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AN INTERTEMPORAL ANALYSIS OF TAXATION AND WORK DISINCENTIVES: AN ANALYSIS OF THE DENVER INCOME MAINTENANCE EXPERIMENT

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An Intertemporal Analysis of Taxation and Work Disincentives: An Analysis of the Denver Income Maintenance Experiment

ABSTRACT

This paper formulates an empirical model of consumption and labor supply that explicitly incorporates income taxes in a multiperiod setting. This model relies on few assumptions and provides a robust framework for estimating parameters needed to predict the response of consumption and hours of work to changes in a consumer's lifetime resource constraints. The empirical specifications developed here apply when a consumer is uncertain about future prices, taxes, income, and tastes, and the estimation of these specifications does not require explicit modeling of either a consumer's expectations or the history of a consumer. The empirical model accommodates both progressive and regressive tax schemes. Estimation of the model involves no complicated procedures; a full set of parameter estimates can be obtained with the application of standard two-stage least squares techniques. The final section of the paper estimates a particular specification of the model using data from the Denver Income Maintenance Experiment. The empirical formulations proposed here are particularly well suited to deal with the kinds of tax schemes used in NIT experiments and the limited duration of those programs.

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Introduction

Any analysis of the full impact of alternative tax policies must consider the varying burden of taxation over time. Tax and income transfer programs applicable to one period or part of the lifetime can be expected to influence behavior in other periods as well. The effects of these programs also are likely to have a varying impact on hours of work over the life cycle as the level and the composition of a consumer's income changes. Without considering these intertemporal issues, no analysis of labor supply can distinguish between the effects of transitory and permanent changes in tax and welfare programs nor can it evaluate the effect of anticipated changes in these programs on current behavior. Without making such a distinction, any evaluation of alternative tax policies is incomplete.

This paper formulates an empirical model of consumption and hours of work that explicitly incorporates income taxes in an intertemporal setting. A theoretical model of life cycle behavior in an uncertain environment provides the underpinning of this empirical model. The close link between theory and empirical specifications leads to the development of an economically interpretable statistical model. This empirical model is applicable in the presence of very general specifications of uncertainty, including uncertainty about future wages, prices, income, tax schedules, and "tastes." In addition, it accommodates both progressive and regressive tax schemes, as well as tax schemes that involve income averaging. It does not require the explicit specification of a tax function, but it does assume that the tax scheme may be approximated by a differentiable function. The econometric framework is designed for the analysis of prime age males, and it does not consider problems associated with labor force participation (i.e., corner solutions for hours of work).

The estimation strategy followed in this study differs from previous work. Instead of fitting labor supply or consumption equations, this paper specifies a functional form for lifetime preferences and estimates the parameters of this function directly using relations derived from the theoretical model. Estimation of these relations requires nothing more than standard linear and nonlinear two-stage least squares procedures. These estimation procedures are simple to implement and do not require explicit modeling of cohort effects or of consumers' expectations and history. While panel data are needed to estimate the most general specifications of lifetime preferences, much of the estimation can be carried out using data from a single cross section. This approach provides all the estimates needed to predict the work disincentive effects of alternative income tax and welfare reforms. In particular, for any set of constraints faced by the consumer (e.g., the lifetime wage path, tax functions, interest rates), the estimates of the lifetime preference function can be used to solve the consumer's dynamic optimization problem explicitly and to compute the consumer's response to changes in constraints.

The final section uses data from the Denver Income Maintenance Experiment to obtain a full set of estimates for the parameters of the lifetime preference function. The empirical formulation is particularly well suited for dealing with the kinds of tax schemes used in NIT experiments and the limited duration of these programs.

I. Characterizations of Consumption and Labor Supply Behavior

This section identifies simple conditions that characterize life cycle consumption and labor supply decisions. We start with the derivation of these conditions using a life cycle model of consumption and labor supply in a world without taxes where the future is uncertain. Next, we extend the model to allow for the existence of taxes. The following sections use the characterizations of behavior derived here as the basis for an empirical model.

A Life Cycle Model of Consumption and Labor Supply under Uncertainty

In the multiperiod model of lifetime consumption and hours of work developed below, there are three sources of uncertainty in future periods: real rates of interest, wages, and preferences. Thus, a consumer is uncertain about his future income, tastes, and the relative future prices of both consumption and leisure. In each period the consumer chooses his consumption, hours of work, and savings to maximize expected utility subject to asset accumulation constraints.

To characterize consumption and labor supply behavior in this model, we require specifications for preferences and asset accumulation constraints. The lifetime preference function of a consumer is assumed to be strongly separable over time with utility in period t given by the function U(C(t), L(t), Z(t)) where C(t) is a Hicks' composite commodity of all market goods, L(t) is the number of hours spent in nonmarket activities, and Z(t) is a vector of "taste shifter" variables at age t. Z(t) may include observed variables such as the number of children or unobserved variables such as "tastes for work." A lifetime is assumed to consist of at most T periods with τ being the total number of hours in each period. When discounting the value of future utility, the consumer uses a rate of time preference equal to ρ. It is assumed that a consumer can hold any combination of g different assets. The variables $A_j(t)$ and $A_j^*(t)$ denote the real value of asset j owned by the consumer at the beginning and the end of period t, respectively. Thus, $\sum_{j=1}^{g} A_j(t)$ equals the real value of a consumer's nonhuman wealth at the start of period t, $\sum_{j=1}^{g} A_j^*(t)$ is real wealth held at the end of the period, the quantity $A_j^*(t) - A_j(t)$ represents the consumer's savings in asset j in period t, and $S(t) = \sum_{j=1}^{g} (A_j^*(t) - A_j(t))$ is total savings in the period. Assuming each real dollar of asset j held at the end of period t earns a rate of interest equal to $r_j(t+1)$ at the beginning of period $t+1$, property income earned on asset j equals $Y_j(t+1) = r_j(t+1)A_j^*(t)$, and total property income in period $t+1$ is $Y(t+1) = \sum_{j=1}^{g} Y_j(t+1)$. At each age t the consumer faces a real wage rate equal to $W(t)$, and this wage rate is assumed to be unaffected by the consumer's behavior.

The vectors $\nu(t')$ defined by $\nu(t') = (Z(t'), W(t'), r_1(t'),\ldots,r_g(t'))$, $t' \geq t$, contain all the variables that are uncertain prior to period t. The random vector $\nu(t)$ is realized at the beginning of period t and is unknown prior to this time. The unknown future $\nu(t')$'s constitute the source of uncertainty in this model; so, future tastes, wages, and interest rates are uncertain. Except for the existence of several moments, the following analysis assumes nothing about the form of the distribution generating these random vectors. "Taste shifter" variables, wages, and rates of interest may be contemporaneously or serially correlated in any fashion. Whatever the form of the distribution, however, it is assumed that the consumer knows it exactly.

Formally, at each age t the consumer's problem is to choose policies for C(k), L(k), and assets $A_1^*(k),\ldots,A_g^*(k)$ for $k \geq t$, to maximize the expected value of the time-preference-discounted sum of total utility

$$(1)\qquad E_t\left\{\sum_{k=t}^{T}\frac{1}{(1+\rho)^{k-t}}U(C(k),\ L(k),\ Z(k))\right\}$$

$$= U(C(t),\ L(t),\ Z(t)) + \frac{1}{1+\rho}E_t\left\{\sum_{k=t+1}^{T}\frac{1}{(1+\rho)^{k-t-1}}U(C(k),\ L(k),\ Z(k))\right\}$$

subject to the budget constraint

$$(2)\qquad S(k) = \sum_{j=1}^{g}(A_j^*(k) - A_j(k)) = W(k)h(k) - C(k)$$

and the asset accumulation constraints

$$(3)\qquad A_j(k+1) = A_j^*(k)(1 + r_j(k+1)) \qquad j = 1,\ldots,g;\quad k = t,\ldots,T$$

where $h(k) \equiv \tau - L(k)$ is hours of work, the asset levels $A_1(t)\cdots A_g(t)$ are predetermined variables, and the "t" subscript associated with the expectation operator E_t indicates that the consumer accounts for all information available in period t when calculating expected values. The consumer must end life with nonnegative wealth, i.e., $\sum_{j=1}^{g}A_j^*(T) \geq 0.$[1] Expectations are calculated over

[1]Adding a bequest function to the lifetime preference function given by (1) avoids the need for this nonnegativity constraint, but it implies no changes in the analysis.

the random vectors $\nu(t+1),\ldots,\nu(T)$. Equation (2) represents the consumer's period k budget constraint; savings, S(k), must equal labor earnings, W(k)h(k), minus expenditure on goods consumption, C(k). Equation (3) indicates that the endowment of tangible wealth in asset j at the beginning of period k+1, $A_j(k+1)$, equals the wealth held at the end of period k, $A_j^*(k)$, plus property income earned at the beginning of period k+1, $A_k^*(k) \cdot r_j(k+1)$.

A dynamic programming or functional equation formulation of this optimization problem provides a convenient framework for characterizing period t consumption and labor supply decisions. Define the value function corresponding to period t+1 by

$$(4) \qquad V(A(t+1),\ t+1) = \max\left[U(C(t+1),\ L(t+1),\ Z(t+1)) + \frac{1}{1+\rho} E_{t+1}\left\{\sum_{k=t+2}^{T} \frac{1}{(1+\rho)^{k-t-2}} U(C(k),\ L(k),\ Z(k))\right\}\right]$$

where $A(t+1) \equiv (A_1(t+1),\ldots,A_G(t+1))$ denotes a vector indicating the consumer's holdings of each asset at the start of period t+1, and the maximization is carried out satisfying the appropriate budget and asset accumulation constraints. Given initial assets A(t+1), V(A(t+1), t+1) shows the maximum expected lifetime utility a consumer can attain in period t+1 and is analogous to an indirect utility function. Because $V(\cdot)$ depends on the realized value of $\nu(t+1)$, it is uncertain in period t. Formally, $V(\cdot)$ is a function of many other variables such as the parameters of the distribution generating future $\nu(k)$'s, but they are supressed as arguments for simplicity. As an alternative to the above formulation of the lifetime optimization problem, one can view the consumer as acting to maximize

$$(5) \qquad U(C(t),\ L(t),\ Z(t)) + \frac{1}{1+\rho} E_t\{V(A(t+1),\ t+1)\}$$

instead of (1).

For decision variables C(t), L(t), and $A_1^*(t),\ldots,A_G^*(t)$, optimization of (5) subject to equations (2) and (3) for period t (i.e., for k = t) implies the following first order conditions

$$(6) \qquad U_C(C(t),\ L(t),\ Z(t)) = \lambda(t)$$

$$(7) \qquad U_L(C(t),\ L(t),\ Z(t)) = \lambda(t)W(t)$$

$$(8) \qquad \lambda(t) = \frac{1}{1+\rho} E_t\{\lambda(t+1)(1 + r_j(t+1))\} \qquad j = 1,\ldots,g$$

where $\lambda(k)$ is the Lagrangian multiplier associated with the period k budget constraint (i.e., condition (2)), and the subscripts on U denote partial derviatives. The derivation of (8) uses the fact that $\frac{\partial V(A(t+1),\ t+1)}{\partial A_j(t+1)} = \lambda(t+1)$ for all j which follows from a straightforward application of the envelope theorem. $\lambda(t)$ is the marginal utility of wealth in period t. According to condition (6), consumption is choosen so that the marginal utility of consumption equals the marginal utility of wealth. Equation (7) determines the choice of leisure, and it indicates that at the optimum the marginal utility of leisure equals the product of the marginal utility of wealth times the real wage rate. Equation (7), of course, assumes an interior solution for hours of work in period t which is the only case considered in this paper. The equations given by (8) determine the consumer's savings allocation rule and determines how resources are allocated over time. According to (8), the consumer chooses a portfolio to equate the marginal utility of the last dollar invested in each asset.

Two relations characterize consumption: labor supply and savings behavior. Substituting (6) into (7) yields

$$(9) \qquad U_L(t) = U_C(t)W(t)$$

where $U_L(t) = U_L(C(t),\ L(t),\ Z(t))$ and $U_C(t) = U_C(C(t),\ L(t),\ Z(t))$. This condition, of course, represents the well known proposition that in equilibrium a consumer sets the marginal rate of substitution between leisure and consumption equal to the real wage rate. Using results presented in MaCurdy (1978), it is possible to show that conditions (6) and (8) imply the relation

$$(10) \qquad U_C(t) = \frac{1 + \mu(t+1)}{1 + \rho} E_t\{U_C(t+1)\}$$

where $\mu(t+1)$ is a measure of the riskless rates of interest in period t+1.[1] An optimal savings or portfolio policy, then, requires the marginal utility of wealth to follow a martingale. While conditions (9) and (10) alone do not uniquely determine the consumption and labor supply decisions in period t, they do provide simple characterizations of these decisions. These conditions form the basis for the empirical model developed in the next section.

Income Taxes

Below we incorporate income taxes into the above life cycle model of consumption and labor supply under uncertainty and determine the consequences

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[1] $\mu(t+1)$ is the minimum real rate of interest an investor requires to hold an asset at the end of period t whose rate of return is certain. Formally,

$$\mu(t+1) = E_t\{r_j(t+1)\} + \sigma_t\left(\frac{\lambda(t+1)}{E_t\{\lambda(t+1)\}},\ r_j(t+1)\right)$$

where $\sigma_t(\cdot,\cdot)$ denotes the covariance between two variables calculated using all information available in period t. This covariance term can be interpreted as the risk premium the consumer assigns to asset j.

of introducing taxes on the characterizations of behavior given by (9) and (10).

For the following theoretical development to be applicable in analyzing experimental negative income tax data, we consider tax schemes in which the payment of taxes depends on some function of the consumer's current and past incomes. In particular, suppose the payment of taxes in period t, denoted $M(t)$, is determined according to the function $M(t) = M(I(t), I(t-1), \psi(t))$ where $I(k) = W(k)h(k) + Y(k)$ is total income received in period k, and $\psi(t)$ is a vector of parameters of the tax function (including exemption parameters, etc.). The following analysis assumes only that M is continuously differentiable so that marginal tax rates associated with current and past hours of work always exist. Nothing is assumed about the progressivity or regressivity of the tax program. Since tax functions are likely to be uncertain in future periods, it is assumed that $\psi(k)$ is a random variable which is realized at the beginning of period k. In terms of the above framework, $\psi(k)$ is included as one of the elements of the random vector $\nu(k)$. $M(\cdot)$ need not be a positive tax function; it can also encompass negative income taxes or welfare payments. In negative income tax experiments, taxes are commonly computed using some average of current and past incomes. In this case the tax function simplifies to $M^*(\omega I(t) + (1-\omega)I(t-1), \psi(t))$ where ω is a fixed weight. Such a tax scheme is also relevant in the case of federal income taxes when a consumer income-averages. It is straightforward in the following analysis to generalize $M(\cdot)$ so that it also depends on incomes prior to period t-1 (e.g., $I(t-2)$, $I(t-3)$, etc.).[1]

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[1] It is also possible to modify $M(\cdot)$ so that earnings, $W(t)h(t)$, are taxed differently from property income $Y(t) = \sum_{j=1}^{g} Y_j(t)$.

Modifying constraints and the value function to account for the presence of taxes permits the derivation of the implied optimality conditions. The existence of taxes does not alter the form of the asset accumulation constraints given by (3), but it does imply new budget constraints. Instead of (2), we have

$$(11) \qquad S(t) = W(t)h(t) - C(t) - M(t)$$

for each t.

The value function relevant for the consumer's optimization problem is different from the one given by (4). In addition to the arguments A(t+1), one must also add I(t) to the list. This adjustment reflects the fact that the consumer's past income influences current tax payments and, thus, affects the maximum expected lifetime utility a consumer can attain in period t+1. The consumer's objective function, then, is

$$(12) \qquad U(C(t),\ L(t),\ Z(t)) + \frac{1}{1+\rho} E_t\{V(A(t+1),\ I(t),\ t+1)\}$$

where V(A(t+1), I(t), t+1), or simply V(t+1), is computed by maximizing the right-hand side of (4) subject to the appropriate budget and asset accumulation constraints.

Optimization of (12) subject to budget constraint (11) and asset accumulation constraints (3) with respect to C(t), L(t), and $A_1^*(t),\ldots,A_g^*(t)$ yields first order conditions given by (6) and

$$(13) \qquad U_L(t) = [\lambda(t)(1 - m_t(t)) - \frac{1}{1+\rho} E_t\{\lambda(t+1)m_t(t+1)\}]W(t)$$

$$(14) \qquad \lambda(t) = \frac{1}{1+\rho} E_t\{\lambda(t+1)(1 + r_j^*(t+1))\} \qquad j = 1,\ldots,g$$

where $m_k(j) = \frac{\partial M(j)}{\partial I(k)}$ is the marginal tax rate in period j associated with an increase in period k income, and $r_j^*(t+1) \equiv [1 - m_{t+1}(t+1) - \frac{1}{(1+\rho)\lambda(t+1)} \cdot E_{t+1}\{\lambda(t+2)m_{t+1}(t+2)\}]r_j(t+1)$. The derivation (13) and (14) uses the facts that $\frac{\partial V(t+1)}{\partial I(t)} = -\lambda(t+1)m_t(t+1)$ and $\frac{\partial V(t+1)}{\partial A_j(t+1)} = \lambda(t+1)[1 - m_{t+1}(t+1)\frac{r_j(t+1)}{1+r_j(t+1)}] - \frac{1}{(1+\rho)} E_{t+1}\{\lambda(t+2)m_{t+1}(t+2)\}\frac{r_j(t+1)}{1+r_j(t+1)}$ which follow from applications of the envelope theorem. The variable $r_j^*(t+1)$ may be interpreted as the real, after-tax rate of return the consumer associates with asset j in the sense that a consumer is indifferent on the margin between holding asset A_j and another asset that earns a tax free interest rate equal to $r_j^*(t+1)$.

Using results presented in MaCurdy (1978), it is possible to derive characterizations of behavior that closely resemble those obtained above. The martingale property for the marginal utility of consumption given by (10) still holds with $\mu(t+1)$ reinterpreted as a measure of a tax-free, riskless rate of interest. One may write condition (13) as

$$(15) \qquad U_L(t) = U_C(t)[1 - m_t(t) - R(t+1)\ E_t\{m_t(t+1)\}]W_t$$

where $R(t+1)$ is a discount rate equal to $\frac{1}{1+\bar{r}(t+1)}$ and $\bar{r}(t+1)$ is a real, after-tax, risk adjusted rate of interest. In the case where the future marginal tax rate $m_t(t+1)$ is known with certainty in period t, direct substitution of (6) and (10) into (13) yields (15) with $\bar{r}(t+1) = \mu(t+1)$. In the general case, however, when the marginal tax rate is uncertain, $\bar{r}(t+1)$ equals $\mu(t+1)$ plus a risk premium that accounts for the marginal riskiness of $m_t(t+1)$.[1] In any case, the term $\left[1 - m_t(t) - R(t+1)E_t\{m_t(t+1)\}\right]W(t)$

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[1]Formally, $\bar{r}$ is defined as $\bar{r} = \mu + \sigma_t(-\frac{\lambda(t+1)}{E_t(\lambda(t+1))}, \frac{m_t(t+1)}{\bar{m}_t(t+1)})$ where $\sigma_t(\cdot,\cdot)$ denotes the covariance between two variables calculated using all information available in period t, and $\bar{m}_t(t+1) = \frac{1}{1+\bar{r}(t+1)} E_t\{m_t(t+1)\}$. This covariance term represents a marginal risk premium. See MaCurdy (1978) for details.

represents the marginal wage rate, or more precisely, the expected present value of the real marginal, after-tax wage rate which is suitably discounted for risk. Condition (15) simply indicates that a consumer chooses hours of work and consumption to equate the marginal rate of substitution to the marginal wage rate.

In a world with taxes and uncertainty, then, conditions (10) and (15) characterize consumption and labor supply decisions in period t. These conditions provide the basis for the empirical models developed below. It is important to emphasize that equations (10) and (15) do not, in general, imply a unique solution for consumption and hours of work in the regressive tax case. At each point where these equations are satisfied, the consumer must compare the implied expected lifetime utilities in order to determine which point is optimal. The following analysis only uses the fact that (10) and (15) are necessary conditions.

II. Estimation Strategies

This section outlines alternative strategies for estimating parameters needed to describe behavior in the model proposed above, and it presents an overview of the procedure used in this paper.

To simplify the discussion in this section, it is assumed that the tax function is intertemporally separable or, equivalently, that period t taxes depend only on period t income. As a consequence of this assumption, $m_t(t+1) = 0$ in (15) and the marginal wage rate becomes $\omega(t) = (1 - m_t(t))W(t)$ which is assumed to be directly observable in the following discussion. We return to the general case in the next section where a complete specification of the empirical model estimated in this paper is presented.

The approach typically found in demand analyses for estimating the parameters required to predict a consumer's behavior is to esimate directly the parameters of demand functions. These functions relate consumption and leisure to all of the "exogenous" or nonchoice variables of the optimization problem, which in a static analysis includes such variables as income and current prices, and in a life cycle analysis also includes past and future prices as well as measures of wealth.

An approach of this nature is infeasible for the problem outlined in the previous section for two reasons. First, it is not possible to derive explicit expressions relating consumption and leisure at any age to the "exogenous" variables of the model (e.g., the parameters of the distribution of future wages and income), even for the most elementary specifications of preferences, taxes, and uncertainty. Without such expressions, one cannot obtain explicit relations for the familiar type of demand functions, and it is not clear what approximations for these demand functions are even appropriate.

A second major difficulty encountered in the implementation of existing life cycle approaches relates to the unavailability of data on both the retrospective and prospective information used by each consumer to determine observed current decisions. At best, data sets provide very limited information on variables needed to construct a consumer's lifetime constraints (e.g., future and past wages, and initial values of wealth). Without this sort of information, it is not possible to use existing empirical models to obtain a full set of parameter estimates needed to describe consumption or labor supply behavior in a lifetime framework. Invariably, the generation of information on a consumer's lifetime wage and income paths involves the imposition of inherently arbitrary assumptions.

The empirical models discussed below avoid both the above difficulties. Estimation does not require explicit expressions for the demand functions of the sort described above. Furthermore, the following empirical specifications and econometric techniques rely on minimal assumptions concerning the constraints faced by consumers outside the sample period on which an analyst has no data. One needs no information on the consumer's realized or anticipated opportunities in past or future periods, and one can permit the existence of general forms of cohort effects. Estimation can be carried out using only data that is directly observed within the sample period.

The strategy followed in this paper is to specify a functional form for preferences and to use the equations characterizing optimal behavior given by (10) and (15) to estimate the unknown parameters of U and the rate of preference directly. These estimates provide all the information a researcher requires to construct the lifetime preference function whose expected value is given by (1). Given this function and assumptions needed to formulate the constraints faced by a consumer (e.g., the lifetime wage path, interest rates, tax functions, the form of uncertainty, etc.), a researcher can literally solve the consumer's optimzation problem and can compute the consumer's response to various changes in constraints. Given a consumer's measured characteristics, then, the estimation procedure discussed below provides essentially all the estimates a researcher requires to describe or to analyze any aspect of consumption or labor supply behavior in the theoretical setting outlined above.

<u>The Use of Pseudo Demand Functions</u>

Work on "two stage budgeting" and "decentralizability" in the literature on consumer demand and separability of preferences provides one strategy for estimating the parameters of the period specific preference function U(·) up to a monotonic transformation.[1] Following this literature one defines demand functions for consumption and leisure that depend only on variables observed within a single period. In the absence of taxes, this approach uses equation (9) and the budget constraint given by (2) (i.e., $S(t) = W(t)h(t) - C(t)$ when there are no taxes) to solve for C(t) and L(t) as functions of the form $C(t) = C(W(t), S(t), Z(t))$ and $L(t) = L(W(t), S(t), Z(t))$. These demand functions are completely analogous to those obtained if the consumer's optimization problem were purely static with S(t) held fixed. Using one of the many parameterizations of static demand functions available in the literature, one can develop empirical specifications for C(·) and L(·) with

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[1]For a concise survey of the theoretical work in this literature, see Blackorby-Primont-Russell (1975).

$-S(t)$ playing the role of exogenous property income.[1] The only implication of life cycle theory that affects the estimation of these specifications concerns the fact that $S(t)$ is a choice variable. Thus, to account for life cycle factors in estimating the parameters of $C(\cdot)$ and $L(\cdot)$, one must apply simultaneous equation techniques that treat $S(t)$ as an endogenous variable.

To estimate the parameters of $C(\cdot)$ and $L(\cdot)$ when income taxes are present, essentially the same approach applies. Observing that budget constraint (11) can be written as $\tilde{S}(t) = \omega(t)h(t) - C(t)$ where $\tilde{S}(t) = S(t) + M(t) - (W(t) - \omega(t))h(t)$ and $\omega(t) = (1 - m_t(t))W(t)$, it is easy to verify that the implied solution for $C(t)$ and $L(t)$ with income taxes is $C(t) = C(\omega(t), \tilde{S}(t), Z(t))$ and $L(t) = L(\omega(t), \tilde{S}(t), Z(t))$. There is no change in the functional form of $C(\cdot)$ and $L(\cdot)$, but the arguments of these functions do change. Estimation in this case requires one to treat both $\omega(t)$ and $\tilde{S}(t)$ as endogenous. It is important to emphasize that the parameters of the demand functions $C(\cdot)$ and $L(\cdot)$ are not of much interest in their own right unless $S(t)$ is in fact exogenous. The coefficient in $L(\cdot)$ associated with the wage rate, for example, does not measure the response of $L(\cdot)$ to a change in $W(t)$ because $S(\cdot)$ also simultaneously adjusts. Using the parameter estimates and the specification of $C(\cdot)$ and $L(\cdot)$, however, it is possible to construct the implied period specific utility functions $U(\cdot)$ up to a monotonic transformation which provides an essential component of the information needed to form the lifetime preference function.

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[1] This, of course, is not a new idea. The ability to define demand functions of this sort is known as decentralizability in the literature on consumer demand and separability of preferences (see Blackorby-Primont-Russell (1975) and comment that follows by Phlips). Discussion of this point originates with the work of Strotz (1957, 1959) and Gorman (1959) on two-stage optimization. In the literature on labor supply, this point is discussed by Heckman (1974b). To my knowledge, there is no study that estimates demand functions of the sort considered above using micro data.

This treatment of taxes is equivalent to procedures found in many recent static empirical studies of taxes and labor supply. Hall (1973) noted that, in the presence of nonlinear tax programs, one can represent the consumer as facing a linear budget constraint that is tangent to his actual nonlinear budget set at the observed hours of work. Thus, one can in effect solve for a structural labor supply function which assumes a linear budget constraint where the slope and the intercept of the linearized constraint play the roles of the wage rate and property income, respectively. In terms of the above notation, $\omega(t)$ represents the slope and $-\tilde{S}(t)$ corresponds to the intercept. Using this structural labor supply function as the basis for an empirical specification to estimate parameters needed to describe hours of work behavior significantly simplifies the estimation problem because it does require the analyst to obtain a closed form solution to the consumer's optimization problem.[1, 2] This approach for dealing with taxes in a labor supply analysis is used by Hausman-Wise (1976), Hurd (1976), Rosen (1976), Johnson-Pencavel (1980), Burtless-Hausman (1978) and Hausman (1979). Even if

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[1]A closed form solution here refers to solving for hours of work as a function of all the exogenous variables of the optimization problem including those variables determining the budget set such as the gross wage rate, income, and parameters of the tax function. Due to nonlinearities in the budget constraint, it is difficult to obtain closed form solutions of the type for the labor supply function except for the most trivial specifications of preferences and tax functions.

[2]It is important to emphasize that estimates of the parameters of the structural labor supply function which assumes a linear budget constraint cannot be used directly to predict a consumer's response to shifts in the budget set; the arguments of this function are themselves choice variables. As noted above, however, the estimates of this labor supply function can be used to construct the implied utility function which provides all the information needed to predict responses to any change in opportunities.

the assumptions of the static model are satisfied and $W(t)$ and $S(t)$ are exogenous, it is important to recognize that one must treat both $\omega(t)$ and $\tilde{S}(t)$ as endogenous variables in estimation since these variables are functions of marginal tax rates which directly depend on hours of work. Hall (1973), Hausman-Wise (1976), and Rosen (1976) attempt to account for this endogeneity by evaluating marginal tax rates at a fixed number of hours of work for everyone in the sample and using these tax rates to compute $\omega(t)$ and $\tilde{S}(t)$. As a consequence, the values of $\omega(t)$ and $\tilde{S}(t)$ used in their empirical analyses are not those observed for individuals in the sample which induces a nonclassical errors in the variables problem and inconsistent parameter estimates. Burtless-Hausman (1978), Hausman (1979), Hurd (1976), and Johnson-Pencavel (1980) do properly account for the endogeneity of $\omega(t)$ and $\tilde{S}(t)$. Burtless-Hausman (1978) and Hausman (1979) develop estimation techniques based on models of discrete choice and maximum likelihood to solve this endogeneity problem when budget constraints may be characterized as a set of linear segments. Hurd (1976) and Johnson-Pencavel (1980) account for the endogeneity of marginal tax rates in the case of differentiable constraints by means of instrumental variable and maximum likelihood procedures which are typically found in the analysis of simultaneous equations. All of the above studies fail to account for decision making in an intertemporal setting because they use "exogenous" nonwage income (i.e., $Y(t)$) when constructing the linearized income term $\tilde{S}(t)$ rather than using the net savings variable $S(t)$ which is implied by the above theory.

Standard two-stage least squares procedures provide the easiest methods for estimating the parameters of the functions $C(\cdot)$ and $L(\cdot)$. To apply these procedures, one needs only cross section data on consumption, hours of work, marginal tax rates, wage rates, and a valid set of instruments for the variables $\omega(t)$ and $\tilde{S}(t)$. Using these data, it is possible to construct the variables $\omega(t)$ and $\tilde{S}(t)$ so that they are in fact observed. Thus, one may treat the relation $C(t) = C(\omega(t), \tilde{S}(t), Z(t))$ and $L(t) = L(\omega(t), \tilde{S}(t), Z(t))$

as structural equations and estimate their parameters using standard simultaneous equation techniques. If these equations are linear in parameters but nonlinear in variables, then one can apply linear two-stage least squares procedures of the sort considered by Kelijian (1971).[1] If, on the other hand, there are nonlinearities both in parameters and in variables, then one can apply the nonlinear two-stage procecures of Amemiya (1974, 1977). An attractive feature of these limited information estimation methods is that they do not require the imposition of any assumptions concerning a consumer's expectations or history in order to carry out estimation. Furthermore, these approaches do not require the explicit specification of tax functions, and they can be applied when tax schemes are either regressive and progressive. Appendix A describes these estimation procedures in some detail and compares them with more familiar procedures found in the empirical literature on taxes and labor supply.

Estimating the Marginal Rate of Substitution Function

Assuming that the period specific utility function, $U(\cdot)$, is a monotonic transformation of $U^*(t)$ which has a known functional form, a second method for estimating the parameters of $U^*(t)$ is to formulate an empirical framework using equation (15) which relates the marginal rate of substitution to the marginal wage rate. Taking natural logs of (15) yields

$$(16) \qquad \ln U_L(t) - \ln U_C(t) = \ln \omega(t)$$

[1]As long as a structural equation is linear in parameters, Kelijian (1971) shows that any set of instruments may be used to predict endogenous variables in the application of two-stage least squares assuming that one includes all the exogenous variables appearing in the structural equation as instruments.

where $\omega(t) = (1 - m_t(t))W(t)$ is an observed variable for the case considered here. Given $U^*(t)$, it is possible to derive an explicit parameterization for the log of the marginal rate of substitution function, $\ell n\ U_L(t) - \ell n\ U_C(t)$, or, equivalently, $\ell n\ U_L^*(t) - \ell n\ U_C^*(t)$. Besides the parameters of $U^*(t)$, $\ell n\ U_L^*(t) - \ell n\ U_C^*(t)$ will also be a function of $C(t)$, $L(t)$ and $Z(t)$ which includes both measured and unmeasured characteristics that affect a consumer's "tastes."[1]

Equation (16) will, in general, be in the form of a nonlinear simultaneous equation. Assuming a specification of $U^*(t)$ that implies $\ell n\ U_L^*(t) - \ell n\ U_C^*(t)$ is linear in disturbances, it is possible to estimate the parameters of this equation and of $U^*(t)$ using standard linear or nonlinear two-stage least squares procedures of the sort mentioned above. In the application of these estimation procedures the variables $L(t)$, $C(t)$ and $\ell n\ \omega(t)$ all must be considered as endogenous. The next section introduces explicit functional forms for $U^*(t)$ and discusses the estimation of equation (16) in greater detail. Estimation procedures are also developed to allow for a tax function that is not intertemporally separable. For a complete discussion and justification of the applicability of two-stage least squares procedures as a method for estimating equation (16), the interested reader should refer to Appendix A.

The use of equation (16) as the basis for an empirical specification to estimate the parameters of $U^*(t)$ offers many attractive features. As in the case of the pseudo demand functions discussed above, estimation can be carried out using a cross section of individuals and procedures that are simple to implement. There is no need to assume anything about the consumer's past or future. The estimation

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[1] It is important to sharply distinguish the marginal rate of substitution (i.e., MRS) function referred to here from those appearing in previous labor supply studies, such as Heckman (1974a, 1974b). Assuming interior solutions for hours of work, the MRS function in previous studies is equivalent to an inverted labor supply function; the wage rate or the MRS is written as a function of hours of work, income, and other prices. Here, the MRS function is simply the ratio of marginal utilities and has the same arguments as the utility function.

procedure does not require the explicit specification of a tax function, and it can be applied no matter what the shape of this tax function (e.g., it may be convex or concave).[1] The use of equation (16) does not require one to solve for the demand functions for either consumption or leisure to carry out the estimation procedure; and, as a result, it avoids several of the difficulties associated with the alternative approach discussed above which estimates demand functions $C(\cdot)$ and $L(\cdot)$. Whereas this alternative estimation strategy starts with specifications of $C(\cdot)$ and $L(\cdot)$ and uses the estimates of these functions to construct $U^*(t)$, the approach employed below starts with a specification of $U^*(t)$ and estimates its parameters directly using a transformation of equation (16). Since the demand functions $C(\cdot)$ and $L(\cdot)$ are not of much interest in their own right with regard to predicting a consumer's behavior in an intertemporal setting, there are obvious advantages to using an estimation procedure that does not require their specification. In particular, one is not restricted to consider only preference functions for which analytical solutions for demand equations exist or for which the implied demand equations are linear in disturbances. Thus, it is possible to consider a richer class of preference functions. This is particularly true when one enlarges the number of commodities admitted into the analysis.

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[1]For the regressive tax case, one requires conditions in addition to those provided in the previous section in order to characterize a consumer's equilibrium fully. In particular, one must compare lifetime utility at each point of "potential equilibrium" (i.e., where (16) is satisfied). The empirical model of this study only uses the fact that (16) must be satisfied at the equilibrium. See Appendix A for details.

Choosing a Monotonic Transformation

To construct the lifetime preference function whose expected value is given by (1), we require an explicit parameterization for discounted utility associated with each period or age k. We know that this utility is given by the function $G(U^*(k), Z(k)) \equiv \frac{1}{(1-\rho)^{t-k}} F(U^*(k), Z(k))$ where G and F are monotonic increasing transformations of $U_i^*(k)$, and Z(k) represents a consumer's characteristics that affect tastes. In contrast to the typical single period-static analysis, the choice of the transformation G(·) or F(·) is not irrelevant. Changing the form of G(·) in general alters preferences and implies different behavior.[1]

If one is willing to assume a known form for F(·) and a known value for the rate of time preference ρ, then either of the two estimation strategies outlined above providesall the information required to construct the lifetime preference function whose expected value is given by (1). Under these assumptions, it is possible to obtain a full set of estimates of the lifetime preference function using data from a single cross section only. For general specifications of G(·) in which there are unknown parameters, one must turn to panel data in order to obtain a complete set of estimates. Using panel data, it is possible to estimate the parameters of G(·) without resorting to any assumptions concerning the absence of cohort effects.

The condition that identifies the appropriate functional form for the monotonic transformation is equation (10) which characterizes savings behavior and the intertemporal allocation of resources. According to this condition, we have

$$F'(t)U_C^*(t) = \frac{1+\mu(t+1)}{1+\rho} E_t\{F'(t+1)U_C^*(t+1)\}$$

where F'(k) is the derivative of F(·) with respect to U*(k), and $U_C^*(k)$ is the partial derviative of U*(k) with respect to C(k). This condition implies the

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[1]Obviously, behavior is unaffected by taking monotonic transformations of the entire lifetime preference function.

existence of the relation

(17) $$F'(t+1)U^*_C(t+1) = \frac{1+\rho}{1+\mu(t+1)} F'(t)U^*_C(t)[1 + \upsilon(t+1)]$$

where $\upsilon(t+1)$ is a forecast error which is uncorrelated with $\frac{1+\mu(t+1)}{1+\rho} F'(t)U^*_C(t)$.[1] Taking the natural logs of (17) yields

(18) $$\ln(F'(t+1)U^*_C(t+1)) = b_{t+1} + \ln(F'(t)U^*_C(t)) + \eta(t+1)$$

where $b_{t+1} = \ln \frac{1+\rho}{1+\mu(t+1)} + E_t\{\ln(1 + \upsilon(t+1))\}$, and $\eta(t+1)$ is an error term with zero mean. Given the special assumptions that $\mu(t+1)$ and the distribution generating the forecast errors, $\upsilon(t+1)$, are the same for different consumers,[2] then b_{t+1} is constant across consumers and may be considered a parameter.

Given an assumed functional form for $F(\cdot)$, it is in principle possible to use equation (17) or (18) as the basis for an empirical specification to estimate the parameters of F and $\frac{1+\rho}{1+\mu(t+1)}$ or b_{t+1} given the availability of panel data. One can compute values for the variables $U^*(t)$ and $U^*_C(t)$ for each individual in the sample using the parameter estimates and the residuals obtained from the estimation of equation (16). When these estimated variables are substituted into equations (17) and (18), the only unknown parameters of these

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[1] Thus, taking the expected value of the left-hand side of (17) yields

$$\begin{aligned} E_t\{F'(t+1)U^*_C(t+1)\} &= E_t\{\frac{1+\rho}{1+\mu(t+1)} F'(t)U^*_C(t)[1+\upsilon(t+1)]\} \\ &= \frac{1+\rho}{1+\mu(t+1)} F'(t)U^*_C(t)E_t\{(1+\upsilon(t+1))\} \\ &= \frac{1+\rho}{1+\mu(t+1)} F'(t)U^*_C(t) \end{aligned}$$

which is the result required by condition (10).

[2] Assuming the same distribution generates forecast errors implies that $E_t\{\ln(1+\upsilon(t+1))\}$ is constant across consumers. This is undoubtedly a strong assumption. One would in general expect the moments of the prediction errors associated with equation (17) to depend on a consumer's measured characteristics and on C(t) and L(t).

equations are $\frac{1+\rho}{1+\mu(t+1)}$ or b_{t+1} and those needed to construct F. Assuming a functional form for F(·) which implies that either (17) or (18) is linear in disturbances, one can apply a standard nonlinear two stage least squares procedure to estimate the parameters of his equation. In the following empirical analysis, F is assumed to depend on a consumer's unmeasured characteristics (i.e., disturbances) and to possess a functional form so that equation (18) is linear in disturbances, but equation (17) is not. While estimation of equation (18) under these circumstances provides estimates needed to construct F(·), it does not supply an estimate of the rate of time preference ρ. Thus, in addition to the estimates obtained from the estimation of equations (16) and (18), we require a priori information on the value of ρ in order to formulate a complete specification for the lifetime preference function given by (1).

III. Specifying an Empirical Model

This section introduces explicit specifications for U*(t) and the transformation F(·) which are needed to implement the estimation strategy outlined above, and it discusses methods of estimation and the consequences of measurement error.

A Specification for the Marginal Rate of Substitution Function

Assume that consumer i at age t has utility given by any monotonic transformation of the function

$$U_i^*(t) \equiv \gamma_i(t) \frac{(C_i(t) + \theta_1)^{\alpha_1} - 1}{\alpha_1} - \frac{(h_i(t) + \theta_2)^{\alpha_2}}{\alpha_2} \tag{19}$$

where $\gamma_i(t)$ is an age specific modifier of "tastes," and θ_1, θ_2, α_1 and α_2 are fixed parameters. The following analysis assumes that $\gamma_i(t)$ is related to a consumer's characteristics according to the relation $\gamma_i(t) = \exp\{-X_i(t)\beta - \varepsilon_i(t)\}$ where $X_i(t)$ is a vector of measured characteristics, β is a vector of parameters, and $\varepsilon_i(t)$ is a disturbance reflecting the contribution of unmeasured characteristics.[1] $C_i(t)$ is assumed to be a Hicks' composite measure of consumption of the form $C_i(t) = C_{1i}(t) + \kappa D_i(t)$ where $C_{1i}(t)$ represents the expenditure on nondurable goods, $D_i(t)$ is the value of durables owned by consumer i at age t, and κ is the real rate of return associated with durables.

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[1] In the following analysis, one could replace the assumption that α_1, α_2, θ_1, and θ_2 are constants with one in which α_1, α_2, θ_1, and θ_2 are nonrandom functions of $X_i(t)$.

The natural log of the marginal rate of substitution between leisure and consumption associated with (19) is

$$(\alpha_2 - 1)\ln(h_i(t) + \theta_2) - (\alpha_1 - 1)\ln(C_i(t) + \theta_1) - \ln \gamma_i(t).$$

Combining this specification with equation (16) and the above assumptions concerning "tastes" implies the empirical relations

$$\text{(20)} \qquad \ln \omega_i(t) = X_i(t)\beta + (\alpha_2 - 1)\ln(h_i(t) + \theta_2) - (\alpha_1 - 1)\ln(C_i(t) + \theta_1) + \varepsilon_i(t)$$

where

$$\text{(21)} \qquad \ln \omega_i(t) = \ln\left[1 - m_{ti}(t) - R \quad E_t\{m_{ti}(t+1)\}\right] + \ln W_i(t),$$

and one may treat the discount rate R as either a parameter or a known constant. Equation (20) constitutes an explicit parameterization for equation (16). Estimation of (20) yields a complete set of parameter estimates for the preference function $U_i^*(t)$ given by (19).

A Treatment for Taxes

Consider first the case in which the tax function is intertemporally separable and $m_{ti}(t+1) = 0$ in (21). Few problems arise for this case because data exist for marginal tax rates and for wages, so $\omega_i(t)$ is directly observed.[1] Accounting for the endogeneity of $\omega_i(t)$ in the estimation of equation (20) obviously does not require the introduction of an explicit empirical specification for this variable. Given cross section data on hours of work, consumption,

[1] When data do not exist for marginal tax rates, they can typically be computed or estimated using data on total taxes and income.

marginal wages, a consumer's characteristics, and a valid set of instruments, one can apply a standard nonlinear two-stage least squares procedure to estimate the parameters of equation (20) treating the variables $h_i(t)$, $C_i(t)$, and $\ell n\ \omega_i(t)$ as endogenous. If one is willing to assume that the parameters θ_1 and θ_2 are known, then a standard two-stage least squares procedure can be employed using an arbitrary set of instruments to predict the variables $\ell n(h_i(t) + \theta_2)$ and $\ell n(C_i(t) + \theta_1)$.

When the tax function is not separable and $m_{ti}(t+1) \neq 0$, one must introduce a prediction equation for future marginal tax rates in order to carry out the estimation of equation (20). The problem, of course, is that $\omega_i(t)$ cannot be directly observed in this case. The variable $E_t\{m_{ti}(t+1)\}$ must be replaced by a measured quantity or by an estimable relationship in order to apply any estimation scheme. The following empirical analysis assumes that $E_t\{m_{ti}(t+1)\} = \pi\ Q_i(t)$ where π is a parameter vector, and $Q_i(t)$ is a vector of measured variables observed prior to the future period t+1 for consumer i, including the individual's current and past tax rates, choices of leisure and consumption, wage rates, and interactions involving these variables. In short, $Q_i(t)$ represents the information set available to consumer i in period t, and the linear combination $\pi\ Q_i(t)$ is exactly how the consumer uses this information set to predict future tax rates. Given rational expectations, we have

(22) $$m_{ti}(t+1) = \pi\ Q_i(t) + e_i(t+1)$$

where $e_i(t+1)$ is a forecast error which is uncorrelated with $Q_i(t)$. Since data exist on $m_{ti}(t+1)$ and on $Q_i(t)$, it is possible to estimate equation (22) by least squares and form an estimated quantity for $E_t\{m_{ti}(t+1)\}$.

Substituting $\hat{\pi} Q_i(t)$ into (21) and assuming that the discount rate R is known allows one to compute a value for $\ell n\ \omega_i(t)$ for each individual. Given data on $\ell n\ \omega_i(t)$, it is possible to apply the two-stage least squares procedures described above for the separable tax case to estimate the parameters of equation (20).[1] This is the estimation method used in the following empirical analysis. While this method produces consistent estimates for the parameters of equation (20) and of the function $U^*(t)$, it is important to emphasize that the standard errors reported by a conventional nonlinear or linear simultaneous equation program are not valid asymptotically because they fail to account for the fact that $\hat{\pi} Q_i(t)$ is an estimated quantity. As a result, the test statistics reported below for the nonseparable tax case must be interpreted with care.[2]

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[1]Amemiya (1974) in his original article considered nonlinear simultaneous equations of the form $y_1 + f(y_2, x, \theta) = \varepsilon$ where the y's are endogenous variables, x is an exogenous variable, θ is a parameter vector, ε is a classical disturbance, and $f(\cdot)$ is a known function. His analysis, however, goes through without modification if one replaces $y_1 + f(\cdot)$ by a known nonlinear function $g(y_1, y_2, x, \theta)$. Equation (20) has this form if R is assumed to be unknown. It is possible to estimate R along with the other parameters using nonlinear two-stage procedures.

[2]While a method that simultaneously estimates equations (20) and (22) is desirable on the grounds that it will report asymptotically correct standard errors for all parameters, difficulties arise in the implementation of such a procedure. It is important to recognize that $Q_i(t)$ contains valid instruments for equation (22), but not for (20). Thus, in the estimation of (20) one must treat many of the elements of $Q_i(t)$ as endogenous variables. Unless a large set of valid instruments is available, the simultaneous estimation of equations (20) and (22) is not a straightforward task.

A Specification for the Monotonic Transformation

Assume that the period specific utility function for consumer i at age t, $U_i(t) = U(C_i(t), h_i(t), Z_i(t))$, is given by

$$(23) \qquad F_i(U_i^*(t)) = \Omega_i(t) \frac{(U_i^*(t) + \delta)^{\sigma} - 1}{\sigma}$$

where $\Omega_i(t)$ is an age specific modifier of tastes, and σ and δ are fixed parameters. $\Omega_i(t)$ is related to a consumer's characteristics by the equation $\Omega_i(t) = \exp\{X_i(t)\phi + a_i(t)\}$ where $X_i(t)$ is a vector of measured characteristics, ϕ is a parameter vector, and $a_i(t)$ is an error term reflecting the contribution of unmeasured characteristics. Given specifications (19) and (23), equation (18) implies the empirical specification

$$(24) \qquad \ell nU_{Ci}^*(t+1) - \ell nU_{Ci}^*(t) = b - (X_i(t+1) - X_i(t))\phi$$
$$+ (1-\sigma)[\ell n(U_i^*(t+1) + \delta) - \ell n(U_i^*(t) + \delta)]$$
$$+ f_i(t+1)$$

where $U_i^*(k)$ is defined by (19), $U_{Ci}^*(k) = \gamma_i(k)(C_i(k) + \theta_1)^{\alpha_1 - 1}$, and $f_i(t+1)$ is a disturbance.

If a valid set of instruments and data on the variables $U_{Ci}^*(t+1)$, $U_{Ci}^*(t)$, $U_i^*(t+1)$, and $U_i^*(t)$ were available, one could obtain estimates of the parameters b, ϕ, δ, and σ by applying a nonlinear two-stage least squares procedure to (24) treating the variables $U_i^*(t+1)$ and $U_i^*(t)$ as endogenous. While data are not available on these variables, they can be estimated for each individual in the sample using panel data and the parameter estimates from the estimation of equation (20). This approach is used in the following empirical analysis. In particular, given estimates for α_1, θ_1, α_2, and θ_2, an estimate for $\gamma_i(t)$ is constructed using the relation

$$(25) \qquad \gamma_i(t) = \exp\{(\alpha_2 - 1)\ln(h_i(t) + \theta_2) - (\alpha_1 - 1)\ln(C_i(t) + \theta_1) - \ln \omega_i(t)\}$$

which follows from equation (20). These estimates combined with data on consumption and hours of work allows one to form estimated values for $U^*_{Ci}(t)$ and $U^*_i(t)$. Inserting these estimated values into equation (24), this equation is estimated by two-stage least squares. This procedure produces consistent estimates for the parameters of the transformation function $F(\cdot)$ (i.e., ϕ, δ, and σ) but, as in the case described above, it is important to recognize that the standard errors reported by this estimation procedure are asymptotically invalid because there is no adjustment for the fact that the data being used depend on estimated coefficients.[1]

Measurement Error

Introducing the existence of measurement error into the above analysis implies serious consequences. In the estimation of equation (20), one can allow for error in measuring the endogenous variables and still obtain consistent parameter estimates, but this measurement error must enter equation (20) as a classical errors in the variables scheme. Thus, the marginal wage, $\omega_i(t)$, for example, may be measured with error, but it must be related to the true value of the marginal wage $\bar{\omega}_i(t)$ by the equation $\ln \omega_i(t) = \ln \bar{\omega}_i(t) + \xi_i(t)$ where $\xi_i(t)$ is a randomly distributed error term which is distributed independently of all variables used as instruments. The same kinds of schemes

[1] Also similar to the above case, it is a nontrivial task to estimate jointly equations (20) and (24) in order to compute the correct standard errors. Equation (24) is nonlinear in disturbances.

are required for the variables $\ell n(h_i(t) + \theta_2)$ and $\ell n(C_i(t) + \theta_1)$ if consistency of the estimates is to be preserved. This is a strong assumption.[1, 2]

In the estimation of equation (24), it is difficult to admit the presence of any measurement error, especially with regard to wages and marginal tax rates. For two-stage least squares procedures to produce consistent parameter estimates, it is necessary to assume that the error term $\varepsilon_i(t)$ appearing in equation (20) contains no measurement error components and represents only variables omitted from the specification of $U_i^*(t)$. If $\varepsilon_i(t)$ contains such components, then the computed variables $U_{Ci}^*(t)$ and $U_i^*(t)$ used in the estimation of (24) are observed with error, and this error does not vanish asymptotically. In this case a nonlinear errors in the variables problem exists. Two-stage least squares procedures applied to (24) will not yield consistent parameter estimates under these circumstances.

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[1] In the case of the nonseparable tax function, this assumption essentially requires the fitted value $\hat{\pi} Q_i(t)$ of equation (22) to measure $E_t\{m_{ti}(t+1)\}$ exactly. It is not sufficient for $\hat{\pi} Q_i(t)$ to be merely unbiased for $m_{ti}(t+1)$.

[2] Burtless-Hausman (1978) and Hausman (1979) analyze measurement error in marginal tax rates that arises from classical errors in variables for hours of work. Given a nonlinear tax function, this implies a nonlinear measurement error problem for marginal tax rates which, in general, will not satisfy the assumption needed here.

IV. Empirical Analysis

This section reports estimates for the parameters of the marginal rate of substitution function and the monotonic transformation, and it interprets the results in terms of substitution and income effects.

Samples

The samples used in the following analysis are drawn from an original data set of 440 family-based males which is constructed from the Pre-enrollment and the Monthly Labor Supply Files of the Denver Income Maintenance Experiment. This original data set includes control and experimental persons who satisfied the following criteria: classified in the Manpower Program as "control" or "counselling only"; originally enrolled and matched with the pre-enrollment interview; completed the last interview of the assigned experimental program; and age 25 to 60 in 1971. To be included in any of the samples used below, a person had to meet the following additional criteria: married at the time of enrollment; worked in at least one of the years 1972-1975; not assigned to a declining tax rate program (these individuals were eliminated due to the difficulty of computing their marginal tax rates); assigned to the same financial treatment program in the years 1973 and 1974; and not missing data on crucial variables. These additional criteria result in a panel of 299 individuals for the years 1972-1975. Tables 1 and 2 present summary statistics for the annual averages of consumption, wage, and hours of work variables across members of this panel for each year. Table 1 reports statistics for individuals who were classified as controls for each of the years 1972-1975; and Table 2 reports statistics for experimentals who were assigned to fixed tax rate programs in 1973 and 1974.[1] A detailed description of the construction of the data file used in this study can be found in Appendix B.

Two samples were analyzed below: one is designated the "control sample" and the second is referred to as the "full sample." The control sample includes data for the years 1972-1975 on 121 males who were married, working, and a "control" in each of the four years.[2] The full sample includes data for the years 1973-1974 on 255 males who were married, working, and a "control" or an "experimental" in a fixed rate program in each of the two years.[3]

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[1]Notice that the sizes of the "control" and the "experimental" groups indicated in Tables 1 and 2 do not add up to the total sample size of 299. There are 180 individuals who are classified as controls in the years 1973 and 1974, but 24 of these individuals were assigned as "experimentals" in either 1972 or 1975 and were deleted in computing the summary statistics reported in Table 1.

[2]It is possible to carry out the following empirical analysis using less restrictive acceptance criteria for inclusion in the sample. An individual could be included in the control sample for any year in which he was a control, married, and working. Thus, there would be only one observation in the data set on a person who satisfied all the criteria in only one year. Use of this less restrictive acceptance criteria results in a larger data set, but the computation of the standard errors of estimates is complicated because the data set is unbalanced. The empirical results reported below do not appear to be sensitive to the way in which one generates the control sample. In all cases considered, the results computed using the "control" sample could be reproduced using the complete sample of 156 controls reported on in Table 1.

[3]As discussed in footnote 2 on this page, one can impose a less restrictive acceptance criteria at the cost of obtaining an unbalanced data set which complicates the computation of standard errors. In all cases considered, the empirical results based on the "full sample" could be reproduced using the entire sample of 299 individuals.

TABLE 1

SAMPLE STATISTICS FOR CONTROLS

Variable	Mean	Standard Deviation	Minimum Value	Maximum Value
Monthly Hours Worked, 1972	175.40	48.42	7.68	325.33
Monthly Hours Worked, 1973	174.07	45.25	0.43	301.82
Monthly Hours Worked, 1974	168.99	49.33	1.37	299.29
Monthly Hours Worked, 1975	162.14	56.15	1.71	347.81
Fraction Working in 1972	0.98	. . .	. . .	. . .
Fraction Working in 1973	0.96	. . .	. . .	. . .
Fraction Working in 1974	0.94	. . .	. . .	. . .
Fraction Working in 1975	0.94	. . .	. . .	. . .
Hourly Wage Rate, 1972	3.55	1.06	0.49	7.07
Hourly Wage Rate, 1973	3.71	1.07	0.59	7.72
Hourly Wage Rate, 1974	3.64	1.02	0.50	6.68
Hourly Wage Rate, 1975	3.66	1.21	0.53	8.72
Marginal Wage Rate, 1972	2.77	0.80	0.47	5.46
Marginal Wage Rate, 1973	2.80	0.83	0.56	5.66
Marginal Wage Rate, 1974	2.68	0.73	0.47	5.69
Marginal Wage Rate, 1975	2.68	0.91	0.72	8.20
Monthly Consumption, 1972	622.48	198.21	110.42	1181.28
Monthly Consumption, 1973	608.74	194.98	74.48	1091.78
Monthly Consumption, 1974	594.56	202.91	126.28	1269.08
Monthly Consumption, 1975	593.37	232.29	108.90	1616.87
Age in Years in July 1972	38.04	8.59	26.50	59.00
Education at Start of Exp.	10.99	2.72	2.00	18.00
Fraction Black	0.33	. . .	. . .	. . .
Fraction Chicano	0.34	. . .	. . .	. . .
Sample Size 156				

TABLE 2

SAMPLE STATISTICS FOR EXPERIMENTALS

Variable	Mean	Standard Deviation	Minimum Value	Maximum Value
Monthly Hours Worked, 1972	171.35	54.27	6.67	347.61
Monthly Hours Worked, 1973	165.61	47.78	5.90	257.14
Monthly Hours Worked, 1974	162.44	58.00	0.48	304.77
Monthly Hours Worked, 1975	159.09	59.72	0.57	293.93
Fraction Working in 1972	0.99	. . .	. . .	. . .
Fraction Working in 1973	0.92	. . .	. . .	. . .
Fraction Working in 1974	0.92	. . .	. . .	. . .
Fraction Working in 1975	0.89	. . .	. . .	. . .
Hourly Wage Rate, 1972	3.48	1.15	0.99	8.28
Hourly Wage Rate, 1973	3.60	1.23	0.54	7.67
Hourly Wage Rate, 1974	3.45	1.21	0.39	7.13
Hourly Wage Rate, 1975	3.51	1.21	0.42	7.87
Marginal Wage Rate, 1972	2.02	0.86	0.41	5.91
Marginal Wage Rate, 1973	1.98	0.84	0.16	4.52
Marginal Wage Rate, 1974	1.79	0.74	0.19	3.84
Marginal Wage Rate, 1975	2.32	0.91	0.21	5.60
Monthly Consumption, 1972	626.12	205.76	251.86	1342.67
Monthly Consumption, 1973	639.96	193.96	124.32	1193.45
Monthly Consumption, 1974	597.02	210.53	47.18	1143.37
Monthly Consumption, 1975	595.37	192.06	149.66	1087.58
Age in Years in July 1972	36.66	8.16	26.50	58.83
Education at Start of Exp.	10.54	2.82	3.00	20.00
Fraction Black	0.23	. . .	. . .	. . .
Fraction Chicano	0.36	. . .	. . .	. . .
Sample Size 119				

All variables used in the folliwng empirical analysis are average monthly values for the year, and all dollar quantities are measured in terms of November, 1971 dollars. The wage rate is average hourly earnings. Family consumption is the sum of expenditure on nondurables plus an imputed service flow for durables. Nondurable expenditure is computed as total family income and benefits minus taxes paid, alimony, child support, other payments outside the household, and net changes in the holdings of liquid assets and durables including houses, vehicles, property, and other durables. The durable goods service flow is computed as one percent per month times the net value of durables, where this net value is the total worth of the house, vehicles, and other durables minus the house mortgage and nonmortgage debt and is truncated at zero if negative.

Marginal tax rates are computed as follows. For individuals assigned as controls in the experiment, marginal tax rate data are available and are computed using federal, state, and social security taxes. It is assumed that controls do not income average so that taxes in each year only depend on income earned in the year. Thus, in calculating the marginal wage given by (21) for these individuals, $m_{ti}(t+1) = 0$ and $m_{ti}(t)$ is the marginal tax rate associated with period t.

For experimentals, taxes are computed on the basis of a twelve month average of current and past incomes. Since past income is taxed at the same rate as current income, we have $m_{ti}(t+1) = m_{t+1,i}(t+1)$ for each t; and we may denote either of these variables as simply $m_i(t+1)$ which represents the marginal tax rate in period t+1. If an experimental is below the break even point and receiving payments, then $m_i(t)$ depends on the particular financial treatment program to which the individual is assigned. If, on the other hand, the experimental is above the break even point, then $m_i(t)$ is computed as if the individual were a control. Using the above rules to calculate marginal tax rates in each month, the variables $m_i(t)$ and $m_i(t+1)$ appearing in the specification of the marginal wage rate given by (21) are the average of the marginal tax rates faced by the individual in each month over the year.[1]

Estimates of the Marginal Rate of Substitution Function

The discussion below first considered the estimation of equation (20) assuming that the parameters θ_1 and θ_2 are known and equal to $\theta_1 = 0$ and $\theta_2 = 0$. Given this assumption, the parameters α_1, α_2, and β appearing in equation (20) can be estimated using standard two-stage least squares procedures; there is no need for the use of any nonlinear estimation schemes. Estimation is carried out using both the control and the full samples.

For each individual in the control sample, we have four sets of observations or years of data available to estimate equation (20). Combining observations for a given worker into a single system of simultaneous equations creates a model that is well suited for carrying out the estimation. Given that worker i is age t in 1972, t+1 in 1973, etc., stacking observations on equation (20) for worker i in declining order of years yields the system of equations

[1] There is, of course, a time aggregation problem associated with this method of computing marginal tax rates for experimentals.

$$\begin{bmatrix} \ell n\ \omega_i(t+3) \\ \cdot \\ \cdot \\ \cdot \\ \ell n\ \omega_i(t) \end{bmatrix} = \begin{bmatrix} X_i(t+3) \\ \cdot \\ \cdot \\ \cdot \\ X_i(t) \end{bmatrix} \beta + \begin{bmatrix} \ell n\ h_i(t+3) \\ \cdot \\ \cdot \\ \cdot \\ \ell n\ h_i(t) \end{bmatrix} (\alpha_2 - 1)$$

$$+ \begin{bmatrix} \ell nC_i(t+3) \\ \cdot \\ \cdot \\ \cdot \\ \ell nC_i(t) \end{bmatrix} (1 - \alpha_1) + \begin{bmatrix} \varepsilon_i(t+3) \\ \cdot \\ \cdot \\ \cdot \\ \varepsilon_i(t) \end{bmatrix} ;$$

or, in vector notation, we have

(26) $$Y_{1i} = X_i\beta + Y_{2i}(\alpha_2 - 1) + Y_{3i}(1 - \alpha_1) + \varepsilon_i \qquad i = 1,\ldots,N$$

where the definitions of the vectors Y_{1i}, Y_{2i}, Y_{3i} and ε_i and the matrix X_i follow from the specification above, and N is the total number of individuals in the sample. The following analysis assumes that the error vectors ε_i are independently distributed across individuals. No restrictions are imposed on the covariance matrix of ε_i which permits arbitrary forms of serial correlation. The parameters of equation (26), α_1, α_2, and β, are estimated using a two-stage procedure which permits the imposition of equality constraints across equations and treats the vectors Y_{1i}, Y_{2i} and Y_{3i} as endogenous.[1, 2]

The first two rows of Table 3 report estimates of α_1, α_2, and β for several specifications of $X_i(t)$ which represent measured characteristics that influence a consumer's tastes. In the first row, $X_i(t)$ contains only

[1]When computing standard errors, the two-stage least squares procedure used here takes account of the fact that $E(\varepsilon_i\varepsilon_i')$ is not a diagonal matrix.

[2]The set of instruments used to predict the elements of Y_{2i} and Y_{3i} are listed in a footnote at the bottom of Table 3.

an intercept; and, in the second row, the number of children and dummy variables indicating a consumer's race are also included. The estimates of α_1 and α_2 are very similar across the alternative specifications. These estimates also do not change if one includes year dummies in $X_i(t)$.

Rows 3 and 4 of Table 3 report estimates of α_1, α_2, and β using data from the full sample for the same two specifications of $X_i(t)$. To obtain these estimates a simultaneous equation system like (26) is constructed for each individual in the full sample using two years of data rather than four, and the parameters of this model are estimated by constrained two-stage least squares. To compute expected future marginal tax rates needed to calculate the marginal wage rate given by (21) for experimentals, several specifications of equation (22) were tried, all yielding similar results. Similar results were also obtained when actual marginal tax rates were used in place of their expected values (i.e., a perfect foresight assumption), and these are the results reported in Table 1.[1, 2] As in the case of the control sample, one obtains similar estimates for α_1 and α_2 for the alternative specifications. Once again, the inclusion of year dummies in $X_i(t)$ does not change the results. The estimates of α_1 and α_2 for the full sample are higher than those obtained for the control sample for every specification.

Table 4 presents estimates of the marginal rate of substitution function with θ_1 and θ_2 treated as parameters rather than as known constants.

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[1] In calculating the present value of the expected or actual future marginal tax rates, the discount rate R is assumed to be .91.

[2] In computing marginal wages for experimentals in the year 1974, one must distinguish between individuals on three year as opposed to five year programs. Three year experimentals are controls for all or most of 1975, so their 1975 taxes do not depend on their current income. As a result, $m_{it}(t+1) = 0$ when t refers to 1974.

TABLE 3

PARAMETER ESTIMATES FOR THE MARGINAL RATE OF SUBSTITUTION FUNCTION
(standard errors in parentheses)

Sample	α_1	α_2	Intercept	Children	Black	Chicano
Control	.37 (.2)	1.13 (.18)	-3.6 (1.8)	. . .	. . .	. . .
Control	.34 (.2)	1.16 (.19)	-3.9 (1.75)	-.014 (.016)	.015 (.04)	-.04 (.05)
Full	.71 (.2)	1.32 (.18)	-2.5 (1.12)	. . .	. . .	. . .
Full	.67 (.21)	1.37 (.19)	-2.9 (1.18)	-.013 (.02)	.097 (.05)	.005 (.05)

*The set of instruments used to predict endogenous variables include number of children, race dummies, a fully interacted quardatic in age and education, education of father and mother of head, dummy variables indicating national heritage, year dummies, and dummy variables indicating the individual's assigned normal income category and financial treatment or tax program.

To obtain these estimates a simultaneous equations model like (26) is constructed for each individual in the sample with the variables $\ell n(h_i(k) + \theta_2)$ and $\ell n(C_i(k) + \theta_1)$ replacing the variables $\ell n\ h_i(k)$ and $\ell n\ C_i(k)$, respectively, and the parameters of this model are estimated by constrained nonlinear two-stage least squares. In estimation, the parameters θ_1 and θ_2 were restricted to lie above the minimum values of $C_i(k)$ and $h_i(k)$ for all i and k to ensure that the marginal rate of substitution function is defined for all individuals in the sample for all years.[1] Rows 1 and 2 report parameter estimates for the control sample when an intercept, number of children, and race dummies are included in $X_i(t)$. Row 2 presents estimates with θ_2 constrained to equal zero. Rows 3 and 4 report the corresponding estimates for the full sample. According to all the results in Table 4, we can accept the hypothesis that $\theta_1 = 0$ and $\theta_2 = 0$, and thus the marginal rate of substitution function is log linear.

The problems of nonrandom assignment associated with negative income tax experiments are well known. To be a participant in the Denver Income Maintenance Experiment, an individual's family income, or some measure of its "normal" income, must lie below a certain cutoff level at the time of the pre-enrollment interview. Also, once enrolled in the experiment, individuals are nonrandomly assigned to various financial treatment programs depending on assessed values of their "normal" income. As a consequence of this nonrandom sampling and assignment, the empirical results reported above may be subject to biases arising from censoring.[2]

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[1]This is a binding constraint. Experimentation indicated that unconstrained optimization would have certainly violated this restriction.

[2]For a discussion of the censoring problems that arise in analyzing data from neagive income tax experiments, see Cain-Watts (1973, p. 343) and Hausman-Wise (1977).

TABLE 4

PARAMETER ESTIMATES FOR THE MARGINAL RATE OF SUBSTITUTION FUNCTION USING THE NONLINEAR SPECIFICATION (standard errors in parentheses)

Sample	α_1	α_2	θ_1	θ_2	Intercept	Children	Black	Chicano
Control	.38	1.13	-.1	0	-3.6	-.014	.016	-.034
	(.62)	(.20)	(43)	(10)	(4.9)	(.016)	(.05)	(.06)
Control	.37	1.13	-.4	. . .	-3.7	-.013	.017	-.04
	(.29)	(.16)	(3)		(1.6)	(.03)	(.06)	(.05)
Full	.72	1.38	-.1	0	-2.8	-.012	.098	.001
	(.20)	(.23)	(1)	(1)	(3)	(.01)	(.04)	(.09)
Full	.73	1.38	-.1	. . .	-2.8	-.011	.098	.001
	(.19)	(.15)	(.5)		(1.5)	(.01)	(.04)	(.05)

*The set of instruments used in the nonlinear two-stage least squares procedures includes number of children, race dummies, a fully interacted quadratic in age and education, education of father and mother of head, dummy variables indicating national heritage, year dummies, and dummy variables indicating the individual's assigned normal income category and financial treatment or tax program.

While a formal and complete solution to this censoring problem is beyond the scope of this paper, several potential remedies were explored. First, dummy variables indicating each individual's assigned normal income category were included in $X_i(t)$ in the estimation of equation system (26) for both the control and the full samples. The estimates of α_1 and α_2 do not change with the inclusion of these variables for the control sample. For the full sample, however, there is a large increase in the estimates of α_1. In particular, for the specification corresponding to row 4 of Table 3, α_1 goes from .67 to 1.18 which implies that indifference curves are nonconvex for some combinations of consumption and hours. Since the standard error of α_1 also experiences a large increase, it is not clear whether this rise in α_1 is statistically significant or not.

As a second potential remedy, equation system (26) is estimated in first differences.[1] The results are presented in Table 5.[2] Given the special assumption that $\varepsilon_i(t)$ appearing in equation (20) follows a permanent-transitory scheme where the transitory component is independently distributed over time, first differencing avoids biases arising from income truncation and nonrandom assignment. Since individuals are assigned to various normal income categories prior to the sampling period, censoring only occurs for permanent components. Eliminating these permanent effects by working with $\varepsilon_i(t)$ in first differences, then, avoids sample selection biases arising from assignments on the basis of previous income. Comparing the results in Tables 3

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[1] In first differences, the last equation must, of course, be dropped since data are not available for the first year of the sample. Thus, for the control sample the new system consists of three rather than four equations per individual and for the full sample there is only one equation per individual.

[2] Including year dummies as explanatory variables in $X_i(t)$ do not affect the reported results.

TABLE 5

PARAMETER ESTIMATES FOR THE MARGINAL RATE OF SUBSTITUTION FUNCTION BASED ON FIRST DIFFERENCES
(standard errors in parentheses)

Sample	α_1	α_2	Children
Control	.78 (.13)	1.09 (.25)	.01 (.02)
Full	.34 (.16)	1.01 (.12)	.03 (.04)

*The set of instruments used to predict endogenous variables includes change in number of children, a fully interacted quadratic in age and education, education of father and mother of head, dummy variables indicating national heritage, and year dummies.

and 5, we see that a higher value of α_1 is implied for the control sample, and a lower value for both α_1 and α_2 is obtained for the full sample. There does not appear to be a consistent pattern in the change of estimates.

Estimates of the Monotonic Transformation

The following discussion presents estimates for equation (24) assuming that the parameter δ is greater than or equal to that constant $\bar{U}$ which sets $U_i^*(t) + \bar{U} = 0$ when U_i^* is evaluated at zero consumption of goods and leisure.[1] This restriction ensures that the period specific utility function given by (23) is defined over the entire range of consumption and hours of work. Define δ^* as $\delta^* = \delta - \bar{U}$ and observe δ^* must satisfy the constraint $\delta^* \geq 0$. Two specifications of equation (24) are estimated for both the control and the full sample: one imposes the constraint $\delta^* = 0$, and the other treats δ^* as a parameter. Both specifications include the number of children as a shifter of tastes and an element of $X_i(t)$ in (24).

Using the estimates of α_1 and α_2 obtained above and data on an individual's hours of work and consumption for each year, one can compute a value for $\gamma_1(t)$ using formula (25) and thus the values of $U_i^*(t) + \delta$ and $U_{Ci}^*(t)$ for every individual for each year of the sample. Forming the differences in these variables needed to estimate tequation (24) for the control sample yields a set of three observations per individual. One obtains a single observation per individual in the case of the full sample. Stacking the observations on equation (24) for a given worker into a single system of simultaneous equations creates a model similar to (26). Two-stage least squares estimation of this

[1] In evaluating $\bar{U}$, it is assumed that leisure consumption is zero if an individual works 16 hours a day, 30 days a month. Thus,

$$\bar{U} = \frac{1}{\alpha_2}(480)^{\alpha_2}.$$

equation system, treating utilities and marginal utilities as endogenous variables, yields a full set of estimates for the parameters of the monotonic transformation. Nonlinear two-stage least squares methods are required when δ^* is treated as a parameter.

Table 6 presents a set of estimates for these parameters. The values of α_1 and α_2 in the second row of Table 3 (i.e., $\alpha_1 = .34$, $\alpha_2 = 1.16$) were used to compute $\gamma_i(t)$ and the variables $U_i^*(t)$ and $U_{Ci}^*(t)$ for the control sample, and those in the fourth row of Table 3 (i.e., $\alpha_1 = .67$, $\alpha_2 = 1.37$) were used for the full sample. In rows 1 and 3, δ^* is constrained to be zero; it is estimated in rows 2 and 4. According to the results in Table 6, we can easily accept the hypothesis that $\delta^* = 0$. The estimate of σ for the control sample is .86. For the full sample, the estimates of σ range from 1.5 to 2 which implies the period specific utility function is not concave in goods and leisure consumption over the entire feasible space for these decision variables. Once again it is difficult to account for the seemingly large differences in estimates obtained for the control and the full sample.

Implied Income and Substitution Effects

In a complete analysis of consumption and labor supply behavior, one would use the parameter estimates obtained above to construct a preference function and then use this function to calculate a consumer's response to various changes in income, wages, or tax policies. The following analysis is much less ambitious. It uses the above results only to compute income and substitution derivatives. These quantities are of limited interest since they represent responses to only infintesimal changes and are inappropriate for analyzing global variations in income, wages, or tax schemes. Furthermore, these derivatives are computed only for the static case; that

TABLE 6

PARAMETER ESTIMATES FOR THE MONOTONIC TRANSFORMATION
(standard errors in parentheses)[a]

Sample	σ	δ*	Intercept	Children
Control	.86 (.23)	. . .	.01 (.008)	-.04 (.023)
Control	.86 (.83)	0 (11598)	.01 (.008)	-.04 (.02)
Full	1.97 (.33)	. . .	.06 (.03)	-.05 (.04)
Full	1.5 (.25)	.0006 (.002)	.05 (.02)	-.07 (.05)

[a]As noted in the text, these standard errors do not account for the fact that the variables $U^*_{Ci}(k)$ and $U^*_i(k)$ depend on estimated quantities.

*The set of instruments used to predict endogenous variables includes the change in number of children, race dummies, a fully interacted quadratic in age and education, education of father and mother of head, dummy variables indicating national heritage, year dummies, and dummy variables indicating the individual's assigned normal income category and financial treatment or tax program.

is, a consumer is assumed to operate in a one period framework with preferences given by (19). The purpose of the following discussion is simply to translate the parameter estimates obtained above into familiar quantities.

Analytical solutions for consumption demand and labor supply functions do not exist for preferences U*(t) defined by (19) even in the static case. These functions, however, are not needed to evaluate income and substitution derivatives. The decompositions implied by the "fundamental matrix equation" provide an alternative framework for computing these drivatives given knowledge of the utility function and points of evaluation (see Phlips, 1974, pp. 47-50). Let H denote the Hessian matrix associated with the utility function U*(t), and let U_C, ω, and Y denote the implied marginal utility of consumption, the real marginal wage rate, and real property income, respectively. Defining $P' = (1, \omega)$ as the price vector and $q = P' H^{-1} P$, the solutions to the fundamental matrix equation imply the following relations

$$\begin{pmatrix} \frac{\partial C}{\partial Y} \\ -\frac{\partial h}{\partial Y} \end{pmatrix} = \frac{1}{q} U^{-1} P$$

(27)
$$\begin{pmatrix} \left.\frac{\partial C}{\partial \omega}\right|_U \\ -\left.\frac{\partial h}{\partial \omega}\right|_U \end{pmatrix} = U_C H^{-1} - \frac{U_C}{q} P' H^{-1} H^{-1} P$$

$$\begin{pmatrix} \left.\frac{\partial C}{\partial \omega}\right|_Y \\ -\left.\frac{\partial h}{\partial \omega}\right|_Y \end{pmatrix} = \begin{pmatrix} \left.\frac{\partial C}{\partial \omega}\right|_U \\ -\left.\frac{\partial h}{\partial \omega}\right|_U \end{pmatrix} + \begin{pmatrix} \frac{\partial C}{\partial Y} \\ -\frac{\partial h}{\partial Y} \end{pmatrix} h$$

where $|_U$ denotes compensated substitution derivatives, and $|_Y$ denotes uncompensated effects. Given a choice of consumption, C, hours of work, h, and the marginal wage rate, ω, it is possible to calculate H^{-1}, P, U_C and q and use the above formulas to evaluate all income and substitution derivatives. This is the approach followed below. The quantities C, h, and ω are set equal to the means of these variables for the control sample; in particular, C = \$600, h = 170, and W = \$2.75. Given estimates of α_1 and α_2, the value of γ is computed using formula (25) which simply requires the marginal rate of substitution to equation the marginal wage rate.[1]

For the estimates of α_1 and α_2 for the control sample reported in the second row of Table 3 (i.e., α_1 = .34, α_2 = 1.16), the implied income derivatives are $\frac{\partial C}{\partial Y} = .23$ and $\frac{\partial h}{\partial Y} = -.28$; the compensated substitution effects are $\frac{\partial C}{\partial \omega}\Big|_U = 252$ and $\frac{\partial h}{\partial \omega}\Big|_U = 91$; and the uncompensated effects are $\frac{\partial C}{\partial \omega}\Big|_y = 292$ and $\frac{\partial h}{\partial \omega}\Big|_y = 44$. The implied substitution elasticities for hours of work associated with changes in the marginal real wage rate are 1.47 for the compensated and .7 for the uncompensated. For the estimates of α_1 and α_2 for the full sample reported in the fourth row of Table 3 (i.e., α_1 = .67, α_2 = 1.37), the implied derivaties are $\frac{\partial C}{\partial Y} = .58$, $\frac{\partial h}{\partial Y} = -.14$, $\frac{\partial C}{\partial \omega}\Big|_U = 271$, $\frac{\partial h}{\partial \omega}\Big|_U = 98$, $\frac{\partial C}{\partial \omega}\Big|_y = 371$, and $\frac{\partial h}{\partial \omega}\Big|_y = 73$ with a compensated substitution elasticity equal to 1.58 and an uncompensated elasticity equal to 1.1. If one uses instead the estimates of α_1 and α_2 reported in Table 5 for the differenced data to compute these effects, one finds for the control sample that $\frac{\partial h}{\partial Y} = -.2$ and $\frac{\partial h}{\partial \omega}\Big|_y = 116$ which indicates a smaller income effect for hours of

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[1] Thus, $\gamma = (h^{(\alpha_2-1)} G^{(1-\alpha_2)})/\omega$ where h = 170, C 600 and ω = 2.75.

work and a much larger uncompensated effect than obtained above. For the full sample, the estimates in Table 5 imply $\frac{\partial h}{\partial y} = -.35$ and $\left.\frac{\partial h}{\partial \omega}\right|_y = 57$ which represents a slightly lower substitution effect and a much larger income effect for hours of work than obtained above using the estimates from Table 3.

Thus, the implied income and substitution effects for hours of work for both the control and the full sample are much larger than is typically found for prime-age males, no matter what set of estimates is used. Before further examining this finding, it is important to emphasize that the values of these income and substitution derivatives are sensitive to the points of evaluation. If, for example, we evaluate the first set of results for controls at their average wage reported in Table 1 rather than at their average marginal wage (i.e., at \$3.60 instead of at \$2.75), then $\left.\frac{\partial h}{\partial \omega}\right|_y = 18$ which is 40 percent reduction in the uncompensated effect.

One possible explanation for the larger estimated income and substitution effects for hours of work concerns the restrictiveness of the functional form assumptions for preferences. As a direct consequence of the strong separability assumption, for example, it is possible to show that the utility function given by (19) does not simultaneously permit a zero income effect for hours of work and a nonzero cross compensated substitution effect associated with consumption and the real wage; in other words, $\frac{\partial h}{\partial y} = 0$ necessarily implies $\left.\frac{\partial C}{\partial \omega}\right|_U = 0$. This observation is particularly important for the empirical analysis performed above. Estimating the marginal rate of substitution function attempts simultaneously to fit both a consumption and a labor supply function to the data. Thus, in contrast to the typical labor supply analysis, the relationship between consumption and wages receives as much weight in the estimation of parameters as the relationship between hours of work and wages. If, then, one believes that consumption is fairly responsive to a change in wages, holding real income constant (i.e., $\left.\frac{\partial C}{\partial \omega}\right|_U$ is large), then the estimated value of $\frac{\partial h}{\partial Y}$ will necessarily be nonzero and positive. While the function for preferences given by (19) is special, it should be pointed out that it contains many of the functions used in empirical analysis as special cases. When written as a function of leisure rather than of hours of work (a fairly unimportant modification), (19) nests such utility functions as the CES, Cobb-Douglas, Addilog and the Stone-Geary.

Direct Estimation of the Labor Supply Function

As outlined in Section II, there is an alternative method for estimating parameters needed to describe static labor supply behavior which accounts for taxes and decision making in an intertemporal setting. This method directly estimates the parameters of a pseudo labor supply function. Conceptually, to drive this function, one linearizes the budget constraint around the equilibrium hours of work position and solves for labor supply as a function of the slope and the intercept of this linearized budget line. The resulting function may be written as $h(t) = h(\omega(t), -\tilde{S}(t), Z(t))$ where the marginal wage rate $\omega(t)$ represents the slope of the linearized budget constraint, and the variable $-\tilde{S}(t) = C(t) - \omega(t)h(t)$ corresponds to the intercept which may be interpreted as property or nonwage income. Estimating the parameters of such functions provides another approach for computing income and substitution derivatives for hours of work which can then be compared with those calculated above.

Two specifications of this type of labor supply function are estimated for both the control and the full sample. The first is a simple linear specification of the form

$$h_i(t) = X_i(t)\beta + \psi_1 \omega_i(t) - \psi_2 \tilde{S}_i(t) + e_i(t); \tag{28}$$

and the second is a logrithmic specification given by

$$\ell n\ h_i(t) = X_i(t)\beta^* + \psi_1^* \ \ell n\ \omega_i(t) - \psi_2^* \tilde{S}_i(t) + e_i^*(t) \qquad (29)$$

where β, ψ_1, ψ_2, β^*, ψ_1^* and ψ_2^* are parameters, $X_i(t)$ is a vector of "taste shifter" variables, $-\tilde{S}_i(t) = C_i(t) - \omega_i(t)h_i(t)$ is "property" income, and $e_i(t)$ and $e_i^*(t)$ are disturbances. Roughly speaking, one may interpret the above parameters as: $\psi_1 = \frac{\partial h}{\partial \omega}\Big|_y$, $\psi_2 = \frac{\partial h}{\partial y}$, $\psi_1^* = \frac{\partial \ell n\ h}{\partial \ell n\ \omega}\Big|_y = \frac{2.75}{170} \frac{\partial h}{\partial \omega}\Big|_y$, and $\psi_2^* = \frac{1}{170} \frac{\partial h}{\partial y}$ where it is assumed that derivatives are evaluated at the same points used in the above computation of income and substitution effects (i.e., $h = 170$ and $\omega = \$2.75$). For either sample, one can stack equations (28) or (29) into a simultaneous equation model like (26) for each individual and estimate the parameters of these equations using linear two-stage least squares procedures. In applying this estimation technique, the variables $\omega_i(t)$, $\tilde{S}_i(t)$ and $\ell n\ \omega_i(t)$, $\tilde{S}_i(t)$ must be treated as endogenous. Tables 7 and 8 present estimates for equations (28) and (29), respectively.

Consider first the estimates for the control sample. According to row 2 of Table 8, we have $\frac{\partial \ell n\ h}{\partial \ell n\ \omega}\Big|_y = \psi_1^* = .69$ and $\frac{\partial h}{\partial y} = 170 \cdot \psi_2^* = -.27$. These values are almost identical to those obtained above using the estimates of the marginal rate of substitution function. These results then strongly support the implications of the preceding analysis that suggests labor supply responses are large. The results in Table 7 for controls indicate smaller substitution and income effects than those implied by Table 8, but they are still larger than is normally found.

The estimates of the wage and the income coefficients for the full sample both support and contradict the above results. The uncompensated wage elasticities reported in Table 8 are about .4 which indicates a fairly large labor

TABLE 7

PARAMETER ESTIMATES FOR THE LINEAR SPECIFICATION OF THE LABOR SUPPLY FUNCTION
(standard errors in parentheses)

Sample	Wage	Income	Intercept	Children	Black	Chicano
Control	17 (13)	-.15 (.07)	148 (37)	. . .	. . .	. . .
Control	19 (13)	-.17 (.07)	138 (42)	2.9 (2.9)	-6.7 (8.5)	4 (10)
Full	14 (7.6)	.03 (.04)	127 (25)	. . .	. . .	. . .
Full	12 (7.5)	.01 (.04)	133.7 (24.8)	2.7 (2.1)	-13 (6.9)	3.2 (6.7)

*The set of instruments used to predict endogenous variables includes number of children, race dummies, a fully interacted quadratic in age and education, education of father and mother of head, dummy variables indicating national heritage, year dummies, and dummy variables indicating the individual's assigned normal income category and financial treatment or tax program.

TABLE 8

PARAMETER ESTIMATES FOR THE LOGRITHMIC SPECIFICATION OF THE LABOR SUPPLY FUNCTION
(standard errors in parentheses)

Sample	Log Wage	Income	Intercept	Children	Black	Chicano
Control	.64 (.55)	-.0019 (.001)	4.74 (.57)	. . .	. . .	. . .
Control	.69 (.53)	-.0016 (.001)	4.58 (.6)	-.0014 (.04)	.08 (.12)	.17 (.14)
Full	.39 (.22)	.0005 (.0005)	4.6 (.28)	. . .	. . .	. . .
Full	.39 (.2)	.0005 (.0005)	4.59 (.27)	.008 (.035)	-.067 (.11)	.063 (.1)

*The set of instruments used to predict endogenous variables includes number of children, race dummies, a fully interacted quadratic in age and education, education of father and mother of head, dummy variables indicating national heritage, year dummies, and dummy variables indicating the individual's assigned normal income category and financial treatment or tax program.

supply response, but not nearly as large as those based on estimates of the marginal rate of substitution function. As in the case of the control sample, estimates for the linear specification imply smaller substitution effects. All of the income coefficients for the full sample are the wrong sign; they indicate that leisure is an inferior good.

Conclusion

This paper applies two general procedures for estimating parameters needed to describe consumption and labor supply behavior in an intertemporal setting where the future is uncertain and taxes are present. One procedure directly estimates parameters of the preference function using the marginal rate of substitution function as the basis for an empirical specification. The second procedure estimates parameters of a static labor supply function whose arguments are suitably modified to account for taxes and life cycle factors. Both of these estimation methods are easily applied to analyze data from NIT experiments. Empirical results from both procedures suggest that the work disincentive effects of negative income tax programs for males may be much larger than indicated in previous work. Caution is required in interpreting these results given the small samples used in the empirical analysis and the existence of conflicting results for different samples.

APPENDIX A

Use of the Two-Stage Method for Estimating Labor Supply and Marginal Rates of Substitution Functions

The purpose of this appendix is to justify the two-stage least squares estimation procedure employed in this paper and to compare it with alternative procedures. To simplify the discussion, the following analysis assumes a one-period static model and the existence of a linear segmented budget constraint. The regressive tax case is considered in order to illustrate the essential idea.

The consumption-leisure diagram given by Figure 1 illustrates the case considered here. A consumer earns an exogenous wage W and property income S_1. A marginal tax rate m_1 applies to the branch $(0, \bar{h})$. This implies a marginal wage rate equal to $\omega_1 = (1 - m_1)W$. A lower rate m_2 applies to the branch $(\bar{h}, T)$ which implies the marginal wage rate $\omega_2 = (1 - m_2)W$. It is easy to verify that $S_2 = S_1 + (\omega_1 - \omega_2)\bar{h}$. The consumer's preference function may be written as $U(C, h, \nu)$ where C is goods consumption, h is hours of work, and ν is a "taste shifter." For the population of workers, the density of ν is written as $f(\nu)$ which is assumed to have a zero mean. This function induces a distribution on U. Using well known methods, one may form the indirect preference function $V(S, \omega, \nu)$ which is assumed to be positive. The following analysis assumes a functional form for $U(\cdot)$ and a distribution of tastes that implies one never observes an individual at point $\bar{h}$ or at zero hours of work.

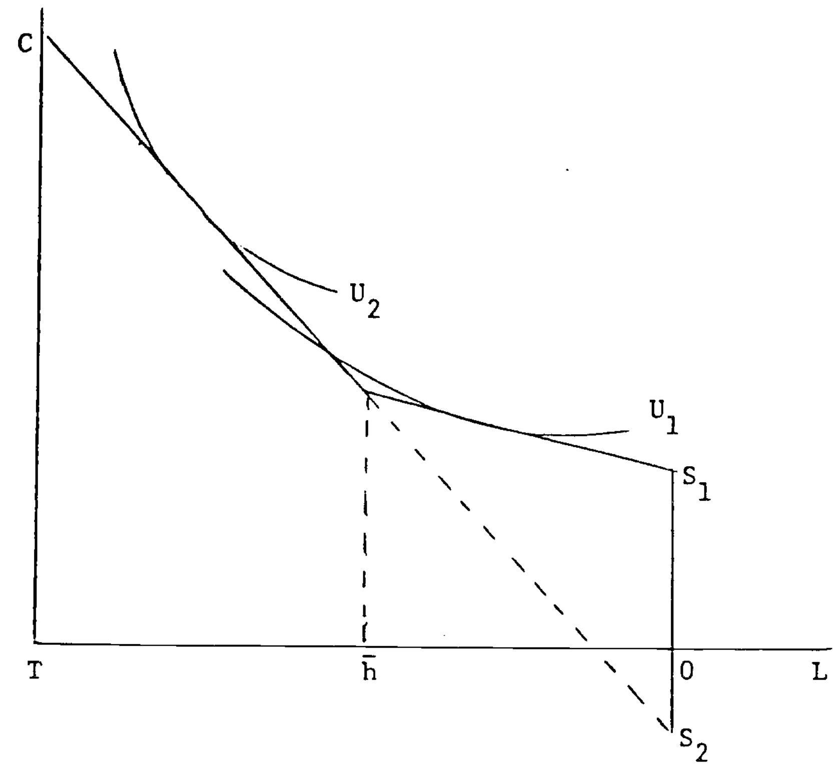

Figure 1

Maximization of utility subject to the budget constraint yields first order conditions

(A.1) $$MRS(C, h, \varepsilon) = \frac{U_L}{U_C} = \omega$$

(A.2) $$\tilde{S} + \omega h = C$$

where subscripts denote partial derivatives. The marginal wage ω and the intercept of the linearized budget constraint S are defined as

(A.3) $$\omega = \omega_1 d + \omega_2(1-d)$$

(A.4) $$\tilde{S} = S_1 d + S_2(1-d)$$

where d takes a value of 1 if $0 < h < \bar{h}$, and 0 otherwise. As shown in Figure 1, these conditions alone do not imply a unique equilibrium; they only identify potential equilibria.

Adding an equation that determines the dummy variable d is required to generate a unique solution. Define labor supply functions for each segment of the budget constraint as

$$h(i) = \frac{V_\omega(S_i,\ \omega_i,\ \nu)}{V_S(S_i,\ \omega_i,\ \nu)} \qquad i = 1,\ 2.$$

A tangency occurs on branch 1 if $0 < h_{(1)} < \bar{h}$, and one occurs on branch 2 if $h \leq h_{(2)} < T$. If

$$V_{(1)} = \begin{cases} V(S_1,\ \omega_1\ \nu) & 0 < h_{(1)} < \bar{h} \\ 0 & \text{otherwise} \end{cases}$$

and

$$V_{(2)} = \begin{cases} V(S_2,\ \omega_2\ \nu) & \bar{h} \leq h_{(2)} < T \\ 0 & \text{otherwise,} \end{cases}$$

it is easy to verify that

$$\text{(A.5)} \qquad d = \begin{cases} 1 & \text{if} \quad \nu \in \Theta = \{\nu:\ \ V_{(1)} \geq V_{(2)}\} \\ 0 & \text{if} \quad \nu \notin \Theta. \end{cases}$$

In other words, the consumer chooses d in order to attain the highest level of utility. The set Θ defines those values of the "taste component" ν for which utility is greater for a tangency point on branch 1 than on branch 2. In the case of Figure 1, $\nu \in \Theta$. The probability we observe a consumer on branch 1 is

(A.6) $\quad \text{Prob}(d=1) = \int_{\Theta} f(\nu)d\nu$

where $\int_{\Theta}$ indicates integration over the set Θ.

Equations (A.1) - (A.6) constitute a complete econometric model that fully characterizes consumption and labor supply. The only source of randomness in this model is the disturbance term ν. The endogenous variables are C, h, ω, $\tilde{S}$ and d, and the exogenous variables are W and S_1..

This study estimates the parameters of the utility function using a limited information procedure. Assuming that ℓn MRS is linear in ν, equation (A.1) in logs is estimated using two-stage least squares, treating the variables C, h, and ω as endogenous. Given the assumptions of this appendix, polynomials in the variables W and S_1 may be used as instruments. To generate consistent parameter estimates, this procedure only uses the facts that condition (A.1) is necessarily satisfied at the equilibrium and ν has zero mean (which is assured given the random sampling scheme assumed here). The fact that condition (A.1) alone does not imply a unique solution to the consumer's optimization problem is irrelevant in proving consistency of the estimators produced by this estimation scheme. In contrast to full information procedures which involve the estimation of reduced forms, the procedure used here does not require the analyst to characterize fully the consumer's equilibrium and to solve for consumption demand and labor supply functions. Thus, explicit specifications of the tax function or the budget constraint or the set Θ are not needed to carry out the estimation scheme.

To compare this limited information scheme with more familiar estimation procedures, it is convenient to consider the estimation of the labor supply function rather than the marginal rate of substitution function. To simplify the discussion, suppose that equations (A.1) and (A.2) imply a labor supply function of the form

$$h = \beta_0 + \beta_1 \omega + \beta_2 \tilde{S} + \nu \tag{A.7}$$

where β_0, β_1 and β_2 are parameters. Given estimates of β_0, β_1, and β_2, it is possible to construct the preference function $U(\cdot)$. Now equations (A.2) - (A.7) constitute a complete characterization of labor supply behavior.

The estimation scheme used in this study amounts to estimating the parameters of equation (A.7) using two-stage least squares procedures. The variables ω and $\tilde{S}$ are observed, and they can be predicted using polynomials in W and S_1 as instruments. There is no need to distinguish between the regressive and the progressive tax case. To prove the consistency of estimators, one only requires the assumptions that ν is randomly distributed with a zero mean and (A.7) always holds in equilibrium.

To apply alternative estimation procedures such as maximum likelihood, one generally requires the explicit specification of reduced forms for consumption or hours of work. To obtain these reduced forms, an analyst must, in principle, solve the consumer's problem and derive consumption demand and labor supply functions. Taking the unconditional expected value of (A.7) to obtain the reduced form for h reveals that one must compute $E(d) = \text{Prob}(d=1)$ which requires the complete specification of the set Θ and $f(\nu)$. The distinction between such factors as regressive and progressive taxes is obviously crucial in constructing Θ. This sort of estimation procedure uses the information that:

(1) the prediction equations for ω and $\tilde{S}$ are directly implied by the behavioral model (i.e., see (A.3), (A.4) and (A.6)); and, (2) parameter restrictions exist across equations. Thus, this procedure yields more efficient parameter estimates than the procedure based on the naive use of two-stage least squares proposed above, assuming, of course, that all the assumptions maintained in computing Θ and Prob(d=1) are valid. The major advantages of the two-stage least squares procedure relate to its computational simplicity and its robustness in the sense that it does not rely on a correct specification of Θ and Prob(d=1).

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