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EMPLOYEE VALUATION OF PENSION CLAIMS  
AND THE IMPACT OF INDEXING INITIATIVES

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ABSTRACT

There is discussion in both Canada and the United States of the government's requiring private pension plans to provide contractual cost-of-living protection. This paper employs both an auction and an implicit contract model to identify the compensating wage differentials required of possible indexing initiatives. The contract model, motivated by the prevalence (especially in Canada) of ad hoc cost-of-living adjustments to pensions in pay, presumes that workers have a call option on the investment earnings in excess of the interest rate assumption used to value the plan. The case for policy action would appear to rest on either (1) the assumption that workers misperceive the value (and, possibly, the security) of pension benefits or (2) the presumption that society should subsidize pension income by providing to pension plans an investment vehicle (such as an index bond) whose risk-return characteristics cannot be duplicated by portfolios of existing assets.

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## 1. INTRODUCTION

In both Canada and the United States, the benefits provided by occupational pension plans in the private sector typically contain no contractual cost-of-living protection. To many advocates of reform, this is the foremost limitation of such plans. As is well known (Bodie (1976), Pesando and Rea (1977), Feldstein (1980)), the real return to traditional plan assets - fixed-income securities and common stocks - is adversely affected by unanticipated inflation. Thus the absence of contractual indexing is typically ascribed to the "open-ended commitment" faced by plan sponsors in the absence of an index bond or an equivalent investment vehicle. Those who consider the possibility of requiring private plans to index pension benefits frequently cite the need for a parallel government initiative, such as selling price-indexed annuities to sponsors of eligible pension plans (Economic Council (1979)) or issuing index bonds (President's Commission (1980)).<sup>1</sup> Alternatively (Task Force (1979)), the government could mandate performance indexing, in which inflation-augmented or "excess" investment earnings above a statutory real rate would be applied to escalating the value of pensions in force.

Those who advocate a reform initiative are concerned with the fact that members of private pension plans are exposed to a great deal of inflation risk. This exposure stems from the fact that accrued pension benefits have the investment characteristics of a long-term bond. Significantly, advocates of reform fail to consider the possibility that workers may prefer not to have fully indexed pension benefits. Bodie (1980) demonstrates that the expected return on the portfolio whose real return

has minimum variance - treasury bills hedged against unanticipated inflation by a small (long) position in a diversified portfolio of commodity futures - is approximately zero. If this portfolio adequately approximates an asset with a risk-free real rate of return, and if workers have full information, then the explanation for the absence of contractual indexing must be restated. Employees are not sufficiently risk averse that they are willing to earn a zero real return on foregone wages in order to obtain fully and contractually indexed pension benefits. Alternatively, employees may hold portfolios (inclusive of pension wealth) in which they have diversified away the inflation risk associated with the accrued value of their pension benefits.

Leaving aside (for the moment) the question of whether a reform initiative can be justified, this paper seeks to identify the compensating wage differentials required if alternative indexing initiatives were mandated by the government. This issue, in turn, draws attention to the question of how members of defined benefit plans<sup>2</sup> value their pension benefits in the context of current versus deferred wage tradeoffs. The primary purpose of the paper is to illustrate that the wage offsets likely to accompany an indexing initiative are potentially quite large, may be difficult to predict, and may vary substantially across firms. In so doing, the paper highlights the information required of workers if they are to assess correctly the accruing value of their pension benefits. The rationale for proposed indexing initiatives, although not necessarily so stated, would appear to depend heavily on the presumption that workers have incomplete information.

To some, the choice of an appropriate model to value pension benefits may seem apparent. If the pension is not indexed, but is payable with certainty, then the accruing value of the contractual benefit is determined using the risk-free nominal rate of interest. The prevalence of ad hoc cost of living adjustments to pensions in pay in both Canada and the United States suggests, however, that this procedure may understate the value to workers of their accruing pension benefits. The paper thus uses two models to assess the impact of possible indexing initiatives. In the auction model, employees receive compensation equal to the value of their marginal product in each period and only benefits legally due under the terms of the plan are factored into employee valuation of their accruing benefits. In the contract model, workers have a call option on the investment earnings in the pension plan above the assumed valuation rate. In spite of the defined benefit formula, workers thus have an equity interest in the pension plan's assets. This model is motivated by the evidence (especially in Canada) that many firms use "excess" investment earnings to finance cost-of-living adjustments to pensions in pay. Because pension benefits in this contract model are - in effect - partially indexed, the wage offsets required of possible reform initiatives are less than those implied by the auction model.

The paper is organized as follows. First, the auction model is reviewed and its salient features noted. Since indexing initiatives might enhance the security as well as the contractual value of pension benefits, explicit treatment is accorded the options inherent in defined benefit plans. Second, evidence regarding ad hoc cost-of-living adjustments in Canada and the United States is briefly reviewed in order to motivate

the contract model. The contract model is next presented, and its economic rationale and empirical validity are briefly discussed. The impact of possible indexing initiatives is then analyzed, with the auction and contract models identifying the probable upper and lower bounds to the offsets required elsewhere in the compensation package. Throughout the analysis, features of Canada's occupational pension plans provide the main institutional background, although most of the analysis is directly applicable to plans in the United States as well.

## 2. THE AUCTION MODEL

In a spot auction market, the employee's valuation of his accruing pension benefits is quite straightforward. He receives in each period total compensation equal to the value of his marginal product and must value only the benefits legally due under the terms of the plan. The valuation is complicated solely by the need to consider the put options implicit in the existence of defined benefit plans.

In Canada, as in the United States prior to the passage of ERISA in 1974, there is no plan termination insurance.<sup>3</sup> Further, most employer-sponsored plans contain explicit provisions which permit the sponsor to terminate the plan with the employer's obligation equal to the lesser of vested benefits or the assets in the plan. Treynor (1977) argues that the implicit liability of the firm, so long as it is an on-going concern, is to honour all vested pension claims. Sharpe (1976), by contrast, analyzes the question of optimal funding policy on the explicit assumption that employers stand prepared to terminate a plan if it is in their interest

to do so. As noted by Bulow (1979), the absence of plan terminations is not sufficient to refute this view of employer behaviour since the threat of plan termination can presumably be used to extract appropriate wage concessions. On the other hand, the widespread absence of plan terminations may reduce and ultimately eliminate the credibility of such a threat. Both views as to employer willingness to terminate unilaterally an occupation pension plan are incorporated into the subsequent analysis.

Assume that economic agents are rational and have access to the same capital market, that the relevant option contracts are available in the capital market (or would be so if agents desired), and that there are no taxes.<sup>4</sup> Assume first that employers will terminate their plans if it is in their interest to do so. Following Sharpe (1976), the firm promises at the beginning of each period to pay its employees a nominal pension claim  $L_1$ , known at present with certainty, at the end of the period. The firm establishes a pension fund ( $A_0$ ) to help meet this obligation, which will have a (presently uncertain) value  $\tilde{A}_1 = (1+\tilde{r})A_0$ , where  $\tilde{r}$  is the (uncertain) return on the plan's assets. At the end of the year, the pension liabilities will be discharged in full if  $\tilde{A}_1 \geq L_1$ , with any excess  $(\tilde{A}_1 - L_1)$  reverting to the firm. If  $\tilde{A}_1 < L_1$ , then employees receive only  $\tilde{A}_1$  and the firm is not liable for the difference. In effect, the firm has a call option on the assets with a striking price  $L_1$ . Employees, in turn, hold a contract to receive the certain pension payment  $L_1$ , and have sold a put option on the assets in the fund with a striking price  $L_1$ . Let  $w_t$  denote the current wage paid in period  $t$ , let  $VMP_t$  denote the value of the worker's marginal product, and - for simplicity - assume that  $\sigma(\tilde{r})$  is the measure of risk pertinent to the pricing of the relevant options.

Then, under the auction model, the following obtains.

$$w_t + PV_t(L_1) - PV_t(\text{PUT}(A_o, L_1, \sigma(\tilde{r}))) = \text{VMP}_t \quad (1)$$

The compensation package consists of three components: the current wage plus the contractual value of the pension claim, less the value of the "pension put". The latter component recognizes, in effect, that the pension benefit actually paid may fall short of its contractual value. The lower the degree of funding ( $A_o$ ) and the higher the degree of risk ( $\sigma(\tilde{r})$ ) of the plan's assets, given  $L_1$ , the greater will be the value of the put option and cet. par. the higher must be  $w_t$  if (1) is to obtain.<sup>5</sup> If the plan's funds are invested solely in the risk-free asset and thus earn the risk-free rate ( $r_f$ ), and if the plan is fully funded when valued at the risk-free rate (i.e.  $A_o(1 + r_f) = L_1$ ), then the value of the put option is zero. In general, this will not be the case.

If, as argued by Treynor (1977), the implicit obligation of an on-going firm is to honour all (vested) pension benefits, then the optioned assets expand to include the equity ( $E_o$ ) in the firm. So long as  $\tilde{E}_1 + \tilde{A}_1 \geq L_1$ , then the pension liability will be discharged in full. Let  $\tilde{e}$  denote the return on the firm assets inclusive of the funds in the pension plan and let  $\sigma(\tilde{e})$  denote the measure of risk relevant to option pricing. Then (1) becomes:

$$w'_t + PV_t(L_1) - PV_t(\text{PUT}(A_o + E_o, L_1, \sigma(\tilde{e}))) = \text{VMP}_t \quad (1)'$$

Since  $E_o > 0$ , the value of the put option cet. par. is less in (1)' than in (1) and thus  $w_t > w'_t$ . Further, in (1)' unlike (1), the employee



must factor the likelihood of firm insolvency into the valuation of the pension put and thus into the valuation of his pension claim.

Finally, if the pension benefit  $L_1$  is fully insured, either by a private insurance company or through public termination insurance such as that required by ERISA, both (1) and (1)' reduce to:

$$w_t'' + PV_t(L_1) = VMP_t \quad (1)''$$

Clearly,  $w_t' > w_t''$ . If benefits are fully insured, then the informational requirements imposed on the employee falls since he no longer has to value the pension put in assessing the value of his accruing pension benefit.

This auction model has two important implications. First, the reported interest rate assumptions used in formal plan valuations are just a "shell" and convey no information regarding employee valuation of accruing pension benefits.<sup>6</sup> If the pension benefit is nominal (i.e. non-indexed), then rational employees use the risk-free nominal rate - not the reported valuation assumption - to calculate the present value of the contractual benefit  $L_1$ . If this nominal rate includes an inflation premium, then employees value this contractual benefit on the implicit assumption that it will be eroded by the inflation - in this simple model - expected to occur during its accrual period. If the contractual benefit  $L_1$  is viewed as the lump sum necessary to purchase a requisite annuity, then its valuation will be premised as well on the erosion of the real value of the pension in force.

Second, under (1), the decision by the employer to underfund the plan will raise the value of the pension put. Since the interest rate assumption represents a means by which the employer can either accelerate or retard

the degree of funding, given the benefit formula, this is the only avenue through which the reported interest rate assumption can influence employee valuation of accruing pension benefits. Under (1)', there can be no presumption that additional payments to the pension fund will enhance the security of the benefit and thus reduce the value of the pension put. If the firm had unencumbered access to assets in the plan, then additional debt-financed contributions which were used to acquire additional bonds for the pension fund would leave the value of the pension put unchanged.<sup>7</sup> If these same contributions were used to acquire additional stocks in the pension fund, the riskiness of the firm would increase and the value of the pension put would rise. For purposes of the subsequent analysis, it will be assumed that altering the level of contributions under (1)' will leave the value of the pension put unchanged.

### 3. "AD HOC" COST-OF-LIVING ADJUSTMENTS: LIMITATIONS OF THE AUCTION MODEL

There are a number of concerns regarding the validity of the auction model described above. With the possible exception of tax considerations,<sup>8</sup> there is no reason why a firm would choose to provide a pension plan. In addition, discontinuities associated with (1) cliff vesting rules and (2) early retirement options render suspect the prediction that in every period the current wage will adjust to offset fully any change in the accruing value of the pension benefit.<sup>9</sup> Of particular relevance to this paper, however, is the tendency for large employers in the non-union sector to make cost-of-living adjustments to pensions in pay.

There are no official data compiled by the Government of Canada on either the magnitude or frequency of these adjustments. Two private surveys,

however, provide evidence on this issue. The Report by Tomenson-Alexander (1978) notes that 117 of the 149 large (active membership of at least 500) plans surveyed provided some form of cost-of-living adjustments to pensions in pay during the period 1971-1975. For the majority (101 respondents), the adjustments were not required under the terms of the plan and thus were entirely ad hoc. For those plans reporting adjustments, the payments averaged two-thirds of the amount necessary to offset fully the impact of inflation as measured by the consumer price index. The most recent Report on Survey of Pension Plans in Canada (March, 1980) by the Financial Executives Institute Canada (FEIC) indicated that 75% of the respondents had taken some action to offset the impact of inflation. Again, in the great majority of cases, this action was taken unilaterally by the plan sponsor. In the United States, data compiled by the Bankers Trust Company (1980) indicate that approximately 70% of large employers made cost-of-living adjustments during the period December 1974 to December 1979, typically on an ad hoc basis. These increases appear to have been somewhat less generous than those made in Canada, averaging about 25% to 30% of the amount required to fully offset the impact of inflation. The key point, of course, is that since employees can never be paid less than the value of their marginal product in the auction model, ad hoc cost-of-living payments represent gratuitous transfers of wealth from employers/shareholders to retired employees.

Of equal importance, there is evidence that a significant proportion of these payments is being financed out of "excess" investment earnings, or investment earnings in excess of the assumed valuation rate. The FEIC reports that 59.9% (by plan respondent) of the cost increases were funded

out of plan assets, while 31.8% were "expensed as paid". It would appear that "expensed as paid" refers to payments out of general corporate revenues, although this is not clear from the questionnaire. In the Tomenson-Alexander survey, the ad hoc adjustments were financed - in whole or in part - through general corporate revenues for only 32 of the 117 firms which made these adjustments. The majority of firms thus financed the ad hoc payments by creating new unfunded liabilities which were discharged through a combination of plan surpluses and special payments. At least in Canada, the likelihood that many firms are using "excess" investment earnings to finance ad hoc cost-of-living adjustments and/or nominal benefit enrichments (career average plans) is widely acknowledged within the private pension industry.

#### 4. THE IMPLICIT CONTRACT MODEL

The essence of the contract model suggested by the stylized facts noted previously can be illustrated using the one-period model introduced earlier. Again, let  $L_1$  represent the nominal value of a pension claim due at the end of the period. Then  $PV_t(L_1)$ , calculated at the risk-free nominal rate  $r_f$ , identifies the contribution  $A_f$  necessary to fully fund the plan. If the plan sponsor values the plan on the basis of an interest rate assumption  $r_v < r_f$ , then the required contribution  $A_v$  (where  $A_v(1+r_v) = L_1$ ) will exceed  $A_f$ . Assume (for simplicity) that the funds are invested at the risk-free rate. Then the plan will experience an actuarial surplus equal to  $(A_v - A_f)(1 + r_f)$ . Under the auction model, this surplus would revert to the employer. Under the contract model, this surplus would be returned to the employee, in this case

in the form of an ad hoc enrichment to the nominal benefit  $L_1$ .

Since these adjustments are (1) the result of an implicit contract and (2) paid later than the period when they are earned, it is logical to assume that the large firms who make these payments also behave according to (1)'. For simplicity, it will be assumed that these large firms have no probability of going bankrupt and hence that the value of the pension put is equal to zero. In general, the return to the plan's assets will be uncertain. The contract model requires that if  $\tilde{r}_1 \leq r_v$ , the employee receives the contractual benefit  $L_1$ . If  $\tilde{r}_1 > r_v$ , then the employee receives the augmented benefit  $\tilde{L}_1 = A_v(1+\tilde{r}_1) > L_1$ . Since the contractual benefit is nominal, the risk-free nominal rate  $r_f$  is still used to calculate its present value. Thus the employee's compensation is:

$$w_t''' + PV_t(L_1) + PV_t(\text{CALL}(A_v, A_v(1+r_v), \sigma(\tilde{r}))) = VMP_t \quad (2)$$

The employee has a call option on the investment earnings in excess of those implied by the valuation rate. If the employee retires at the end of the period, he receives an "ad hoc" increase in his pension. If he is still an active worker at the end of the period, he receives either an amendment to the benefit formula or an appropriate increase in his next period's wage. Note that the degree of risk in the pension fund - and thus the value of the call option - is an integral part of the implicit contract. Note also that  $w_t''$  (in (1)')  $> w_t'''$ .

The rationale for firms and workers entering into this type of implicit contract may be tied to a risk-sharing arrangement. Employees may be concerned with the preservation of the real value of their pension

benefits. They may be unwilling, however, to assume the full cost - a very low and perhaps zero real return on foregone wages - of full and contractual indexing. Instead, employers (because  $\tilde{r}_1 < r_v$  does not lower the contractual benefit  $L_1$ ) and employees share the investment risk associated with the plan's assets. To the extent that workers who terminate prior to retirement are not granted ad hoc adjustments when their deferred pensions become payable,<sup>10</sup> this contract may also serve as a turnover penalty in situations where workers have firm-specific human capital. The fact that these contracts are not written explicitly into the pension plan may be of significance in this regard, since the firm is thus not bound by statutory vesting provisions.

As noted, there is informal evidence which suggests that this contract model may have widespread validity, especially in Canada. More formally, the model predicts that cet. par. those plans which use a lower valuation rate will grant more generous cost-of-living adjustments to pensions in pay. In principle, this prediction is testable, although the requisite data are not readily available. The FEIC data that are available do provide evidence that actuarial experience and ad hoc adjustments are linked. For those final earnings<sup>11</sup> plans in which actuarial deficits (called "experience deficiencies") were large, or between 5.0% and 7.5% of accrued liabilities, no annual or biannual payments were made. For firms in which experience deficiencies were small, or less than 2.5% of accrued liabilities, a full 50% had made annual cost-of-living adjustments during the past few years. If - as seems probable - the frequency of the ad hoc payments is a good proxy for their generosity, the results are consistent with the prediction of the contract model.

## 5. THE IMPACT OF INDEXING INITIATIVES

If the government were to legislate an indexing requirement, it could take at least three forms: (a) full and contractual indexing without the simultaneous introduction of an index bond or its equivalent by the government; (b) full and contractual indexing with the introduction of such an instrument; and (c) performance indexing, in which "excess" investment earnings above a statutory real interest rate are used to make cost-of-living adjustments to pensions in pay.<sup>12</sup> If workers have full information, then option (a) is clearly inefficient since workers subject to today's market opportunities have not opted for fully indexed benefits. Option (b) might be efficient, but only if the new instrument provided by the government (e.g. an index bond offering a real return of 3%) possessed risk-return characteristics that could not be reproduced with existing assets. Option (c), with the statutory real rate replacing the valuation rate, is effectively the implicit contract model sketched earlier in the paper.

Consider first the impact of (a) or (b). Assuming that the minimum-variance portfolio now available is an adequate approximation to an asset with a risk-free real rate of return, they differ only in that the risk-free real interest rate ( $i_f$ ) is presumably higher in (b) than in (a). In the auction model,  $i_f$  replaces  $r_f$  in calculating  $PV_t(L_1)$  in (1)-(1)', thus cet. par. raising the value of accruing pension benefit and requiring either a reduction in  $w_t$  and/or  $L_1$ .<sup>13</sup> To the extent that sponsors of career average and flat benefit plans are forced in effect to pre-fund the retroactive, nominal benefit improvements that typically occur, the funded status of these plans will improve.<sup>14</sup> Under (1), the value of

the put option will fall, thus cet. par. re-inforcing the effects noted above. Under (1)', the value of the put option does not necessarily fall, at least in the case in which the firm finances its additional pension contributions by issuing additional debt. Under (1)", the fully insured case, the value of the put option is zero and any change in the rapidity of funding occasioned by the indexing initiative will exert no impact on the employment contract. In the contract model, options (a) and (b) require that  $i_f$  replace  $r_f$  in calculating  $PV_t(L_1)$  in (2). The call option in (2) no longer exists now that pension benefits are fully indexed. So long as workers - as part of the implicit contract - never receive more than full cost-of-living protection, then the increase in the value of  $PV_t(L_1)$  must exceed the value of the call option if  $r_v > i_f$ . Since this is likely to be the case, both (a) and (b) effectively require a reduction in  $w_t$  and/or  $L_1$ . Consistent with the earlier analysis, the value of the pension put is assumed equal to zero and thus any impact on the funded status of plans requires no compensating wage differential.

Let  $i_f$  now designate the statutory real interest rate in the performance indexing initiative. For the auction model,  $r_f$  continues to be used to determine  $PV_t(L_1)$ , but workers now have a call option on investment earnings in excess of  $i_f$ . Thus, again, a reduction in  $w_t$  and/or  $L_1$  must occur. Since  $i_f$  would be the rate which plan sponsors would be required to use to establish contribution rates, the pre-funding of nominal, retroactive benefit improvements in career average and flat benefit plans would also impact on the employment contract via the valuation of the put options as discussed previously. For the contract model, performance indexing would simply lead to the substitution of  $i_f$  for  $r_v$



in the call option in (2). For  $i_f < r_v$ , which is typical, the value of the call option cet. par. increases and thus an offsetting reduction in  $w_t$  and/or  $L_1$  is required.

Thus, the introduction of any of the initiatives - under both models - is likely to require a reduction in  $w_t$  or  $L_1$  or both. Unless workers are willing to devote a higher fraction of their lifetime earnings to pensions, a reduction in  $L_1$  is inevitable. Policy-makers, in short, must recognize that the initiatives are likely to force workers to take "low start, escalating" as distinct from "high start, non-escalating" pensions. If workers prefer "high start, escalating" pensions because they are liquidity constrained and/or concerned with the security of less than fully insured benefits, the initiatives would be welfare reducing. Finally, in both models, the introduction of a retroactive initiative would arbitrarily redistribute wealth from shareholders to workers with past service credits under the terms of the plan. In the auction model, these windfall gains would exceed the new unfunded liabilities created when plan sponsors valued accrued benefits at the statutory real rate rather than at their previous valuation rate. This result reflects the fact that nominal interest rates typically exceed valuation rates. Under the full information assumption, there is no rationale for requiring that any indexing initiative be made retroactive.

How large will the wage offsets actually be? Assume that the value of the pension put is equal to zero. Under the auction model, the increase in the value of the pension benefit when full and contractual indexing is introduced is easy to calculate. It is simply equal to the present value of the benefit calculated at  $i_f$  less its present value when calculated

at  $r_f$ . Under the contract model, the calculation is complicated by the presence of the call option in (2). Assume that (i) the plan is fully invested at  $i_f$ , (ii) the statutory real rate is  $i_f$ , and (iii) the rate of inflation is always greater than or equal to  $r_v - i_f$ . In this special case, the increase in the value of the pension benefit is equal to the present value of the required annuity calculated at  $i_f$  less its value when calculated at  $r_v$ . This simple result occurs because, in the absence of the indexing initiative, the real value of the benefit declines with certainty at a rate equal to  $r_v - i_f$ .<sup>15</sup> In general, this exact result will not obtain and the use of the difference between the annuity valued at  $i_f$  and  $r_v$  to measure its increased value will only be approximate. Subject to this approximation, however, an important result emerges. As in the case of a defined contribution plan, the value to the employee of his accruing pension benefit is measured by the value of the employer's contribution to the plan.

Consider the simple case of a pension benefit (annuity) that is payable with certainty for 15 years. Assume that  $i_f$  and  $r_f$  equal 2% and 12%, respectively. Assume that  $r_v$  equals 6%, which is in line with current practice in both Canada (Table 1) and the United States. Under the auction model, the value (at the date the annuity becomes payable) of the benefit increases by a full 89% if full and contractual indexing is introduced. Under the contract model, the value of the benefit rises by 32%. The smaller increase in value under the contract model is due to the fact that the benefit is, in effect, partially indexed prior to the introduction of the government's initiative.

To provide a more complete assessment of the impact of a possible indexing initiative, one must consider the stylized features of alternative plan designs. If post-retirement indexing (only) is introduced, then its impact will be the same for all plans under the auction model. Under the contract model, its impact will vary with the plan's valuation assumption. If pre- and post-retirement indexing is introduced, its impact - even in the auction model - will vary with the plan's benefit formula. Assume that the government mandates full and contractual indexing and issues index bonds with a real interest rate of 2% (3%). Assume, again, that the value of the pension put is zero. So long as the risk-free nominal rate exceeds the (maximum) valuation assumption, the impact of the indexing initiative will always be greater under the auction model. It is thus instructive, for the range in interest rate assumptions reported in Table 1, to note the magnitude of the wage offsets required even under the contract model.

The exercises (Table 2) illustrate the impact of (i) post-retirement and (ii) pre- and post-retirement indexing on the required contribution rates for a flat benefit, a career average and a final earnings plan. As noted, the contribution rates established by the choice of the interest rate assumption approximate the value of accruing pension benefits in the contract model. Because of the widespread use of projected benefit funding, contribution rates are calculated using both accrued and projected benefits methods.<sup>16</sup> The benefit in the career average plan is 2% of each year's earnings; in the final earnings plan, 2% of earnings in the final year of employment times the number of years of service; in the flat benefit plan, a fixed amount equal to 2% of earnings in the first year of employment (projected benefit funding) or 2% of the earnings of a representative

member at age 40 (accrued benefit funding), both times the number of years of service. Plan members are assumed to remain with certainty in the plan until age 65 and to draw their pensions for exactly 15 years. The issues of termination and vesting, as well as the more complex benefit structures of actual plans, are thus ignored.

For post-retirement indexing, the contribution rates must be recalculated with the annuities payable under the terms of the plans now valued at the real interest rate of 2% (3%). For pre- and post-retirement indexing, the recalculation of the contribution is more complicated. Because the benefit is tied to earnings in the very last year of employment in the final earnings plan, and since - by construction - the inflation factor is the same in both the interest rate and salary scale assumptions, there is no change in the contribution rate that is established when only post-retirement indexing is required.<sup>17</sup> In the flat benefit and career average plans, where inflation does erode the real value of accruing pension benefits, pre-retirement indexing does require further increases in the contribution rate. Details of all calculations are reported in Appendix A.

The results, while only suggestive, highlight the magnitude of the increases in contribution rates - and thus the required wage offsets - if an indexing initiative is introduced. If post-retirement indexing (at 3%) is introduced, the contribution rate (projected benefit funding) rises from 5.69% of covered earnings to 7.90% for career average plans initially valued at an interest rate assumption of 7.5% and from 11.14 to 12.09 for plans valued at an interest rate of 4.2%.<sup>18</sup> If both pre- and post-retirement indexing (at 3%) are introduced, the contribution rate rises to 14.47% of earnings for both plans. Flat benefit plans are typically renegotiated -

and the amendments made retroactive - at each contract renewal. The sharp increase in contribution rates under pre- and post-retirement indexing thus illustrate the impact of requiring that plan sponsors cost the benefits in real terms on a continuing basis, rather than periodically revising the benefit formula and thus establishing new unfunded liabilities. Finally, it is straightforward using the formulae presented in Appendix A to calculate the reduction in the benefit formula which will hold the required contribution rate constant. If post-retirement indexing (at 3%) is introduced, the benefit must be reduced from 2.0% to 1.49% per year of service in both the final average and career average plans originally valued at an interest rate of 7.5%.

## 6. SUMMARY AND CONCLUSION

Reform advocates have proposed a variety of initiatives to improve the cost-of-living protection provided by private pension plans. The most straightforward would require that plans provide benefits that are fully indexed to the consumer price index, and would be accompanied by the government's issuing index bonds earmarked for pension plans.

This paper has demonstrated that the wage offsets (which could take the form of reductions in initial pension benefits) required of this or similar initiatives are potentially quite large. These offsets may be difficult to predict in view of the uncertainty which exists concerning the appropriate model with which to value accruing pension benefits, and they may vary sharply across firms. Wage offsets are largest in the auction model, where the impact of the initiatives is to transfer purely nominal into real pension benefits. In the contract model, wage offsets are smaller

because pension benefits are already partially indexed in spite of the fact that benefits under the formal terms of the plan are purely nominal. If workers are rational, and thus do not (for example) misperceive nominal to be real pension benefits, the contract model is likely to provide the lower bound to the wage offsets required of an indexing initiative. Further, as emphasized in the text, the introduction of an indexing initiative may - by accelerating the degree of funding (especially) in flat benefit and career average plans - enhance the security of contractual benefits and thus require further wage offsets on this account.

If rational agents enter into voluntary employment contracts, then externally imposed constraints on the form of these contracts will be inefficient. This possibility is raised by Pesando and Rea (1977) with regard to the elimination of deferred vesting, by Lazar (1979) with regard to a ban on mandatory retirement, and by Arnott and Gersovitz (1980) with regard to the requirement that all pension benefits be fully funded. In the present context, contractual indexing accompanied (say) by the issuance of index bonds by the government may be efficient, but only if the index bonds possess risk-return characteristics that cannot be duplicated by portfolios of existing assets. Under the full information assumption, a policy initiative would appear to require the explicit decision to subsidize retirement income that flows through private pension plans.

In fact, advocates of reform would appear to implicitly reject the full information assumption. Many reform advocates (Task Force (1979)) imply that workers, for example, have not appropriately discounted the eroding impact of anticipated inflation on their nominal pension benefits. Although this concern may be misplaced if labour markets are truly competitive,<sup>19</sup>

the fact that this and related concerns do exist must be acknowledged. If the full information assumption is indeed suspect, a preferred policy response might be to ensure, for example, that employers and workers alike understand that plans must be valued on the basis of a real interest rate if the contribution rates so determined are to be sufficient - at least in principle - to provide fully indexed benefits.

Finally, the analysis in the text draws attention to two additional points regarding private pension plans. First, under the auction model, the use of employer contributions to measure the value of accruing pension benefits in empirical tests of current versus deferred wage tradeoffs is generally inappropriate. In view of the substantial variation in reported valuation assumptions, the contribution rates established for a given benefit formula may differ sharply. This fact may receive too little attention from empirical researchers.<sup>20</sup> Second, workers in the contract model have an equity interest in the plan's assets and the distinction between a defined benefit and a defined contribution plan thus becomes blurred. The requirement that pension liabilities be independent of pension fund assets (and hence pension fund performance), which underlies the augmented balance sheet employed recently by Black (1980) and Tepper (1981) in their analysis of optimal funding and pension asset allocation, is clearly violated and results so obtained may require modification.

FOOTNOTES

<sup>1</sup> Analysis of household demand for index bonds (Fischer (1975)), Blinder (1977), and Siegel and Warner (1977)) has already established that borrowers could issue index bonds at lower ex ante real interest rates than those required on traditional bonds. Attention (Fischer (1979) and Levhari and Leviatan (1977)) has shifted to the conundrum posed by the nonissuance of index bonds by private borrowers.

<sup>2</sup> Defined benefit plans are those in which the benefit is defined by a formula (e.g. 2% of final earnings for each year of service) and contributions are varied so as to provide the indicated benefit. In defined contribution plans, the pension is determined by the accumulated value of the contributions made by or on behalf of the employee. For defined contribution plans, there is no ambiguity in determining the value of the worker's pension claim. It is simply equal to the employer's contribution to the plan. In Canada, almost 95% of plan members are in defined benefit plans. Unless otherwise specified, the term "pension plan" in the text will refer to a defined benefit plan.

<sup>3</sup> In December of 1980, the Province of Ontario introduced a termination insurance scheme, although neither the federal government nor any other province has followed suit.

The Employee Retirement Income Security Act (ERISA) established the Pension Benefit Guarantee Corporation to provide plan termination insurance to private sector plans. Pension plans provided by state and local governments, however, are exempted from the provisions of ERISA.



- <sup>4</sup> Employees in Canada can make tax deductible contributions to their own defined contribution plans (called Registered Retirement Savings Plans), and the funds accumulate on a tax free basis. The present ceiling is \$5,500 per year. Employees in the United States can make contributions to Individual Retirement Accounts (IRAs), which offer comparable tax advantages, but only to a limit of \$1,500 per year. Assume that either (1) employees can access the tax subsidies accorded pension plans without requiring that they be a member of an occupational pension plan or (2) labour is supplied perfectly inelastically in a competitive labour market, in which case competition among firms will ensure that the tax subsidy associated with pensions will benefit only workers. Then the use of a before-tax, risk-free nominal interest rate to value accruing benefits in an occupational plan would be appropriate on the assumption that the (nominal) benefits are payable with certainty.
- <sup>5</sup> In the non-union sector, the existence of a poorly funded plan may not be stable since terminating workers cannot be forced to assume the costs of future contributions which enhance the value of all benefits payable under the terms of the plan. See Bulow (1979).
- <sup>6</sup> Note that projected as distinct from accrued benefit funding (Barnow-Ehrenberg (1979)) is also a "shell". The reported interest rate assumption - like the actuarial method - is simply a means of accelerating or retarding (tax deductible) payments into the plan.
- <sup>7</sup> In the debt capacity model analyzed by Black (1980), the firm maximizes its access to the tax subsidy associated with issuing debt by fully funding its plan and holding only bonds in the pension portfolio.

- <sup>8</sup> If the firm could overfund the plan and gain the tax advantages so offered (Bulow (1981)), then it would have an incentive to provide a (defined benefit) plan. See also footnote 4.
- <sup>9</sup> In Canada, statutory provisions typically require that an employee's pension benefits vest (i.e., he becomes legally entitled to a pension payable under the terms of the plan) after he has reached age 45 and completed 10 years of service. Although many plans have more liberal provisions, 74.3% of plan members in 1978 were still in plans which had a vesting requirement of ten or more years of service (Statistics Canada (1980)). The "45 and 10" rule implies that an unrealistically large offset in the current wage may be required in the year that the employee's benefits actually vest. If an employee attains age 45 after completing 10 years of service in a plan whose benefit is 2% of final earnings for each year of service, and if it is assumed that the employee lives with certainty for 15 years after the normal retirement age of 65, then - at an interest rate of 6% (8%) - the value of his accruing benefit rises from zero in the preceding year to 60% (37%) of his current salary in this the year in which his benefit vests. If the employee's productivity is unchanged, and if the value of the relevant pension put is sufficiently close to zero that it can be ignored, then a corresponding reduction in his current wage would be required. In fact, there is no evidence that discrete wage reductions of this order of magnitude actually take place. A similar argument applies to the date at which an employee becomes eligible for an (actuarially unfair) early retirement benefit under the terms of the plan.

- 10 If the worker forfeits the ad hoc enrichment if he terminates prior to retirement, equation (2) must be modified accordingly. In general, evidence on this issue is hard to obtain, although - in Canada - there is reason to believe that many terminated (vested) workers do not receive ad hoc adjustments when their pensions become payable. This is clearly the case when the firm discharges its obligation by purchasing a deferred annuity from a life insurance company. The result is less clear when the firm retains the worker "on the books" and begins to make pension payments when the terminated worker reaches normal retirement age under the terms of the plan.
- 11 Final earnings plans are plans in which the benefit is tied to the employee's earnings at or near the time of his retirement. Detailed data were compiled by the FEIC only for final earnings plans.
- 12 For a detailed discussion of performance indexing, together with simulation experiments designed to illustrate its potential effectiveness, see Task Force (1979).
- 13 Feldstein (1981) also emphasizes this point.
- 14 In flat benefit plans, which typify the union sector, members receive a fixed amount (e.g. \$10.00 per month) for each year of service. In career average plans, members accrue a benefit in each year equal to a fraction (e.g. 2%) of that year's earnings. For both types of plans, but especially for flat benefit plans, retroactive amendments are common. In Canada, the unfunded liabilities created when retroactive amendments are made are amortized over a 15 year period. In an inflationary climate, the succession of such amendments - designed in large part to offset the eroding

impact of inflation on the real value of accrued benefits - virtually guarantees that such plans will be poorly funded. For a discussion of the funded status of flat benefit plans in the United States, most of which is directly applicable to Canada, see Bulow (1979).

- <sup>15</sup> Note, for example, that if the inflation rate ( $\pi$ ) is exactly equal to  $r_v - i_f$ , then the certain return on the plan's assets is  $r_f = i_f + \pi = r_v$  and the value of the call option is zero. The real value of the nominal benefit  $L_1$  thus declines (instantaneously) at the rate  $\pi$ .
- <sup>16</sup> For a detailed discussion of implicit contracts in the context of projected benefit funding, see Bulow (1979). In essence, this method is valid only if there is an implicit contract that the more rapid rate of benefit accruals which occurs as the worker ages will not be offset by slower wage growth relative to that which would occur in a defined contribution plan.
- <sup>17</sup> Although an exogenous salary growth assumption is used in these illustrations, it should not be inferred that all inflation risk has been transferred from the worker to the firm in the final earnings plan. So long as indexing only applies in the post-retirement period, the worker's accrued benefit during his active work years remains nominal and thus retains the investment characteristics of a long-term bond. From this perspective, the mechanical nature of the exercises discussed in the text merits emphasis.
- <sup>18</sup> Under the contract model, the different contribution rates are interpreted as follows. Assume that the real rate is 3% and that the nominal rate is 7.5%, so that the expected rate of inflation is 4.5%. Then the

plan valued at 7.5% is premised on the real erosion of accrued benefits at 4.5% per year during the pre-retirement period, compared to  $4.2 - 3.0 = 1.2\%$  for the plan valued at 4.2%. The plan valued at the lower rate, in effect, contains an implicit commitment to escalate the value of accrued benefits at 3.3% per year, which necessitates the higher contribution rate.

- <sup>19</sup> If labour is inelastically supplied, then competitive behaviour by firms will ensure that the current wages paid to workers will discount this anticipated erosion in the appropriate manner even if workers do not fully perceive the true situation.
- <sup>20</sup> Woodbury (1980) uses the value of employer contributions in his tests of preferences for wage and non-wage benefits without acknowledging the implications cited in the text. Smith (1980), although using the present value of accruing benefits in his tests, accepts the values as calculated by the actuaries of each individual plan. In view of the acknowledged variation in interest rate assumptions across plans, his procedure is essentially equivalent to using employer contributions (exclusive of amortization payments for unfunded prior service liabilities) to value accruing pension benefits.

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Table 1

THE INTEREST RATE ASSUMPTION: PLANS SURVEYED BY FINANCIAL EXECUTIVES INSTITUTE CANADA, FALL 1979

Benefit Formula	No. of Plans	Interest Rate (%): Number of Plans										Mean
		less than 3	3.0-3.9	4.0-4.4	4.5-4.9	5.0-5.4	5.5-5.9	6.0-6.4	6.5-6.9	7.0-7.4	7.5 or above	
Final Earnings	117	1	1	8	19	18	37	12	12	8	1	5.59
Career Earnings	38	0	0	4	3	10	16	1	3	0	1	5.51
Flat Benefit	26	0	1	3	7	7	6	1	1	0	0	5.10

Source: Report on Survey of Pension Plans in Canada (Toronto: Financial Executives Institute Canada, March 1980).  
Appendix J.

Table 2

INTERFIRM VARIATION IN VALUATION ASSUMPTIONS: IMPACT ON ESTIMATED  
COSTS OF PRE- AND POST-RETIREMENT INDEXING

		<u>CONTRIBUTION RATE (% COVERED EARNINGS)</u>				
		<u>Not Indexed</u>	<u>Indexed: Real Interest Rate (i)</u>		<u>Pre- and Post Retirement</u>	
			Post-Retirement (only)	Pre- and Post Retirement	i=2.0	i=3.0
<u>Projected Benefit Funding (Entry Age 30)</u>			i=2.0	i=3.0	i=2.0	i=3.0
Career Average	r=7.5, g=6.4	5.89	8.48	7.90	18.24	14.47
	r=4.2, g=3.1	11.14	12.97	12.09	18.24	14.47
Final Earnings	r=7.5, g=6.4	14.76	21.23	19.80	21.23	19.80
	r=3.0, g=1.9	19.80	21.23	19.80	21.23	19.80
Flat Benefit	r=6.7, g=5.6	2.18	2.99	2.78	15.50	10.18
	r=3.5, g=2.4	8.26	9.17	8.55	15.50	10.18
<u>Accrued Benefit Funding (Age 40)</u>						
Career Average	r=7.5 —	2.55	3.67	3.44	15.72	11.41
	r=4.2 —	7.79	9.07	8.46	15.72	11.41
Final Earnings	r=7.5, g=6.4	13.68	19.69	18.35	19.69	18.35
	r=3.0, g=1.9	18.35	19.69	18.35	19.69	18.35
Flat Benefit	r=6.7 —	3.54	4.85	4.19	15.72	11.41
	r=3.5 —	9.73	10.80	10.07	15.72	11.41

- Notes:
1. Range of interest rate (r) assumptions, by benefit formula, drawn from Report on Survey of Pension Plans in Canada (Toronto: Financial Executives Institute Canada, March 1980), Appendix J. Salary scale assumptions (g) determined by subtracting grand mean interest rate/salary scale "spread" of 1.1% from corresponding interest rates.
  2. Benefit in career average plans is 2% of each year's earnings; in final earnings plans, 2% of earnings in final year of employment; in flat benefit plans, a fixed amount equal to 2% of earnings in first year of employment (projected benefit funding) or 2% of earnings of representative member at age 40 (accrued benefit funding). Plan members remain with certainty in plan until age 65, and draw pensions for exactly 15 years.
  3. Real interest rate of 2%, combined with nominal rate of 7.5%, implies an inflation rate of 5.5%, etc. The constant interest rate/salary scale "spread" of 1.1% implies that real salary growth is 0.9% when the real interest rate is 2% and 1.9% when the real interest rate is 3%.

APPENDIX A

CALCULATION OF CONTRIBUTION RATES UNDER PROJECTED AND ACCRUED  
BENEFIT FUNDING

Projected Benefit Funding

1. Present value of benefits at member's retirement age:

$$\text{Final Earnings Plan} \quad V_R = (kR)W_o e^{gR} \int_R^D e^{-r(A-R)} dA = (kR)W_o e^{gR} \left( \frac{1}{r} (1 - e^{-r(D-R)}) \right)$$

$$\begin{aligned} \text{Career Average Plan} \quad V_R &= kR \left( \frac{1}{R} \int_0^R W_o e^{gs} ds \right) \int_R^D e^{-r(A-R)} dA \\ &= (kR)W_o \left( \frac{e^{gR} - 1}{gR} \right) \left( \frac{1}{r} (1 - e^{-r(D-R)}) \right) \end{aligned}$$

$$\text{Flat Benefit Plan} \quad V_R = bR \int_R^D e^{-r(A-R)} dA = bR \left( \frac{1}{r} (1 - e^{-r(D-R)}) \right)$$

2. Accumulated value of contributions at member's retirement age (all plans):

$$C_R = \int_0^R \bar{c}W_o e^{gs+r(R-s)} ds = \bar{c}W_o \left( \frac{e^{gR} - e^{rR}}{g-r} \right)$$

3. Set  $V_R = C_R$  and solve for contribution rate ( $\bar{c}$ ).
4. Calculation of the impact on the contribution rate of valuing the annuity payable under the plan at a real interest rate ( $i$ ) rather than a nominal interest rate ( $r$ ):

$$\text{Final Earnings Plan} \quad V_R = (kR)W_o e^{gR} \int_R^D e^{-i(A-R)} dA = (kR)W_o e^{gR} \left( \frac{1}{i} (1 - e^{-i(D-R)}) \right)$$

etc.

5. Calculation of the impact on the contribution rate of full indexing, both pre- and post-retirement, with real interest rate  $(i)$  and inflation rate  $p$  ( $p=r-i$ ):

Final Earnings Plan:

Same as in 4.

Career Average Plan:

$$V_R = kR \left( \frac{1}{R} \int_0^R W_0 e^{gs+p(R-s)} ds \right) \int_R^D e^{-i(A-R)} dA$$

$$= (kR)W_0 \left( \frac{e^{pR}}{R(g-p)} \right) (e^{(g-p)R} - 1) \left( \frac{1}{i} (1 - e^{-i(D-R)}) \right)$$

Flat Benefit Plan:

$$V_R = bR e^{pR} \int_R^D e^{-i(A-R)} dA = bR e^{pR} \left( \frac{1}{i} (1 - e^{-i(D-R)}) \right)$$

### Accrued Benefit Funding

1. Calculation of premium ( $p_t$ ) required in period  $t$  to purchase benefit which accrues in period  $t$ :

Final Earnings Plan

$$p_t = kW_t e^{g(R-t)-r(R-t)} \int_R^D e^{-r(A-R)} dA =$$

$$= kW_t e^{(g-r)(R-t)} \left( \frac{1}{r} (1 - e^{-r(D-R)}) \right)$$

Career Average Plan

$$p_t = kW_t e^{-r(R-t)} \int_R^D e^{-r(A-R)} dA = kW_t e^{-r(R-t)} \left( \frac{1}{r} (1 - e^{-r(D-R)}) \right)$$

Flat Benefit Plan

$$p_t = be^{-r(R-t)} \int_R^D e^{-r(A-R)} dA = be^{-r(R-t)} \left( \frac{1}{r} (1 - e^{-r(D-R)}) \right)$$

2. Contribution rate ( $c_t$ ) equals  $p_t$  divided by  $W_t$ .

3. Calculation of the impact on the contribution rate of valuing the annuity payable under the plan at a real interest rate (i) rather than a nominal interest rate (r):

$$\begin{aligned}
 \text{Final Earnings Plan} \quad p_t &= kW_t e^{g(R-t)-r(R-t)} \int_R^D e^{-i(A-R)} dA \\
 \text{etc.} & \\
 &= kW_t e^{(g-r)(R-t)} \left( \frac{1}{i} (1 - e^{-i(D-R)}) \right)
 \end{aligned}$$

4. Calculation of the impact on the contribution rate of full indexing, both pre- and post-retirement, with real interest rate (i) :

Final Earnings Plan: Same as in 3.

$$\text{Career Average Plan:} \quad p_t = kW_t e^{-i(R-t)} \int_R^D e^{-i(A-R)} dA = kW_t e^{-i(R-t)} \left( \frac{1}{i} (1 - e^{-i(D-R)}) \right)$$

$$\text{Flat Benefit Plan:} \quad p_t = be^{-i(R-t)} \int_R^D e^{-i(A-R)} dA = be^{-i(R-t)} \left( \frac{1}{i} (1 - e^{-i(D-R)}) \right)$$

Note: Member receives wage  $W_0$  at time he enters plan, works for R periods, receives pension credit k for each period of service to be applied to earnings base under the plan (or a periodic payment b in the case of a flat benefit plan), and draws pension for (D-R) periods. Salary grows at the nominal rate g and the nominal interest rate is r .