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EXCHANGE RATE DETERMINATION AND THE DEMAND FOR MONEY

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ABSTRACT

This paper examines the conventional monetary equation of exchange rate determination. Under certain exogeneity conditions, one can write the price level, at home and abroad, as the ratio of the nominal money supply to the demand for real money balances. Then, since the exchange rate is the domestic price of foreign exchange, one can equate the exchange rate to the ratio of domestic to foreign prices. This then allows one to write, and estimate, the exchange rate as a function of the money supply differential, income differential and interest rate differential. If the domestic and foreign money demand errors are autocorrelated, and if deviations from purchasing power parity are autocorrelated, tests based on the above model may be invalid. Only if all autoregressive parameters are equal will test results be valid. A full information maximum likelihood procedure is used to estimate and test the assumptions necessary for the conventional procedure to be correct. Finally, two alternative models of exchange rate determination are considered to illustrate the importance of introducing the error terms at the beginning of the analysis.

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Conventional models of the monetary approach to exchange rate determination assume a demand for money function for each country, with the same parameters, and that purchasing power parity holds.[1] These two assumptions, along with a semi-log functional form for money demand, allows one to write the exchange rate as a function of the money supply differential, income differential and interest rate differential, with an error term added at the end:

$$s_t = (m_t - m_t^*) + \alpha + \beta_1(y_t - y_t^*) + \beta_2(i_t - i_t^*) + v_t \tag{1a}$$

where s_t = log of the exchange rate

m_t = log of the money supply

y_t = log of real income

i_t = nominal interest rate

$*$ denotes foreign variables

It is then assumed that v_t is an autoregressive process, AR(1):

$$v_t = \gamma v_{t-1} + \nu_t \tag{1b}$$

In this paper, we examine under what conditions estimating equations (1a) and (1b) is appropriate and we provide an interpretation of the parameters γ and σ_ν^2. In particular, we derive equations (1a) and (1b) from a general model that begins with specifying a conventional domestic and foreign money demand function and allowing for deviations from purchasing power parity to be autoregressive.

I. Derivation of the Conventional Exchange Rate Equation.

Let us begin by assuming conventional money demand functions for the home and foreign country (denoted by an '*'):

$$m_t - p_t = \beta x_t + u_t \tag{2a}$$

$$m_t^* - p_t^* = \beta^* x_t^* + u_t^* \tag{2b}$$

where

$$u_t = \rho u_{t-1} + \varepsilon_t$$

$$u_t^* = \rho^* u_{t-1}^* + \varepsilon_t^*$$

$$x_t = (1\ y_t\ i_t)' = \text{exogenous variables}$$

$$y_t = \text{log of real income}$$

$$i_t = \text{interest rate}$$

$$m_t = \text{log of the money supply}$$

$$p_t = \text{log of the price level}$$

$$\beta = (\alpha\ \beta_1\ \beta_2)$$

$$E\varepsilon_t \varepsilon_t^* = \sigma_{\varepsilon\varepsilon^*}$$

Next, we impose a purchasing power parity (PPP) condition. We assume that the deviations from PPP follow an AR(1) process[2] (where s_t = log of the exchange rate):

$$s_t - (p_t - p_t^*) = \theta[s_{t-1} - (p_{t-1} - p_{t-1}^*)] + \mu_t \tag{3a}$$

so that we can write $p_t - p_t^*$ as

$$(p_t - p_t^*) = s_t - (1-\theta L)^{-1}\mu_t \tag{3b}$$

From equation (2) we can solve for $p_t - p_t^*$ to obtain

$$\begin{aligned} p_t - p_t^* &= \rho p_{t-1} + (1-\rho L)m_t - \beta(1-\rho L)x_t \\ &\quad -\rho^* p_{t-1}^* - (1-\rho^* L)m_t^* - \beta^*(1-\rho^* L)x_t^* \\ &\quad + \varepsilon_t^* - \varepsilon_t \end{aligned} \tag{4}$$

We next equate equations (3b) and (4) to obtain an expression for s_t:

$$\begin{aligned} s_t &= (\rho p_{t-1} - \rho^* p_{t-1}^*) + [(1-\rho L)m_t - (1-\rho^* L)m_t^*] \\ &\quad - [\beta(1-\rho L)x_t - \beta^*(1-\rho^* L)x_t^*] \\ &\quad - (\varepsilon_t - \varepsilon_t^*) + (1-\theta L)^{-1}\mu_t \end{aligned}$$

Multiplying through by (1-θL) and rearranging yields

$$\begin{aligned} s_t = {} & (m_t - m_t^*) - [(\theta+\rho)m_{t-1} - (\theta+\rho^*)m_{t-1}^* \\ & - \theta\rho m_{t-2} + \theta\rho^* m_{t-2}^*] - (\beta x_t - \beta^* x_t^*) \\ & + [\beta(\theta+\rho)x_{t-1} - \beta^*(\theta+\rho^*)x_{t-1}^* - \beta\theta\rho x_{t-2} + \beta^*\theta\rho^* x_{t-2}^*] \\ & + \theta s_{t-1} + \rho p_{t-1} - \rho^* p_{t-1}^* - \theta\rho p_{t-2} + \theta\rho^* p_{t-2}^* + w_t \end{aligned} \tag{5}$$

where $w_t = (\varepsilon_t^* - \varepsilon_t) - \theta(\varepsilon_{t-1}^* - \varepsilon_{t-1}) + \mu_t$

Assuming that ε_t, ε_t^* and μ_t are serially uncorrelated at all leads and lags, but contemporareously correlated, we can show that w_t is MA(1), with

$$Ew_t^2 = (1+\theta^2)\sigma_{\varepsilon *}^2 + (1+\theta^2)\sigma_\varepsilon^2 + \sigma_\mu^2 - 2(1+\theta^2)\sigma_{\varepsilon\varepsilon *} + 2\sigma_{\varepsilon *\mu} - 2\sigma_{\varepsilon\mu}$$

$$Ew_t w_{t-1} = -\theta[\sigma_{\varepsilon *}^2 + \sigma_\varepsilon^2 - 2\sigma_{\varepsilon *\varepsilon} + \sigma_{\varepsilon *\mu} - \sigma_{\varepsilon\mu}]$$

$$Ew_t w_{t-k} = 0 \quad \text{for all } k \geq 2 .$$

We see, therefore, that for the general model (equations (2) and (3)), the implied exchange rate equation (equation (5)) is quite complex. Equation (5) includes a lagged dependent variable, second order lags of p_t, p_t^* and the exogenous variables, nonlinear restrictions and a moving average error term. In this case, an OLS regression of s_t on $\{(m_t-m_t^*), m_{t-1}, m_{t-2}, m_{t-1}^*, m_{t-2}^*, x_t, x_{t-1}, x_{t-2}, x_t^*, x_{t-1}^*, x_{t-2}^*, p_{t-1}, p_{t-2}, p_{t-1}^*, p_{t-2}^*, s_{t-1}\}$ would yield inconsistent parameter estimates, since the error term in (5) is correlated with a right-hand side variable.

A special case of this model, often assumed by researchers, is when the parameters of the money demand function in both countries are equal: $\rho = \rho^*$ and $\beta = \beta^*$, but still allow $\theta \neq 0$. Then, it is easy to show that we can rewrite (5) as

$$\phi(L)s_t = \phi(L)(m_t - m_t^*) - \beta\phi(L)(x_t - x_t^*) + w_t - \rho\mu_{t-1} \tag{6}$$

$$\begin{aligned}\phi(L) &= 1 - (\theta+\rho)L + \theta\rho L^2 \\ &= (1-\theta L)(1-\rho L)\end{aligned}$$

Multiply both sides of (6) by $\phi^{-1}(L)$ to obtain

$$s_t = (m_t - m_t^*) - \beta(x_t - x_t^*) + v_t \tag{7a}$$

$$v_t = (1-\rho L)^{-1}(\varepsilon_t^* - \varepsilon_t) + (1-\theta L)^{-1}\mu_t \tag{7b}$$

Defining $e_t = \varepsilon_t^* - \varepsilon_t$, it can be shown that the autocorrelation function of v_t can be written as

$$Ev_t v_{t-k} = \frac{\rho^k}{1-\rho^2}\sigma_e^2 + \frac{\theta^k}{1-\theta^2}\sigma_\mu^2 + \frac{\rho^k + \theta^k}{1-\rho\theta}\sigma_{e\mu} \tag{8a}$$

Under the usual assumption of $\beta=\beta^*$ and $\rho=\rho^*$, the appropriate estimation procedure for the monetary approach to exchange rates is to estimate equations (7a) and (7b). In addition to assuming $\beta=\beta^*$ and $\rho=\rho^*$, it is often assumed that deviations from PPP are random, that is, $\Theta=0$. In this case, the appropriate model is equation (7a) and an error process with an autocorrelation function given by (8b) (obtained by substituting $\Theta=0$ into 8a):

$$Ev_t v_{t-k} = \frac{\rho^k}{1-\rho^2}\sigma_e^2 + \rho^k\sigma_{e\mu} \tag{8b}$$

Alternatively, it is sometimes assumed that PPP holds as an identity, in which case the error process has an autocorrelation function given by (8c):

$$Ev_t v_{t-k} = \frac{\rho^k}{1-\rho^2}\,\sigma_\nu^2 \tag{8c}$$

Under the assumption that $\beta=\beta^*$ and $\rho=\rho^*$, we are led to equation (7a), which is the standard monetary equation (1a). However, the implied error process for v_t, given by (7b), need not correspond with the process given by (1b). The autocovariance for v_t implied by (1b) is

$$Ev_t v_{t-k} = \frac{\gamma^k}{1-\gamma^2}\,\sigma_\nu^2 \tag{1c}$$

When is the standard approach, estimating (1a) - (1c), correct?[3] That is, when do there exist γ and σ_ν^2, such that

$$\frac{\gamma^k}{1-\gamma^2}\,\sigma_\nu^2 = \frac{\rho^k}{1-\rho^2}\,\sigma_e^2 + \frac{\theta^k}{1-\theta^2}\,\sigma_\mu^2 + \frac{\rho^k+\theta^k}{1-\rho\theta}\,\sigma_{e\mu} \tag{9}$$

for all $k = 0, \pm 1, \pm 2, \ldots$, given $\{\rho,\theta,\sigma_e^2,\sigma_\mu^2,\sigma_{e\mu}\}$?

There are several situations in which equation (9) will hold. Table I lists the four assumptions on $\{\rho,\Theta,\sigma_e^2,\sigma_\mu^2,\sigma_{e\mu}\}$ in which (9) holds. The first is when deviations from PPP are random ($\Theta=0$). Then, if we set $\gamma=\rho$ (where ρ is the autoregressive parameter in the money demand function) and set σ_ν^2 equal to a linear combination of the money demand error variances, σ_e^2 (where $e_t = \varepsilon_t^*-\varepsilon_t$), and covariances with PPP, $\sigma_{e\mu}$, equation (9) will hold. The second case (which is a special case of the first) is when PPP holds as an identity ($\Theta=0$ and $\sigma_{e\mu} = 0$). Then, if we set $\gamma=\rho$ and σ_ν^2 equal to a different linear combination of variances and covariances, equation (9) will hold. The third case is when the deviations from PPP are AR(1) ($\Theta\neq0$). Then, if we set $\gamma=\rho=\Theta$ (where Θ is the autoregressive parameter in the PPP equation) and σ_ν^2 equal to a still different linear combination of the underlying stochastic specification, equation (9) will hold. A fourth case is when the money demand functions are

Table 1

Summary of Conditions for Equation (9) to Hold

Assumption	$\gamma =$	σ_ν^2
1. $\Theta=0$	ρ	$\sigma_\varepsilon^2 + \sigma_{\varepsilon*}^2 - 2\sigma_{\varepsilon\varepsilon*} + (1-\rho^2)(\sigma_{\varepsilon*\mu} - \sigma_{\varepsilon\mu})$
2. $\Theta=0$ and $\sigma_{e\mu} = 0$	ρ	$\sigma_\varepsilon^2 + \sigma_{\varepsilon*}^2 - 2\sigma_{\varepsilon\varepsilon*}$
3. $\Theta\neq0$	$\Theta=\rho$	$\sigma_\varepsilon^2 + \sigma_{\varepsilon*}^2 - 2\sigma_{\varepsilon\varepsilon*} + \sigma_\mu^2 + 2(\sigma_{\varepsilon*\mu} - \sigma_{\varepsilon\mu})$
4. $\rho=0$	Θ	$\sigma_\mu^2 + (1-\Theta^2)(\sigma_{\varepsilon*\mu} - \sigma_{\varepsilon\mu})$.

NOTES: In all cases, $\beta=\beta^*$ and $\rho=\rho^*$. ε_t is the domestic money demand residual, ε_t^* is the foreign money demand residual, and μ_t is the PPP residual. $e_t = \varepsilon_t^* - \varepsilon_t$.

not autoregressive ($\rho=0$). Then, if we set $\gamma=\theta$ and σ_{ν}^{2} equal to the expression in Table 1, equation (9) will hold. Therefore, if the deviations from PPP are not random (see Frenkel (1981)) and we are willing to assume a priori that all autoregressive coefficients are equal ($\rho=\rho^{*}=\theta$) and money demand parameters are equal ($\beta=\beta^{*}$), then estimating the standard equation ((1a) and (1c)) is appropriate. However, to test these assumption one must consider a simultaneous estimation procedure. Estimating equation (5) by nonlinear least squares, assuming w_t is white noise, and testing $\beta=\beta^{*}$ and $\rho=\rho^{*}$ is incorrect, since w_t and s_{t-1} are correlated.

II. Estimation of a Standard Exchange Rate Model.

In order to test these assumptions, consider the following simultaneous equation system. The system consists of equations (2a), (2b) and (3a), rewritten here as

$$m_t - p_t = \alpha(1-\rho) + \beta_1 y_t + \beta_2 i_t - \beta_1 \rho y_{t-1} - \beta_2 \rho i_{t-1} + \rho(m_{t-1} - p_{t-1}) + \varepsilon_t \quad (10a)$$

$$m_t^{*} - p_t^{*} = \alpha^{*}(1-\rho^{*}) + \beta_1^{*} y_t^{*} + \beta_2^{*} i_t^{*} - \beta_1^{*} \rho^{*} y_{t-1}^{*} - \beta_2^{*} \rho^{*} i_{t-1}^{*} + \rho^{*}(m_{t-1}^{*} - p_{t-1}^{*}) + \varepsilon_t^{*} \quad (10b)$$

$$s_t = \theta s_{t-1} + (p_t - p_t^{*}) - \theta(p_{t-1} - p_{t-1}^{*}) + \mu_t \quad (10c)$$

where

$$E \begin{pmatrix} \varepsilon_t \\ \varepsilon_t \\ \mu_t \end{pmatrix} (\varepsilon_t \ \varepsilon_t^{*} \ \mu_t) = \Omega = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \quad (10d)$$

and y_t = log of real income

i_t = interest rate (short term)

m_t = log of money supply

p_t = price level

$*$ = foreign country variable

This system can be efficiently estimated by Full Information Maximum Likelihood (FIML). In particular, we are interested in testing several hypotheses:

$$H_o^{(1)}: \quad \beta_i = \beta_i^* \ (i = 1,2)$$
$$\rho = \rho^*$$
$$H_o^{(2)} \quad \beta_i = \beta_i^* \ (i = 1,2)$$
$$\rho = \rho = \Theta$$
$$H_o^{(2)} \quad \beta_i = \beta_i^* \ (i = 1,2)$$
$$\Theta = 0$$
$$H_o^{(4)} \quad \beta_i = \beta_i^* \ (i = 1,2)$$
$$\Theta = 0$$
$$\sigma_{3i} = \sigma_{i3} \ (i = 1,2,3)$$

In most monetary exchange rate equations, $H_o^{(1)}$ and $H_o^{(4)}$ are explicitly taken as maintained hypotheses. $H_o^{(1)}$ corresponds to the hypothesis that domestic and foreign money demand parameters are equal and $H_o^{(4)}$ corresponds to the additional assumption that PPP holds identically. $H_o^{(2)}$ assumes, in addition to $H_o^{(1)}$, that all autoregressive parameters are equal. $H_o^{(3)}$ assumes, in addition to $H_o^{(1)}$, that deviations from PPP are random.

Equation system (10) was estimated using quarterly data, from 1973-3 to 1979-4.[4] Four currencies, relative to the dollar were used - the U.K. pound, the German mark, the Swedish kronor and the Canadian dollar (see Data Appendix for more details). The period was chosen due to a possible structural change in the international monetary system in the first quarter of 1973, arising from the collapse of the EEC agreement to stabilize the dollar value of their currencies. The system was estimated using the FIML procedure in Version 3.3 of TSP. The results are summarized in Tables 2 to 5. Table 6 summarizes the results of testing $H_o^{(1)}$ through $H_o^{(3)}$, and Table 7 presents an estimate of the correlation matrix of the residuals.

Several results can be seen in Tables 2 to 6. For the U.K., the income elasticity of the demand for dollars was significantly negative for Model 1, but significantly positive for Models 2 to 5. In addition the domestic autoregressive parameter, ρ, was negative (equal to -.138) for Model 1, but positive for Models 2 to 5. A problem arose in estimating the model with Germany. The log likelihood value for the restricted Model 2 was greater than the log likelihood value for the unrestricted Model 1. For Canada, the income elasticity of the demand for U.S. dollars was negative. The autoregressive parameter in the PPP equation, for Canada and Sweden, Θ, was (insignificantly) greater than 1.0, which indicates nonstationarity.[5] (Consequently the test results should be viewed with caution.)

Table 7 presents estimates of the correlation matrix of residuals for Model 1, derived from Ω in equation (10d). We notice that for Germany, Canada and Sweden the correlation between domestic and foreign money demand residuals is quite large - .859, .708, .903, respectively. In general, the correlation between PPP residuals and money demand residuals is small and negative.

Table 2

United Kingdom

Quarterly Data, 73-3 to 79-4

	Model 1	Model 2	Model 3	Model 4	Model 5
α	3.947 (.832)	-1.784 (.801)	-3.060 (.807)		-2.387 (.822)
β_1	- .392 (.114)	.381 (.109)	.535 (.102)		.463 (.112)
β_2	- .001 (.004)	- .003 (.003)	- .003 (.003)		- .003 (.004)
α^*	3.865 (.345)	3.869 (.349)	3.251 (.359)		3.603 (.358)
β_1^*	.384 (.108)				
β_2^*	- .003 (.004)				
ρ	- .138 (.179)	.688 (.095)	.958 (.015)		.724 (.093)
ρ^*	.702 (.112)				
θ	.962 (.015)	.963 (.015)			
$\ell n\ L$	145.782	138.443	133.897		

NOTES: The models correspond to equation (10) and restricted versions of (10). The models are

Model 1: unrestricted

Model 2: $\beta_1 = \beta_1^*$, $\beta_2 = \beta_2^*$, $\rho = \rho^*$.

Model 3: $\beta_1 = \beta_1^*$, $\beta_2 = \beta_2^*$, $\rho = \rho^* = \theta$.

Model 4: $\beta_1 = \beta_1^*$, $\beta_2 = \beta_2^*$, $\rho = \rho^*$, $\theta = 0$.

Model 5: $\beta_1 = \beta_1^*$, $\beta_2 = \beta_2^*$, $\rho = \rho^*$, $\theta = 0$, $\sigma_\mu^2 = 0$.

ℓnL is the log of the likelihood function (except for a constant). Asymptotic standard errors are in parentheses.

Table 3

Germany

Quarterly Data, 73-3 to 79-4

	Model 1	Model 2	Model 3	Model 4	Model 5
α	- .949	- .486	- .439	.692	- .002
	(3.315)	(3.306)	(3.233)	(3.185)	(3.248)
β_1	.281	.180	.182	.089	.127
	(.449)	(.418)	(.419)	(.420)	(.422)
β_2	- .016	- .006	- .006	- .006	- .006
	(.008)	(.004)	(.004)	(.004)	(.004)
α^*	-10.945	.002	- .234	.704	.175
	(1.553)	(3.535)	(3.092)	(3.091)	(3.230)
β_1^*	1.647				
	(.222)				
β_2^*	- .016				
	(.004)				
ρ	.715	.983	.976	.951	.976
	(.151)	(.032)	(.012)	(.032)	(.032)
ρ^*	.474				
	(.150)				
Θ	.975	.975			
	(.013)	(.013)			
ℓn L	137.427	138.144	138.121	71.256	

NOTES: See notes to Table 2.

Table 4

Canada

Quarterly Data, 73-3 to 79-4

	Model 1	Model 2	Model 3	Model 4	Model 5
α	3.370 (1.097)	3.335 (.980)	7.289 (4.741)	3.400 (.970)	3.330 (.979)
β_1	- .318 (.150)	- .311 (.134)	- .847 (.619)	- .320 (.133)	- .310 (.134)
β_2	.004 (.005)	.001 (.004)	- .001 (.008)	.001 (.004)	.001 (.004)
α^*	.021 (.714)	.114 (.675)	2.893 (3.372)	.160 (.668)	.111 (.675)
β_1^*	-.288 (.143)				
β_2^*	-.002 (.004)				
ρ	.085 (.170)	.042 (.142)	.972 (.044)	.028 (.141)	.041 (.142)
ρ^*	-.004 (.157)				
Θ	1.022 (.044)	1.023 (.044)			
$\ln L$	162.513	161.568	145.876	122.902	

NOTES: See notes to Table 2.

Table 5

Sweden

Quarterly Data, 73-3 to 79-4

	Model 1	Model 2	Model 3	Model 4	Model 5
α	-1.992 (1.670)	-1.641 (.292)	-6.482 (54.964)	-6.482 (638.6)	-6.482 (638.6)
β_1	.423 (.227)	.193 (.067)	.286 (.064)	.286 (.064)	.286 (.064)
β_2	- .016 (.005)	- .015 (.522)	- .007 (.007)	- .007 (.007)	- .007 (.007)
α^*	-2.638 (.332)	-1.641 (.292)	12.991 (142.20)	12.991 (1773.)	12.991 (1773.)
β_1^*	.269 (.068)				
β_2^*	.053 (.017)				
ρ	.505 (.127)	.759 (.066)	.999 (.007)	.999 (.085)	.999 (.085)
ρ^*	.440 (.133)				
Θ	1.001 (.007)	1.001 (.007)			
ℓn L	134.725	127.675	122.700	37.765	

NOTES: See notes to Table 2.

Table 6

Likelihood Ratio Tests

Country	$H_o^{(1)}$	$H_o^{(2)}$	$H_o^{(3)}$
United Kingdom	14.678	23.770	
Germany[1]	-	.046	133.776
Canada	1.89	33.274	79.222
Sweden	14.1	24.05	193.92
5% critical value	7.815	9.488	9.488
1% critical value	11.345	13.277	13.277

NOTES: The number reported is $\lambda = 2(\ln L_u - \ln L_r)$, where L_u is the unrestricted likelihood value and L_r is the restricted likelihood value. The hypotheses tested are

$$H_o^{(1)}: \quad \beta_1=\beta_1^*,\ \beta_2=\beta_2^*,\ \rho=\rho^*,\quad k = 3.$$

$$H_o^{(2)}: \quad \beta_1=\beta_1^*,\ \beta_2=\beta_2^*,\ \rho=\rho^* = \Theta,\ k=4.$$

$$H_o^{(3)}: \quad \beta_1=\beta_1^*,\ \beta_2=\beta_2^*,\ \rho=\rho^*,\ \Theta=0, k=4.$$

$\lambda \sim \chi^2(k)$, where k = number of restrictions. For Germany, ln L for Model 1 was less than ln L for the restricted Model 2. The numbers reported are for testing Model 3 against Model 2 and Model 4 against Model 2.

Table 7

Correlation Matrix of Model 1

Correlation Matrix

Country	L	L^*	PPP
United Kingdom	1.000		
	.424	1.000	
	-.319	-.484	1.000
Germany	1.000		
	.859	1.000	
	-.064	-.037	1.000
Canada	1.000		
	.708	1.000	
	-.039	.071	1.000
Sweden	1.000		
	.903	1.000	
	-.153	-.062	1.000

NOTES: Correlation matrix is derived from Ω, of equation (10d). L denotes domestic money demand equation, L^* denotes foreign money demand equation and PPP denotes the PPP equation.

As can be seen from Table 6, the hypothesis that $\beta_1 = \beta_1^*$, $\beta_2 = \beta_2^*$ and $\rho = \rho^*$ is rejected for the U.K. and Sweden and accepted for Germany and Canada. Only for Germany can we, in addition, accept $\rho = \rho^* = \Theta$. Therefore, estimating the standard monetary equation of

$$s_t = (m_t - m_t^*) + \alpha + \beta_1(y_t - y_t^*) + \beta_2(i_t - i_t^*) + v_t \tag{1a}$$

by CORC is appropriate only for the case of Germany. For Canada, while (1a) is the appropriate equation, v_t is <u>not</u> AR(1). For the U.K. and Sweden, equation (1a) is inappropriate, since $(\beta_1, \beta_2, \rho) \neq (\beta_1^*, \beta_2^*, \rho^*)$. However, for completeness, estimates of equation (1a) by CORC are reported in Table 8. On the whole, the results look reasonable. Recall, the coefficient on $(y-y^*)$ is expected to be negative and the coefficient on $(i-i^*)$ is expected to be positive.

As is evident from the previous sections, the final version (reduced form) of the exchange rate equation depends on certain assumptions about money demand parameters and properties of error terms. In fact, the assumption that the parameters of the domestic and foreign money demand is not made simply for expository purposes - it is made so that one can obtain consistent estimates of the model. In addition, the hypothesis is not always accepted.

III. Alternative Exchange Rate Models

This paper has argued that it is essential to specify the error processes at the initial stage of writing the money demand functions, and then deducing the error process of the exchange rate equation. Two other models of exchange rates can also be considered. In the first alternative model, Bilson (1978, pp. 89-90) begins by assuming a partial adjustment model and autoregressive errors for the money demand functions:

Table 8

CORC Estimation of Monetary Equation

1973-4 to 1979-4

Country	Constant	$(y-y^*)$	$(i-i^*)$	γ	R^2	s.e.r.	D.W.
United Kingdom	3.855 (.919)	.358 (.211)	.010 (.007)	.966 (.053)	.90	.050	1.83
Germany	-.551 (.385)	-.885 (.911)	.004 (.008)	.950 (.064)	.96	.042	1.92
Sweden	-1.480 (.318)	-.572 (.096)	.003 (.013)	.897 (.090)	.80	.049	2.22
Canada	1.088 (2.211)	-1.639 (.984)	-.013 (.016)	.614 (.161)	.55	.049	1.88

NOTES: Standard errors are in parentheses. s.e.r. is the standard error of the regression. γ is the autoregressive coefficient.

$$m_t - p_t = \alpha + \beta_1 y_t + \beta_2 i_t + \delta(m_{t-1} - p_{t-1}) + u_t \tag{11a}$$

$$m_t^* - p_t^* = \alpha^* + \beta_1 y_t^* + \beta_2 i_t^* + \delta(m_{t-1}^* - p_{t-1}^*) + u_t \tag{11b}$$

$$u_t = \rho u_{t-1} + \varepsilon_t \tag{11c}$$

$$u_t^* = \rho u_{t-1}^* + \varepsilon_t^* \tag{11d}$$

and the assumption of purchasing power parity:

$$s_t = (p_t - p_t^*) + \mu_t \tag{12}$$

It is straightforward to verify that we can write the exchange rate equation as

$$\begin{aligned}(1-\rho L)s_t = &-(1-\rho)(\alpha-\alpha^*) + (1-\rho L)(m_t-m_t^*) \\ &- \beta_1(1-\rho L)(y_t-y_t^*) - \beta_2(1-\rho L)(i_t-i_t^*) \\ &+ \delta(1-\rho L)s_{t-1} - \delta(1-\rho L)(m_{t-1}-m_{t-1}^*) + w_t\end{aligned} \tag{13a}$$

$$w_t = \varepsilon_t^* - \varepsilon_t + \mu_t - \rho\mu_{t-1} \tag{13b}$$

It is clear that w_t is an MA(1) error process. First, we notice that estimating equation (13a) by OLS would yield inconsistent estimates due to the presence of a lagged dependent variable and an MA(1) error term (w_t is correlated with s_{t-1}). If we assumed that PPP held identically, so that $\mu_t = 0$, then the error term in (13b) could be written as $w_t = \varepsilon_t^* - \varepsilon_t$. In this case, OLS estimation of equation (13a) would yield consistent estimates. However, Bilson (1978) estimates the following equation, by CORC:

$$\begin{aligned}s_t = &-(\alpha-\alpha^*) + (m_t-m_t^*) - \beta_1(y_t-y_t^*) \\ &-\beta_2(i_t-i_t^*) + \delta s_{t-1} + \tilde{w}_t\end{aligned} \tag{14a}$$

$$\tilde{w}_t = -\delta(m_{t-1}-m_{t-1}^*) + (1-\rho L)^{-1}w_t \tag{14b}$$

CORC applied to equation (14a) is inconsistent, since $\tilde{w}_t$ will, in general, be correlated with $(m_t-m_t^*)$, if $(m_t-m_t^*)$ is autoregressive. In addition, applying the CORC procedure to an equation with a lagged dependent variable may be misleading since there may be more than one estimate of ρ.[6]

The results from estimating equation (13a), for Germany, by nonlinear least squares, assuming $\mu_t \equiv 0$, yields

$$\alpha-\alpha^* = \underset{(6.892)}{-.856}$$

$$\beta_1 = \underset{(.946)}{.818} \qquad \text{(income elasticity)}$$

$$\beta_2 = \underset{(.008)}{-.006} \qquad \text{(interest semi-elasticity)}$$

$$\rho = \underset{(.045)}{.986}$$

$$\delta = \underset{(.191)}{-.127}$$

$R^2 = .92 \quad$ D.W. - 1.85 $\quad$ s.e.r. = .043

In this case, only ρ, the autoregressive parameter, is significant.

Another alternative exchange rate model begins by assuming a partial adjustment to the long run exchange rate $\hat{s}_t$ (see Frenkel (1978), p. 181). In this model we assume that the domestic and foreign money demand functions are given by equations (10a) and (10b). To obtain the PPP equation, we assume

$$\hat{s}_t = a + b(p_t-p_t^*) \tag{15a}$$

$$s_t-s_{t-1} = \delta(\hat{s}_t-s_{t-1}) + u_t \tag{15b}$$

Solving for s_t we obtain

$$s_t = a\delta + b\delta(p_t-p_t^*) + (1-\gamma)s_{t-1} + u_t \tag{15c}$$

Substituting from equations (10a) and (10b), assuming $\beta_1 = \beta_1^*$, $\beta_2 = \beta_2^*$ and $\rho = \rho^*$, we obtain

$$s_t = a\delta + b\delta[(m_t - m_t^*) - \rho(m_{t-1} - m_{t-1}^*)$$

$$-(1-\rho)(\alpha-\alpha^*) - \beta(y_t - y_t^*) + \beta_1\rho(y_{t-1} - y_{t-1}^*)$$

$$-\beta_2(i_t - i_t^*) + \beta_2\rho(i_{t-1} - i_{t-1}^*) + \rho(p_{t-1} - p_{t-1}^*)]$$

$$+(1-\delta)s_{t-1} + u_t + b\delta(\varepsilon_t^* - \varepsilon_t) \tag{16}$$

Estimating equation (16), for Germany, yields

$a = .314$
(.2*10**14)

$b = 1.213$
(.840)

$\delta = .440$
(.157)

$\alpha-\alpha^* = .010$
(.6*10**14)

$\beta = 1.659$
(2.005)

$\gamma = -.019$
(.016)

$\rho = 1.227$
(.563)

$R^2 = .91$ D.W. = 1.53   s.e.r., .053

We see that the long run elasticity, b, is close to one, but imprecisely estimated. The short run elasticity, $b\delta$, is equal to .534. The income elasticity and interest semi-elasticity are of the "right" sign, but imprecisely estimated.

IV. Summary

In this paper we have reexamined the estimation of monetary exchange rate equations. Beginning with conventional money demand functions and assuming deviations from purchasing power parity are serially correlated (AR(1)), we derived a reduced form exchange rate equation. Without further assumptions, OLS (or nonlinear least squares) estimation is inconsistent. To derive the conventional exchange rate equation, the most plausible set of assumptions are that the parameters of domestic and foreign money demand functions are the same. In this case estimation of the conventional equation will yield consistent estimates, however the standard errors are incorrect. To obtain correct standard errors from the conventional equation, one must assume, in addition, that the autoregressive parameters in the money demand equations <u>and</u> the PPP equation are equal. Only for Germany does this hypothesis appear warranted, for the U.K., Sweden and Canada, the hypothesis is rejected.

Next, we looked at two different models of exchange rates. In one model, we assume partial adjustment in the money demand function. In the other model, we assume partial adjustment in the PPP equation. Two different exchange rate equations were then derived and estimated. However, the estimates in both alternative models were imprecise.

Footnotes

1. See, for example, Bilson (1978) or Dornbusch (1980).

2. Frenkel (1981, pp. 698-699) finds that the deviations from PPP appear to follow an AR(1) Process (ρ about .90) for the Pound, Franc and DM for the period June 1973 to July 1979.

3. It is true that OLS estimation of (72) will yield consistent estimates of β (under the assumption that x_t is exogenous). However, the standard errors will be biased.

4. Initial values for Model 1 were taken from OLS estimates of (10a) and a regression of $(s_t - p_t + p_t^*)$ on $(s_{t-1} - p_{t-1} + p_{t-1}^*)$. Initial estimates for models 2 to 5 were taken from the final estimates from Models 1 to 4, respectively.

5. However, when the model was reestimated, using different initial values, ρ was less than 1.0. The results were approximately the same.

6. Betancourt and Kelyian (1981) show that there may be several estimates of ρ when estimating a model with a lagged dependent variable and AR(1) error by CORC (due to multiple roots). However, Dufour and Gaudy (1981) point out that this will occur with probability zero. However, from a practical point of view, one must worry about multiple roots since they do appear in practice (Betancourt and Kelyian (1981), pp. 1076-77).

Data Appendix

The data was obtained from the December 1979 and April 1981 International Financial Statistics (IFS) tapes. The variable and its IFS codes is given, by country, in the following table.

	Variable				
Country	Exchange Rate	Income	Interest Rate	Money	Prices
United Kingdom	AE	99B.P	60C	34	64
Germany	AE	99A.R	60B	34	64
Canada	AE	99A.R	60C	34	64
Sweden	AE	99B.P	61	34	64
United States		99A.R	60C	34	64

where the IFS codes stand for:

AE = end of period exchange rate
99B.P = real GDP, 1975 prices
99A.R = real GNP, 1975 prices
60C = Treasury bill
60B = call money rate
61 = government bond yield
34 = money,
64 = consumer price index

References

1. Betancourt, Roger and Harry Kelejian. "Lagged Endogenous Variables and the Cochrane-Orcutt Procedure." Econometrica 49 (July 1981): 1073-1078.

2. Bilson, John F.O. "Rational Expectations and the Exchange Rate." In The Economics of Exchange Rates, edited by Frenkel, Jacob A. and Harry G. Johnson. Addison-Wesley, Reading, Mass. 1978.

3. Dornbusch, Rudiger. "Exchange Rate Economics: Where do we Stand?" BPEA 1:1980.

4. Dufour, Jean-Marie and Marc J.I. Gaudy. "Fixed Points and Minima: A Comment on Betancourt and Kelejian." University of Montreal, Working Paper #8117. May 1981.

5. Frenkel, Jacob A. "Purchasing Power Parity. Doctrinal Perspective and Evidence from the 1920s." Journal of International Economics 8 (May 1978): 169-191.

6. Frenkel, Jacob A. "Flexible Exchange Rates, Prices, and the Role of 'News': 665-705.

7. Frenkel, Jacob A. "Exchange Rates, Prices, and Money: Lessons from the 1920s." AER 70 (May 1980): 235-42.