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WAGE - EMPLOYMENT CONTRACTS:  
GLOBAL RESULTS

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Wage - Employment Contracts: Global Results

ABSTRACT

This paper studies the efficient agreements about the dependence of workers' earnings on employment, when the employment level is controlled by firms. The firms' superior information about profitability conditions is responsible for this form of contract governance. Under plausible assumptions, such agreements will cause employment to diverge from efficiency as a byproduct of their attempt to mitigate risk. It is shown that, if leisure is a normal good and firms are risk neutral, employment is always above the efficient level. Such a one-period implicit contracting model cannot, therefore, be used to "explain" unemployment as a rational byproduct of risk sharing between workers and a risk neutral firm under conditions of asymmetric information.

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## 1. Introduction

Most labor agreements specify the relationship between total compensation and level of employment, but leave the latter under the firm's control. Such a provision for contract governance may be necessary because information about the value of the firm's short-run production is not easily perceived and verified by labor. This asymmetry sets up a potential conflict between the goals of risk-sharing and productive efficiency. In this paper we attempt to analyze the solution to this problem by looking at some properties of the labor contracts that are optimal in a model where the firm will choose the employment level after it ascertains some relevant random parameters.

The results can be roughly characterized as follows, subject of course to assumptions whose innocence and plausibility we will later espouse:

- 1) There is more employment fluctuation under the optimal contract than would be observed if employment were chosen to maximize profits subject to the constraint that worker's utility be held constant in all situations. There is less income fluctuation.
- 2) There is less employment fluctuation and more income fluctuation than in the contract that would be implemented if all information could be directly verified by both parties.
- 3) The level of employment realized is in all cases one of "involuntary overtime." If workers could recontract with the firm ex post under conditions of symmetric information, the level of employment would be lower. In other words, the value of the marginal product of labor is always less than workers' marginal valuation of their leisure.

- 4) Finally, although levels of employment are higher and firms are more profitable in states in which labor is more productive, workers' utility will be monotonically decreasing in the firm's profitability. "Good times" are not shared by all.

These results show that the asymmetry of information that has been suggested as a source of suboptimal employment policies results in the opposite bias. It cannot be used as a foundation for a theory of involuntary unemployment.

Risk sharing between firms and workers has been a central focus of the literature on implicit contracts.\* In addition to a random profitability of firms, other features treated in various papers include: private rather than common knowledge of this random fluctuation, risk aversion by firms as well as workers, income effects in the demand for leisure, and random parameters in workers' utility functions as well as in firms' profit functions. The maintained assumptions of this paper are:

- i) Workers are risk averse and firms are risk neutral.
- ii) Firms have complete control of employment, ex post, because the information about their profitability is not publicly available.
- iii) The worker's welfare is represented by a single collective utility function, as if a union with well-specified risk preferences were to strike the bargaining agreement. The actual implementation of the agreements within the group of workers -- for example, seniority rules and the wage structure for different categories of workers -- is not addressed.

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\* We cannot attempt any reasonable summary of this interesting and rapidly expanding literature here. The papers most closely related to this one are Phelps-Calvo (1977) and Hall-Lillien (1979). Their results are discussed below. An excellent survey of the research on implicit contracts is Azariadis (1979). He mentions the problem treated here on pp. 28-30.

- iv) The preferences of labor are assumed non-stochastic over the life of the contract. The relevant uncertainty affects only the value of the firm's output.
- v) Finally, the form of feasible contracts is highly simplified. Compensation can be made to depend only upon the firm's contemporaneous choice of employment. More complicated arrangements in which compensation is allowed to depend upon the duration of unemployment, for example, are not considered.

These assumptions characterize the structure of the model. The qualitative result of overemployment will be the byproduct of the positive income effect on leisure.

The model is presented in Section 2. The main results are derived in Section 3. In Section 4 we offer some intuitive remarks and compare our results to those obtained under different specifications. We also briefly examine the relation of this problem to models from the principal-agent and the optimal-taxation literatures.

## 2. The Model

The relevant uncertainty is parameterized by  $\theta$ , and affects only the value of the firm's output. If  $l$  is the employment level, then  $f(l, \theta)$  is this value. The contract specifies the wage paid,  $w(l)$ , as a function of employment. The net payoff to the firm is thus  $f(l, \theta) - lw(l)$ . With the relevant uncertainty present in this general form it is hard to derive specific results. Therefore we will treat the special case of multiplicative uncertainty

$$(2.1) \quad f(l, \theta) = \theta g(l)$$

where  $g$  is an increasing concave function, and  $\theta$  is a positive random variable with a positive continuous density over an interval. The firm is assumed to be risk neutral, and therefore maximizing the mathematical expectation of  $\theta g(l) - lw(l)$  is its objective.

Workers' utility is an increasing function of earnings  $lw(l)$  and a decreasing function of level of employment. Because workers are risk averse we write their objective as

$$(2.2) \quad Eu(lw(l), l)$$

where  $u$  is a concave function. The expectation in (2.2) is taken with respect to the distribution of  $l$ . However,  $l$  is chosen by firms. Its distribution will therefore depend on the form of the entire contract and on the distribution of  $\theta$ .

Under any contract  $w(l)$  in any state  $\theta$ , the firm chooses the level of employment  $l(\theta)$  and pays the associated wage  $w(l(\theta))$ . It is notationally simpler to work with total compensation than with the

wage rate; thus we define

$$(2.3) \quad r(\theta) = \ell(\theta) w(\ell(\theta)) \quad .$$

The problem is to choose  $w(\cdot)$  so as to maximize

$$(2.4) \quad E u(r(\theta), \ell(\theta))$$

subject to

$$(2.5) \quad E \theta g(\ell(\theta)) - r(\theta) \geq c$$

where  $\ell(\theta)$  is defined by the solution to

$$(2.6) \quad \max_{\ell} f(\ell, \theta) - \ell w(\ell)$$

and  $r(\theta)$  is given by (2.3). By varying  $c$  parametrically, the family of efficient contracts will be delineated.

We will examine the characteristics of solutions to this problem and show that overemployment is the typical outcome. By comparing our solution with solutions to related problems, we will ascertain some of the qualitative implications of informational asymmetry and differential attitudes towards risk. Specifically we ask whether and to what extent profits, employment and labor compensation are more stable in this problem than when these features are absent.

Before proceeding further let us look at three simpler versions of this problem that will be useful as benchmarks.

First, consider the maximization of (2.4) subject to (2.5), but where  $\ell(\theta)$  can be chosen arbitrarily. This corresponds to that part of the implicit contracts literature in which the realization of uncertainties can be verified by both parties and therefore can be used explicitly to condition the outcomes.

In this case the solution can be characterized by the two equations

$$(2.7) \quad -\theta g'(\ell(\theta)) = \frac{u_{\ell}(r(\theta), \ell(\theta))}{u_r(r(\theta), \ell(\theta))}$$

$$(2.8) \quad u_r(r(\theta), \ell(\theta)) = K, \text{ a constant.}$$

The former is the condition for productive efficiency. That is, in all states  $\theta$ , marginal productivity of an extra unit of labor is equal to the marginal disutility of that unit. The latter equation is the condition for efficiency in risk-bearing (Borch's equation where one of the two parties is risk neutral).

Next, we can consider the original problem in the case when utility takes the particular form

$$(2.9) \quad u(r, \ell) = v(r - h(\ell))$$

where  $h$  is an increasing function describing the marginal disutility of labor and  $v$  is an arbitrary increasing concave function. The utility functions (2.9) are precisely those in which the income elasticity of leisure demanded (or labor supplied) is zero.

Hall and Lillien studied implicit contracting under (2.9) in the case when  $v$  is linear. The solution they found applies to the case of concave  $v$  as well. It is to set  $w(\cdot)$  and thus  $r(\cdot)$  so as to implicitly describe an indifference curve; that is



$$(2.10) \quad r(\ell) - h(\ell) = \bar{u}$$

It is easy to see why the firm's solution to its problem automatically satisfies (2.8). Regardless of the choice of  $\ell$ , (2.10) guarantees that (2.8) will hold because the argument of  $v(\cdot)$  is fixed.

The firm's choices in each state will also automatically satisfy the productive efficiency condition. For this particular utility function, condition (2.7) becomes

$$(2.11) \quad \theta g'(\ell) = h'(\ell)$$

In each state the firm chooses the point of  $r(\ell)$  such that the marginal cost of hiring labor is just equal to the marginal revenue product. Thus

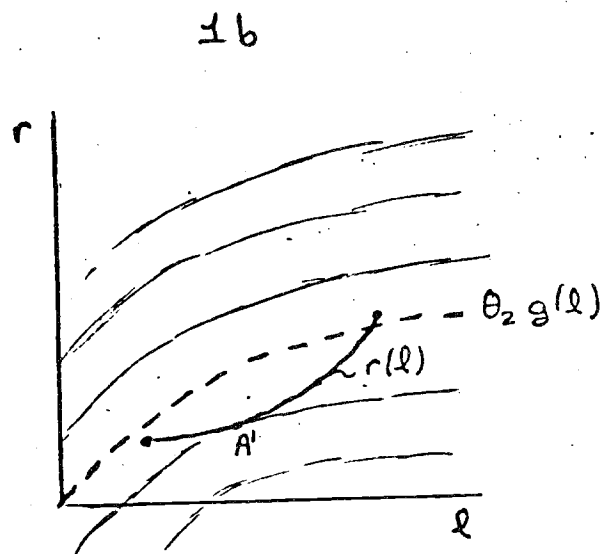
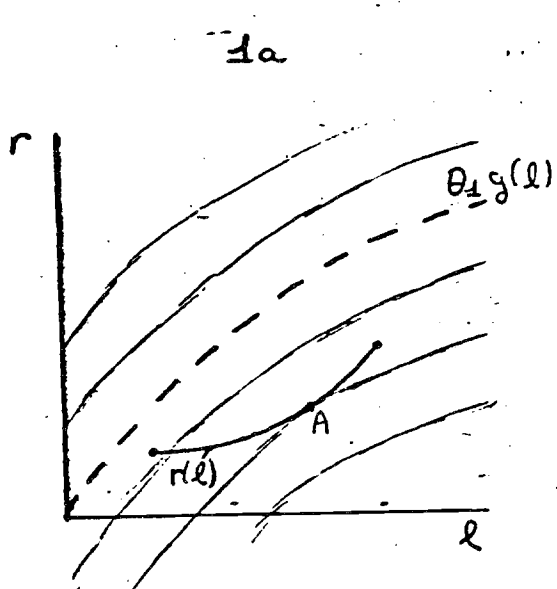
$$(2.12) \quad \theta g'(\ell) = r'(\ell)$$

And from (2.10), since  $\bar{u}$  is constant along the contract, we have

$$(2.13) \quad r'(\ell) = h'(\ell)$$

Combining (2.12) and (2.13), we see that the profit-maximizing choice is invariably the productively efficient choice.

This same argument can be shown graphically. Graphs (1a) and (1b) show the firm's isoprofit curves for two different values of  $\theta$  and specify a particular contract  $r(\ell)$ . Profits increase to the southeast. Points A, A' are the profit-maximizing points, satisfying condition (2.12). For a profit maximizing firm to choose the productively efficient points this requires that the slope of the contract always be equal to the worker's marginal rate of substitution. In other words,  $\bar{u}$  should be constant along  $r(\ell)$ . Efficiency



in risk-bearing requires that  $u_r$  should be constant along  $r(l)$ . For utility functions of the form (2.9), there is no conflict between productive efficiency and risk-sharing, and thus, no loss due to the private nature of observation of  $\theta$ . With constant  $u_r$  no further income smoothing is desirable, thus there is no gain to be had from further insurance by the firm. With efficient production in all periods, there is no Pareto-improvement to be had from re-contracting.\*

For the first-best contract to be incentive compatible, utility functions must be of the form (2.9). Our final example is a simple instance of what can go wrong when (2.9) does not hold.

Suppose the worker's utility function is additively separable:

$$(2.14) \quad u(r, l) = m(r) - n(l)$$

with  $m(\cdot)$  concave and  $n(\cdot)$  convex. Now condition (2.8) becomes

$$(2.15) \quad m'(r) = K$$

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\* Hall and Lillien also consider the consequences of random effects in the utility function. In this case they show that a contract administered by firms cannot implement the full-information optimum even when utility functions are of the form (2.9).

In other words, in this case the optimal contract would involve paying the worker a fixed amount in all states of the world. The labor required should vary smoothly according to

$$(2.16) \quad K/n'(\ell) = \theta g'(\ell)$$

which is the version of condition (2.7) for this particular utility function. It is easy to see that this contract could not be enforced under differential information. Because the contract does not provide for any variation in salary with respect to working time, the firm would always require the maximal amount of labor.

In subsequent sections of the paper we will examine the general solution to the problem when (2.9) does not hold and when in particular the income elasticity of leisure demanded is positive rather than zero. As this third example indicates, in such problems (2.7) and (2.8), the risk-sharing and productive efficiency conditions, will be in conflict. Thus devising a contract which can be implemented despite differential information will be a second-best problem. Its solution will entail overemployment for all  $\theta$  (except the highest and lowest possible values, where efficiency will hold).

### 3. Solution

The method of solution to be used below is novel in models of implicit contracting, drawing heavily on some techniques first developed in the literature on incentive compatibility and optimal auction design. \* \*\*

The idea is to regard the problem as the choice of two functions of  $\theta$ ,  $r(\theta)$  and  $\ell(\theta)$ , instead of the single relation  $w(\ell)$ . Thus we have

$$(3.1) \quad \max E u(r(\theta), \ell(\theta))$$

subject to

$$(3.2) \quad E \theta g(\ell(\theta)) - r(\theta) \stackrel{\geq}{=} c$$

and that, for each  $\theta$ ,

$$(3.3) \quad \max_{\tilde{\theta}} \theta g(\ell(\tilde{\theta})) - r(\tilde{\theta}) \text{ occurs at } \tilde{\theta} = \theta$$

The second set of constraints corresponds to (2.6).

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\* See Wilson (1977), Riley and Samuelson (1979), for an introduction to the auction design problem. Stochastic auction designs have been treated by Maskin, Riley and Weitzman (1979).

\*\* On incentive compatibility see Green and Laffont (1979) and Laffont and Maskin (1980), where the treatment of the continuous-parameter problem is closest to what will be used here.

It is clear that given any solution of the original problem we can define  $r(\theta)$  and  $\ell(\theta)$  by the values these variables actually take on for each value of  $\theta$ , and then  $r(\theta)$ ,  $\ell(\theta)$  will solve (3.1) - (3.3). Conversely, if we can arrange for a "truthtelling" solution  $r(\theta)$ ,  $\ell(\theta)$  to (3.1) - (3.3), then the implicit relation

$$(3.4) \quad r(\ell) \equiv r(\ell^{-1}(\ell))$$

where  $\ell^{-1}(\ell)$  is the value of  $\theta$  such that  $\ell(\theta) = \ell$ , gives us a solution to the original problem. It must only be insured that this inverse is well defined. We will see below that this is not a problem because any solution to (3.1) - (3.3) will satisfy

$$(3.5) \quad \ell'(\theta) > 0$$

by virtue of the second order conditions necessary for (3.3) to hold.\*

The next step is to replace (3.3) by the statement that the first and second order conditions for that problem hold as identities in  $\theta$  at  $\tilde{\theta} = \theta$ . These are

$$(3.6) \quad \theta g'(\ell(\theta)) \ell'(\theta) - r'(\theta) = 0$$

and

$$(3.7) \quad \theta g''(\ell(\theta)) (\ell'(\theta))^2 + \theta g'(\ell(\theta)) \ell''(\theta) - r''(\theta) < 0$$

Since (3.6) is an identity in  $\theta$ , we can differentiate it to obtain an

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\* There is no a priori reason to restrict contracts  $w(\ell)$  to functions; for some problems correspondences might work better. Furthermore, two contracts  $w_1(\ell)$  and  $w_2(\ell)$  which differ only on portions which are never chosen in any state are to all intents and purposes equivalent. Thus in cases (unlike the present one) where there is not an exact equivalence between the  $w(\ell)$  formulation and the  $(w(\theta), \ell(\theta))$  formulation it would seem that it is the latter that is the more fundamental specification.

expression for  $r''(\theta)$ . Substituting this in (3.7) we can then rewrite the second order conditions as

$$g'(\ell(\theta))\ell'(\theta) > 0$$

or simply  $\ell'(\theta) > 0$  by the monotonicity of  $g$ . In this way we see that (3.5) is automatically satisfied, and can be dropped as an explicit constraint in the maximization.

The problem we solve is to maximize (3.1) subject to (3.2) and (3.6).<sup>\*</sup> To simplify subsequent calculations we let

$$(3.8) \quad y(\theta) = g(\ell(\theta))$$

and

$$(3.9) \quad v(r,y) = u(r, g^{-1}(y)) \quad .$$

In this notation the problem is

$$(3.10) \quad \max E v(r,y)$$

subject to

$$(3.11) \quad E\{\theta y - r\} \geq c$$

$$(3.12) \quad \theta y' - r' = 0. \quad \text{for all } \theta$$

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<sup>\*</sup> For the purposes of this exposition, we are simply assuming that the constraint (3.5) is nowhere binding. A complete solution, taking this constraint into account is considerably messier. Such a solution will be composed of two types of subportions. In regions of  $\theta$  over which (3.5) is not binding, the contract will continue to satisfy equations of the form of (3.14)-(3.16). In regions in which (3.5) is binding, both  $l$  and  $r$  will be constant. The resultant contract curves will be similar to those described in the text, but they will be kinked at certain points. The conclusions we derive will not be affected.

We are also ignoring the possibility of discontinuous contract curves. It turns out that having income a normal good is sufficient for a continuous contract to be optimal.

These issues will be discussed more fully in a subsequent paper.

Because  $v$  is concave and because the restrictions are linear, the first order conditions and the transversality conditions are sufficient for a maximum, (see Ewing (1969), pp. 129-131).

Writing the Lagrangian expression

$$(3.13) \quad E\{v(r(\theta), y(\theta)) + f(\theta) (r'(\theta) - \theta y'(\theta)) + \kappa (\theta y(\theta) - r(\theta) - c)\}$$

we obtain the first-order conditions:

$$(3.14) \quad v_r - \kappa = \frac{d}{d\theta} f(\theta) = f'$$

$$(3.15) \quad v_y + \kappa \theta = \frac{d}{d\theta} (-f \cdot \theta) = -\theta f' - f$$

$$(3.16) \quad r' - \theta y' = 0$$

and the transversality conditions

$$(3.17) \quad f(a) = f(b) = 0$$

$$(3.18) \quad -f(a) \cdot a = -f(b)b = 0$$

where  $a$  and  $b$  are the endpoints of the support of the distribution of  $\theta$ . Under reasonable smoothness assumptions, these equations will yield unique continuous, smooth solutions  $f(\theta)$ ,  $g(\theta)$ ,  $r(\theta)$ . Expressions (3.14) and (3.15) can be combined to yield

$$(3.19) \quad v_y + v_r \theta + f = 0$$

As efficiency requires  $v_y + v_r \theta = 0$ , the bias of employment away from the efficient level depends solely on the sign of the function  $f$ . If  $f > 0$  we have overemployment: the value of the marginal product of labor,  $\theta g'$ , falls short of the rate at which labor must be compensated on the margin  $-u_\ell/u_r$ .

By definition  $u_\ell/u_r \equiv v_y g'/v_r$  and thus  $f > 0$  implies  $v_y + v_r \theta < 0$ , or,

$$\theta g' < \frac{-u_\ell}{u_r}$$

We now turn to a proof of this main result -- that indeed  $f > 0$ , except at  $a$  and  $b$  where  $f = 0$ , and thus that overemployment always obtains.

Differentiating (3.14) with respect to  $\theta$  we have

$$f'' = v_{rr} r' + v_{ry} y'$$

using (3.16)

$$f'' = (v_{rr} \theta + v_{ry}) y'$$

using (3.19)

$$\begin{aligned} (3.20) \quad f'' &= [v_{rr} \left( \frac{-f - v_y}{v_r} y \right) + v_{ry}] y' \\ &= [v_{rr} \left( \frac{-v_y}{v_r} y \right) + v_{ry} - \frac{v_{rr}}{v_r} f] y' \end{aligned}$$

The condition that leisure demand be a normal good is just

$$(3.21) \quad \left( u_{rr} \left( \frac{-u_\ell}{u_r} \right) + u_{r\ell} \right) < 0$$

Since  $u_{rr} \equiv v_{rr}$ ,  $u_\ell = v_y g'$ ,  $u_{r\ell} = v_{ry} g'$  and  $g' > 0$ , the first two bracketed terms above can be signed by this assumption:

$$(3.22) \quad v_{rr} \left( \frac{-v_y}{v_r} y \right) + v_{ry} < 0$$

Moreover, since  $y' > 0$ , we know that if  $f \leq 0$  then  $f'' < 0$ . It follows that if  $f$  has a zero between  $a$  and  $b$ , one of the two transversality conditions must fail. (For example, if  $f(e) = 0$  for  $a < e < b$  and  $f'(e) < 0$ , then



$f(b) < (b-e)f'(e) < 0$ .) Thus  $f \geq 0$  and, in particular,  $f > 0$  for all  $\theta$  in the interior of  $[a,b]$ . A corollary is that  $u$  decreases as  $\theta$  increases along the contract.

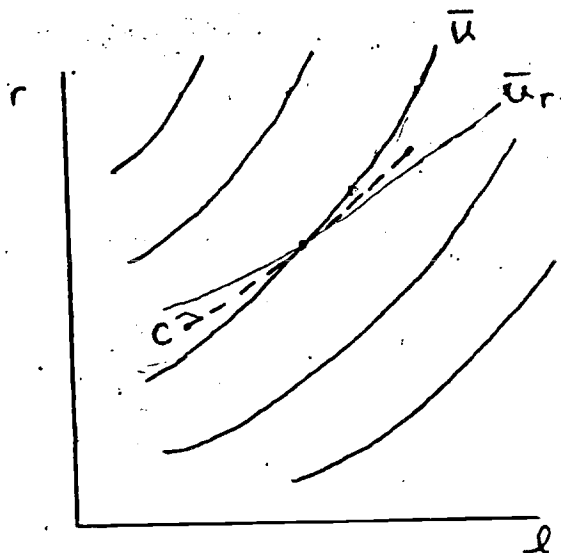
The function  $f$  can also be used to derive information about  $u_r$  along the contract. From (3.14) we know that  $v_r = f' + \kappa$  and we can show that  $\kappa = E v_r$ . Since we know from the above theorem that  $f'(a) > 0$  and  $f'(b) < 0$ , these relations are sufficient to show that for  $\theta$  near  $a$   $u_r$  is greater than its average value and for  $\theta$  near  $b$   $u_r$  is less than its average value.\*

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\* If we knew that  $f''$  were less than zero everywhere, we could easily show that  $u_r$  declines along the entire length of the contract. But this need not hold in general, and so the claim can only be made near the endpoints.

4. Intuition and Comparison with Other Results

The discussion above has been quite abstract, yet the intuition behind the overemployment result is actually very clear. The diagram below represents the utility function of workers, increasing to the northwest.

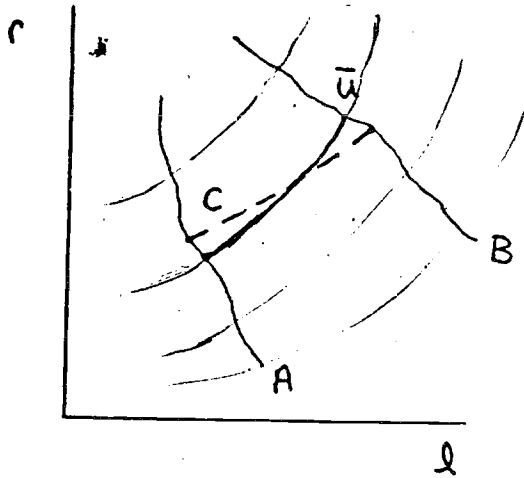


The curve  $\bar{u}$  is an indifference curve, and  $\bar{u}_r$  is a constant marginal utility of income locus. When leisure is a normal good  $\bar{u}_r$  must have a smaller slope (algebraically) than  $\bar{u}$ . Moving northeastward along  $\bar{u}$ , the marginal utility of income declines.

If  $\bar{u}$  were implemented as the contract we would always have productive efficiency but  $u_r$  would not be constant. The first-best contract would lie along  $\bar{u}_r$ , but if we left it to a profit-maximizing firm to implement  $\bar{u}_r$  as a contract we would have efficiency in risk bearing, but not in production. The firm would profit maximize by setting marginal product equal to the slope of the contract not of the indifference curve. Because of the relationship between these two slopes, the level of employment is too high under the  $\bar{u}_r$  contract.

The solution of our problem,  $C$ , will produce a compromise between  $\bar{u}$  and  $\bar{u}_r$ .<sup>\*</sup> But, as this will still be less than  $\bar{u}$ , it will still be characterized by overemployment for all  $\theta$ .

We can now justify claims 1) and 2) of the introduction. First let us compare the optimal contract  $C$  with any constant utility contract whose path it crosses:



Let A and B be the locus of points  $(r, l)$  such that  $u_r/u_l = a$  and  $b$  respectively. As long as leisure is a normal good these curves move leftward with increases in utility. We know from the transversality conditions that under contract  $C$  the endpoints are on these loci. Similarly, in a constant utility contract, since productive efficiency is achieved at all times, the firm's choices at  $\theta = a$  and  $\theta = b$  also lie on these loci. Thus when leisure is a normal good, the spread between  $l(b)$  and  $l(a)$  is greater in the optimal contract than in the constant utility contract.

When income is a normal good (so that A and B move upward with increases in  $u$ , as they do in the diagram), then a similar argument demonstrates that

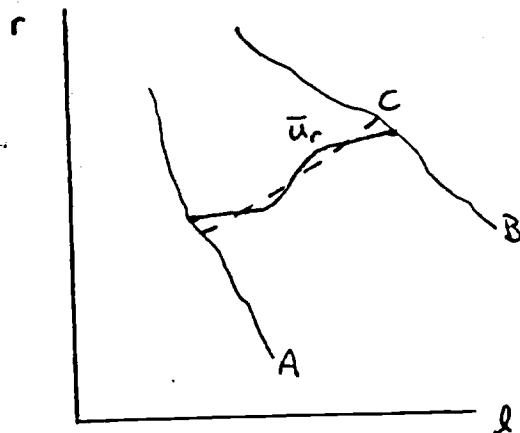
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\* If  $f''$  is always negative, then the slope of the optimal contract at any point lies between the slopes of the  $\bar{u}$  curve and of the  $u_r$  curve through that point.

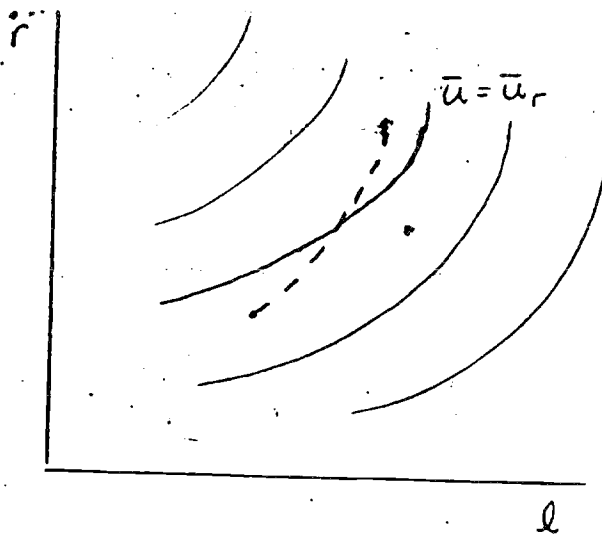
$r(b) - r(a)$  is smaller in the optimal contract than in the constant utility contract.

Because  $u_r$  is not necessarily monotonic, we cannot make quite as general a claim for arbitrary  $u_r$  crossed by the optimal contract, but we can make an analogous argument if we stick to the locus of constant  $u_r$  at a value equal to  $Eu_r$  along the contract curve.

Compare the second-best optimal contract and its associated  $Eu_r$  level with a first-best contract at which  $u_r$  is identically equal to this  $Eu_r$ . Efficiency once again guarantees that the endpoints of the contract lie on A and B. And from the conclusions of the previous section we know that the contract C must start with a higher  $u_r$  at a and end with a lower  $u_r$  at b. Thus if leisure is a normal good, the variation of employment is greater along the first-best than along the second-best contract. If income is a normal good, the variation of income is less along the first-best than along the second-best.



In the papers by Grossman and Hart (1980) and Azariadis (1980), under-employment is shown to be the rule. These papers use the no income effect utility function but introduce risk aversion on the part of firms. The same diagram is useful to explain these results:



Now  $\bar{u}$  and  $\bar{u}_r$  coincide. But efficiency in risk-bearing requires  $u_r/\phi'$  to be constant, where  $\phi'$  is the marginal utility of profit. Profit is increasing in  $\theta$ , and  $\ell'(\theta) > 0$ , so we know that  $\phi'$  will be decreasing as we move north-eastward along the contract. To keep  $u_r/\phi'$  constant,  $u_r$  must decrease with  $\ell$  as well. This means that the locus where  $u_r/\phi'$  is constant must cut  $\bar{u}_r$  from below. A contract with  $u_r/\phi'$  constant is thus one with underemployment. Combining both goals in the second-best problem will still produce underemployment.

This seems to be the appropriate point at which to relate this model to the principal-agent literature and to the problem of optimal income taxation. In our problem the "agent" is the firm who has proprietary information. With a risk neutral agent we expect full efficiency to be feasible. But here, the "effort" of the agent, choosing  $\ell$ , enters directly into the principal's welfare and not only indirectly through its influence on "output", which here is total

revenue to be shared ( $\theta g(l(\theta))$ ). It is this composition of an externality problem with an incentive problem that gives the model its second-best character.

Comparison with the optimal income tax literature is more difficult.\* There are, indeed, many more similarities than differences. If we think of  $\theta$  as distinguishing various types of individuals according to their productivity, then the optimal tax problem is to find a schedule of taxes to maximize

$$\int u(r, l) d\theta \quad \text{such that} \quad E\{\theta l - r\} \geq C$$

Here  $r$  is net income and so  $\theta l - r$  is tax received from individuals of type  $\theta$ .

The firm in our problem under different circumstances  $\theta$  is like the workers in an optimal income tax problem with different levels of ability. The constraint of keeping workers' expected utility above a fixed level corresponds to the constraint of raising a fixed amount of revenue from the income tax. The firm's choice of  $l$  along a fixed  $(r, l)$  schedule is like the workers' choice of when faced with a fixed relation between before and after tax income.

In the taxation literature there is no direct way of observing the individual's type and thus tax functions must rely on charging according to observable characteristics. This creates an incentive problem analogous to the one we have discussed. We must allow the firm to choose its preferred combination of  $w$  and  $l$  in each state along the contract given it; the government presents a tax schedule to its citizens and then must allow the individuals each to choose the level of work and net income they prefer along it.

Thus the problems are extremely close formally. Where then are the differences? What is the special structure of our

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\* Mirrlees (1971), (1979). Our use of the transversality condition mirrors the investigation by Seade (1977).

problem that causes overemployment to result? Why is this result sensitive to the income elasticity of leisure demand whereas it is the price elasticity that determines the departure of optimal income taxation from the first-best of lump-sum taxation of ability?

The details of the optimal tax problem differ because the parameters which are controlled by the schedule are different: In the tax system the schedule specifies net income not as a function of hours worked  $\ell$ , but as a function of gross income  $w\ell$ . This single difference is sufficient to make the tax problem sensitive not to the income elasticity of leisure, but to the price elasticity of leisure. Indeed if we considered the optimal form of a tax not on income but on hours worked, the results would be completely analogous to ours. We could say that the second-best optimal labor tax, adjusted to the incentive problem would always result in underemployment.

## 5. Conclusion

Since its beginnings, the implicit contracts literature has had the explanation of unemployment and wage rigidity as its goal. The intention was to offer a structure under which wage rigidity is optimal, and in which unemployment follows as a result. To some extent these goals were achieved, but, it is safe to say, always by introducing some special features in the contracting process that were not obviously an essential part of the model. For example, a common device is a two-period structure in which the contract operates somewhat differently in the second period than in the first.

In this paper we have given what we believe to be the first results using the implicit contracts theme which does not rely on any of these structural conditions. Paradoxically, the interaction of differential risk aversion and incomplete information is precisely the opposite of the original intention. Long term relationships between employers and workers increase employment variability, resulting in more employment that would be ex post efficient when profitability conditions are adverse. Thus, the implicit contracts theory may not yield the underpinnings for a theory of macroeconomic fluctuations.



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