected changes in R&D expenditures start a chain reaction which leads to more R&D expenditures far into the future, then $\gamma(\tau)$ seems to be describing a situation where firms patent around the links of this chain almost as quickly as they are completed. There is also a long slim tail of the distributed lag from r to p which probably represents the effect of the basic research done in the past on current patented innovations.[19]

[19]The results on the form of the lag between R&D and patents are quite similar to what Zvi Griliches and I, in joint preliminary work, have suggested for that lag structure, see Pakes and Griliches (1980b). Here again the reader should be cautioned not to interpret this lag structure as representing a production-type relationship between past R&D and patentable output (see Section I).

Figure 2: The Distributed Lag From r to p [$\gamma(\tau)$] and From η_3 to p [$b_3(\tau)$].



The estimates of $g(\tau)$ indicate that interfirm differences in the propensity to patent are not as stable over time as one might have expected. Thus, recalling that g_t is the propensity to patent $(g_t = \sum_{\tau=0}^{\infty} b_{3,\tau}\eta_{3,t-\tau})$ we find that the correlation of g_t and $g_{t-\tau}$ is only about 0.75 for $\tau = 1$, going down to around 0.6 for $\tau = 2$, 3, and 4, and decaying at a fairly constant rate of 0.9 thereafter.

A question of general interest is: how good a measure of inventive output can be derived from the recently computerized U.S. Patent Office data base? Here we are associating inventive output with those events that cause differences in the ε's, that is, with those events that are related to R&D activity and cause changes in the stock market value of the firm. The data suggest that some differences in patent applications approximate differences in inventive output quite closely, while others do not.

Consider constructing a cross section of patent applications by firm in order to study the causes of interfirm differences in inventive output (or their effects). The estimates indicate that 76 percent of the interfirm variance in patents is caused by the ε_t while the rest is noise. If one were to ask what proportion of the variance in p_t is caused by the events determining current research demand the answer would be a little, but not much, less. To see this we consider the projection of p_t onto r_t, i.e., $p_t = \phi r_t + g'_t$ where $\text{cov}(g'_t, r_t) = 0$.[20] Appropriate calculations indicate that $\phi = 1.12$ while $\text{var}(\phi r_t)/\text{var}(p_t) = 0.74$. A 1 percent difference in R_t will, therefore, lead to a 1.12 percent difference in patent applications, while about 74 percent of the interfirm variance in p_t can be attributed to interfirm variance in r_t. Inverting these cal-

[20] Here $\phi = \Sigma c_{rr}(\tau)\gamma(\tau)/c_{rr}(0)$, where $c_{rr}(\tau) = \text{cov}(r_t, r_{t-\tau})$ and $\gamma(\tau)$ is the τ-th lag coefficient in the distributed lag from r to p.

culations one finds that, on the average, a 1 percent difference in current patent applications is associated with factors that have led to a 0.66 percent difference in R_t;[21] or (on evaluating derivatives at the sample means of all variables) a difference of one patent is associated with events that, on average, lead to a 304 thousand dollar difference in current R&D activity.

Unfortunately intrafirm differences in patent applications do not seem to be as good an indicator of intrafirm differences in inventive output as interfirm differences. The proportion of the variance in $p_{it} - p_{it-1}$ caused by the ε is about 8 percent, with 45 percent of this 8 percent being caused by research-related and patent-related events that changed the market value of the firm in the given period (by ε_t). These ratios do, however, increase significantly when one takes intrafirm differences in patent applications that are farther apart. The proportion of the variance in $p_{it} - p_{it-5}$ caused by the ε is 15 percent, with over 75 percent being caused by events that occurred during the 5-year period. For ten-year differences the figures move to over 20 and 85 percent respectively. Thus if one were to use intrafirm differences in patent applications to study the effect of changes in a firm's inventive output on, say, its investment policy or its share of a given market, then one ought, probably, to stick to longer-term changes in all variables.[22]

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[21]That is $r_t = \phi' p_t + g_t''$, where $\text{cov}(g_t'', p_t) = 0$, and $\phi' = .66$.

[22]This difference between the interfirm and intrafirm results is a function of the form of the lag structures which the data prefers, that is of the relative stability of $\gamma(L)r_t$ when compared with the instability of g_t.

IV. Concluding Remarks

Empirical work on either the causes or the effects of inventive activity has had difficulty in finding variables which are able to indicate when and where changes in inventive output have occurred. The recent computerization of the U.S Patent Office's data base may provide some help in this context but there is the problem that *a priori* one does not know the relationship between successful patent applications and any economically meaningful measure of this output. To provide a partial answer to this question this paper investigates the relationship between successful patent applications, a measure of the inputs into the inventive process (R & D expenditures) and a variable which provides a measure of, among other diverse factors, the economic value of the output from this process (movements in the stock-market value of the firm's equity). The model used to interpret these relationships was relatively simple. The firm was assumed to choose its R & D expenditures to maximize the expected discounted value of the net cash flows from its activities; the market was assumed to evaluate this expectation (subject to error) on the basis of information available at different dates; and patents were assumed to be an error-ridden indicator of the market value of the events that lead to changes in the firm's research activities. The error process in the patent equation was assumed to be noise in the economic sense that it never affected either the value of the firm or its R & D expenditures. In the model, then, the complexities and the randomness inherent in both the inventive process and in the patenting decision are left to be captured by stochastic processes.

A distinct advantage of considering the model in detail is that it places refutable restrictions on the process generating p, r, and q, which in turn allow one to test whether the interpretation we are giving to the parameter estimates is consistent with the observed behaviour of the data. Here the empirical results supported our interpretation, quite strongly, as our parameters were estimated with much more precision than is characteristic of most studies of technological change, and, as the appendix shows, they are quite robust to the simplifying assumptions used in the text. There are two empirical implications of these test results which, though discussed in detail only in the appendix, are likely to be of some interest to future research in this area. First, there seems to be very little measurement error in R&D. Second, though in principle the availability of data on both p and r ought to allow one to investigate events that have a direct effect on patentable output separately from those events which only affect patents through the R&D expenditures they induce, to implement this research strategy one is likely to require an additional variable which discrimintes between these two types of events more sharply than R&D does (perhaps investment expenditures).

Our major interest is in the relationship between patent application's and the market's evaluation of the output of the firm's research activities. An understanding of this relationship would allow us to use the patent data to study the causes and effects of a firm's formal inventive endeavours. The precise parameter estimates are presented in the last section but the general character of the results can be summarized quite succinctly. Interfirm differences in patent applications seem to follow interfirm differences in the market value of a firm's research output quite closely, but intertemporal differences in a firm's patent applications are largely a result of intertemporal differences in its propensity to patent. This last statement must be modified when one considers longer-term differences in the patents applied for by a firm, since a larger portion of their variance is caused by events which lead the market to re-evaluate the firm's inventive output during the period.

Appendix: Generalizations and Robustness Tests

In this appendix I consider two generalizations of the model presented in the text and show why the data indicate they are not necessary. The first is to allow for measurement error in r (by measurement error I mean a stochastic process which affects r but does not affect p or q). The second is to allow for two dynamic factors. The discussion of the relevance of these generalizations will focus attention on two of the empirical findings which supported the model presented in the paper.[1] These findings are that the bivariate process generating $\{r_t, p_t\}$ exhibits a Granger ordering from r to p, i.e., $r \Rightarrow p$;[2] and that the covariance matrix formed by the disturbances from the projection of (q_t, r_t, p_t) on their past values has the form given in equation (9) (see p.16). The special form of this covariance matrix can be given an intuitive explanation in terms of the model used in the text. The innovation (or disturbance) in r should reflect unexpected

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[1]There are actually three empirical findings which underlie the interpretation given in the text to movements in (q, r, p). The third is that movements in q cannot be predicted by a linear function of the observable variables which describe the firm's history. This point, however, has been investigated fairly thoroughly elsewhere and has received substantial support in many different contexts (see Fama, 1970, for a review of the literature).

[2]$r \Rightarrow p$ iff, in a regression of r on lagged r and lagged p, the lagged p do not help in predicting current r, see Granger (1969). Our finding is that $r \Rightarrow p$ but $p \not\Rightarrow r$.

events that change the expected discounted value of the firm's R&D activity and should therefore be highly correlated with q; while that part of the innovation in p which cannot be predicted from the innovation in the r series is noise in the economic sense that it is not related to q. Thus, if one projects q on the innovations in both r and p, the coefficient of the innovation in r should be positive and significant while that of the innovation in p should be zero. This zero restriction is equivalent to the restriction on the covariance matrix in equation (9).

The appendix concludes with a set of tests showing the robustness of these two empirical results to some of the simplifying assumptions used in the text.

Interest in the two factor model stems from replacing the patent equation given in the text with a more structural model of the patenting process [see equation (5) and the discussion preceding it]. To build such a model we would introduce an additional sequence of random variables, $\{F_t\}$, which determined the productivity of current and past R&D expenditures in producing valuable output, and retain $\{G_t\}$ as the sequence determining the amount of patents applied for given the output produced. Thus, one would replace equation (5) in the text by $P_t = P(F_t, R_t, R_{t-1}, \ldots, G_t)$. Clearly, current and past values of F would be among the determinants of R&D demand so that if one were to use this patenting equation, the equation which sets R_t [equation (3)] would have to be replaced by $\max_{R_t} H(R_t, R_{t-1}, \ldots, F_t, F_{t-1}, \ldots, S_t) - R_t$. S_t is introduced into this equation to allow for factors which have an effect on R&D demand but have no independent effect on patentable output; that is, for factors which only affect patents through the change in R&D expenditures they induce. To allow for measurement error in R&D one simply reinterprets R_t to be the (latent) optimal value of R&D expenditures derived from the model and introduces the sequence $\{W_t\}$ to represent measurement error in R&D. That is, observed R&D expenditures, say R_t^0, is given by, $R_t^0 = W_t R_t$. Assuming $w_t = \log W_t$, $s_t = \log S_t$, and $f_t = \log F_t$, evolve as three stationary processes and solving for the moving average representation of this model (see page 13) we have:

$$q_t = \varepsilon_{1,t} + \varepsilon_{2,t} + \eta_{1,t}$$

$$\text{(A1)} \qquad r_t = \sum_{\tau=0}^{\infty} c_{2,1,\tau}\varepsilon_{1,t-\tau} + \sum_{\tau=0}^{\infty} c_{2,2,\tau}\varepsilon_{2,t-\tau} + \sum_{\tau=0}^{\infty} b_{2,\tau}\eta_{2,t-\tau}$$

$$p_t = \sum_{\tau=0}^{\infty} c_{3,1,\tau}\varepsilon_{1,t-\tau} + \sum_{\tau=0}^{\infty} c_{3,2,\tau}\varepsilon_{2,t-\tau} + \sum_{\tau=0}^{\infty} b_{3,\tau}\eta_{3,t-\tau}$$

where $b_{2,0} = b_{3,0} = 1$, $r_t = \log R_t^0$, $w_t = \sum_{\tau=0}^{\infty} b_{2,\tau}\eta_{2,t-\tau}$, $s_t = \sum_{\tau=0}^{\infty} s_\tau \varepsilon_{1,t-\tau}$, $f_t = \sum_{\tau=0}^{\infty} f_\tau \varepsilon_{2,t-\tau}$, and the rest of the variables are as defined in the text. It is assumed that $\varepsilon_{1,t}$, $\varepsilon_{2,t}$, $\eta_{1,t}$, $\eta_{2.t}$, and $\eta_{3,t}$ represent realizations of mutually uncorrelated white noise processes, and that all sequences of the form $\{\alpha_j\}$ (here, and in the discussion below) are square summable.

The question which arises then is under which conditions can the model in (A1) generate the empirical results referred to above. First, consider the conditions required of (A1) in order for there to be a Granger ordering from r to p. It is easiest to investigate this issues by using a result due to Sims (1972, Theorem 2); $r \Rightarrow p$ is equivalent to there being a one-sided distributed lag from r to p. Let the distributed lag coefficients from a two-sided regression of p or r be given by the sequence $\{h_j\}_{j=-\infty}^{+\infty}$ and let their z-transform be $h(z) = \sum_{j=-\infty}^{+\infty} h_j z^j$. Then h(z) can be found from the appropriate covariance-generating functions as

$$h(z) = g_{p,r}(z) g_{r,r,}(z)^{-1}$$

(A2)
$$= [c_{3,1}(z)c_{2,1}(z^{-1}) + c_{3,2}(z)c_{2,2}(z^{-1})k_2]$$
$$\cdot [c_{2,1}(z)c_{2,1}(z^{-1}) + c_{2,2}(z)c_{2,2}(z^{-1})k_2 + b_2(z)b_2(z^{-1})k_\eta]^{-1} \quad ,$$

where $g_{x,y}(z) = \sum_{j=-\infty}^{+\infty} E(x_t, y_{t-j})z^j$; that is $g_{x,y}(z)$ is the cross-variance generating function of x and y, $k_2 = Var(\varepsilon_{2,t})/Var(\varepsilon_{1,t})$, $k_\eta = Var(\eta_{2,t})/Var(\varepsilon_{1,t})$, and $c_{1,1}(z) = \sum_{\tau=0}^{\infty} c_{1,1,j} z^j$ (the z-transform of the sequence $\{c_{1,1,j}\}$), and so on for other sequences.

With this notation the Sim's resultis r =>p is equivalent to h(z) being one-sided in non-negative powers of z ($h_j = 0$ for all $j < 0$). Note that if $k_\eta = k_2 = 0$, one returns to the model used in the text, and the proof of the one-sidedness of h(z) in this model can be obtained by multiplying (A2) by $c_{2,1}(z)^{-1}c_{2,1}(z) = 1$, since then, $h(z) = c_{3,1}(z)c_{2,1}(z)^{-1}[c_{2,1}(z)c_{2,1}(z^{-1})]$ $[c_{2,1}(z)c_{2,1}(z^{-1})]^{-1} = c_{3,1}(z)c_{2,1}(z)^{-1}$, which involves only non-negative powers of z. Our question is, however, can h(z) be one-sided if either k_η or $k_2 \neq 0$? The case where $k_2 = 0$ but $k_\eta \neq 0$ is the dynamic factor analysis model with a single dynamic factor and error processes affecting each variable. The question of whether there can be a Granger ordering in this model has been investigated, in a slightly different context, by Sims (1977) and Geweke (1977b). Clearly for $h(z)$ to be one-sided in this case we require a relationship between $c_{1,1}(z)$ and $b_{1,1}(z)$. Though I know of no necessary condition that this relationship must satisfy, the condition given by both Sims and Geweke which insures that $h(z)$ is one-sided with $k_\eta \neq 0$ is $c_{1,1}(z) = \lambda b_{1,1}(z)$ for some scalar, λ [that this is sufficient can be verified by direct substitution into (A2)]. To see that this condition will not satisfy the covariance restriction on the disturbances from (9) we need only note the following fact. The coefficients from the projection of any one error-ridden indicator of $\varepsilon(q = \varepsilon + v)$ on any two other error-ridden indicators of ε (the innovations in r and in p) will, if all errors are mutually uncorrelated, both be nonzero; unless, of course, one of the latter indicators (in our case the innovation in r) contains no measurement error ($k_\eta = 0$), in which case it equals a scalar times ε.

Now consider the case where $k_\eta = 0$ but $k_2 \neq 0$. This is a model with two dynamic factors but no measurement error in r. For there to be a one-sided $h(z)$ in this model one requires a relationship between $c_{2,1}(z)$ and $c_{2,2}(z)$, which is, perhaps, a little more likely since both these z-transforms summarize the reaction of the same decision variable (R&D expenditures) to events which cause unexpected changes in the firm's maximand (its stock market value). In particular, consider the special case of the proportionality constraint where $c_{2,1}(z) = c_{1,1}(z)$. Not only will it generate a one-sided $h(z)$, but also, in this special case, the innovation in r $[c_{2,1,0}(\varepsilon_{1,t}+\varepsilon_{2,t})]$ is an exact multiple of the stock market value of all the events which lead to changes in either R&D or patenting in a given year; thus, the innovation in p will have no effect on q, which is independent of the innovation in r. In this case, then, we will not be able to distinguish between the two-factor and the one-factor models.[3] That is in order for one to distinguish between $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ the time pattern of the reaction of R&D to the two different kinds of events must be different. Alternatively, if one were considering a research strategy based on a two factor model, a fourth variable which does react differently to ε_1 than to ε_2 (e.g., investment expenditures) could be added to the model.

There are two other sets of conditions which I am aware of which can generate a one-sided $h(z)$ and the covariance matrix of disturbances given by equation (9); but both of these imply further restrictions which the data will not accept. One set is $\sigma_\eta^2 = 0$, $c_{2,1}(z) = \kappa c_{3,1}(z)$, and $c_{2,2}(z) = \kappa c_{3,2}(z)$ for the same scalar, κ. The other is, $c_{2,1,\tau} = c_{2,2,\tau} = 0$ for $\tau > \ell_1$, $c_{3,1,\tau} = c_{3,2,\tau} = 0$ for $\tau < \ell_1+\ell_2+1$, and r_t has a univariate autoregressive representation of order ℓ_2.[4] In the first case $h(z) = \kappa$, while in the second p_t is uncorrelated with $q_{t-\tau}$ and the innovation in $r_{t-\tau}$ for $\tau < \ell_1+\ell_2+1$. None of these constraints are accepted by the data.

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[3]If $c_{2,1}(z) = c_{2,2}(z)$, then there is an autoregressive representation of the two factor model which is identical to the autoregressive representation of the (q_t, r_t, p_t) process provided in equation (9).

[4]The proof of both of these assertions is obtained by direct substituion. Note that in the second case if we let $\phi_1(z) = c_{3,1}(z)c_{2,1}(z^{-1})+c_{3,2}(z)c_{2,2}(z^{-1})k_2$ and $\phi_2(z) = [c_{2,1}(z)c_{2,1}(z^{-1}) + c_{2,2}(z)c_{2,2}(z^{-1})k_2 + b_2(z)b_2(z^{-1})k_\eta]^{-1}$ then $\phi_{1,\tau} = 0$ for $\tau < \ell_2 + 1$ and $\phi_{2,\tau} = 0$ for $|\tau| > \ell_2$, which proves that $h(z)$ is one-sided. There will be, in this case, a particular value of k_2 which satisfies the covariance restriction.

Table A-1 summarizes the additional tests I have performed to ensure the robustness of the $r \Rightarrow p$ finding. Lines 1 present the test statistics for the joint significance of four lagged p and four lagged r in the p and r equations (here, as in lines 2 and 3, all estimated parameters are quite similar to those presented in Table 2). Lines 2 present similar test results after first-differencing both the p and the r series. These lines were meant primarily to test whether the result that $r \Rightarrow p$ was an artifact of our sample selection criteria (our sample was necessarily limited to large patentees; see Pakes and Griliches, 1980a for more details on the sample). Since the selection probabilities are constant over the sample period, any sample selection bias ought to be nearly eliminated in the within dimension. Lines 3 present the appropriate test statistics after weighting each equation by the firm's mean R & D expenditures over the sample period. Here there was a danger of heteroskedasticity in the process generating $\{r_t, p_t\}$; in particular I was concerned about the possibility that the relationship between p and r might be specially noisy at low levels of R&D expenditures. Since the logarithmic transformation makes the associated deviations of p from its sample mean large in absolute value, it might well induce zero coefficients on lagged p in the r equation.

Line 4 deserves slightly more attention. All results so far presented have assumed that the bivariate process generating $\{r_t, p_t\}$ is jointly covariance stationary, or stationary after some simple transformation of these variables. Statistical tests performed for joint stationarity reject it, though it is interesting to see that stationarity of the univariate

Table A-1: Additional Test Results on the Relationship Between[a/] p and r: Test Statistics for Joint Significance

	Dependent variant	Actual values: four lagged values of r	Actual values: four lagged values of p	Critical values, percent 5	Critical values, percent 1
1. Granger test	p	10.29	345.00	2.40	3.40
	r	2,128.23	1.47	2.40	3.40
2. Granger test on first differences b/	p	5.83	27.89	2.43	3.80
	r	3.26	1.74	2.43	3.80
3. Weighted Granger tests c/	p	7.56	442.00	2.40	3.40
	r	1,303.00	1.50	2.40	3.40
4. Hosoya generalization of Granger condition d/	p	3.37	86.24	1.70	2.10
	r	590.71	1.34	1.70	2.10

a/ The distributions of the test statistics are $F^{4,468}$ in models 1 and 3; $F^{4,352}$ in model 2, and $F^{16,444}$ in model 4.

b/ Granger tests performed on first differences of p and r series.

c/ Each firm is weighted according to its mean R & D expenditures over the sample period and Granger tests performed.

d/ The coefficients in the projection of p and r on their lagged values are not constrained to be the same in different years.

process generating r could be accepted. One could allow for non-stationarity by permitting the parameters of the estimated model to differ from year to year. On investigating this possibility we found that though the yearly differences were statistically significant, their economic implications, as summarized in the text, were not (which explains why the text only discusses the stationary case). Line 4 presents the test results for $r \Rightarrow p$ and $p \not\Rightarrow r$ when all coefficients are allowed to vary between the years of the sample. This is, in fact, the test for Hosoya's (1977) generalization of the Granger condition for the ordering of two variables. Hosoya's condition does not require that the bivariate process be covariance-stationary, linearly indeterministic or have an autoregressive representation; and it is shown to be equivalent to the Sim's (1972) condition without these assumptions.

A similar set of robustness tests was performed on the restriction on the covariance matrix of disturbances from the model's autoregressive form (maximum-likelihood estimation techniques were used). In no case was the test statistic greater than its expected value under the null hypothesis that the restriction was indeed appropriate.

REFERENCES

Arrow, Kenneth J. "Economic Welfare and the Allocation of Resources for Invention." In *The Rate and Direction of Inventive Activity: Economic and Social Factors*. Edited by R.R. Nelson. (National Bureau of Economic Research: Universities-National Bureau Conference Series No. 13.) Princeton: Princeton University Press, 1962, Pp. 609-25.

Comanor, William S., and F.M. Scherer. "Patent Statistics as a Measure of Technical Change," *Journal of Political Economy*, LXXVII (May/June 1969), 392-98.

Crisp Master File. Chicago: Graduate School of Business, University of Chicago, 1975.

Fama, Eugene F. "Efficient Capital Markets: A Review of Theory and Empirical Work," *Journal of Finance*, XXV (May 1970), 383-417.

Geweke, John. "Wage and Price Dynamics in U.S. Manufacturing." In *New Methods in Business Cycle Research: Proceedings from a Conference*. Edited by Christopher A. Sims. Minneapolis: Federal Reserve Bank of Minneapolis, 1977. Pp. 111-56. (1977a)

———. "The Dynamic Factor Analysis of Economic Time-Series Models." In *Latent Variables in Socio-Economic Models*. Edited by D.J. Aigner and A.S. Goldberger. Amsterdam, New York, Oxford: North-Holland, 1977. Pp. 365-83. (1977b)

Granger, C.W.J. "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods," *Econometrica*, XXXVII (July 1969), 424-38.

Griliches, Zvi. "Issues in Assessing the Contribution of Research and Development to Productivity Growth," *Bell Journal of Economics*, X (Spring 1979), 92-116.

Griliches, Zvi. "Research Expenditures and Growth Accounting," in B.R. Williams (ed.), Science and Technology in Economic Growth, London: MacMillan, 1973, pp. 59-95.

———, and J. Schmookler. "Inventing and Maximizing," *American Economic Review*, LIII (September 1963), 725-29.

Hosoya, Yuzo. "On the Granger Condition for Non-Causality," *Econometrica*, XLV (October 1977), 1735-36.

Lucas, Robert E. Jr, and Edward C. Prescott. "Investment Under Uncertainty," *Econometrica*, XXXIX (September 1971), 659-81.

Marschak, Jacob, and W.H. Andrews, Jr. "Random Simultaneous Equations and the Theory of Production," *Econometrica*, XII (July-October 1944), 143-205.

Meese, Richard. "Dynamic Factor Demand Schedules for Labor and Capital Under Rational Expectations," *Journal of Econometrics*, XIV (1980), 141-58.

Nelson, Richard R. (ed.). *The Rate and Direction of Inventive Activity: Economic and Social Factors*. (National Bureau of Economic Research: Universities-National Bureau Conference Series No. 13.) Princeton: Princeton University Press, 1962.

Pakes, Ariél and Zvi Griliches. "Patents and R&D at the Firm Level: A First Look," National Bureau of Economic Research Working Paper, No. 561, 1980a.

______________. "Patents and R&D at the Firm Level: A First Report," Economics Letters, 5, 377-381, 1980b.

Pakes, Ariél and M.A. Schankerman, " An Exploration into the Determinants of Research Intensity," National Bureau of Economic Research Working Paper, No. 438, 1980.

Rosenberg, Nathan. "Science, Invention and Economic Growth," Economic Journal, LXXXIV, March 1974, 90-108.

Sargent, Thomas J. "Estimation of Dynamic Labor Demand Schedules under Rational Expectations," *Journal of Political Economy*, LXXXVI (December 1978), 1009-44.

———. *Macroeconomic Theory*. New York and London: Academic Press, 1979.

———, and Christopher A. Sims. "Business Cycle Modeling without Pretending to Have Too Much *A Priori* Economic Theory." In *New Methods in Business Cycle Research: Proceedings from a Conference*. Edited by Christopher A. Sims. Minneapolis: Federal Reserve Bank of Minneapolis, 1977. Pp. 45-109.

Scherer, Frederic M. "Corporate Inventive Output, Profits, and Growth," *Journal of Political Economy*, LXXIII (June 1965), 290-97. (1965a)

———. "Firm Size, Market Structure, Opportunity, and the Output of Patented Inventions," *American Economic Review*, LV (December 1965), 1097-125. (1965b)

Schmookler, Jacob. *Invention and Economic Growth*. Cambridge, Mass.: Harvard University Press, 1966.

———, and Oswald H. Brownlee. "Determinants of Inventive Activity," *American Economic Review*, LII (May 1962), 165-76.

Sims, Christopher A. "Money, Income, and Causality," *American Economic Review*, LXII (September 1972), 540-52.

———. "Exogeneity and Causal Ordering in Macroeconomic Models." In *New Methods in Business Cycle Research: Proceedings from a Conference*. Edited by Christopher A. Sims. Minneapolis: Federal Reserve Bank of Minneapolis, 1977. Pp. 23-43.

Taylor, C.T., and Z.A. Silberston. *The Economic Impact of the Patent System*. Cambridge: Cambridge University Press, 1973.

U.S.D.C. Patent and Trademark Office. *General Information Concerning Patents*. (003-004-00555-6.) Washington, D.C.: U.S. Government Printing Office, 1978.

U.S.D.C. Patent and Trademark Office, Office of Technology and Assessment. *Reports 1-9*. Washington, 1973-1979.

von Hippel, Eric. "Appropriability of Innovation Benefit as a Predictor of the Functional Locus of Innovation," *Research Policy* forthcoming.

Wilson, Robert Woodrow. "The Sale of Technology Through Licensing." Unpublished Ph.D. thesis, Yale University, 1975.

Wold, Herman O.A. "On Prediction in Stationary Time Series," *Annals of Mathematical Statistics*, XIX (1948), 558-67.